Post-reheating Cosmology in a dilaton-Einstein-Gauss-Bonnet scenario of modified gravity

Stefano Scopel



Based on:

"<u>WIMPs in dilatonic Einstein Gauss-Bonnet cosmology</u>", Anirban Biswas, Arpan Kar, Bum-Hoon Lee, Hocheol Lee, Wonwoo Lee, S. S., Liliana Velasco-Sevilla and Lu Yin, JCAP 08 (2023) 024, e-Print: <u>2303.05813</u> [hep-ph]

"<u>Gauss-Bonnet Cosmology: large-temperature behaviour and bounds from Gravitational Waves</u>", Anirban Biswas, Arpan Kar, Bum-Hoon Lee, Hocheol Lee, Wonwoo Lee, Stefano Scopel, Liliana Velasco-Sevilla, Lu Yin, JCAP 09 (2024) 007, e-Print: 2405.15998 [hep-ph]



- hard to fit GR with other fundamental interactions
- naturalness problem (stabilitation of the Weak Scale under Quantum Corrections)

 \rightarrow both General Relativity (GR) and the Standard Model of Particle Physics believed to be incomplete

- a new era of precision tests of GR from discovery of Gravitational Waves (GW) and direct measurement
 of merger events of compact objects
- an effective approach to probe extensions of GR using observational data is the use of <u>effective</u> models

Horndeski's theory: most general scalar-tensor theory having equations of motion with second-order time derivatives in four-dimensional spacetime, in which the theory does not have a ghost state **Einstein-Gauss-Bonnet (EGB):** one of the simplest Horndeski's theories obtained by adding the Gauss-Bonnet term, a specific quadratic combination of the curvature \rightarrow topological invariant not affecting the dynamics, unless the Gauss-Bonnet term is non-minimally coupled to a scalar field

→ <u>dilatonic Einstein Gauss-Bonnet (dEGB) scenario</u>

Dilatonic Einstein Gauss-Bonnet (dEGB) scenario

$$S = \int_{\mathcal{M}} \sqrt{-g} \ d^4x \left[\frac{R}{2\kappa} - \frac{1}{2} \nabla_{\mu} \phi \nabla^{\mu} \phi - V(\phi) + \begin{array}{c} f(\phi) R_{\text{GB}}^2 + \mathcal{L}_m^{\text{rad}} \\ \\ \text{scalar field} \\ \text{coupled GB term} \end{array} \right], \qquad \kappa \equiv 8\pi G = 1/M_{PL}^2$$

non-minimal coupling encoded in the f(ϕ) function (if $\phi' \equiv df/d\phi = 0$ the GB term does not affect the dynamics). In the following:

$$f(\phi) = \alpha e^{\gamma \phi}$$

(exponential form arises within theories where gravity is coupled to the dilaton)

\rightarrow study the phenonology of WIMP decoupling in terms of the two free parameters α and γ

WIMP thermal decoupling

Boltzmann's equation

$$rac{dn}{dt} + 3Hn = - < \sigma v > ig(n^2 - n_{eq}^2ig)$$
 (Lee-Weinberg)

$$\frac{dY}{dx} = -\frac{xs}{H(T=m)} < \sigma v > \left(Y^2 - Y_{eq}^2\right)$$

x=m/T, m=WIMP mass, Y=n/s, n=WIMP number density, s=entropy density

$<\sigma v>$ =annihilation cross section time velocity of WIMP to SM particles

- at high temperature the WIMP number density follows its equilibrium value $n_{\rm eq}$
- when T<m n_{eq} is exponentially suppressed and the WIMP decouples from the plasma
- from that moment Y=n/s almost constant until today
- today get $n_0 = Y_0 s_0$ with s_0 the present entropy density



x_f ~20

decoupling happens at T~m/20 → Cold Dark Matter, i.e. the WIMP is non relativistic at decoupling and can start forming structures before baryons



Relic density for Dark Matter through thermal decoupling in non-standard Cosmology



 T_f T_f A>1 \rightarrow earlier decoupling at same $\langle \sigma v \rangle \rightarrow$ larger relic density

