Center for Theoretical Physics of the Universe

Cosmology, Gravity and Astroparticle Physics

Primordial Black Hole Mergers as a Cosmological Probe

Qianhang Ding

IBS CTPU-CGA

Reference: 2312.13728, QD
2410.02591, QD, Minxi He, Volodymyr Takhistov

High1 Workshop on Particle, String and Cosmology
Jan 13

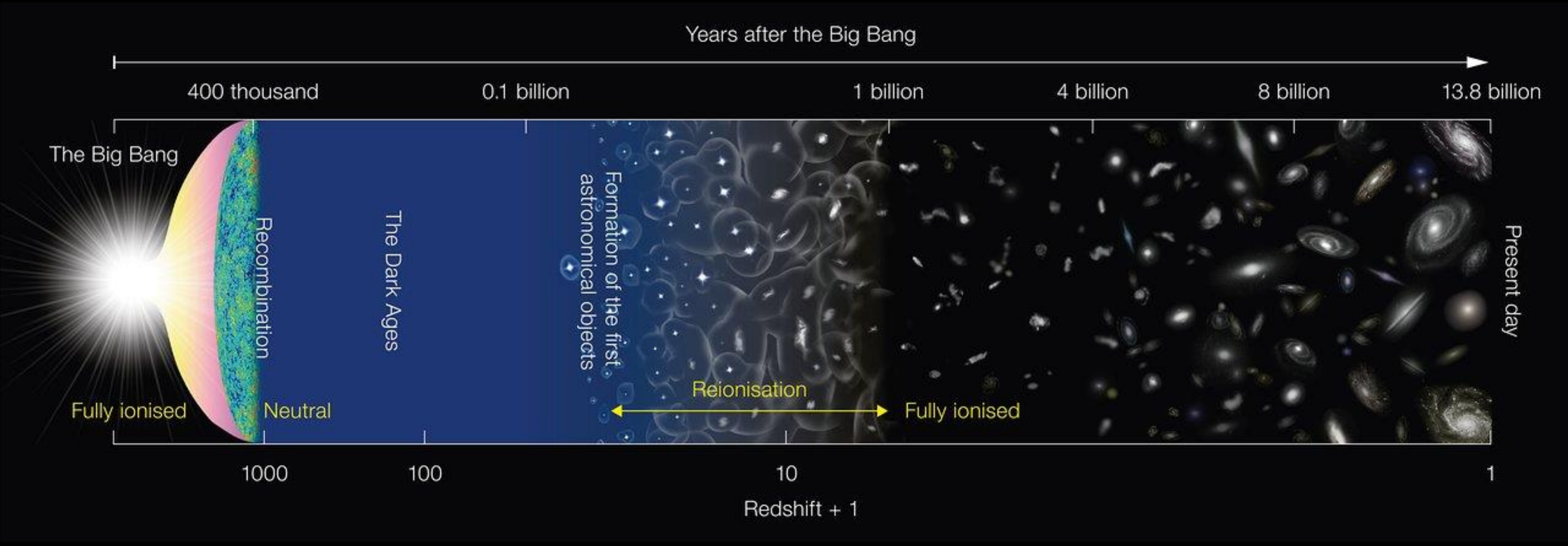
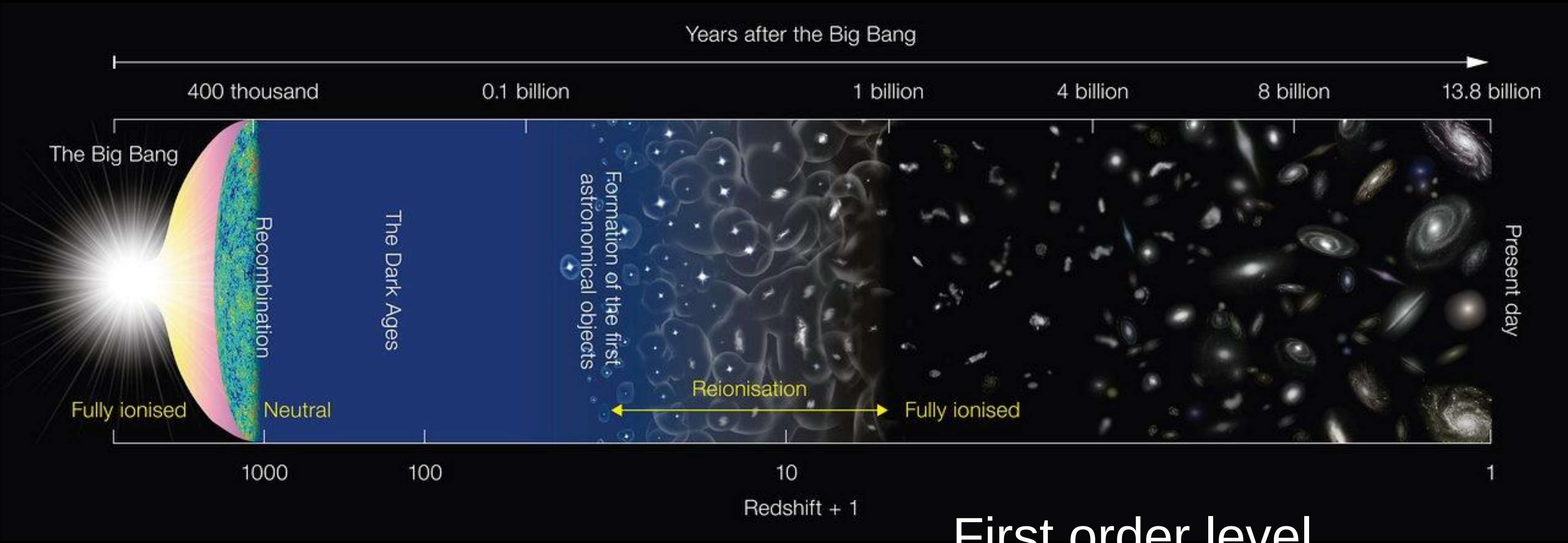


Image Credit: NAOJ

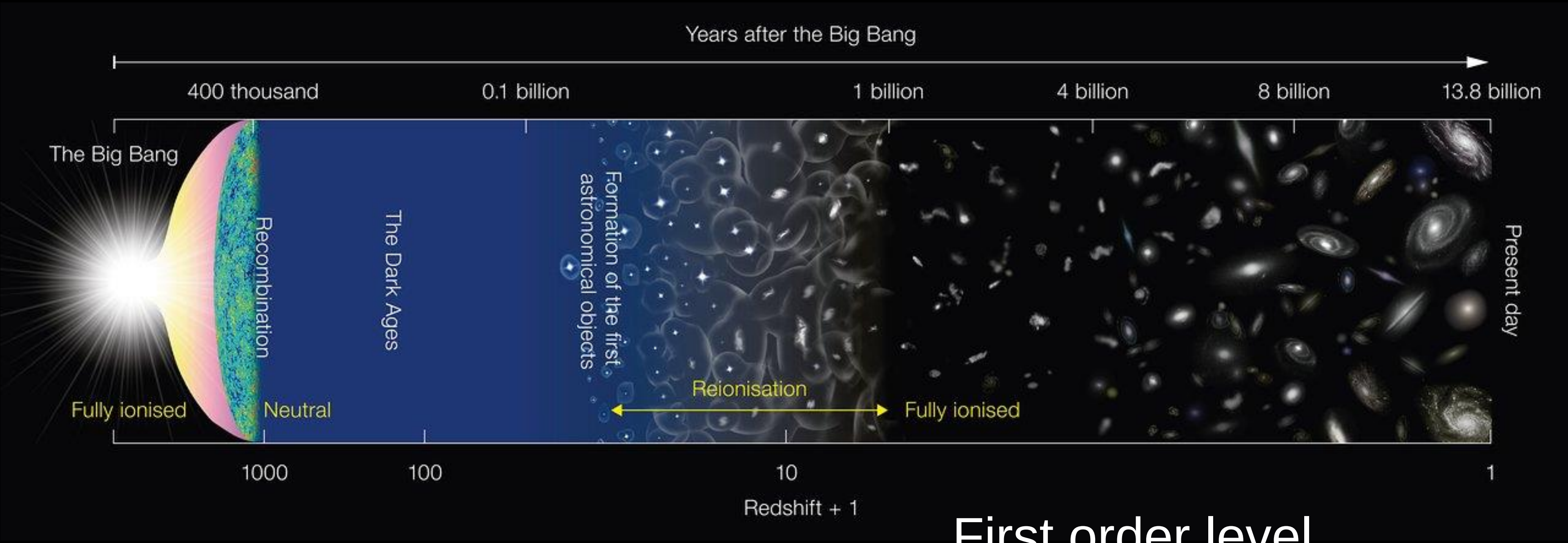
Background level
Hubble expansion rate



First order level
Structure formation

Image Credit: NAOJ

Background level
Hubble expansion rate

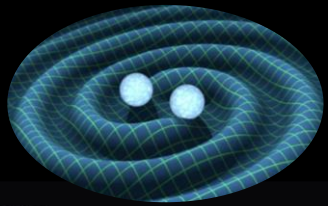


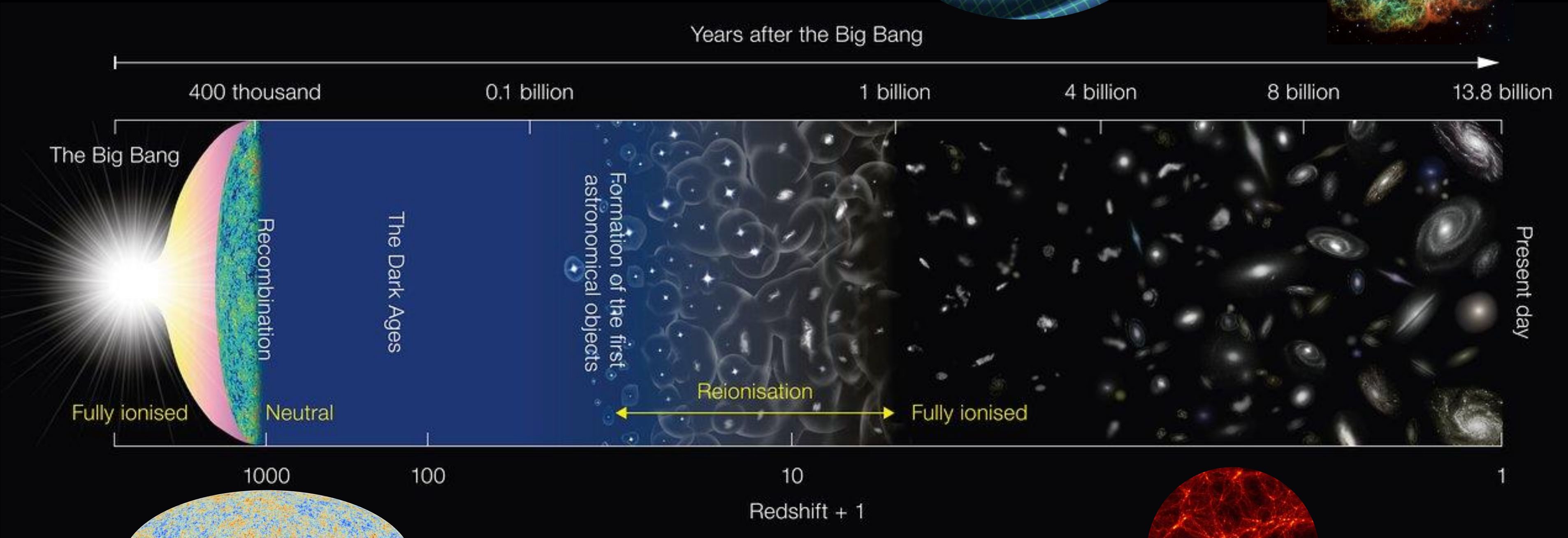
First order level
Structure formation

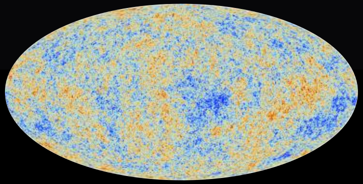
2312.13728, QD

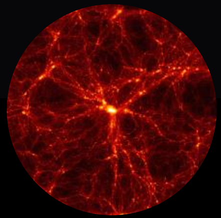
Image Credit: NAOJ

73.04 ± 1.04
km/s/Mpc



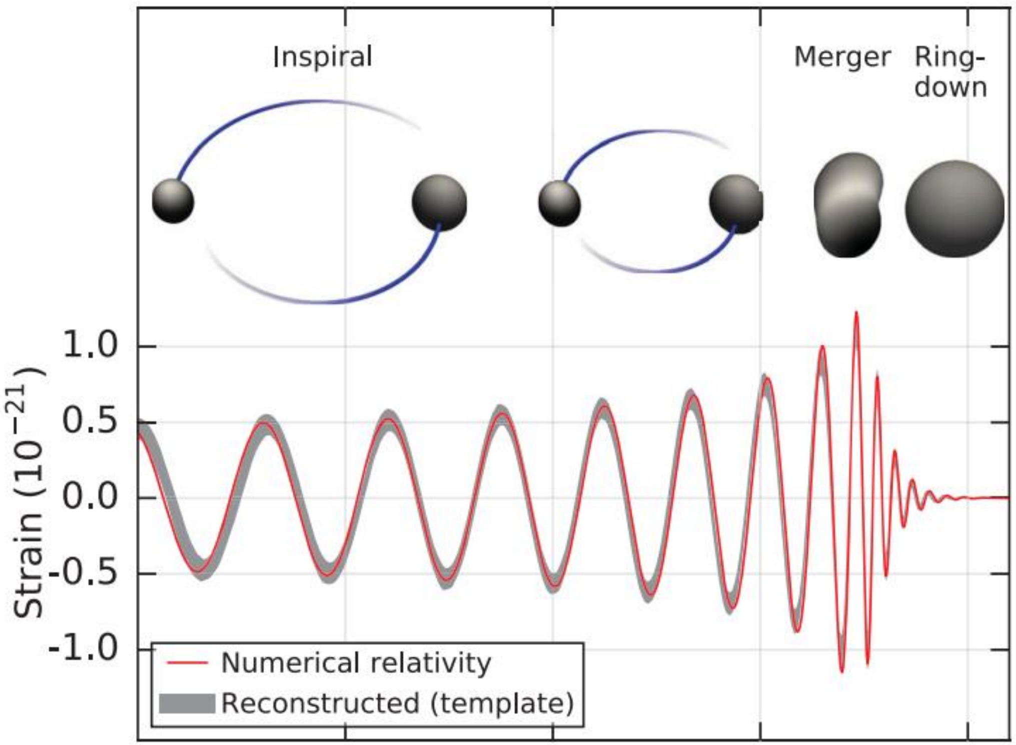






67.4 ± 0.5
km/s/Mpc

Image Credit: NAOJ

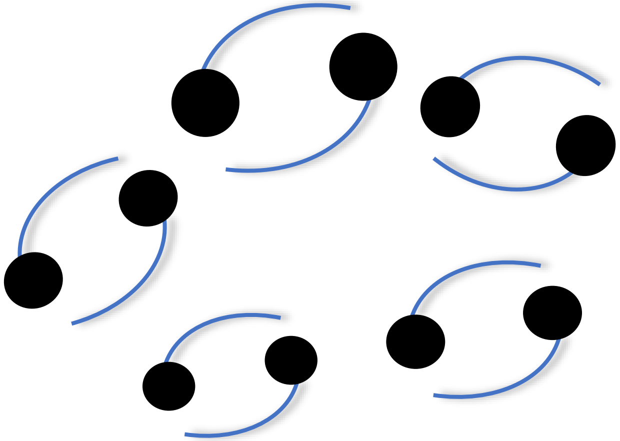


$$\mathcal{M}_z = (1+z)\mathcal{M}$$

$$d_L = \frac{1+z}{H_0}\int_0^z \frac{c}{E(z')}dz'$$

B. P. Abbott et al. Observation of Gravitational Waves from a Binary Black Hole Merger. Phys. Rev. Lett., 116(6):061102, 2016.

$$h(t) = \frac{4}{d_L(z)}\left(\frac{G\mathcal{M}_z}{c^2}\right)^{5/3}\left(\frac{\pi f(t)}{c}\right)^{2/3}\cos\Phi(t)$$



