Dark matter from inflationary quantum fluctuations

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KIAS-IBS-CKC Joint Workshop on Particle, String and Cosmology

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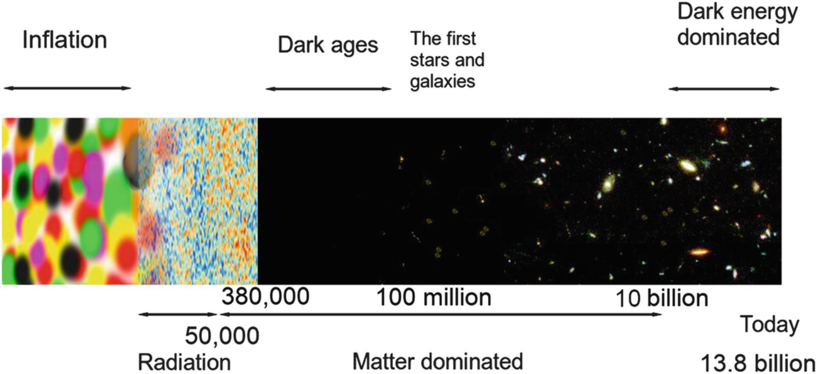
January 14, 2025

Institute for Basic Science



Malik & Matravers, How Cosmologists Explain the Universe to Friends and Family

- Inflation generates the seed for the **large scale structures**.
- Most of the proposed scenarios deal with the production of DM during the radiation domination.
- **Why not during inflation?**
  One difficulty is, even if DM particles produce during inflation, the energy density exponentially dilutes $\cdots$

Superhorizon modes freeze and survive!

DM from inflationary superhorizon modes

Spin 0: Scalar DM

- Polarski, Starobinsky, PRD (1994)
- Graham, Mardon, Rajendran, PRD (2016)
- ...
- Firouzjahi, MAG, Mukohyama, Talebian, PRD (2022)
- ...

Spin 1: Vector DM

Vector modes do not freeze at superhorizon due to conformal symmetry. Direct interaction between DM and inflaton is needed.

- Bastero-Gil, Santiago, Ubaldi, Vega-Morales, JCAP (2019)
- Nakai, Namba, Wang, JHEP (2020)
- Salehian, MAG, Firouzjahi, Mukohyama, PRD (2021)
- Firouzjahi, MAG, Mukohyama, Salehian, JHEP (2021)
- ...

Spin 2: Tensor DM

Spin-2 fields cannot have mass smaller than Hubble parameter during inflation due to the Higuchi bound. Direct interaction between the DM field and inflaton is needed.

- MAG, JCAP (2023)

Massive modes in an expanding universe

There are three independent scales:

- Physical momentum $\frac{k}{a(t)}$
- Hubble parameter $H(t)$
- Mass m

Superhorizon
$H \gg \frac{k}{a}$

Subhorizon
$H \ll \frac{k}{a}$

Non-relativistic
$m \gg \frac{k}{a}$

Relativistic
$m \ll \frac{k}{a}$

Massive
$m \gg H$

Massless
$m \ll H$

What do we need for dark matter?

- **During inflation**:
  $H \gg \frac{k}{a}$ & $H \gg m \Rightarrow$ Massless modes freeze at superhorizon
- **After inflation before matter-radiation equality:**
  $m \gg \frac{k}{a}$ & $m \gg H \Rightarrow \rho \propto a^{-3}$

$$\boxed{H_{\mathrm{eq}} \lesssim m \ll H_{\mathrm{inf}}}$$

A free spectator scalar field during inflation

$$S = \frac{1}{2}\int \mathrm{d}^3x\,\mathrm{d}t\, a^3 \left[\dot{X}^2 - (\nabla_i X)^2 - m^2 X^2\right]$$

Going to Fourier space, quantizing $X_{\mathbf{k}}(\tau) = X_k(\tau)\,\hat{a}_{\mathbf{k}} + X_k^*(\tau)\,\hat{a}^\dagger_{-\mathbf{k}}$, the mode function satisfies

$$\ddot{X}_k + 3H\,\dot{X}_k + \left(k^2/a^2 + m^2\right) X_k = 0$$

- **During inflation:**
  $H \gg \frac{k}{a}$ & $H \gg m \Rightarrow$
  $X_k \sim \mathrm{constant}$
  Bunch-Davies initial condition

$$\mathcal{P}_X(k,\tau) = \frac{k^3}{2\pi^2}\left|X_k\right|^2 = \left(\frac{H_{\rm inf}}{2\pi}\right)^2$$

