Decoherence of Primordial Perturbations and Maldacena's Consistency Condition

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Based on ongoing work with **Junsei Tokuda** (McGill University)

Outline

- **Introduction**

- **Decoherence in cosmology**
  - Wavefunction formalism
  - Decoherence rate

- **Local observer's effect**
  - Maldacena's consistency conditions
  - Regularizing IR divergence

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Inflation as a Source for Cosmological Perturbations

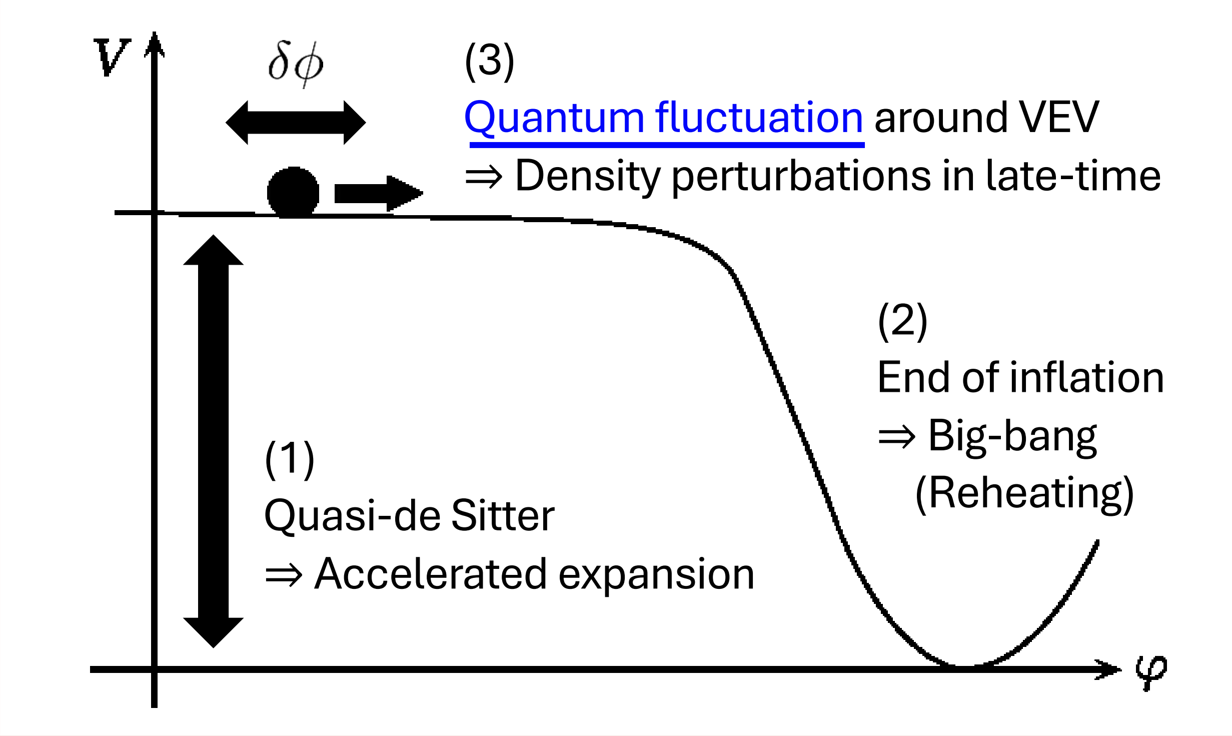
- **Slow-roll inflation can be an answer for ……**

$$\mathcal{L}_\mathrm{m} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$: Inflaton

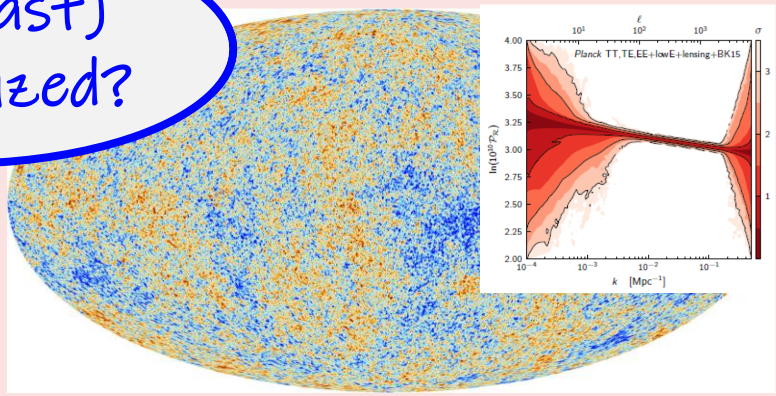
$$\epsilon = M_\mathrm{pl}^2\left(\frac{V'}{V}\right)^2 \ll 1, \quad |\eta| = M_\mathrm{pl}^2\left|\frac{V''}{V}\right| \ll 1$$

