Theoretical Tools for Quantum Batteries:

Basic Concepts and Their Applications

Lecture 4

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Lecture 4: Our recent results

I. Dephasing enabled fast charging of quantum batteries

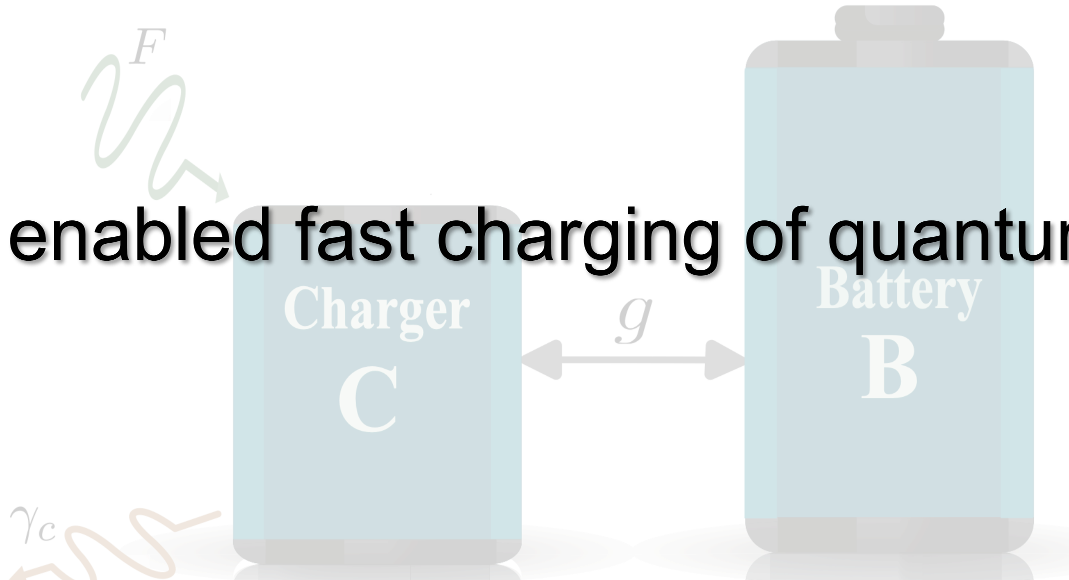
Shastri, Jiang, Xu, Venkatesh & GW, npj Quantum Inf. **11**, 9 (2025)

Purkait, Venkatesh & GW, arXiv:2508.13497 (2025)

II. Collectively enhanced charging via QHEs

Ito, Purkait, GW, arXiv:2008.07089 (2020)

Dephasing enabled fast charging of quantum batteries



Shastri, Jiang, Xu, Venkatesh & GW, npj Quantum Inf. **11**, 9 (2025)

Purkait, Venkatesh & GW, arXiv:2508.13497 (2025)

Collaborators

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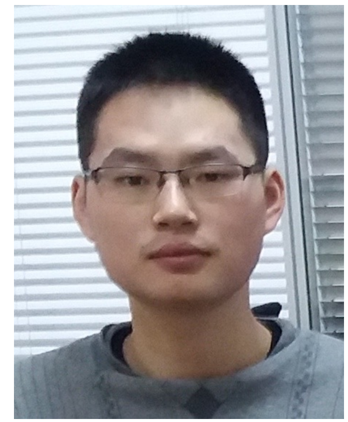

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Dr. Chayan Purkait

Shastri, Jiang, Xu, Venkatesh & GW, npj Quantum Inf. **11**, 9 (2025)

Purkait, Venkatesh & GW, arXiv:2508.13497 (2025)

Quantum coherence: Nonzero off-diag. elements of $\hat{\rho}$ in $\widehat{H}$-basis.

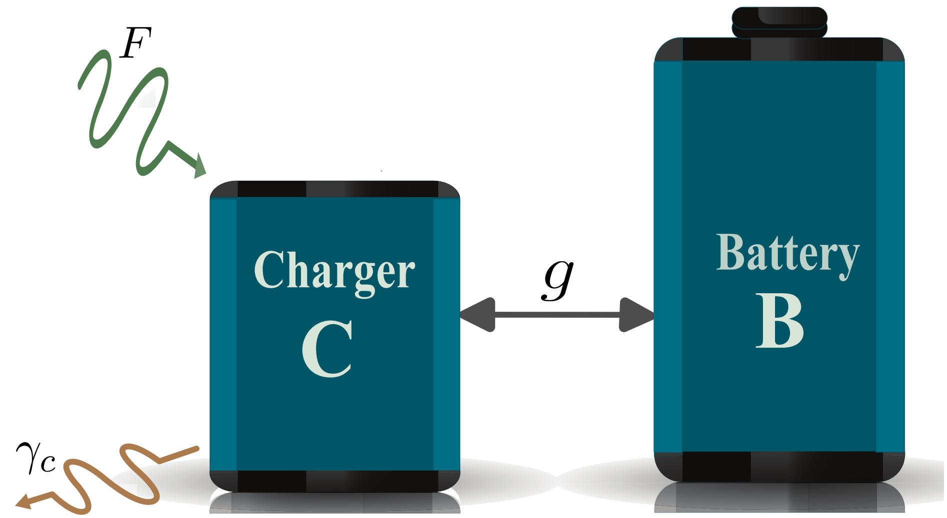
Originated from superposition of energy eigenst.

Such superposition is easy to destroyed by the interaction with environment.

Decoherence
- Dissipation: Involving energy exchange with environment.
- Dephasing: Without energy exchange with environment.

Open models of quantum batteries.

Farina *et al*., PRB **99**, 035421 (2019)
Saha *et al*., arXiv:2309.15634 (2023)
Shaghaghi *et al*., Entropy **25**, 430 (2023)



Usually, decoherence is "thing to avoid".

Decoherence (dephasing) as a resource?

Shastri, Jiang, Xu, Venkatesh & GW, npj Quantum Inf. **11**, 9 (2025)

Dephasing: Decay of off-diagonal elements in $\hat{H}$-basis.

(coherence: nonzero off-diag. elements in $\hat{H}$-basis.)

Example: For a TLS

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|e\rangle + e^{i\theta}|g\rangle\right) \qquad \{|e\rangle, |g\rangle\} : \text{ basis of } \hat{H}$$

$$\hat{\rho} = |\psi\rangle\langle\psi| = \frac{1}{2}\begin{pmatrix} 1 & e^{-i\theta} \\ e^{i\theta} & 1 \end{pmatrix} \quad : \text{pure st.}$$

Dephasing →

$$\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad : \quad \text{statistical mix.} \begin{cases} |e\rangle & 50\% \\ |g\rangle & 50\% \end{cases}$$

$$\langle \hat{H}^n \rangle \equiv \mathrm{Tr}\left[\hat{\rho}\, \hat{H}^n\right] \text{ unchanged.}$$

Dephasing

Dephasing: Decay of off-diagonal elements in $\hat{H}$-basis.

(coherence: nonzero off-diag. elements in $\hat{H}$-basis.)

Corresponding jump op. $\hat{L} \propto \hat{H} \quad \left(\hat{L} = \hat{H}/\varepsilon\right)$.

$$\frac{d\hat{\rho}}{dt} = -i\left[\hat{H}, \hat{\rho}\right] + \frac{\tilde{\Gamma}}{2}\left(2\hat{H}\hat{\rho}\hat{H}^{\dagger} - \hat{H}^{\dagger}\hat{H}\hat{\rho} - \hat{\rho}\hat{H}^{\dagger}\hat{H}\right) \qquad \left(\tilde{\Gamma} \equiv \Gamma/\varepsilon^2\right)$$

$$\hat{\rho}(t) = \sum_{i,j} \rho_{ij}(t)\left|\varepsilon_i\right\rangle\left\langle\varepsilon_j\right|$$

$$\frac{d\rho_{ij}}{dt} = -i\left(\varepsilon_i - \varepsilon_j\right)\rho_{ij} - \frac{\tilde{\Gamma}}{2}\left(\varepsilon_i - \varepsilon_j\right)^2 \rho_{ij}$$

→ off-diag. $(i \neq j)$: $\rho_{ij}(t) \propto \rho_{ij}(0)\, e^{-\frac{\tilde{\Gamma}}{2}\left(\varepsilon_i - \varepsilon_j\right)^2 t}$

diag. $(i = j)$: $\rho_{ii}(t) = \rho_{ii}(0)$

Master eq. describing dephasing:

$$\frac{d\hat{\rho}(t)}{dt} = \frac{\gamma}{2}\left(2\hat{L}\hat{\rho}(t)\hat{L} - \left\{\hat{L}^2, \hat{\rho}(t)\right\}\right) \quad \text{with} \quad \left[\hat{L}, \hat{H}\right] = 0$$

For a TLS with $\hat{H} = \omega\hat{\sigma}^+\hat{\sigma}^-$

$\hat{L} = \hat{\sigma}^+\hat{\sigma}^-$

Example:

1. Continuous weak energy measurement

   Initial st. with coherence reduces to one of the energy eigenst.

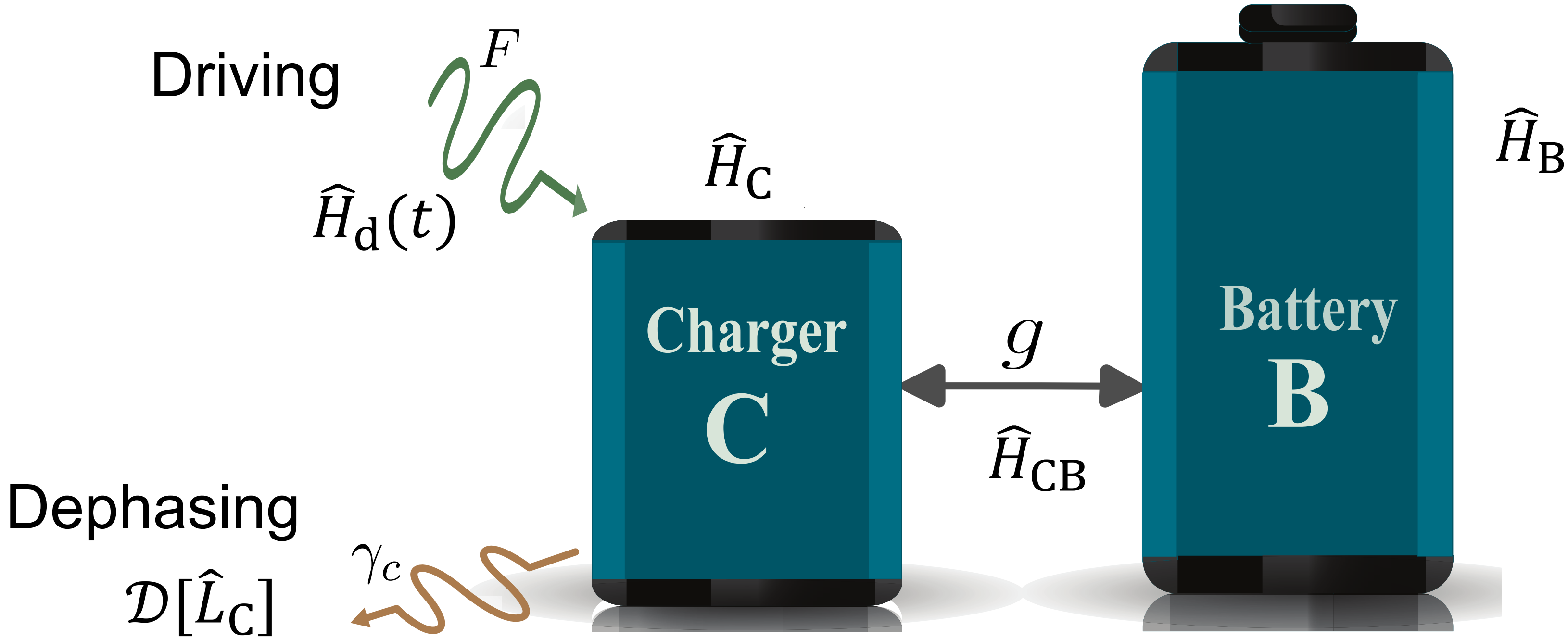
2. Spins under a noisy magnetic field

   $\hat{H}'(t) = B(t)\,\hat{\sigma}_z$ with $B(t) = \sqrt{\gamma/2}\,\xi(t)$

   Relative phase in the initial superposition is randomized stochastically.

   Ensemble avr. of $\hat{\rho} = \frac{1}{2}\begin{pmatrix} 1 & e^{-i\theta} \\ e^{i\theta} & 1 \end{pmatrix}$ with random θ.

