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Machine Learning Riemannian Metrics with Special Holonomy

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Recent advances in machine learning are giving pure mathematicians powerful "numerical laboratories." Geometric deep networks, reinforcement-learning agents, and large-language-model copilots now verify intricate calculations, uncover hidden symmetries, and suggest plausible conjectures in complex differential and algebraic geometry.

This tutorial demonstrates how these tools can be combined into end-to-end pipelines that numerically approximate Riemannian metrics with special holonomy. Building on neural-network approximations of Ricci-flat metrics on Calabi–Yau manifolds, we turn to G_{2}-geometry as a working example. A torsion-free G_{2}-structure—specified by a special 3-form on a 7-manifold—yields a metric whose holonomy lies in G_{2}, yet explicit examples remain rare and difficult to obtain. Focusing on 7-manifolds arising as Calabi–Yau links, we present exploratory ML pipelines that probe the geometry and topology of G_{2}-structures and could offer data-driven clues for locating torsion-free cases.

Finally, we argue that the same methodology can serve as a proof of concept for applying machine learning to investigate other H-structures — reductions of the frame bundle to any closed, connected subgroup $H \subset \mathrm{SO}(n)$—on oriented n-manifolds whose topology admits such reductions.