Constraints on inelastic scattering of low-mass WIMP using Migdal effect

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Based on
arXiv: 2407.16187
in collaboration with Stefano Scopel and Gaurav Tomar

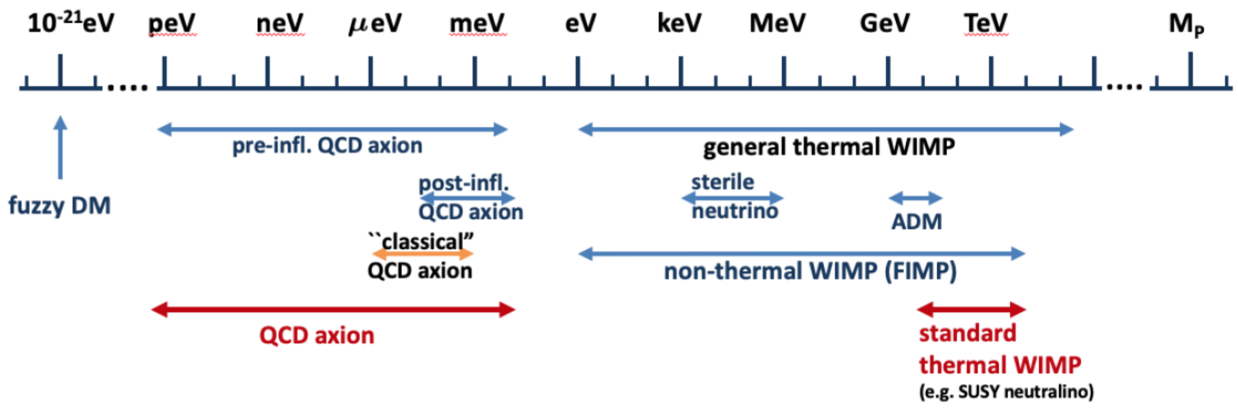
2025 HIGH1 WORKSHOP ON PARTICLE, STRING AND COSMOLOGY

SOGANG UNIVERSITY

CQUeST
Center for Quantum SpaceTime

Dark Matter and WIMPs

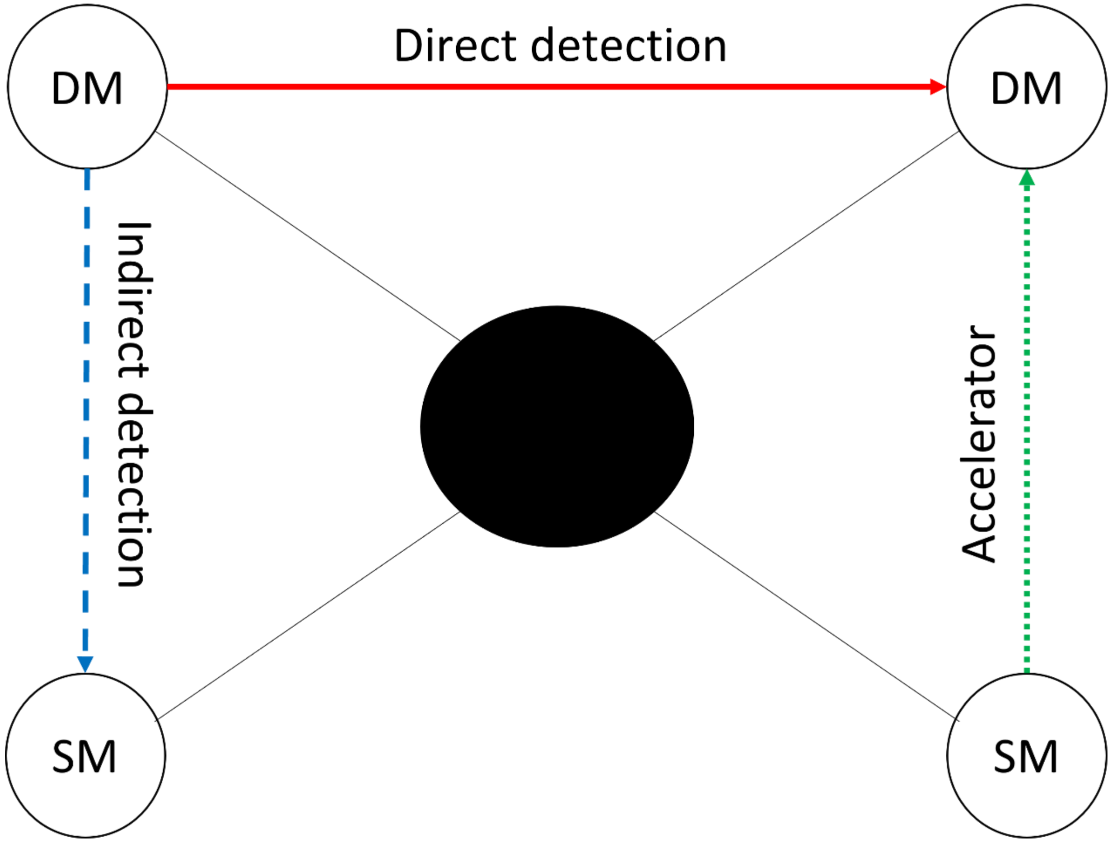
APPEC DM report



- Many evidences of Dark Matter
  - Galaxy rotational curve
  - CMB
  - Lensing effect

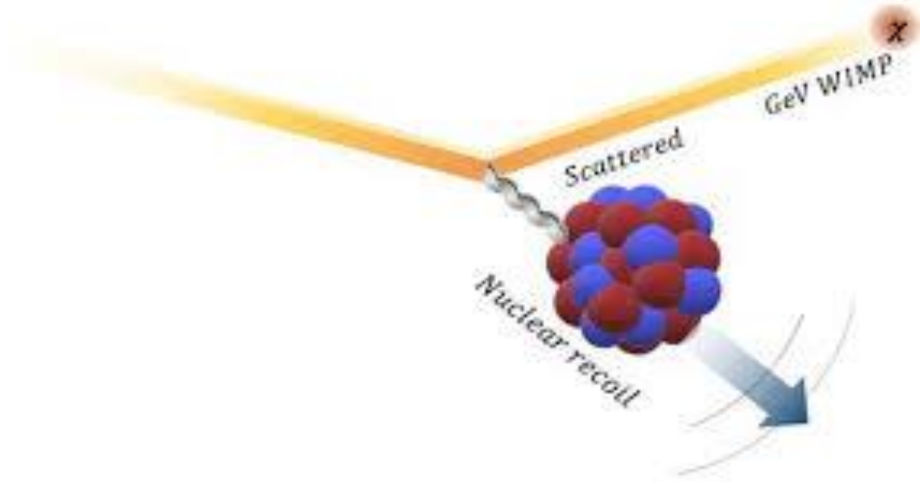
- Many candidates
  - Neutrino
  - Cold Dark Matter (CDM)
  - Weakly Interacting Massive Particle (WIMP)
  - Weak-type interaction
    - no electric charge, no color
  - Mass range in GeV-TeV range
  - WIMP miracle
    - correct relic abundance is obtained at WIMP $<\sigma v> = weak\ scale$
    - most extensions of SM are proposed independently at that scale.

