23rd December 2025 KAIS-SNU winter school

Ultracold atomic quantum simulator

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National Research Foundation of Korea

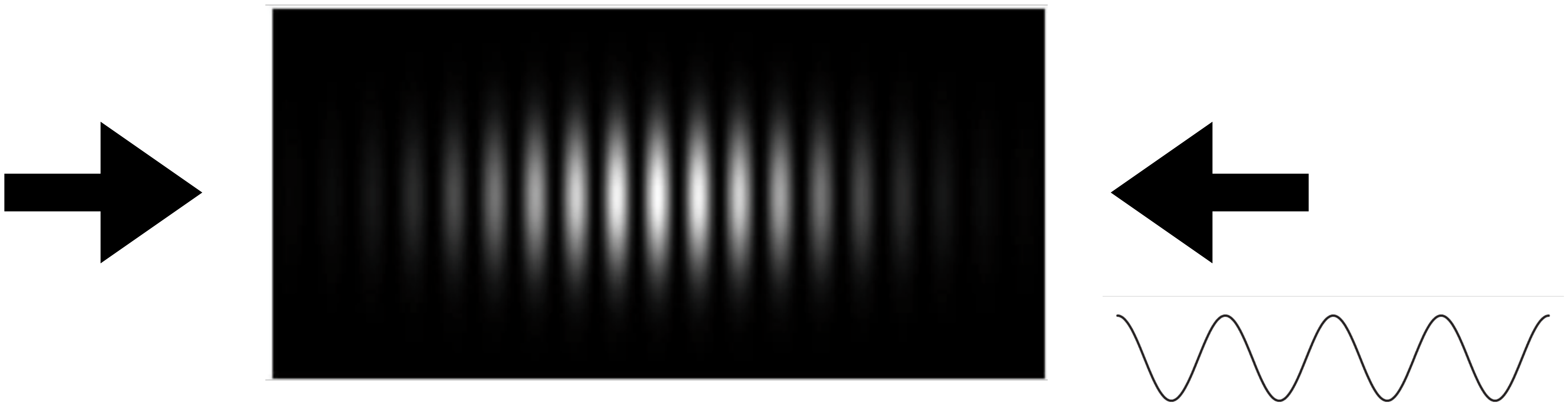
KAIST

How do we build a quantum simulator using Ultracold Atoms?

Loading the atoms in optical lattices

Nature Physics, **1**, 23 (2005)
Review of Modern Physics. **80**, 885 (2008)
Nature Reviews Physics **2**, 411 (2020)

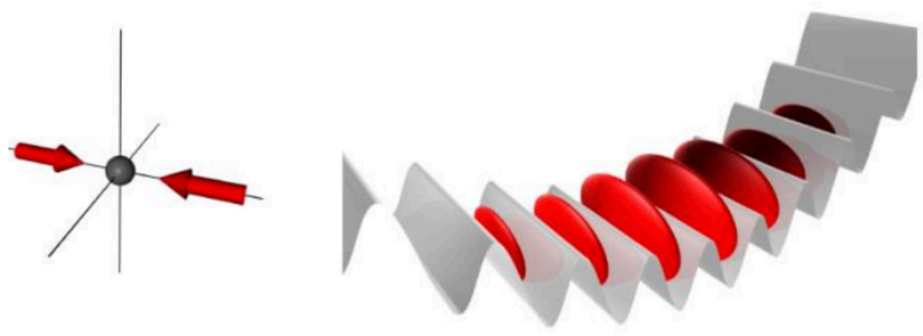
Optical lattice

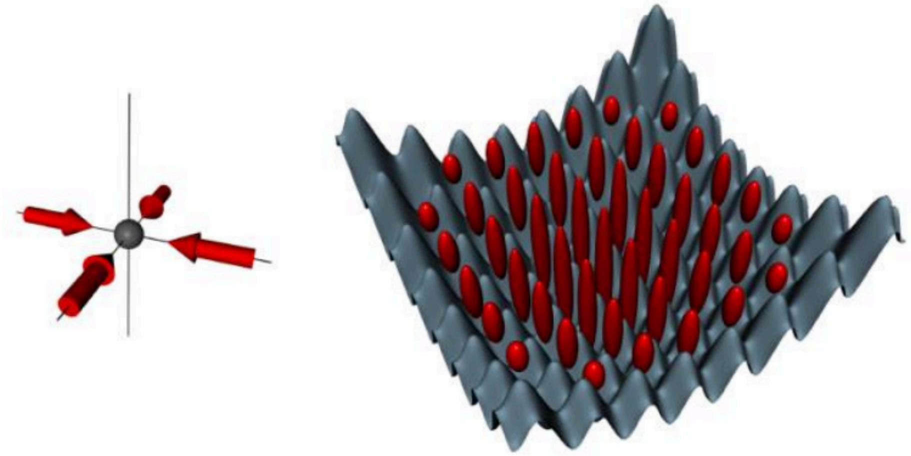


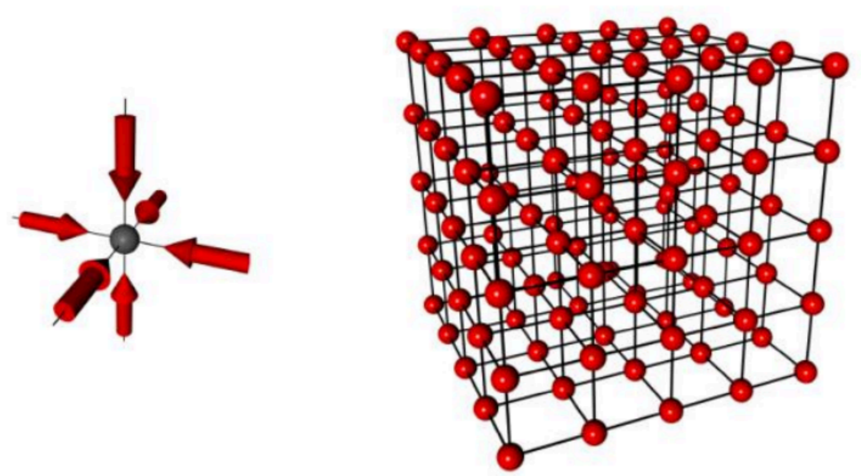
Defect free & perfect artificial crystal !!

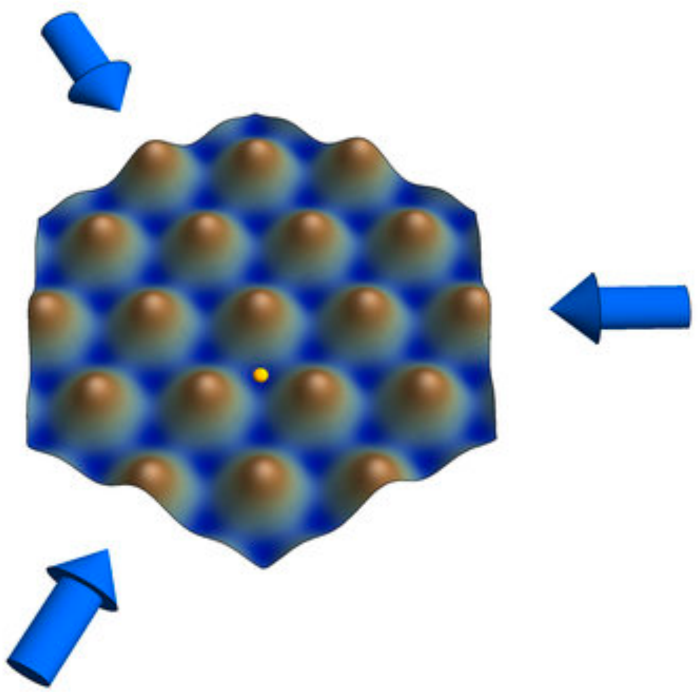
Optical lattice

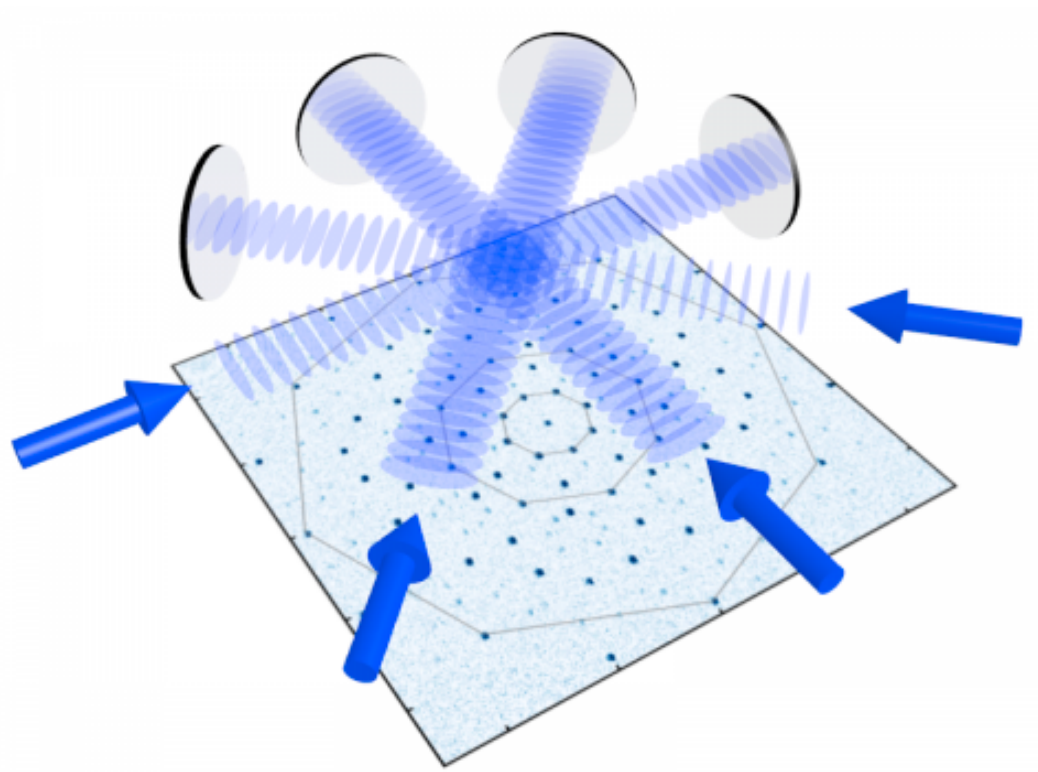
Various lattice geometry from interference patterns

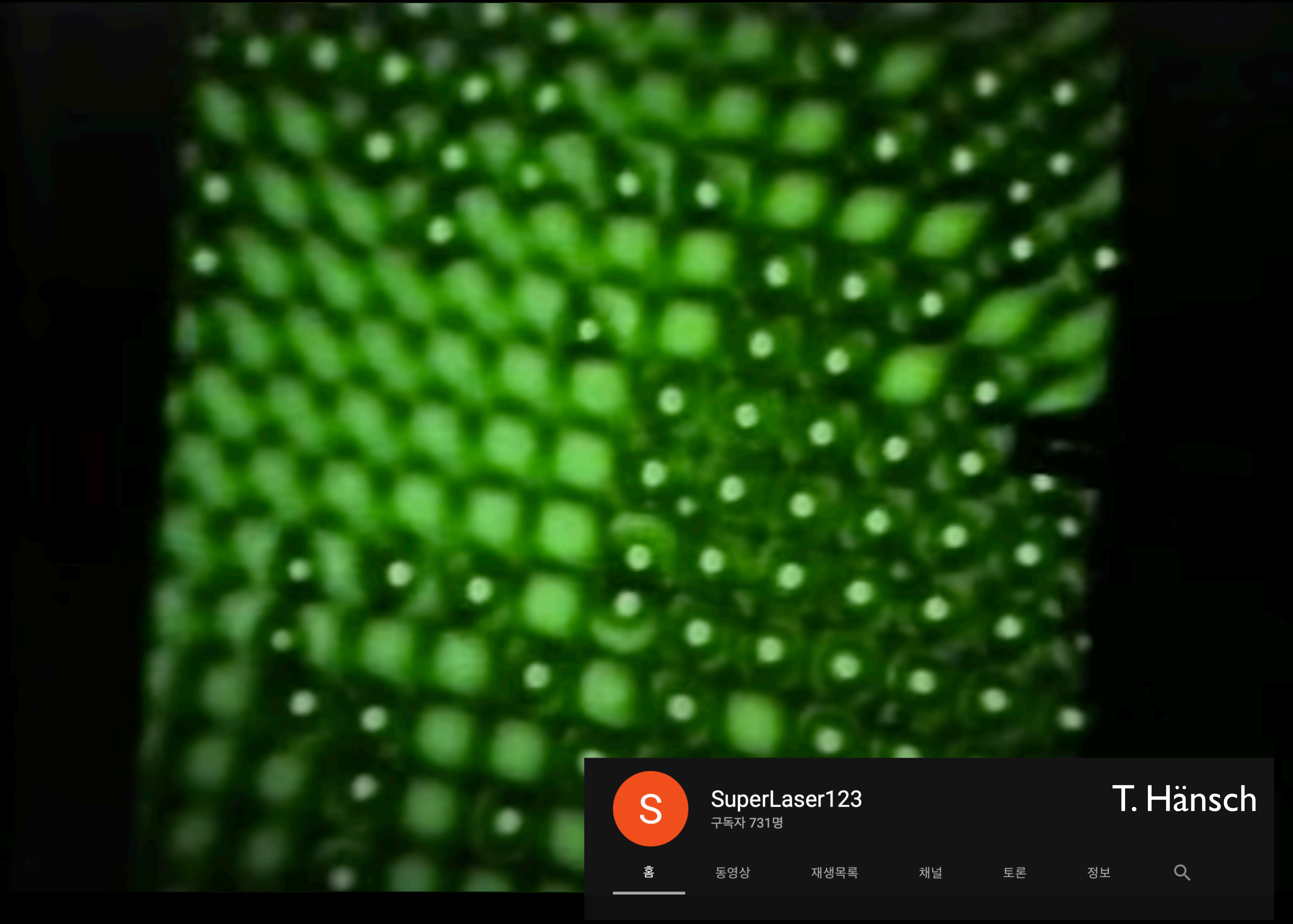








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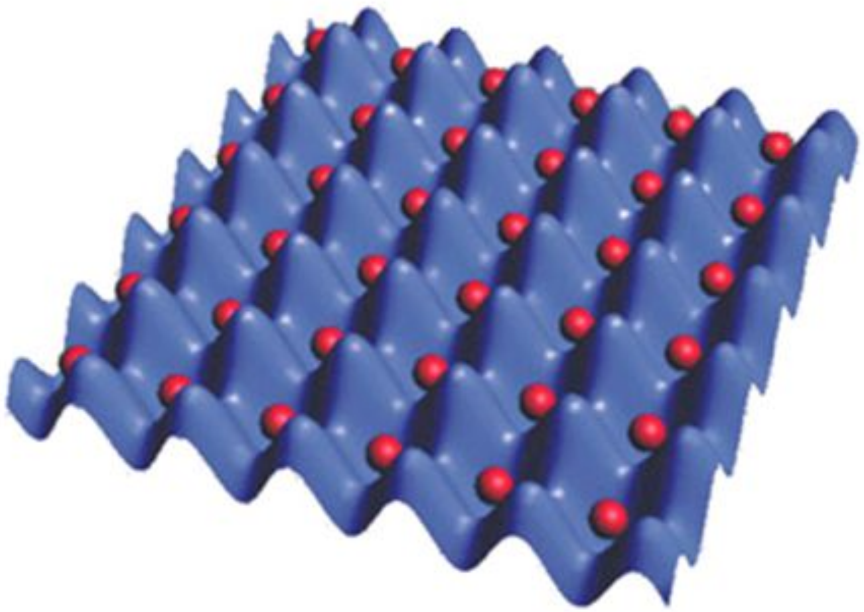
SuperLaser123
구독자 731명
T. Hänsch
홈
동영상
재생목록
채널
토론
정보

Simulating condensed matter systems

Ultracold atomic matter

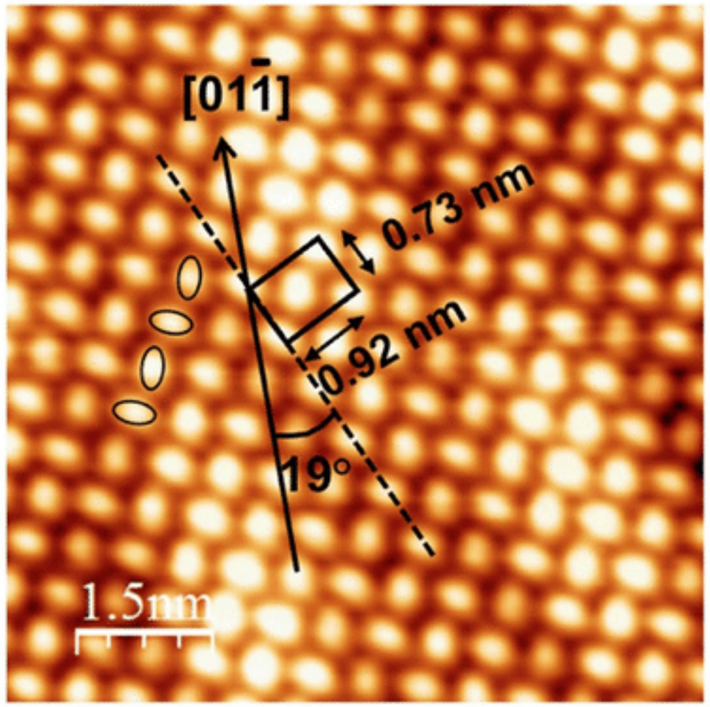


How do the atoms behave under the optical lattices?

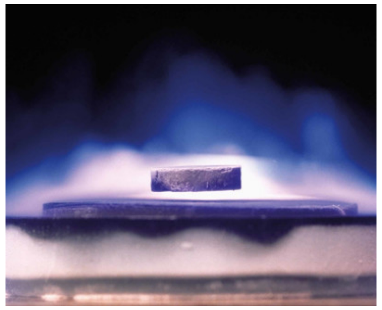
How does it relate to the solid-state materials?

Simulating condensed matter systems

Real material

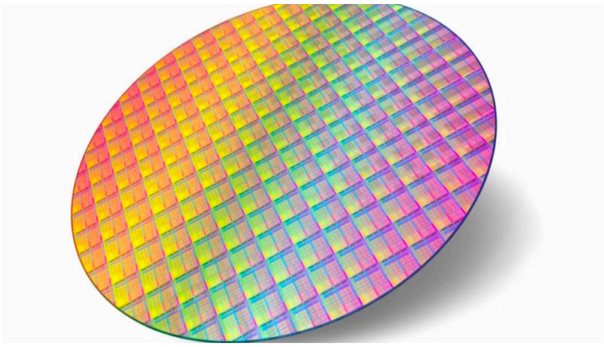


How do the physicist understand solid-state materials?



Superconductors

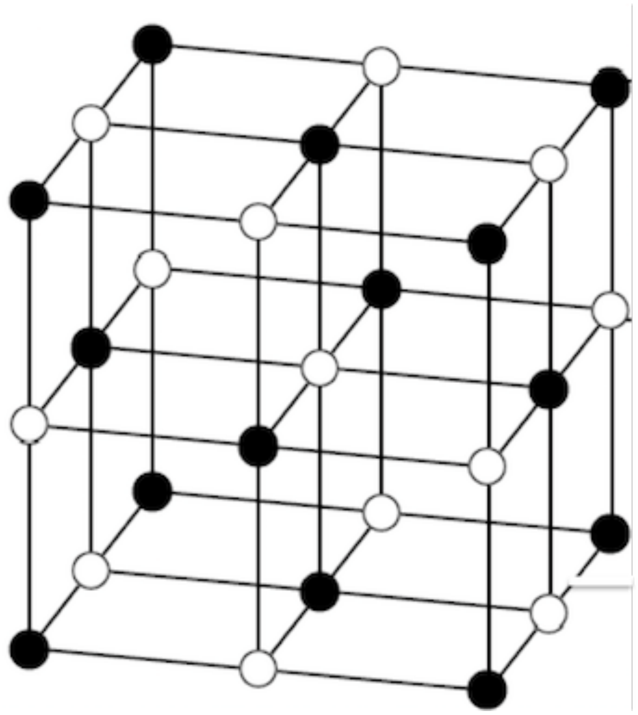
Magnets



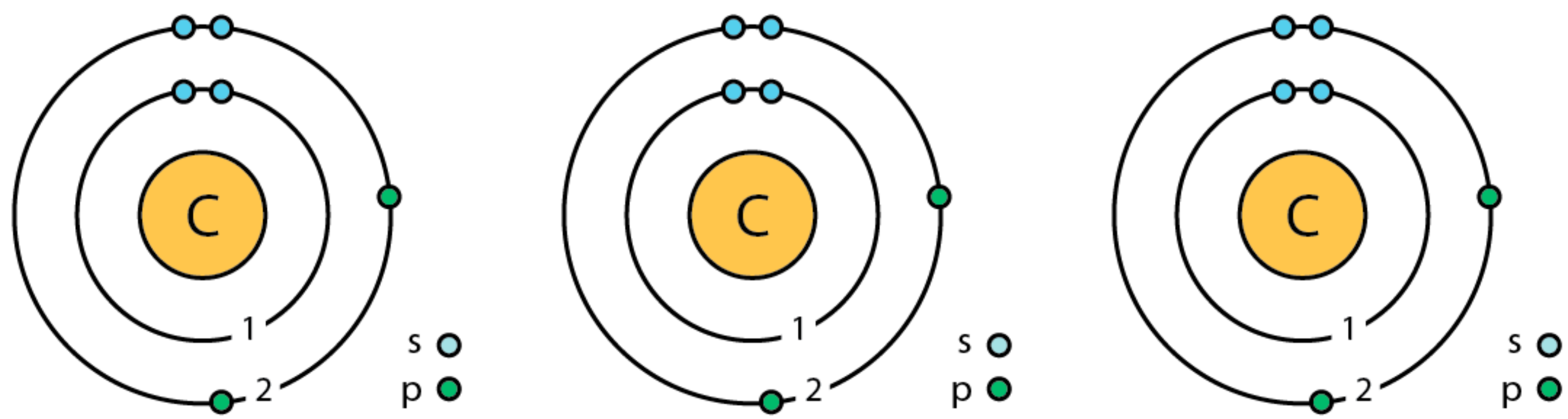
Semi-conductors

Simulating condensed matter systems

Solid-state materials have crystal structures



Atoms (nucleus and electrons)

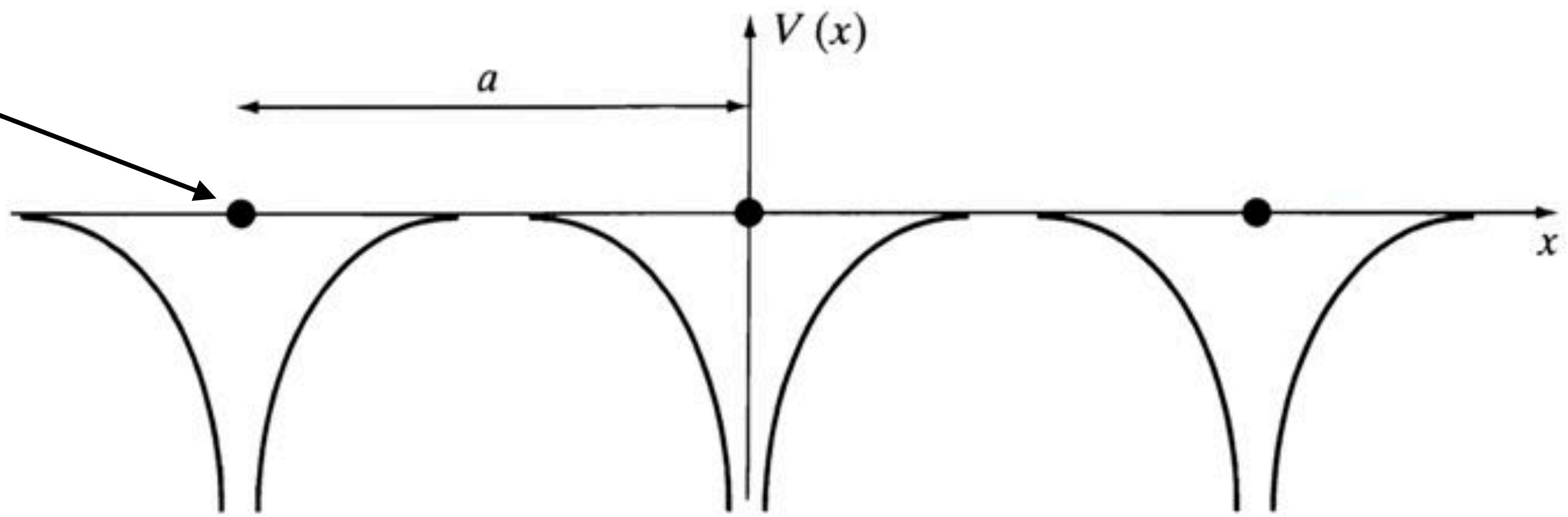


Arrange the atoms very closely

Simulating condensed matter systems

Problems of electron's eigenstate under periodic potential

Atomic ion (nucleus)



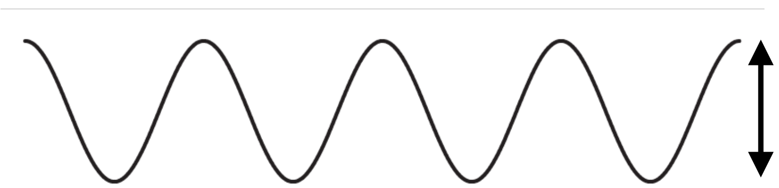
What are the eigenstates of the electron?

$$i\hbar \frac{\partial}{\partial t}\psi(x,t) = \left[\frac{p^2}{2m} + V_0 \sin(k_x x) \right] \psi(x,t)$$