$$\phi = \phi_0(t) + \delta\phi(x)$$

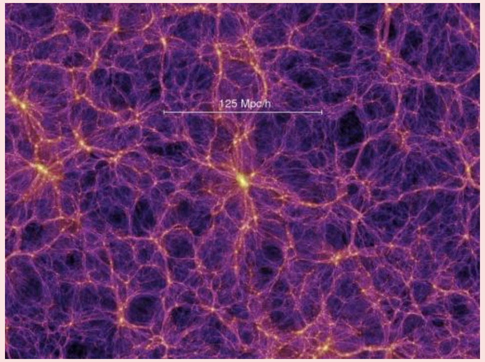
1) Flatness, horizon problem etc. in late time
2) Transition from inflation to big-bang
3) **Origin of cosmological structures**



How (fast) classicalized?



[Planck 1807.06211]



[Millennium Simulation 2005]

Late time
Classical anisotropy
and inhomogeneity

Quantization of Curvature Perturbations

❑ Curvature perturbations

$$\phi = \phi_0(t) + \delta\phi(x) \quad \Longleftrightarrow \quad h_{ij} = (e^{\zeta(x)}a(t))^2(\delta_{ij} + \gamma_{ij})$$

Perturbations of density — Perturbations of scale factor

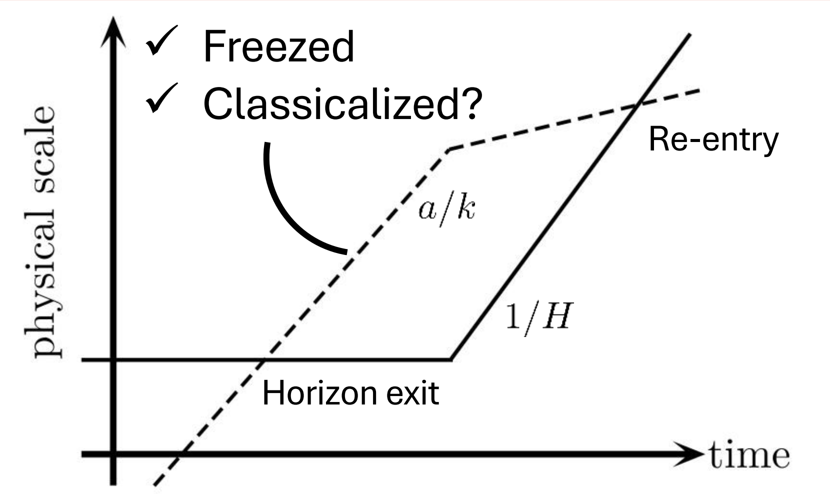
$\zeta(x)$: Curvature perturbations

$\gamma_{ij}(x)$: Tensor perturbations

❑ Canonical quantization with Bunch-Davies vacuum

$$\begin{cases} S^{(2)} = M_{\rm pl}^2 \int d\tau \frac{d^3k}{(2\pi)^3} a^2\epsilon \left[\zeta_{\mathbf{k}}'^2 - k^2\zeta_{\mathbf{k}}^2\right] \qquad d\tau = \frac{dt}{a}, \quad \zeta' = \frac{d\zeta}{d\tau} \\ \zeta_{\mathbf{k}} \equiv a_{\mathbf{k}} u_k(\tau) + a_{-\mathbf{k}}^\dagger u_k^*(\tau), \qquad a u_k(\tau) \underset{k\tau\to-\infty}{\propto} e^{-ik\tau} \\ \pi_{\mathbf{k}} \equiv \frac{\delta S^{(2)}}{\delta \zeta_{\mathbf{k}}'} = a^2\epsilon\zeta_{-\mathbf{k}}', \qquad [\zeta_{\mathbf{k}}(\tau), \pi_{\mathbf{k}'}(\tau)] = i(2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}') \end{cases}$$