General setup



$$[\hat{L}_\mathrm{C}, \hat{H}_\mathrm{C}] = 0$$

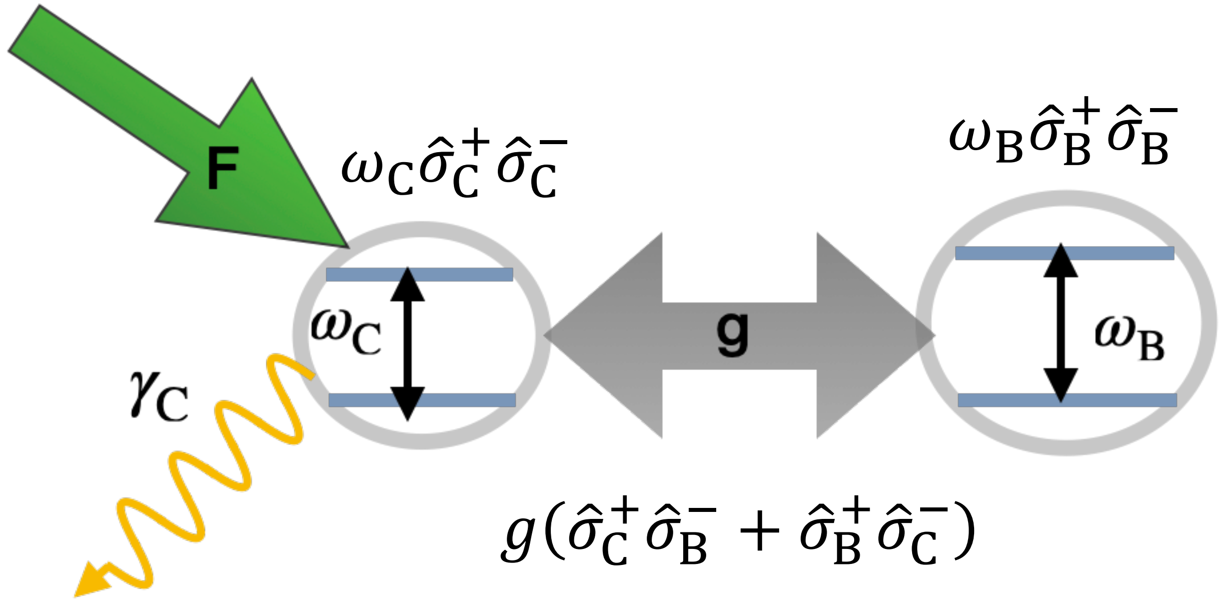
See also:
Farina *et al*. PRB **99**, 035421 (2019)
Saha *et al*. arXiv:2309.15634 (2023)

$$\hat{H}(t) = \hat{H}_\mathrm{C} + \hat{H}_\mathrm{d}(t) + \hat{H}_\mathrm{B} + \hat{H}_\mathrm{CB}$$

$$\frac{d\hat{\rho}(t)}{dt} = -i[\hat{H}, \hat{\rho}(t)] + \frac{\gamma_C}{2}\left(2\hat{L}_\mathrm{C}\hat{\rho}(t)\hat{L}_\mathrm{C} - \{\hat{L}_\mathrm{C}^2, \hat{\rho}(t)\}\right)$$

$$\equiv -i[\hat{H}, \hat{\rho}(t)] + \mathcal{D}[\hat{L}_\mathrm{C}]\,\hat{\rho}(t)$$

$$F\left(\hat{\sigma}_{\mathrm{C}}^{-} e^{i\omega_{\mathrm{d}} t} + \hat{\sigma}_{\mathrm{C}}^{+} e^{-i\omega_{\mathrm{d}} t}\right)$$



$$g\left(\hat{\sigma}_{\mathrm{C}}^{+}\hat{\sigma}_{\mathrm{B}}^{-} + \hat{\sigma}_{\mathrm{B}}^{+}\hat{\sigma}_{\mathrm{C}}^{-}\right)$$

$$\mathcal{D}[\hat{L}_{\mathrm{C}}]$$

$$\hat{L}_{\mathrm{C}} = \hat{\sigma}_{\mathrm{C}}^{+}\hat{\sigma}_{\mathrm{C}}^{-} = (\mathbb{I} + \hat{\sigma}_{\mathrm{C}}^{z})/2$$

initial st.: $\hat{\rho}(0) = |g\rangle\langle g|_{\mathrm{B}} \otimes |g\rangle\langle g|_{\mathrm{C}}$

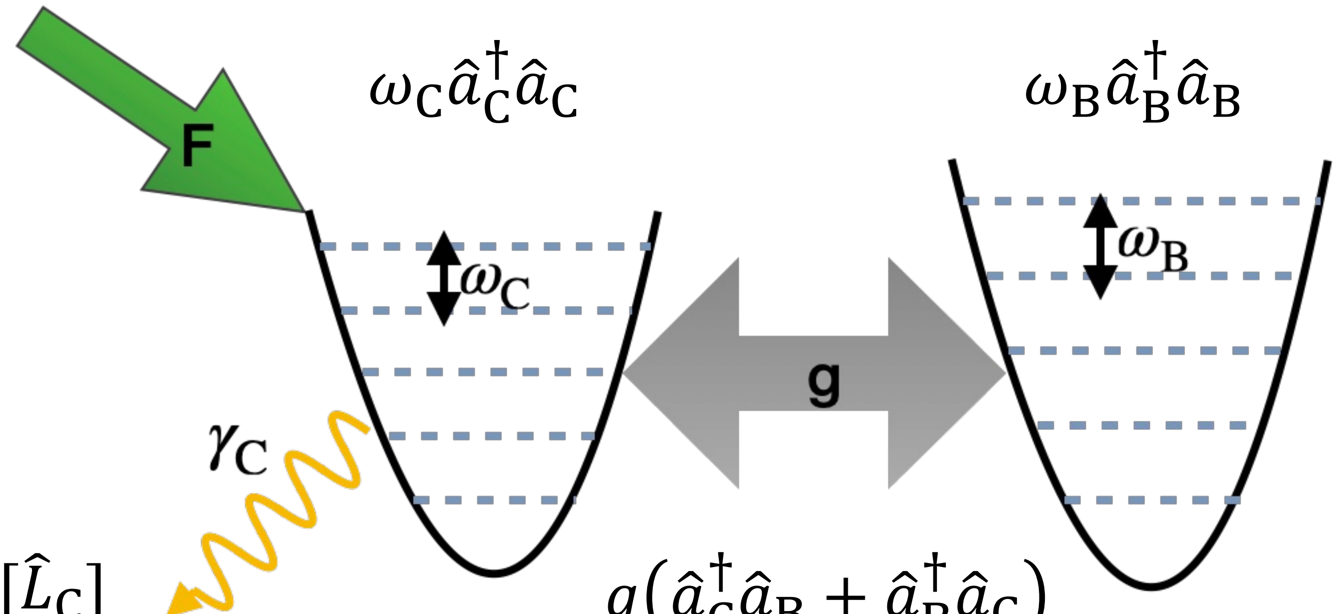
$$\hat{H} = \omega_{\mathrm{C}}\hat{\sigma}_{\mathrm{C}}^{+}\hat{\sigma}_{\mathrm{C}}^{-} + \omega_{\mathrm{B}}\hat{\sigma}_{\mathrm{B}}^{+}\hat{\sigma}_{\mathrm{B}}^{-} + g\left(\hat{\sigma}_{\mathrm{C}}^{+}\hat{\sigma}_{\mathrm{B}}^{-} + \hat{\sigma}_{\mathrm{B}}^{+}\hat{\sigma}_{\mathrm{C}}^{-}\right)$$
$$+F\left(\hat{\sigma}_{\mathrm{C}}^{-} e^{i\omega_{\mathrm{d}} t} + \hat{\sigma}_{\mathrm{C}}^{+} e^{-i\omega_{\mathrm{d}} t}\right)$$

$$\omega_{\mathrm{C}} = \omega_{\mathrm{B}} \implies \left[\hat{H}_{\mathrm{C}} + \hat{H}_{\mathrm{B}}, \hat{H}_{\mathrm{CB}}\right] = 0$$

Harmonic oscillators (HOs)

$F\left(\hat{a}_{\mathrm{C}} e^{i\omega_{\mathrm{d}} t} + \hat{a}_{\mathrm{C}}^{\dagger} e^{-i\omega_{\mathrm{d}} t}\right)$

$\omega_{\mathrm{C}} \hat{a}_{\mathrm{C}}^{\dagger} \hat{a}_{\mathrm{C}}$ $\omega_{\mathrm{B}} \hat{a}_{\mathrm{B}}^{\dagger} \hat{a}_{\mathrm{B}}$



γ_{C}

$\mathcal{D}[\hat{L}_{\mathrm{C}}]$

$g\left(\hat{a}_{\mathrm{C}}^{\dagger} \hat{a}_{\mathrm{B}} + \hat{a}_{\mathrm{B}}^{\dagger} \hat{a}_{\mathrm{C}}\right)$

$\hat{L}_{\mathrm{C}} = \hat{a}_{\mathrm{C}}^{\dagger} \hat{a}_{\mathrm{C}}$

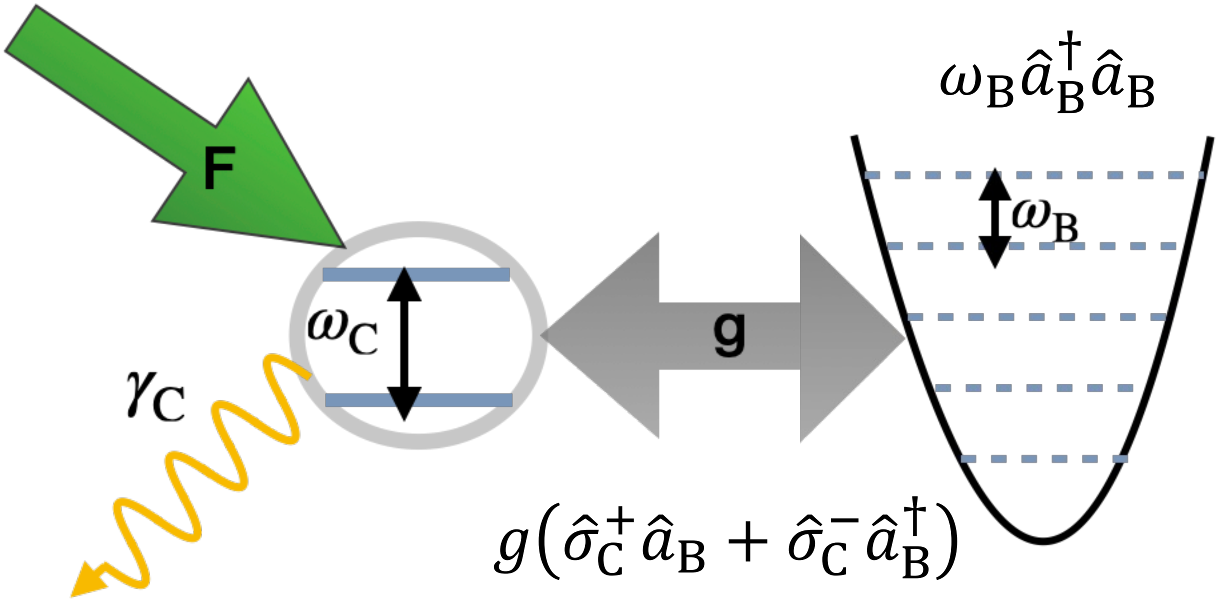
$$\begin{aligned}\widehat{H} = {} & \omega_{\mathrm{C}} \hat{a}_{\mathrm{C}}^{\dagger} \hat{a}_{\mathrm{C}} + \omega_{\mathrm{B}} \hat{a}_{\mathrm{B}}^{\dagger} \hat{a}_{\mathrm{B}} + g\left(\hat{a}_{\mathrm{C}}^{\dagger} \hat{a}_{\mathrm{B}} + \hat{a}_{\mathrm{B}}^{\dagger} \hat{a}_{\mathrm{C}}\right) \\ & + F\left(\hat{a}_{\mathrm{C}} e^{i\omega_{\mathrm{d}} t} + \hat{a}_{\mathrm{C}}^{\dagger} e^{-i\omega_{\mathrm{d}} t}\right)\end{aligned}$$

$$\omega_{\mathrm{C}} = \omega_{\mathrm{B}} \implies \left[\widehat{H}_{\mathrm{C}} + \widehat{H}_{\mathrm{B}}, \widehat{H}_{\mathrm{CB}}\right] = 0$$

Hybrid: TLS−HO

$$F\left(\hat{\sigma}_{\mathrm{C}}^{-}e^{i\omega_{\mathrm{d}}t}+\hat{\sigma}_{\mathrm{C}}^{+}e^{-i\omega_{\mathrm{d}}t}\right)$$

$$\omega_{\mathrm{B}}\hat{a}_{\mathrm{B}}^{\dagger}\hat{a}_{\mathrm{B}}$$



$$\mathcal{D}[\hat{L}_{\mathrm{C}}]$$

$$g\left(\hat{\sigma}_{\mathrm{C}}^{+}\hat{a}_{\mathrm{B}}+\hat{\sigma}_{\mathrm{C}}^{-}\hat{a}_{\mathrm{B}}^{\dagger}\right)$$

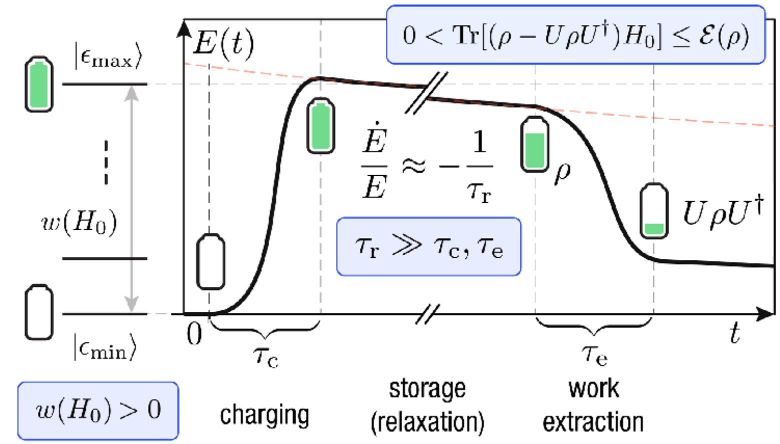
$$\hat{L}_{\mathrm{C}}=\hat{\sigma}_{\mathrm{C}}^{+}\hat{\sigma}_{\mathrm{C}}^{-}=(\mathbb{I}+\hat{\sigma}_{\mathrm{C}}^{z})/2$$

$$\begin{aligned}\hat{H}=\omega_{\mathrm{C}}\hat{\sigma}_{\mathrm{C}}^{+}\hat{\sigma}_{\mathrm{C}}^{-}+\omega_{\mathrm{B}}\hat{a}_{\mathrm{B}}^{\dagger}\hat{a}_{\mathrm{B}}+g\left(\hat{\sigma}_{\mathrm{C}}^{+}\hat{a}_{\mathrm{B}}+\hat{\sigma}_{\mathrm{C}}^{-}\hat{a}_{\mathrm{B}}^{\dagger}\right)\\+F\left(\hat{\sigma}_{\mathrm{C}}^{-}e^{i\omega_{\mathrm{d}}t}+\hat{\sigma}_{\mathrm{C}}^{+}e^{-i\omega_{\mathrm{d}}t}\right)\end{aligned}$$

$$\omega_{\mathrm{C}}=\omega_{\mathrm{B}}\ \Longrightarrow\ \left[\hat{H}_{\mathrm{C}}+\hat{H}_{\mathrm{B}},\hat{H}_{\mathrm{CB}}\right]=0$$

Energy of the battery E_B

$$E_\mathrm{B} = \mathrm{Tr}_\mathrm{B}\left[\hat{\rho}_\mathrm{B}\hat{H}_\mathrm{B}\right]$$



[Campaioli *et al.*, ROMP **96**, 031001 (2024)]

Ergotropy of the battery $\mathcal{E}_\mathrm{B}$

$$\mathcal{E}_\mathrm{B} = E_\mathrm{B} - \min_{\hat{U}_\mathrm{B}\in \mathrm{SU}(d)} \mathrm{Tr}_B\left[\hat{U}_\mathrm{B}\hat{\rho}_\mathrm{B}\hat{U}_\mathrm{B}^\dagger\hat{H}_\mathrm{B}\right]$$

[Allahverdyan *et al*. EPL **67**, 565 (2004)]

Maximum extractable energy by all possible unitary.