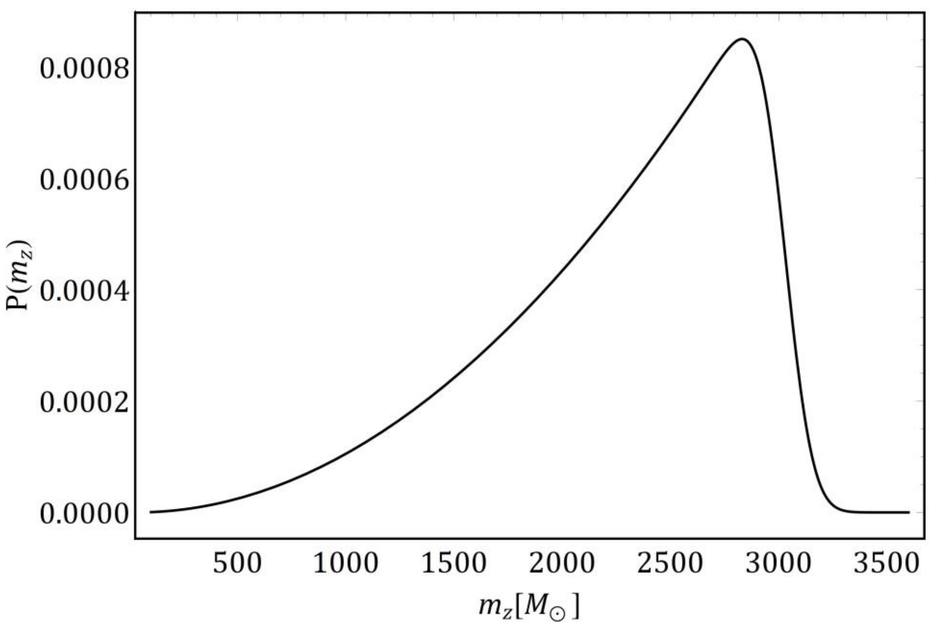
$$(m_z^i, d_L^i)$$

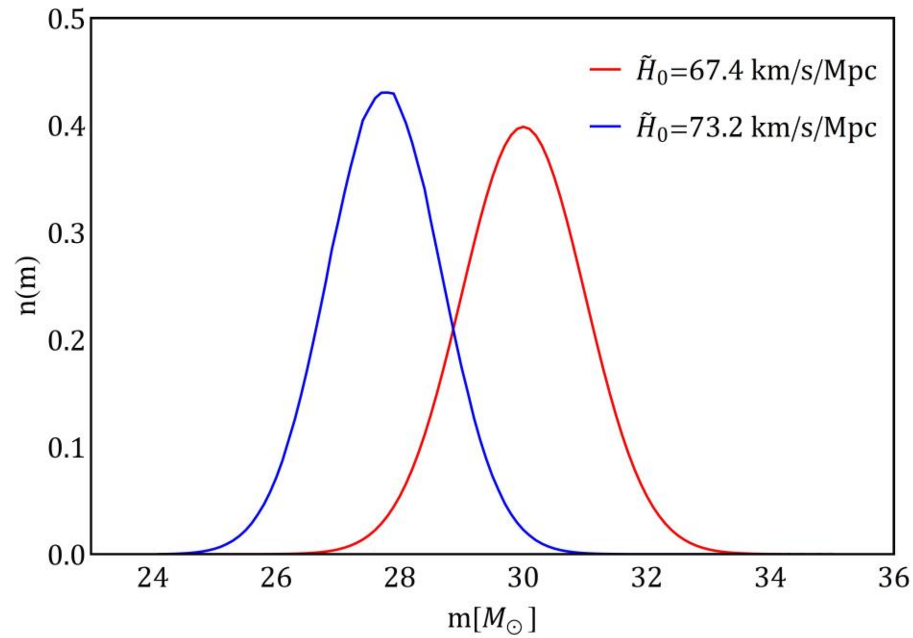
$$d_L^i = \frac{1+z_i}{H_0}\int_0^{z_i}\frac{c}{E(z')}dz'$$

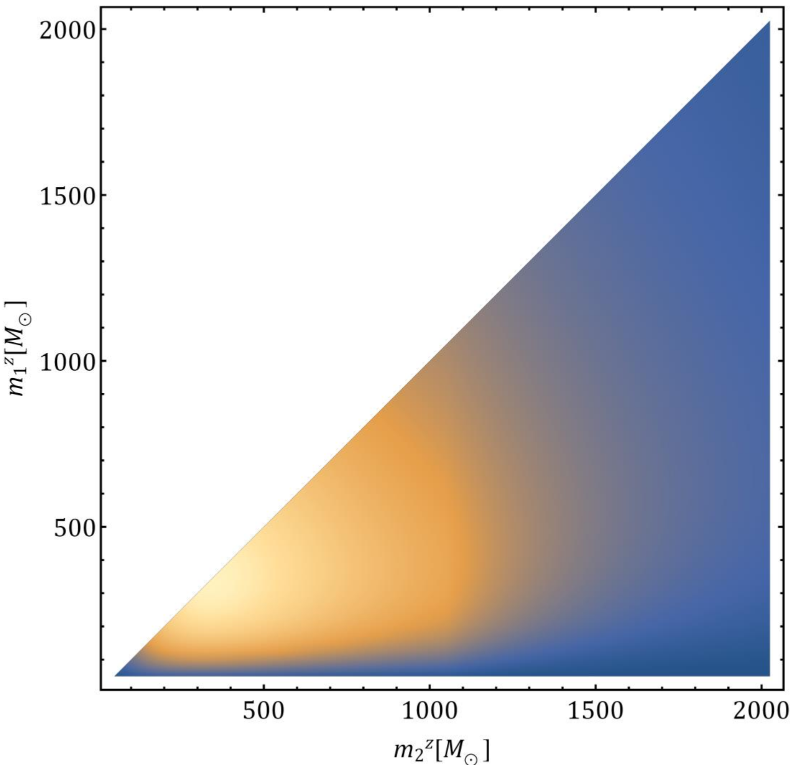
Assume a Hubble parameter $\widetilde{H}_0$

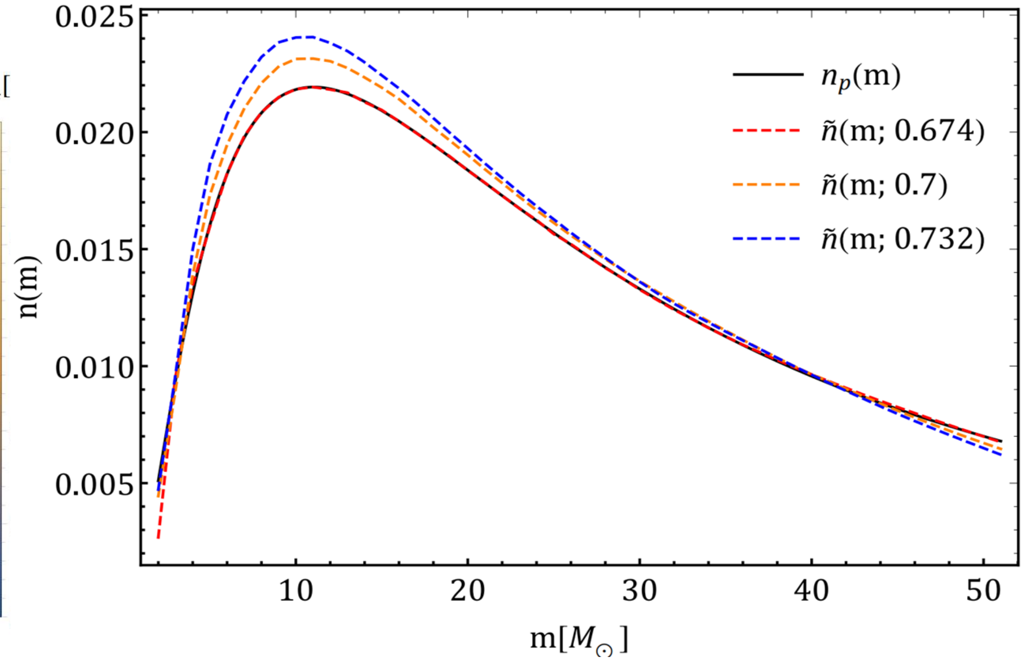
$$z_i = z(d_L^i; \widetilde{H}_0)$$

$$m^i = m_z^i/(1+z_i)$$



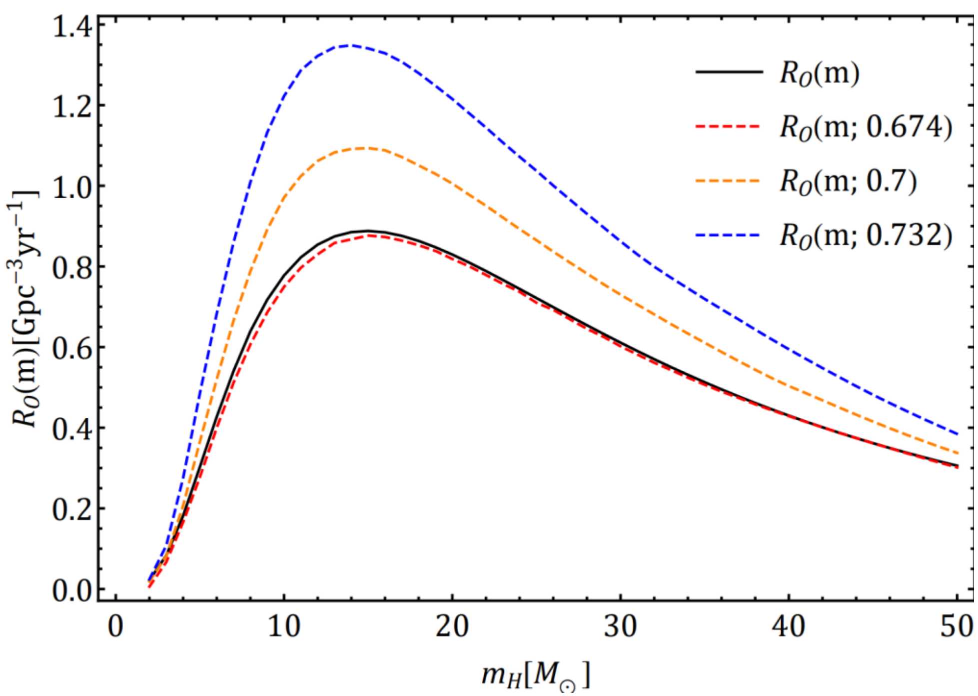




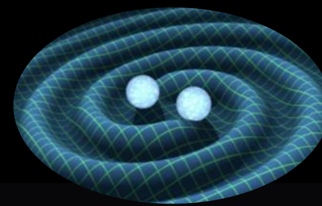


$$n(m) = \frac{1}{\sqrt{2\pi}\sigma m} \exp\left[-\frac{\ln^2(m/m_{pk})}{2\sigma^2}\right]$$

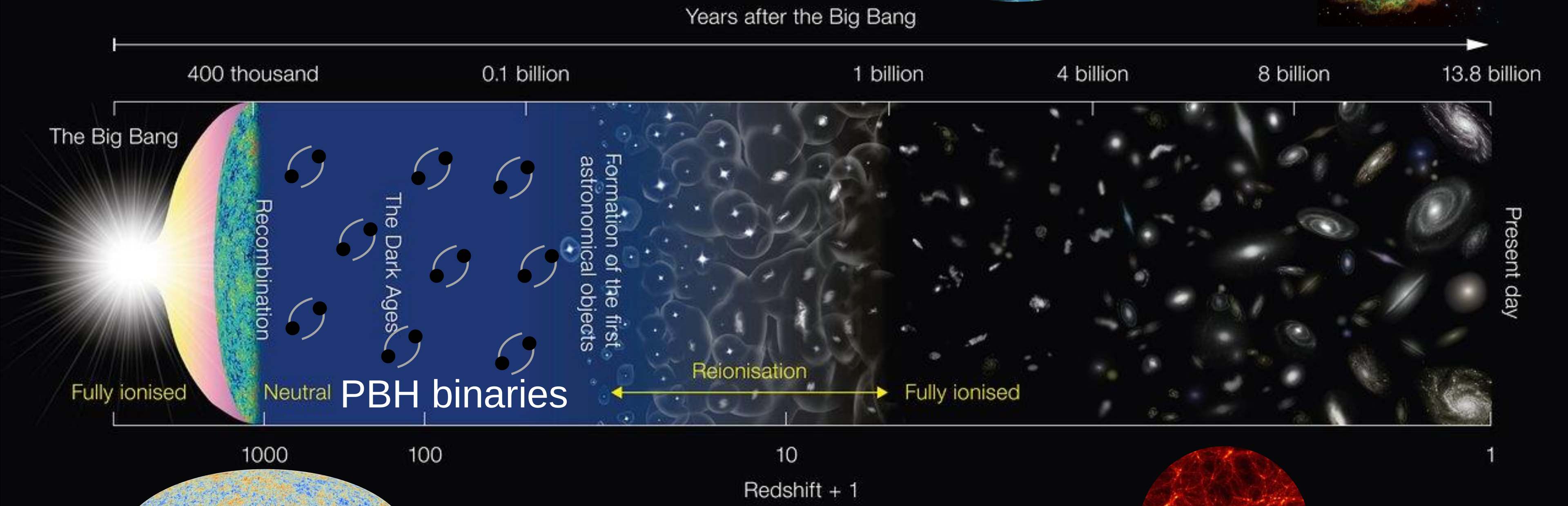
$$R_{ij} = \rho_{\mathrm{PBH}} \min\left(\frac{n(m_i)}{m_i}, \frac{n(m_j)}{m_j}\right) \Delta_m \frac{dP}{dt}$$

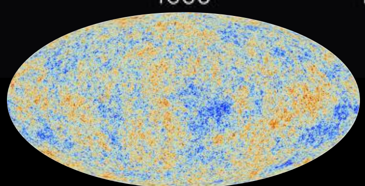


Background level
Hubble expansion rate



73.04 ± 1.04 km/s/Mpc





67.4 ± 0.5 km/s/Mpc

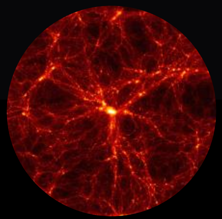
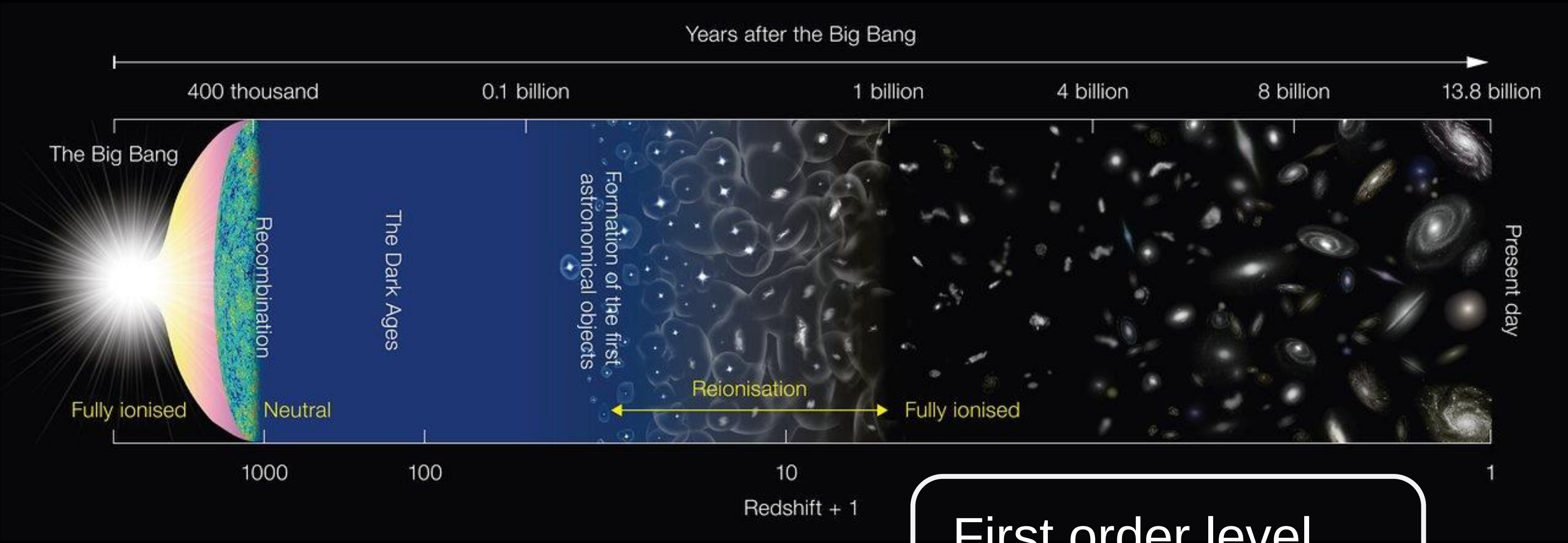