$$\Rightarrow \quad u_k(\tau) = \frac{H}{2\epsilon M_{\rm pl}\sqrt{k^3}}(1 + ik\tau)e^{-ik\tau} \quad \left(\underset{|k\tau|<1}{\longrightarrow} \frac{H}{2\epsilon M_{\rm pl}\sqrt{k^3}} = \text{const.} \right)$$



❑ "Classicalization" of super-horizon mode

$$\zeta = \frac{H}{2\epsilon M_{\rm pl}\sqrt{k^3}}(a_{\mathbf{k}} + a_{-\mathbf{k}}^\dagger) \quad \Rightarrow$$

Minimal uncertainty?

Time-independent eigenstates of ζ?
(deterministic)

Formally speaking?

Quantization of Curvature Perturbations

Quantumness: Coherence, Entanglement, Incompatible observations etc.

- Unique in quantum theory
- How interesting in cosmology?

Inflationary phenomenology

✓ Quantum-to-classical transition

➡ Coherence [Polarski et al. gr-qc/9504030, Nelson 1601.03734, etc.]

➡ ✓ Stochastic formalism (Yesterday's talk)

$$\phi = \phi_{\mathrm{UV}} + \phi_{\mathrm{IR,cl}}$$

✓ Open EFT of inflation

[Hongo et al. 1805.06240, Salcedo et al. 2404.15416]

New shape in spectrum?

(Bottom-up) quantum gravity

✓ Bell test for primordial perturbations

[Sou et al. 2405.07141]

➡ ✓ Test for QG pheno.?

✓ Holography, quantum cosmology,

Hawking radiation, multiverse, …

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Quantum Interference and Decoherence

❑ Closed system

✓ Measuring state $|\Psi\rangle = \alpha\,|\psi_1\rangle + \beta\,|\psi_2\rangle$, target physical quantity $\widehat{A}$

➡ Expectation value $\langle\Psi|\widehat{A}|\Psi\rangle = |\alpha|^2\,\langle\psi_1|\widehat{A}|\psi_1\rangle + |\beta|^2\,\langle\psi_2|\widehat{A}|\psi_2\rangle + (\alpha\beta^*\,\langle\psi_2|\widehat{A}|\psi_1\rangle + \text{c.c.})$

Quantum interference

e.g., $\langle\psi_2|a_n\rangle\,\langle a_n|\psi_1\rangle \sim e^{ia_n(\psi_1-\psi_2^*)}$

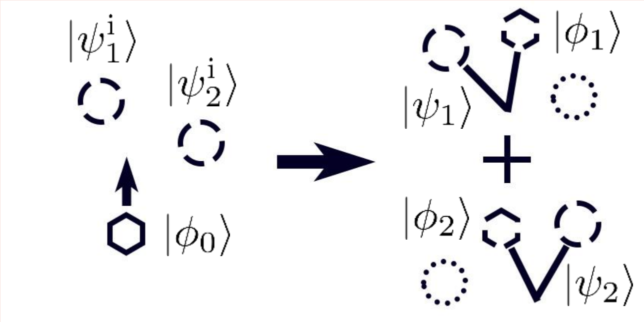
✓ Density matrix

$$\rho = |\Psi\rangle\,\langle\Psi| = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$$

➡ Diagonal: classical probability

Off-diagonal: quantum superposition

❑ Decoherence in open system



System (observed)

Environment (unobserved)

✓ Quantum interactions between $|\psi\rangle$ and $|\phi\rangle$

$$|\Psi_{\mathrm{i}}\rangle = (\alpha_{\mathrm{i}}\,|\psi_1^{\mathrm{i}}\rangle + \beta_{\mathrm{i}}\,|\psi_2^{\mathrm{i}}\rangle)\,|\phi_0\rangle \;\;➡\;\; |\Psi\rangle = \alpha\,|\psi_1\rangle\,|\phi_1\rangle + \beta\,|\psi_2\rangle\,|\phi_2\rangle$$

$$➡\;\; \rho_{\mathrm{S}} = \mathrm{Tr}_{\mathrm{E}}[|\Psi\rangle\,\langle\Psi|] = \begin{pmatrix} |\alpha|^2 & \alpha\beta^*\langle\phi_2|\phi_1\rangle \\ \alpha^*\beta\langle\phi_1|\phi_2\rangle & |\beta|^2 \end{pmatrix}$$

✓ $\langle\phi_2|\phi_1\rangle \sim 0$ if scattered to independent states. More scattering, more independent, less interference.