Detection strategies



- Direct detection: DM interacts with SM particles (left to right)
- Indirect detection: DM annihilation (top to bottom)
- Accelerator: DM creation (bottom to top)

Direct Detection (DD)

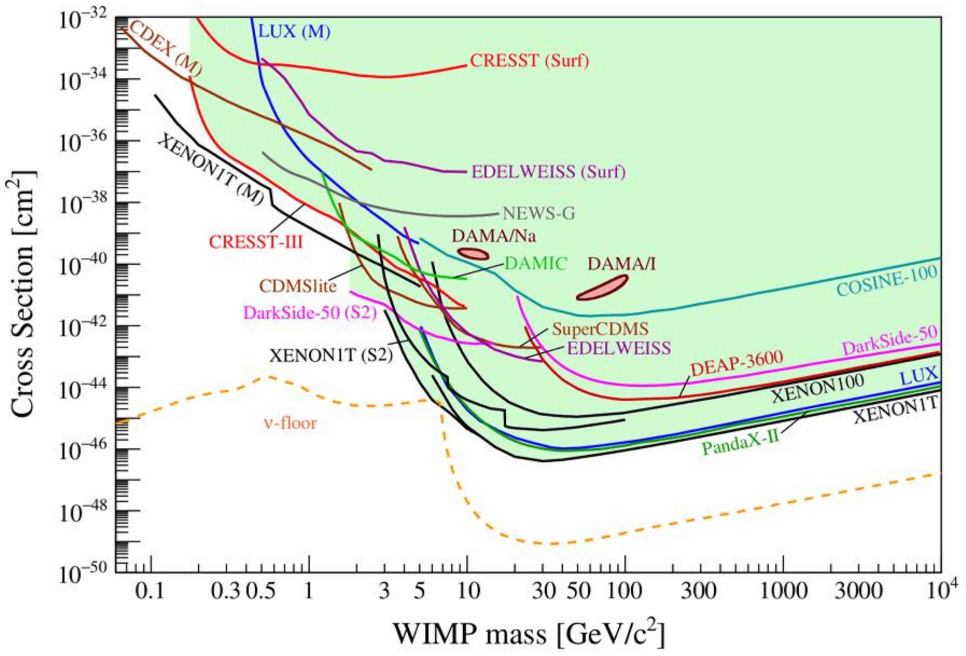


- The signals are WIMP-nucleus recoil events

- Low probability requires high exposure
- Underground to avoid background

- Depend on features of targets and experimental set-ups
- Different nuclear targets and background subtraction:
    - COSINE, ANAIS, DAMA, LZ, PandaX-4T, XENON-nT, PICO-60 and ect.

Migdal effect



- We still have not observed DM

- We tried to optimize DD experiments to the search of sub-GeV DM

- One of main challenges in detecting low-mass DM is the small deposited energy below the threshold of detectors

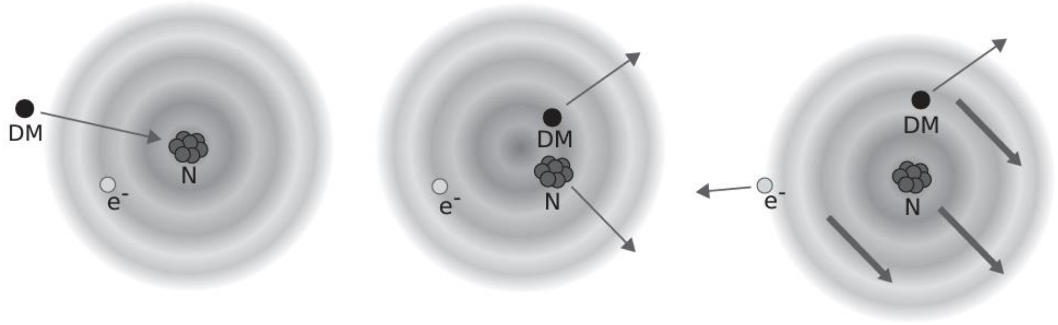
- Migdal effect can help to overcome this problem

Migdal effect

- During DM-nucleus scattering, the electrons inside the atom can become excited or de-orbit in a delayed reaction

- Generating additional(secondary) signals as electronic recoil energy

- This allows to detect low-mass DM in unexplored regions



Matthew J. Dolan, Felix Kahlhoefer, Christopher McCabe, *Directly Detecting Sub-GeV Dark Matter with Electrons from Nuclear Scattering*, Physical Review Letters 121, 101801 (2018)

Non-Relativistic Effective Theory (NREFT)

- WIMP is slow, so that the recoil events are non-relativistic
- NREFT provides a general and efficient way to characterize results with mass of WIMP and coupling constants

Operators spin up to 1/2

- Hamiltonian: $\Sigma_{i=1}^{N}\left(c_i^n \mathcal{O}_i^n + c_i^p \mathcal{O}_i^p\right)$
- Non-relativistic process
  - all operators must be invariant by Galilean transformations ($v \sim 10^{-3}c$ in galactic halo)
- Building operators using: $i\dfrac{\vec{q}}{m_N},\ \vec{v}^{\perp},\ \vec{S}_\chi,\ \vec{S}_N$

$$\mathcal{O}_1 = 1_\chi 1_N; \quad \mathcal{O}_2 = (v^\perp)^2; \quad \mathcal{O}_3 = i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$$

$$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N; \quad \mathcal{O}_5 = i\vec{S}_\chi \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp); \quad \mathcal{O}_6 = (\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$$

$$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp; \quad \mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp; \quad \mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N})$$

$$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp)$$

$$\mathcal{O}_{13} = i(\vec{S}_\chi \cdot \vec{v}^\perp)(\vec{S}_N \cdot \frac{\vec{q}}{m_N}); \quad \mathcal{O}_{14} = i(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^\perp)$$

$$\mathcal{O}_{15} = -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}),$$

Non-Relativistic Effective Theory (NREFT)

- Scattering amplitude:
$$\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\Sigma_{spins}|M|^2 \equiv \Sigma_k \Sigma_{\tau=0,1}\Sigma_{\tau'=0,1} R_k^{\tau\tau'}\left(\vec{v}_T^{\perp\,2}, \frac{\vec{q}^{\,2}}{m_N^2}, \left\{c_i^\tau, c_j^{\tau'}\right\}\right) W_k^{\tau\tau'}(y)$$

- $R_k^{\tau\tau'}$: WIMP response function
  - Velocity dependence: $\mathcal{R}_k^{\tau\tau'} = \mathcal{R}_{k,0}^{\tau\tau'} + \mathcal{R}_{k,1}^{\tau\tau'}\left(v^2 - v_{min}^2\right)$

- $W_k^{\tau\tau'}$: nuclear response function
  - $y = (qb/2)$^2
  - b: harmonic oscillator size parameter
  - $k = M,\ \Delta,\ \Sigma',\ \Sigma'',\ \widetilde{\Phi}'$ and Φ''
  - allowed responses assuming nuclear ground state is a good approximation of P and T

Non-Relativistic Effective Theory (NREFT)

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- Differential cross section : $\frac{d\sigma}{dE_R} = \frac{1}{10^6}\frac{2m_N}{4\pi}\frac{c^2}{v^2}\left[\frac{1}{2j_\chi+1}\frac{1}{2j_N+1}\,\Sigma_{spin}|M|^2\right]$

- Differential rate : $\frac{dR}{dE_R} = N_T \int_{v_{min}}^{v_{esc}} \frac{\rho_\chi}{m_\chi}\, v\, \frac{d\sigma}{dE_R}\, f(v)\, dv$

- With $E_R = \frac{\mu_{\chi N}^2\, v^2}{m_N}$, $\quad v_{min} = \frac{1}{\sqrt{2m_N E_R}}\left|\frac{m_N E_R}{\mu_{\chi N}} + \delta\right|$