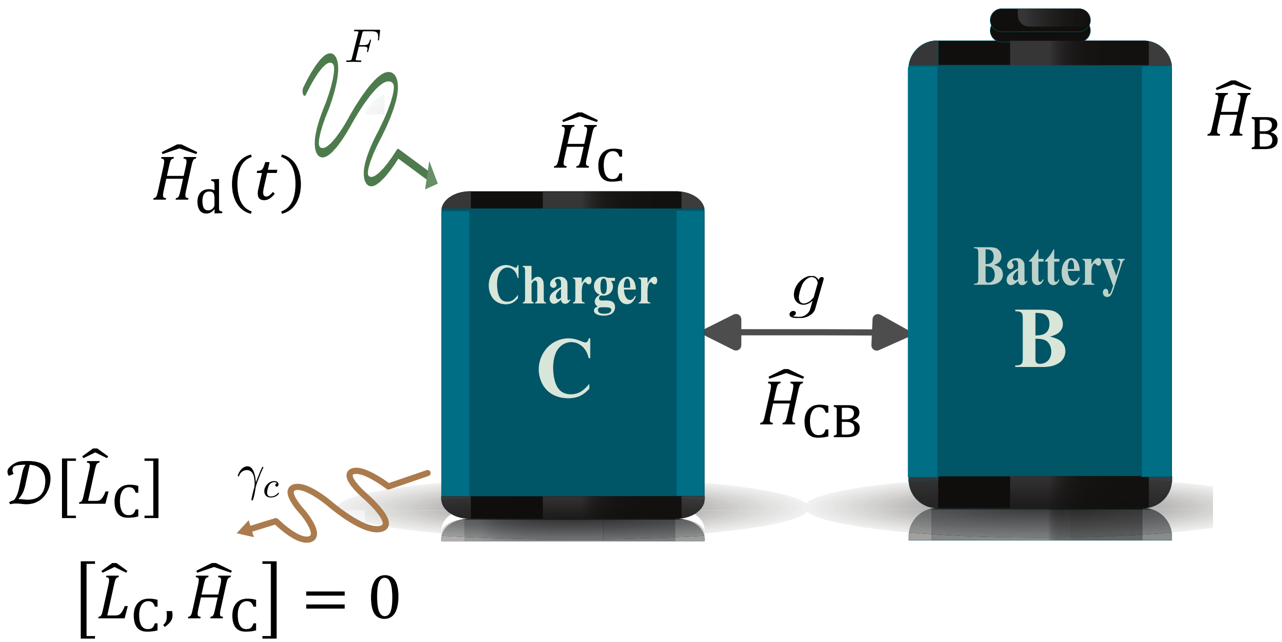
“quality” of the energy

Charging time τ

$$\left|\frac{E_\mathrm{B}(\tau) - E_\mathrm{B}(\infty)}{E_\mathrm{B}(0) - E_\mathrm{B}(\infty)}\right| = e^{-n}$$

(standard exp decay: $n = 1$)

Setup of the numerical simulations



Initial state: $\hat{\rho}(0) = |g\rangle\langle g|_{\mathrm{B}} \otimes |g\rangle\langle g|_{\mathrm{C}}$

Both battery & charger are “empty”.

Evolution: $$\frac{d\hat{\rho}(t)}{dt} = -i\left[\hat{H}(t), \hat{\rho}(t)\right] + \mathcal{D}\left[\hat{L}_{\mathrm{C}}\right]\hat{\rho}(t)$$

Battery will be charged up by the charger under driving $\hat{H}_{\mathrm{d}}(t)$.

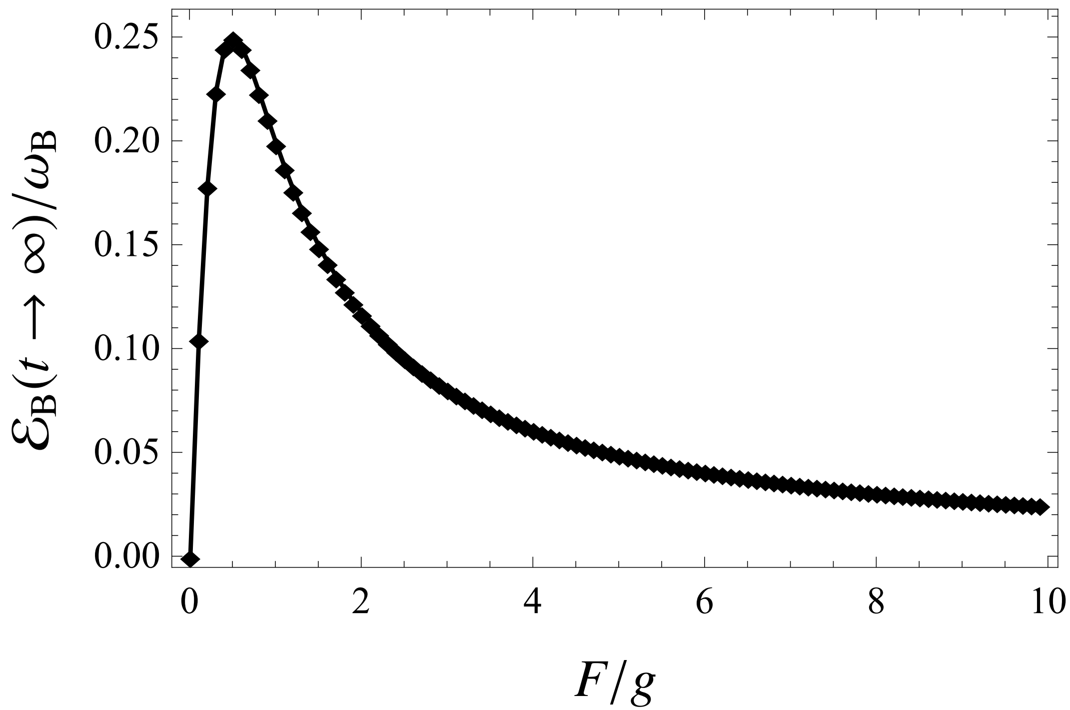
Results

Shastri, Jiang, Xu, Venkatesh & GW, npj Quantum Inf. **11**, 9 (2025)

Purkait, Venkatesh & GW, arXiv:2508.13497 (2025)

TLSs at resonance: Steady state

At resonance: $\omega_\mathrm{B} = \omega_\mathrm{C} = \omega_\mathrm{d}$



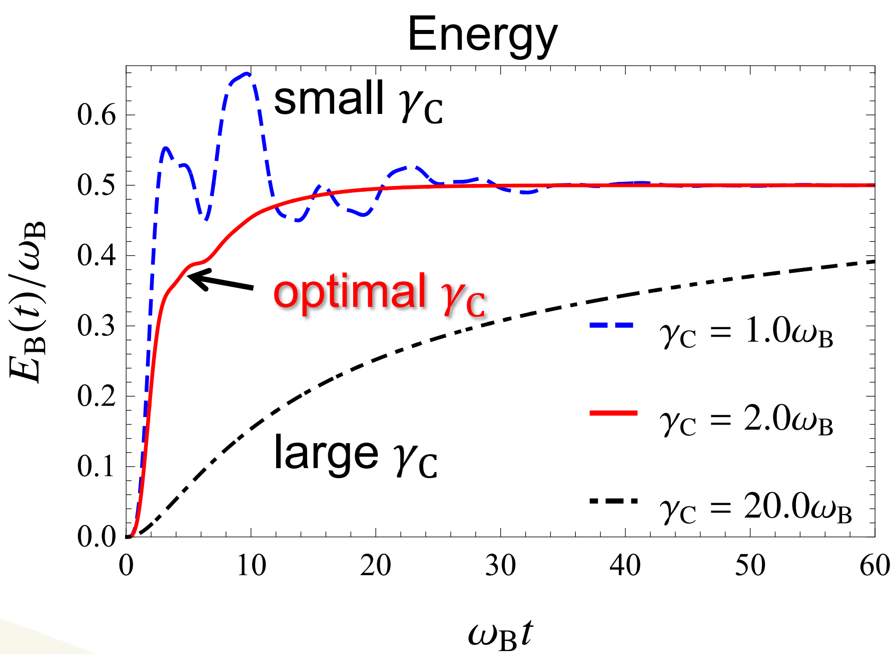
Steady st. E_B & $\mathcal{E}_\mathrm{B}$ are indep. of γ_C.

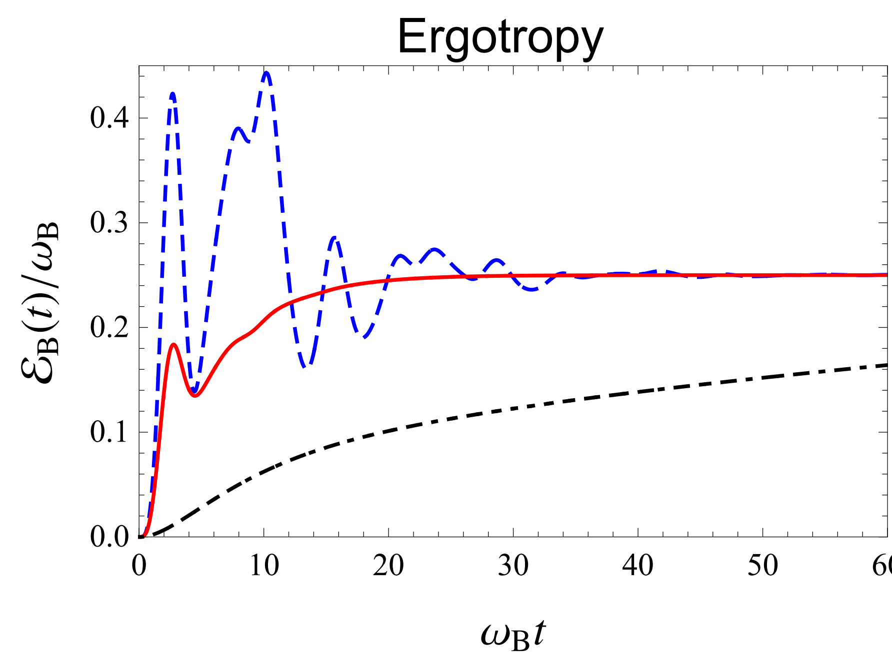
$$E_\mathrm{B}(\infty)/\omega_\mathrm{B} = \frac{1}{2} \qquad \mathcal{E}_\mathrm{B}(\infty)/\omega_\mathrm{B} = \frac{F/g}{1+(F/g)^2}$$

$\mathcal{E}_\mathrm{B}(\infty)$ takes max $(0.25\omega_\mathrm{B})$ when $F/g = 0.5$.

TLSs at resonance: Charging dynamics

For intermediate (optimal) driving: $F/g = 0.5$ $(g = 1.0\omega_B)$





Moderate dephasing leads to fast stabilization to steady state!

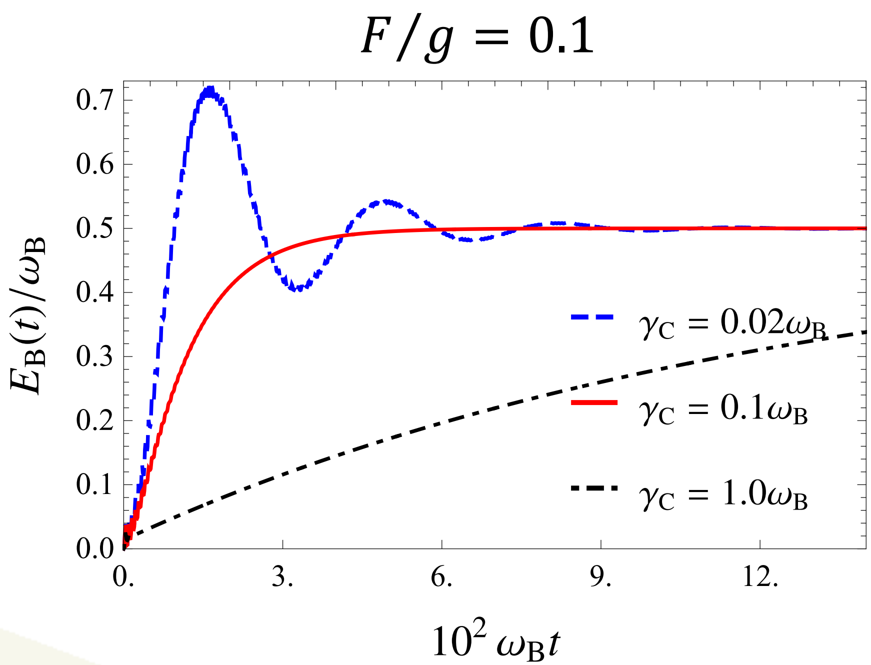
Transient maxima are **impractical**:

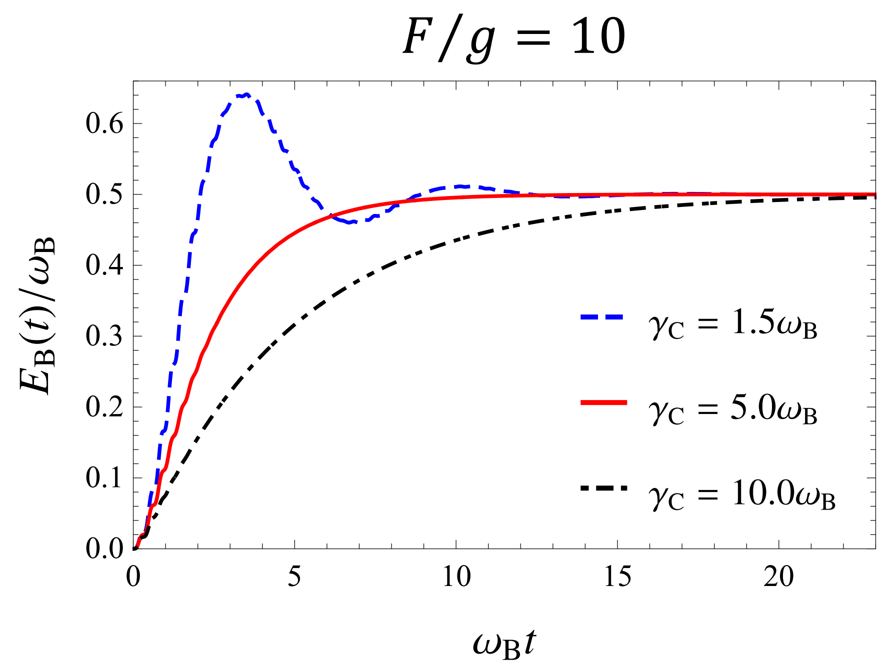
1. Fine temporal control is required.
2. Disturbance by sudden parameter change for decoupling is inevitable.