✓ L1 norm of coherence $C_1 \equiv \min_{\sigma:\ \mathrm{diag}} ||\rho-\sigma||_1 = \sum_{i\neq j} |\rho_{ij}|$ is used as a measure of coherence.
[Baumgratz et al.,1311.0275]

* Basis independent measure of coherence: Rényi entropy, purity, quantum discord, etc.
[Streltsov et al. 1612.07570, Henderson and Vedral quant-ph/0105028, etc.]

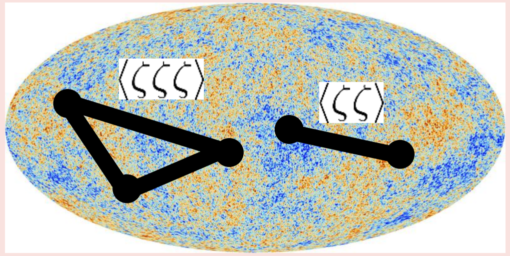
Wavefunction Formalism

[De Witt 1967, Wheeler 1987, etc.]

❑ Observables: correlation functions

$$\langle\Omega|\widehat{\zeta}_{k_1}\widehat{\zeta}_{k_2}(\tau)|\Omega\rangle = \int \mathcal{D}\zeta(\tau)\langle\Omega|\zeta;\tau\rangle\,\langle\zeta;\tau|\Omega\rangle\zeta_{k_1}\zeta_{k_2} \equiv \int \mathcal{D}\zeta(\tau)|\Psi[\zeta(\tau)]|^2\zeta_{k_1}\zeta_{k_2}$$

$$\widehat{\zeta}(\tau)\,|\zeta;\tau\rangle = \zeta(\tau)\,|\zeta;\tau\rangle$$



❑ Wavefunction of the Universe (WFU)

Gaussian

Gravitational non-linearity

$$\Psi[\zeta(\tau)] \equiv \langle\zeta;\tau|\Omega\rangle = \exp\left[-\frac{1}{2}\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3}\psi_2\zeta_{k_1}\zeta_{k_2} - \frac{1}{3!}\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3}\frac{d^3\mathbf{k}_3}{(2\pi)^3}\psi_3\zeta_{k_1}\zeta_{k_2}\zeta_{k_3} - \cdots\right]$$

$\psi_n(\mathbf{k}_1,\ldots,\mathbf{k}_n)$: wavefunction coefficient, $\mathrm{Im}[\psi_n]$: phases in WFU

✓ $\psi_2\zeta^2 \sim -i\mathcal{L}^{(2)}[\zeta], \quad \psi_3\zeta^3 \sim -i\mathcal{L}^{(3)}_{\rm int}[\zeta], \ \ldots$ at semi-classical level $\Psi \simeq e^{iS_{\rm cl}[\zeta(\tau)]}$

✓ $\langle\zeta_1\zeta_2\rangle' = \dfrac{1}{2\,\mathrm{Re}[\psi_2'(k_1)]}, \quad \langle\zeta_1\zeta_2\zeta_3\rangle' = -\dfrac{2\,\mathrm{Re}[\psi_3']}{\prod_{i=1}^3 2\,\mathrm{Re}[\psi_2'(k_i)]}$

$\langle\{\pi_1,\zeta_2\}\rangle' = -\dfrac{\mathrm{Im}[\psi_2']}{2\,\mathrm{Re}[\psi_2']}, \quad \langle\pi_1\zeta_2\zeta_3\rangle' = \dfrac{2\,\mathrm{Im}[\psi_2'(k_1)\psi_3'^*]}{\prod_{i=1}^3 2\,\mathrm{Re}[\psi_2'(k_i)]}$ etc.