Image Credit: NAOJ

Background level
Hubble expansion rate

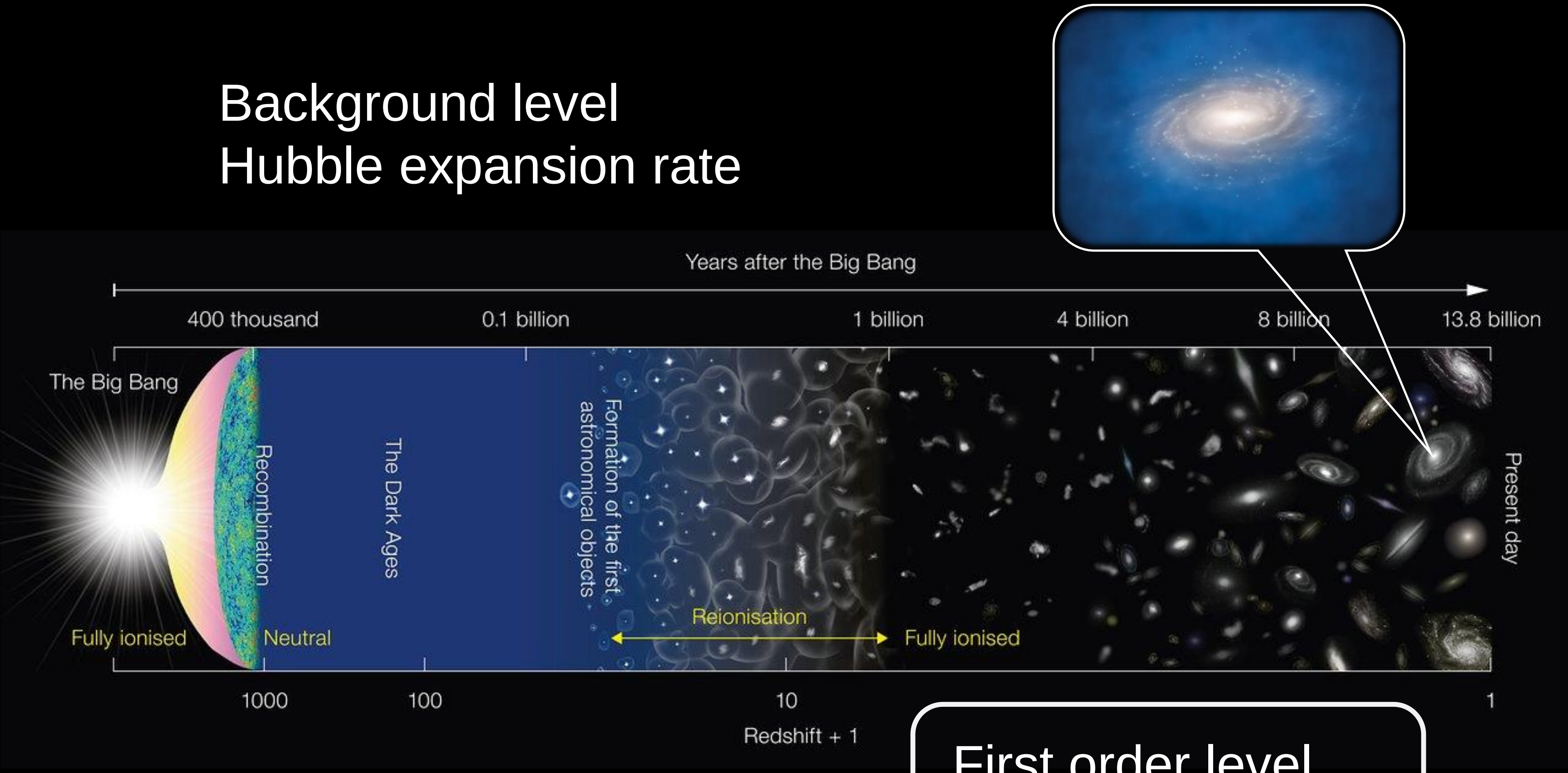


First order level
Structure formation

2410.02591, QD, Minxi He, Volodymyr Takhistov

Image Credit: NAOJ

Background level
Hubble expansion rate



First order level
Structure formation

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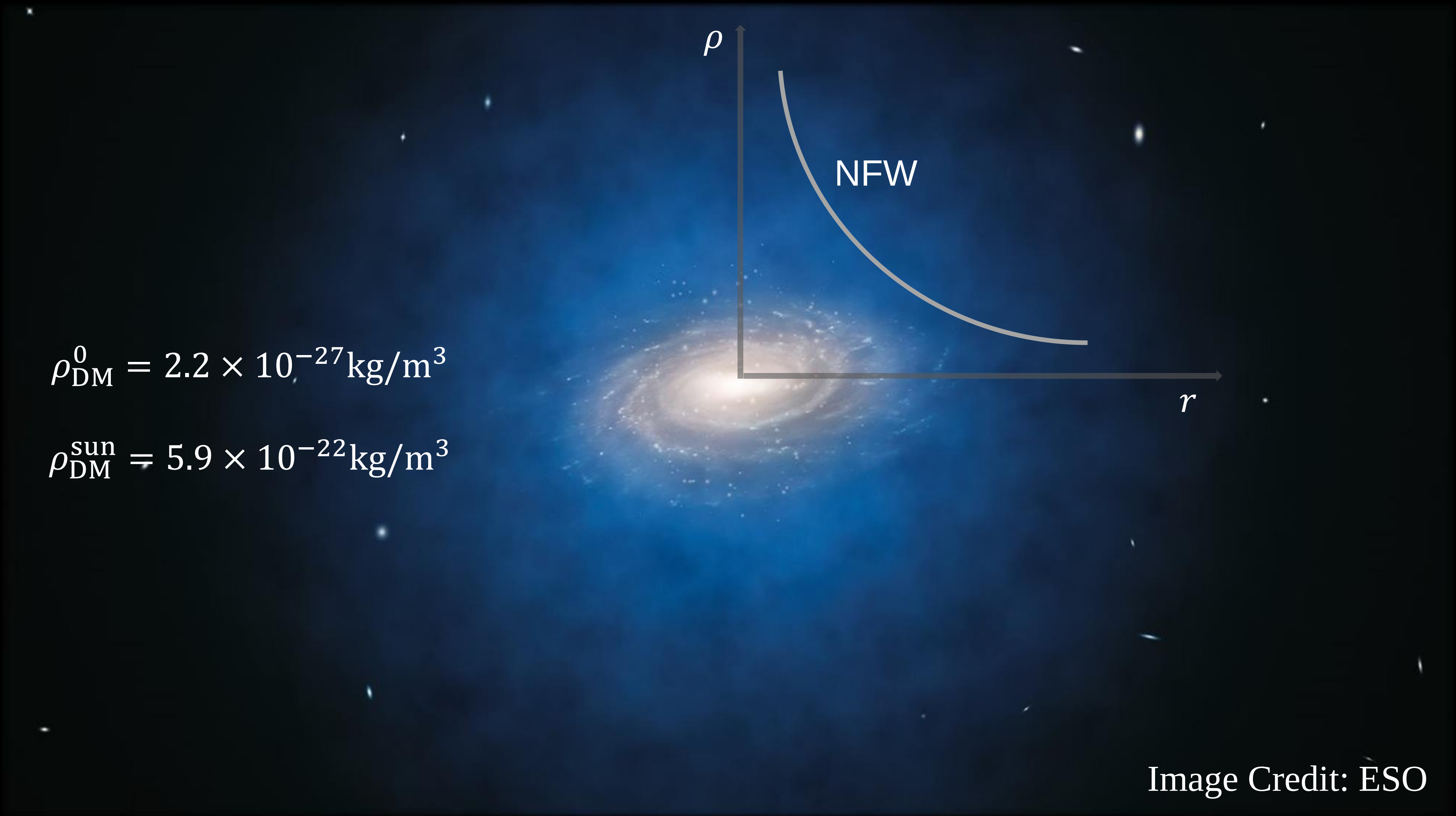
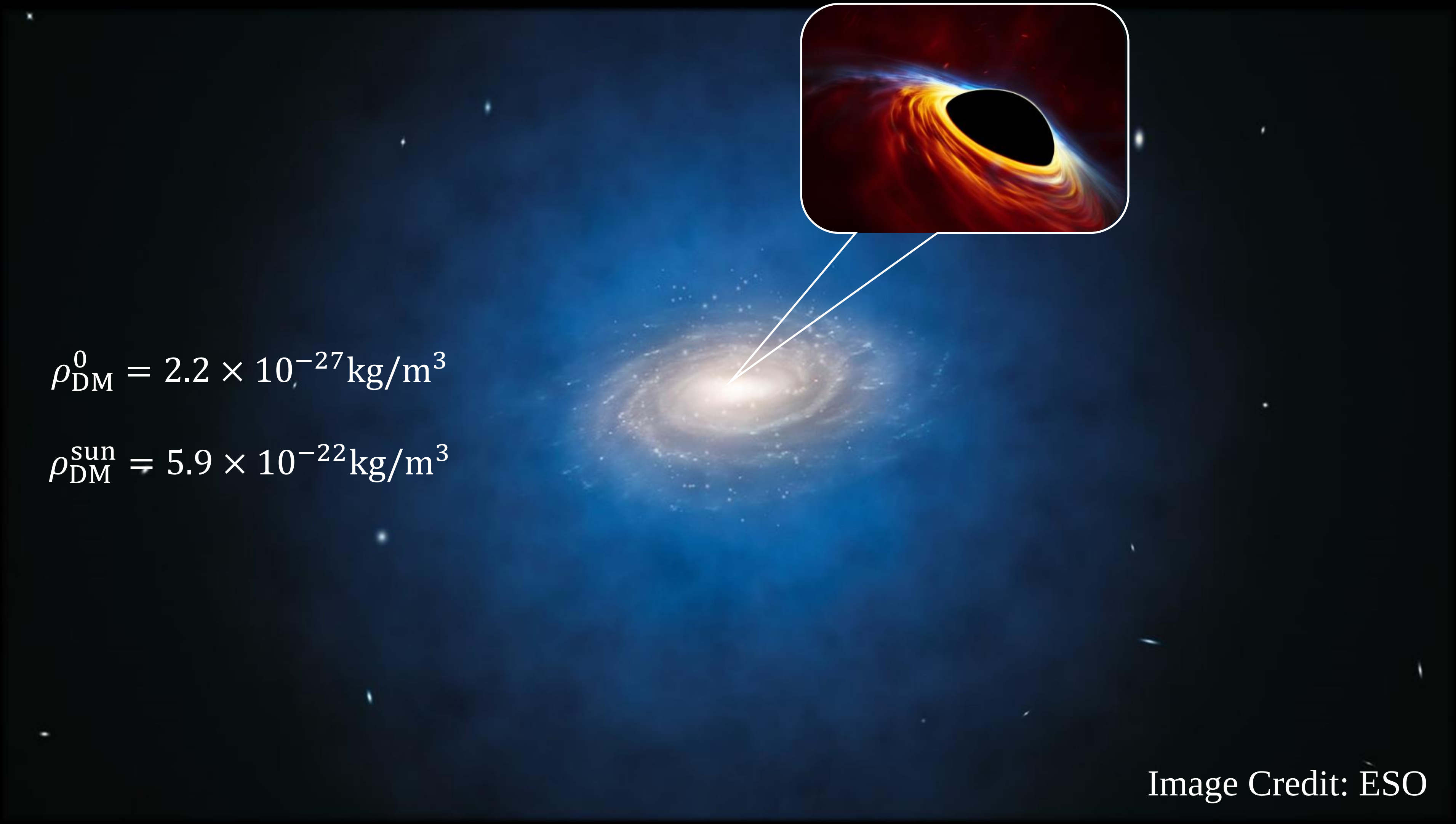


$$\rho_{\mathrm{DM}}^{0} = 2.2 \times 10^{-27} \mathrm{kg/m^3}$$

$$\rho_{\mathrm{DM}}^{\mathrm{sun}} = 5.9 \times 10^{-22} \mathrm{kg/m^3}$$

Image Credit: ESO



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$$\rho_{\mathrm{DM}}^{\mathrm{sun}} = 5.9 \times 10^{-22}\mathrm{kg/m^3}$$

Image Credit: ESO


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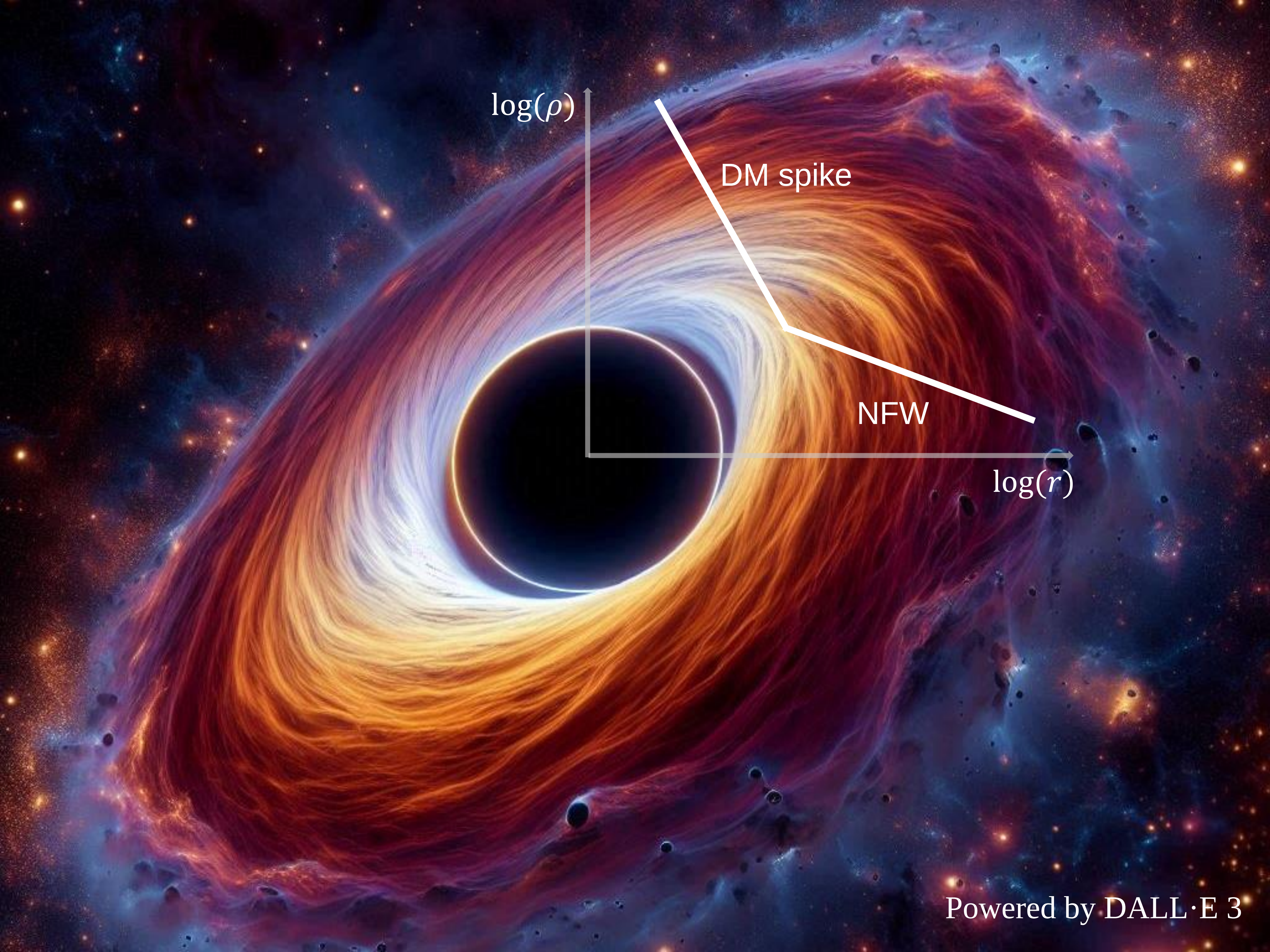
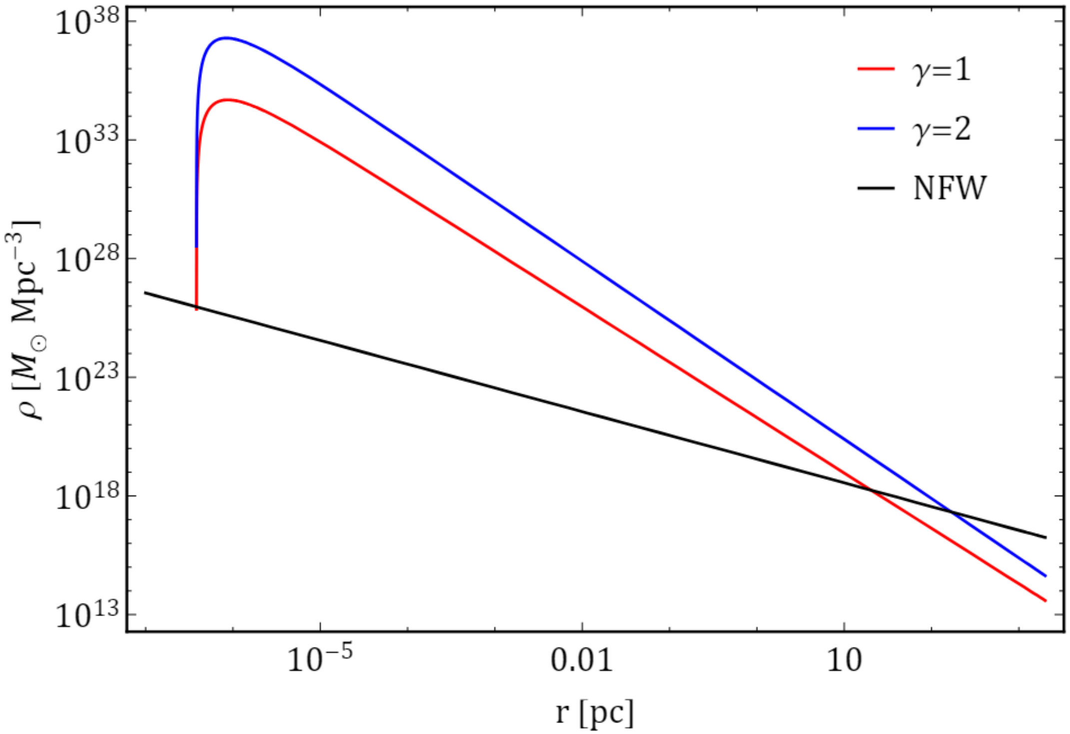