TLSs at resonance: Charging dynamics

Weak & strong driving

$(g = 1.0\omega_B)$





Moderate dephasing leads to fast stabilization to steady state!

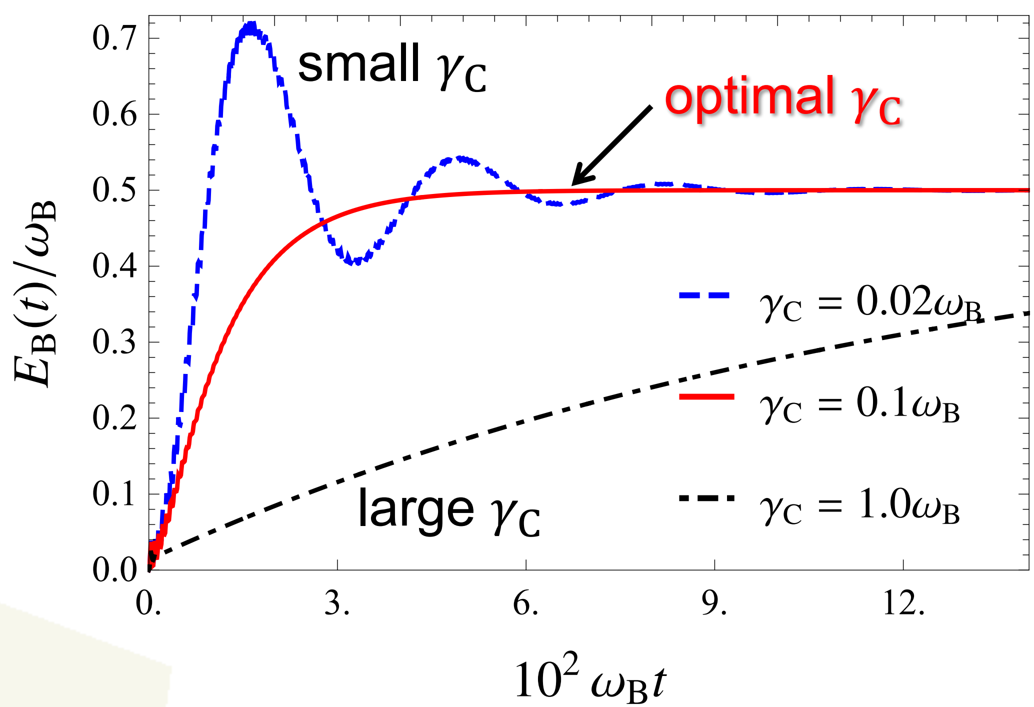
Transient maxima are **impractical**:

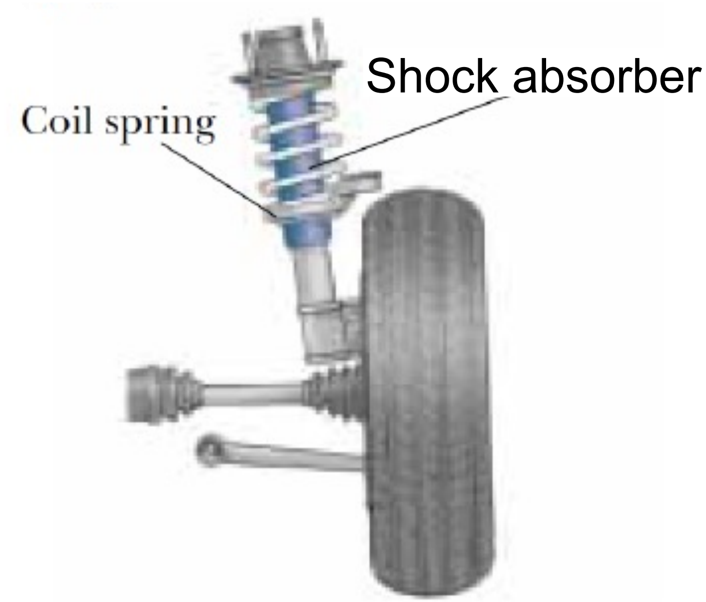
1. Fine temporal control is required.

2. Disturbance by sudden parameter change for decoupling is inevitable.

Coherent osc. vs quantum-Zeno

Moderate dephasing leads to fast charging!





weak dephasing $(\gamma_C \lesssim F, g)$: underdamped coherent oscillation

strong dephasing $(\gamma_C \gg F, g)$: quantum Zeno effect ➡ slow energy flow

➡ Optimal γ_C with fast relaxation.

Dephasing works as something similar to a shock absorber of the car.

Universal competition under dephasing

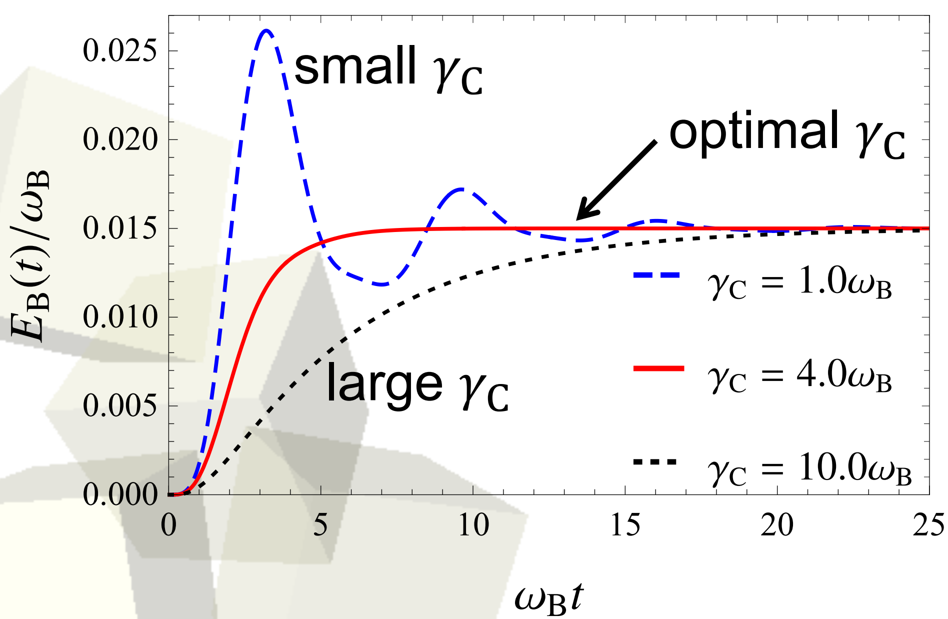
Competition:

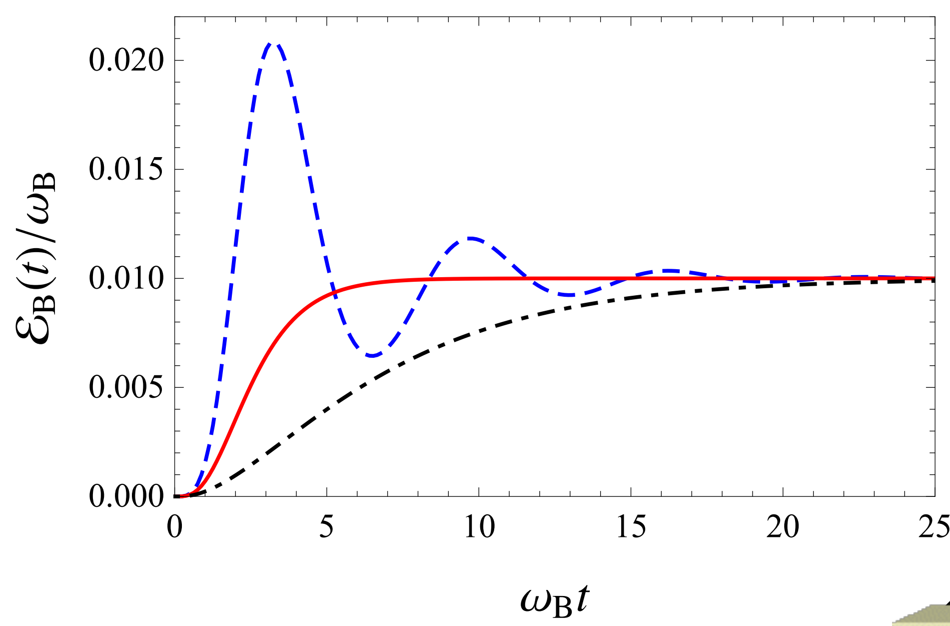
Coherent oscillation vs Quantum Zeno freezing

Universal for systems under dephasing.

HOs at resonance

$(F/g = 0.1, g = 1.0\omega_B)$





Universal competition under dephasing

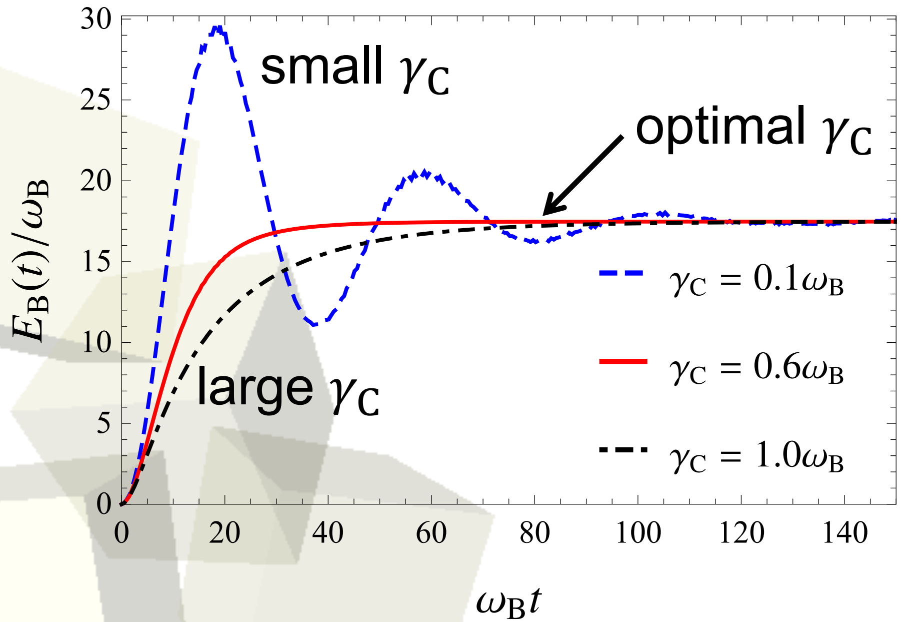
Competition:

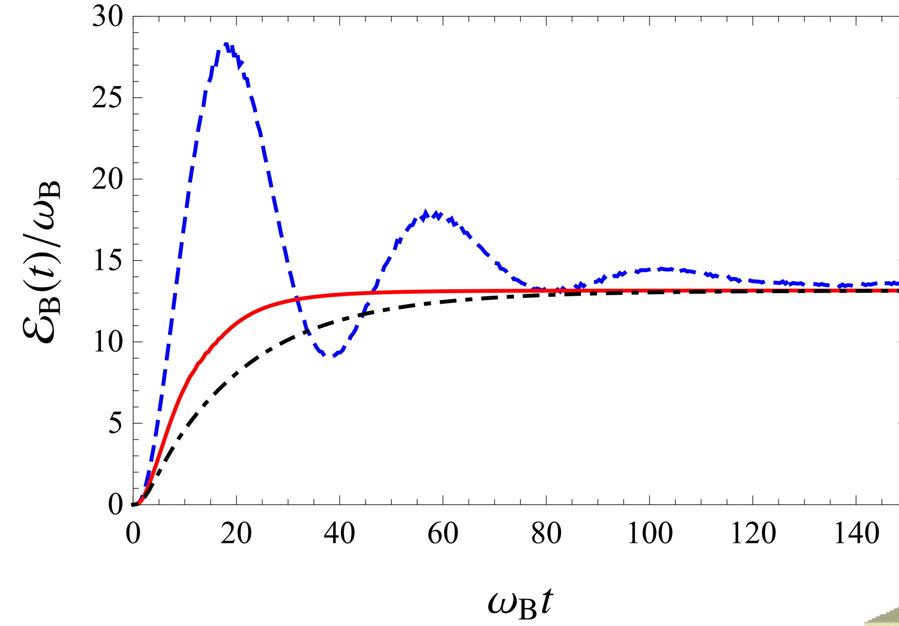
Coherent oscillation vs Quantum Zeno freezing

Universal for systems under dephasing.