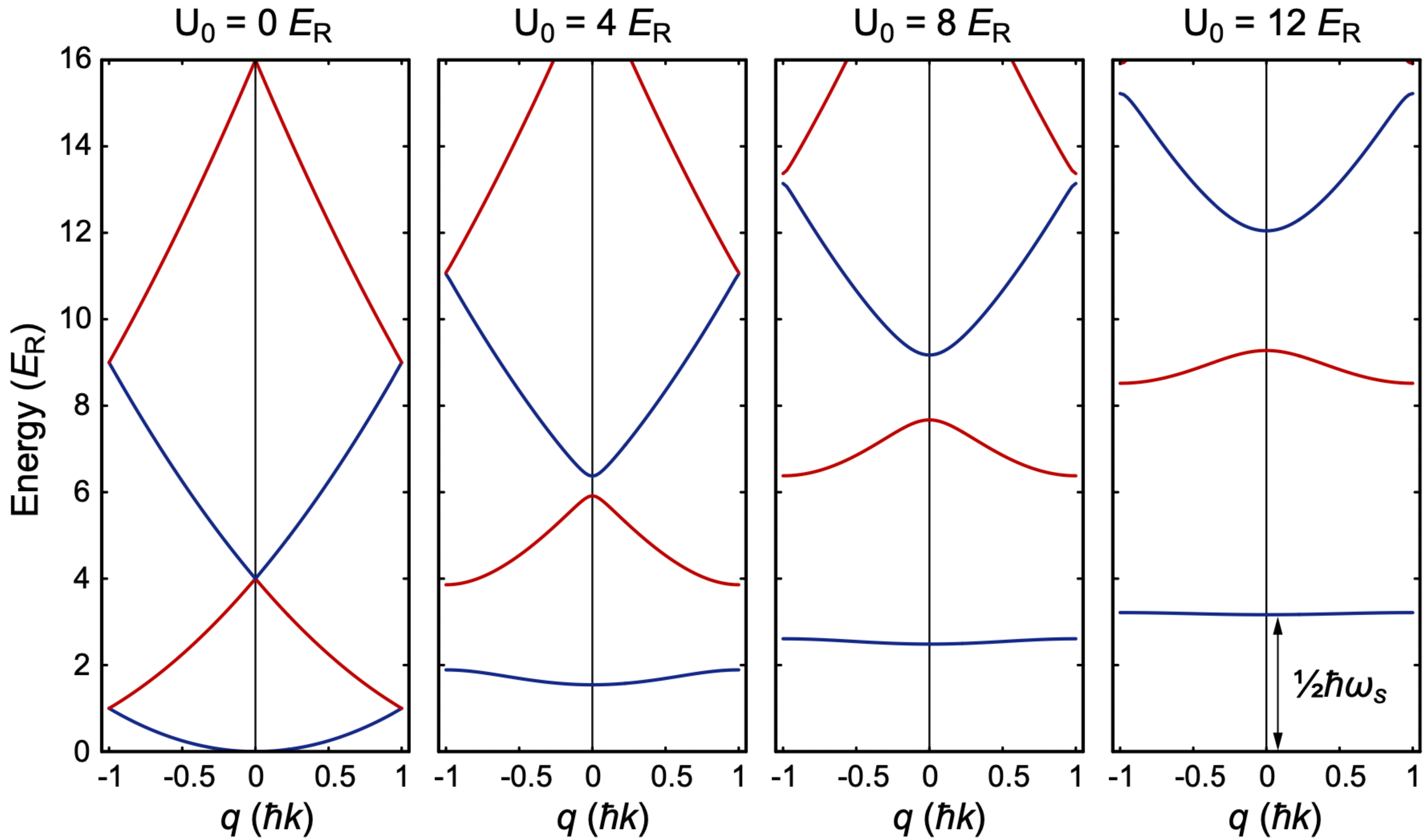
$$\psi_{q,n} = \sum_j e^{iqx_j}\, w_n(x - x_j)$$

Band structure

U_0: Depth of the lattice

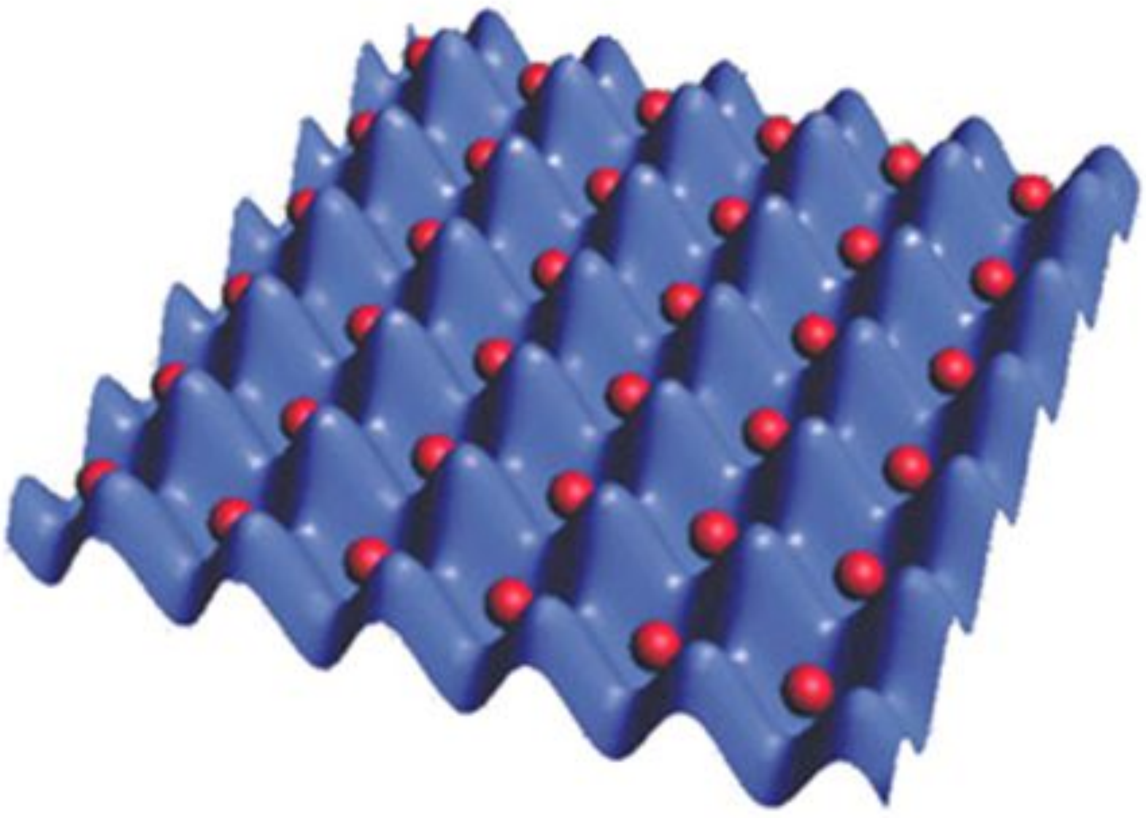


$$E_R = \frac{\hbar k^2}{2m}$$



Simulating condensed matter systems

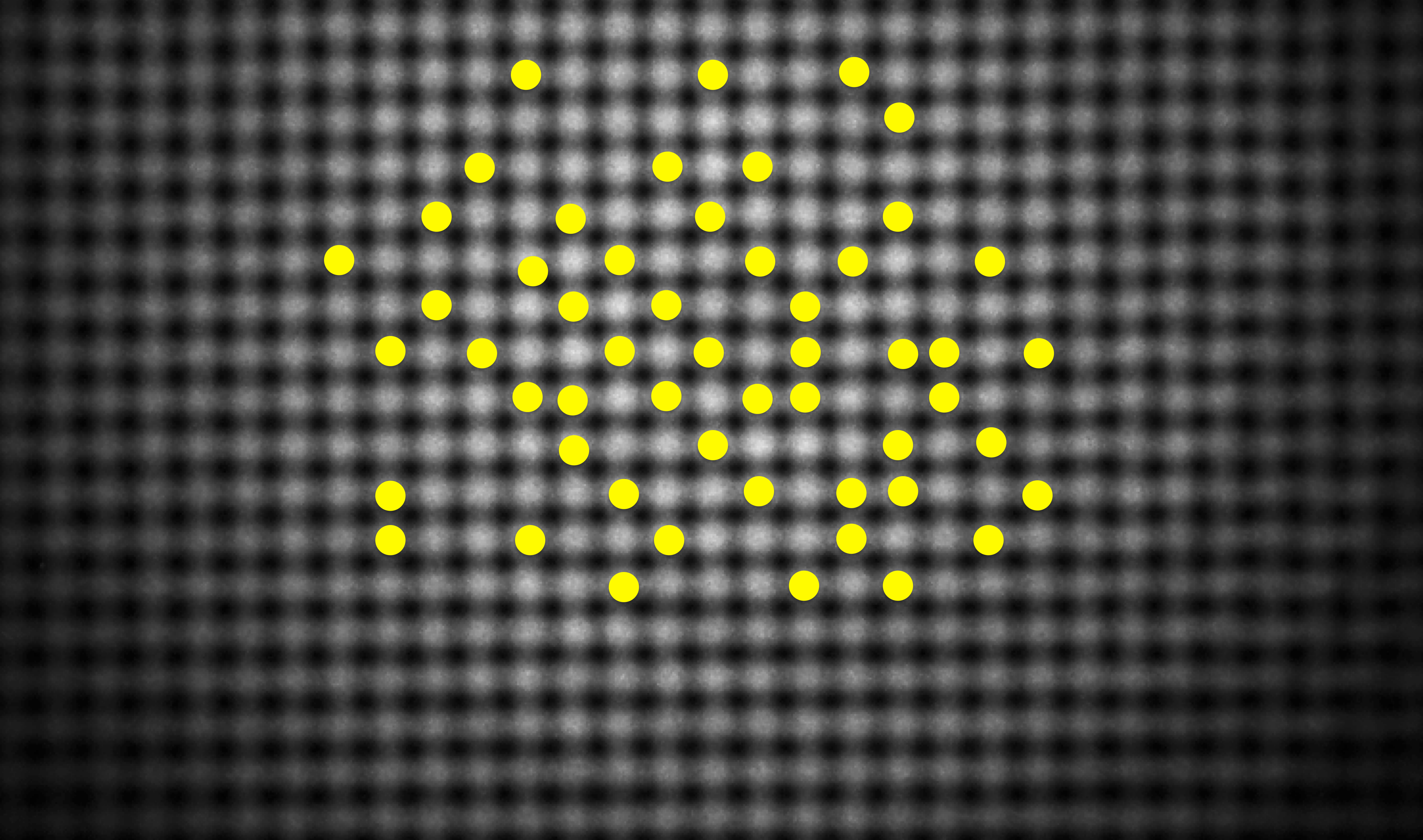
Ultracold atomic matter



$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left[\frac{p^2}{2m} + V_0\sin(k_x x)\right]\psi(x,t)$$

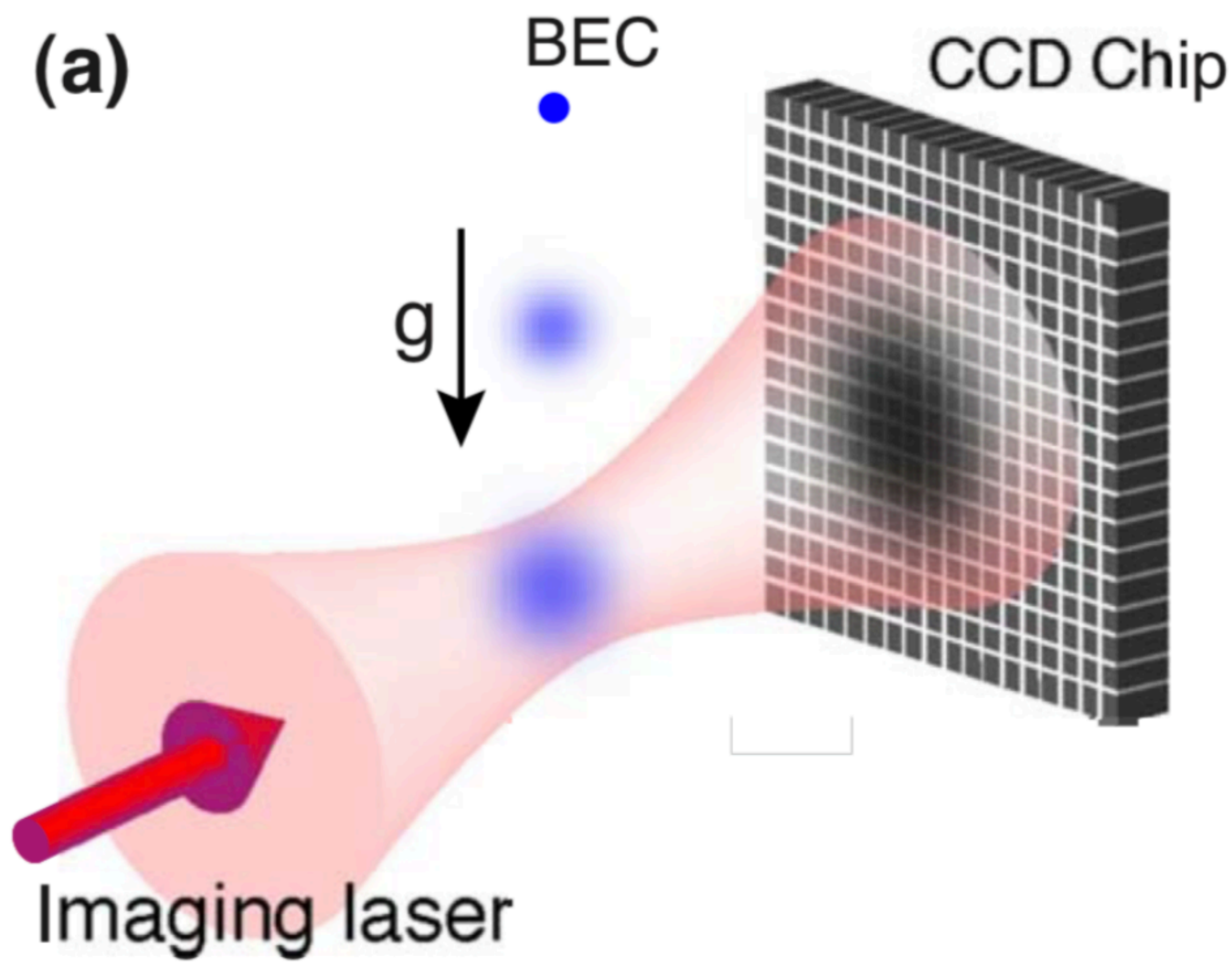
This periodic potential is provided by optical lattice

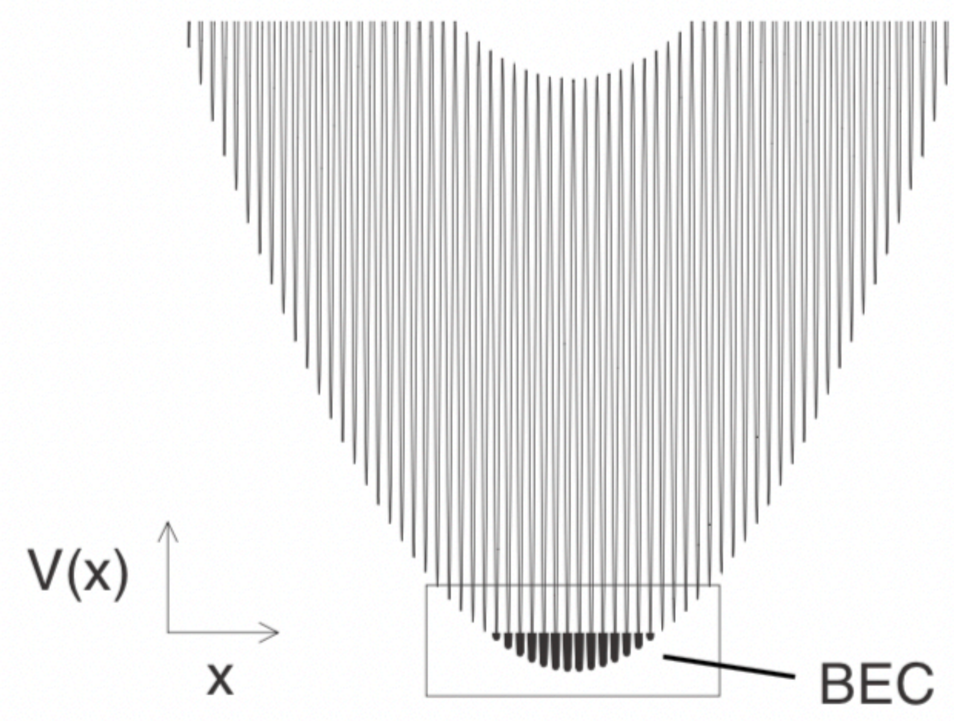
- N ~ 1,000 - 10,000 particles
- Bosons, Fermions, and mixture
- Classically trackable & intractable regime

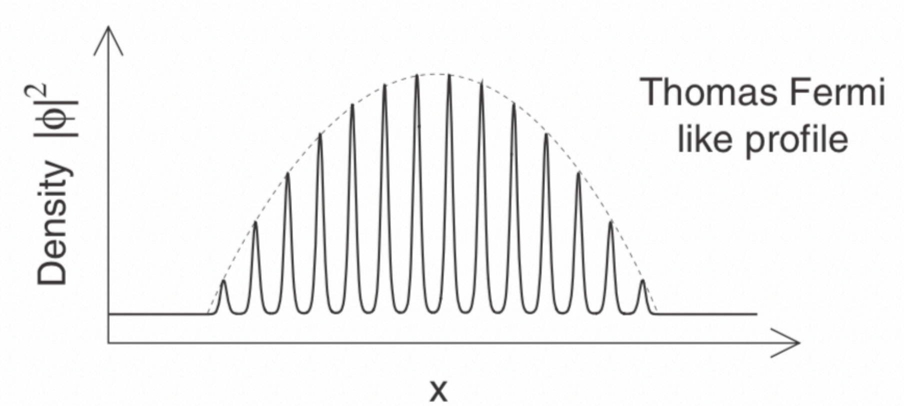


Time-of-flight of BEC in lattice

Q:What happens when we suddenly turn off the lattice?



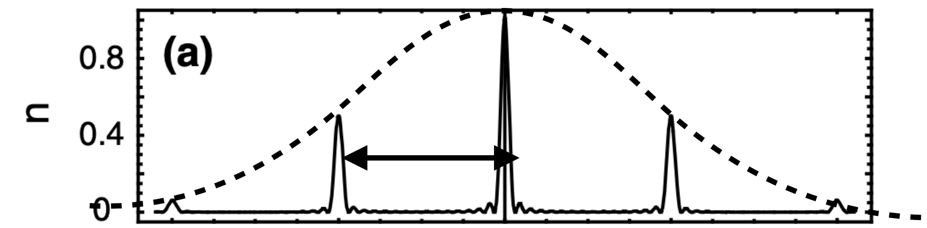




Time-of-flight of BEC in lattice

Q: Three different length scales

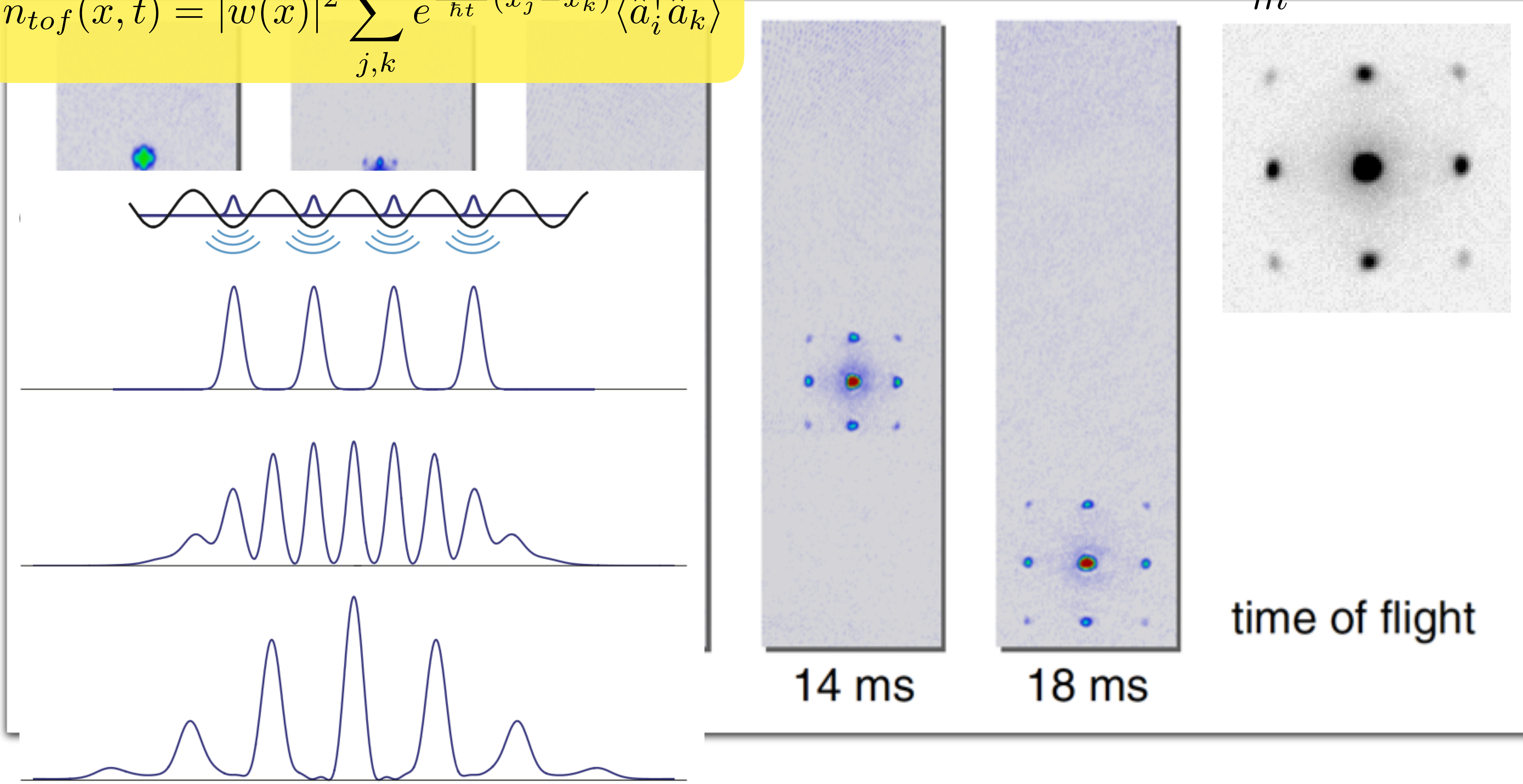
1) Peak distance
2) Peak width
3) Overall envelop



Wannier envelop   Grating interference

$$n_{tof}(x,t) = |w(x)|^2 \sum_{j,k} e^{\frac{imx}{\hbar t}(x_j - x_k)} \langle \hat{a}_i^\dagger \hat{a}_k \rangle$$

$$l = \frac{2\hbar k t}{m}$$



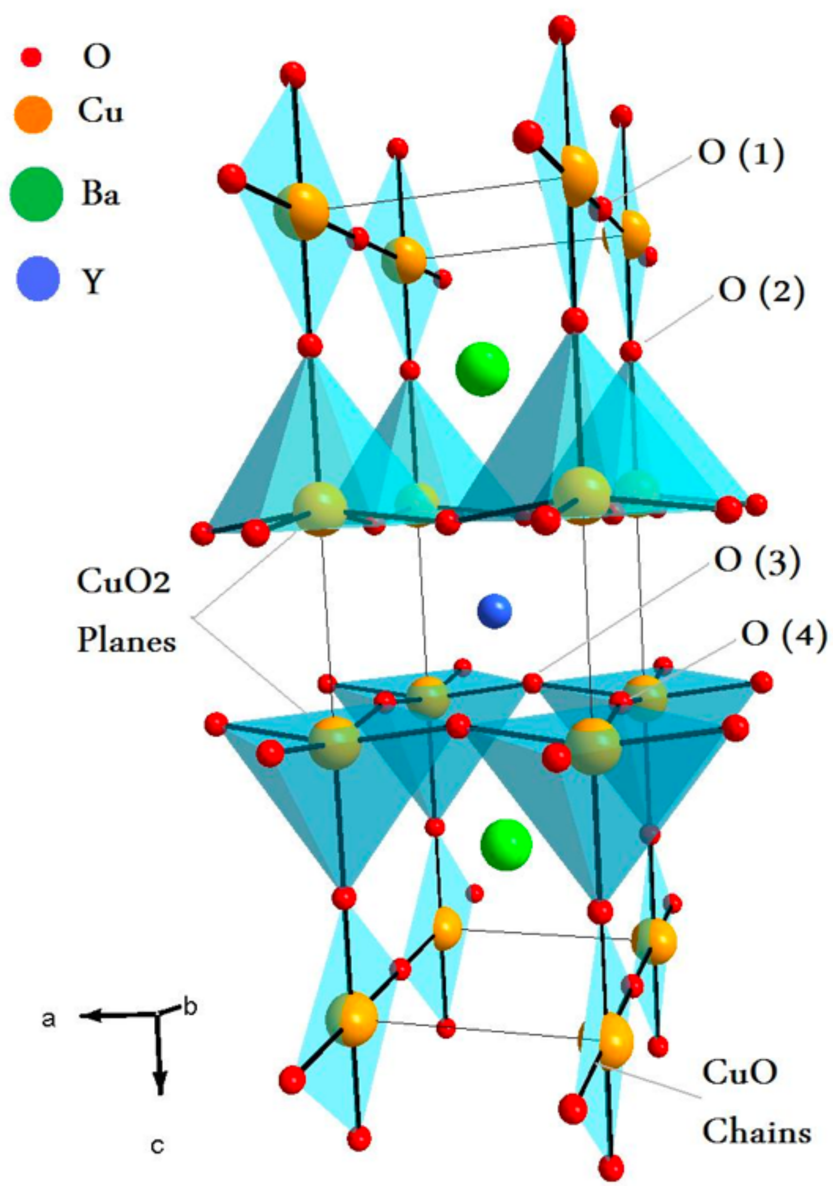
Bose-Hubbard model in optical lattice

D. Jaksch *et al*., Phys. Rev. Lett. **81**, 3108 (1998).
M. Greiner *et al*., Nature **415**, 39 (2002).

Motivations

Why it is difficult?