Nuclear recoil event rate

- Nuclear recoil event rate

$$\mathrm{R_{NR}} = M\tau_{exp}\frac{\rho_\chi}{m_\chi}\int_{v_{min}}^{v_{esc}} dv_T\ f(v_T)\,v_T\,\Sigma_T\,N_T\int_{E_{R,th}}^{E_R^{max}} dE_R\,\zeta_{exp}\frac{d\sigma}{dE_R}$$

  - $M\tau_{exp}$: exposure
  - N_T : the number of targets per unit mass
  - E_R^{max} : maximum recoil energy
  - $E_{R,th}$: experimental energy threshold
  - ζ_{exp} : experimental features such as quenching, resolution, efficiency, etc.
  - $f(v_T)$: velocity distribution function (assumed as Maxwellian)

Migdal effect signal

- $\chi T \to \chi' T$
  - energy conservation: $\frac{1}{2}\mu_{\chi T} v_T^2 = E_{\chi'} + E_T + \Delta$
  - $\Delta_{max} = \frac{1}{2}\mu_{\chi T} v_T^2$
  - $E_R = \frac{\mu_{\chi T}^2}{m_T} v^2 \left[1 - \frac{\Delta}{\mu_{\chi T} v^2} - cos\theta \sqrt{1 - \frac{2\Delta}{\mu_{\chi T} v^2}}\right]$
  - $v_{min}(E_R) = \frac{1}{\mu_{\chi T}\sqrt{2 m_T E_R}} \left|m_T E_R + \mu_{\chi T}\Delta\right|$

- $\Delta = \mathrm{E_{EM}} + \delta$: amount of lost kinetic energy of DM particle
- $\delta = m_{\chi'} - m_\chi$: mass splitting of DM particle
- E_{EM} : electromagnetic energy deposited by the ionization process

Migdal effect signal

- Migdal event rate

$$\mathrm{R}_{\mathrm{Migdal}} = \int dE_{det} \int dE_R \int dv_T \, \frac{d^3 R_{\chi T}}{dE_R dv_T dE_{det}}$$

$$E_{det} = QE_R + E_{EM} + \delta \cong E_{EM} + \delta$$
$$E_{EM} = E_e + E_{nl}$$

- E_{det} : total deposited energy
- E_e : outgoing electron energy
- E_{nl} : atomic de-excitation energy
- Q : quenching factor

Migdal effect signal

- Migdal event rate

$$\frac{d^3R_{\chi T}}{dE_R dv_T dE_{det}} = \frac{d^2R_{\chi T}}{dE_R dv_T} \times \frac{1}{2}\Sigma_{n,l}\ \frac{d}{dE_e} p^c_{q_e}\big(nl \to (E_e)\big)$$

$$\frac{d^2R_{\chi T}}{dE_R dv_T} = M\tau_{exp}\frac{\rho_\chi}{m_\chi}\ f(v_T)v_T\ \Sigma_T\ N_T\ \zeta_{exp}\frac{d\sigma}{dE_R}$$

  - $p^c_{q_e}$: ionization probability

	Target	E_{det} interval (keV)	n (from 3 up to)
XENON1T	Xe	[0.186, 3.8]	5
DS50	Ar	[0.083, 0.106]	3
SuperCDMS	Ge	[0.07, 2]	3

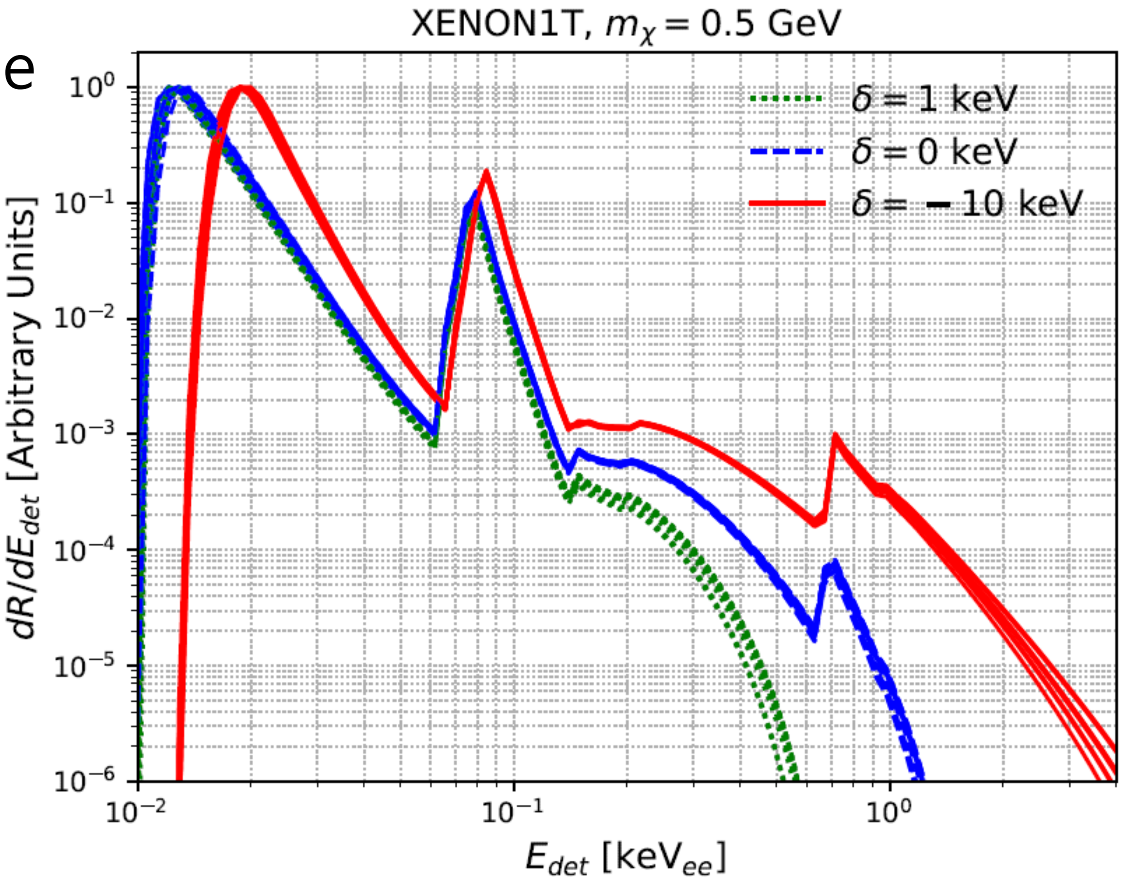
Migdal effect signal

- Impulse approximation (time scale)
  - DM-nucleus collisions happen rapidly compared to the time taken for the atom to traverse its potential
  - time of collision and emission: $t \cong 1/E_R < 1/\omega_{ph}$
  - $t \approx 10^{-12} s \rightarrow E_{cut} \approx 50$ meV
  - $E_R = \frac{\mu_{\chi T}^2}{m_T} v^2 \left[1 - \frac{\Delta}{\mu_{\chi T} v^2} - cos\theta \sqrt{1 - \frac{2\Delta}{\mu_{\chi T} v^2}}\right]$

m_χ **cut**	$\delta = 0$ **keV**	$\delta = -10$ **keV (exothermic)**	$\delta = +10$ **keV (endothermic)**
XENON1T	0.02 GeV	0.6 MeV	3 GeV
DS50	0.01 GeV	0.2 MeV	3 GeV
SuperCDMS	0.016 GeV	0.34 MeV	3 GeV

Migdal spectrum

- Normalized Migdal Spectrum
  - NREFT interaction has mild effect: every interaction has almost same shape
  - because it's mainly determined by ionization probability
  - the type of interaction can affect the magnitude