TLS-HO at resonance

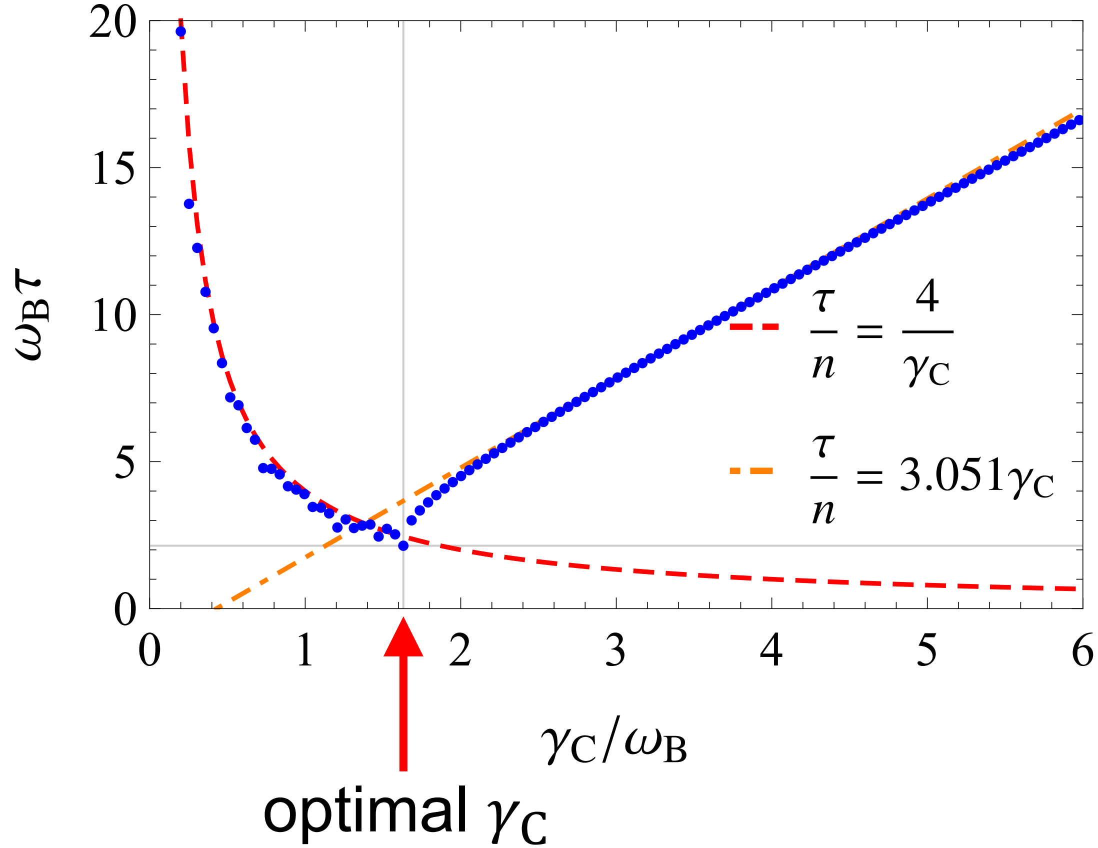
$(F/g = 3.0, g = 1.0\omega_B)$





Charging time (TLSs at resonance)

For intermediate driving: $F/g = 0.5$



weak dephasing ($\gamma_C \ll g, F$) : $\tau \sim 1/\gamma_C$

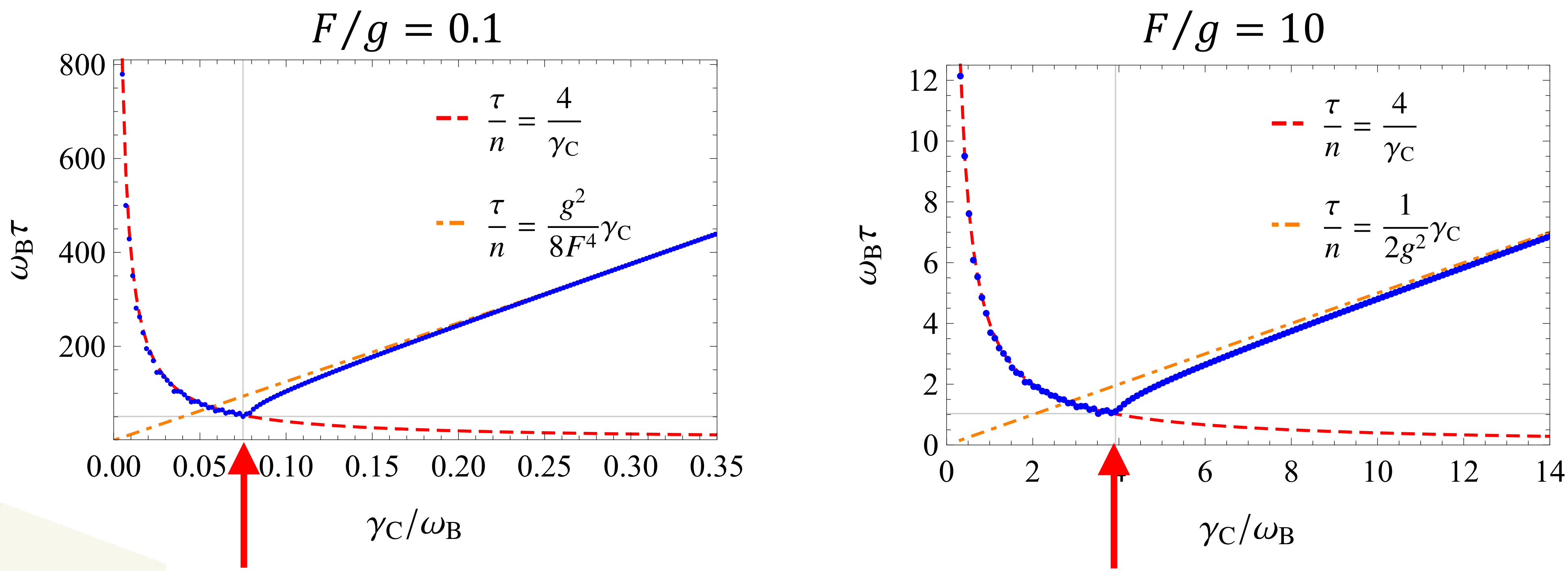
strong dephasing ($\gamma_C \gg g, F$) : $\tau \sim \gamma_C/[E^2]$

Minimum τ when

$$\frac{1}{\gamma_C} \sim \frac{\gamma_C}{[E^2]}$$

Charging time (TLSs at resonance)

Weak & strong driving



optimal γ_C optimal γ_C

weak dephasing ($\gamma_C \ll g, F$) : $\tau \sim 1/\gamma_C$

strong dephasing ($\gamma_C \gg g, F$) : $\tau \sim \gamma_C/[E^2]$

Minimum τ when

$$\frac{1}{\gamma_C} \sim \frac{\gamma_C}{[E^2]}$$

Charging time & optimal dephasing

	$F/g \gg 1$	$F/g \ll 1$
$\gamma_C \ll g$	$\tau \sim \frac{4}{\gamma_C}$	$\tau \sim \frac{4}{\gamma_C}$
$\gamma_C \gg g, F$	$\tau \sim \frac{1}{2g^2}\gamma_C$	$\tau \sim \frac{g^2}{F^4}\gamma_C$
Optimal dephasing	$\gamma_C^* \approx 2\sqrt{2}\, g$	$\gamma_C^* \approx \frac{8F^2}{\sqrt{2}g}$

Charging strategy

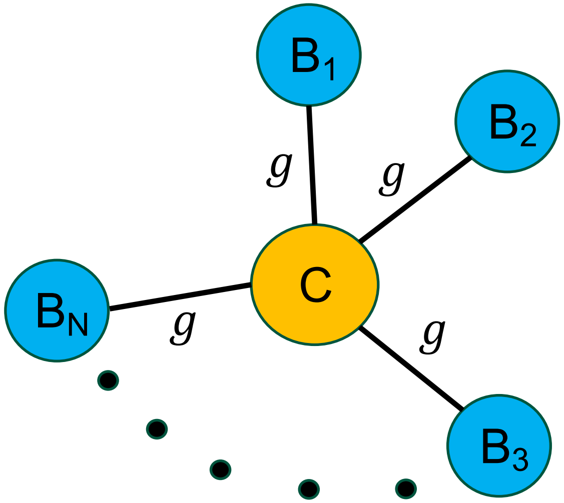
1. Set $F/g = 0.5$, maximizing the steady state $\mathcal{E}_B$.

2. Since $\gamma_C^* \approx \frac{8}{\sqrt{2}}\frac{F}{g}F = 2\sqrt{2}F$ and $\tau \sim \frac{4}{\gamma_C} \approx \sqrt{2}/F$, make F, g as large as possible.

Batteries in star configuration

To enhance steady-state ergotropy:

➡ **Multiple batteries in star configuration**



$$\hat{H}_{\mathrm{C}} = \omega_{\mathrm{C}} \hat{\sigma}_{\mathrm{C}}^{+} \hat{\sigma}_{\mathrm{C}}^{-}$$

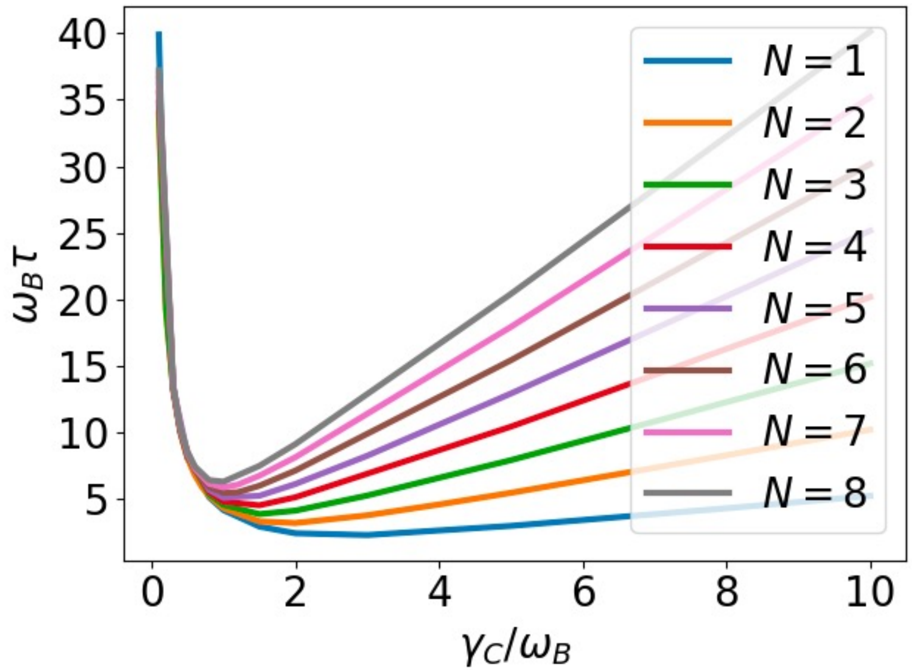
$$\hat{H}_{\mathrm{B}} = \sum_{i=1}^{N} \hat{H}_{\mathrm{B}_i} \quad \text{with} \quad \hat{H}_{\mathrm{B}_i} = \omega_{\mathrm{B}} \hat{\sigma}_{\mathrm{B}_i}^{+} \hat{\sigma}_{\mathrm{B}_i}^{-}$$

$$\hat{H}_{\mathrm{CB}} = g\left(\hat{\sigma}_{\mathrm{C}}^{-} \sum_{i=1}^{N} \hat{\sigma}_{\mathrm{B}_i}^{+} + \hat{\sigma}_{\mathrm{C}}^{+} \sum_{i=1}^{N} \hat{\sigma}_{\mathrm{B}_i}^{-}\right) \quad : \quad \text{“Star” config.}$$

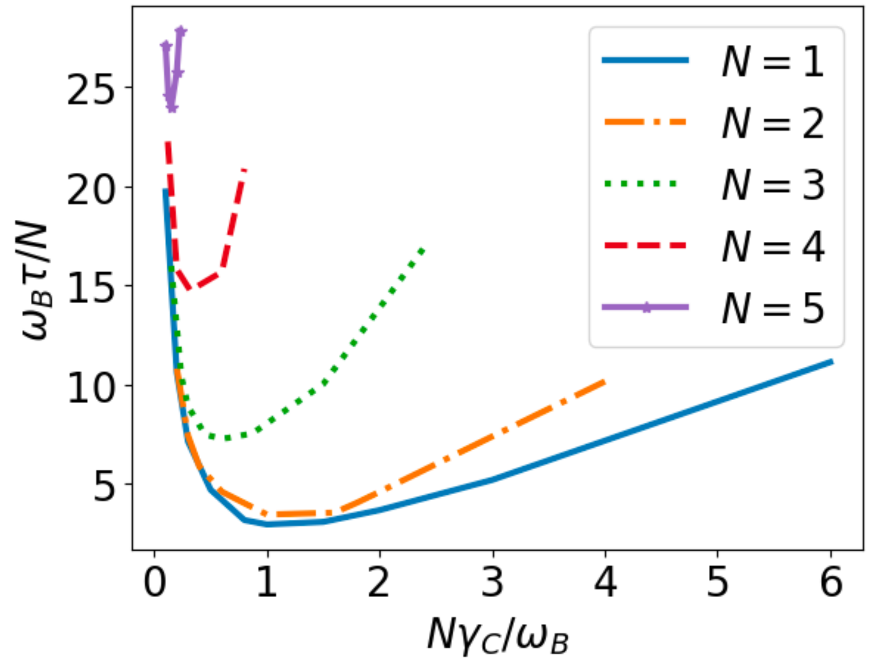
$$\hat{H}_{\mathrm{d}} = F\left(\hat{\sigma}_{\mathrm{C}}^{-} e^{i\omega_{\mathrm{d}} t} + \hat{\sigma}_{\mathrm{C}}^{+} e^{-i\omega_{\mathrm{d}} t}\right)$$

Charging time τ

Strong drive ($F/g = 10$)



Intermed. drive ($F/g = 0.5$)