For a non-standard cosmology with A>1 the annihilation cross section $\langle \sigma v \rangle$ <u>needs to be larger</u> that the standard value 2.1e-26 cm³s⁻¹ to explain the data, corresponding to $\Omega h^2 \langle 0.1 \text{ for a standard cosmology}$

Assuming s-wave annihilation <ov> is the same in the early Universe and today





In a non-standard Cosmology the value of **<σv>** corresponding to the correct relic abundance can be driven beyond the observational limits from **DM indirect searches**



Friedman equation in dEGB Cosmology:

$$\begin{split} H^2 &= \frac{\kappa}{3} \left(\frac{1}{2} \dot{\phi}^2 + V - 24 \dot{f} H^3 + \rho_{\rm rad} \right) \,, \\ \dot{H} &= -\frac{\kappa}{2} \left(\dot{\phi}^2 + 8 \frac{d(\dot{f} H^2)}{dt} - 8 \dot{f} H^3 + \rho_{\rm rad} + p_{\rm rad} \right) \\ \ddot{\phi} &+ 3H \dot{\phi} + V' - 24 f' H^2 (\dot{H} + H^2) = 0 \,. \end{split}$$

Continuity equations:

$$\dot{\rho}_{\rm rad} + 3H(\rho_{\rm rad} + p_{\rm rad}) = 0 \qquad \mbox{(radiation)}$$

$$\dot{\rho}_{\{\phi+{\rm GB}\}} + 3H\left(\rho_{\{\phi+{\rm GB}\}} + p_{\{\phi+{\rm GB}\}}\right) = 0 \qquad \mbox{(scalar field + GB)}$$

N.B. Identify the term

$$\rho_{\rm GB} = -24\dot{f}H^3 = -24f'\dot{\phi}H^3 = -24\alpha\gamma e^{\gamma\phi}\dot{\phi}H^3.$$

as an energy density, although it can be negative (plot absolute value). Of course the total density of the Universe $\rho = \rho_{\phi} + \rho_{rad} + \rho_{GB}$ remains positive.

We evolve the Friedmann equations numerically from Big Bang Nucleosinthesis to high temperatures - WIMP phenomenology is affected for T[~]m/20. For a WIMP mass 10 GeV<m<1 TeV this corresponds to <u>500 MeV<T<50 GeV</u>.

Boundary conditions at $T_{BBN} \simeq 1$ MeV:

- $\dot{\phi}_{\text{BBN}} \equiv \dot{\phi}(T_{BBN})$ so that $\rho_{\phi,\text{BBN}} \equiv \frac{1}{2} (\dot{\phi}_{\text{BBN}})^2 \leq 3 \times 10^{-2} \rho_{\text{rad},\text{BBN}}$ (from N_{eff} = 2.99 ± 0.17, radiation must dominate at T_{BBN} not to spoil BBN) $(\dot{\phi}_{\text{BBN}} \rightarrow - \dot{\phi}_{\text{BBN}}$ equivalent to $\gamma \rightarrow -\gamma$, can fix $\dot{\phi}_{\text{BBN}} > 0$ and study both signs of γ)
- fixing ϕ_{BBN} corresponds to a redefinition of the α parameter (gauge) in the dEGB function:

$$\phi'_{\rm BBN} = \phi_{\rm BBN} + \phi_0, \ \alpha' = \alpha e^{-\gamma \phi_0}, \ \gamma' = \gamma$$

so, without generality, can choose the gauge $\phi_{BBN} = 0$ or, equivalently, can express the results in terms of the gauge-invariant parameter:

$$\tilde{\alpha} = \alpha \, e^{\gamma \phi_{\rm BBN}}$$

Example of evolutions of energy densities and equations of state for $\dot{\phi}_{BBN} = 0$ (in this case the results are invariant when $\gamma \rightarrow \gamma$)



energy densities



 \widetilde{lpha} =-1 km²

 $\gamma = 1$

 $\tilde{\alpha}$ =1 km²

Example of evolutions of energy densities and equations of state for $\dot{\phi}_{\rm BBN}$ >0

energy densities

equation of state



N.B. for f'(ϕ)=0 (no GB term, only kination) WIMP indirect detection already excludes $\rho_{\phi,BBN} = 3 \times 10^{-2} \rho_{rad,BBN}$ for any WIMP mass m



what is the effect of the dEGB term?







