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Discovering Rotating Black Holes in a Viable Lorentz-Violating Qauntum Gravity

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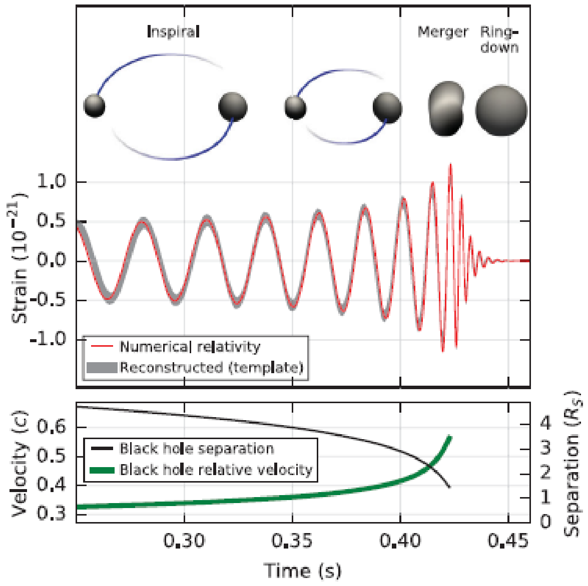
Basen on MIP, Deniz O. Devecioglu
[arXiv:2402.02253v2] [EPJC]+work in progress

- 1. Rotating BH is believed to be real!



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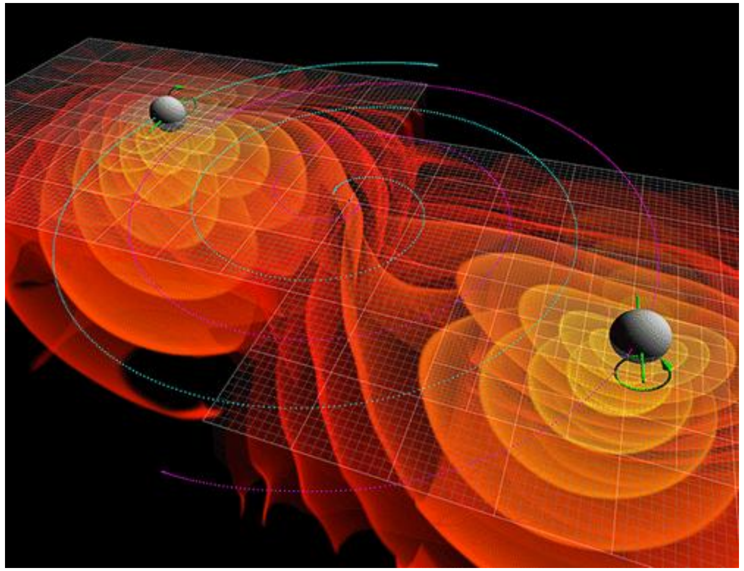


- 2. The final state of collapse/collision is described Kerr solution (no-hair/uniqueness theorem)!!
- This is the importance of finding exact solutions in gravity theories.

- 3. But there are several evidences that GR is not enough for a (UV) complete description of our universe: Non-renormalizability, dark energy (and dark matter ?), Hubble tension (?), etc.

- 4. If there are Kerr-like rotating black hole solutions in another viable gravity model, we can use those as a laboratory for testing the gravity model, in comparison with Kerr in GR.

- 5. But, it is extremely hard to get an “exact” rotating solution other than in GR (or its relativistic cousins), for example, in a UV-complete gravity with Lorentz-violation (Horava gravity).

- The only known solutions (D=4) are
- (1) D=4 non-rotating solutions (Lu-Mei-Pope, Kehagias-Sfetsos, Park (2009), Kiritsis-Kofinas (2010))
- (2) D=4 slowly rotating solution (Lee-Kim-Myung, Aliev-Senturk (2010))
- (3) D=4 numerical rotating solution for Einstein-Aether gravity [Adam et al., CQG 39, 125001 (2022)].

- 6. Today, we are going to propose a general procedure/strategy for finding exact rotating solutions but without (or less) tears!

- And, following the procedure, we will show the exact (Kerr-like) rotating black hole solutions in low-energy Horava gravity.
- (cf. complementary to numerical solutions in EA (2022)!)