Many-body quantum problem is usually intractable with classical computers



a. Complex structures and lacks of experimental controllabilities

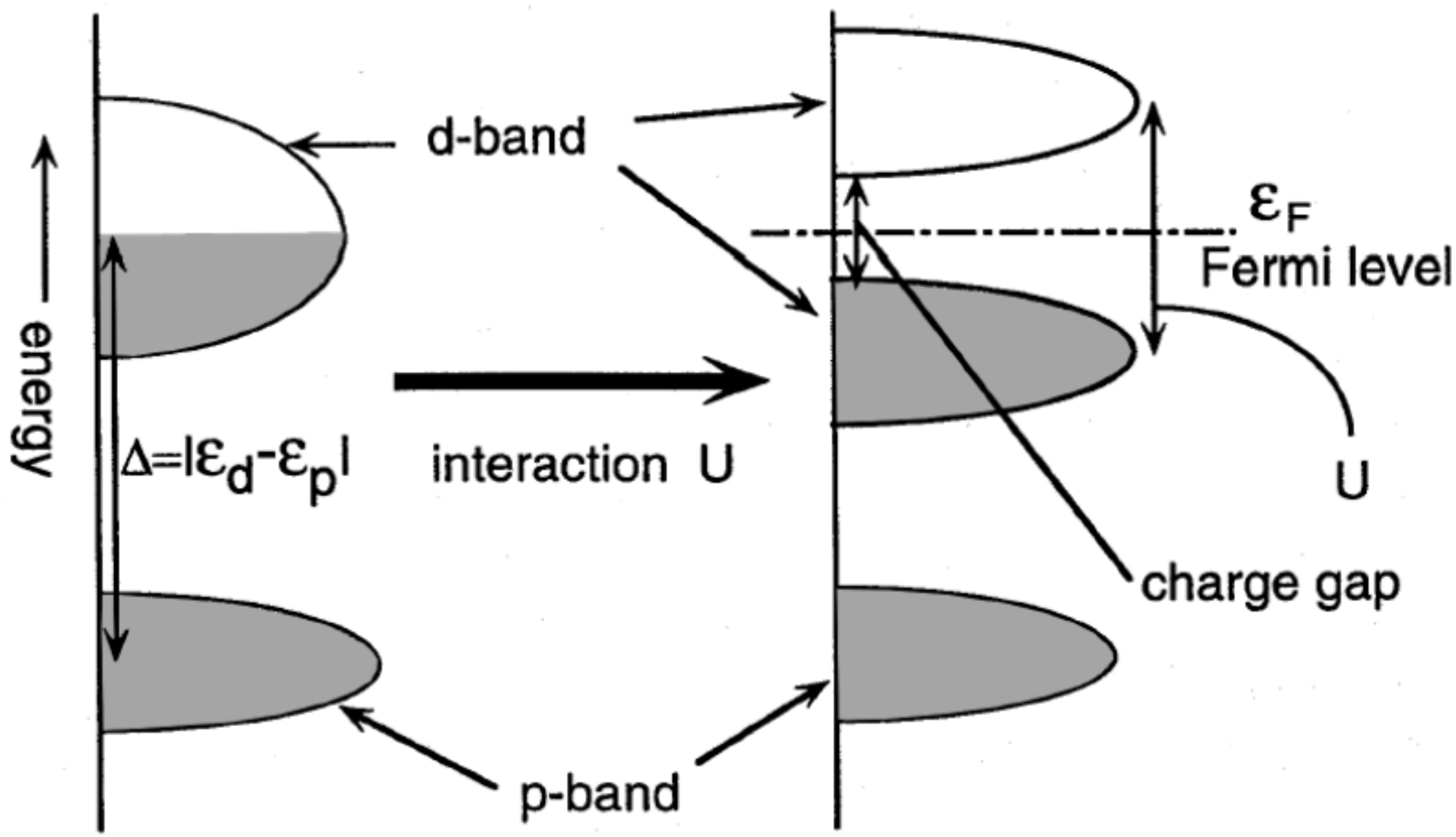
b. Strong interactions and limits of perturbative approaches

c. Even the simple model has no exact solution

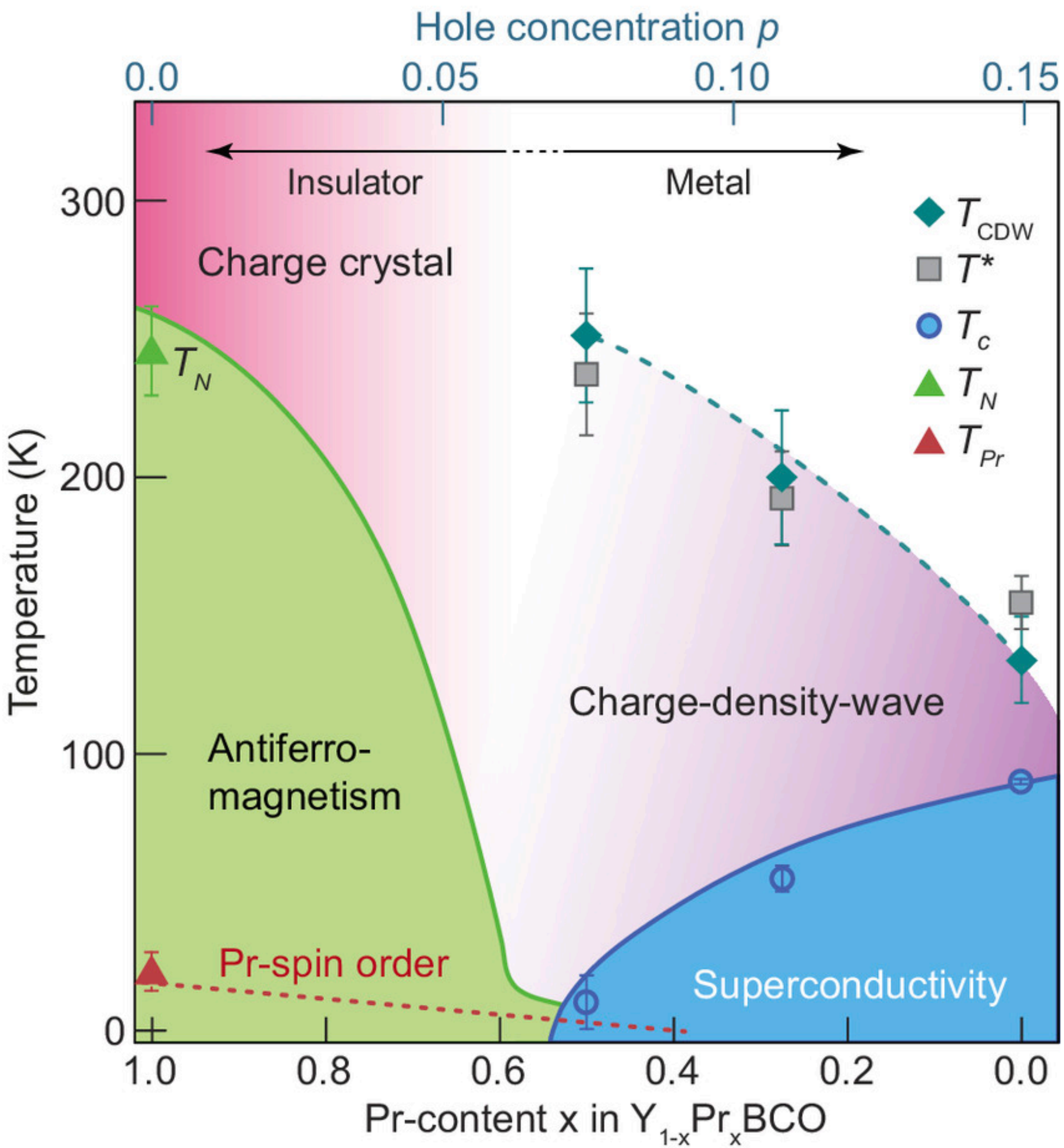
$$H = -t \sum_{(i,j),\sigma} \hat{c}_{i\sigma}^{\dagger} \hat{c}_{j\sigma} + V \sum_{i,\sigma,\sigma'} \hat{c}_{i\sigma}^{\dagger} \hat{c}_{i\sigma'}^{\dagger} \hat{c}_{i\sigma'} \hat{c}_{i\sigma}$$

Fermi-**Hubbard model**

Mott insulator



Mott insulator



Bose-Hubbard Hamiltonian

$$\hat{H} = -J \sum_{<i,j>} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i \varepsilon_i \hat{n}_i$$

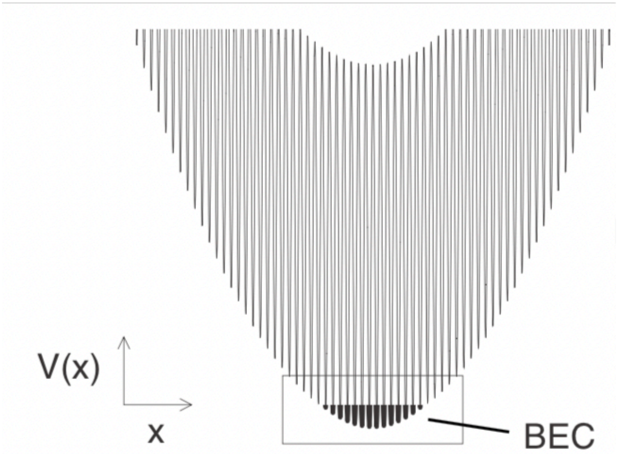
1) Tunneling matrix (hopping) element

$$J = -\int d^3x \; w(\mathbf{x} - \mathbf{x}_i) \left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{x}) \right) w(\mathbf{x} - \mathbf{x}_j)$$

2) Onsite interaction matrix element

$$U = \frac{4\pi\hbar^2 a_s}{m} \int d^3x \; |w(\mathbf{x})|^4$$

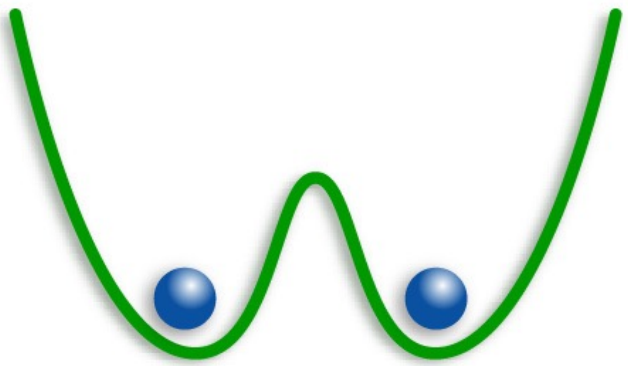
3) External confinement energy



Bose-Hubbard Hamiltonian

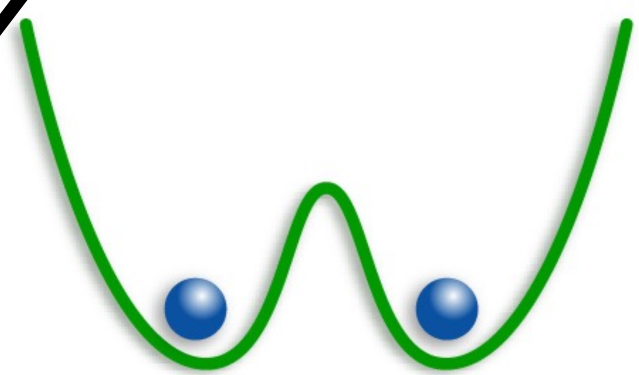
Many-body ground state of the Bose-Hubbard Hamiltonian

$$\frac{\text{Interaction Energy } U}{\text{Kinetic Energy } J}$$



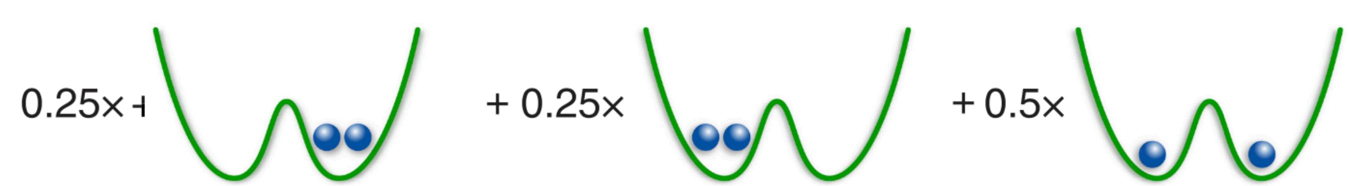
Many-body ground state of the Bose-Hubbard Hamiltonian

$$\frac{\text{Interaction Energy } U}{\text{Kinetic Energy } J} \ll 1$$



$$\Psi = \psi_1 \otimes \psi_2$$

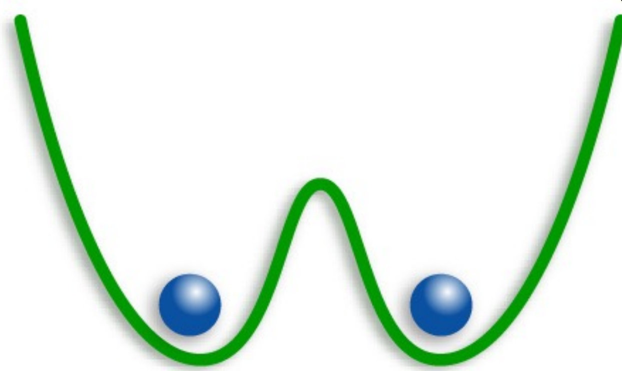
$$= \left(\frac{1}{\sqrt{2}} (\psi_l + \psi_r) \right)^2$$



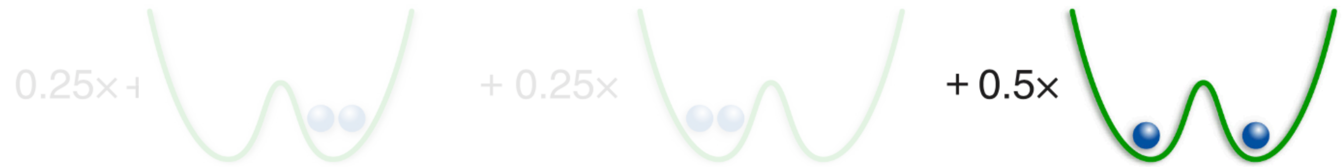
Bose-Hubbard Hamiltonian

Many-body ground state of the Bose-Hubbard Hamiltonian

$$\frac{\text{Interaction Energy } U}{\text{Kinetic Energy } J} \gg 1$$



$$\Psi = |1,1\rangle = \frac{1}{\sqrt{2}}(\psi_l \otimes \psi_r + \psi_r \otimes \psi_l)$$

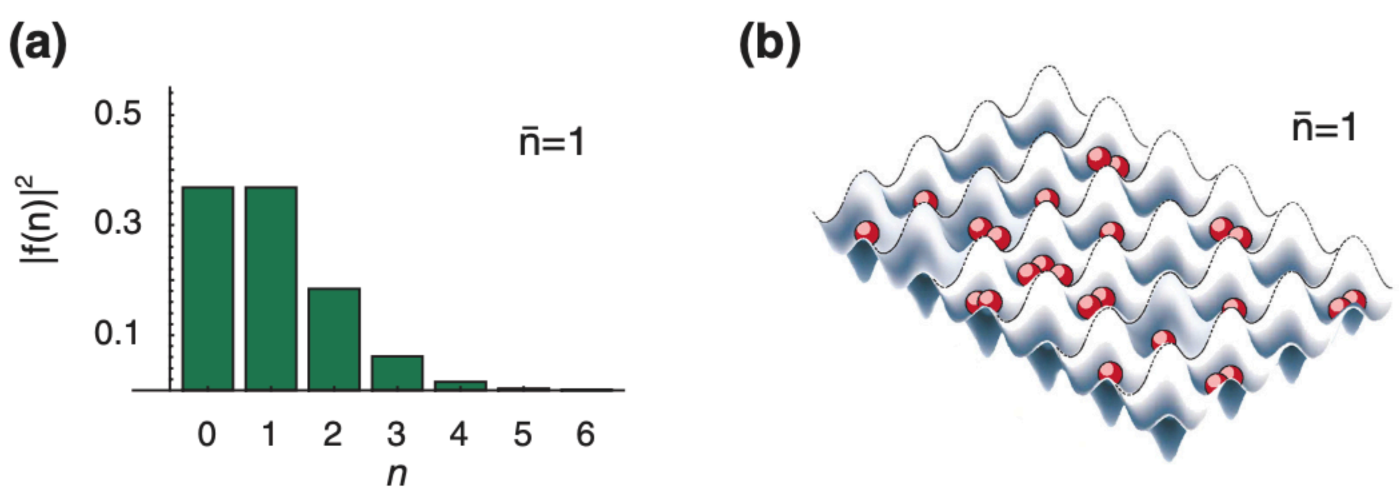


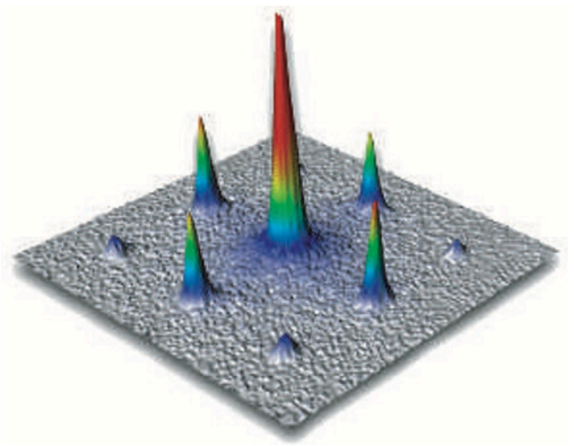
Bose-Hubbard Hamiltonian

Many-body ground state of the Bose-Hubbard Hamiltonian

$$\hat{H} = -J \sum_{<i,j>} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i \varepsilon_i \hat{n}_i$$

Weakly interacting regime ($J \gg U$) $\quad \Psi_{SF} \propto \left(\sum_{i=1}^{M} \hat{a}_i^\dagger \right)^N |0\rangle$

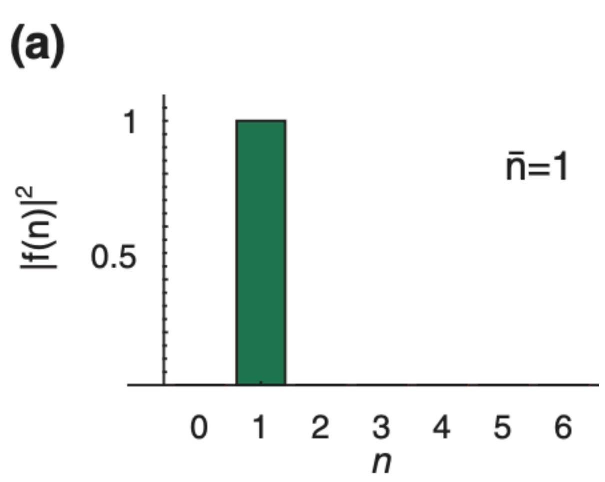


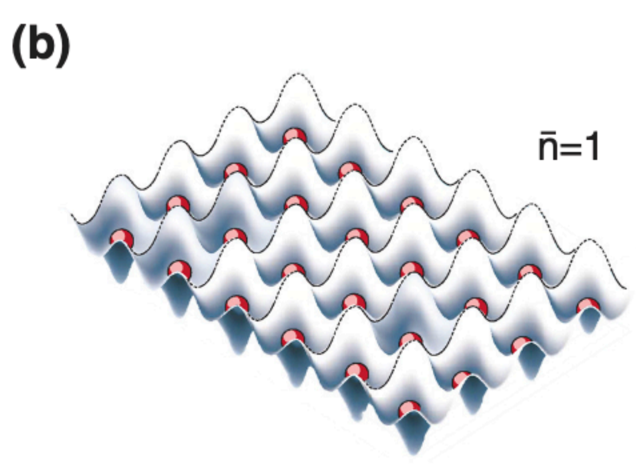


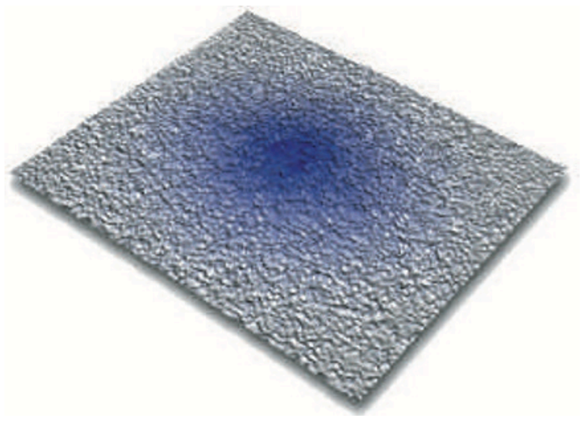
Many-body ground state of the Bose-Hubbard Hamiltonian

$$\hat{H} = -J \sum_{<i,j>} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i \varepsilon_i \hat{n}_i$$

Strongly interacting regime ($J \ll U$) $\Psi_{MI} \propto \prod_{i=1}^{M} \hat{a}_i^\dagger |0\rangle$





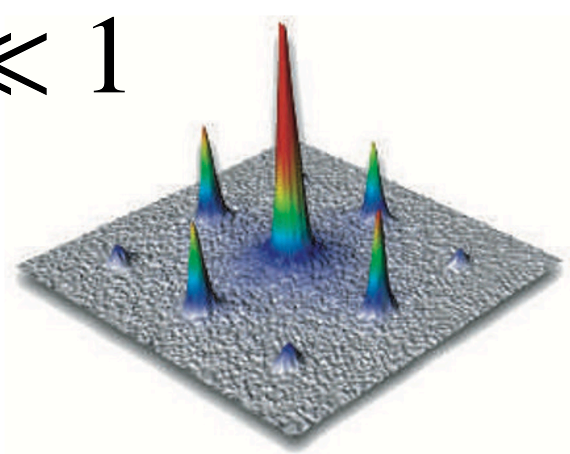


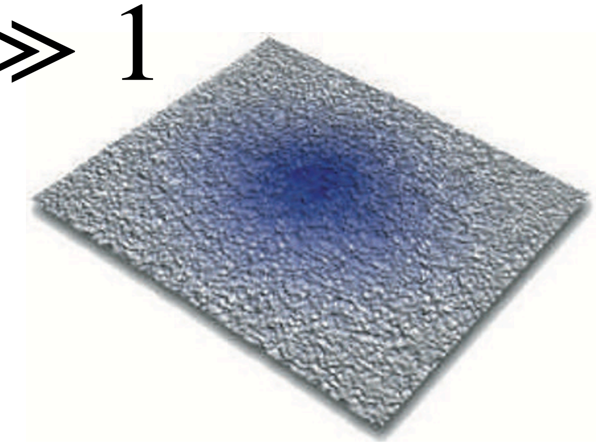
Many-body ground state of the Bose-Hubbard Hamiltonian

$$\frac{\text{Interaction Energy } U}{\text{Kinetic Energy } J}$$

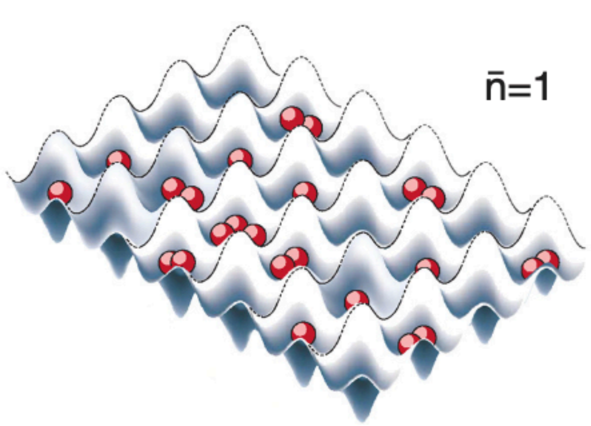
Superfluid **Mott insulator** U/J

$\ll 1$ $(U/J)_c$ $\gg 1$

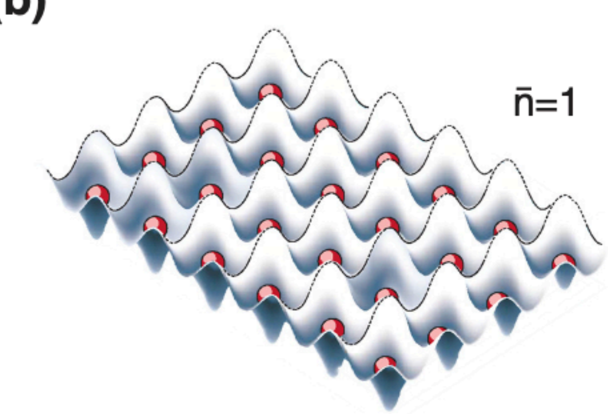




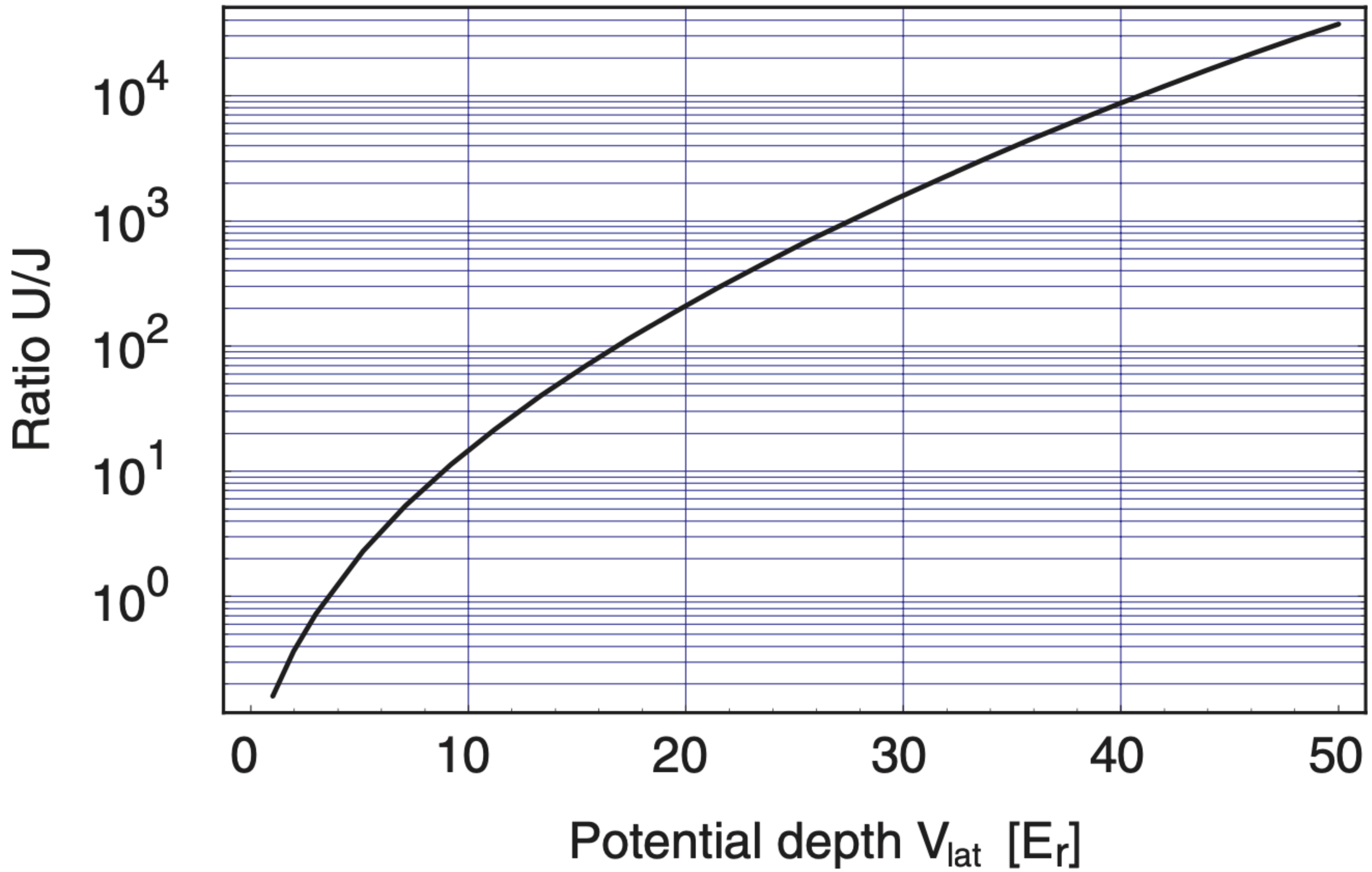
(b)