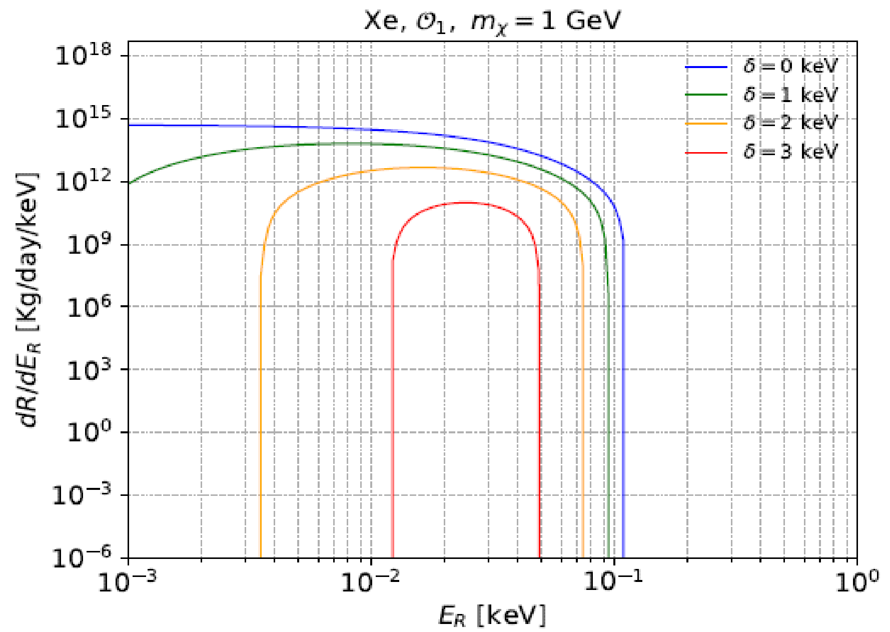
S. Kang, Stefano Scopel, Gaurav Tomar, *Low-mass constraints on WIMP effective models of inelastic scattering using the Migdal effect*, arXiv: 2407.16187

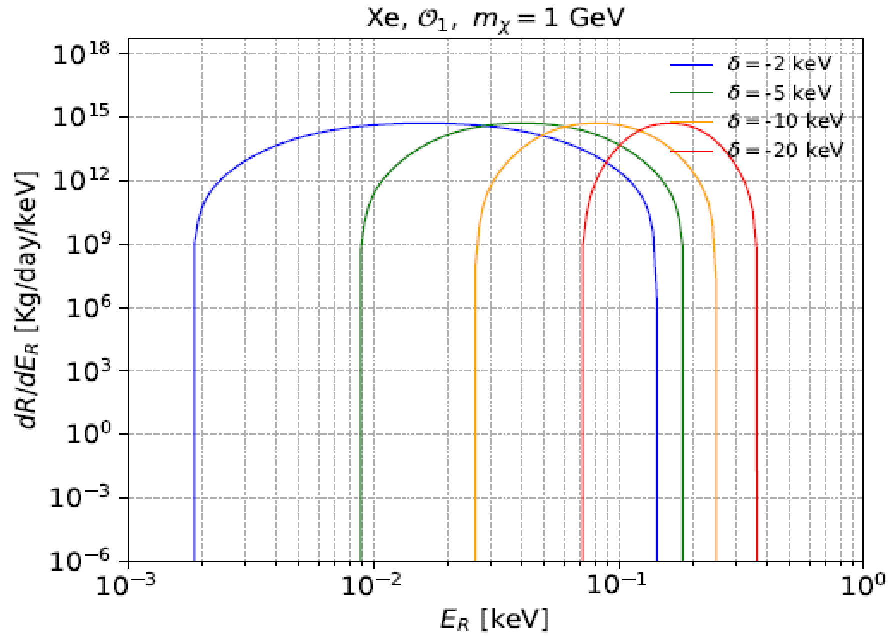
Nuclear recoil spectrum

- Endothermic ($\delta > 0$):
  - normalization decreasing
  - energy interval shrinking

- Exothermic ($\delta < 0$):
  - normalization same
  - energy interval shifting to higher range

- Migdal signal is enhanced for $\delta < 0$
  - p_{qe}^{c} is growing linearly with E_R





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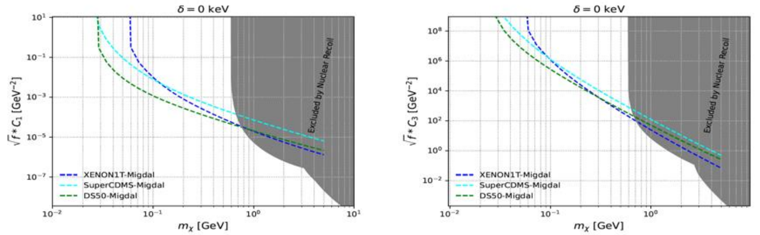
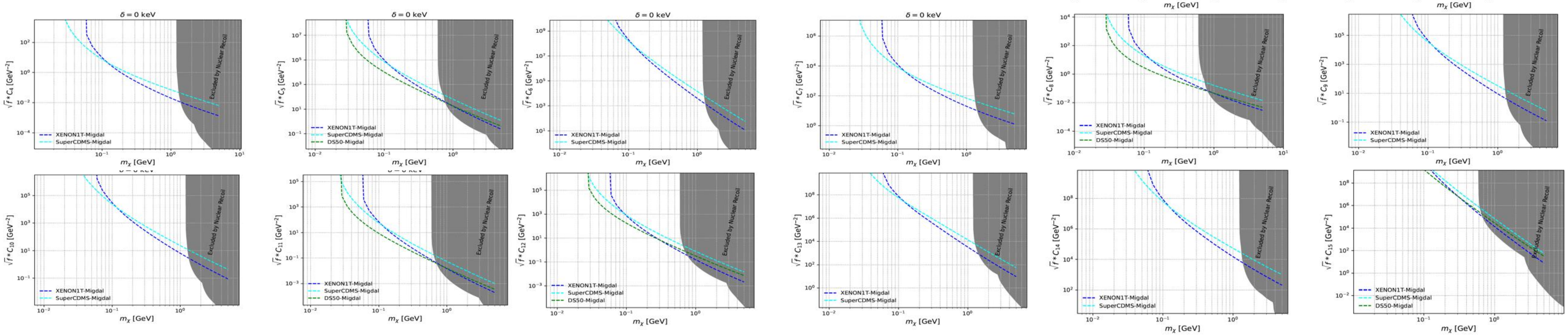
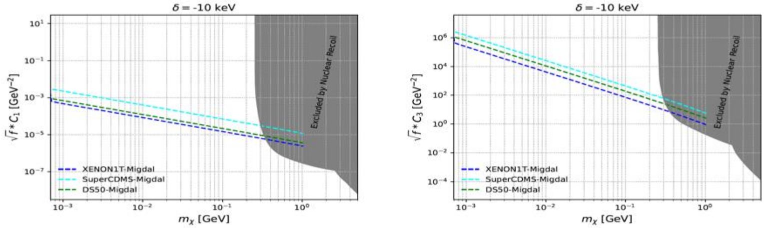
$m_\chi - c$ plane