$\log(\rho)$
DM spike
NFW
$\log(r)$
Powered by DALL·E 3

Dark Matter Spike



Fokker-Planck Equation

$$\frac{\partial[f(E,t)g(E)]}{\partial t} = -\frac{\partial \mathcal{F}(E,t)}{\partial E},$$

$$-\mathcal{F}(E,t) \equiv D_{EE}(E)\frac{\partial f(E,t)}{\partial E} + D_E(E)f(E,t).$$

$$\rho(r) \simeq \rho_0(r_0/r)^\gamma$$

$$\rho(r) = 4\pi\int_{\Phi(r)}^{0} dE\, f(E)\, \sqrt{2[E-\Phi(r)]}$$

$$\rho_{\rm sp}(r) = \rho_R\left(1-\frac{4R_{\rm s}}{r}\right)^3\left(\frac{R_{\rm sp}}{r}\right)^{\gamma_{\rm sp}}$$

$$\gamma_{\rm sp} = \frac{9-2\gamma}{4-\gamma}$$

OJ287 $M_{\rm BH} = 1.8 \times 10^{10} M_{\odot}$ $P = 12.067\ yr$

$m_{\rm BH} = 1.5 \times 10^{8} M_{\odot}$ $\dot{P} = -0.00099$

Video Credit: NASA JPL

The Astrophysical Journal Letters, 962:L40 (5pp), 2024 February 20 https://doi.org/10.3847/2041-8213/ad2465

The First Robust Evidence Showing a Dark Matter Density Spike Around the Supermassive Black Hole in OJ 287

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Abstract

Black hole dynamics suggests that dark matter would redistribute near a supermassive black hole (SMBH) to form a density spike. However, no direct evidence of a dark matter density spike around an SMBH has been identified. In this Letter, we present the first robust evidence showing a dark matter density spike around an SMBH. We revisit the data of the well-known SMBH binary OJ 287 and show that the inclusion of the dynamical friction due to a dark matter density spike around the SMBH can satisfactorily account for the observed orbital decay rate. The derived spike index $\gamma_{\rm sp} = 2.351^{+0.032}_{-0.045}$ gives an excellent agreement with the value $\gamma_{\rm sp} = 2.333$ predicted by the benchmark model assuming an adiabatically growing SMBH. This provides a strong verification of the canonical theory suggested two decades ago modeling the gravitational interaction between collisionless dark matter and SMBHs.

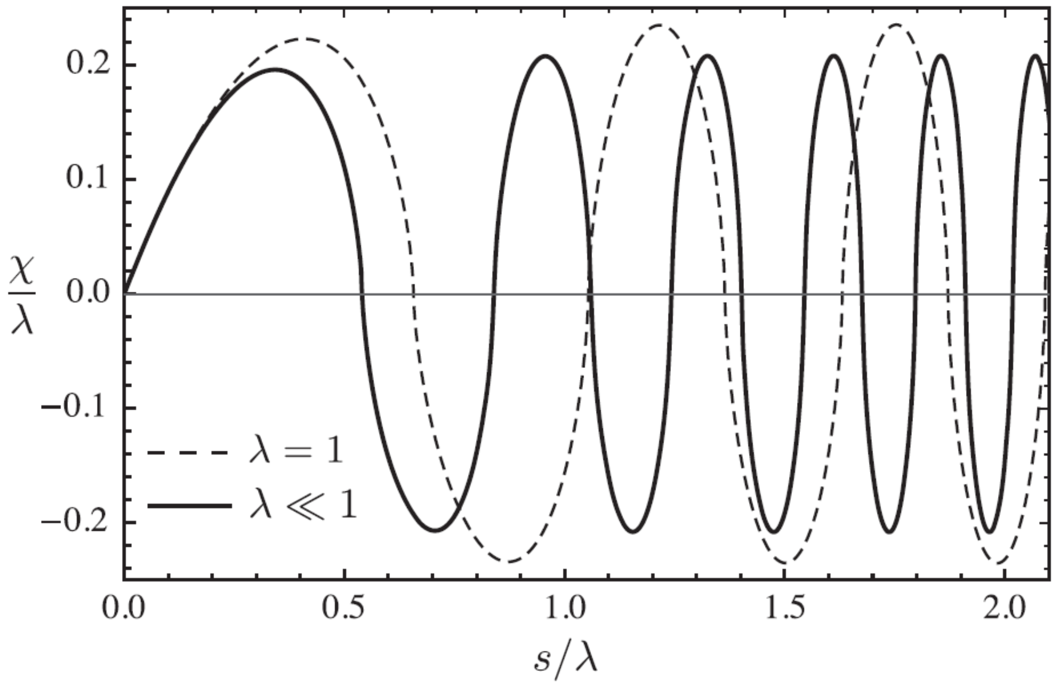
$$\gamma_{\rm sp} = 2.333$$

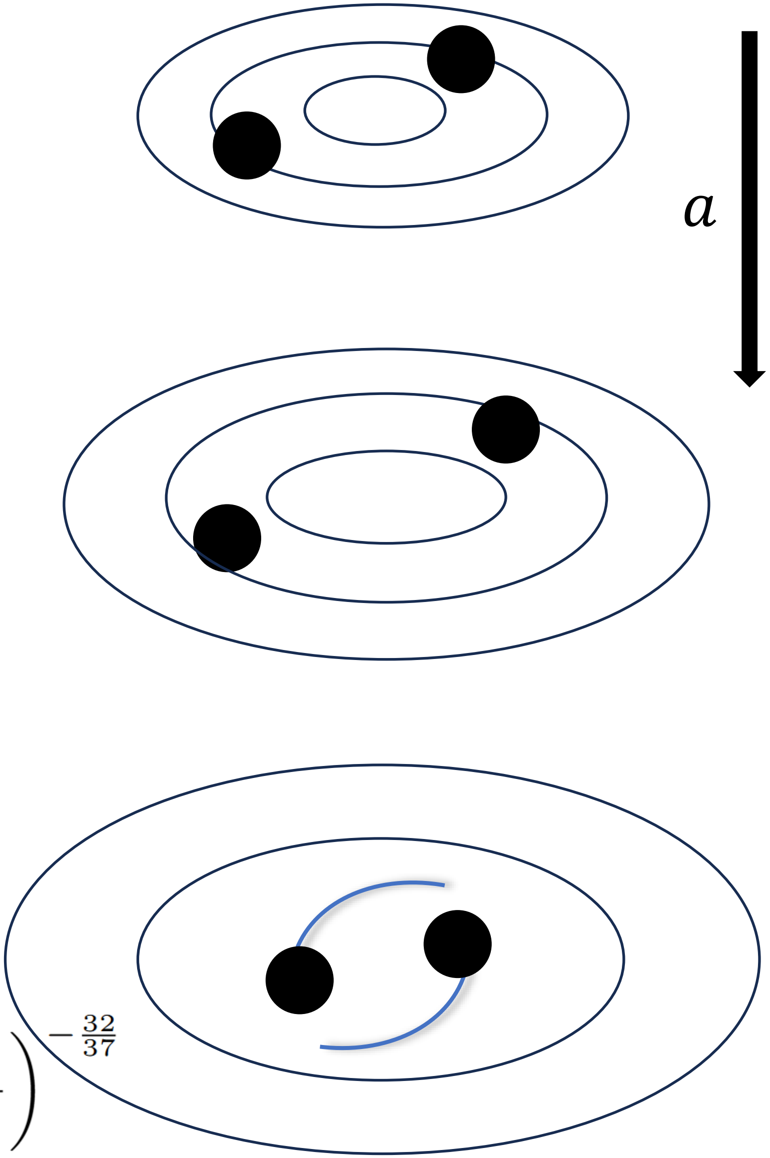
How about primordial black holes in DM spike?

How about primordial black holes in DM spike?

PBH merger rate would be enhanced!

Early PBH Binary





Ali-Haïmoud, Yacine, Ely D. Kovetz, and Marc Kamionkowski. "Merger rate of primordial black-hole binaries." *Physical Review D* 96.12 (2017): 123523.

$$\frac{dR}{dm_1 dm_2} = \frac{1.6 \times 10^6}{\mathrm{Gpc^3 yr}} f_{\mathrm{PBH}}^{\frac{53}{37}} \left(\frac{t(z)}{t_0}\right)^{-\frac{34}{37}} \eta^{-\frac{34}{37}} \left(\frac{M}{M_\odot}\right)^{-\frac{32}{37}}$$
$$\times S(M, f_{\mathrm{PBH}}) \psi(m_1) \psi(m_2)$$

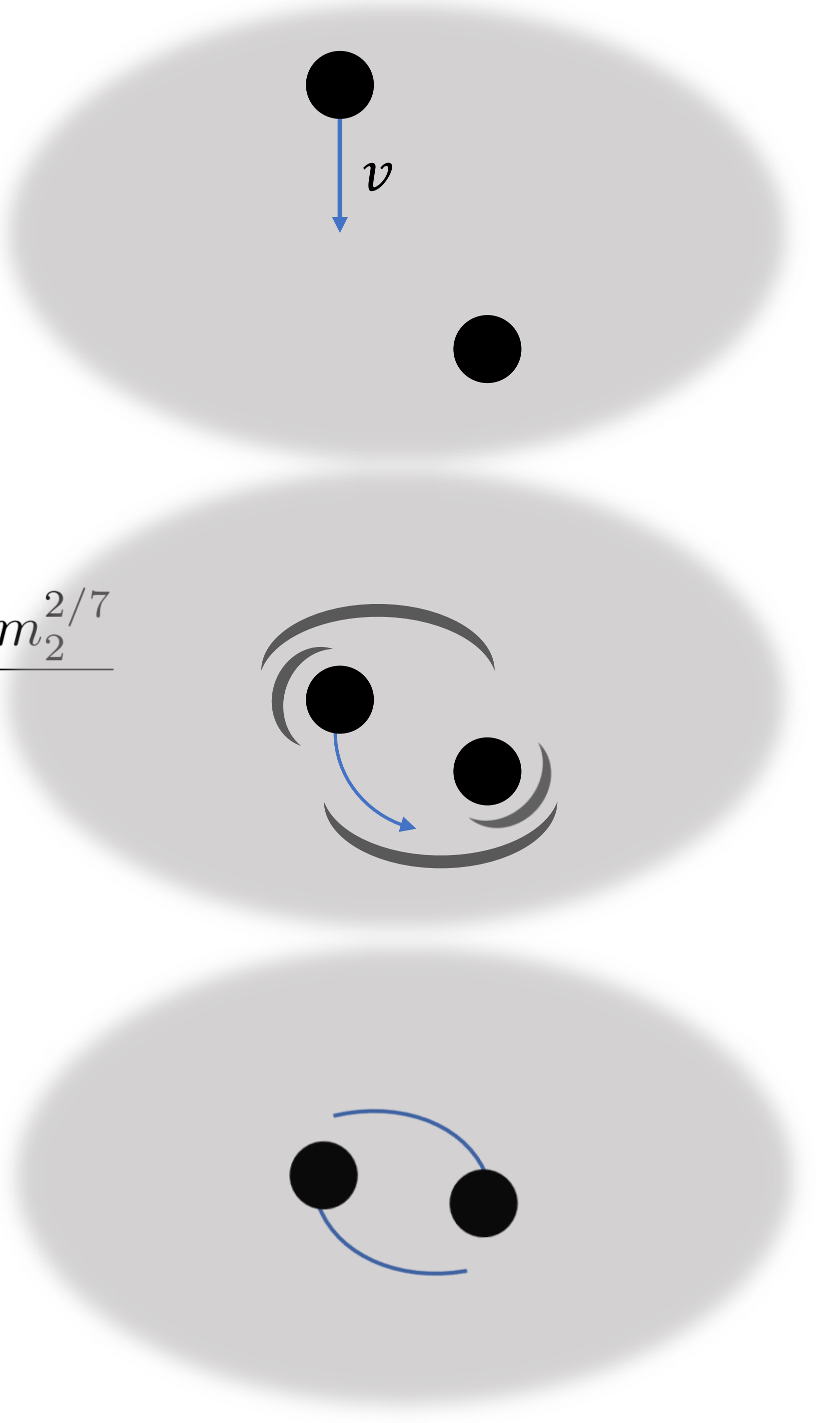
Late PBH Binary

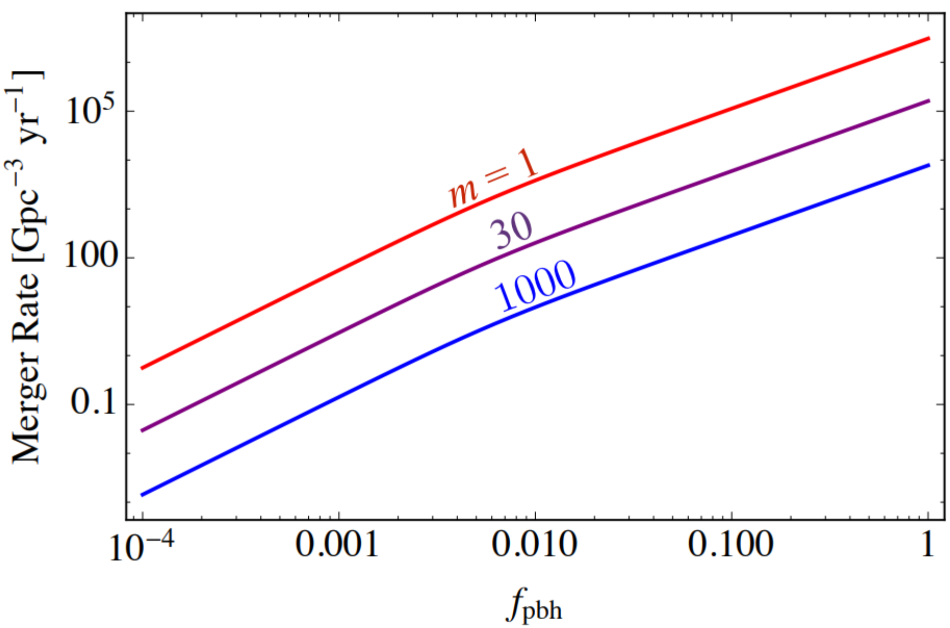
$$R_{\rm cap} = \int \frac{1}{2}\left(\frac{\rho_{\rm PBH}}{M_{\rm PBH}}\right)^2 \sigma_{\rm cap} v_{\rm rel} dV$$

$$\sigma_{\rm cap} = 2\pi\left(\frac{85\pi}{6\sqrt{2}}\right)^{2/7} \frac{G^2(m_1+m_2)^{10/7} m_1^{2/7} m_2^{2/7}}{c^{10/7} v_{\rm rel}^{18/7}}$$

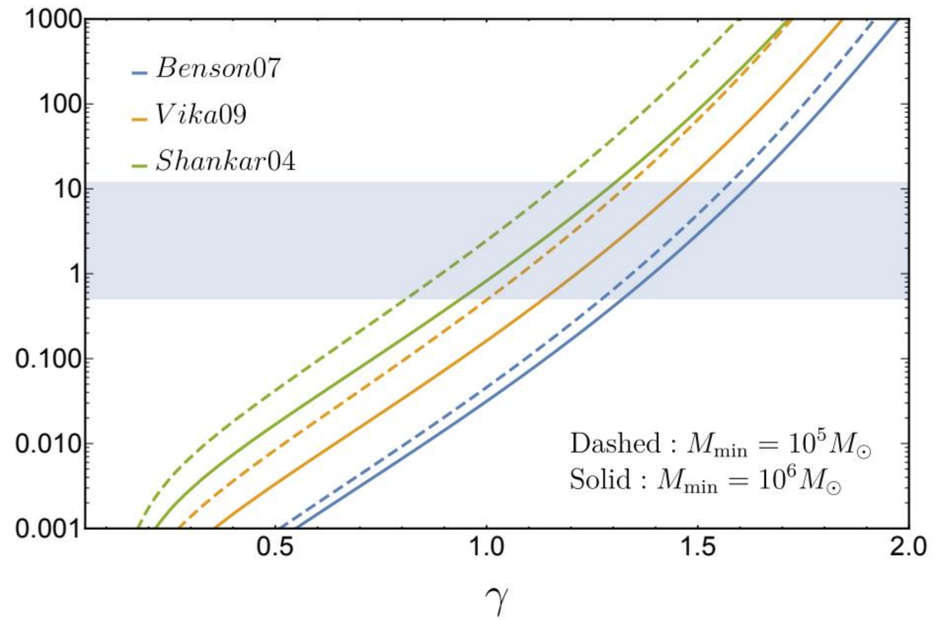
$$t_{\rm cap} \gg t_{\rm merge}$$

$$R_{\rm merge} \simeq R_{\rm cap}$$





Ali-Haïmoud, Yacine, Ely D. Kovetz, and Marc Kamionkowski. "Merger rate of primordial black-hole binaries." *Physical Review D* 96.12 (2017): 123523.