(b)



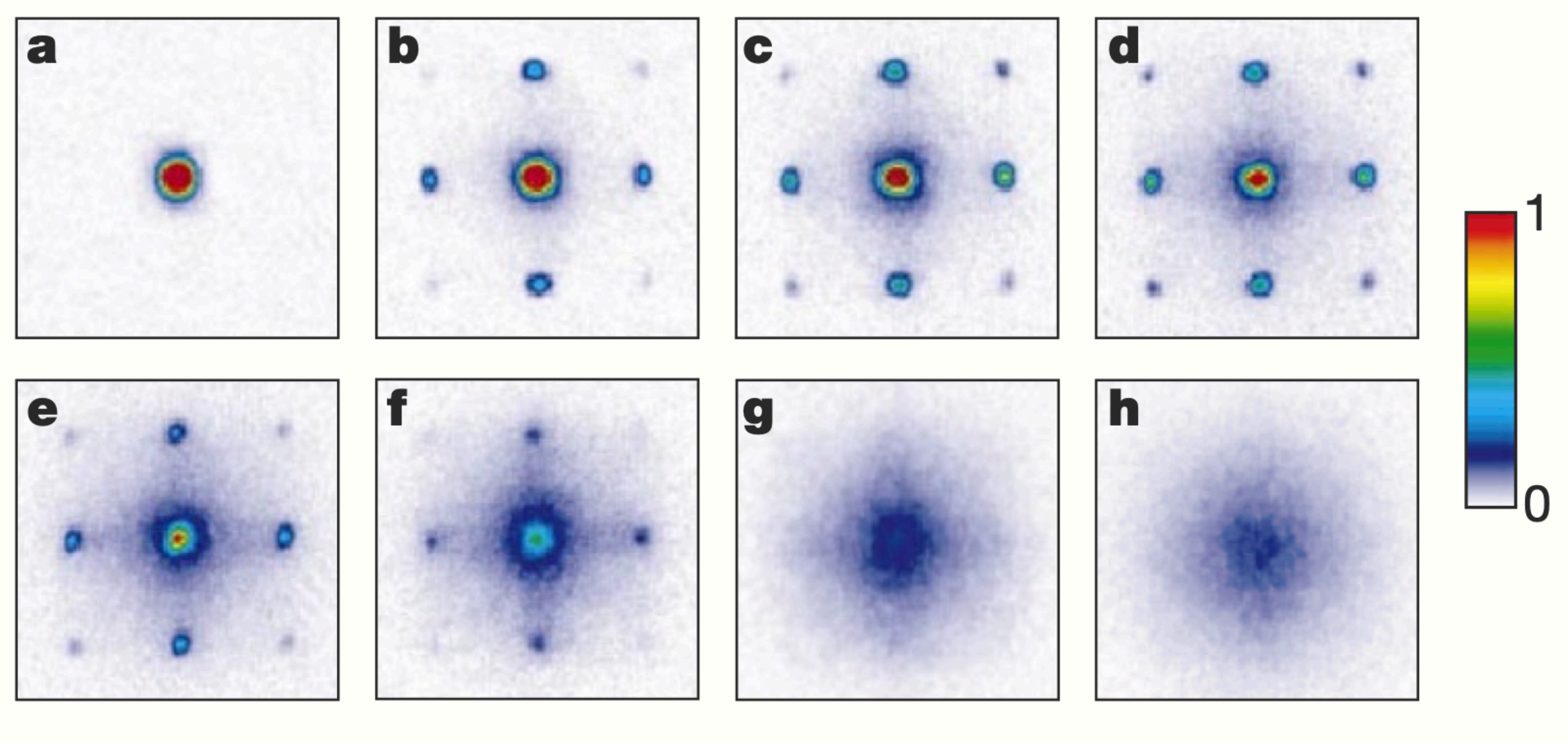
Superfluid-Mott insulator transition in optical lattice



Increasing the laser power of the optical lattices, we can tune the U/J

Bose-Hubbard Hamiltonian

Superfluid-Mott insulator transition in optical lattice



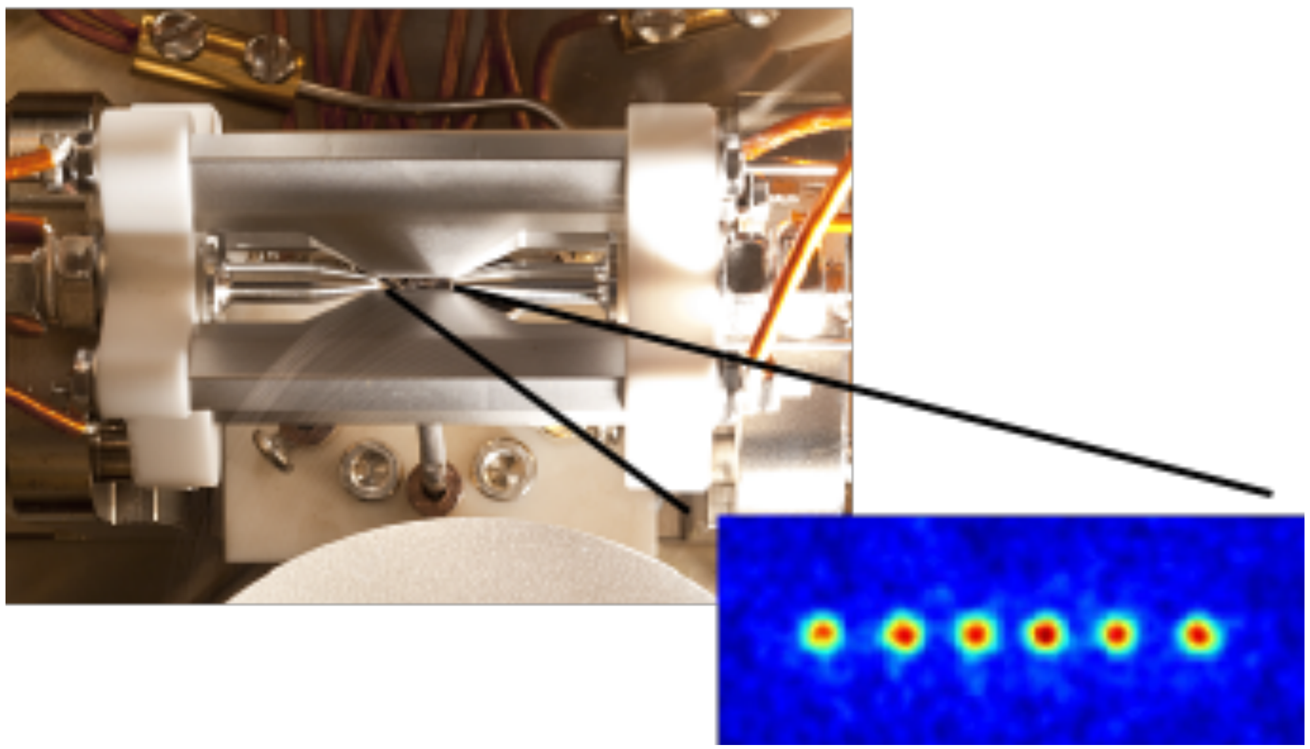
M. Greiner et al., Nature 415, 39 (2002).

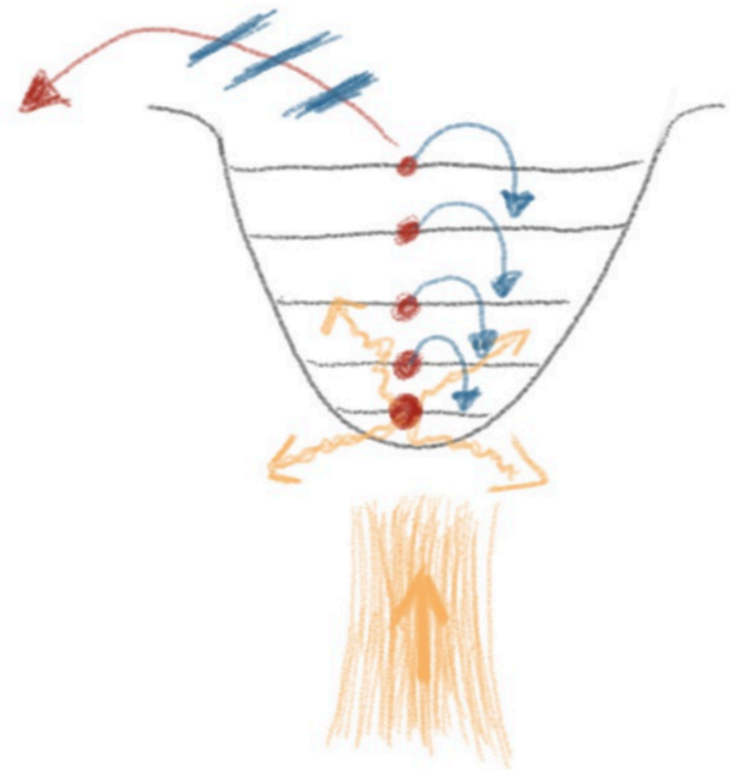
Single-atom imaging and addressing

Bakr *et al*., Nature **5**, 462 (2009).
Bakr *et al*., Science **329**, 547 (2010).
Sherson *et al*., Nature **467**, 68 (2010).
Weitenberg *et al*., Nature **471**, 319 (2011).

High resolution imaging system

Trapped ion system



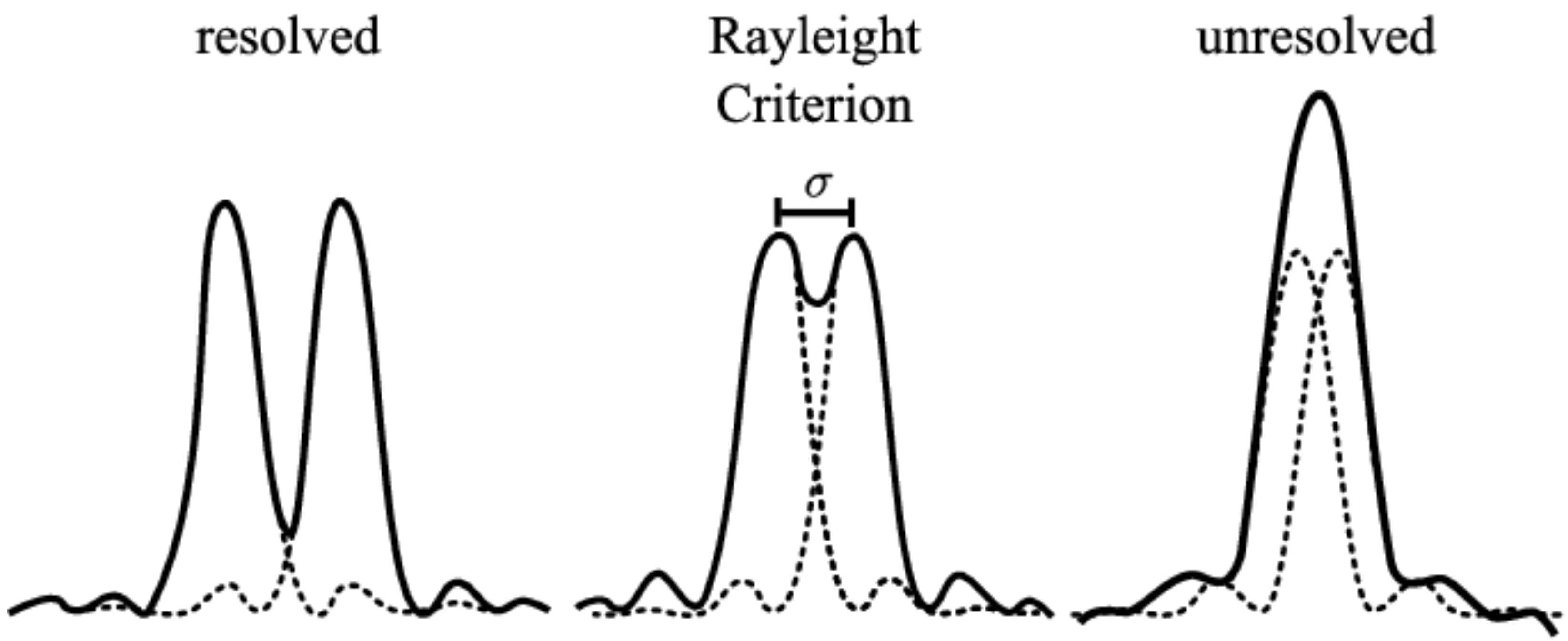


We need to collect enough photon before the atom leaves or moves !!

The short distance in the lattice is a central challenge for single-atom detection

High resolution imaging system

Challenges: diffraction limited imaging system



$$d_{res} = 1.22\frac{\lambda}{2NA}$$

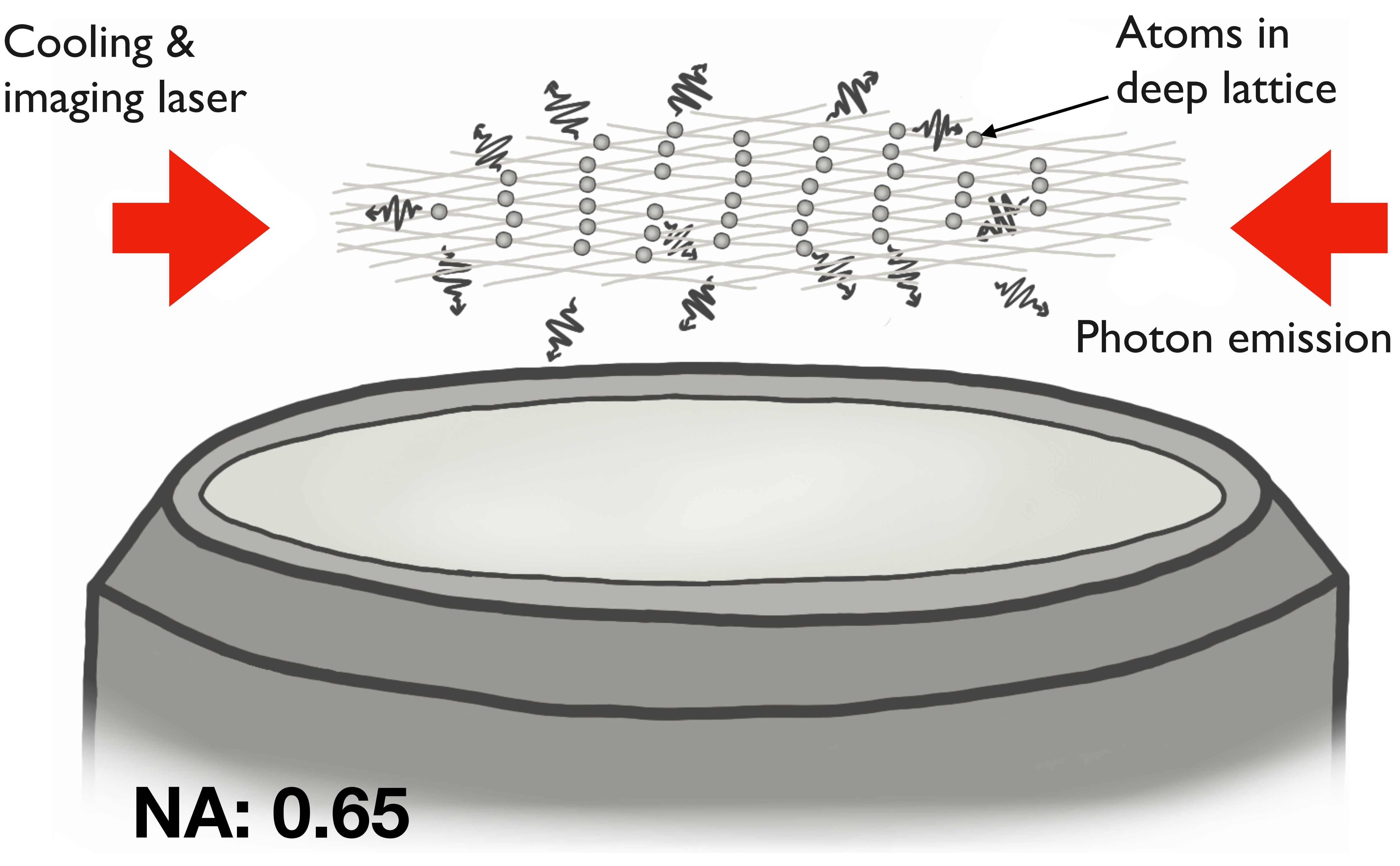
$d_{res} \geq a_{lat}$

$\lambda \sim 600 - 800$ nm

$$NA \geq 0.5$$

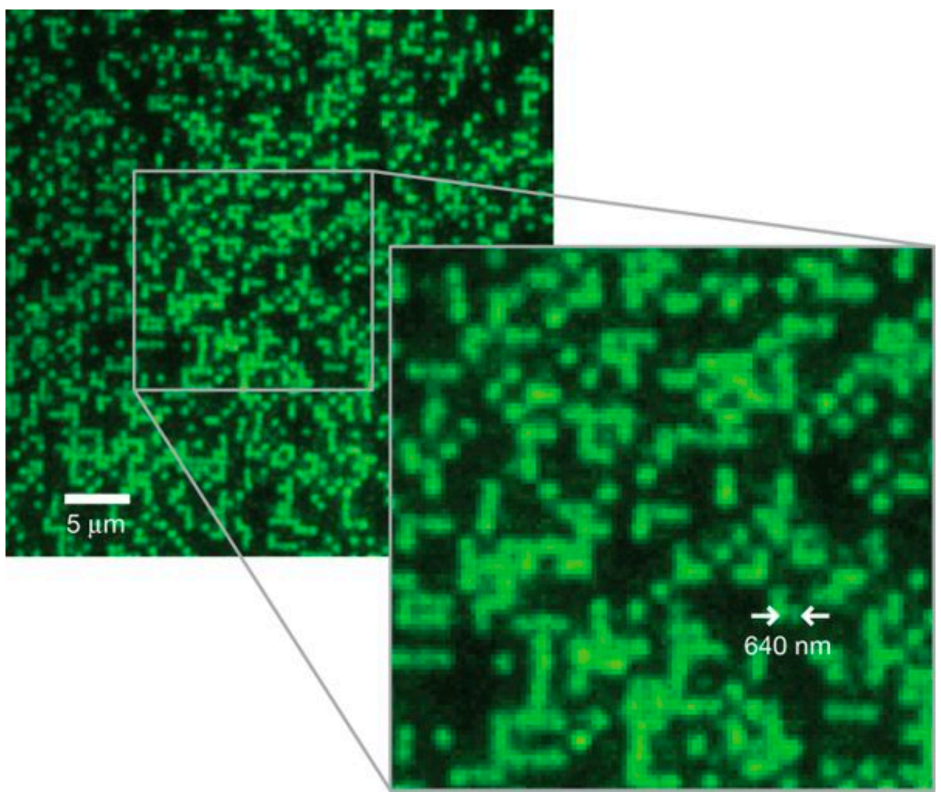
Quantum gas microscope

Place a high NA objective lens near the atoms & collect photons



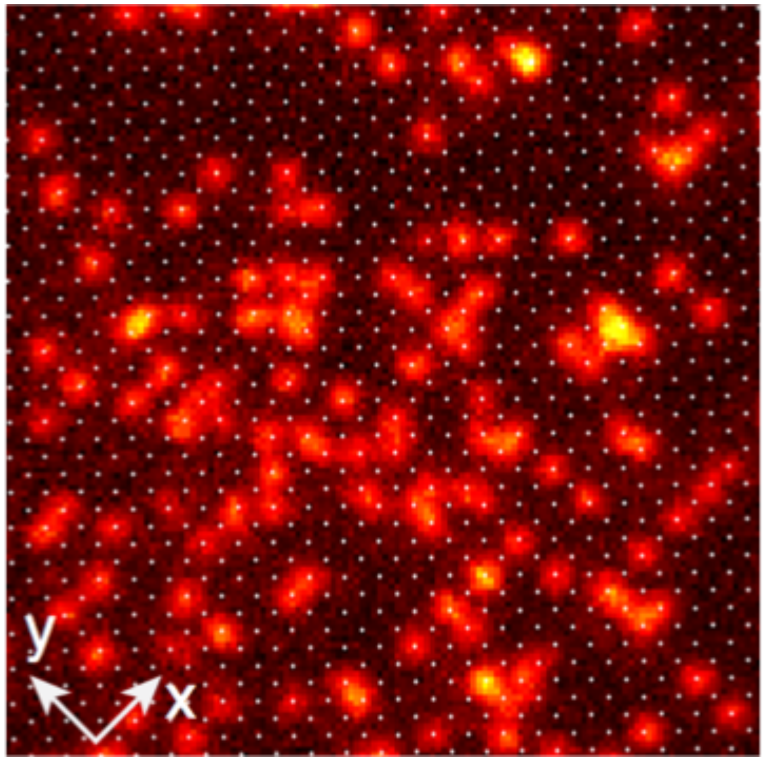
Quantum gas microscope

The first single-atom images in two dimensional optical lattices



Markus Greiner (Harvard)

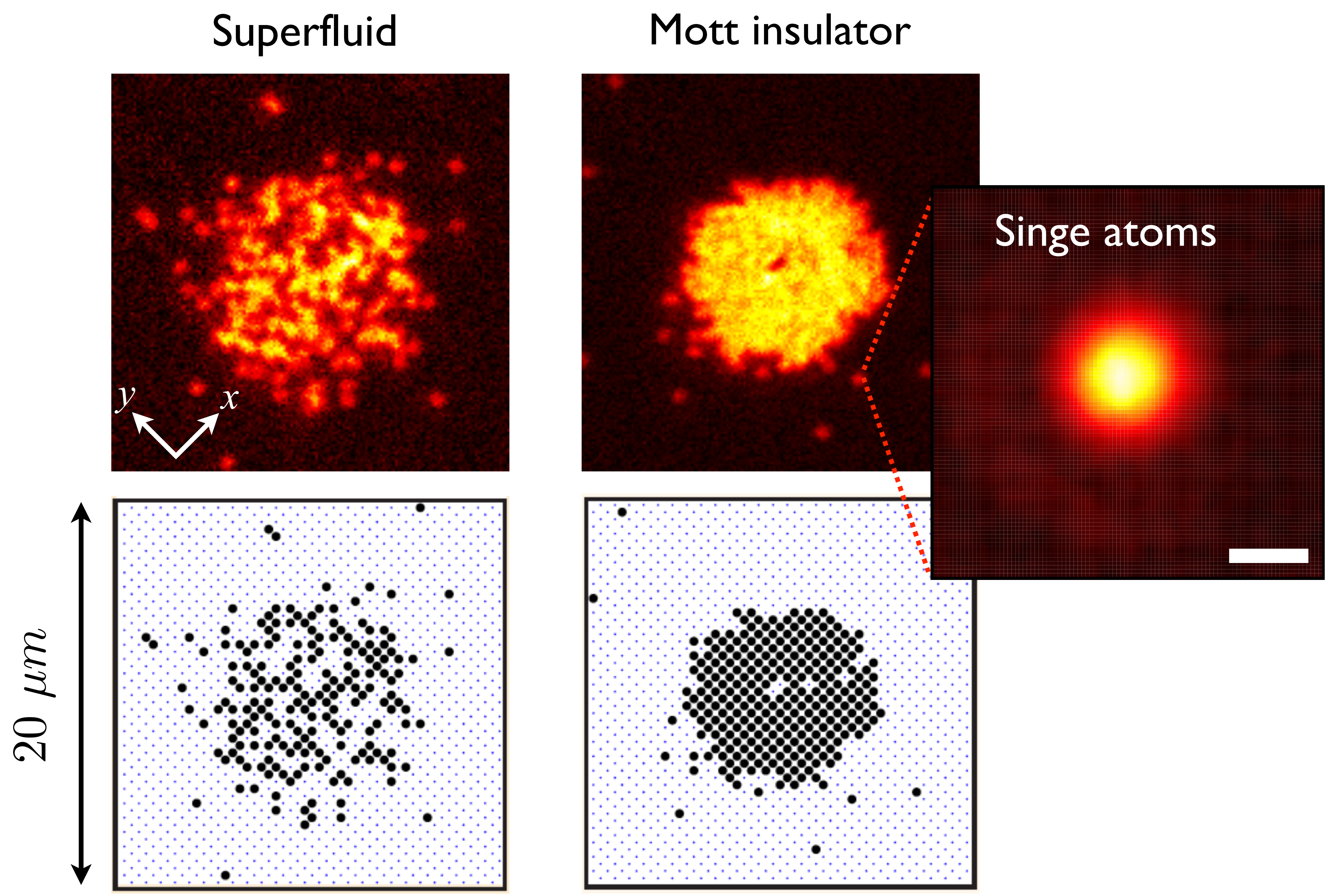
Nature 462, 74 (2009).