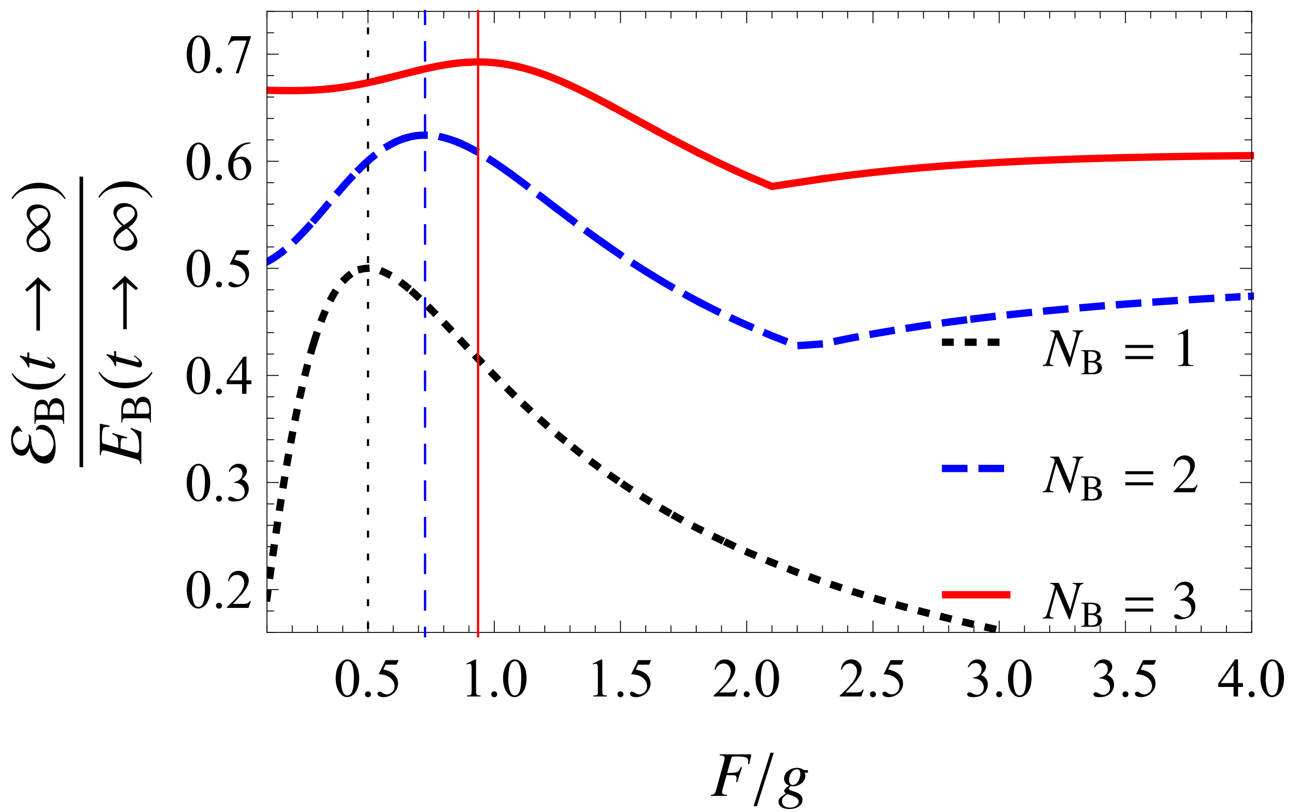


In all the cases, there is an optimum γ_C for fast charging.

Strong driving regime: minimum $\tau \sim N$

Intermediate driving regime: minimum $\tau \sim N^{3.2}$

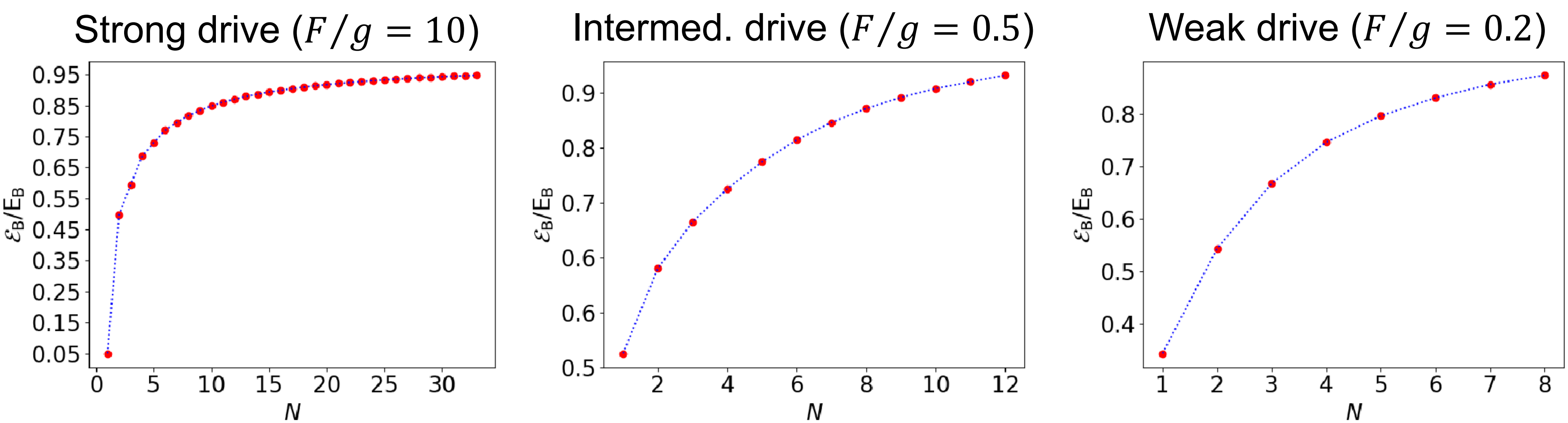
Enhancement of steady-state ergotropy



$$E_{\mathrm{B}}(\infty) = N_B \frac{\omega_{\mathrm{B}}}{2}$$

$$\frac{\max[\mathcal{E}_{\mathrm{B}}(\infty)]}{E_{\mathrm{B}}(\infty)} = 0.5\ \ (N_B = 1);\ \ 0.62\ \ (N_B = 2);\ \ 0.69\ (N_B = 3)$$

Asymptotic freedom



$$\frac{\mathcal{E}_B}{E_B} \sim 1 - \frac{a}{N} \quad \text{as } N \to \infty \quad \text{with a const. } a = O(1)$$

Asymptotic freedom: proof

Bounds of the passive energy: $\omega_B\delta \le (E_B - \mathcal{E}_B) \le 2\omega_B$

1. Dimensional reduction because of the permutation symmetry of the Liouvillian & initial state. $[\, 2^N \longrightarrow (N+2)^2/4$ (for N even)$]$

$$\mathrm{Tr}\big[\hat{H}_B \hat{\rho}^{\downarrow}\big] = \sum_{i=1}^{(N+2)^2/4} r_i^{\downarrow}\varepsilon_i^{\uparrow} \le \frac{4}{(N+2)^2}\sum_{i=1}^{(N+2)^2/4} \varepsilon_i^{\uparrow}$$

$$= \frac{4}{(N+2)^2}\left[0\binom{N}{0} + \omega_B\binom{N}{1} + 2\omega_B\left(\frac{(N+2)^2}{4} - \binom{N}{0} - \binom{N}{1}\right)\right]$$

Upper bound: $(E_B - \mathcal{E}_B) \le 2\omega_B$

2. Nonzero asymptotic passive energy because of the mixture.

Mixed st.: $r_1^{\downarrow} < 1$ and some $r_{i\neq 1}^{\downarrow} > 0$ ⟹ Define $\delta \equiv 1 - r_1^{\downarrow}\big|_{N\to\infty} > 0$

$$\mathrm{Tr}\big[\hat{H}_B \hat{\rho}^{\downarrow}\big] = \sum_{i=1} r_i^{\downarrow}\varepsilon_i^{\uparrow} = \sum_{i=2} r_i^{\downarrow}\varepsilon_i^{\uparrow} \ge \varepsilon_2^{\uparrow}\delta$$

Lower bound: $\omega_B\delta \le (E_B - \mathcal{E}_B)$

Together with $E_B = N\omega_B/2$, we get: $\frac{\mathcal{E}_B}{E_B} \sim 1 - \frac{a}{N}$ with $2\delta \le a \le 4$

Physical mechanism of asymptotic freedom

Asymptotic steady state is mixed st.

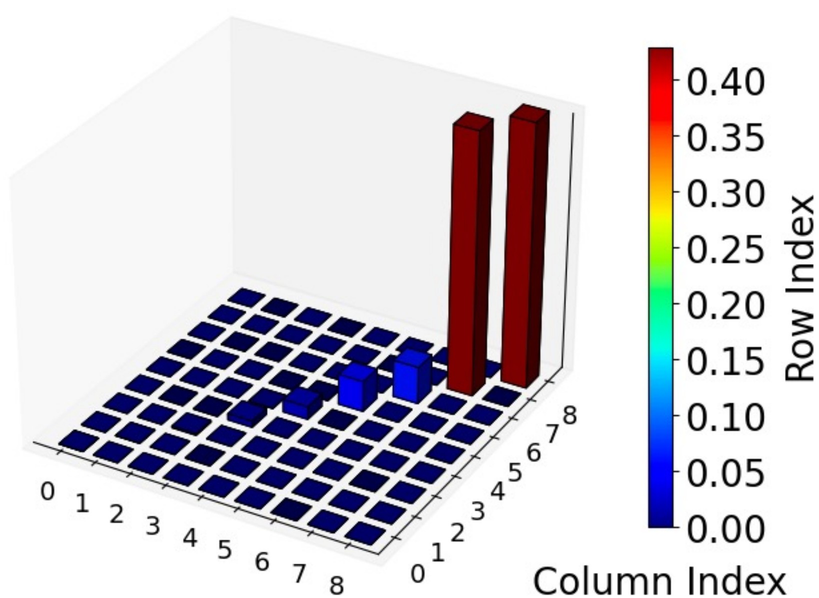
Ground state is unique: $|0\rangle^{\otimes N}$

Q: Why asymptotic free even though the state is mixed?

A: Emergent approximate ground st. degeneracy in $N \to \infty$.

Relevant quantity for discussion of $\mathcal{E}_B/E_B$: Δ_g/E_B

Δ_g: energy gap btwn. ground & 1st excited st.



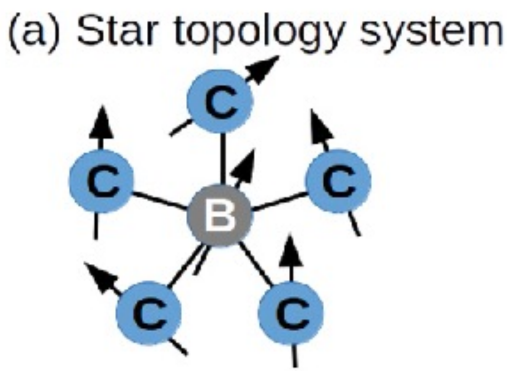
$\Delta_g \sim \omega_B,\ E_B \sim N\omega_B$ → $\Delta_g/E_B \sim 1/N$

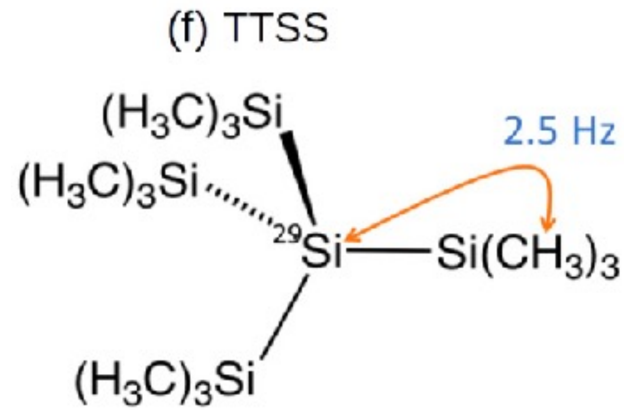
Gap becomes negligible as $N \to \infty$.

Experimental feasibility

Joshi & Maheshi, PRA **106**, 042601 (2022)

Implementation of charger-battery sys. in NMR spin sys.





1 battery coupled to multiple chargers

Battery: central nuclear spin $\left(I = \frac{1}{2}\right)$

Charger: ^{1}H nuclear spin $\left(I = \frac{1}{2}\right)$

$$\hat{H}_{\mathrm{BC}} = J(\hat{S}_x \hat{I}_x + \hat{S}_y \hat{I}_y)$$

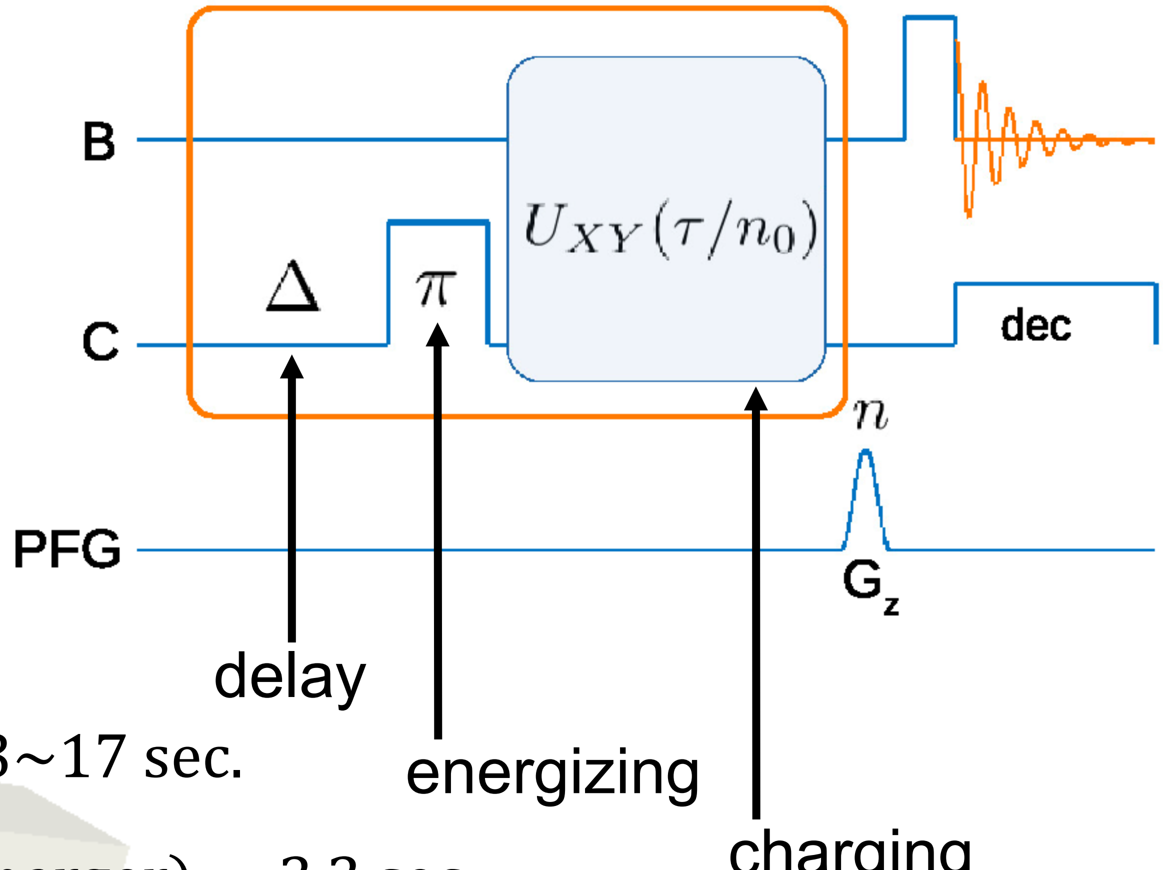
$\hat{S}_{x,y,z}$: battery spin

$\hat{I}_{x,y,z} = \sum_i \hat{I}^i_{x,y,z}$: battery spins

Experimental feasibility

Joshi & Maheshi, PRA **106**, 042601 (2022)

Mimic dephasing by iteratively reenergizing chargers after delay.