Nishikawa, Hiroya, et al. "Primordial-black-hole mergers in dark-matter spikes." *Physical Review D* 99.4 (2019): 043533.

PBH merger rate from early binaries

PBH merger rate from late binaries in DM spike

PBH Merger Rate in DM Spike

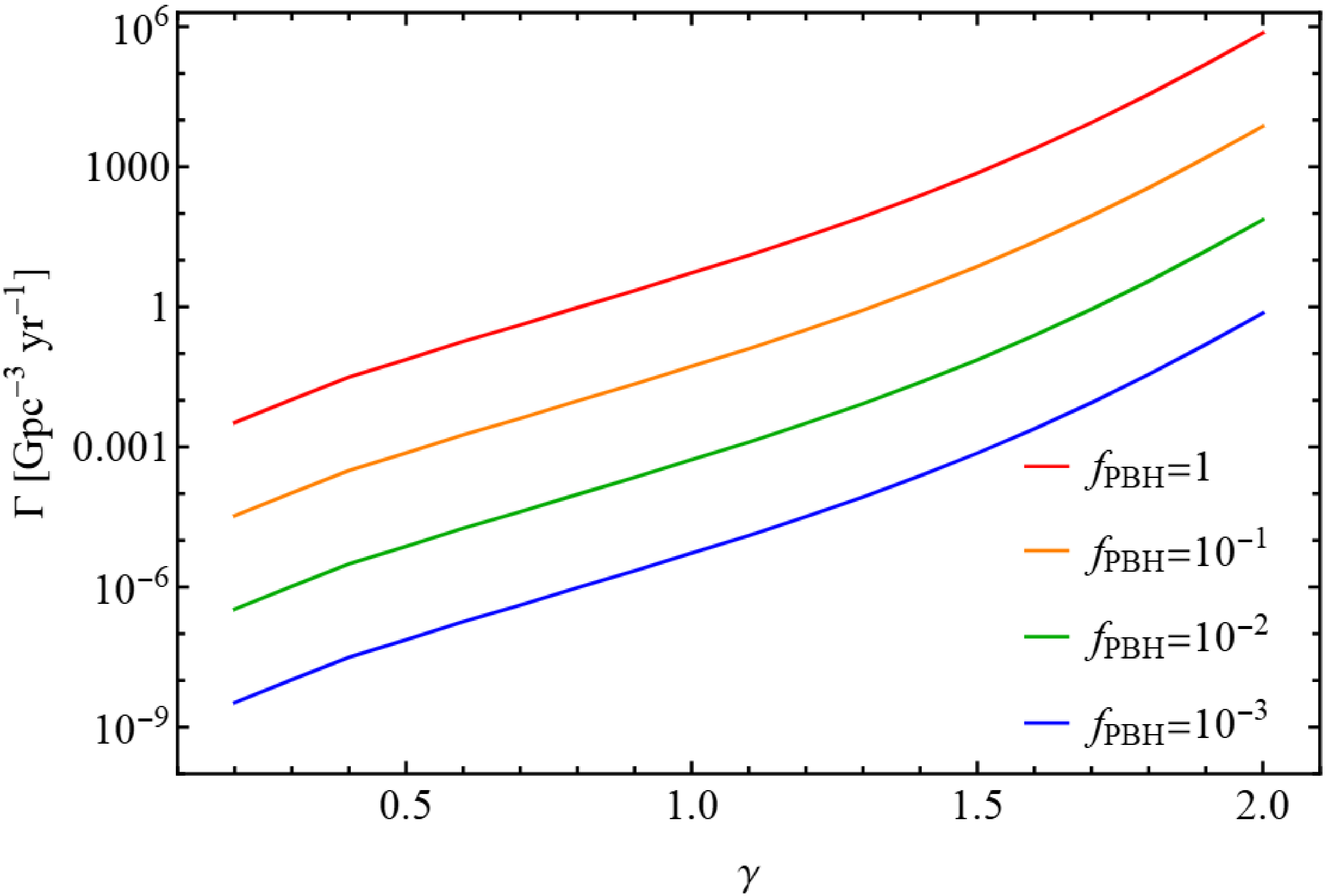
$$N_{\rm sp} = \int_{4R_s}^{R_{\rm sp}} \frac{1}{2} \left(\frac{f_{\rm PBH}\rho_{\rm sp}(r)}{M_{\rm PBH}} \right)^2 \sigma_m(r) v_{\rm rel}(r) d^3 r$$

$$\sigma_m(r) = 1.4 \times 10^{-14} \left(\frac{M_{\rm PBH}}{30\, M_\odot} \right)^2 \left(\frac{v_{\rm rel}(r)}{200\,{\rm km\,s^{-1}}} \right)^{-\frac{18}{7}} {\rm pc}^2$$

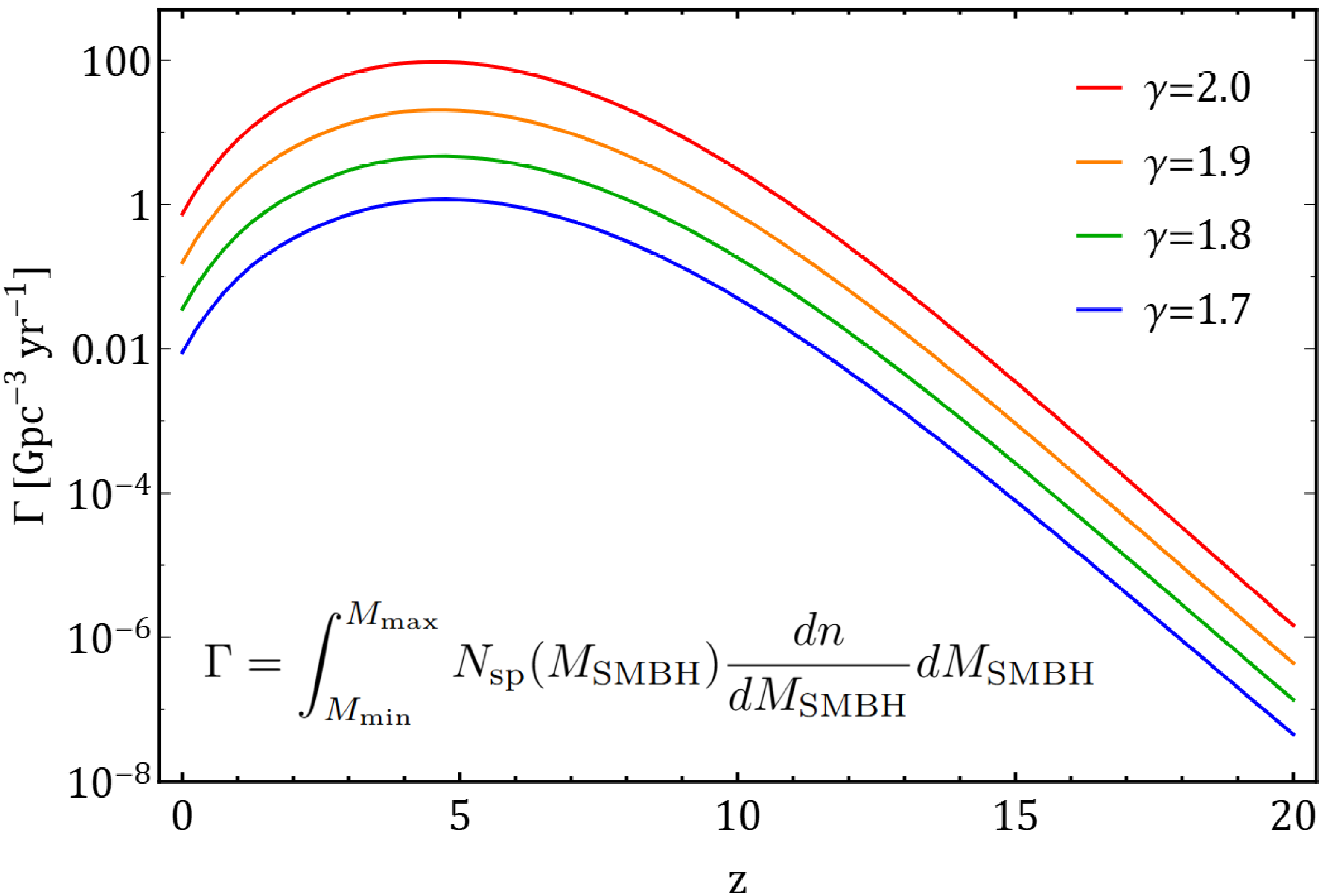
$$\Gamma = \int_{M_{\rm min}}^{M_{\rm max}} N_{\rm sp}(M_{\rm SMBH}) \frac{dn}{dM_{\rm SMBH}} dM_{\rm SMBH}$$

$$\frac{dn}{dM_{\rm SMBH}} = \frac{dn}{dM_{\rm vir}} \frac{dM_{\rm vir}}{dM_{\rm SMBH}} = f(\sigma) \frac{\rho_m}{M_{\rm vir}} \frac{d \log(\sigma^{-1})}{dM_{\rm vir}} \frac{dM_{\rm vir}}{dM_{\rm SMBH}}$$

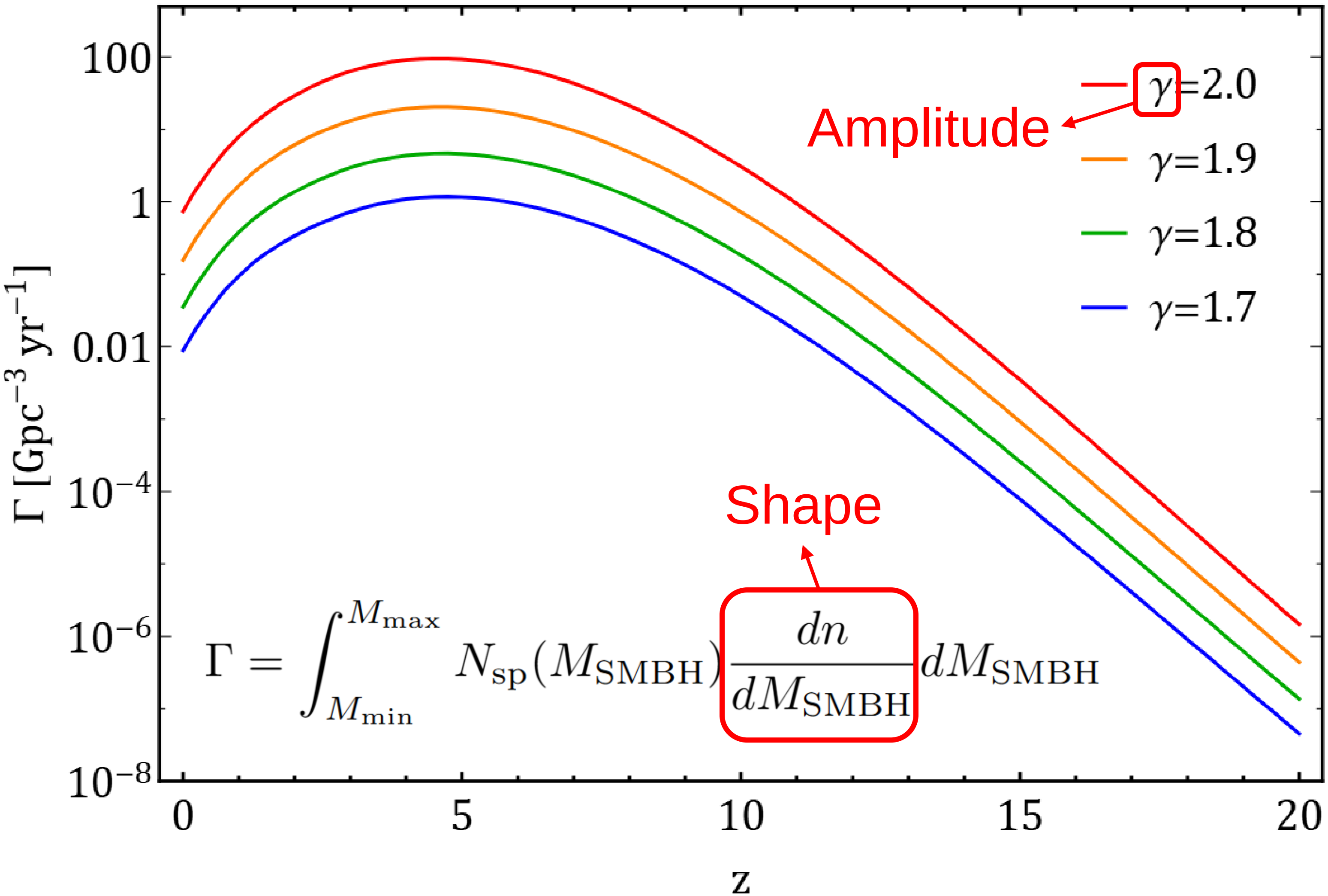
$$\log_{10} \left(\frac{M_{\rm SMBH}}{M_\odot} \right) = a + b \log_{10} \left(\frac{\sigma}{200\,{\rm km\,s^{-1}}} \right)$$



10^6
1000
1
0.001
10^{-6}
10^{-9}
Γ [Gpc^{-3} yr^{-1}]
0.5
1.0
1.5
2.0
γ
$f_{\rm PBH}=1$
$f_{\rm PBH}=10^{-1}$
$f_{\rm PBH}=10^{-2}$
$f_{\rm PBH}=10^{-3}$

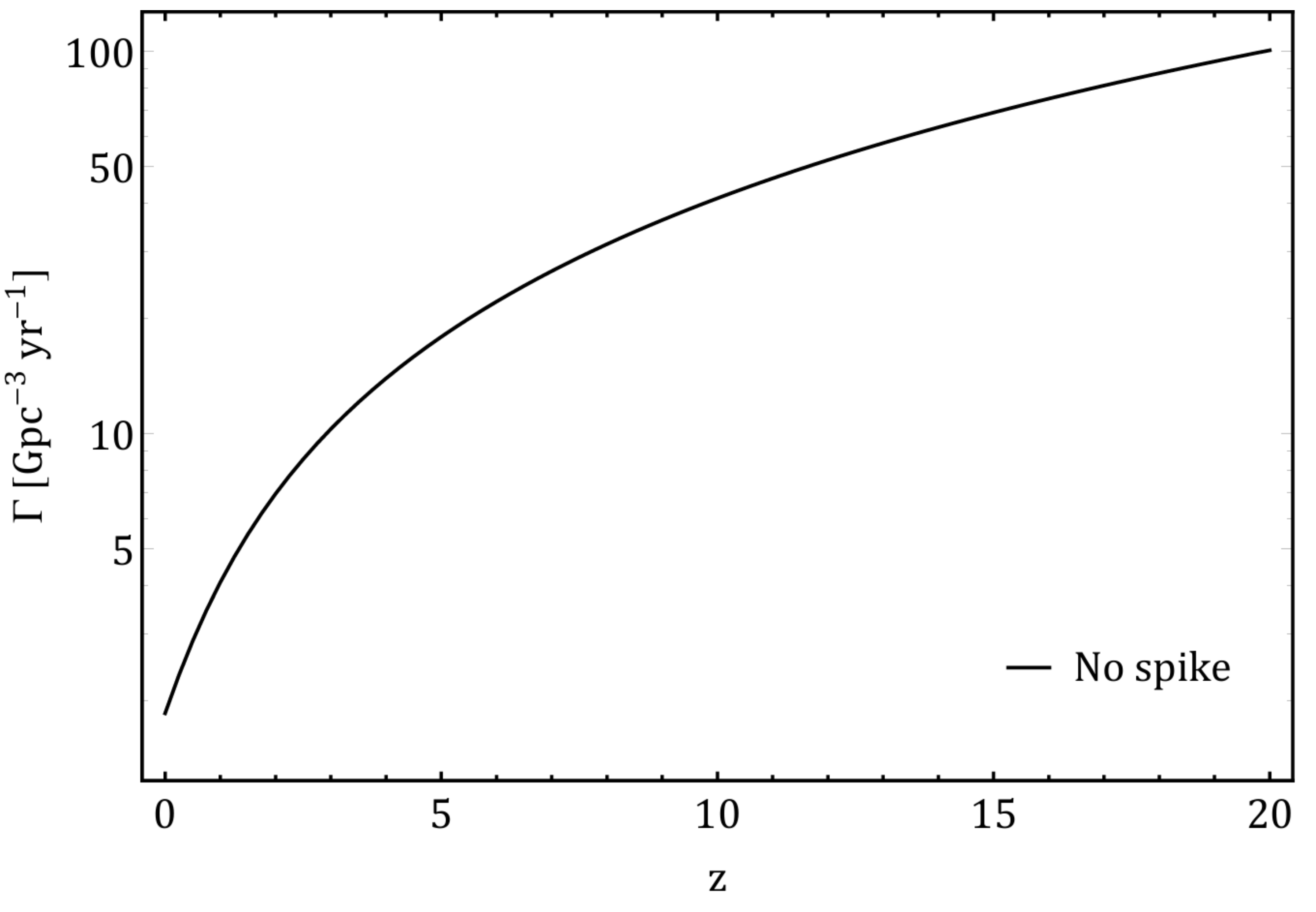


Γ [Gpc^{-3} yr^{-1}]
100
1
0.01
10^{-4}
10^{-6}
10^{-8}
0
5
10
15
20
z
γ=2.0
γ=1.9
γ=1.8
γ=1.7
$\Gamma = \int_{M_{\rm min}}^{M_{\rm max}} N_{\rm sp}(M_{\rm SMBH}) \frac{dn}{dM_{\rm SMBH}} dM_{\rm SMBH}$