at tree level

* $(2\pi)^3\delta^3(\sum_i \mathbf{k}_i)$ is omitted in $(\cdots)'$

* Same result in all other ordering since $\langle\zeta\rangle = 0$

Calculation of Decoherence Rate

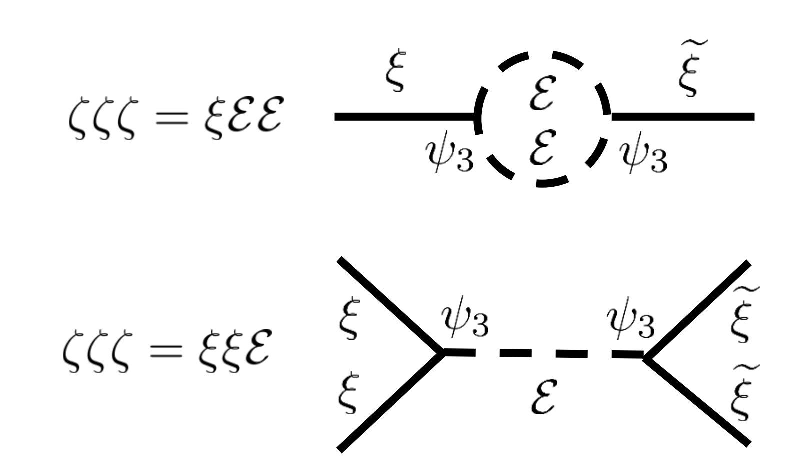
[Nelson 1601.03734]

❑ Interaction between system and environment

$$\langle \rho[\xi, \tilde{\xi}] \rangle_\xi \propto \left\langle \int \mathcal{D}\mathcal{E} |\Psi_{\mathrm{G}}[\mathcal{E}]|^2 e^X \right\rangle_\xi = \exp\left[\sum_n \frac{1}{n!} \langle X^n \rangle_{\mathrm{G}} \right] \propto e^{-\Gamma} \quad , \quad X = -\frac{1}{6} \int \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}_3}{(2\pi)^9} (\zeta_1 \zeta_2 \zeta_3 \psi_3 + \tilde{\zeta}_1 \tilde{\zeta}_2 \tilde{\zeta}_3 \psi_3^*)$$

Cumulant expansion

✓ $n = 1: \langle \zeta^3 \rangle_{\mathrm{G}} = 0\,, \quad n = 2: \int \prod_j d^3\mathbf{k}_j d^3\tilde{\mathbf{k}}_j \, \langle \zeta^3 \tilde{\zeta}^3 \rangle_{\mathrm{G}} \, \psi_3(\mathbf{k}_i) \psi_3^*(\tilde{\mathbf{k}}_i)$



✓ Demonstration: only one mode k_{S} is system

$$\Gamma \sim P_{k_{\mathrm{S}}} \int_{\mathrm{Env}} \frac{d^3\mathbf{k}_{\mathrm{E}} d^3\mathbf{k}'_{\mathrm{E}}}{(2\pi)^6} (2\pi)^3 \delta^3_{\mathbf{k}_{\mathrm{E}} + \mathbf{k}'_{\mathrm{E}} + \mathbf{k}_{\mathrm{S}}} P_{k_{\mathrm{E}}} P_{k'_{\mathrm{E}}} \, \mathrm{Im}[\psi_3]^2 + P_{k_{\mathrm{S}}}^2 P_{2k_{\mathrm{S}}} \, \mathrm{Im}[\psi_3]^2$$

"one-loop" "tree"

✓ Result: [Nelson 1601.03734 , Sou et al. 2207.04435]

$$\Gamma \sim \frac{H^2}{M_{\mathrm{pl}}^2} \left[\frac{1}{\epsilon^3} \left(\frac{aH}{k_{\mathrm{S}}} \right)^6 + \epsilon \left(\frac{aH}{k_{\mathrm{S}}} \right)^3 \right]$$

Boundary one-loop · Bulk one-loop

WITH IR divergence and UV divergence $\Gamma \supset \log \frac{k_{\mathrm{S}}}{k_{\mathrm{IR}}}, \Lambda_{\mathrm{UV}}$

Local observer's effect (This talk)

[cf. Unruh 1110.2199]

Time smeared observables (Stay tuned for our paper)

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Maldacena's Consistency Condition

[Maldacena astro-ph/0210603, Pimentel 1309.1793, etc.]