 $\tilde{\alpha}$ (km²)

white regions: excluded by WIMPs; shaded regions: excluded by GW from compact-binary mergers; non-white regions: allowed; A. Biswas, A. Kar, B.H. Lee, H. Lee, W. Lee, S. S., L. Velasco-Sevilla and L. Yin, JCAP 08 (2023) 024

Enhancement factor A=H/H_{GR}



A can be very large (10⁵), however the corresponding enhancement of $\langle \sigma v \rangle$ is more moderate because annihilations can remain sizeable after freeze-out when $\Gamma/H^{T^{3-\alpha}}$ and $\alpha > 3$ (*F*. D'Eramo, N. Fernandez and S. Profumo, JCAP 05 (2017) 012, [1703.04793]).

- the presence of the dEGB term slows down the scalar field evolution when $w \rightarrow -1/3$ (non-accelerated Cosmology, "slow ϕ roll") recovering allowed configurations when $\tilde{\alpha}$, $\gamma < 0$ and $\tilde{\alpha} * \gamma < 0$

in spite of a high degree of non linearity and phenomenological complexity at low temperatures, at large-enough temperatures dEGB exhibits only <u>very few</u> <u>asymptotic behaviours</u> – attractor solutions?

- w=1
- w=-1/3
- 1<w<7/3



Introduce the following non-dimensional variables:

$$\begin{split} x &\equiv \frac{\rho_{\phi}}{\rho} = \frac{\kappa}{6} \left(\frac{\dot{\phi}}{H}\right)^{2}, \\ y &\equiv \frac{\rho_{\rm rad}}{\rho} = \frac{\kappa g_{*} \pi^{2}}{90} \frac{T^{4}}{H^{2}}, \\ z &\equiv \frac{\rho_{\rm GB}}{\rho} = -8\kappa \dot{f}H = -8\kappa \frac{\partial f}{\partial \phi} \dot{\phi}H \end{split}$$

with the constraint:

$$x + y + z = 1$$

can express the Friedmann equations in terms of two independent variables (x and z):

$$\begin{aligned} x' &= 2 \left[\epsilon(x,z) - 3 \right] x + \left[\epsilon(x,z) - 1 \right] z \equiv F(x,z), & \text{autonomous equations} \\ z' &= 2x + \left[\epsilon(x,z) - 3 \right] z + 2 \left[2 - \epsilon(x,z) \right] \equiv G(x,z) \end{aligned}$$

$$(' &= \frac{d}{dN} = \frac{d}{d\ln a} = \frac{1}{H} \frac{d}{dt}) & \text{N=\# of e-foldings ("time" parameter)} \\ \epsilon &\equiv -\frac{\dot{H}}{H^2} = 1 + q \quad \text{and} \quad \frac{4x^2 + 8x + z^2 + 2\sqrt{\frac{6}{\kappa}} \text{sign}(\alpha) |\gamma| |z| x^{3/2}}{4x - 4xz + z^2} \\ q &= -\frac{da}{a} = \text{deceleration parameter} \end{aligned}$$

N.B. the ϵ parameter is directly related to the equation of state:

$$w = \frac{2}{3}\epsilon - 1$$

A. Biswas, A. Kar, Bum-Hoon Lee, Hocheol Lee, Wonwoo Lee, S. S., L. Velasco-Sevilla, Lu Yin, JCAP 09 (2024) 007

N)

the two equations describe a velocity field in the x-z plane

critical points: x'=z'=0

three types of critical points:

- stable \rightarrow attractors
- unstable \rightarrow saddle points
- repelling nodes

In a stable critical point (attractor) at $N \rightarrow -\infty$ the differential equations saturate at $x, z \rightarrow x_c, z_c$ and so also the equation of state since $\varepsilon = \varepsilon(x_c, z_c)$

this explains the asymptotic behaviours at large T

A. Biswas et al., JCAP 09 (2024) 007



Critical points for finite x,z:



$$x_c = \frac{\epsilon - 1}{5 - \epsilon},$$
$$z_c = -2\frac{\epsilon - 3}{5 - \epsilon}$$