X has a scale-invariant power spectrum

- **After inflation:**
  $m \gg \frac{k}{a}$ & $m \gg H \Rightarrow$
  $X_k \sim a^{-3/2}\cos(mt)$

$$\rho_X = \frac{1}{2}\left[\dot{X}^2 + (\nabla_i X)^2 + m^2 X^2\right]$$

$$\bar{\rho}_X(t) = \langle \rho_X \rangle \approx m^2 \langle X \rangle^2/2 \propto a^{-3}$$

The averaged energy density scales like dark matter

Large fluctuations in DM energy density

Using Gaussianity of X ($\langle X^3 \rangle = 0, \cdots$) & $\langle X \rangle^2 \ll \langle X^2 \rangle$, for DM density contrast $\delta_X = (\rho_X - \bar{\rho}_X)/\bar{\rho}_X$, we find

$$\langle \delta_X^2 \rangle = 2, \qquad \langle \delta_X^3 \rangle = 8, \qquad \cdots$$

X has scale-invariant power spectrum and there will be $O(1)$ fluctuations at all scales. In particular, there will be large isocurvature perturbations at CMB scales $k \sim k_{\rm eq}$!!

We need a model which only enhances small scales

$$\boxed{k \gg k_{\rm eq} \quad (k_{\rm eq} \sim 10^{-2}\mathrm{Mpc}^{-1})}$$

Simple case: Non-minimal coupling (e.g. interaction with inflaton)

$$S = \frac{1}{2} \int \mathrm{d}^3x \, \mathrm{d}t \, a^3 f(t)^2 \left[\dot{X}^2 - (\nabla_i X)^2 - m^2 X^2 \right]$$

Choosing $f(t)$ such that the initial power spectrum becomes

$$\mathcal{P}_{X,i}(k) = \mathcal{A} \left(\frac{H_{\rm inf}}{2\pi} \right)^2 \delta \left[\ln \left(k/k_p \right) \right] ; \qquad k_p \gg k_{\rm eq}$$

Evolution after inflation

The time evolution is given by $X_k(t) = T_m(k,t) X_{k,i}$ where, assuming $f = 1$ after inflation, transfer function satisfies

$$\ddot{T}_m + 3H\,\dot{T}_m + \left(k^2/a^2 + m^2\right) T_m = 0$$

The above equation can be analytically solved (see [MAG, M. Sasaki, T. Suyama, arXiv:2501.03444]).

In the limit $m \gg \frac{k}{a}$ & $m \gg H$, the spectral energy density $\Omega_X(k,t) \equiv \frac{1}{3M_{\rm Pl}^2 H^2}\frac{\mathrm{d}\bar{\rho}_X}{\mathrm{d}\ln k}$ becomes function of three parameters

$$\Omega_X = \frac{\beta_{\rm inf}}{24}\sqrt{\frac{m}{H}}\,|\mathcal{I}_m(k)|^2 \delta\left[\ln\left(k/k_p\right)\right]; \qquad \mathcal{I}_m(k) \equiv \frac{\sqrt{\frac{\pi}{2}}\,e^{-\frac{\pi k}{8ma_k}}}{\Gamma\left(\frac{3}{4} + \frac{ik}{4ma_k}\right)},$$

where $\beta_{\rm inf} \equiv \mathcal{P}_{\mathcal{R}}\, r\, \mathcal{A}$ with $r = \mathcal{P}_h/\mathcal{P}_{\mathcal{R}}$ is tensor-to-scalar ratio, $\mathcal{P}_h = 2H_{\rm inf}^2/(\pi^2 M_{\rm Pl}^2)$ and $\mathcal{P}_{\mathcal{R}}$ are the power spectra of GWs and curvature perturbation.

Constraints on the model parameters $m, k_p, \beta_{\rm inf}$

We assume that X **constitutes all dark matter:**

$$\bar{\Omega}_X(\tau_{\rm eq}) = \int \Omega_X(\tau_{\rm eq}, k)\,\mathrm{d}\ln k = 1/2$$

- **Freezing at superhorizon:**

$$m \ll H_{\rm inf} \sim \left(\frac{r}{0.03}\right)^{1/2} \left(\frac{\mathcal{P}_{\mathcal{R}}}{3\times 10^{-10}}\right)^{1/2} 10^{13}\,\mathrm{GeV}$$