- **Ward identity in WFU**
  - ✓ Spatial gauge transformation $x^i \to x^i + \xi^i, \quad h_{ij} \to h_{ij} - a^2 D_{\{i}\xi_{j\}}$ with $h_{ij} = e^{2\zeta} a^2 \delta_{ij}$

$$\Rightarrow \quad \Psi[h_{ij} - a^2 D_{\{i}\xi_{j\}}] = \Psi[h_{ij}] \longrightarrow D_i\left[\frac{1}{\sqrt{h}}\frac{\delta\Psi}{\delta h_{ij}(x)}\right] = 0 \quad \cdots\cdots(*)$$

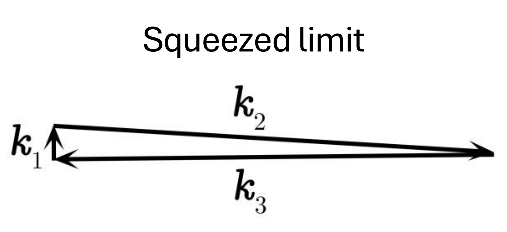
Ward identity
(momentum constraint)

  - ✓ Consistency conditions

$$\frac{\delta}{\delta h_{kl}(y)}(*)\bigg|_{h_{ij}=a^2\delta_{ij}}, \quad \frac{\delta}{\delta h_{mn}(z)}\frac{\delta}{\delta h_{kl}(y)}(*)\bigg|_{h_{ij}=a^2\delta_{ij}}, \ \cdots \quad \text{with} \quad \frac{\delta^n\Psi}{\delta\zeta^n}\bigg|_{\zeta=0} = \psi_n = (2\pi)^3\delta(\textstyle\sum_i \mathbf{k}_i)\psi'_n$$

$$\Downarrow$$

$$\lim_{k_1\to 0}\psi'_3(k_1,k_3) = \left(3 - k_3\frac{d}{dk_3}\right)\psi'_2(k_3)$$



- **Correlation functions**

$$\langle\zeta_1\zeta_2\rangle' = \frac{1}{2\,\mathrm{Re}[\psi'_2(k_1)]} \equiv \mathcal{P}(k_1), \quad \langle\zeta_1\zeta_2\zeta_3\rangle' = -\frac{2\,\mathrm{Re}[\psi'_3]}{\prod_{i=1}^3 2\,\mathrm{Re}[\psi'_2(k_i)]} \equiv \mathcal{B}(k_1,k_2,k_3)$$

$$\Rightarrow \quad \lim_{k_1\to 0}\mathcal{B} = -\mathcal{P}(k_1)\left(3 + k_3\frac{d}{dk_3}\right)\mathcal{P}(k_3) = -(n_s-1)\mathcal{P}(k_1)\mathcal{P}(k_3)$$

Maldacena's consistency condition

where $\mathcal{P}(k) \propto k^{n_s-4}$
for minimal single-field inflation

Local Observer Effect: Coordinate for Free-falling Observer

[Tanaka and Urakawa 1103.1251, Pajer et al. 1305.0824]

❑ Correlation functions in coordinate space

$$\langle \zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle = \int \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}_3}{(2\pi)^9}(2\pi)^3\delta^3_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3}\langle\zeta_1\zeta_2\zeta_3\rangle' \supset \int_{k_1\ll k_3}\frac{k_1^2 dk_1\ k_3^2 dk_3}{k_1^3 k_3^3} \sim \log k_1\Big|_{k_1\to 0}$$

Correlation between IR and UV is dominant

❑ (Conformal) free-falling observer (Conformal Fermi normal coordinate)

$$ds^2 = a^2(-d\tau^2 + e^{2\zeta}d\mathbf{x}^2) = a^2(-d\tau^2 + d\mathbf{x}_\mathrm{F}^2) + \cdots$$