Immanuel Bloch (MPQ)

Nature 467, 68 (2010).

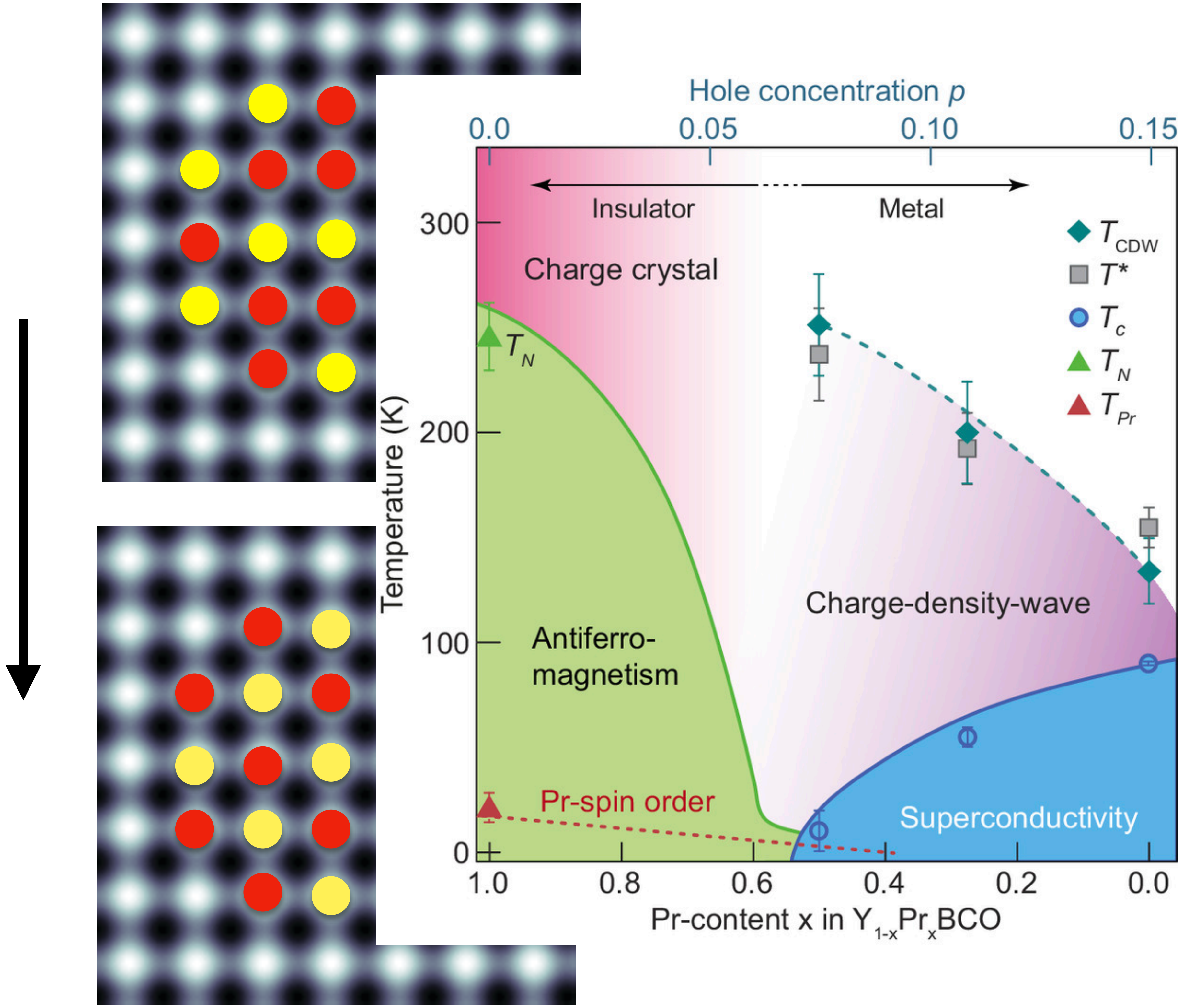
Quantum gas microscope



J. F. Sherson *et al.*, Nature **467**, 68 (2010).

Fermi gas microscope

Reduce temperature



Anti-ferromagnetic spin order in optical lattices

REPORTS

QUANTUM SIMULATION

Site-resolved measurement of the spin-correlation function in the Fermi-Hubbard model

Maxwell F. Parsons, Anton Mazurenko, Christie S. Chiu, Geoffrey Ji, Daniel Greif, Markus Greiner*

QUANTUM SIMULATION

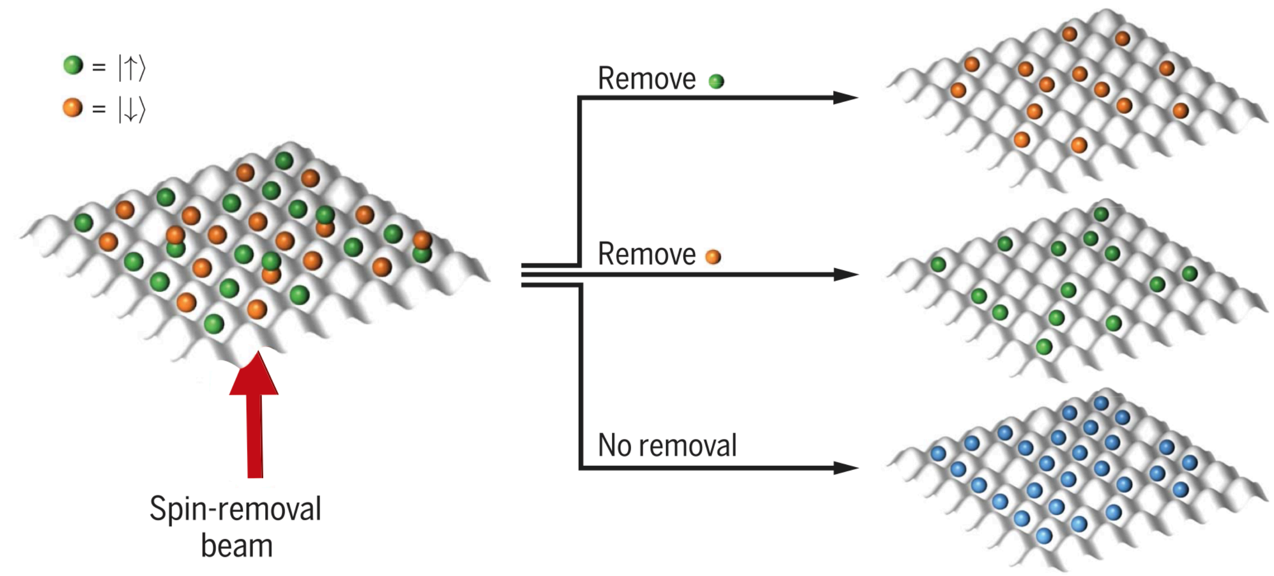
Spin- and density-resolved microscopy of antiferromagnetic correlations in Fermi-Hubbard chains

Martin Boll,[1*] Timon A. Hilker,[1*] Guillaume Salomon,[1*] Ahmed Omran,[1] Jacopo Nespolo,[2] Lode Pollet,[2] Immanuel Bloch,[1,2] Christian Gross[1]†

QUANTUM SIMULATION

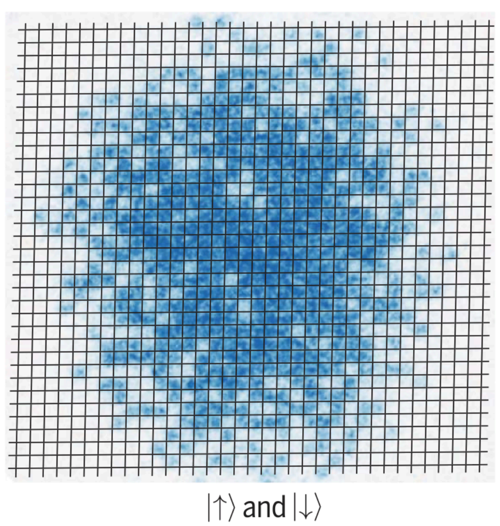
Observation of spatial charge and spin correlations in the 2D Fermi-Hubbard model

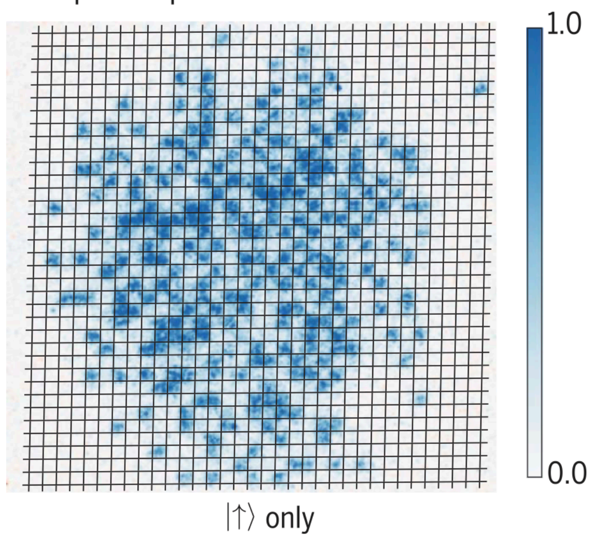
Lawrence W. Cheuk,[1*] Matthew A. Nichols,[1*] Katherine R. Lawrence,[1] Melih Okan,[1] Hao Zhang,[1] Ehsan Khatami,[2] Nandini Trivedi,[3] Thereza Paiva,[4] Marcos Rigol,[5] Martin W. Zwierlein[1]†



Detecting both spins: The image shows a fermionic Mott insulator characterized by a uniform central density.

Detecting one spin: Spin correlations result in the apparent checkerboard pattern after removal of one spin component.





M. F. Parsons *et al*., Science **353**, 1253 (2016).
M. Boll *et al*., Science **353**, 1257 (2016).
L. W. Cheuk *et al*., Science **353**, 1260 (2016).

Fermi gas microscope

QUANTUM SIMULATION

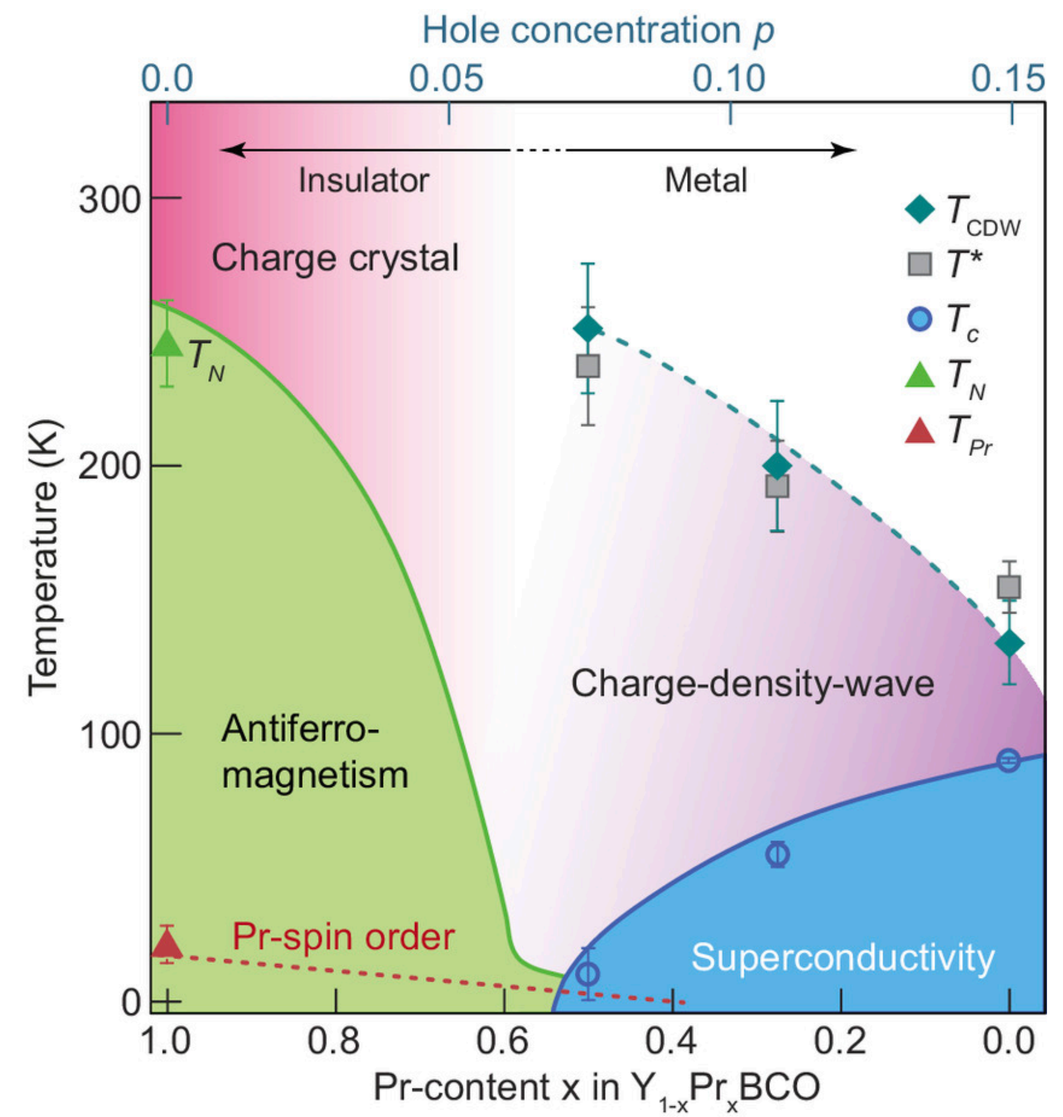
Spin transport in a Mott insulator of ultracold fermions

Matthew A. Nichols[1,2,3], Lawrence W. Cheuk[2,4], Melih Okan[1,2,3], Thomas R. Hartke[1,2,3], Enrique Mendez[1,2,3], T. Senthil[1], Ehsan Khatami[5], Hao Zhang[1,2,3], Martin W. Zwierlein[1,2,3]*

QUANTUM SIMULATION

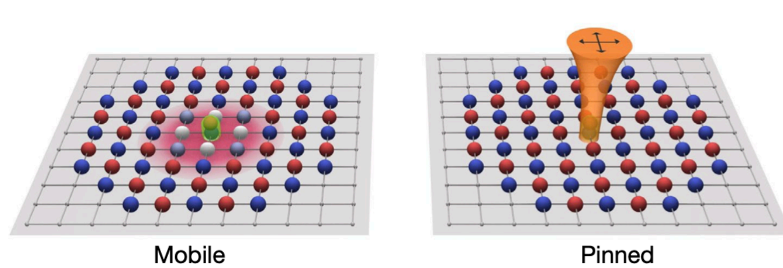
Bad metallic transport in a cold atom Fermi-Hubbard system

Peter T. Brown[1], Debayan Mitra[1], Elmer Guardado-Sanchez[1], Reza Nourafkan[2], Alexis Reymbaut[2], Charles-David Hébert[2], Simon Bergeron[2], A.-M. S. Tremblay[2,3], Jure Kokalj[4,5], David A. Huse[1], Peter Schauß[1]*, Waseem S. Bakr[1]†

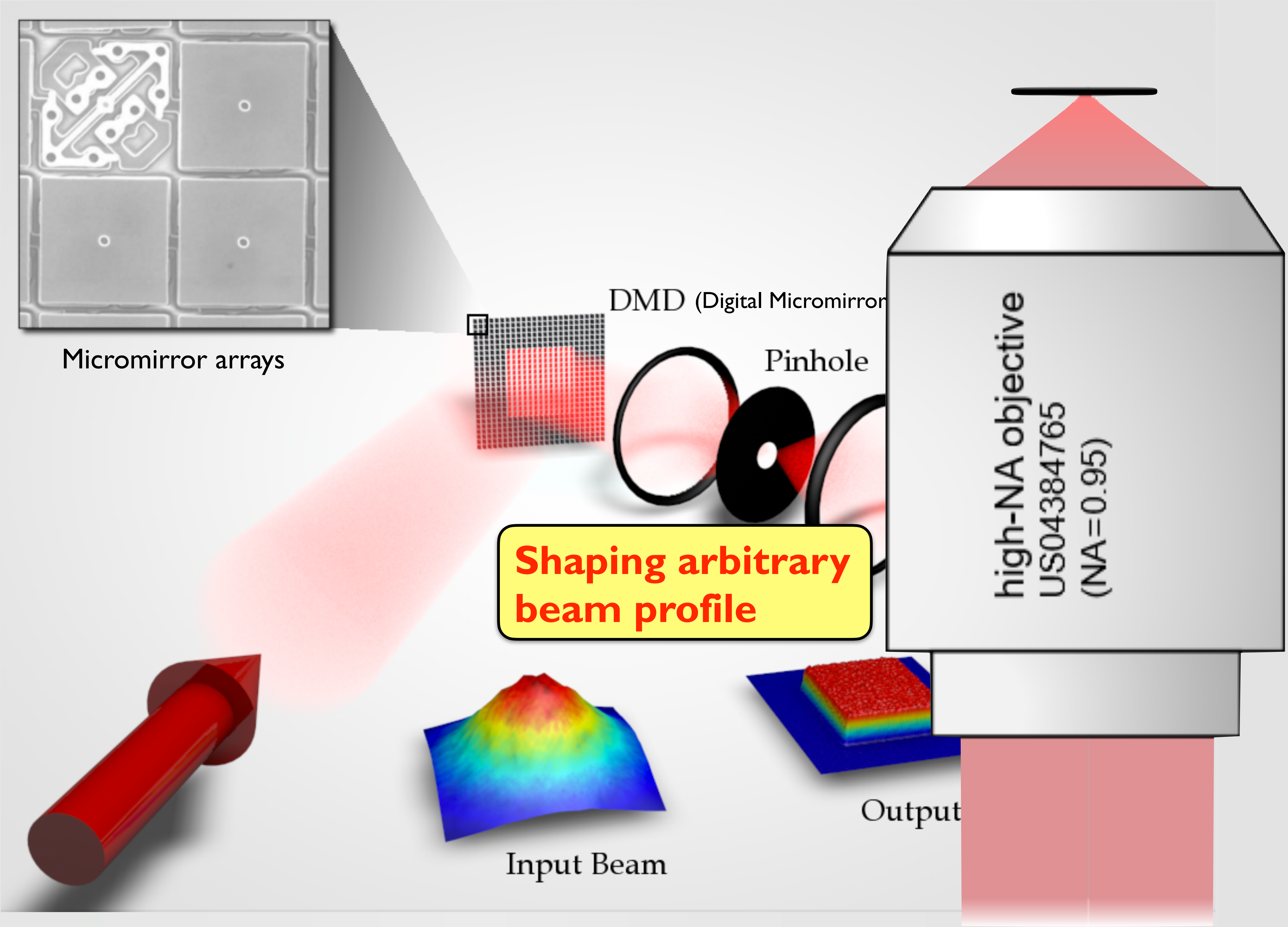


Imaging magnetic polarons in the doped Fermi–Hubbard model

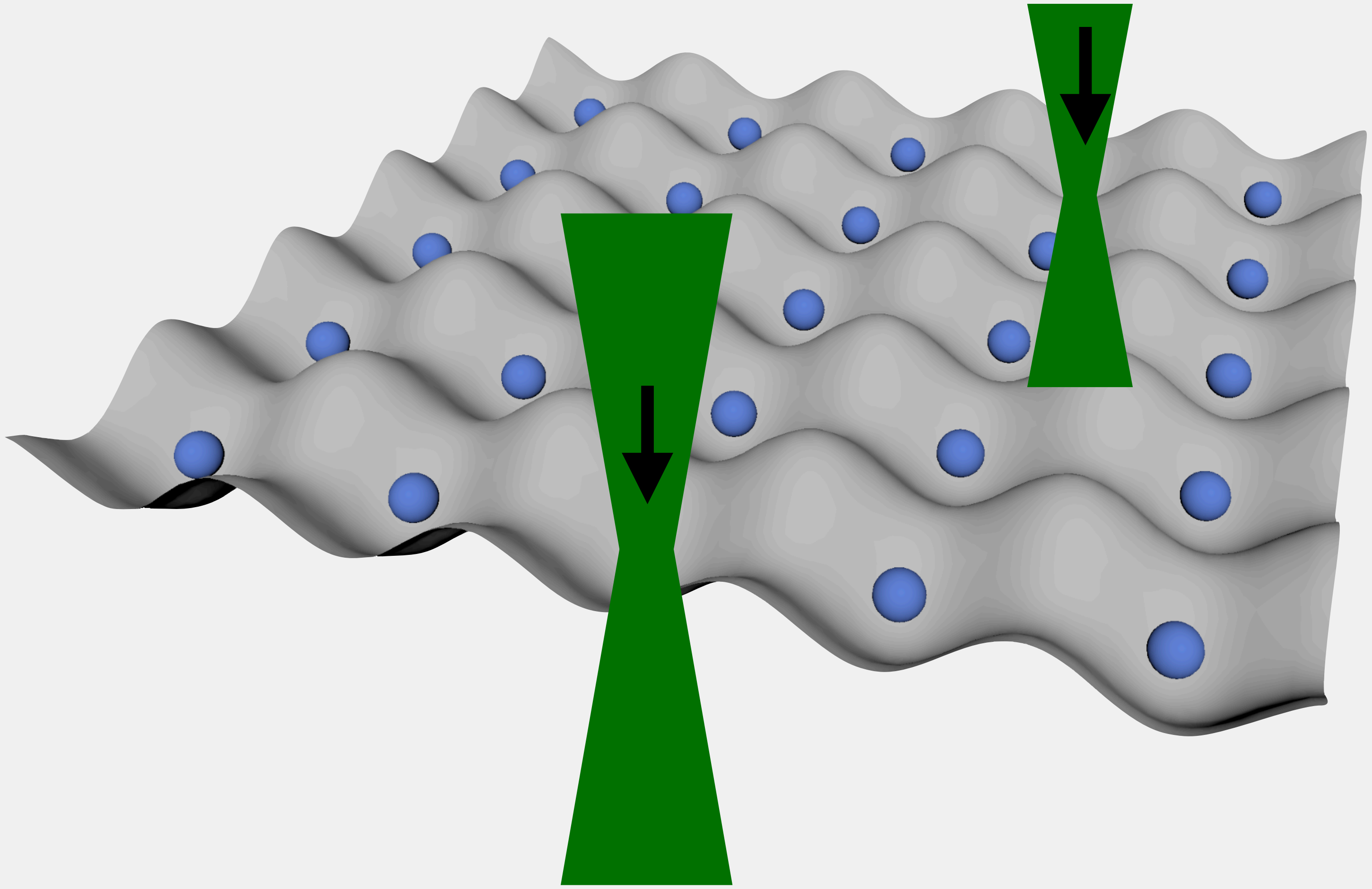
Joannis Koepsell[1]*, Jayadev Vijayan[1], Pimonpan Sompet[1], Fabian Grusdt[2,3], Timon A. Hilker[1,5], Eugene Demler[2], Guillaume Salomon[1], Immanuel Bloch[1,4] & Christian Gross[1]