- **Non-relativistic condition:**

$$\frac{m}{1\,\mathrm{eV}} \gg 10^{-26}\,\frac{k_p}{\mathrm{Mpc}^{-1}}$$

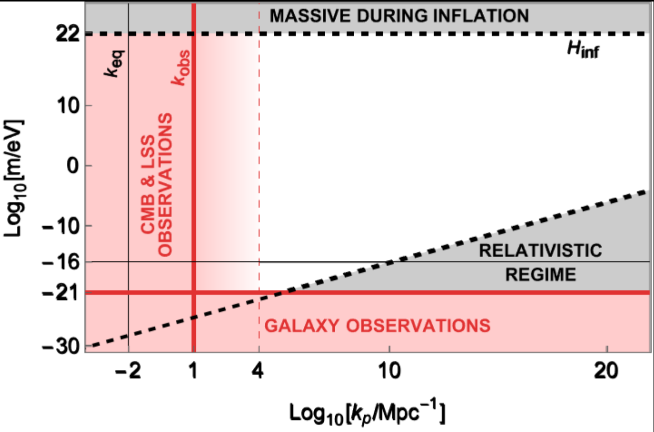
- **CMB and LSS observations:**

$$k_p \gtrsim 10^3 k_{\rm obs} \sim 10\,\mathrm{kpc}^{-1}$$

- **Galaxy observations:** The dark matter's de Broglie wavelength should be smaller than its halo

$$m \gtrsim 10^{-21}\,\mathrm{eV}$$

Constraints on the model parameters m and k_p



We have set $H_{\rm inf} = 10^{13}$ GeV which corresponds to $r \sim 0.03$

Formation of subsolar halos

- t_s: Time that is taken for the wave to propagate at the distance $\lambda_p \propto a/k_p$ ($t_s \sim a^2 m/k_p^2$)
- t_f: Free fall time ($t_f \sim 1/\sqrt{G\rho} \sim H^{-1}$)

Halos form when $t_f < t_s$:

$$\frac{k_p}{1\,\mathrm{kpc}^{-1}} < \mathcal{Z}(z)\left(\frac{m}{10^{-18}\mathrm{eV}}\right)^{1/2}$$

where $\mathcal{Z}(z) \simeq \mathcal{O}(1)$ for $0 < z < 3000$. Equivalently

$$m > H_p\,; \qquad H_p = k_p/a_p$$

The typical size of halos:

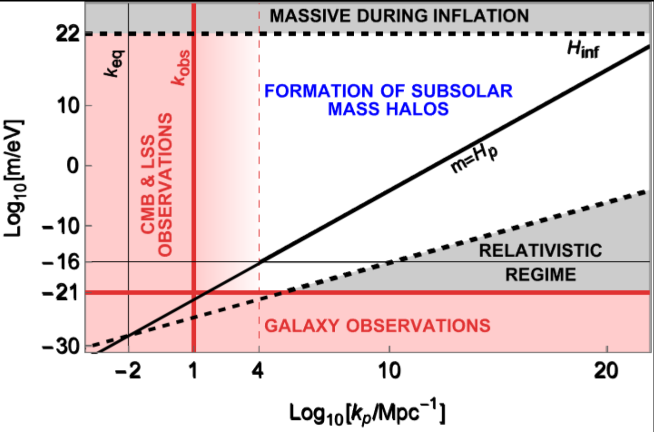
$$M_h \sim \frac{4\pi}{3}\rho_m k_p^{-3} \sim \mathcal{O}(10^2)M_\odot\left(\frac{k_p}{1\ \mathrm{kpc}^{-1}}\right)^{-3}$$

Imposing $k_p \gtrsim 10^3 k_{\mathrm{obs}} \sim 10\,\mathrm{kpc}^{-1}$ to respect CMB and LSS obs.

$$M_h < \mathcal{O}(10^{-1})M_\odot$$

The modes that are non-relativistic at the time of horizon re-entry forms subsolar halos after matter-radiation equality.

Constraints on the model parameters m and k_p



We have set $H_{\rm inf} = 10^{13}$ GeV which corresponds to $r \sim 0.03$

Summary

- Light spectator fields with $m \ll H_{\rm inf}$ may exist during inflation and can play the role of DM.
- To be consistent with CMB and LSS observations, the contribution of the spectator field to the DM power spectrum must be completely suppressed at scales $k \gtrsim 10\,\mathrm{Mpc}^{-1}$.
- The well-known lower bound $m \gtrsim 10^{-21}\,\mathrm{eV}$ from galaxy observations applies to these scenarios.
- The modes that are non-relativistic at the time of horizon re-entry forms subsolar mass halos after matter-radiation equality.
- Apart from some details, these results apply to DM with any spin that originates from inflationary quantum fluctuations.
- While spin-0 and spin-1 spectator fields contribute negligibly to GWs, spin-2 fields can contribute significantly [MAG, Sasaki, PLB (2023), MAG, Sasaki, Suyama, PLB (2023)]. We may find how spin-2 DM differs from spin-0 and spin-1 [MAG, Sasaki, Suyama, work in progress].

Thank you!