$\Delta = 3\sim17$ sec.

$T_1(\text{charger}) = 3.3$ sec.
$T_1(\text{battery}) = 115.4$ sec.

Summary & conclusion

Decoherence can be a resource.

- Moderate dephasing helps fast charging!

- General behavior: Tradeoff between
  Coherent oscillations vs Zeno freezing

- Multiple batteries in star configuration:
  Asymptotic free due to approx. ground st. degeneracy.

Shastri, Jiang, Xu, Venkatesh & GW, npj Quantum Inf. **11**, 9 (2025)

Purkait, Venkatesh & GW, arXiv:2508.13497 (2025)

Collectively enhanced charging via quantum heat engines

Ito, Purkait, GW, arXiv:2008.07089

Motivation

Steady-state QHEs: Stable energy supplier driven by heat.

Low cost, robust, free from complex controls.

An ensemble of QHEs & that of QBs can effectively behave as "identical particles".

→ Collective enhancements expected.

QHEs-QBs setting is attractive both practically and conceptually.

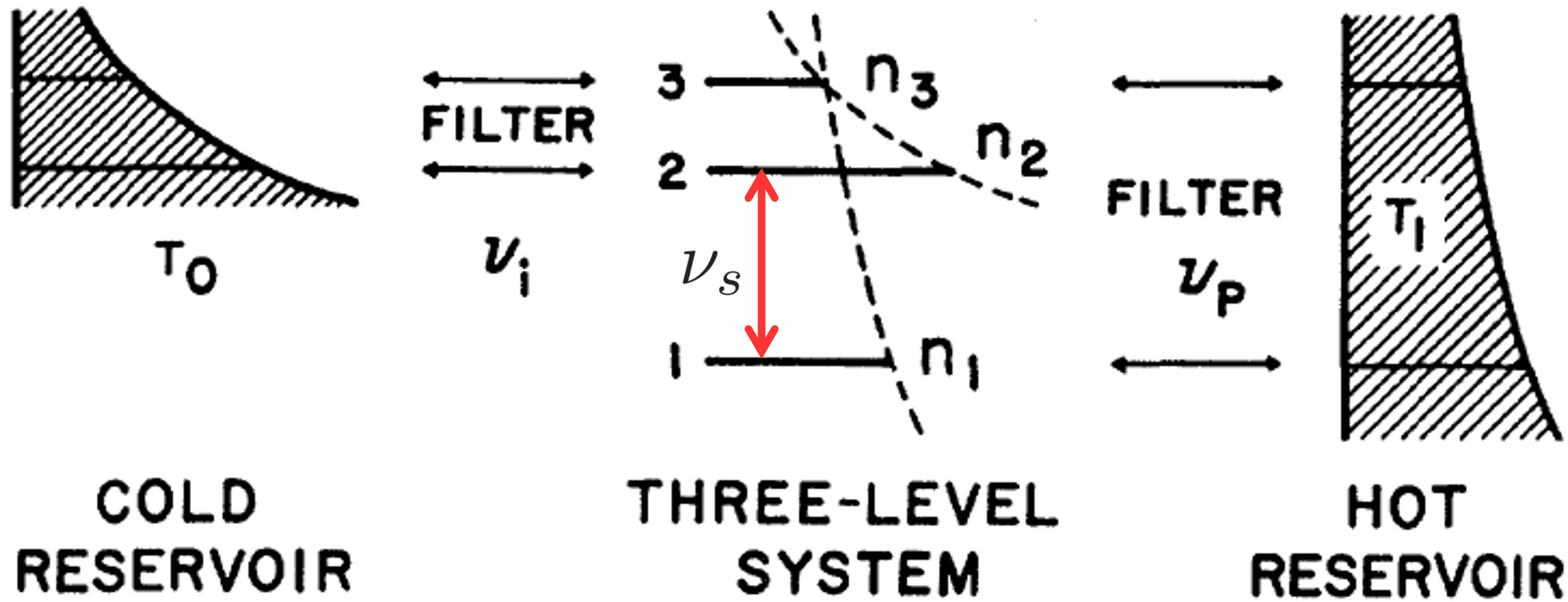
Dr. Kosuke Ito

Dr. Chayan Purkait

Ito, Purkait, GW, arXiv:2008.07089

Three-level masers as heat engines

[Scovil & Schulz-DuBois, PRL **2**, 262 (1959)]



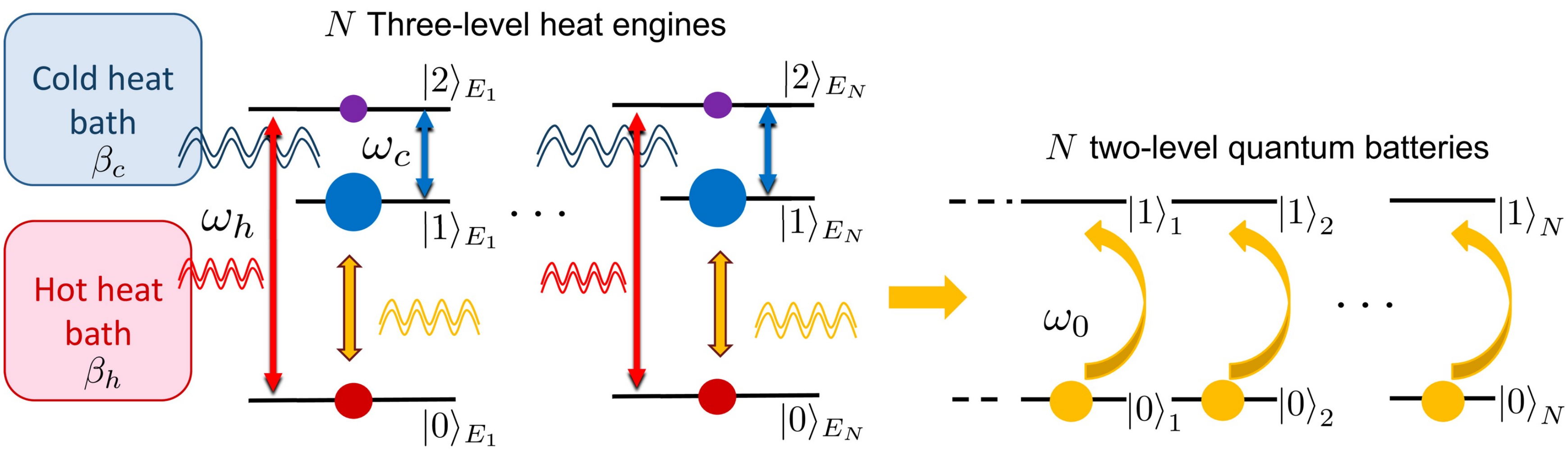
Efficiency: $\eta_M = \nu_s/\nu_p$

$$\frac{n_2}{n_1} = \frac{n_2}{n_3}\frac{n_3}{n_1} = \exp\left(\frac{h\nu_i}{k_B T_0}\right)\exp\left(-\frac{h\nu_p}{k_B T_1}\right) = \exp\left[\frac{h\nu_s}{k_B T_0}\left(\frac{\nu_p}{\nu_s}\frac{T_1 - T_0}{T_1} - 1\right)\right]$$

Condition for maser action: $n_2/n_1 \geq 1$ ⟺ $\eta_M \leq \eta_C \equiv \dfrac{T_1 - T_0}{T_1}$

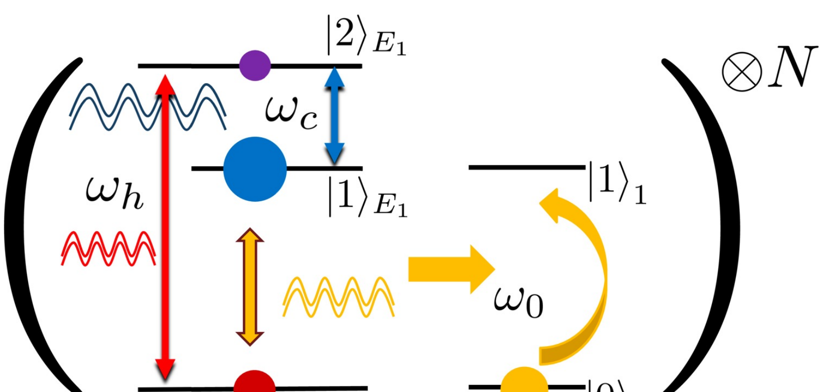
[Refrigeration (reverse process): $n_2/n_1 \leq 1$]

QHEs as chargers: Collective vs individual charging



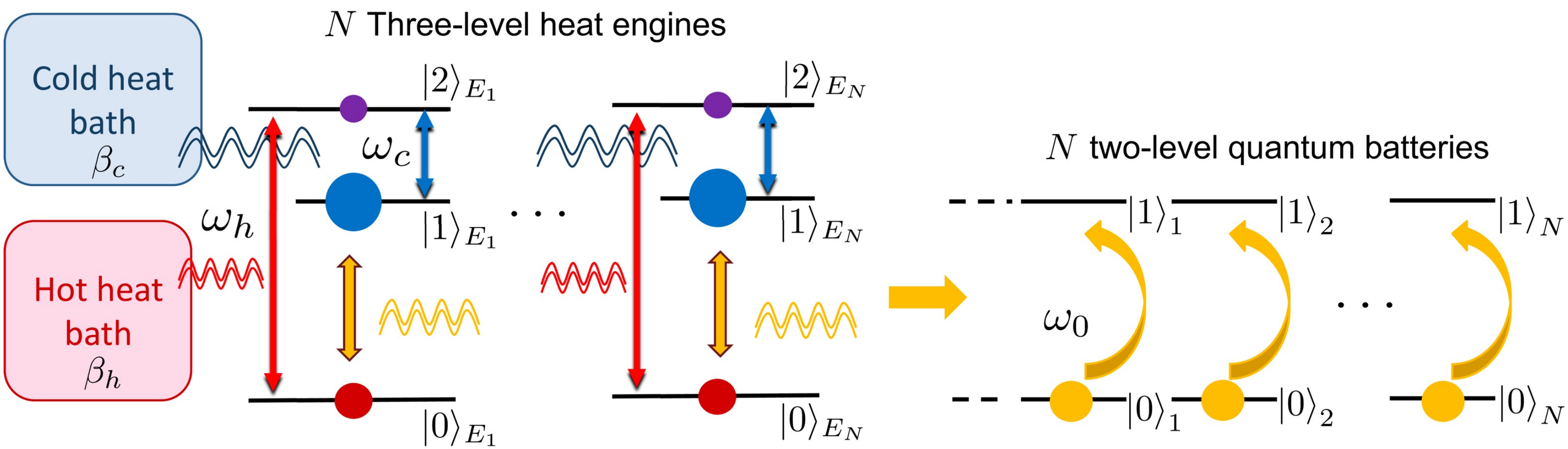
Collective charging

VS



Individual charging

QHEs as chargers: Collective vs individual charging



Collective charging

$$\hat{H}^{c}_{tot} = \sum_{j=1}^{N} \hat{H}_{E_j} + \sum_{k=1}^{N} \hat{H}_{B_k} + \sum_{j,k=1}^{N} \hat{K}^{(N)}_{j,k}$$

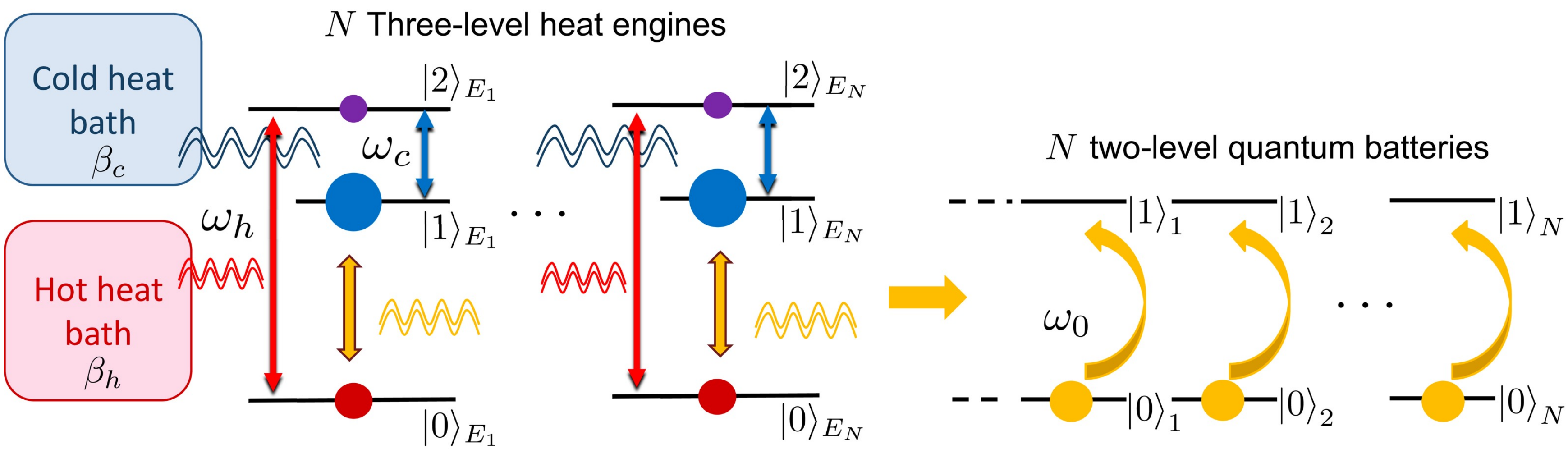
$$= \hat{H}_E + \hat{H}_B + \sum_{j=1}^{N} i\frac{g}{\sqrt{N}}\left(|1\rangle\langle 0|_{E_j} \otimes \hat{S}_B^{-} - |0\rangle\langle 1|_{E_j} \otimes \hat{S}_B^{+}\right)$$

Emergent bosonic statistics due to permutation sym. in Eng.-Batt. int.