1. A TWO-STEP PROCEDURE TO FIND KERR-TYPE STATIONARY SOLUTIONS

- (i) Find an exact "massless" rotating space-time solution, which means it is the exact solution for an arbitrary rotation parameter a.

- (ii) Introduce the "mass"-dependent ansatz functions into the massless seed solution; without loss of much generality but still in a solvable way, i.e., with ODE for the polar angle θ or the radial coordinate r. (so that computer can solve it!)

2. LOW-ENERGY HORAVA GRAVITY AND ROTATING BLACK HOLE SOLUTIONS

- The low energy (non-projectable) Horava gravity

$$S_g = \int_{\mathbf{R}\times\Sigma_t} dt d^3x \sqrt{g} N \left[\frac{1}{\kappa} \left(K_{ij}K^{ij} - \lambda K^2 \right) + \xi R - 2\Lambda + \frac{\sigma}{2} a_i a^i \right]$$

$$K_{ij} = (2N)^{-1} \left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \right)$$

$$a_i = \nabla_i lnN$$

$$ds^2 = -N^2 c^2 dt^2 + g_{ij} \left(dx^i + N^i dt \right) \left(dx^j + N^j dt \right)$$

- 3 LV parameters λ, ξ, and σ

- **EQN for N, N^i, and g^{ij}**

$$\mathcal{H} \equiv \frac{1}{\kappa}\left(K_{ij}K^{ij} - \lambda K^2\right) - \xi R + 2\Lambda - \sigma\left(\frac{1}{2}\frac{\nabla_i N \nabla^i N}{N^2} - \frac{\nabla_k \nabla^k N}{N}\right) = 0\,,$$

$$\mathcal{H}^i \equiv \frac{2}{\kappa}\nabla_j\left(K^{ji} - \lambda\, K g^{ji}\right) = 0\,,$$

$$E_{ij} \equiv \frac{1}{\kappa}\left(E^{(1)}_{ij} - \lambda E^{(2)}_{ij}\right) + \xi E^{(3)}_{ij} + \frac{\sigma}{2}E^{(4)}_{ij} = 0,$$

$$\begin{aligned}
E^{(1)}_{ij} &= N_i \nabla_k K^k{}_j + N_j \nabla_k K^k{}_i - K^k{}_i \nabla_j N_k - K^k{}_j \nabla_i N_k - N^k \nabla_k K_{ij} \\
&\quad -2NK_{ik}K_j{}^k - \frac{1}{2}NK^{k\ell}K_{k\ell}\,g_{ij} + NKK_{ij} + \dot{K}_{ij}\,, \\
E^{(2)}_{ij} &= \frac{1}{2}NK^2 g_{ij} + N_i\partial_j K + N_j \partial_i K - N^k(\partial_k K)g_{ij} + \dot{K}\,g_{ij}\,, \\
E^{(3)}_{ij} &= N\left(R_{ij} - \frac{1}{2}Rg_{ij} + \frac{\Lambda}{\xi}g_{ij}\right) - (\nabla_i\nabla_j - g_{ij}\nabla_k\nabla^k)N\,, \\
E^{(4)}_{ij} &= \frac{1}{N}\left(-\frac{1}{2}g_{ij}\nabla_k N\nabla^k N + \nabla_i N \nabla_j N\right).
\end{aligned}$$

- **With arbitrary λ, ξ, and σ, the manifest symmetry of action is the foliation-preserving diffeomorphism**

$(Diff_{\mathcal{F}})$

$$\begin{aligned} \delta_\xi t &= -\xi^0(t), \quad \delta_\xi x^i = -\xi^i(t, \mathbf{x}), \\ \delta_\xi N &= (N\xi^0)_{,0} + \xi^k \nabla_k N, \\ \delta_\xi N_i &= \xi^0{}_{,0} N_i + \xi^j{}_{,0} g_{ij} + \nabla_i \xi^j N_j + N_{i,0}\xi^0 + \nabla_j N_i \xi^j, \\ \delta_\xi g_{ij} &= \nabla_i \xi^k g_{kj} + \nabla_j \xi^k g_{ki} + g_{ij,0}\xi^0. \end{aligned}$$

Here, the physical (gauge-invariant) quantities are K, $K_{ij}K^{ij}$, R, $R_{ij}R^{ij}$, $K_{ij}R^{ij}$, etc, capturing physical singularities!