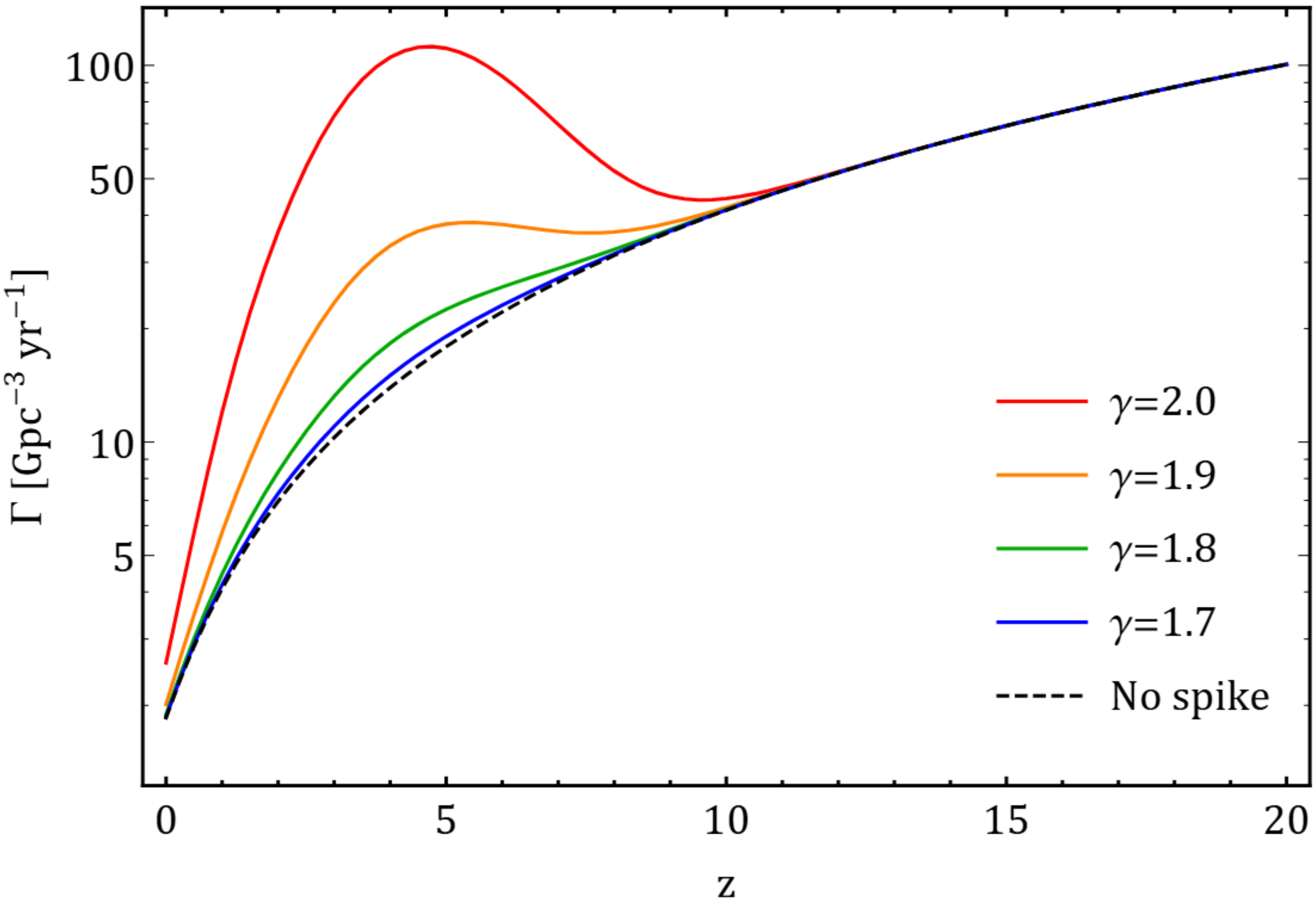


100
1
0.01
10^{-4}
10^{-6}
10^{-8}
Γ [Gpc^{-3} yr^{-1}]
0
5
10
15
20
z
γ=2.0
γ=1.9
γ=1.8
γ=1.7
Amplitude
Shape
$\Gamma = \int_{M_{\min}}^{M_{\max}} N_{\rm sp}(M_{\rm SMBH}) \boxed{\frac{dn}{dM_{\rm SMBH}}} dM_{\rm SMBH}$

PBH Merger Rate Evolution

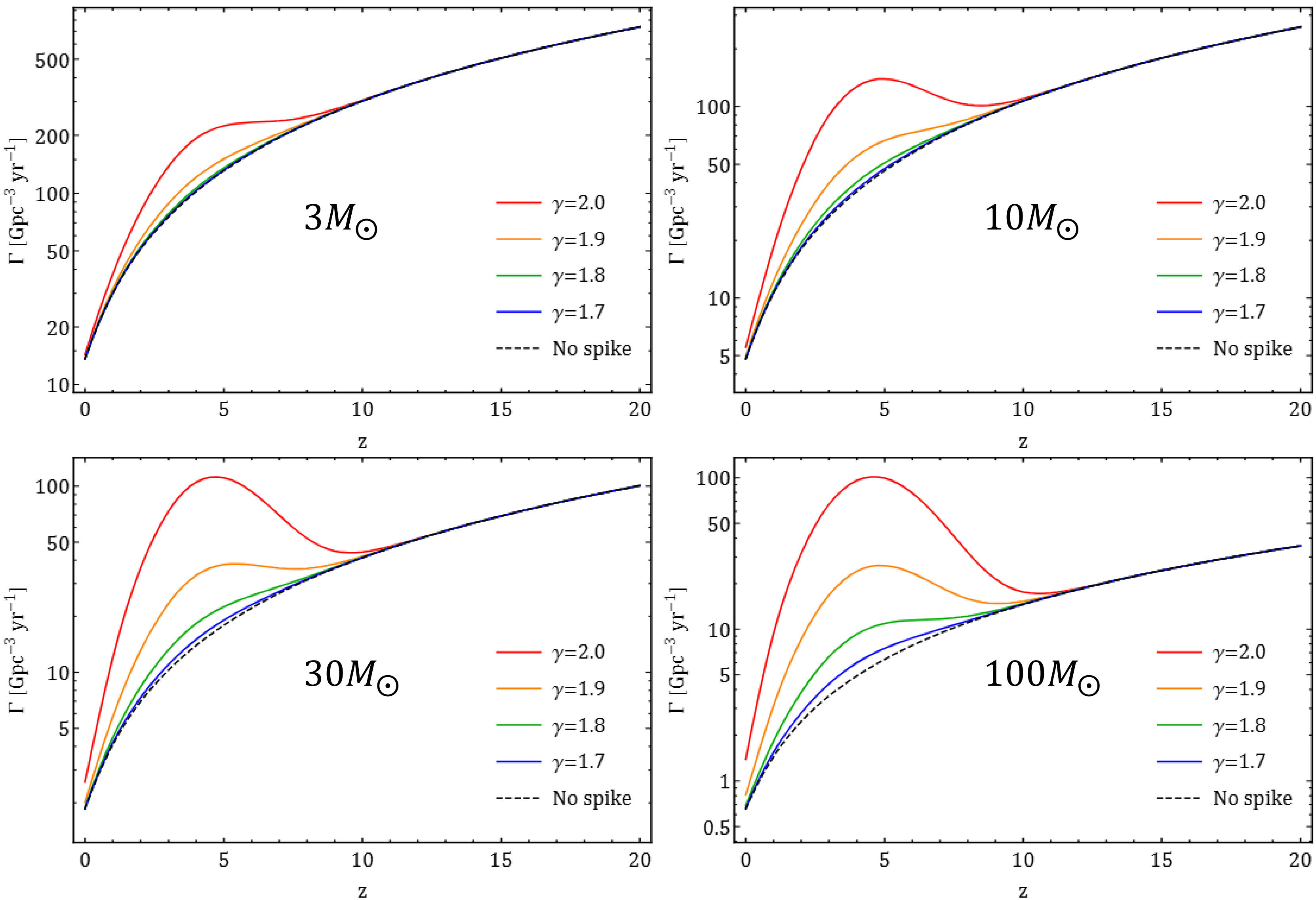


PBH Merger Rate Evolution

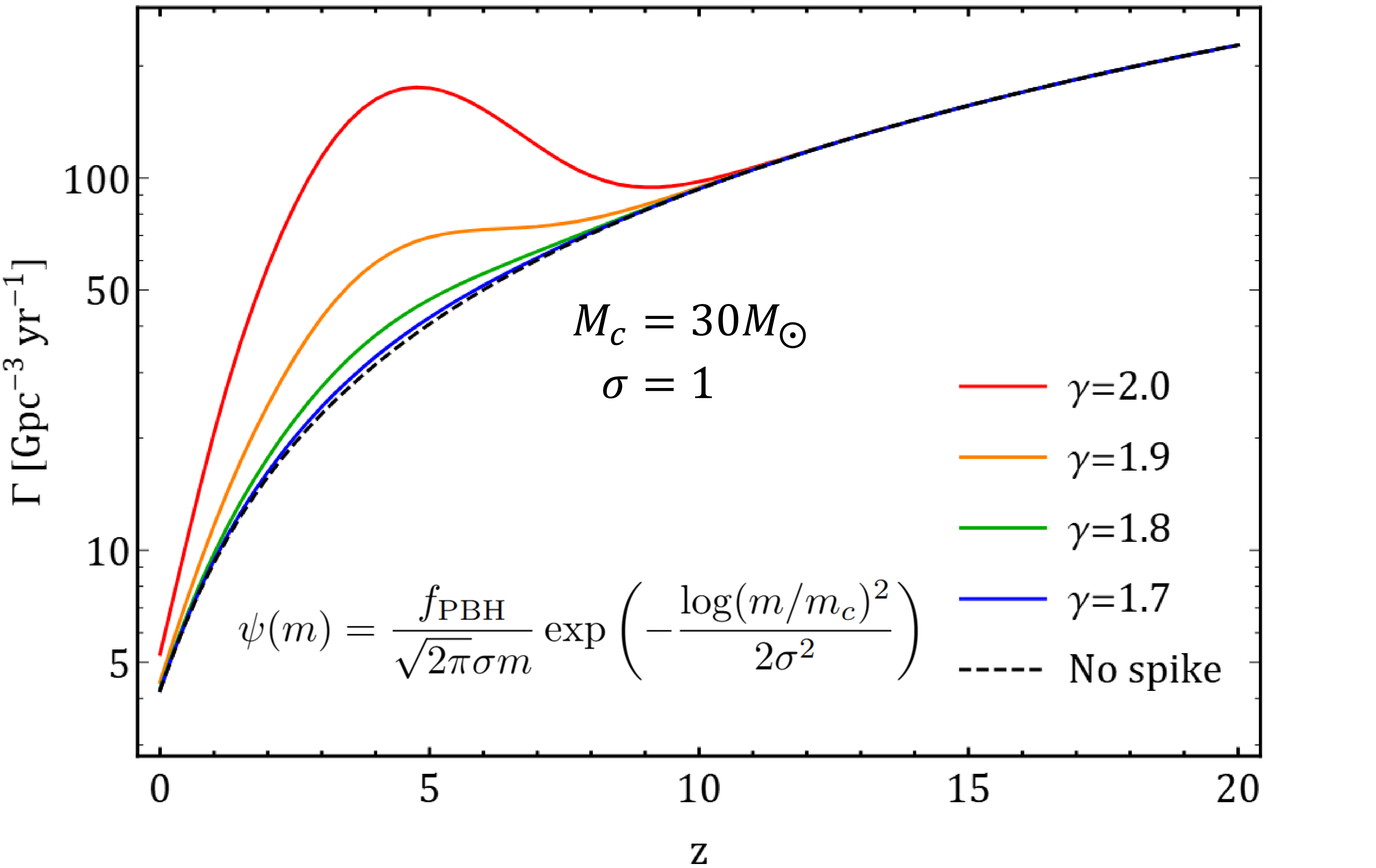


$$R_{\rm tot} = R_{\rm eb} + R_{\rm sp} + R_{\rm halo} + R_{\rm cluster} + \text{etc.}$$
$$\simeq R_{\rm eb} + R_{\rm sp}$$