→ Collective enhancement

High-power, high-capacity, high-quality charging.

QHEs as chargers: Collective vs individual charging



Collective charging

$$\hat{H}^c_{tot} = \sum_{j=1}^N \hat{H}_{E_j} + \sum_{k=1}^N \hat{H}_{B_k} + \sum_{j,k=1}^N \hat{K}^{(N)}_{j,k}$$

$$= \hat{H}_E + \hat{H}_B + \sum_{j=1}^N i\frac{g}{\sqrt{N}}\left(|1\rangle\langle 0|_{E_j} \otimes \hat{S}_B^- - |0\rangle\langle 1|_{E_j} \otimes \hat{S}_B^+\right)$$

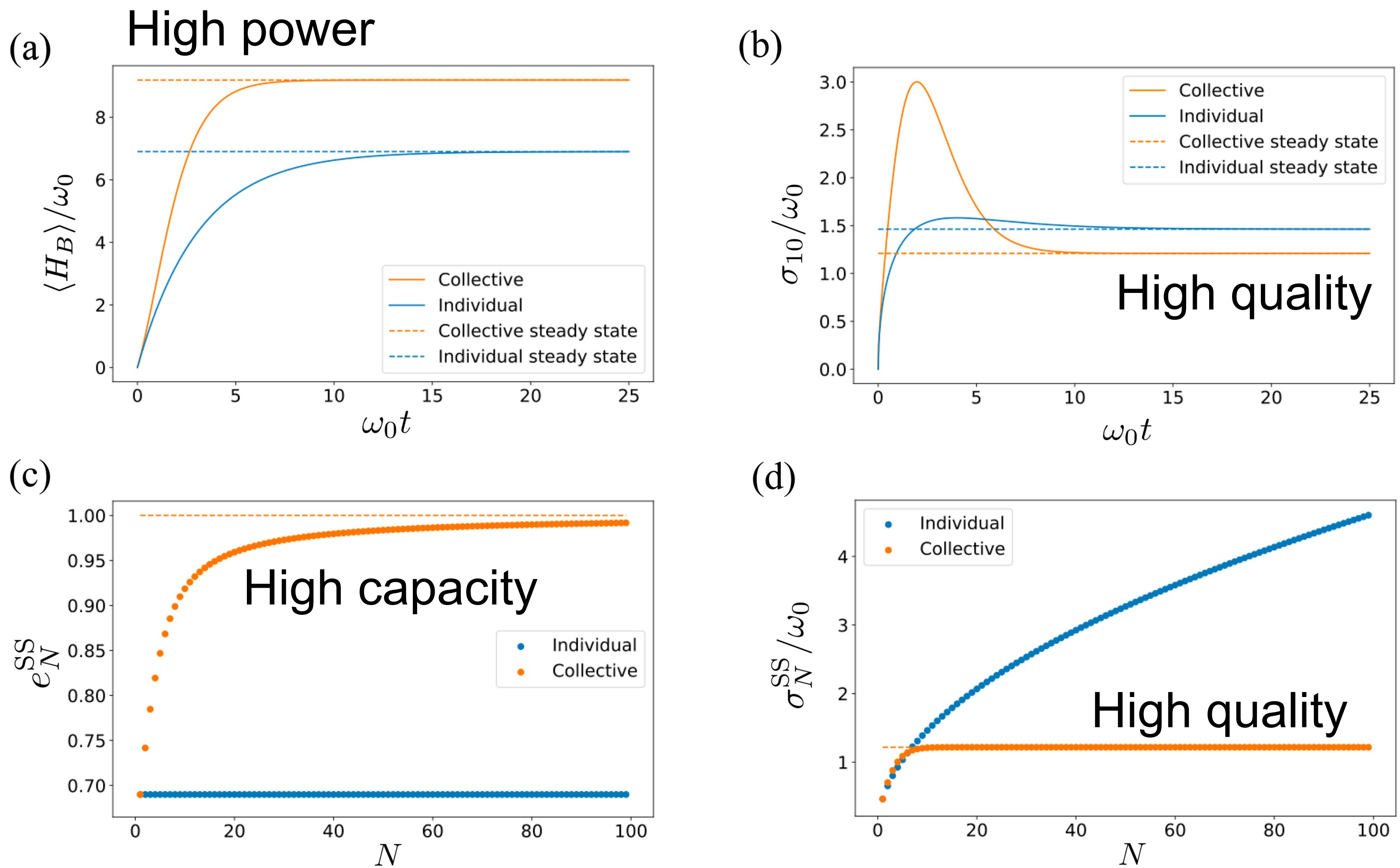
If (thermalization time of engines) << (dyn. time of batteries)

effective ME: $\frac{d\hat{\rho}_B}{dt} = -i[\hat{H}_B, \hat{\rho}_B] + \Gamma_e[\mathcal{D}[\hat{S}_B^-]\hat{\rho}_B + e^{-\beta_e\omega_0}\mathcal{D}[\hat{S}_B^+]\hat{\rho}_B]$

Superradiance-like excitation in the effective negative temp.

Results: High-power, high-capacity, high-quality charging

Parameters: $N = 10$, $\beta_e \omega_0 = -0.8$, $\Gamma_e/\omega_0 = 0.1$

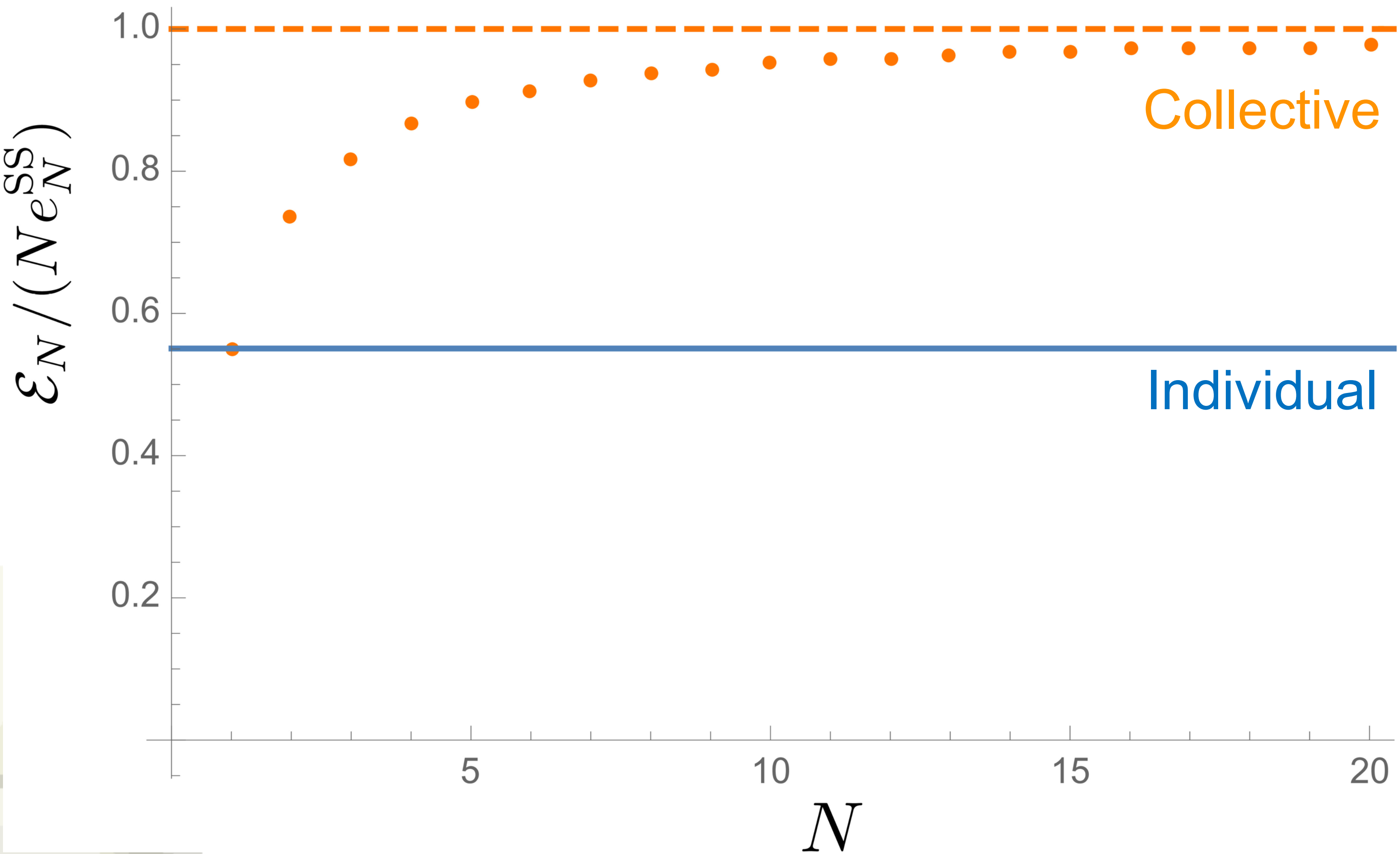


$$e_N \equiv \langle \hat{H}_B \rangle / N\omega_0$$

$$\sigma_N \equiv \sqrt{\left\langle \left(\Delta \hat{H}_B\right)^2 \right\rangle} = \sqrt{\left\langle \hat{H}_B{}^2 \right\rangle - \langle \hat{H}_B \rangle^2}$$

Results: Asymptotic freedom

Parameters: $N = 10$, $\beta_e \omega_0 = -0.8$, $\Gamma_e / \omega_0 = 0.1$



$$\frac{\mathcal{E}_N}{N e_N^{SS}} \sim 1 - \frac{e^{-|\beta_e \omega_0|}}{N} \quad (\text{for } N \to \infty)$$

Potential platform

Zhu *et al.*, Nature **478**, 221 (2011)

LETTER

doi:10.1038/nature10462

Coherent coupling of a superconducting flux qubit to an electron spin ensemble in diamond

Xiaobo Zhu[1], Shiro Saito[1], Alexander Kemp[1], Kosuke Kakuyanagi[1], Shin-ichi Karimoto[1], Hayato Nakano[1], William J. Munro[1], Yasuhiro Tokura[1], Mark S. Everitt[2], Kae Nemoto[2], Makoto Kasu[1], Norikazu Mizuochi[3,4] & Kouichi Semba[1]

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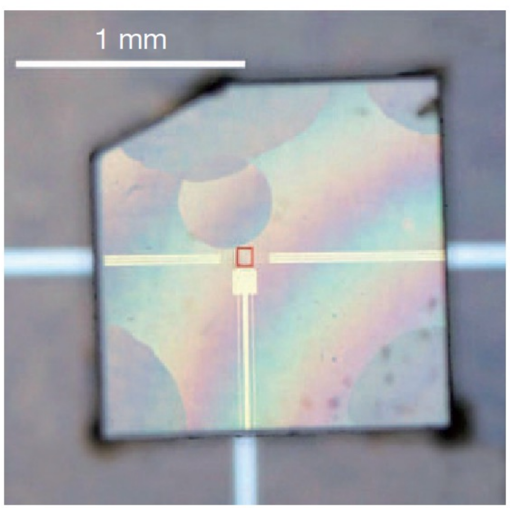

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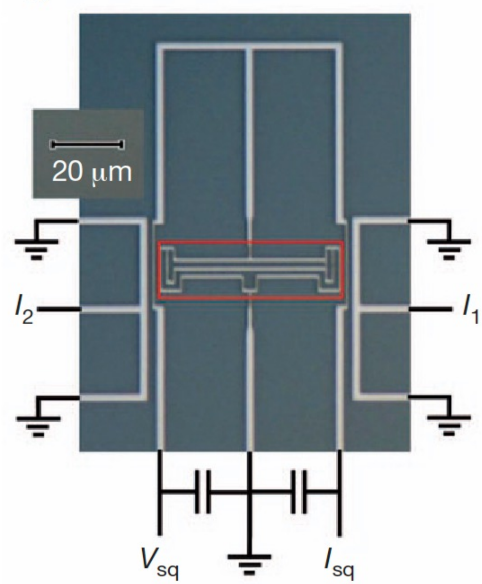

Realization of coherent coupling btwn. a SC flux circuit & diamond NV^- centers.

QHEs: SC flux devices

QBs: NV^- centers

(degenerate NV^- center as an effective TLS)

Summary & conclusion

Steady-state QHEs: High-performance chargers for collective charging

Resource-efficient

thermal energy, free from timing

Indistinguishability

possible to make identical engines

High-power, high-capacity, and high-quality

$\langle \hat{H}_B \rangle / \tau$ $\quad e_N \equiv \langle \hat{H}_B \rangle / N\omega_0$ $\quad$ small σ_N

Experimental feasibility: (SC flux circuit) + (diamond NV^- centers) setup can be a potential platform.

Ito, Purkait, GW, arXiv:2008.07089

Theoretical Tools for Quantum Batteries: Basic Concepts and Their Applications

Lecture 0: Introduction to quantum batteries

Lecture 1: Theory of open quantum systems 101

Lecture 2: Ergotropy & coherence

Lecture 3: Dicke model: A workhorse of QB research

Lecture 4: Our recent results