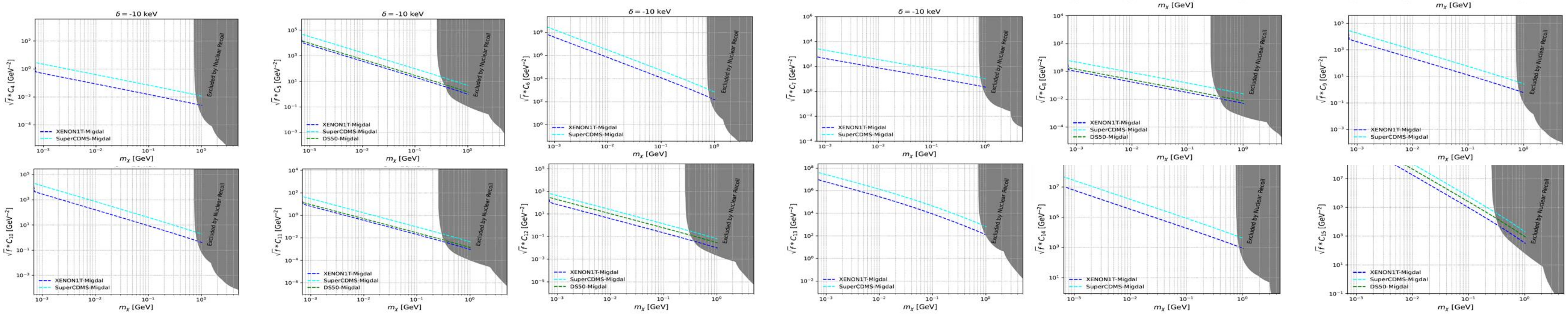
$\delta = 0$ keV





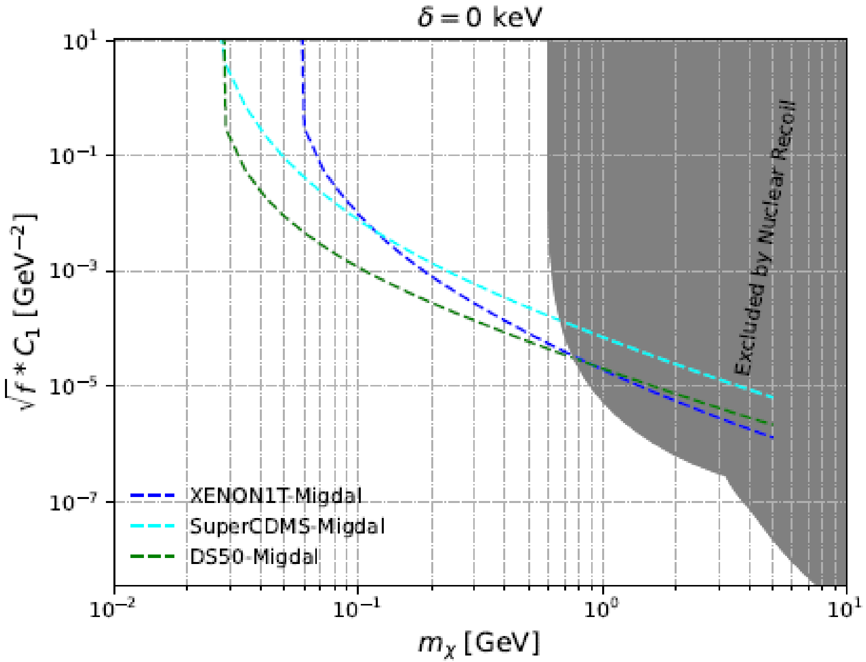
$\delta = -10$ keV

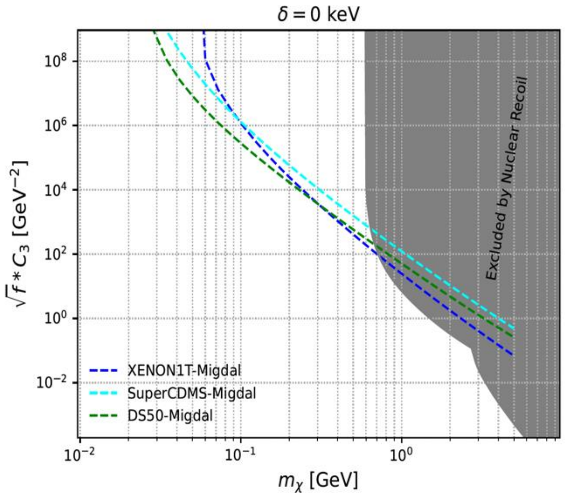


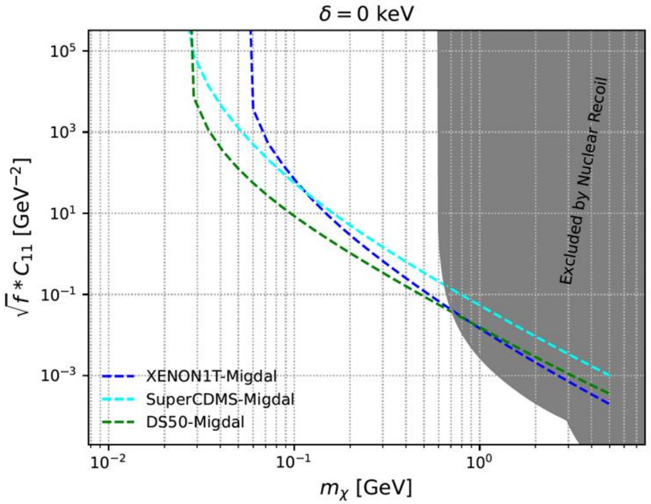


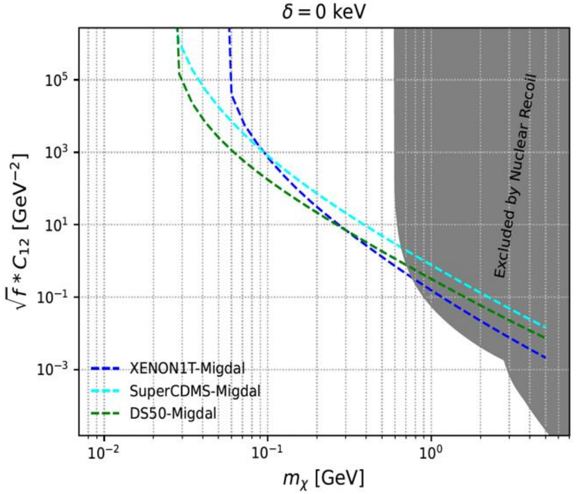
$m_\chi - c$ plane

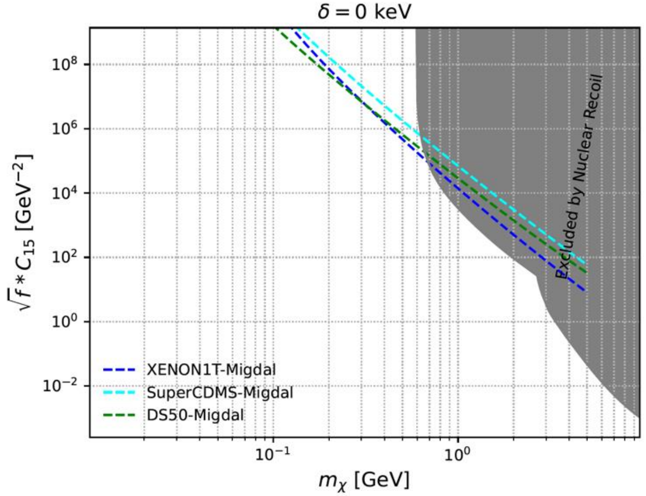
- SI type($O_{1,3,11,12,15}$)
  - W_{TM} : proportional to square of target mass
  - $W_{T\Phi''}$: non-vanishing for all nuclei
    favors heavier elements