3. Step 1: Find massless rotating BH solutions.

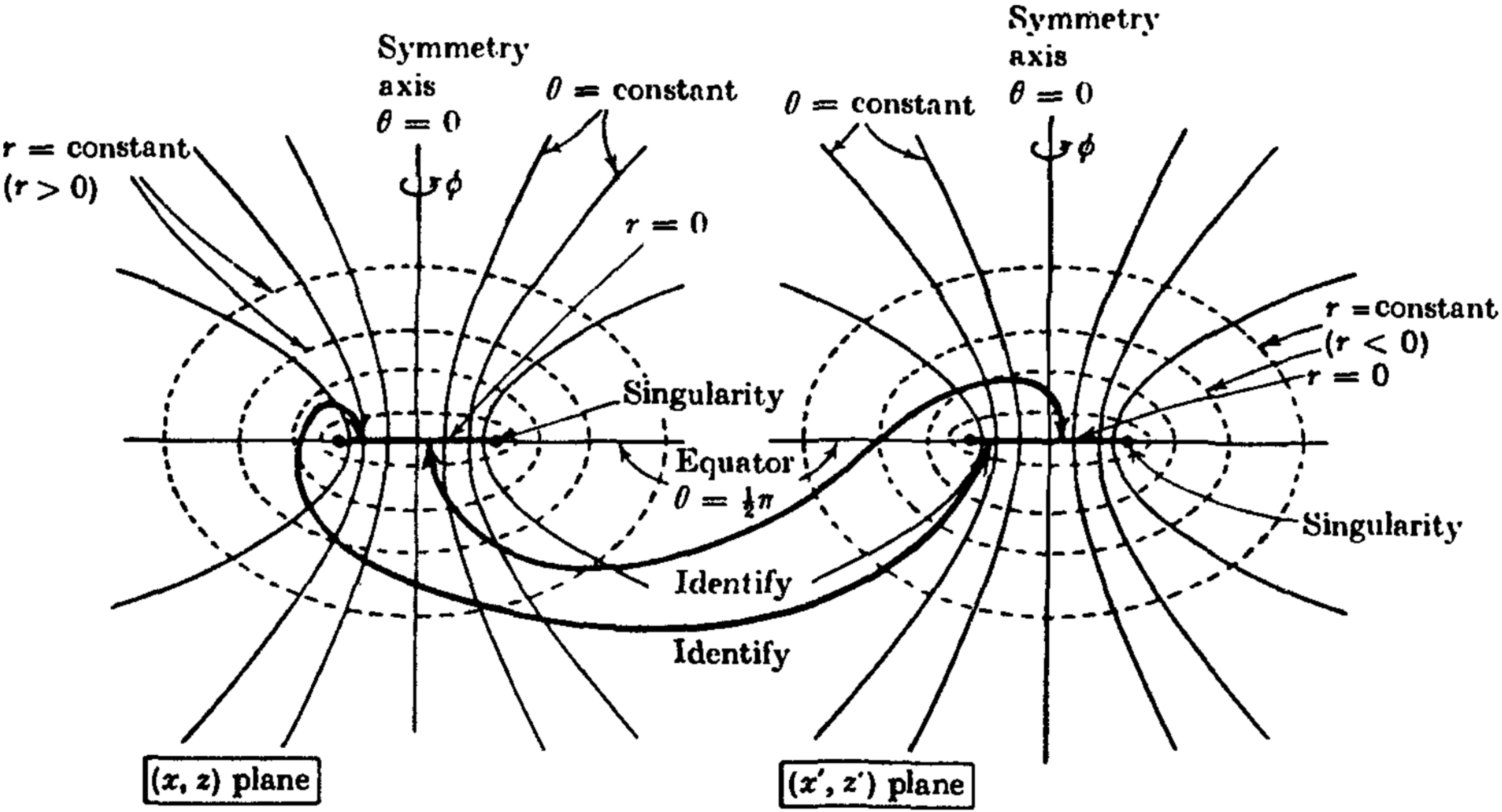
- We consider $\Lambda = 0, \ \sigma = 0$, (asymptotically flat), for simplicity.