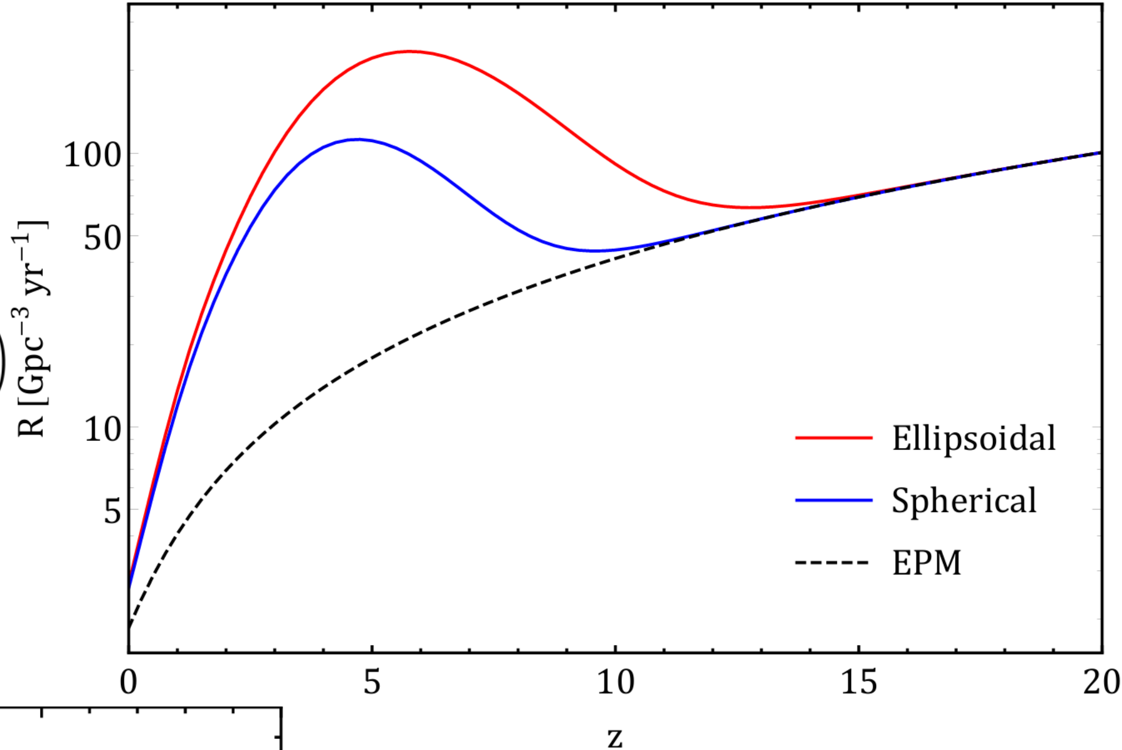
PBH Mass Impact



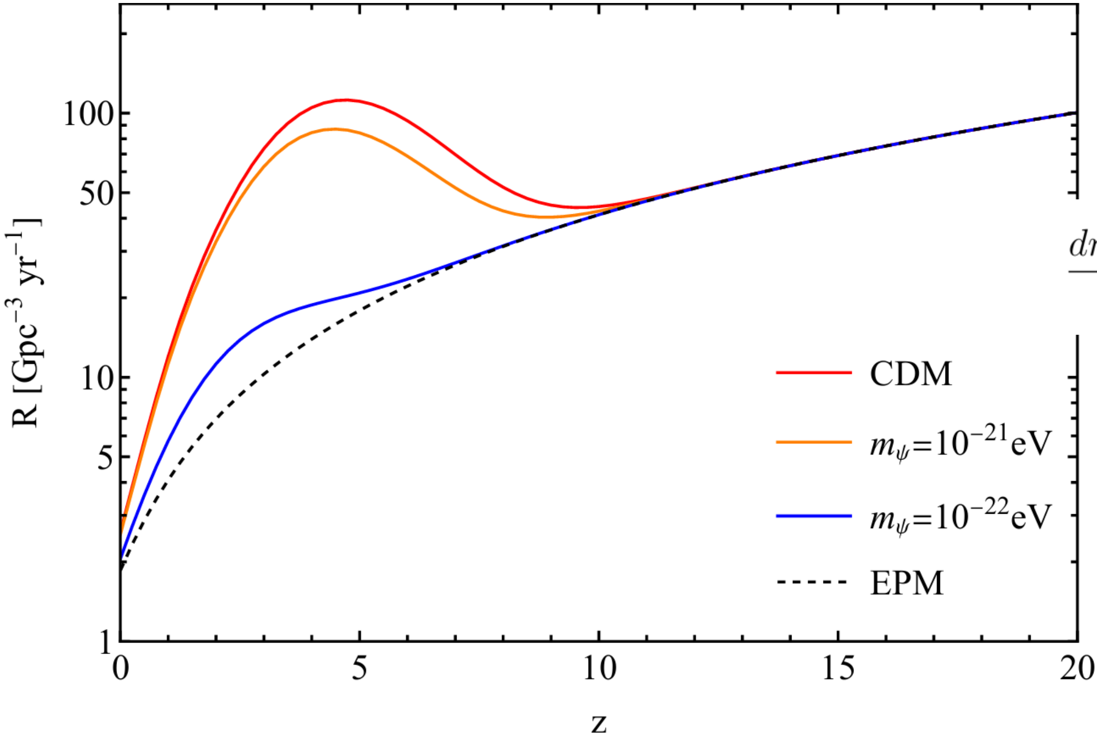
PBH Mass Impact



Halo Impact

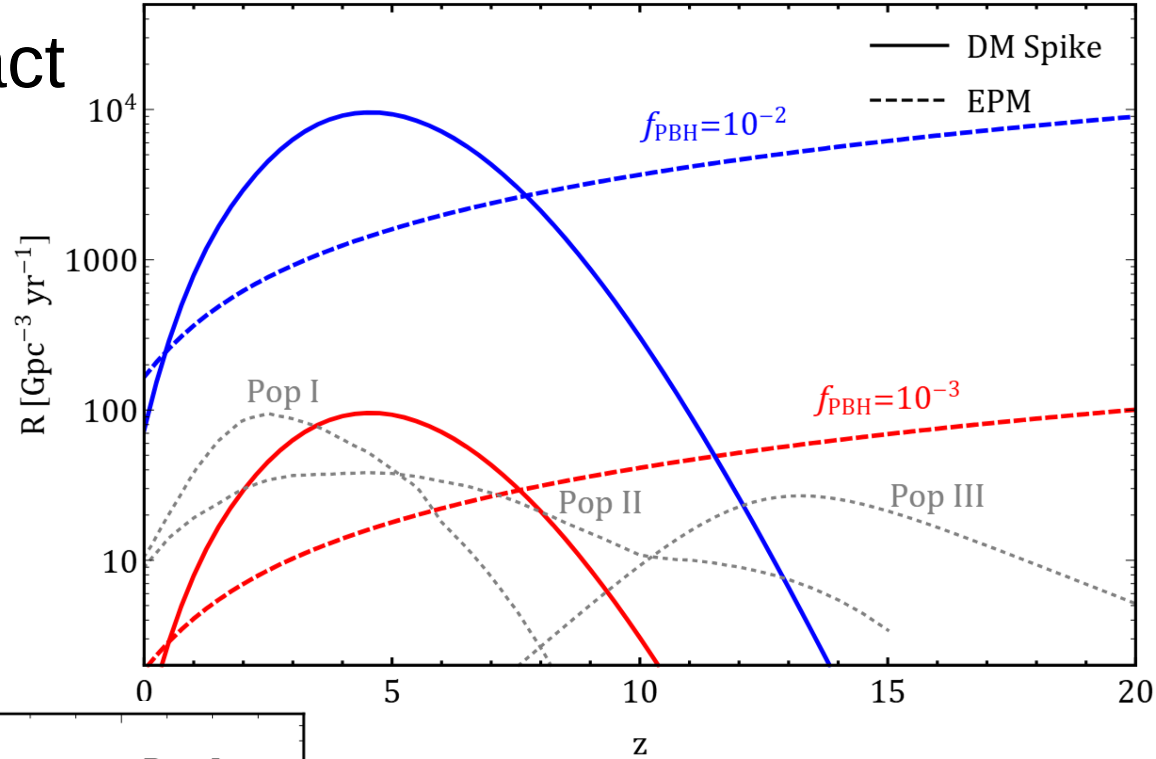


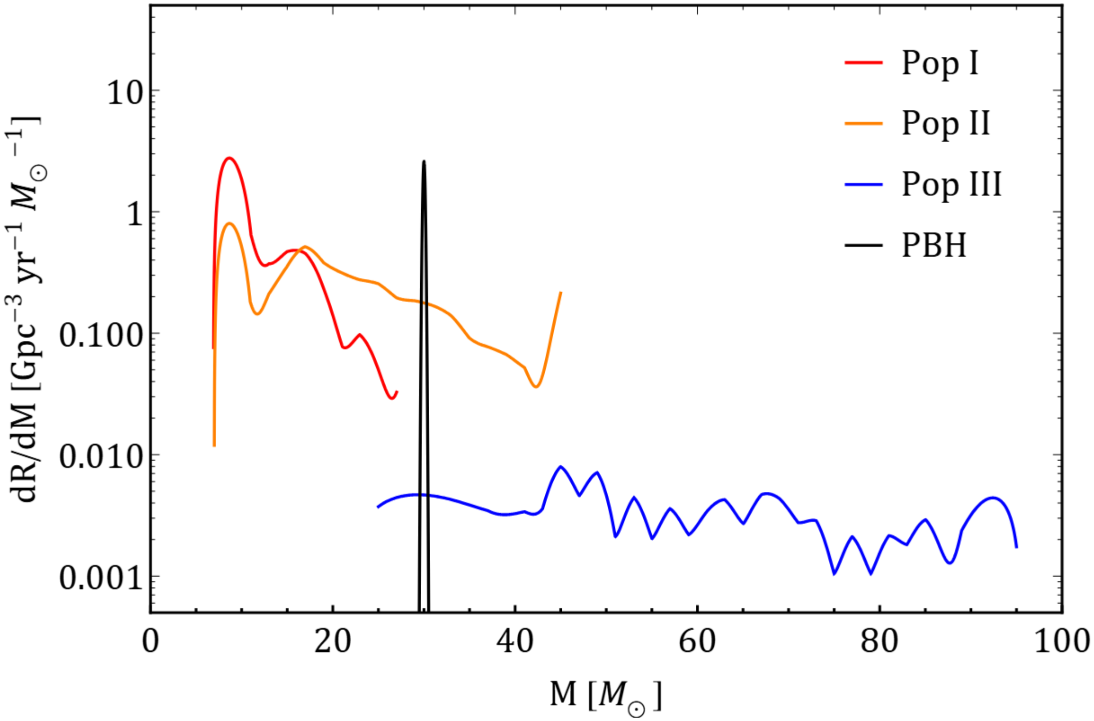
$$f_{\rm ST}(\sigma) = F\sqrt{\frac{2a}{\pi}}\left[1+\left(\frac{\sigma^2}{a\delta_{\rm c}^2}\right)^p\right]\frac{\delta_{\rm c}}{\sigma}\exp\left(-\frac{a\delta_{\rm c}^2}{2\sigma^2}\right)$$



$$\left.\frac{dn(M,z)}{dM}\right|_{\psi\rm DM} = \left.\frac{dn(M,z)}{dM}\right|_{\rm CDM}\left[1+\left(\frac{M}{M_0}\right)^{-1.1}\right]^{-2.2}$$

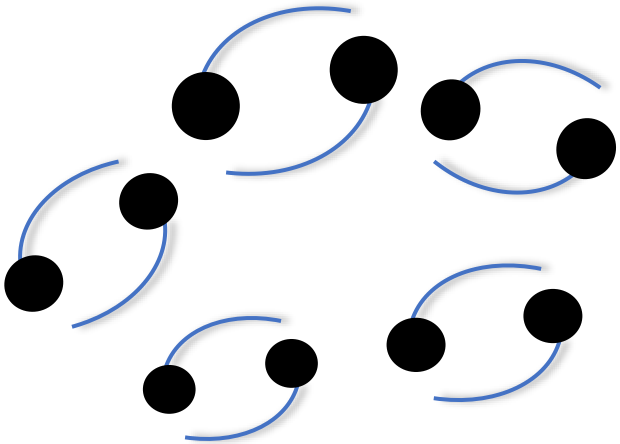
Astrophysical BH Impact



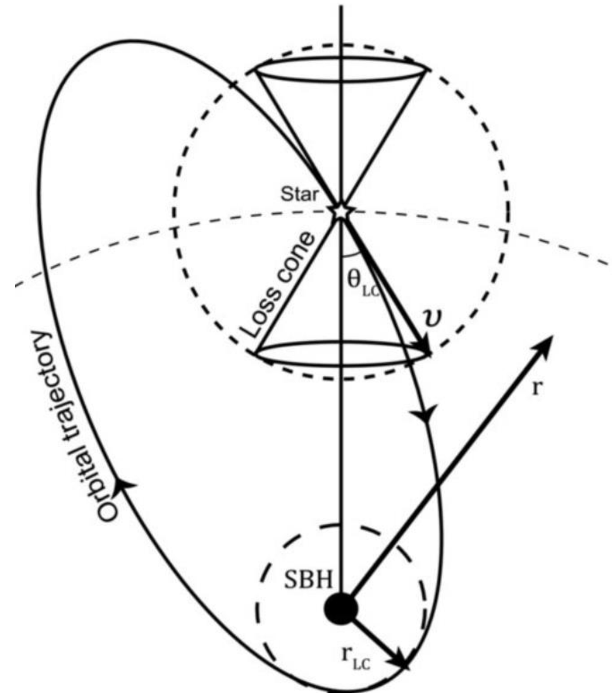


DM Spike Evolution

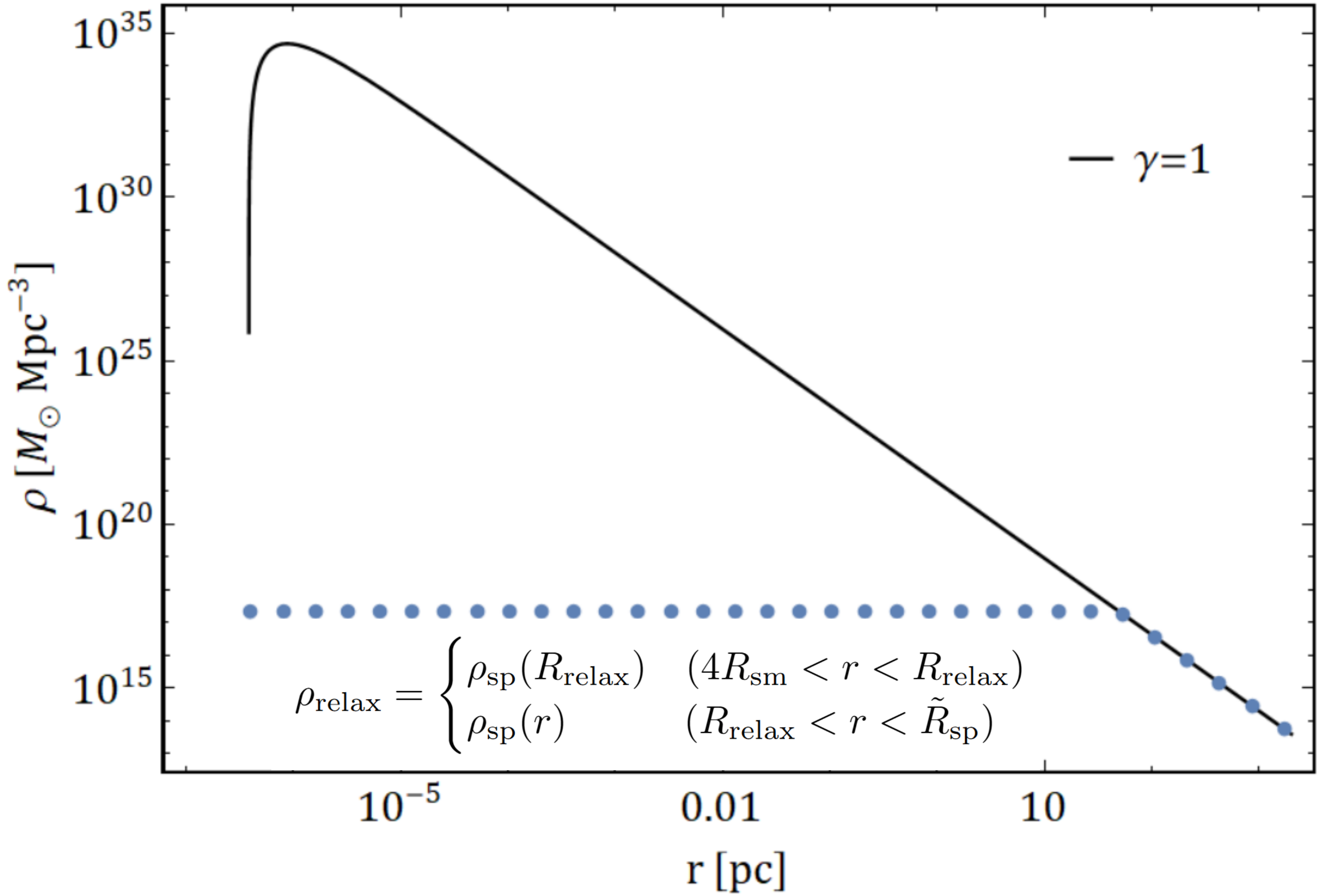
Two-body relaxation



Loss-cone refilling

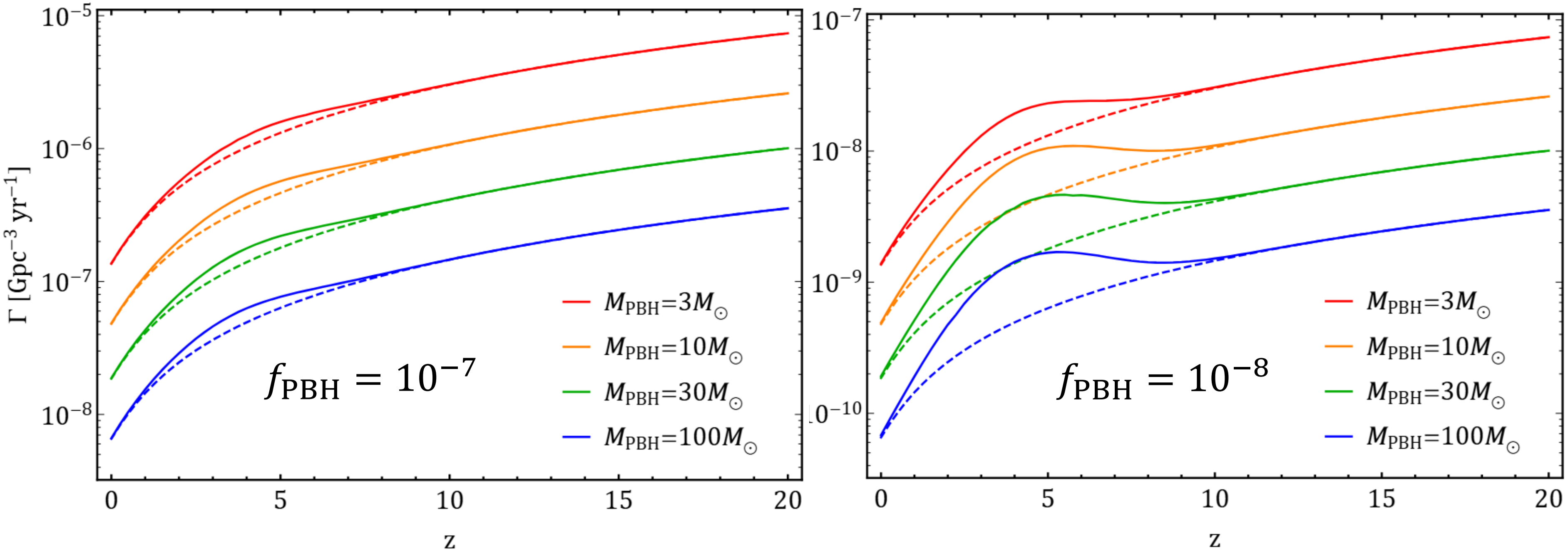


Two-Body Relaxation



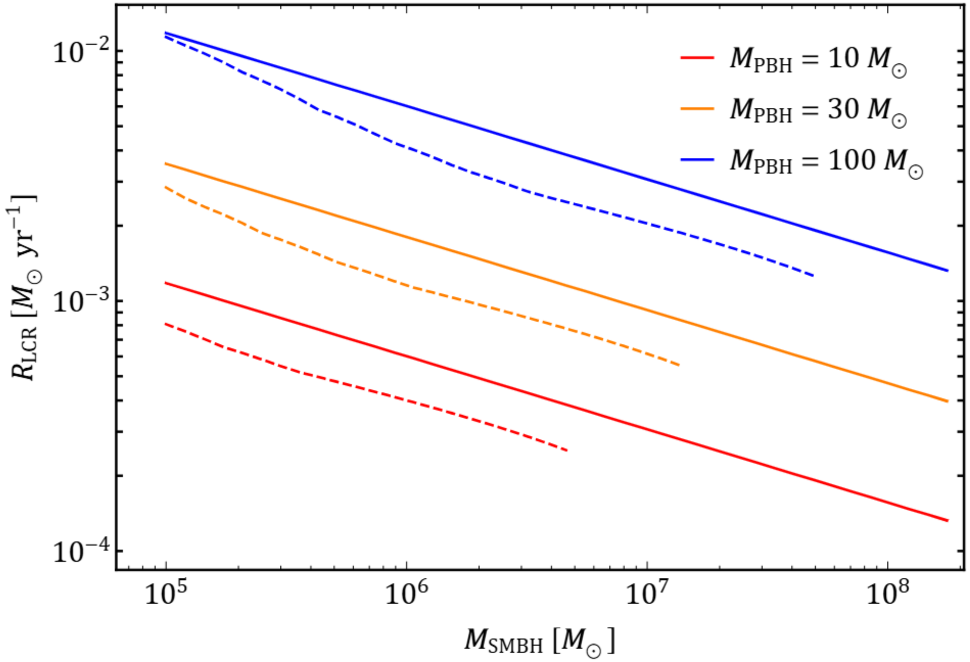
$$t_{\rm relax} = \frac{v_{\rm rel}^3(R_{\rm relax})}{8\pi G^2 M_{\rm PBH} f_{\rm PBH} \rho_{\rm sp}(R_{\rm relax}) \log(b_{\rm max}/b_{\rm min})}$$