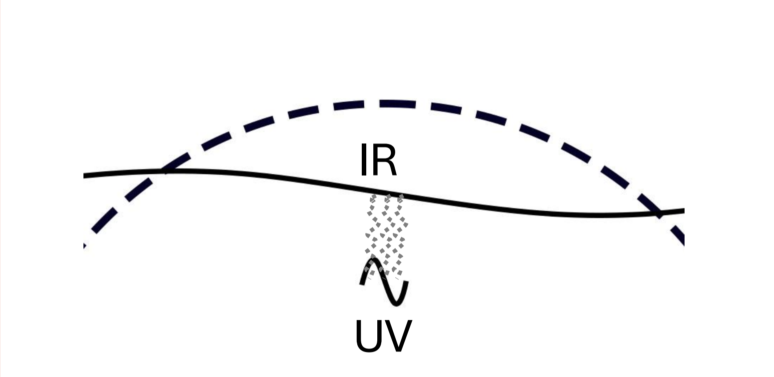
Geodesic distance: physical

where $\mathbf{x}_\mathrm{F} \simeq (1+\zeta(0))\mathbf{x}$

➡ $\zeta_\mathrm{F}(\mathbf{x}_\mathrm{F}) \simeq \zeta(\mathbf{x}_\mathrm{F}) - \zeta(0)\mathbf{x}_\mathrm{F}\cdot\partial_{\mathbf{x}_\mathrm{F}}\zeta(\mathbf{x}_\mathrm{F})$

➡ $\zeta_{\mathrm{F},\mathbf{k}} \simeq \zeta_\mathbf{k} + \zeta(0)(3+\mathbf{k}\cdot\partial_\mathbf{k})\zeta_\mathbf{k}$

$\zeta(0)$: const. mode in Fourier space $\lim_{k\to 0}\zeta_\mathbf{k}$



✓ Correlation functions

$$\lim_{k_1\to 0}\langle\zeta_1\zeta_2\zeta_3\rangle'_\mathrm{F} = \lim_{k_1\to 0}\langle\zeta_1\zeta_2\zeta_3\rangle' + \langle\zeta_1\zeta_1\rangle'\left(3+k_3\frac{d}{dk_3}\right)\langle\zeta_3\zeta_3\rangle' = \underline{0}$$

* Not only leading order $\mathcal{O}(k_1^{-3}k_3^{-3})$, but next-leading $\mathcal{O}(k_1^{-2}k_3^{-4})$ vanishes as well. $\mathcal{O}(k_1^{-1}k_3^{-5})$ is physical.

$$\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle_\mathrm{F} \xrightarrow[\mathrm{IR}]{} \int_{k_1\ll k_3}\frac{k_1}{k_3^3}dk_1\ dk_3$$

Integrand is decaying in $k_1\to 0$, and the result is finite.

Local Observer Effect in Wavefunction Formalism

[Sano and Tokuda ongoing]

❑ Wavefunction in geodesic coordinate

$$\Psi[\zeta_F] = \exp\left[-\frac{1}{2}\int \frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3}\psi_{F,2}\zeta_{F,k_1}\zeta_{F,k_2} - \frac{1}{3!}\int \frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3}\frac{d^3\mathbf{k}_3}{(2\pi)^3}\psi_{F,3}\zeta_{F,k_1}\zeta_{F,k_2}\zeta_{F,k_3} - \cdots\right]$$

✓ Correction to ψ_3 from Gaussian part

$$\zeta(0) \approx \int_{k\to 0}\frac{d^3\mathbf{k}}{(2\pi)^3}\zeta_k$$

$$-\frac{1}{2}\int \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2}{(2\pi)^6}\psi_{F,2}\zeta_{F,1}\zeta_{F,2} = -\frac{1}{2}\int \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2}{(2\pi)^6}\underbrace{\psi_{F,2}}_{=\psi_2}\zeta_1\zeta_2 - \frac{1}{2}\int_{k_1\to 0}\frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}_3}{(2\pi)^9}(2\pi)^3\delta^3\left(\sum \mathbf{k}_i\right)\zeta_1\zeta_2\zeta_3\left(3 - k_3\frac{d}{dk_3}\right)\psi'_{F,2}$$

$$\zeta_{F,\mathbf{k}} \simeq \zeta_{\mathbf{k}} + \zeta(0)(3 + \mathbf{k}\cdot\partial_{\mathbf{k}})\zeta_{\mathbf{k}}$$