Recently, we have found that massless Kerr solution is also an exact solution for (UV-complete) Horava gravity [Park-Lee, arXiv:2309.13859 [hep-th].]

$$ds_0^2 = -\frac{\rho^2 \Delta_r^{(0)}}{\Sigma_{(0)}^2} dt^2 + \frac{\rho^2}{\Delta_r^{(0)}} dr^2 + \rho^2 d\theta^2 + \frac{\Sigma_{(0)}^2 \sin^2\theta}{\rho^2} d\phi^2$$

$$\rho^2 = r^2 + a^2 \cos^2\theta, \Delta_r^{(0)} = \left(r^2 + a^2\right), \Sigma_{(0)}^2 = \left(r^2 + a^2\right) \rho^2$$

$$x = (r^2 + a^2)^{1/2} \sin\theta \ \cos\phi, y = (r^2 + a^2)^{1/2} \sin\theta \ \sin\phi, z = r \ \cos\theta$$



Symmetry
axis
$\theta = 0$
ϕ
r = constant
($r > 0$)
θ = constant
$r = 0$
Singularity
Equator
$\theta = \frac{1}{2}\pi$
Identify
Identify
(x, z) plane
θ = constant
Symmetry
axis
$\theta = 0$
ϕ
r = constant
($r < 0$)
$r = 0$
Singularity
(x′, z′) plane

- **Remarks:**
- **(1) $r = 0$ is not the end of the coordinates but there is another copy of Kerr spacetime in the $r < 0$ regime, with another asymptotic infinity at $r \to -\infty$! ; the massless Kerr metric was interpreted as a wormhole solution** [Gibbons-Volkov (2017)]

- **(2) For massless rotating solution, there is no event horizon. But the genuine spacetime deformation for a rotating object is believed to be naturally encoded in the ellipsoidal coordinates with the rotation parameter a : Mass just deforms the metric in r direction!**

4. Step 2. Consider mass-dependent ansatz for viable ODEs.

- In Kerr metric, the mass term is tightly bounded to the rotation parameter a and it is not easy to separate them (in the component form).

$$ds^2 = -\left(1 - \frac{2GMr}{\rho^2}\right) dt^2 - \frac{2GMar\sin^2\theta}{\rho^2}(dt\, d\phi + d\phi\, dt) + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]d\phi^2,$$

$$\Delta(r) = r^2 - 2GMr + a^2$$

$$\rho^2(r,\theta) = r^2 + a^2\cos^2\theta.$$

- Key point for the ansatz: Consistently separate the spin part and the mass part so that we might crack the rigid structure of Kerr solution!

- **Our ansatz with 3 undetermined functions,**

$f(r), g(r), \text{ and } \Delta_r(r)$ **:**

$$ds_1^2 = -N^2 dt^2 + \frac{\rho^2}{\Delta_r} dr^2 + \rho^2 d\theta^2 + \frac{\Sigma^2 \sin^2\theta}{\rho^2} \left(d\phi + N^\phi dt\right)^2$$

$$\Sigma^2 = \left(r^2 + a^2\right)\rho^2 + f(r) a^2 \sin^2\theta,$$

$$N^2 = \frac{\rho^2 \Delta_r(r)}{\Sigma^2}, \quad N^\phi = -\frac{g(r)}{\Sigma^2}$$

Solution (with the usual choice $N^\phi|_\infty = 0, W(\infty) \equiv N\sqrt{g_{rr}}|_\infty = 1$):

$$f(r) = 2mr, \ \ g(r) = 2amr\sqrt{\kappa\xi}, \ \Delta_r(r) = r^2 + a^2 - 2mr$$

- Remarks:
- 1. It is rather surprising that the Kerr-solution cracking term with the LV factor κξ appears only in N^ϕ .
- 2. But, if we look at the component form, one can easily see the non-trivial LV effect for $\xi \neq 1/\kappa$ in g_{tt} as well as in $g_{t\phi}$ components!

$$ds_1^2 = \left[-\frac{(\Delta_r - a^2\sin^2\theta)}{\rho^2} + \frac{(\kappa\xi - 1)\ (2mr)^2\ a^2\sin^2\theta}{\rho^2\Sigma^2}\right] dt^2 + \frac{\rho^2}{\Delta_r}dr^2 + \rho^2 d\theta^2 + \frac{\Sigma^2\sin^2\theta}{\rho^2}d\phi^2 - \frac{4amr\sqrt{\kappa\xi}\ \sin^2\theta}{\rho^2}dtd\phi$$

- 3. Our solution, as well as the Kerr solution with $\xi = 1/\kappa$, are valid for an arbitrary λ due to $K = 0$, i.e., "maximal" slicing.