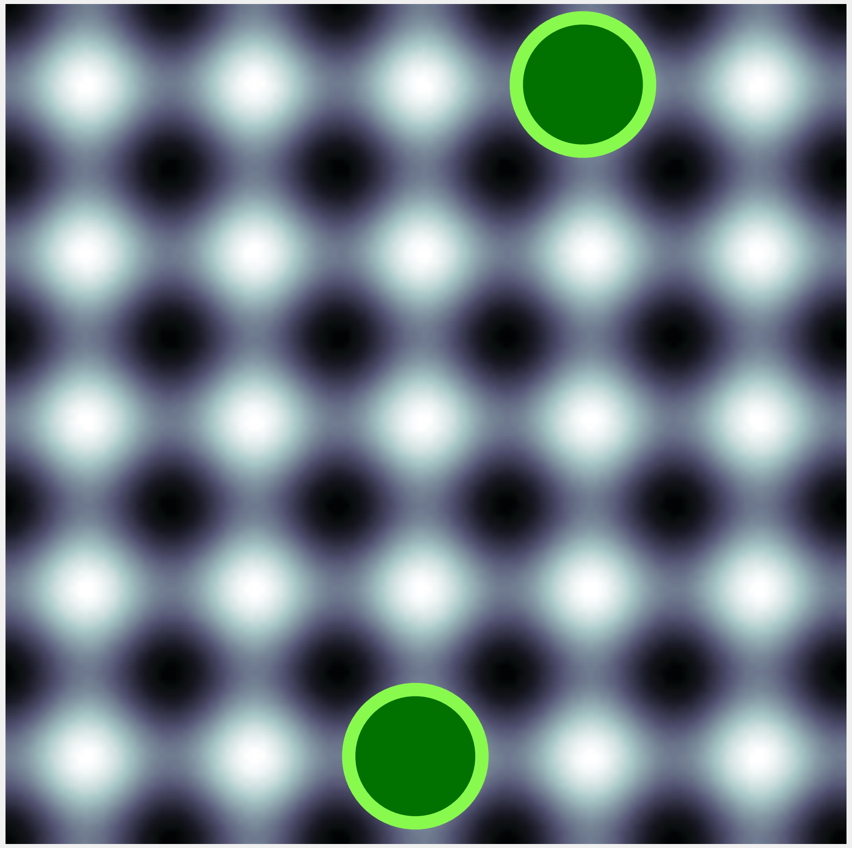


Single-site addressing

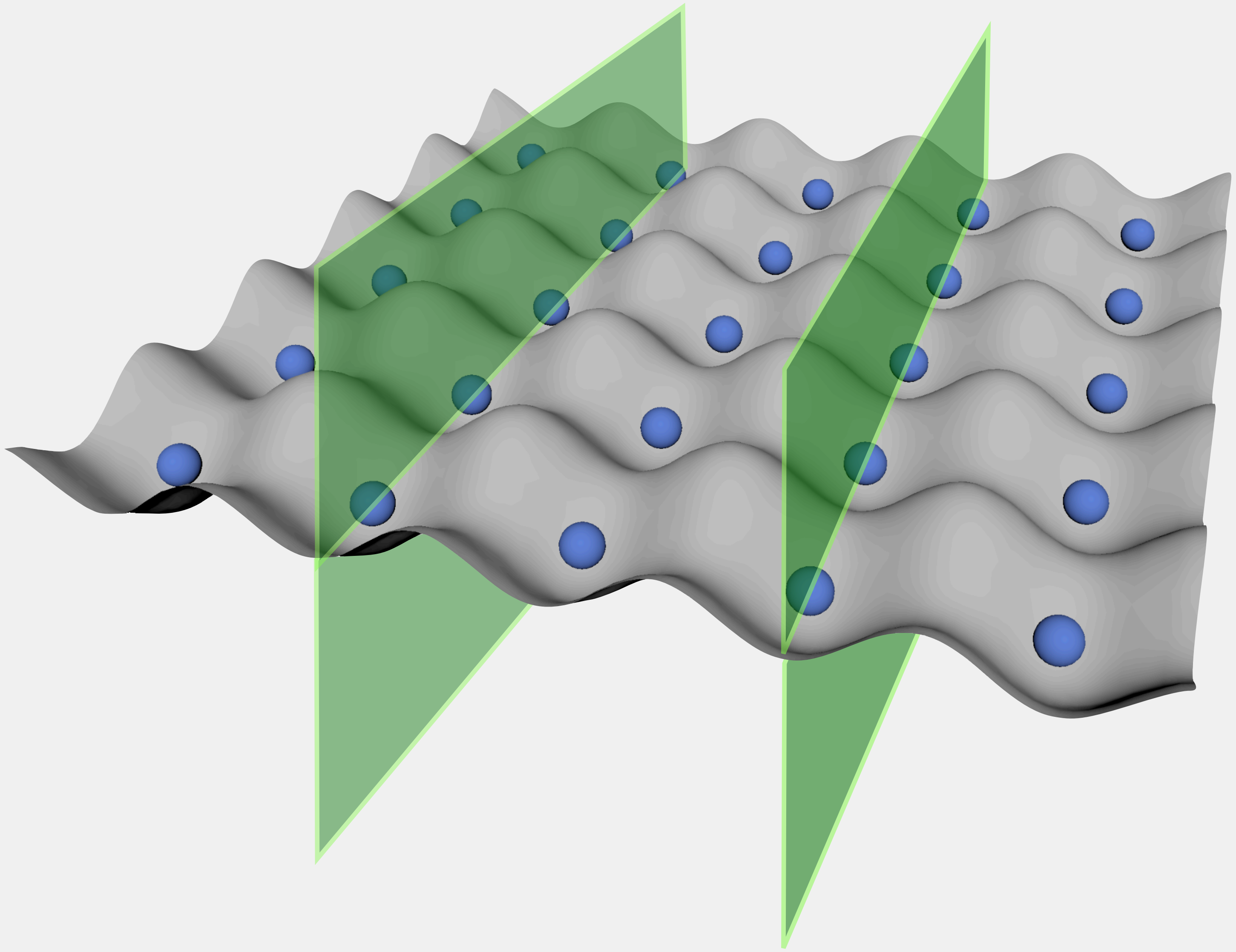


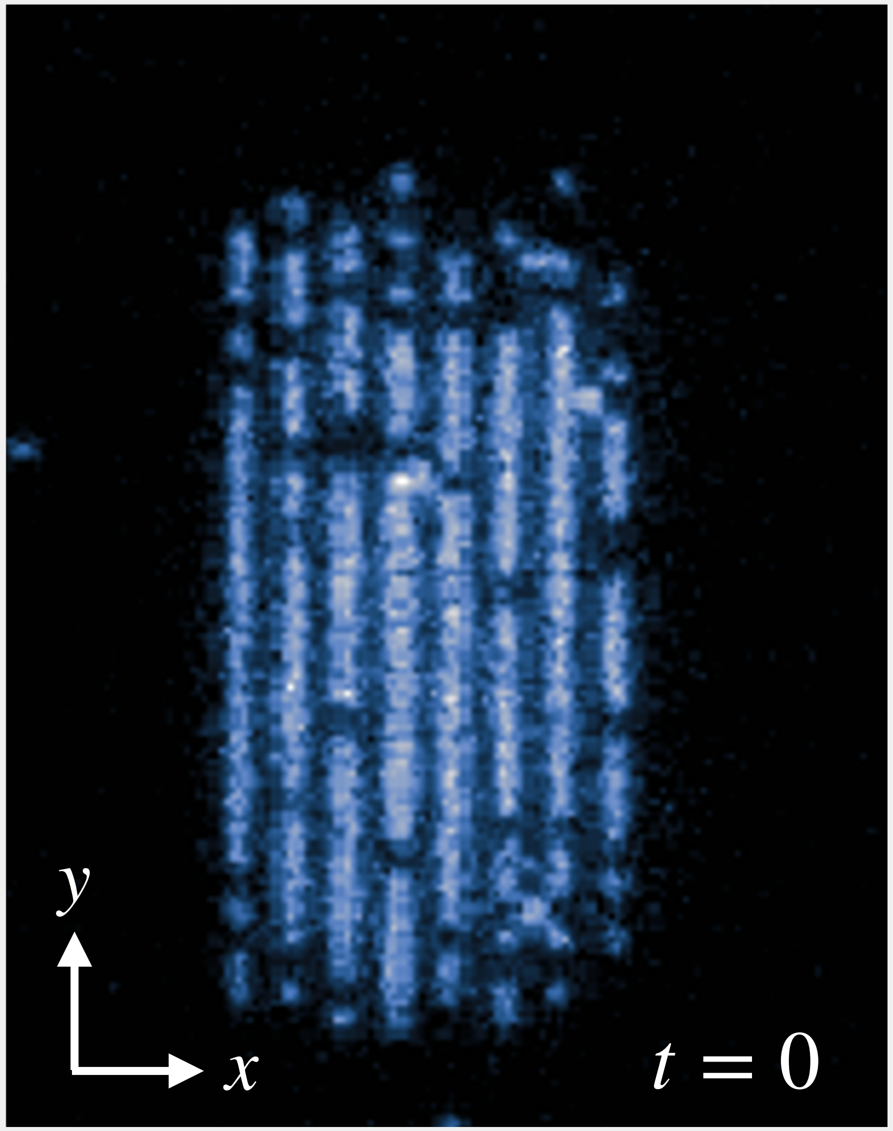
Potential engineering in a single-site resolution





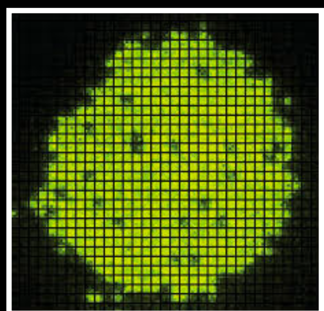
Potential engineering in a single-site resolution



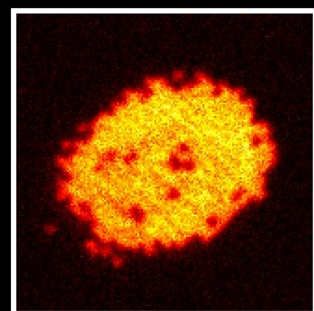


Existing quantum gas microscope (QGM) around the world

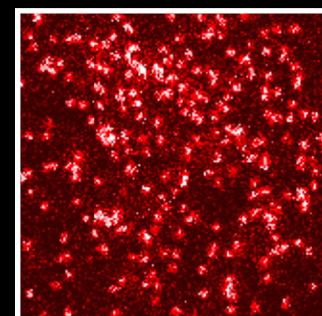
Rubidium-87 (Boson)



M. Greiner
(Harvard)

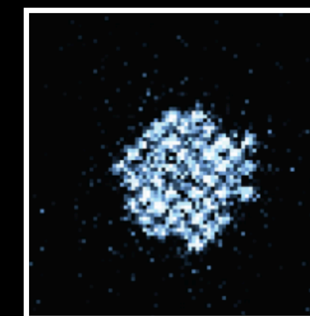


I. Bloch
(MPQ)

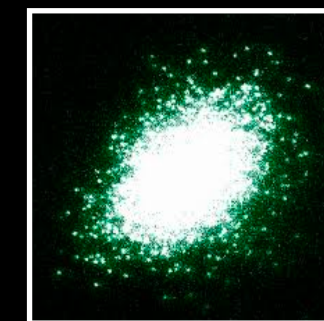


T. Fukuhara
(RIKEN)

Ytterbium-174 (Boson)

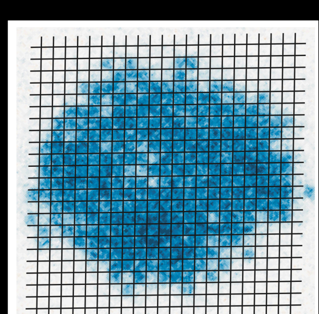


M. Kozuma
(Tokyo Tech)

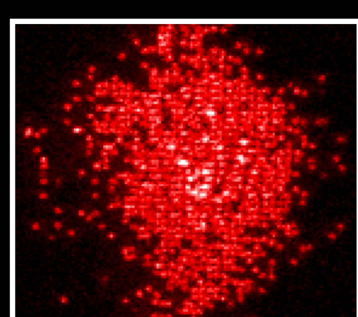


Y. Takahashi
(Kyoto Univ.)

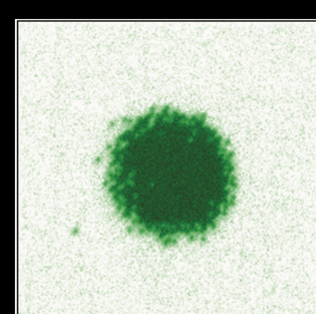
Lithium-6 (Fermion)



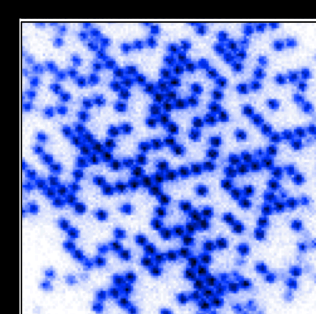
M. Greiner
(Harvard)



I. Bloch
(MPQ)

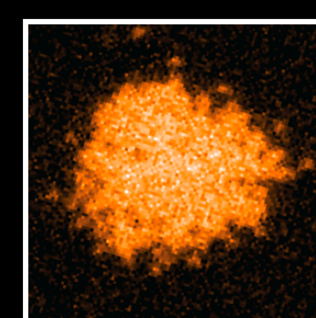


W. Bakr
(Princeton)

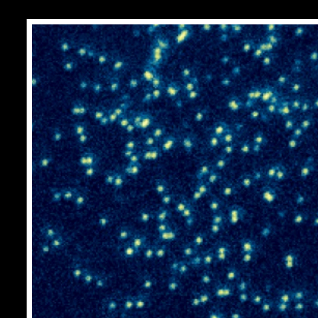


P. Schauß
(Virginia)

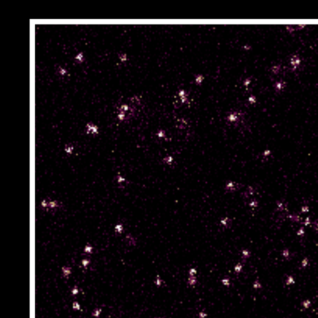
Potassoum-40 (Fermion)



M. Zwierlein
(MIT)

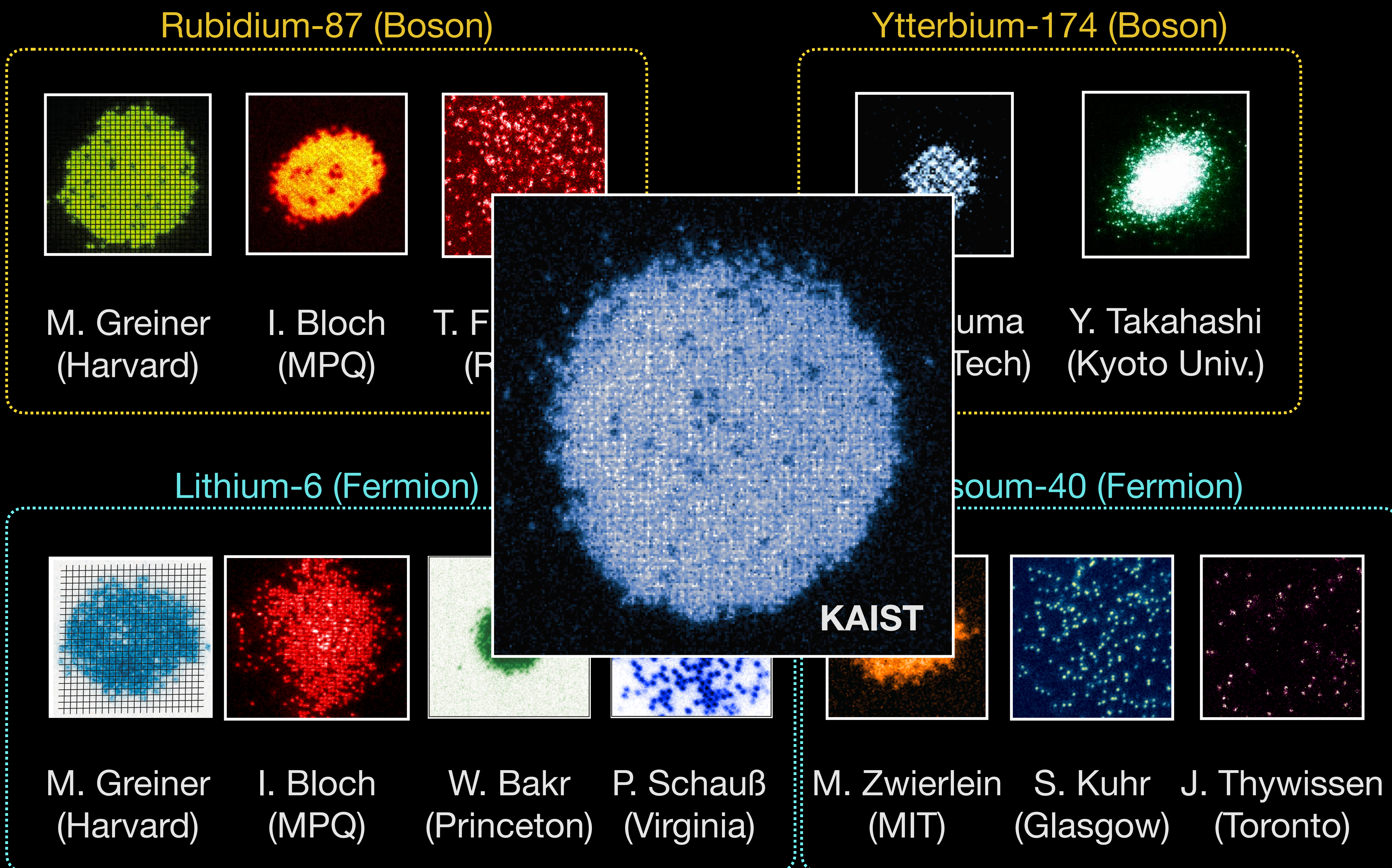


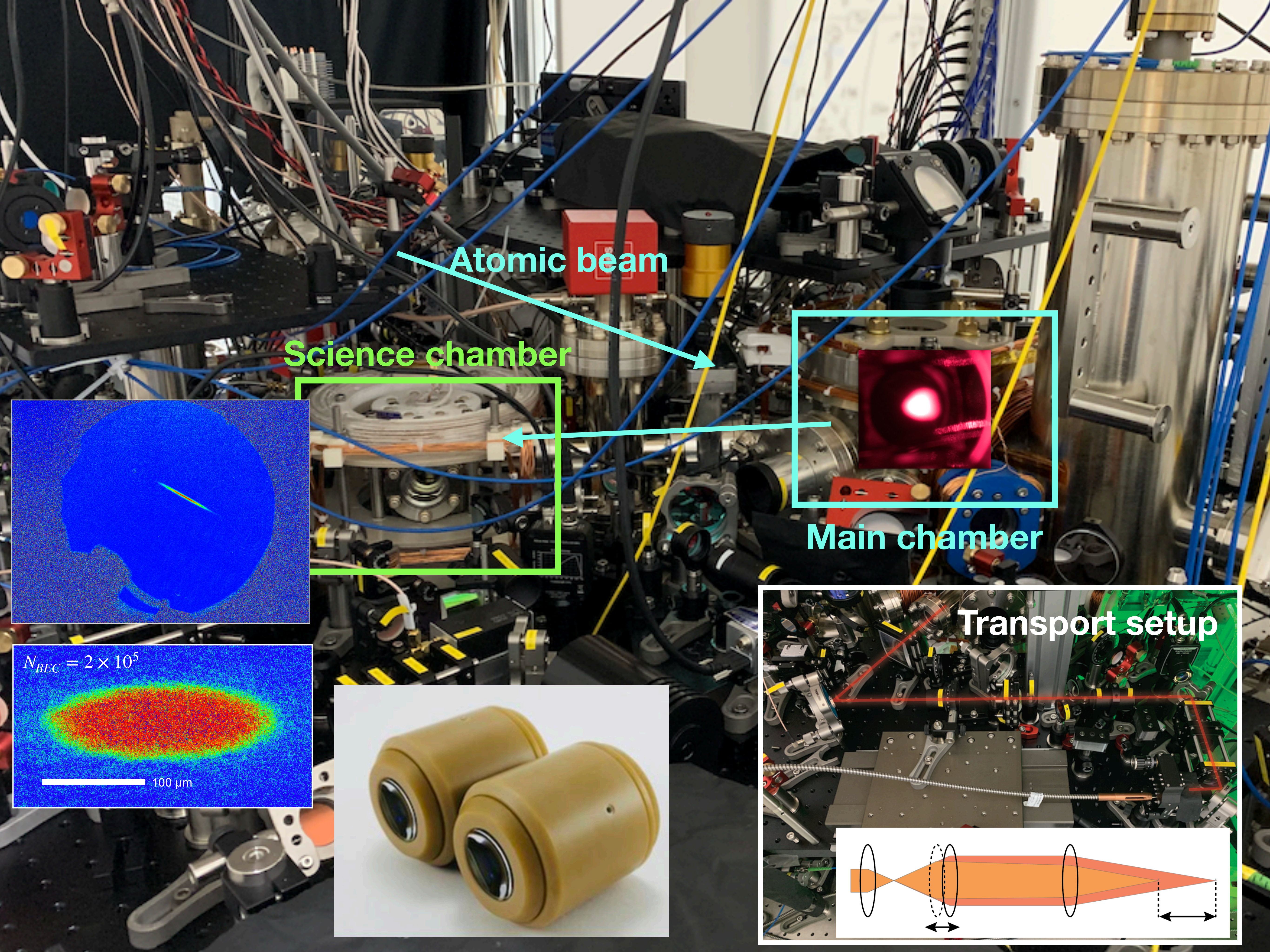
S. Kuhr
(Glasgow)



J. Thywissen
(Toronto)

Existing quantum gas microscope (QGM) around the world





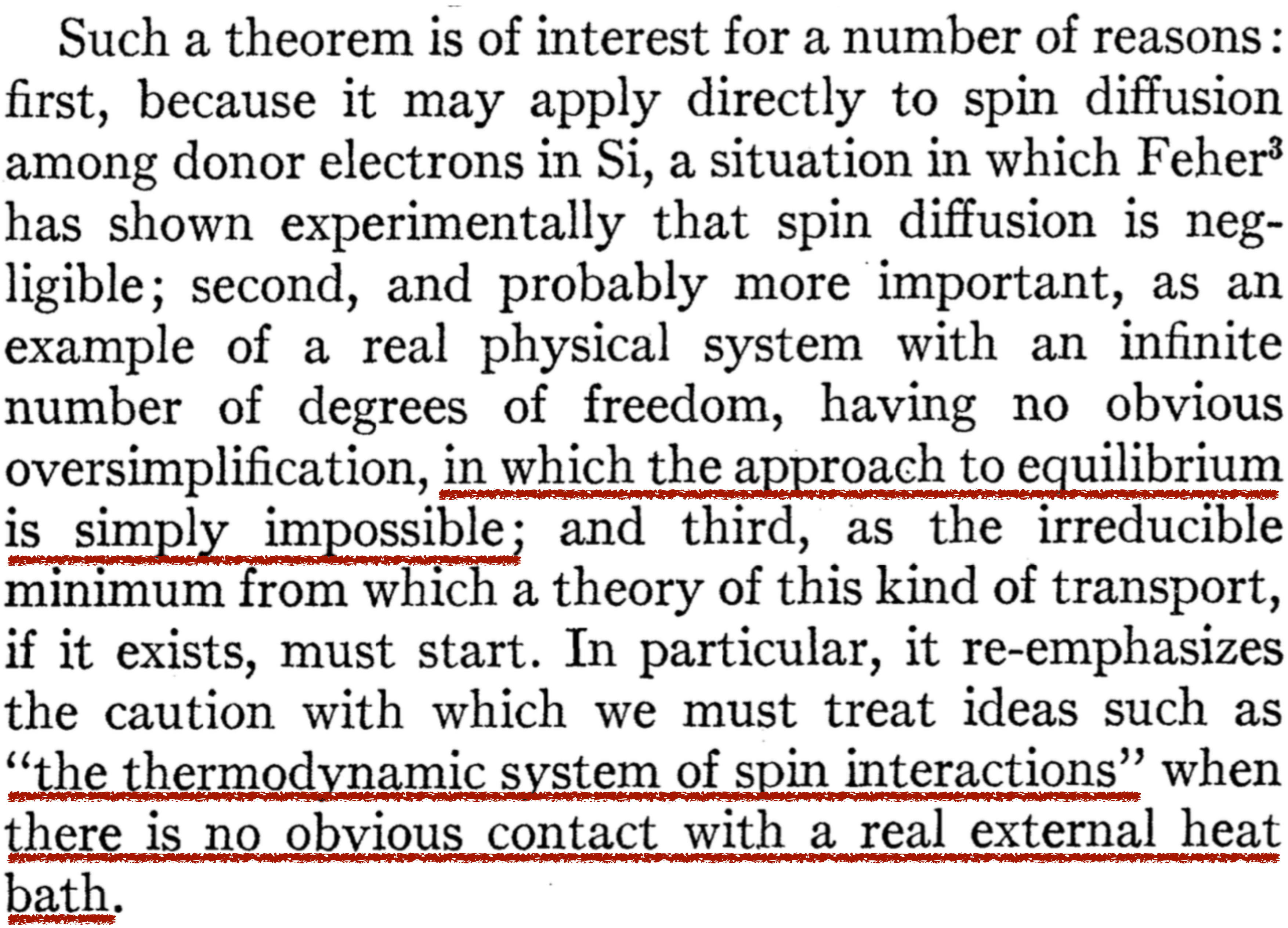
Atomic beam
Science chamber
Main chamber
$N_{BEC} = 2 \times 10^5$
100 μm
Transport setup

Many-body localization in 2D

Phys. Rev. Lett. **78**, 2803 (1997).
Annals of Physics **321**, 1126 (2005).
Annu. Rev. Condens. Matter Phys. **6,**15 (2015).
Annu. Rev. Condens. Matter Phys. **6,**383 (2015).

What is the Many-body localization?

P.W. Anderson


Such a theorem is of interest for a number of reasons: first, because it may apply directly to spin diffusion among donor electrons in Si, a situation in which Feher[3] has shown experimentally that spin diffusion is negligible; second, and probably more important, as an example of a real physical system with an infinite number of degrees of freedom, having no obvious oversimplification, in which the approach to equilibrium is simply impossible; and third, as the irreducible minimum from which a theory of this kind of transport, if it exists, must start. In particular, it re-emphasizes the caution with which we must treat ideas such as "the thermodynamic system of spin interactions" when there is no obvious contact with a real external heat bath.

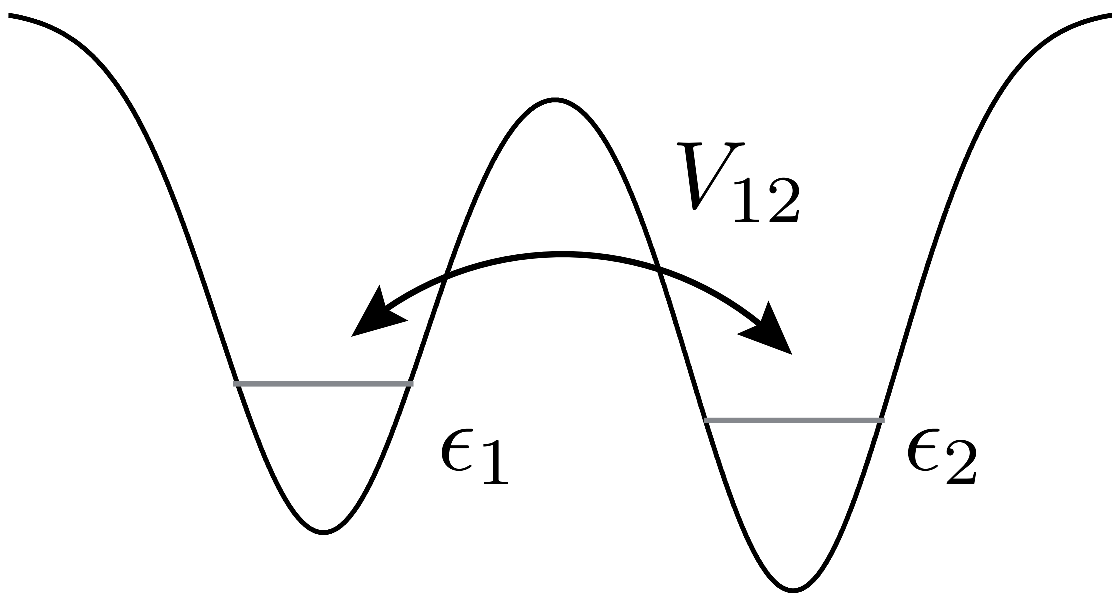
P.W. Anderson, Phys.Rev. **109**, 1492 (1958).