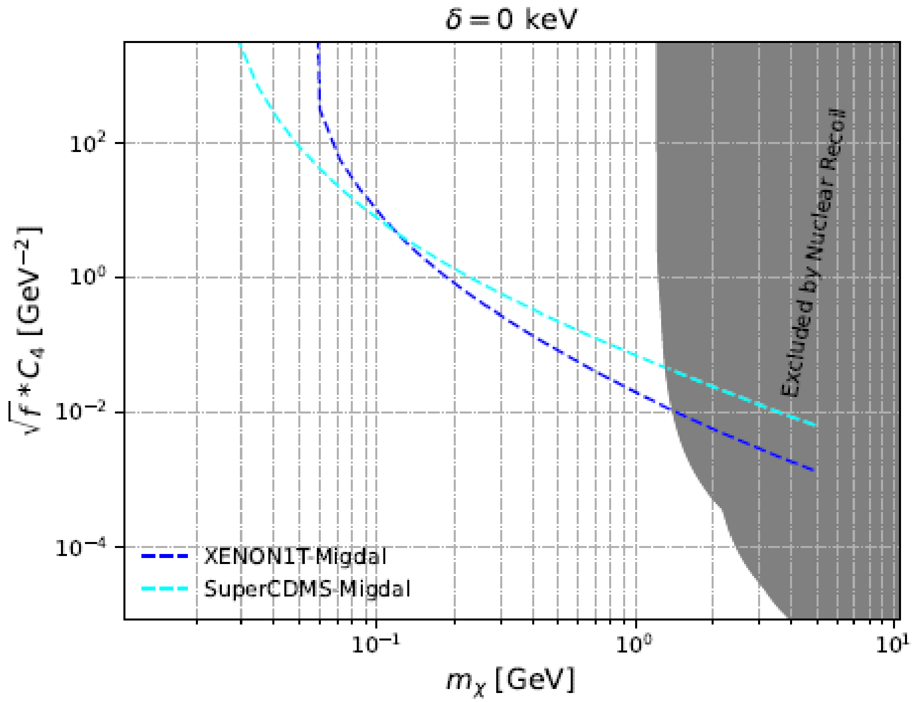


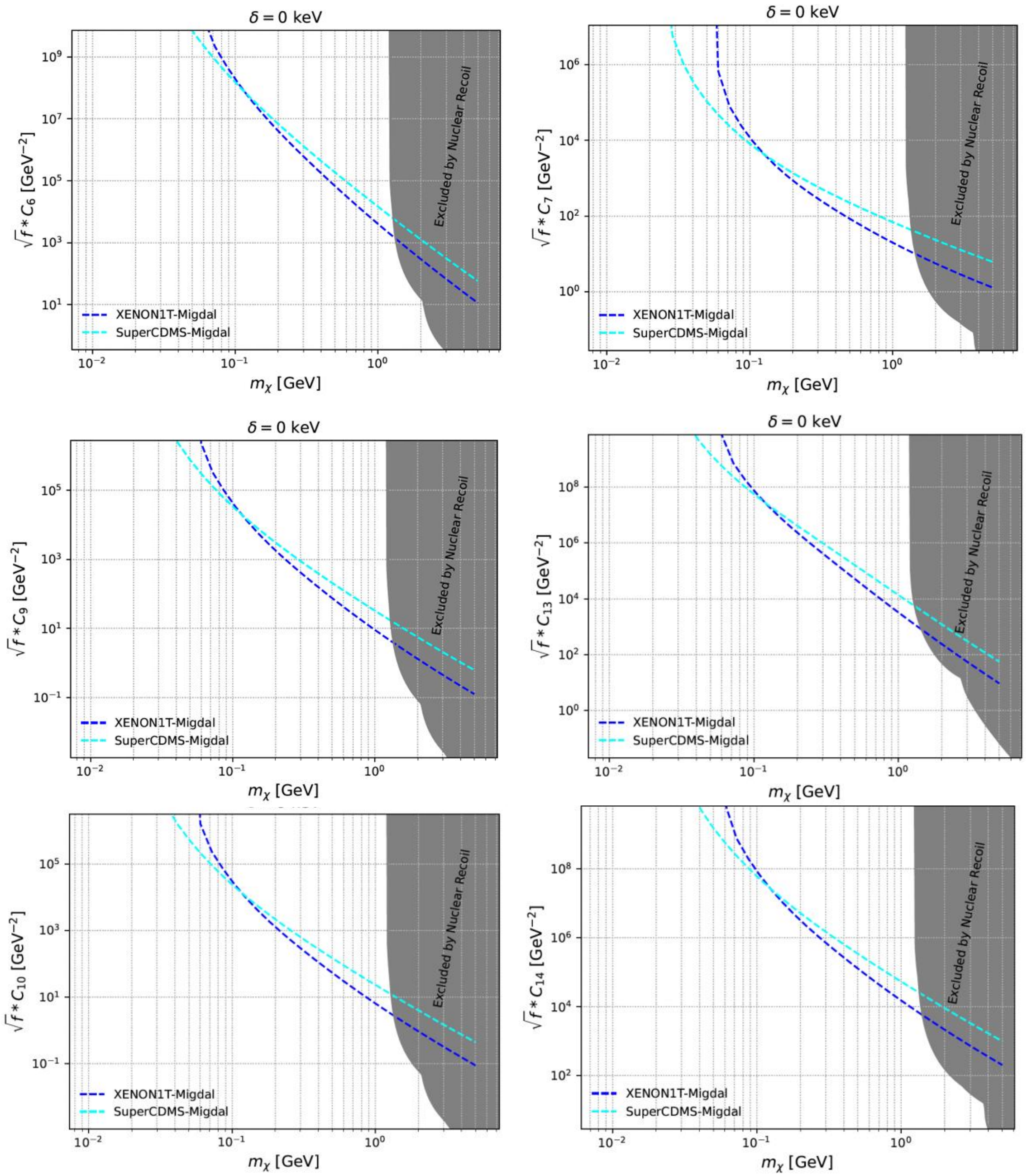


S. Kang, Stefano Scopel, Gaurav Tomar, *Low-mass constraints on WIMP effective models of inelastic scattering using the Migdal effect*, arXiv: 2407.16187

$m_\chi - c$ plane

- SD type($O_{4,6,7,9,10,13,14}$)
  - $W_{T\Sigma'}$ & $W_{T\Sigma''}$: driven by spin of targets
  - $W_{T\widetilde{\Phi}'}$: requires targets spin > 1/2