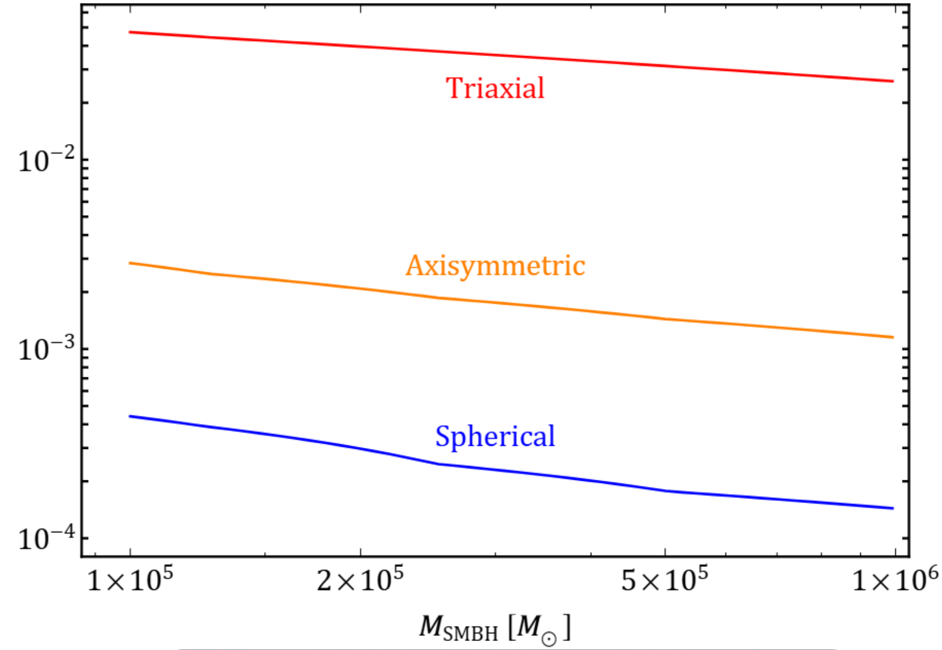
Two-Body Relaxation



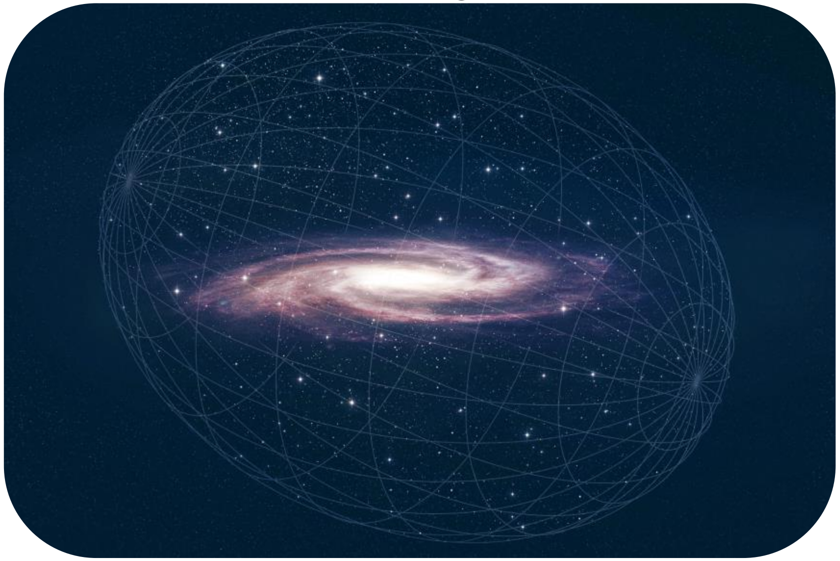
$$t_{\rm relax} = \frac{v_{\rm rel}^3(R_{\rm relax})}{8\pi G^2 M_{\rm PBH} f_{\rm PBH} \rho_{\rm sp}(R_{\rm relax}) \log(b_{\rm max}/b_{\rm min})}$$

Loss-Cone Refilling

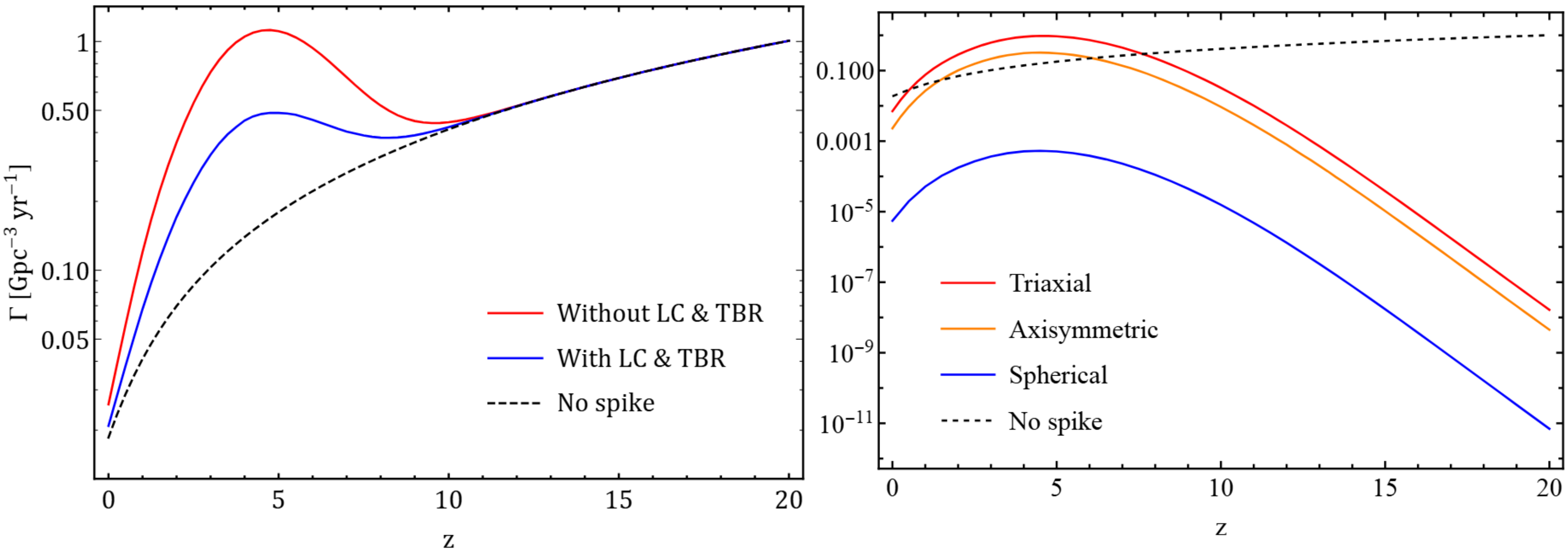




$$\frac{dM}{dt} = f_{\rm PBH}\frac{M_{\rm PBH}}{M_{\rm SMBH}}\frac{\sigma^3}{G}$$

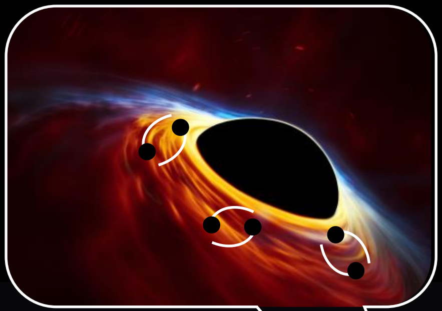


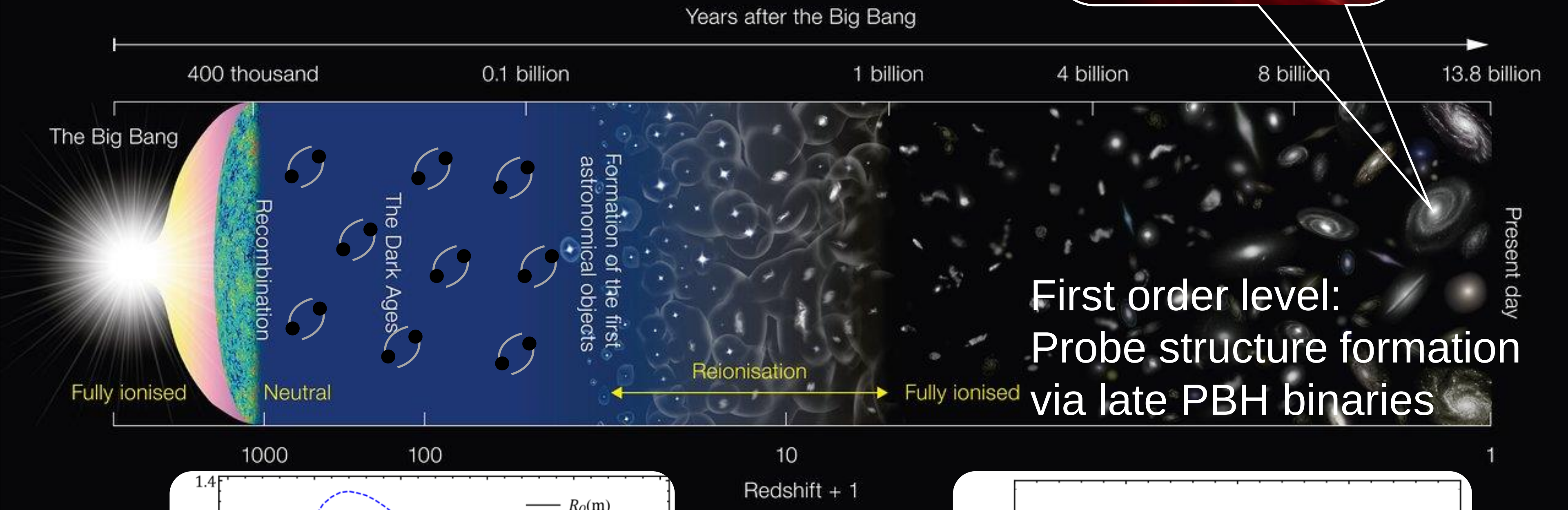
Loss-Cone Refilling



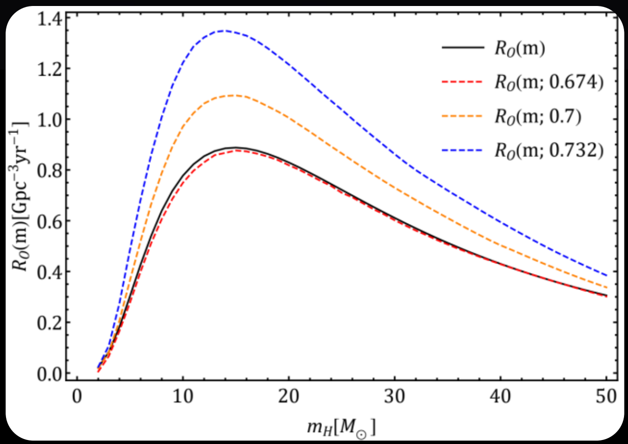
PBH Mergers as a Cosmological Probe

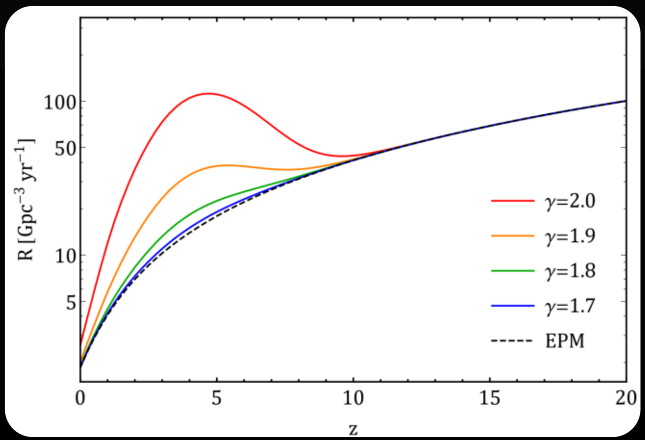
Background level:
Probe Hubble expansion rate
via early PBH binaries



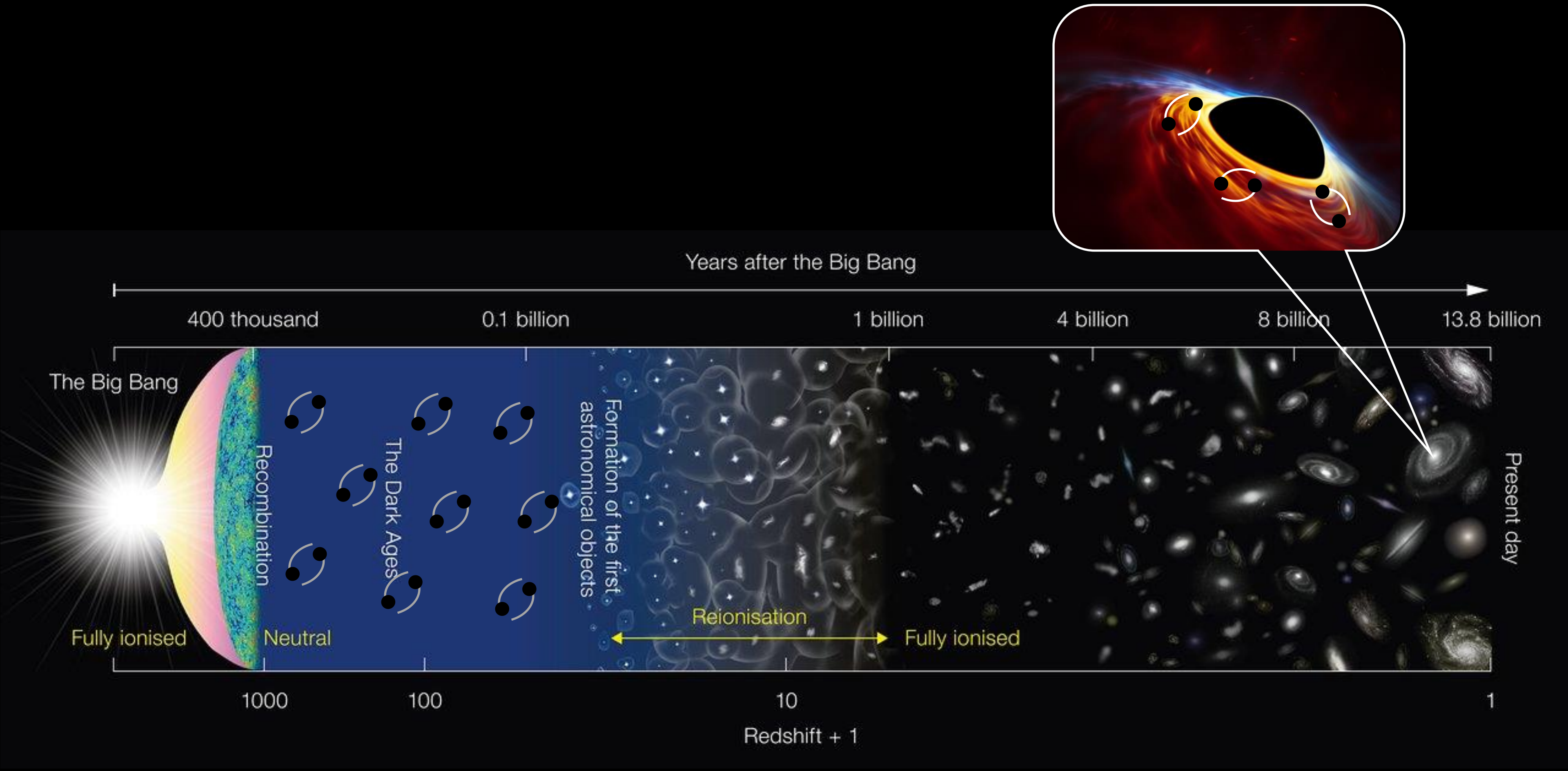


First order level:
Probe structure formation
via late PBH binaries





PBH Mergers as a Cosmological Probe



Thank you!