➡ 3rd

$$-\frac{1}{3!}\int \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}_3}{(2\pi)^9}\psi_{F,3}\zeta_1\zeta_2\zeta_3 + \Delta\Psi_3 = -\frac{1}{3!}\int_{\text{non-soft}}\frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}_3}{(2\pi)^9}\psi_{F,3}\zeta_1\zeta_2\zeta_3$$

$$-\frac{1}{2}\int_{k_1\to 0}\frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}_3}{(2\pi)^9}(2\pi)^3\delta^3\left(\sum \mathbf{k}_i\right)\zeta_1\zeta_2\zeta_3\underbrace{\left(\psi'_{F,3} + \left(3 - k_3\frac{d}{dk_3}\right)\psi'_2\right)}_{\lim_{k_1\to 0}\psi'_3}$$

$$\left.\begin{array}{l} k_1 \to 0 \\ k_2 \to 0 \\ k_3 \to 0 \end{array}\right\} \text{equivalent}$$

➡ $$\lim_{k_1\to 0}\psi'_{F,3} = \lim_{k_1\to 0}\psi'_3 - \left(3 - k_3\frac{d}{dk_3}\right)\psi'_2 = 0$$ Leading (and next leading) cancels

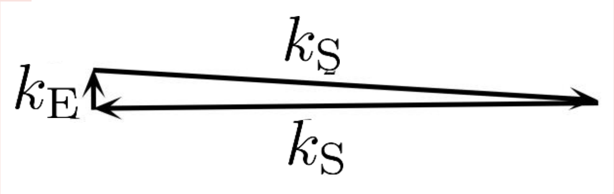
✓ Modification to decoherence rate

Power in $(\mathrm{Im}\,\psi_3)^2$ changes by $(k_E/k_S)^4$ ➡ $\Gamma_{IR} \sim \log(k_{IR}/k_S) \Longrightarrow (k_{IR}/k_S)^4 \sim 0$

Summary and Outlook

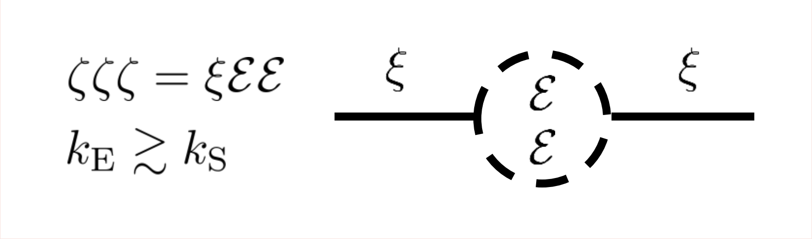
- **Transition from quantum to classical is an open question.**

  ➡ { Wavefunction formalism; Decoherence $C_1 = \sum_{i\neq j}|\rho_{ij}|$ } (Well established in QM)



  ✓ IR divergence in decoherence rate

$$\langle|\rho[\zeta_{\rm S},\tilde{\zeta}_{\rm S}]|\rangle \propto e^{-\Gamma}, \quad \Gamma \propto \int_{k_{\rm E}} P^2_{k_{\rm E}}\,{\rm Im}[\psi_3]^2 \sim \log(k_{\rm IR}/k_{\rm S})$$



- **Local observer effect**

  ✓ Long and short modes do not correlate for local obs.

$$ds^2 = a^2(-d\tau^2 + e^{2\zeta}d\mathbf{x}^2) = a^2(-d\tau^2 + d\mathbf{x}_{\rm F}^2) + \cdots \quad \Longrightarrow \quad \lim_{k_1\to 0}\psi'_{{\rm F},3} = \lim_{k_1\to 0}\psi'_3 - \left(3 - k_3\frac{d}{dk_3}\right)\psi'_2 = \underline{0}$$

  ✓ Modified IR contribution

Correlation function $\quad \langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle \sim \log k_{\rm long} \longrightarrow \int_{k_1\ll k_3}\frac{k_1}{k_3^3}dk_1\ dk_3 \sim 0$

Decoherence $\quad \Gamma_{\rm IR} \sim (k_{\rm IR}/k_{\rm S})^4 \sim 0$