(implicit, ε depends on x,z)

substituting back x_c, z_c in $\varepsilon(x, z)$:

$$(\epsilon - 1)(\epsilon - 3)\left(\sqrt{\frac{6}{\kappa}}|\gamma|\operatorname{sgn}(\alpha z_c)\sqrt{\frac{\epsilon - 1}{5 - \epsilon}} + 2\epsilon\right) = 0,$$

three solutions:

• *ε*=1 , (x_c, z_c)=(1,0) (kination)

always present, do not depend on the parameters α and γ

- *ε*=3 , (x_c,z_c)=(0,1) (Gauss Bonnet)
- $(x_c, z_c) = (x_{fp}, z_{fp})$ solution of cubic equation:

$$2\epsilon^3 - 10\epsilon^2 + \frac{3}{\kappa}\gamma^2\epsilon - \frac{3}{\kappa}\gamma^2 = 0$$

with $x_{fp}+z_{fp} = 1$. Only when sign(αz_c)<0, which implies:

- $|\gamma| < \sqrt{6\kappa}$ when $\alpha > 0$ (in this case $z_{fp} < 0$ and $\epsilon > 3$
- $|\gamma| > \sqrt{6\kappa}$ when $\alpha < 0$ (in this case $z_{fp} > 0$ and $\varepsilon < 3$

A. Biswas et al., JCAP 09 (2024) 007

Critical points at infinity:

expand $\varepsilon(x,z)$ in the leading term in $O(x/z) \rightarrow 0$ ("slow-roll" regime):

$$\epsilon = \frac{1 + \mathcal{O}(\frac{x^{3/2}}{z})}{1 - 4\frac{x}{z} + \mathcal{O}(\frac{x}{z^2})} \simeq 1 + 4\frac{x}{z}$$
$$\Rightarrow \quad (\epsilon - 1)z \to 4x,$$

substituting back in the differential equation one gets x'=0.

N.B. combining x'=0 with x'+y'+z'=0 implies y'+z'=(y+z)'=0However, the evolution of radiation is : $y'=-2y\neq 0$ This implies that $y'=-z'\neq 0$ with a <u>large cancelation</u> between y and z

In this regime $\epsilon \rightarrow 1$ and w=-1/3, so the density of the Universe evolves as $\rho \sim T^2$. However the density of radiation is still evolving as as $\rho_{rad} \sim T^4$. This implies that also $\rho_{GB} \sim T^4$

$\rho^{\sim}\rho_{rad}+\rho_{GB}\rightarrow T^{4}-T^{4}=T^{2}$

Schematic view of the critical points:

w=-1/3, Gauss Bonnet (0,1)

w=1, kination (1,0)

1<w<7/3, fast roll (x_{fp}, z_{fp}) only for sign (αz_{fp}) <0

w=-1/3, slow roll (0,-∞)



A. Biswas et al., JCAP 09 (2024) 007

Stability of critical points

Need to study the sign of the eigenvalues of the Jacobian of the differential equations in the vicinity of the critical point:

expand at linear order close to the critical point:

$$x' = F(x, z) \simeq F(x_c, z_c) + \frac{\partial F}{\partial x}(x - x_c) + \frac{\partial F}{\partial z}(z - z_c)$$
$$z' = G(x, z) \simeq G(x_c, z_c) + \frac{\partial G}{\partial x}(x - x_c) + \frac{\partial G}{\partial z}(z - z_c)$$
$$X' = J(X - X_c)$$

with:

$$J = \begin{pmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial z} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial z} \end{pmatrix}_{(x,z)=(x_c, z_c)}$$













Ζ (0,1) γ>0, z_{BBN}<0 γ<0, z_{BBN}>0 (8) 8 (1,0) Х +x, (0,-∞)