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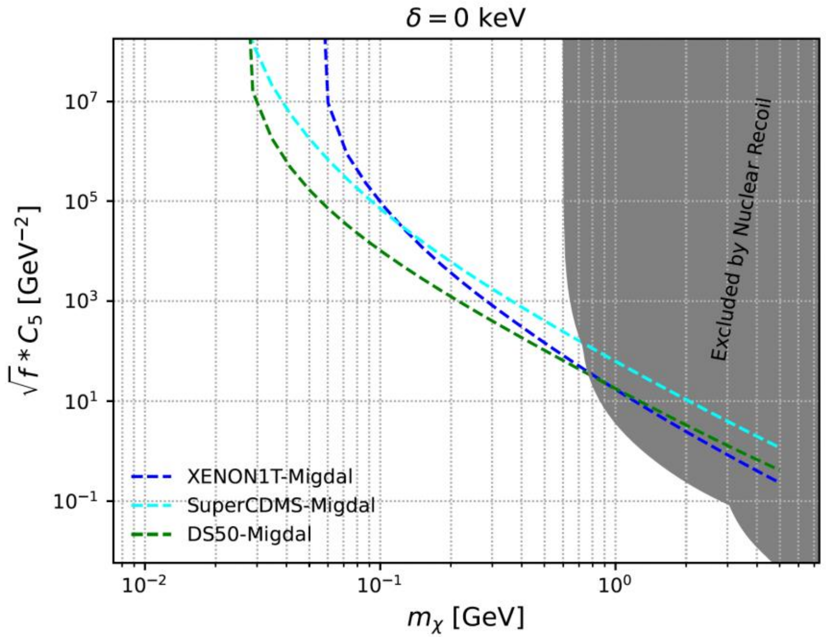
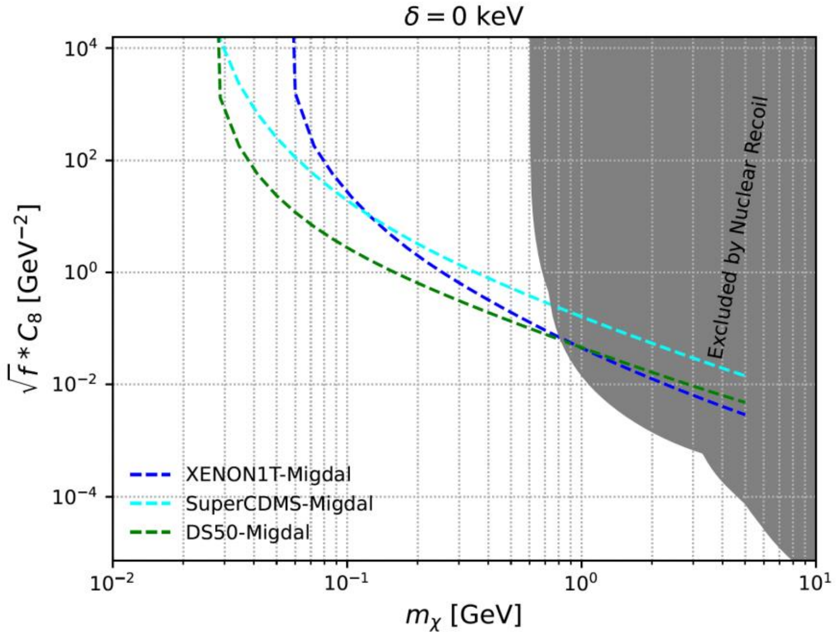
$m_\chi - c$ plane

- Others($O_{5,8}$)
  - velocity independent: $W_{T\Delta}$ related to angular momentum
  - velocity dependent: W_{TM}

- Xe, Ar: W_{TM} dominates → SI type
- Ge: $W_{T\Delta}$ dominates → SD type





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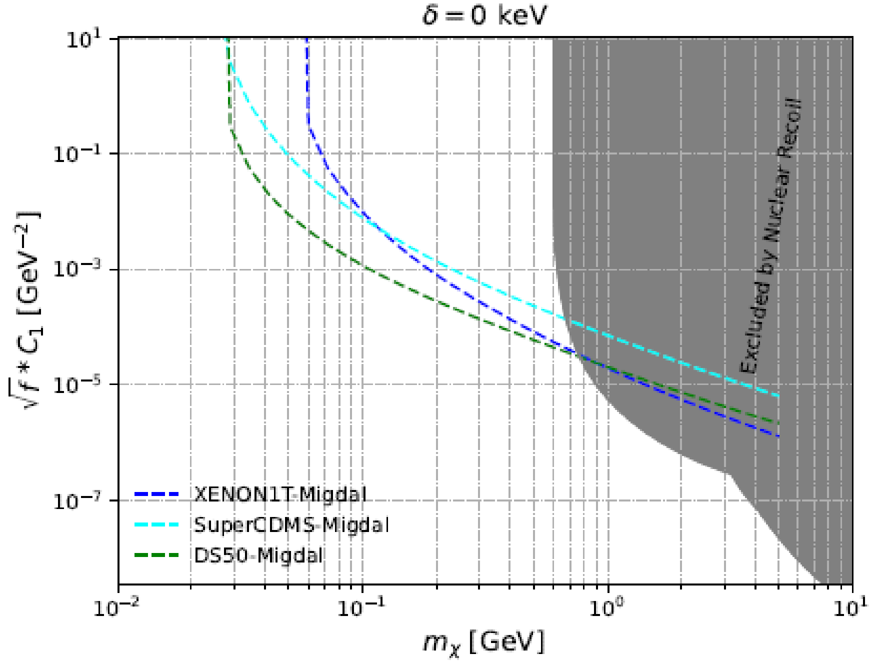
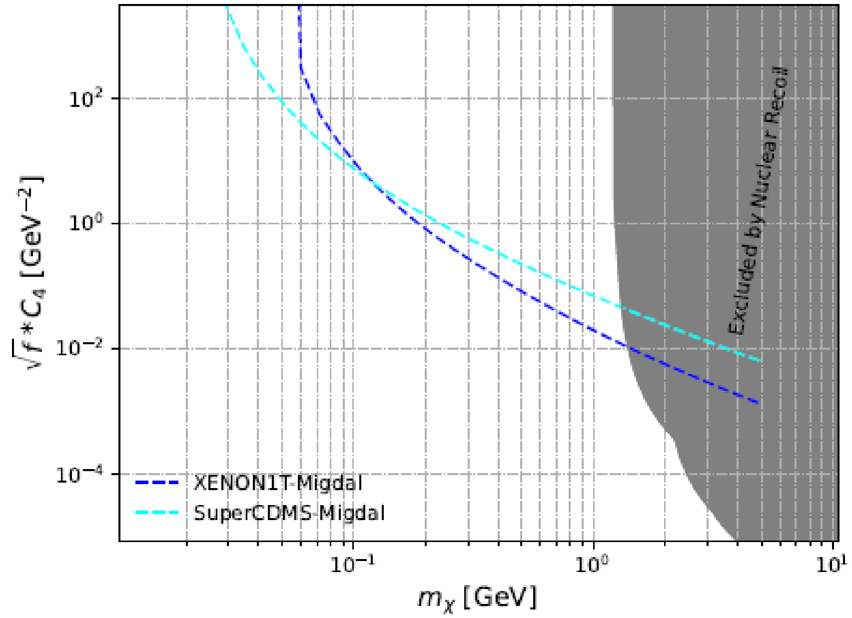
$m_\chi - c$ plane

- Extend sensitivity to low WIMP mass
- $\delta = 0$ keV
  - SI type
    - low m_χ : DS50
    - high m_χ : XENON1T
  - SD type
    - low m_χ : SuperCDMS
    - high m_χ : XENON1T
- $\delta = -10$ keV
  - XENON1T dominates
  - become flat and saturate the m_χ cut