- This makes even Kerr solution (or Schwarzschild solution with a=0) has different notions of singularities, due to the lack of the full Diff with $\lambda \neq 1$.

5. SINGULARITY STRUCTURE

- **Curvature invariants ($K = 0,\ K_{ij}R^{ij} = 0$)**

$$R \sim \frac{a^2 m^2}{\rho^6 \Sigma^4},\quad R_{ij}R^{ij} \sim \frac{m^2}{\rho^{12}\Sigma^8},\quad K_{ij}K^{ij} \sim \frac{\kappa\xi a^2 m^2}{\rho^6 \Sigma^4}$$

show the curvature singularities at $\Sigma^2 = 0$, as well as

$\rho^2 = 0$ (the usual ring singularity at r = 0, θ = π/2).

cf: 4D curvatures

$$R^{(4)} \sim (\kappa\xi - 1)\frac{a^2 m^2}{\rho^6 \Sigma^4},\quad R^{(4)}_{\mu\nu}R^{(4)\mu\nu} \sim (\kappa\xi - 1)^2 \frac{a^4 m^4}{\rho^{12}\Sigma^8},$$

$$R^{(4)}_{\mu\nu\sigma\rho}R^{(4)\mu\nu\sigma\rho} \sim (\kappa\xi - 1)\frac{m^2}{\rho^{12}\Sigma^8} + \frac{m^2}{\rho^{12}}(\cdots),$$

$$\Sigma^2 = \left(r^2 + a^2\right)\rho^2 + f(r)a^2\sin^2\theta$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$