Ergodicity in quantum systems

Ergodic system

Glassy system

Anderson localization



Perturbative limit

$|\epsilon_1 - \epsilon_2| > V_{12}$

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \psi_1 + \frac{V_{12}}{\epsilon_2 - \epsilon_1}\psi_2 \\ \psi_2 - \frac{V_{12}}{\epsilon_2 - \epsilon_1}\psi_1 \end{pmatrix}$$

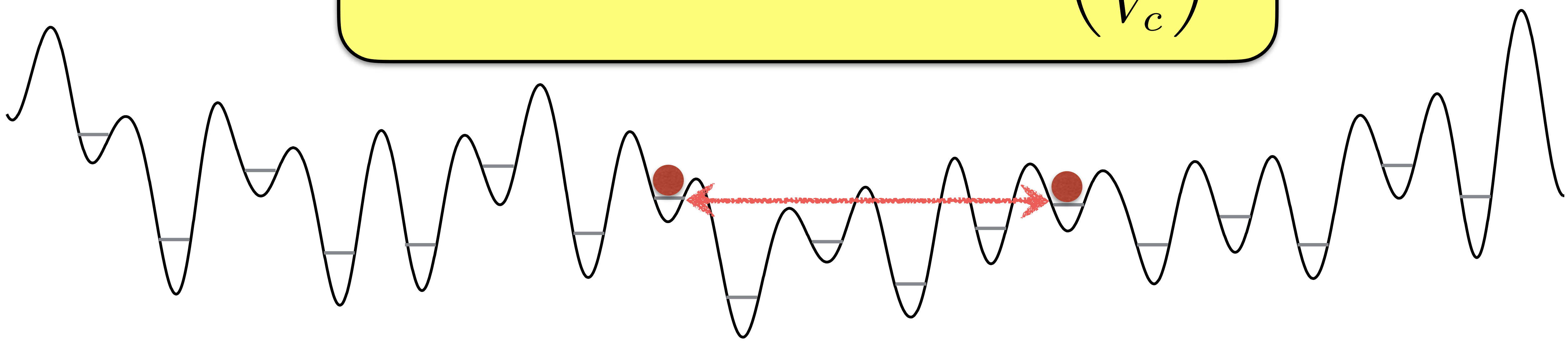
Resonant limit

$|\epsilon_1 - \epsilon_2| < V_{12}$

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_1 + \psi_2 \\ \psi_2 - \psi_1 \end{pmatrix}$$

Anderson localization

Delocalization $|\epsilon_1 - \epsilon_n| < V \left(\frac{V}{V_c} \right)^n$



Scaling analysis - d=1, 2: Always localized (no critical disorder Δ_c)
d=3: Wavefunction can be delocalized below Δ_c

P.W. Anderson, Phys.Rev. **109**, 1492 (1958).

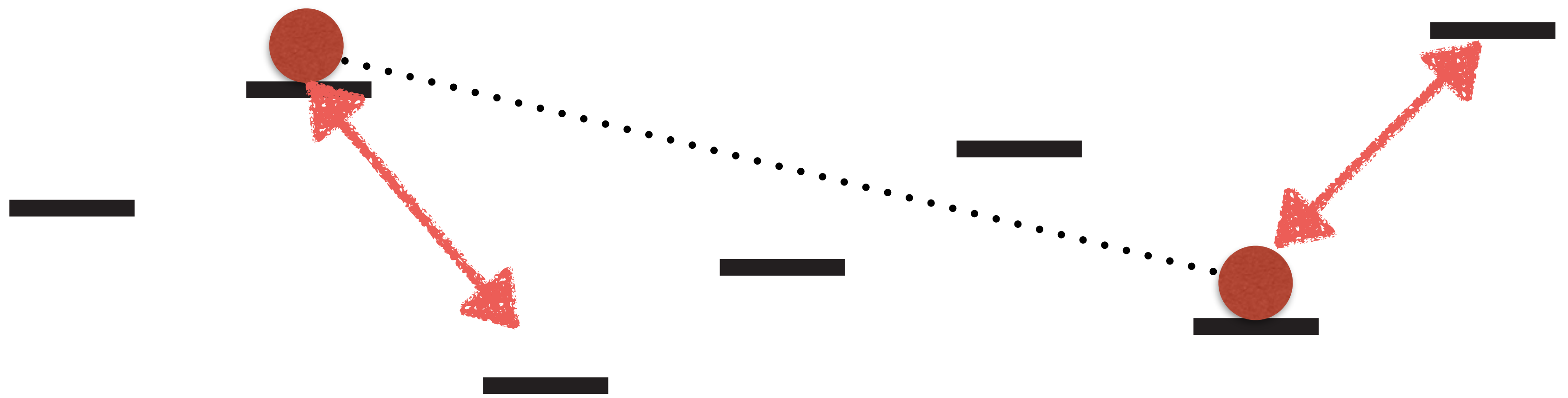
Many-body systems under disorder

Many-body localization: Localization in an interacting systems

$$\hat{H} = \sum_{\alpha} \xi_{\alpha} \hat{c}_{\alpha}^{\dagger} \hat{c}_{\alpha} + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} \hat{c}_{\alpha}^{\dagger} \hat{c}_{\beta}^{\dagger} \hat{c}_{\gamma} \hat{c}_{\delta}$$

What is the role of interaction?

Opens a possible relaxation channels

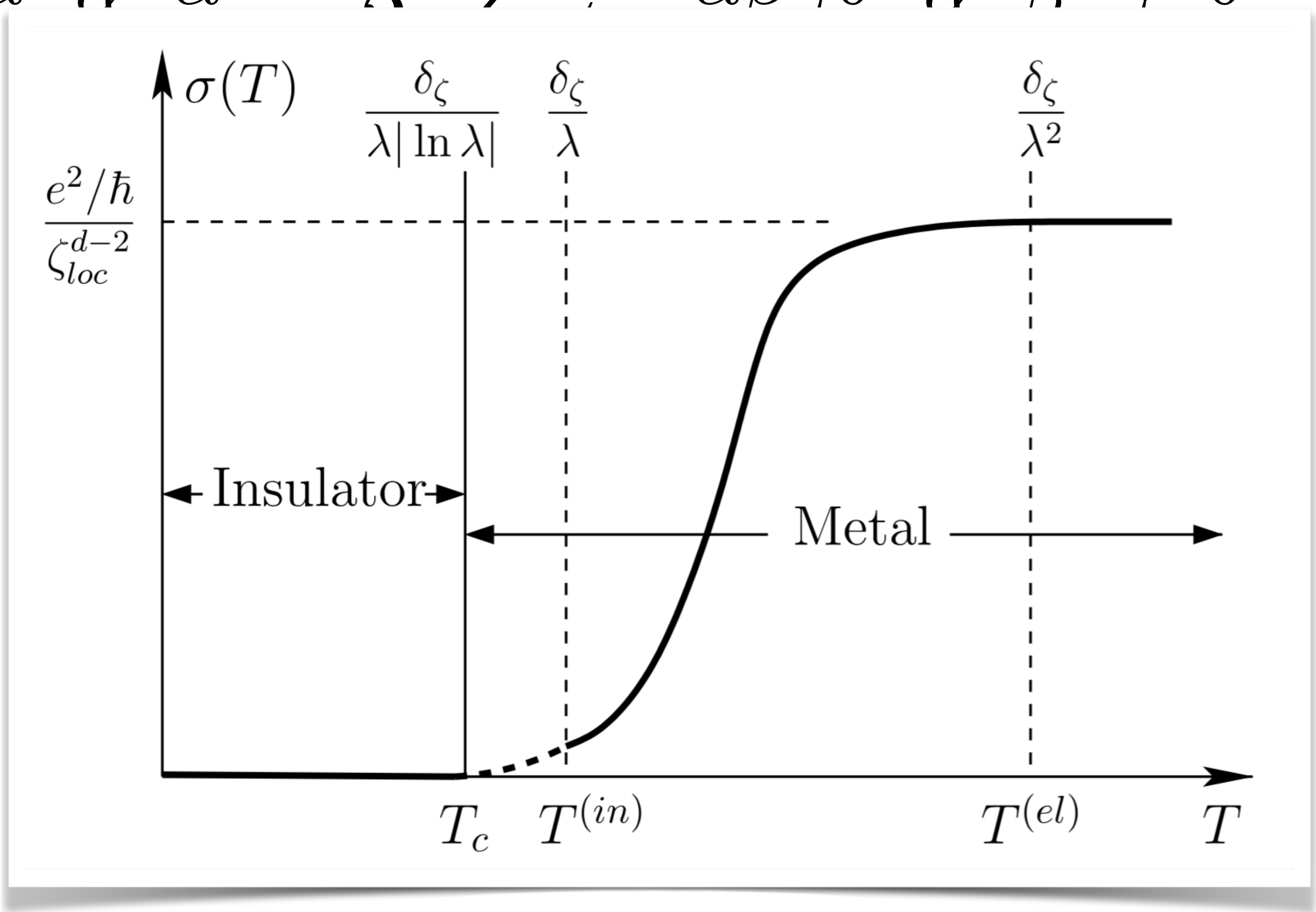


B. L. Altshuler *et al.*, Phys. Rev. Lett. **78**, 2803 (1997).
D. M. Basko *et al.*, Annals of Physics **321**, 1126 (2005).

Many-body systems under disorder

Many-body localization: Localization in an interacting systems

$$\hat{H} = \sum_{\alpha} \xi_\alpha \hat{c}^\dagger_\alpha \hat{c}_\alpha + \frac{1}{2} \sum V_{\alpha\beta\gamma\delta} \hat{c}^\dagger_\alpha \hat{c}^\dagger_\beta \hat{c}_\gamma \hat{c}_\delta$$



B. L. Altshuler *et al.*, Phys. Rev. Lett. **78**, 2803 (1997).
D. M. Basko *et al.*, Annals of Physics **321**, 1126 (2005).

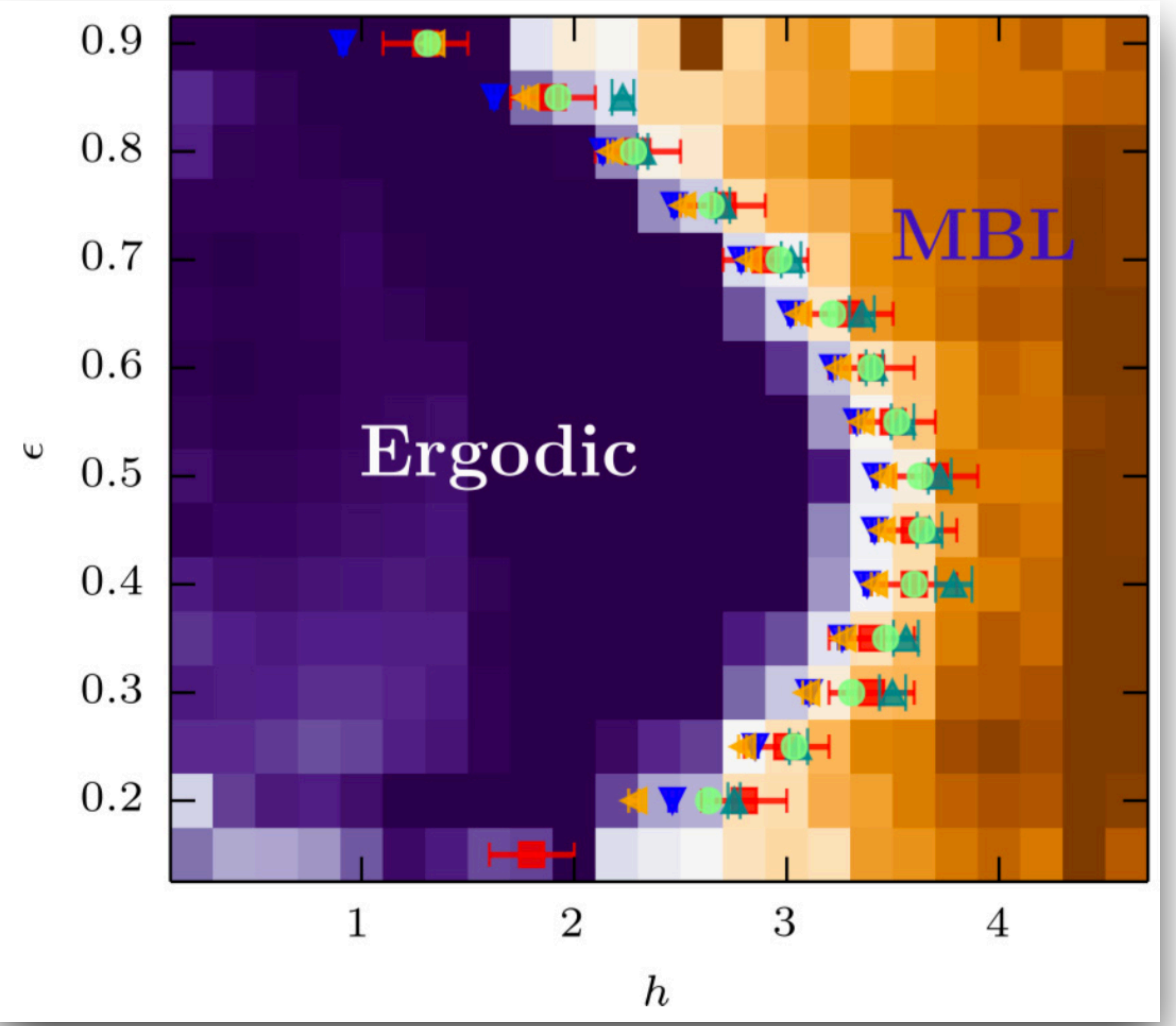
Numerical studies

Pioneering works A. Pal and D.A. Huse Phys.Rev.B **82**, 174411 (2010).

$$\hat{H} = \sum_i h_i \hat{\sigma}_i^z + \sum_{ij} J_{ij} \hat{\sigma}_i \cdot \hat{\sigma}_j$$

h_i Random magnetic field

J_{ij} Short-range interaction



	Ergodic phase	MBL phase
S_{ent}	Volume law	Area law
Level spacing	Gaussian orthogonal ensemble	Poisson distribution
Eigenstate	Satisfy ETH	Violate ETH

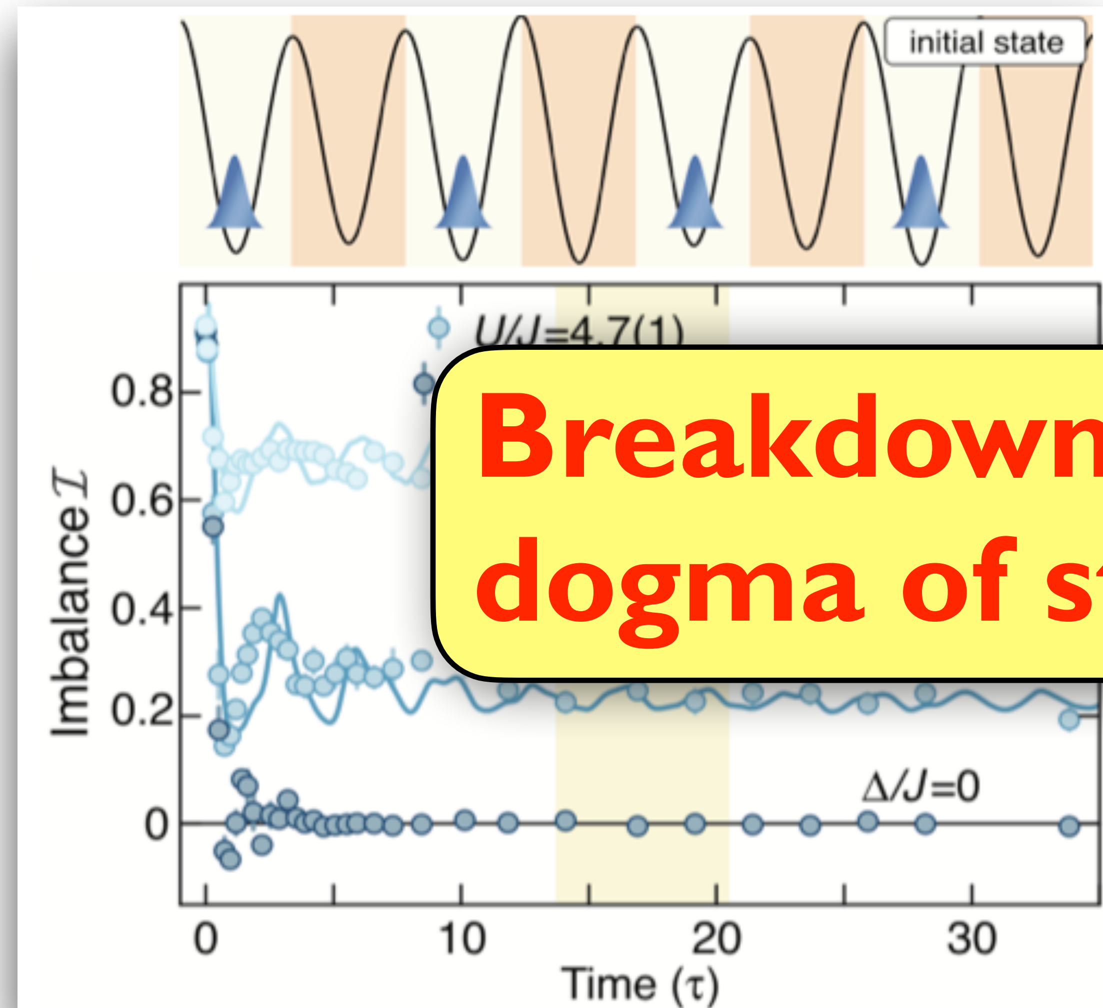
*ETH: Eigenstate Thermalization Hypothesis.

Annu. Rev. Condens. Matter Phys. **6**,15 (2015).
Annu. Rev. Condens. Matter Phys. **6**,383 (2015).

D. J. Luitz *et al.*, Phys.Rev.B **91**, 081103 (2015).

Early experimental results

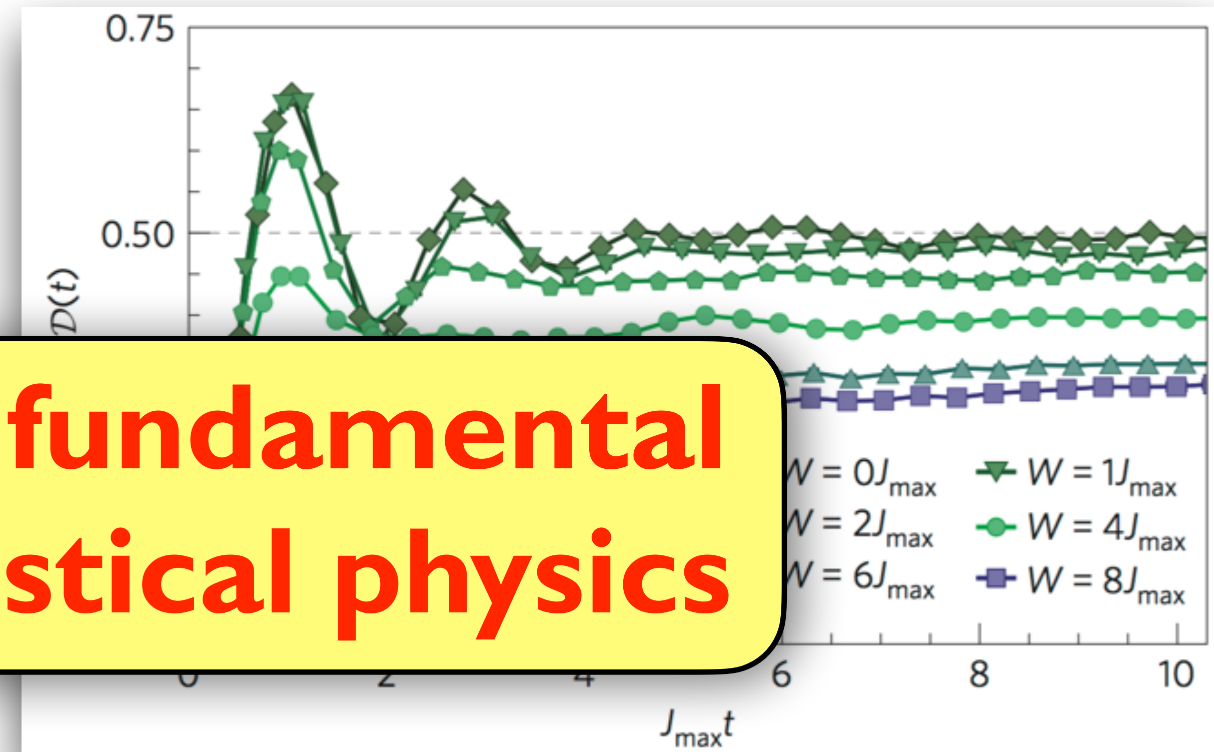
Neutral atoms



M. Schreiber *et al.*, Science **347**, 1229 (2015).

$$\mathcal{I} = \frac{n_{\mathrm{odd}} - n_{\mathrm{even}}}{n_{\mathrm{odd}} + n_{\mathrm{even}}}$$

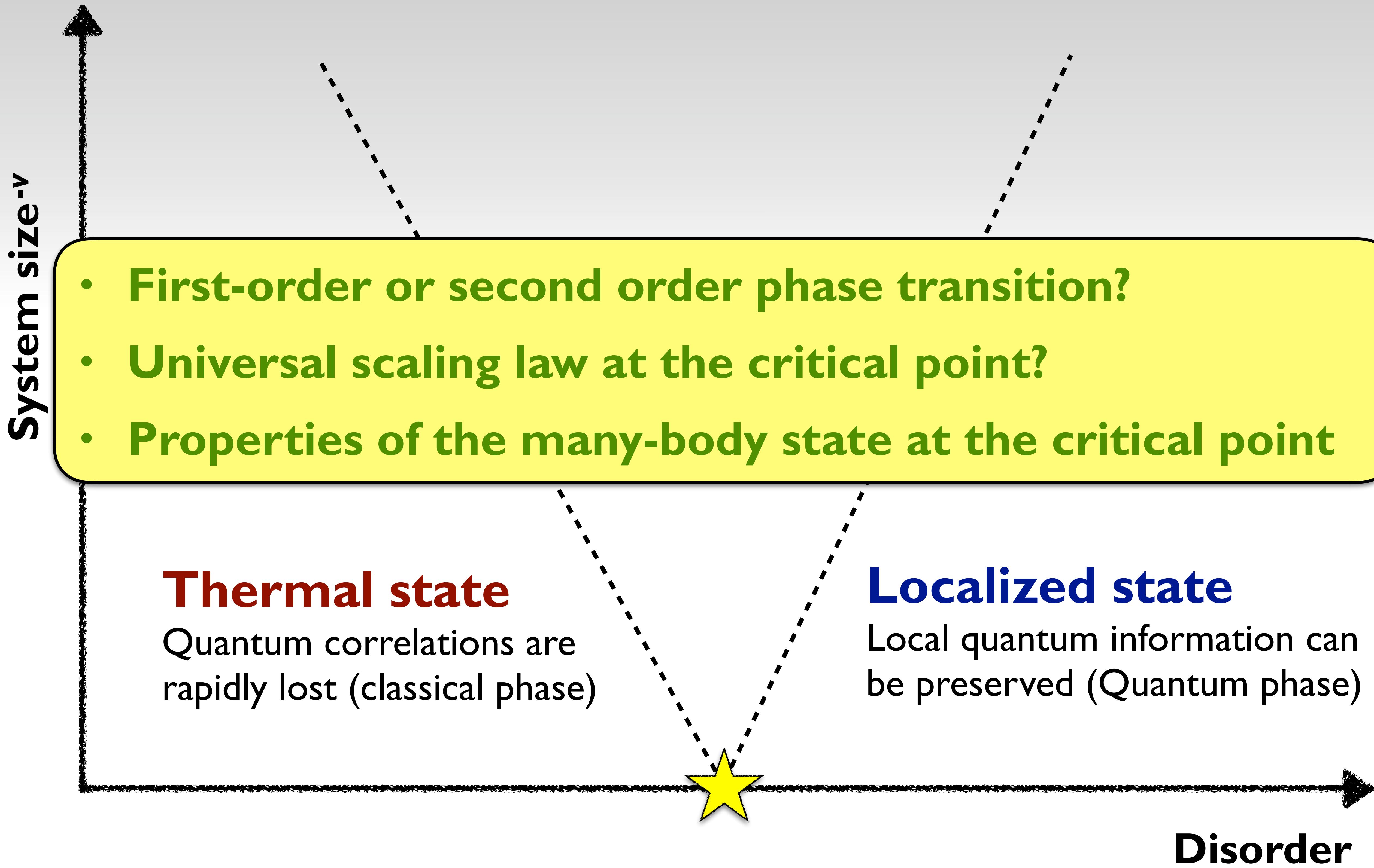
Trapped ions



J. Smith *et al.*, Nat.Phys **12**, 907 (2016).

Breakdown of fundamental dogma of statistical physics

New phases of matter in non equilibrium



- **First-order or second order phase transition?**
- **Universal scaling law at the critical point?**
- **Properties of the many-body state at the critical point**

Exotic phases of matter

Proof (?) of the Many-body localization in one dimension

Selected for a Viewpoint in *Physics*
PRL **117,** 027201 (2016) PHYSICAL REVIEW LETTERS week ending 8 JULY 2016

Diagonalization and Many-Body Localization for a Disordered Quantum Spin Chain

John Z. Imbrie
Department of Mathematics, University of Virginia, Charlottesville, Virginia 22904-4137, USA
(Received 13 April 2016; published 5 July 2016)

We consider a weakly interacting quantum spin chain with random local interactions. We prove that many-body localization follows from a physically reasonable assumption that limits the extent of level attraction in the statistics of eigenvalues. In a Kolmogorov-Arnold-Moser-style construction, a sequence of local unitary transformations is used to diagonalize the Hamiltonian by deforming the initial tensor-product basis into a complete set of exact many-body eigenfunctions.