 $\tilde{\alpha} > 0 \quad |\gamma| > \sqrt{6}$

 $\gamma < 0$ $T = 10^{16} \text{ GeV}, \rho_{\phi}(T_{BBN}) = 10^{-4} \rho_{BBN}$ $\gamma < 0$ $|\gamma| > \sqrt{6\kappa}$ $|\gamma| < \sqrt{6\kappa}$ $|\gamma| < \sqrt{6\kappa}$

α <0

 $|\gamma| > \sqrt{6\kappa}$

-2

0

 $\tilde{\alpha}$ (km²)

2

α >0

8

6

4

- 3.5

- 3.0

- 2.5

- 2.0

1.5

- 1.0

- 0.5

- 0.0

-0.5

10

8

≥

A. Biswas, A. Kar, Bum-Hoon Lee, Hocheol Lee, Wonwoo Lee, S. S., L. Velasco-Sevilla, Lu Yin, JCAP 09 (2024) 007

 $\gamma < 0$

-4

-6

-8

-10

-10

Stochastic background of gravitational waves

 $\eta(\hat{k},T)$

- Any plasma of relativistic particles in thermal equilibrium emits a stochastic background of gravitational • waves, which in the case of the Standard Model is expected to peak at a frequency of around 80 GHz
- present detectors are only sensitive to frequencies of the order of few Hertz, some proposals exist to extend the experimental reach to the GHz range
- such background can be presently constrained using BBN bounds

$$\frac{1}{a^4} \frac{d}{dt} \left(a^4 \rho_{\rm GW}(t) \right) = \left(\frac{\partial}{\partial t} + 4H \right) \rho_{\rm GW}(t) = 4 \frac{T^4}{M_{\rm PL}^2} \int \frac{d^3k}{(2\pi)^3} \eta(T,k)$$

evolution of GW density
according to the
Cosmological model
$$\eta(\hat{k},T) = \begin{cases} \frac{1}{8\pi} \frac{16}{g_1^4 \ln(5T/m_{D_1})}, & k \lesssim \alpha_1^2 T, \\ \eta_{\rm HTL}(\hat{k},T) + \eta^T(\hat{k},T), & k \gtrsim 3T, \end{cases}$$

$$\hat{m}_{D_i} = \frac{m_{D_i}}{T}, \quad m_{D_i}^2 = \begin{cases} \frac{d_1 \frac{11}{16} g_1^2 T^2, \, d_1 = 1, \\ d_2 \frac{11}{16} g_2^2 T^2, \, d_2 = 3 \\ d_3 2 g_3^2 T^2, \, d_3 = 8. \end{cases}$$

$$\Omega_{\rm GW}(f,T_0)h^2 = \Omega_{\gamma_0}h^2 \frac{\lambda}{M_{\rm PL}} \int_{T_{\rm EWCO}}^{T_{\rm Max}} dT \left(\frac{g_{*0}}{g_{*}(T)}\right)^{4/3} T^2 \hat{k}(f,T)^3 \frac{\eta(\hat{k},T)}{\sqrt{\rho(T)}} \beta(T)$$

For a plasma of standard model particles GW production at all epochs is peaked at:

$$\frac{k}{T} = \hat{k}(f,T) = \left[\frac{g_{*s}(T)}{g_{*s}(T_0)}\right]^{\frac{1}{3}} \frac{2\pi f}{T_0} ~~3.92$$

A. Ringwald et al., JCAP 03 (2021), 054



$$\Omega_{\rm GW}(f,T_0)h^2 = \Omega_{\gamma_0}h^2 \frac{\lambda}{M_{\rm PL}} \int_{T_{\rm EWCO}}^{T_{\rm Max}} dT \left(\frac{g_{*0}}{g_{*}(T)}\right)^{4/3} T^2 \hat{k}(f,T)^3 \frac{\eta(\hat{k},T)}{\sqrt{\rho(T)}} \beta(T)$$

- ultra-violet dominated, i.e. the integral is driven by the contribution of the GWs emitted at high temperature
- dominated by the highest temperature where the energy density of the Universe is driven by the plasma of relativistic particles
- In the SM this is usually assumed all the way up to the reheating temperature of the Universe in such case the signal is diluted as a⁻⁴ (scale factor)
- However, in the case of the slow-roll asymptotic solution (w=-1/3) found in GB Cosmology, one has:

$\rho \sim \rho_{rad} + \rho_{GB} \rightarrow T^4 - T^4 = T^2$

radiation domination at high temperatures <u>but dilution as $a^{-2} \rightarrow$ enhanced signal compared to SM!</u>

Bounds on T_{max}~ reheating temperature (cyan/blue) or maximal temperature of GW production (red)



hatched regions: GW bounds from BH-BH and BH-NS mergers

- In Standard Cosmology the GW stochastic background is never at the level of the BBN bounds, even for T_{RH}~10¹⁶ GeV.
- On the other hand, in GB Cosmology BBN puts sensible bounds on $T_{RH} \simeq 10^8 10^9 \text{ GeV} \ll 10^{16} \text{ GeV}$ whenever w=-1/3 ("slow-roll" solutions)
- complementarity between bounds from stochastic background and those from BH-BH and BH-NS mergers

Constraints from Gravitational Waves from BH-BH and BH-NS merger events at T=T_L

(Z. Lyu, N. Jiang and K. Yagi, Phys. Rev. D 105 (2022) 064001 [2201.02543])

TABLE I. Astrophysical bounds on EdGB gravity. We show bounds from a LMXB, NSs (~2 M_{\odot} NSs), GWs from BBHs, and NSBHs (this work). The one in brackets comes from GW190814 assuming that it is a BBH, which has some uncertainty. For NSBH, we present the bound from GW200115 and that by combining NSBHs (GW200115, GW200105, and GW190814; assuming the last one as a NSBH is a conservative choice) and BBHs from [26].

		,	с и			
			GW (BBH)		GW (NSBH) (this work)	
	LMXB	NS	01–02	01–03	GW200115	Combined
$\sqrt{\alpha_{\rm GB}}$ (km)	1.9 [30]	1.29 [45]	5.6 [25], 1.85 [46], 4.3 [47]	1.7 [26], 4.5 [48], (0.4) [48]	1.33	1.18

(linearized perturbation close to the system)

$$f(\phi) = f(\phi(T_L)) + f'(\phi(T_L)) \Delta \phi + \mathcal{O}((\Delta \phi)^2)$$
$$|f'(\phi(T_L))| \le \sqrt{8\pi} \,\alpha_{\rm GB}^{\rm max}$$

N.B. Residual evolution of ϕ after BBN can be non-zero for "fast-roll" solutions (modify cosmological value f($\phi(T_L)$). In this case the value of $\phi(T_L)$ can be displaced compared to ϕ_{RRN} :

$$\phi(T_L) \simeq \phi_{\rm BBN} + \frac{\dot{\phi}_{\rm BBN}}{H_{\rm BBN}}$$

$\tilde{\alpha}$ =-1 km²

γ**=**-1

γ=+1



Conclusions

we analized WIMP relic density and indirect signals in GB Cosmology

- the simple kination case (only scalar field) is already disfavored by WIMP indirect searches unless $\dot{\phi}_{\rm BBN}$
- the dEGB term is even more disfavoured for $\tilde{\alpha}$, $\gamma > 0$, when w=7/3
- however, the presence of the dEGB term can slow down the scalar field evolution when $w \rightarrow -1/3$ (non-accelerated Cosmology, "slow ϕ roll") recovering allowed configurations when $\tilde{\alpha}$, $\gamma < 0$ and $\tilde{\alpha} * \gamma < 0$
- bounds from WIMPs complementary to those from GW from compact-binary mergers
 we also studied the equation of state of the Universe at high temperatures in GB
 Cosmology
- only few asymptotic equations of state at high temperature: w=1 (kination), w=-1/3 (slow roll), 1<w<7/3 (fast roll)
- clear interpretation in terms of three attractors (stable critical points) of a set of autonomous differential equations
- in the "slow-roll" w=-1/3 regime the GW stochastic backround from the plasma of relativistic particles is enhanced compared to standard Cosmology and reaches the level of the BBN bound for T_{RH} ~10⁸-10⁹ GeV