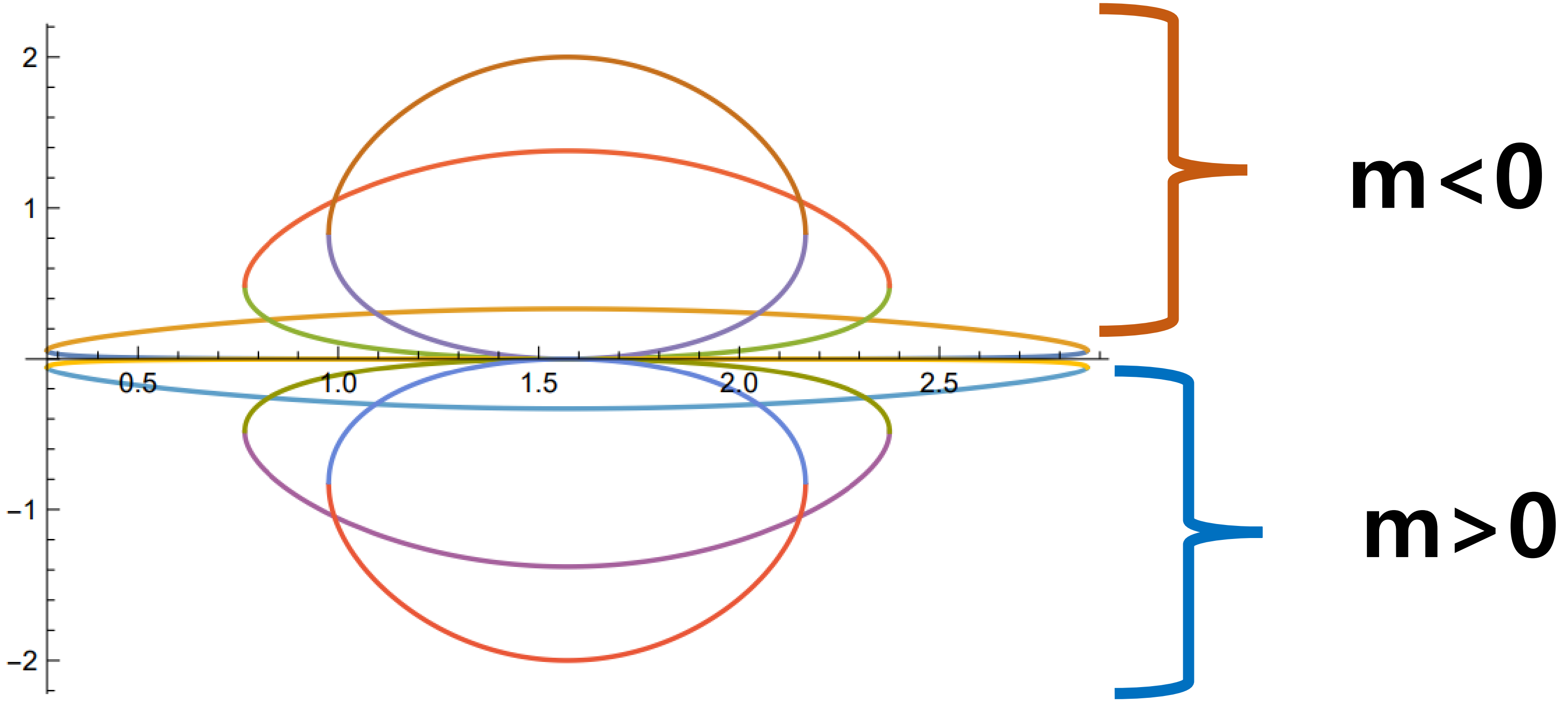
$$f(r) = 2mr$$

- **New singularity surface at $\Sigma^2 = 0$;:** $\Sigma^2 = \left(r^2 + a^2\right)\rho^2 + f(r)a^2\sin^2\theta$

$\rho^2 = r^2 + a^2\cos^2\theta$

$f(r) = 2mr$

$$\cos^2\theta = \frac{2mra^2 + r^2(r^2 + a^2)}{a^2(2mr - (r^2 + a^2))}$$



r vs. θ $[0, \pi]$ of singularity surfaces for $\Sigma^2(r, \theta) = 0$

6. Other Properties.

- 1. There are two **Killing** horizons at

$$\Delta_r = r^2 + a^2 - 2ma = 0$$

as in GR!: The **zeroth-law** is satisfied **(next page!)**

2. The event horizon has the same role for the null and time-like particles due to the unique $Diff_{\mathcal{F}}$. invariant distance $ds^2 = -N^2 c^2 dt^2 + g_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$

$$\begin{aligned} \delta_\xi(ds^2) &= (\xi^0 \partial_0 g_{tt} + \xi^i \partial_i g_{tt}) dt^2 + (2\xi^0 \partial_0 N_j + 2\xi^i \partial_i N_j) dt dx^j \\ &\quad + (\xi^0 \partial_0 g_{ij} + \xi^k \partial_k g_{ij}) dx^i dx^j \\ &= \xi^\mu \partial_\mu (ds^2). \end{aligned}$$

- Properties of Killing horizons:
- (a). Hypersurface-orthogonal.
- (b). $\kappa_\pm$ are constants on the corresponding horizons

$$\chi^{[\mu}\nabla^{\nu]}_{(4)}\kappa_\pm = -\chi^{[\mu}R^{\nu]}{}_{\sigma(4)}\chi^\sigma = -\chi^{[\mu}T^{\nu]}{}_{\sigma(eff)}\chi^\sigma = 0,$$

where we used $R^{(4)}_{\mu\nu} - (1/2)g^{(4)}_{\mu\nu}R^{(4)} = T^{(eff)}_{\mu\nu}$ with

$$T^\mu{}_{\nu(eff)} = \begin{pmatrix} \widehat{\rho} & 0 & 0 & 0 \\ 0 & \widehat{p_1} & \widehat{p_2} & 0 \\ 0 & \widehat{p_3} & -\widehat{p_1} & 0 \\ \widehat{p_4} & 0 & 0 & -3\widehat{\rho}, \end{pmatrix}$$

which violates the DEC $T^0{}_{0(eff)} \geq |T^\mu{}_{\nu(eff)}|$, especially by $|T^3{}_{3(eff)}| = 3|\widehat{\rho}| > \widehat{\rho}$: 0th law of black hole thermodynamics!