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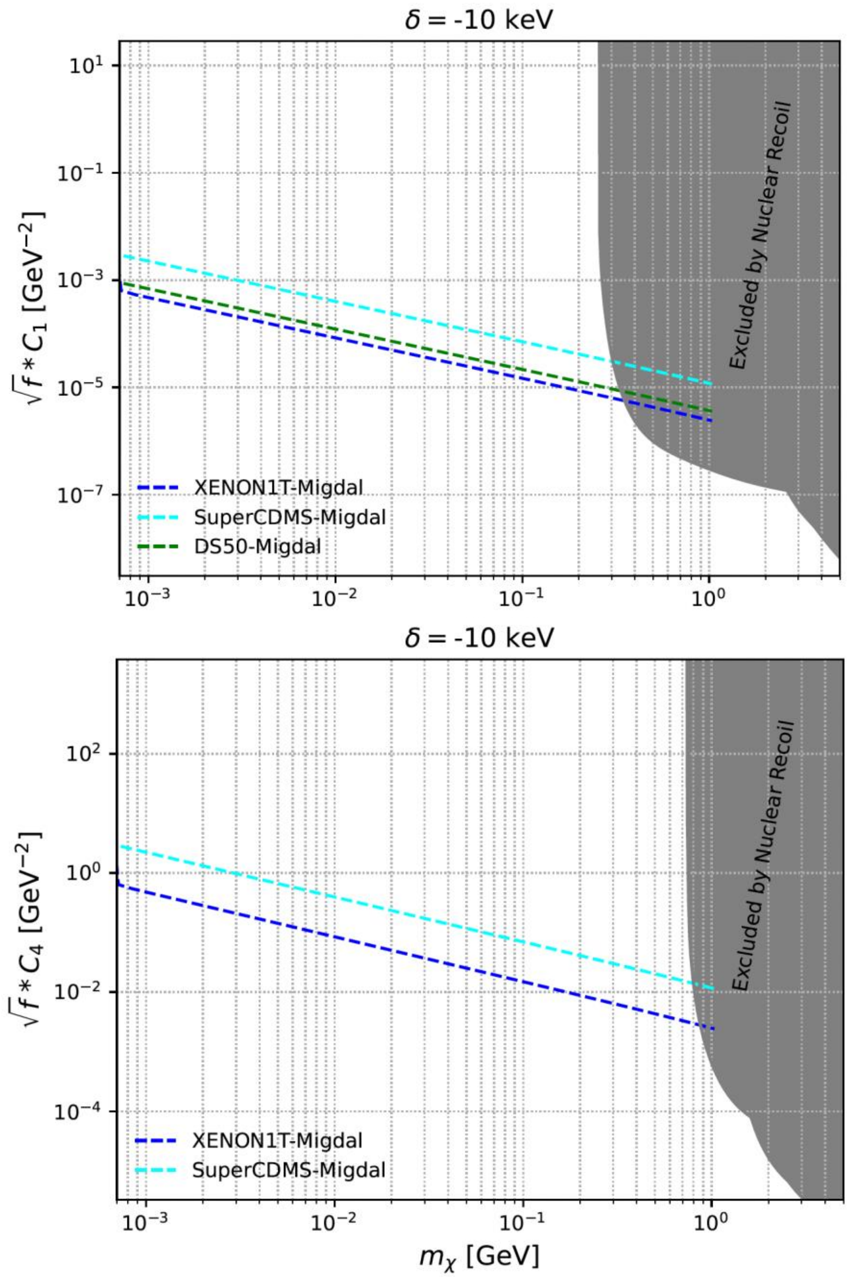

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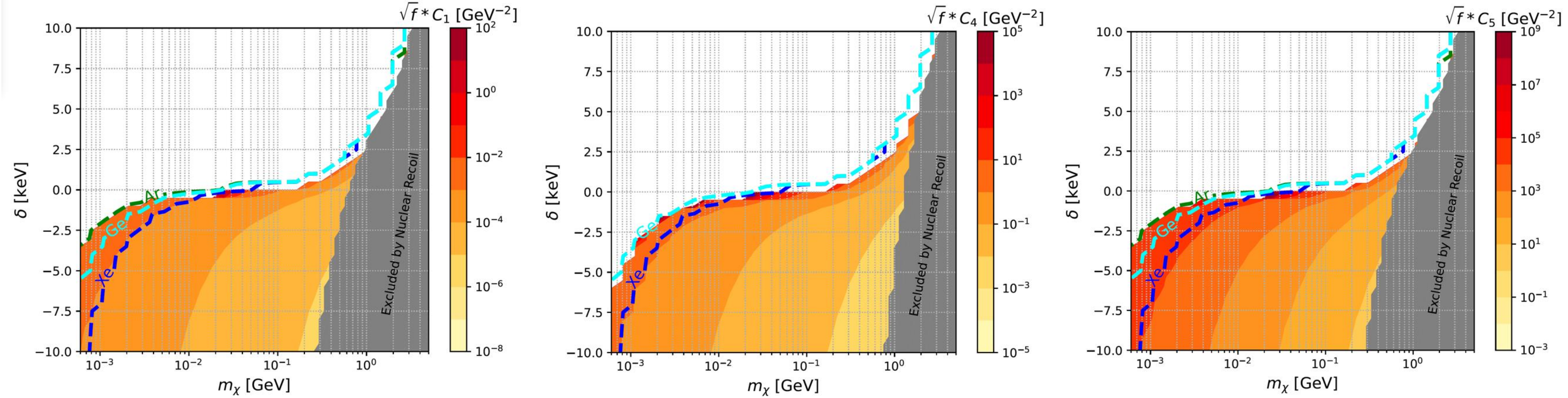
$m_\chi - \delta$ plane

- Endothermic($\delta > 0$): very small region
- Exothermic($\delta < 0$): Bounds are stronger with inceasing $|\delta|$ fixing m_χ



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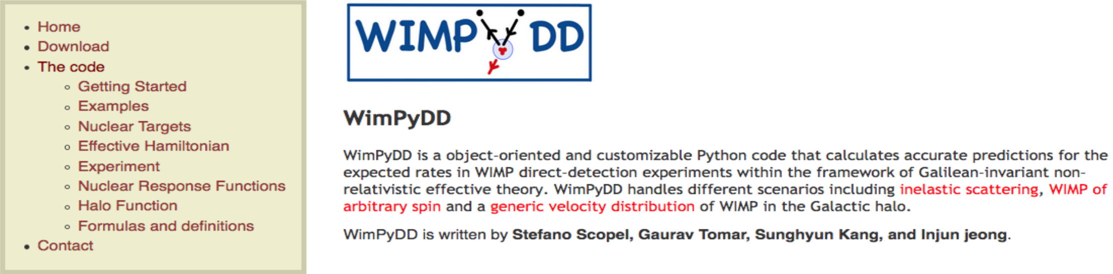
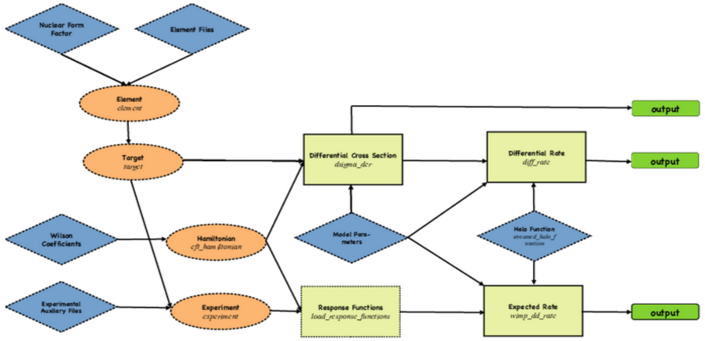
WimPyDD





- User-friendly Python code

- Calculates expected rates in any scenarios:
    - arbitrary spins
    - inelastic scattering
    - generic WIMP velocity distribution

- Published and can be downloaded:
    - https://wimpydd.hepforge.org/

Injun Jeong, S. Kang, Stefano Scopel, Gaurav Tomar, *WimPyDD: An object-oriented Python code for the calculation of WIMP direct detection signals*, Computer Physics Communications, 2022.108342

Summary

- Due to the absence of a signal from WIMPs
  DD experiments have become interested in searching sub-GeV DM

- Migdal effect can help to overcome a problem of small energy deposition below the threshold of detectors

- Using Migdal effect we can significantly extend to low WIMP masses considering especially a down-scattering process

- Complementarity of various experiments can put more stringent bounds at low mass region