- 3. For charged black holes, rotating balck holes exist only for a novel coupling $\zeta\eta^{-1} = \kappa\xi$. with the LV Maxwell action,

$$S_M = \int_{\mathbf{R}\times\Sigma_t} dt d^3x \sqrt{g} N \left[-\frac{2\eta}{N^2} \left(E_i + F_{ij} N^j \right)^2 + \zeta F_{ij} F^{ij} \right]$$

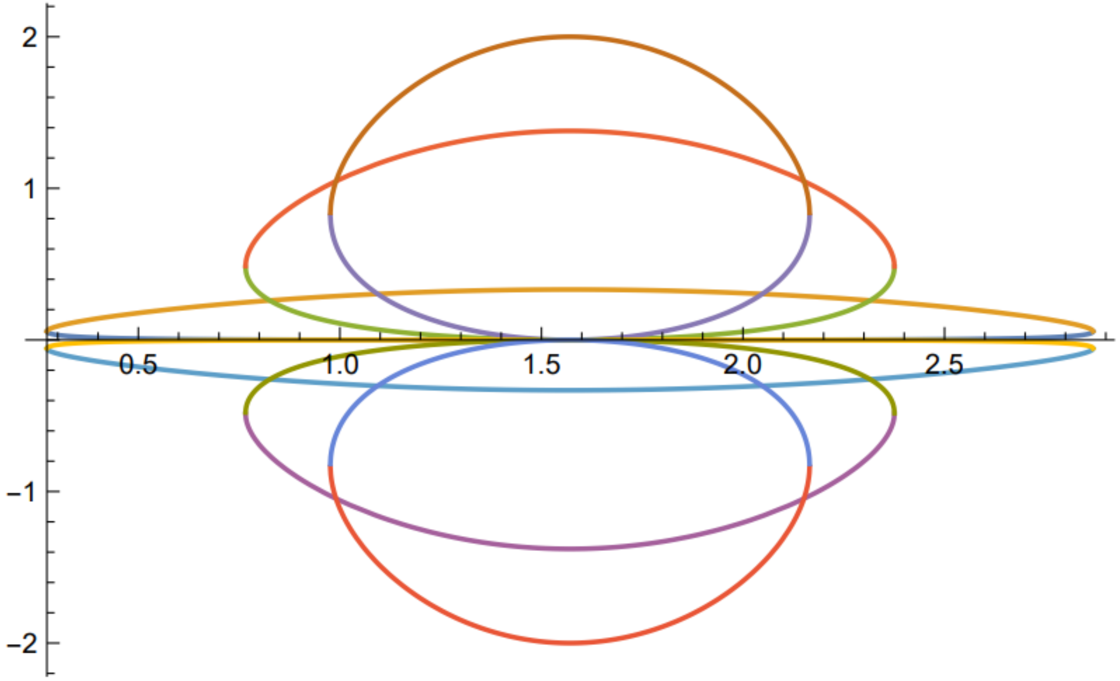
- This produces the same speed of graviton and light (em) $c_g = c_l = \sqrt{\kappa\xi}$
- In this case, the arrival delay of (+1.74 ± 0.05)s in the coincident gravitational waves (GW) and gamma rays (GW170817, GRB170817A)

$$-3 \times 10^{-15} c_l < (c_g - c_l) < 7 \times 10^{-16} c_l,$$

- does not mean $-3 \times 10^{-15} < \Delta c_g / c < 7 \times 10^{-16}$ with $\Delta c_l = 0$ (Here, $c_g = c + \Delta c_g,\ c_l = c + \Delta c_l$.)

7. Closed Time-like Curves (CTC)? (work in progress)

- CTC region is protected by the new singularity at $\Sigma^2 = 0$; Just the mere existence of LV coupling can protect CTC region!!
- Causality is saved in LV gravity, whereas it is not in GR that is Lorentz-invariant !!



8. Summary

- We obtained exact solutions of rotating black holes for a viable low-energy LV quantum gravity, i.e., Horava gravity.

- We use the two-step procedure with undetermined functions from the massless black hole solution as a seed solution.

- (Curvature) Singularity structure is different from GR (a ring singularity) and more interesting due to a new (time-like) singularity at $\Sigma^2 = 0,$: No CTC, i.e., No causality-violating region !!

- Black hole has Killing horizons and the laws of Black Hole thermodynamics (0^{th}, 1^{st}) are satisfied, even with LV terms!

- **The charged rotating solutions (with LV Maxwell action) exist only for a novel coupling such that the speed of graviton and light (EM) are the same, but not necessary to the same as the usual speed of light in vacuum! (cf. GW170817, GRB170817A)**

9. Open Problems

- 1. Uniqueness proof of our solutions ?

- 2. QNMs and Stability?

- 3. 4D Rotating BH solutions in UV complete Horava?

cf. 3D is known [Park (2013, 2020), Sotiriou et al (2014)]