Issues in the MBL studies

Few years later…

PHYSICAL REVIEW B **105**, 224203 (2022)

Editors' Suggestion

Challenges to observation of many-body localization

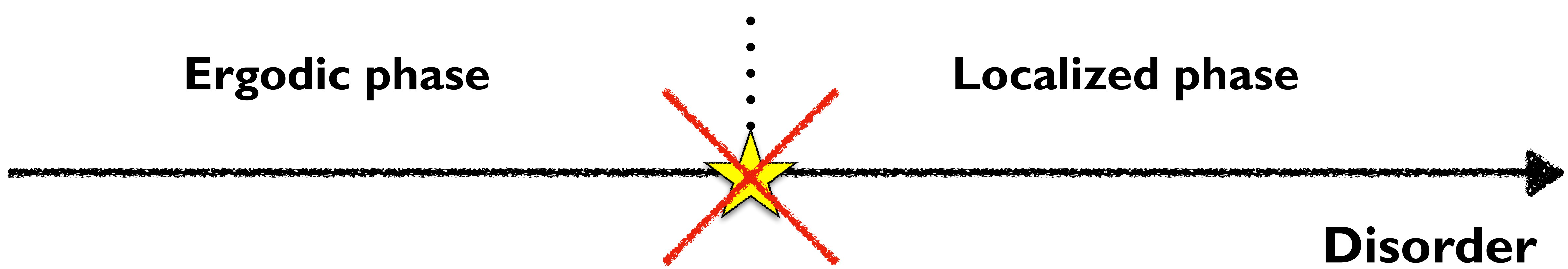
Piotr Sierant[1,2] and Jakub Zakrzewski[2,3,*]

[1]*The Abdus Salam International Center for Theoretical Physics, Strada Costiera 11, 34151, Trieste, Italy*

[2]*Institute of Theoretical Physics, Jagiellonian University in Kraków, Łojasiewicza 11, 30-348 Kraków, Poland*

[3]*Mark Kac Complex Systems Research Center, Jagiellonian University in Krakow, 30-348 Kraków, Poland*

(Received 5 October 2021; revised 6 May 2022; accepted 2 June 2022; published 14 June 2022)



Few years later…

PHYSICAL REVIEW B **105**, 224203 (2022)

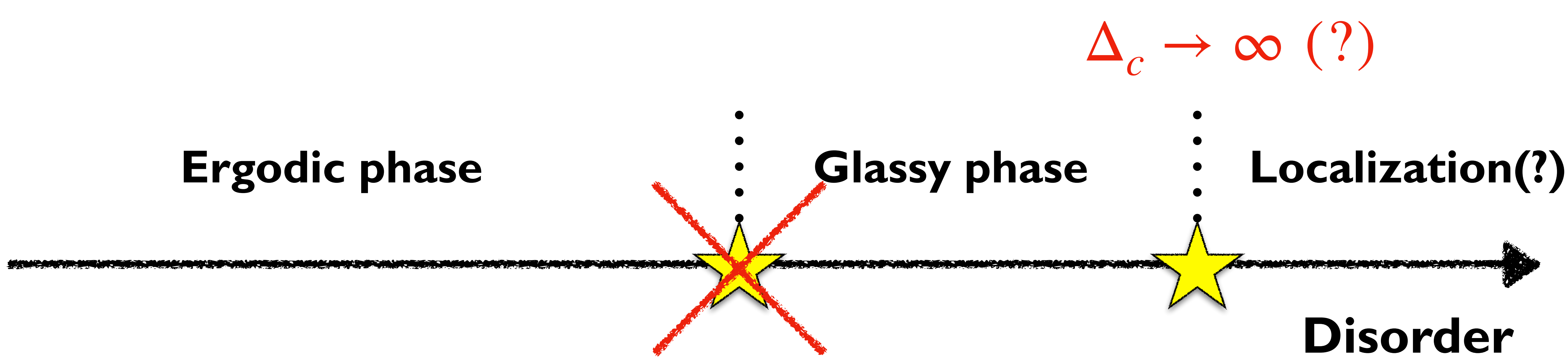
Editors' Suggestion

Challenges to observation of many-body localization

Piotr Sierant[1,2] and Jakub Zakrzewski[2,3,*]

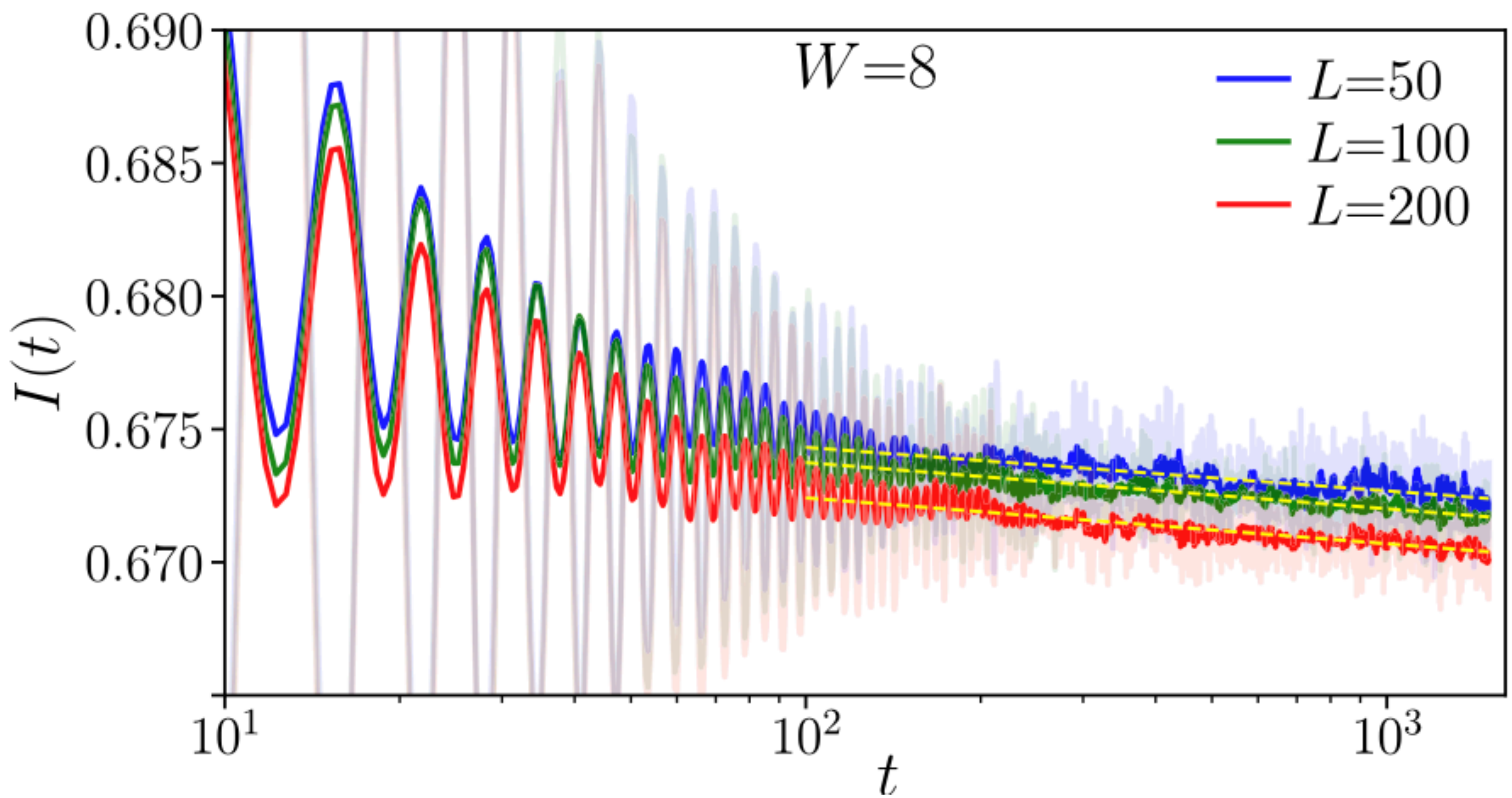
[1]*The Abdus Salam International Center for Theoretical Physics, Strada Costiera 11, 34151, Trieste, Italy*
[2]*Institute of Theoretical Physics, Jagiellonian University in Kraków, Łojasiewicza 11, 30-348 Kraków, Poland*
[3]*Mark Kac Complex Systems Research Center, Jagiellonian University in Krakow, 30-348 Kraków, Poland*

(Received 5 October 2021; revised 6 May 2022; accepted 2 June 2022; published 14 June 2022)



Issues in the MBL studies

Few years later…



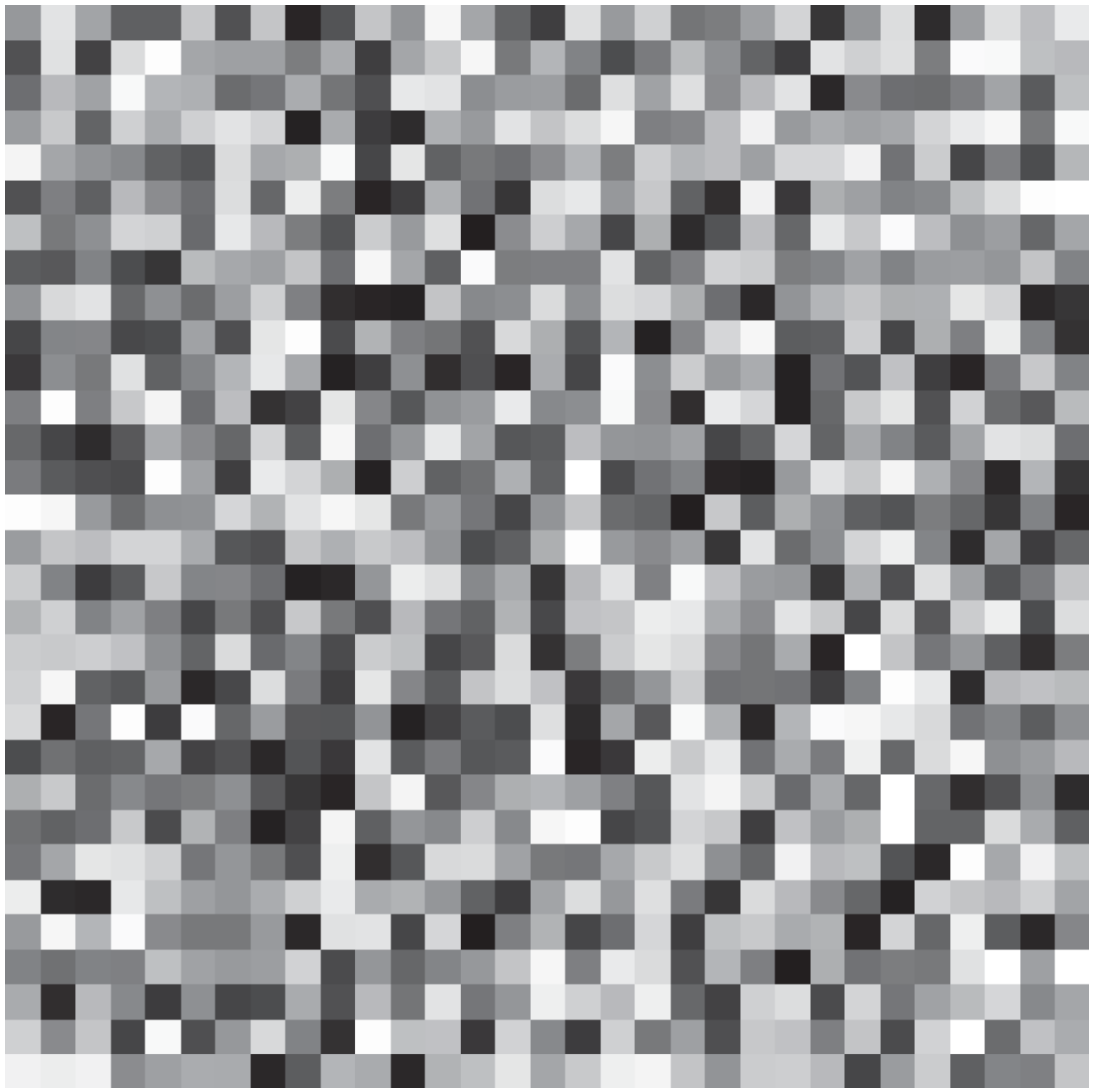
The imbalance decays as the system size increases !!

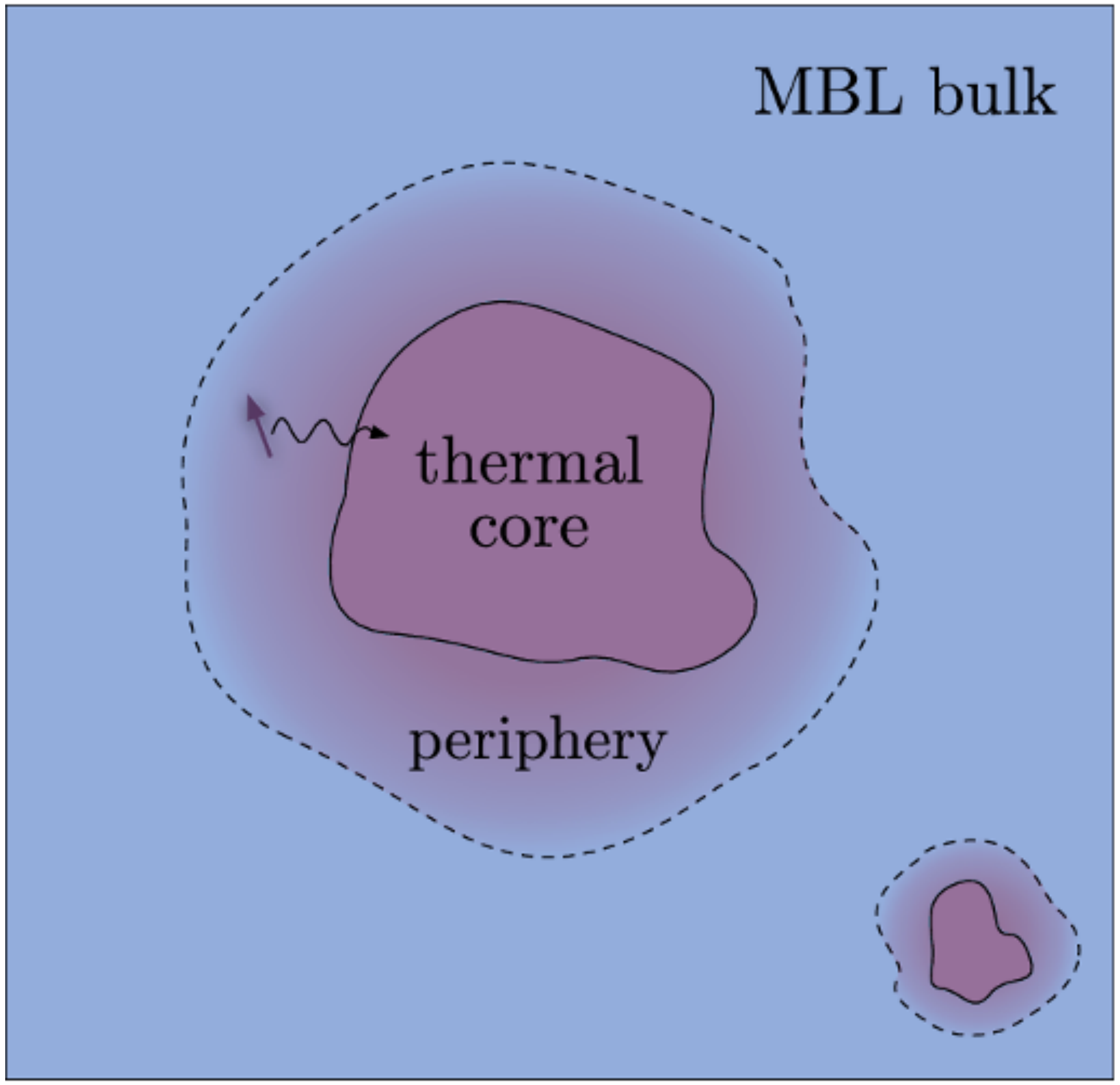
We still observe a finite decay rate even after very long time evolution

$$\lim_{L\to\infty} \mathcal{I}(L) = 0 \ (?)$$

Issues in the MBL studies

Possible scenario: quantum avalanche





Random disorder exhibits rare weak disorder regions.

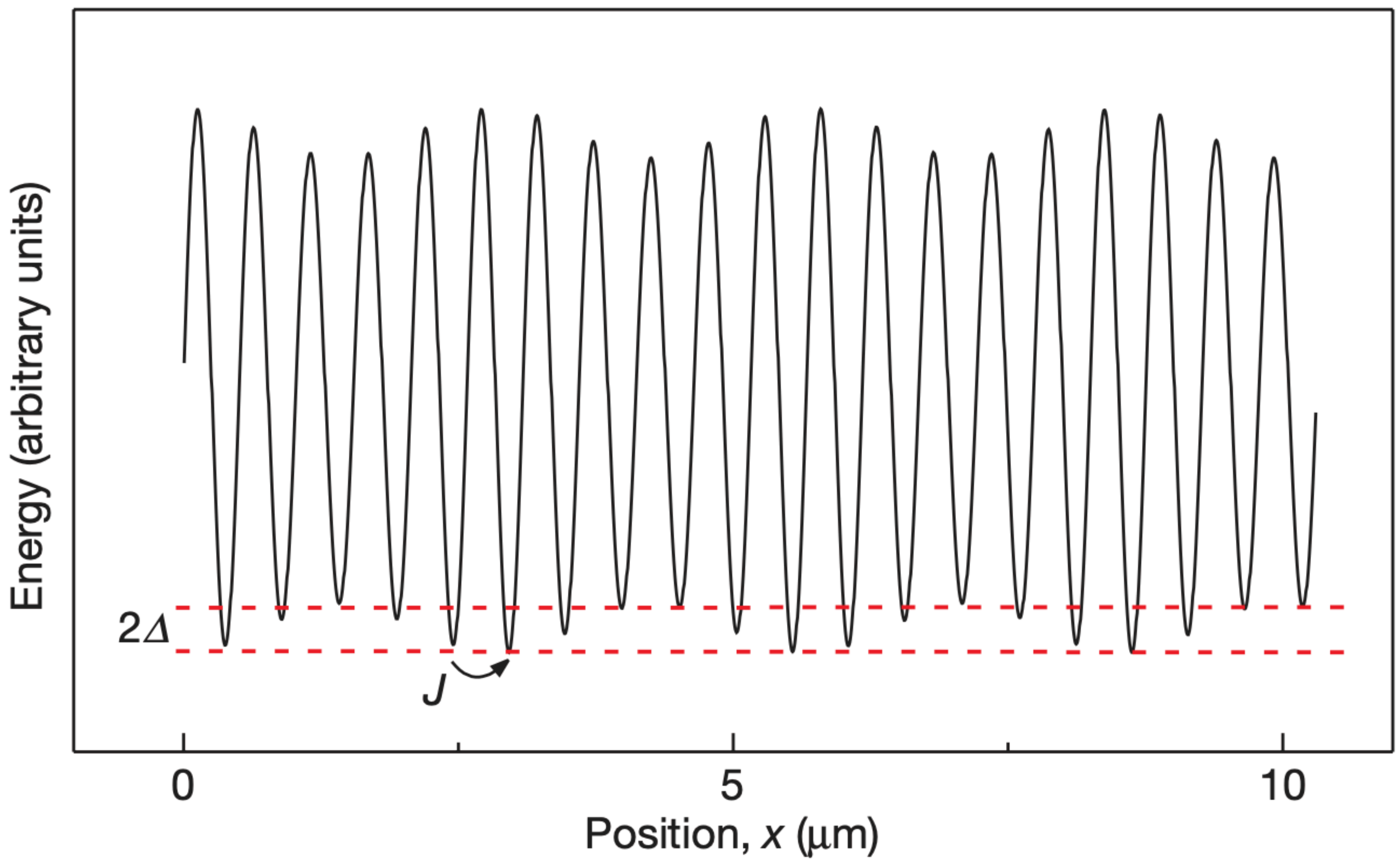
The system can be locally thermalized in this "disorder-free" region.

Issues in the MBL studies

PHYSICAL REVIEW B **106**, 184209 (2022)

Coexistence of localization and transport in many-body two-dimensional Aubry-André models

Antonio Štrkalj[1,*] Elmer V. H. Doggen[2,3] and Claudio Castelnovo[1]
ted Kingdom
rlsruhe, Germany
sruhe, Germany



$$\Delta \sum_m \cos(2\pi\beta m + \phi)$$

$$\beta = (\sqrt{5} - 1)/2$$

Mean-field theory of failed thermalizing avalanches

P. J. D. Crowley[1,*] and A. Chandran[2]
[1]Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
[2]Department of Physics, Boston University, Boston, Massachusetts 02215, USA

Issues in the MBL studies

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Reports on Progress in Physics

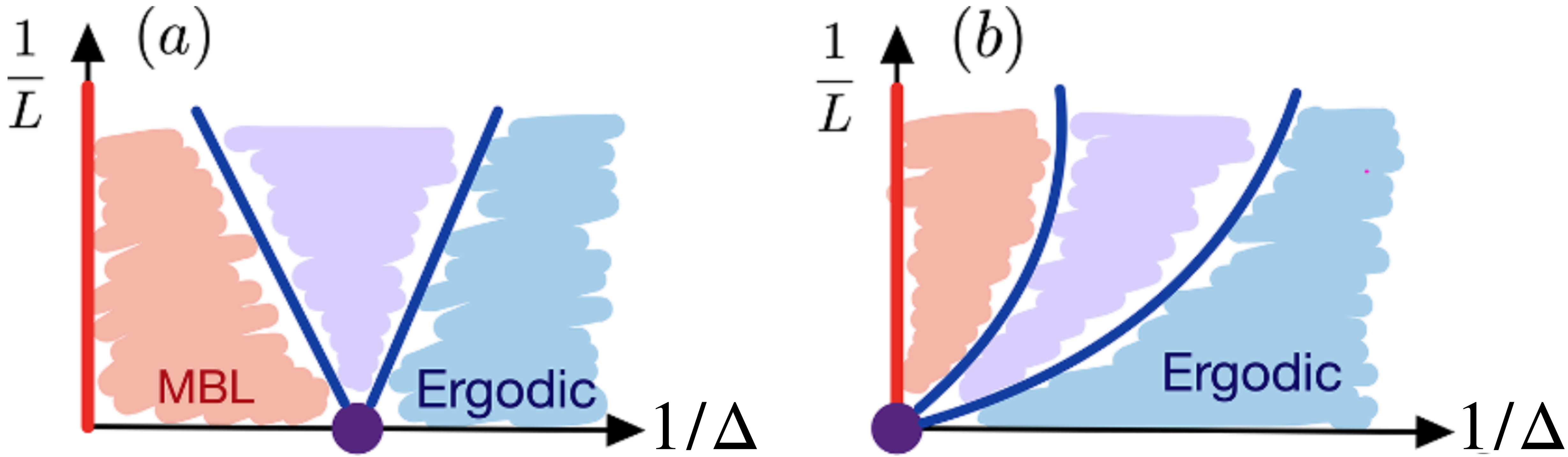
Rep. Prog. Phys. **88** (2025) 026502 (87pp)

https://doi.org/10.1088/1361-6633/ad9756

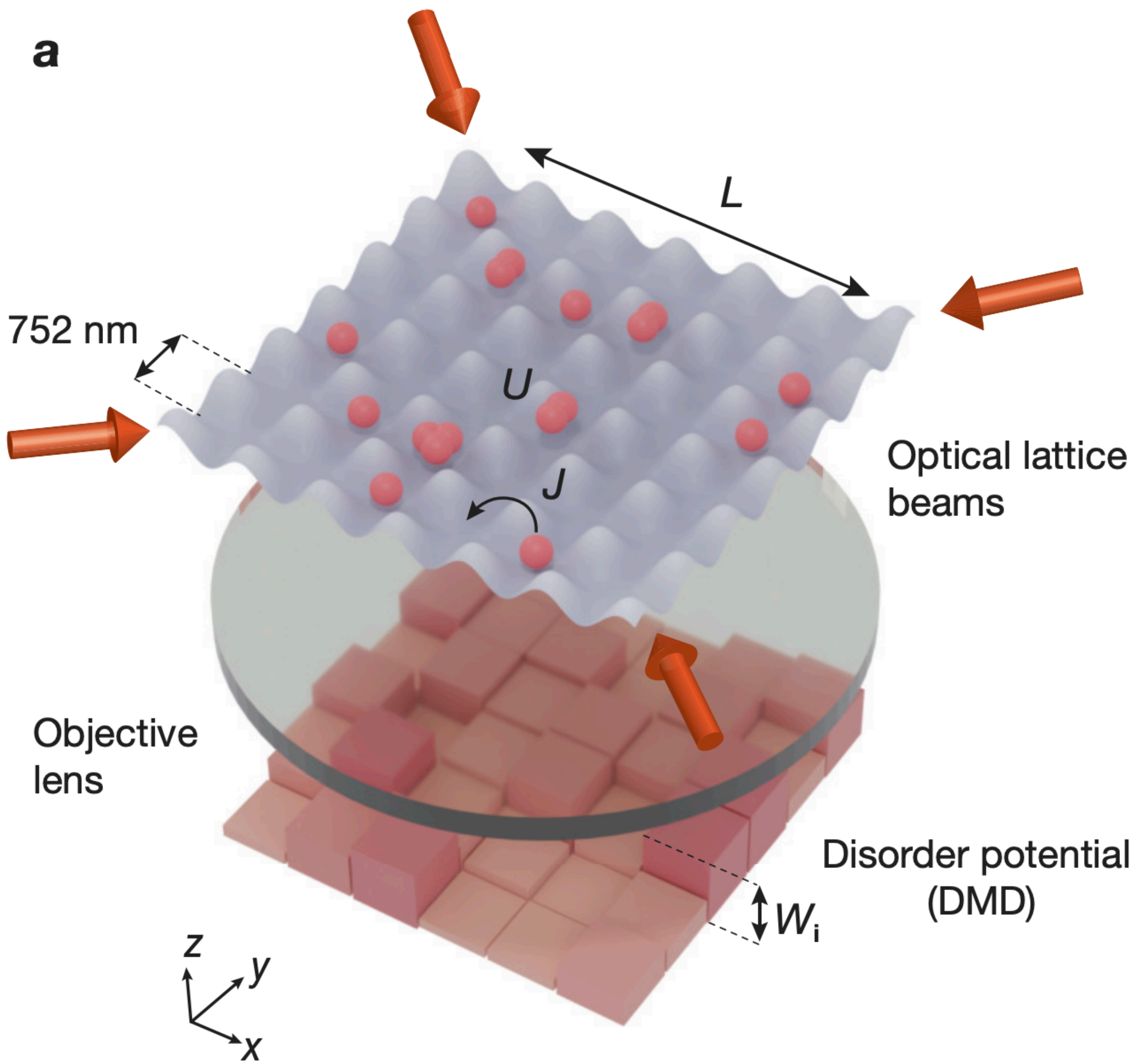
Review

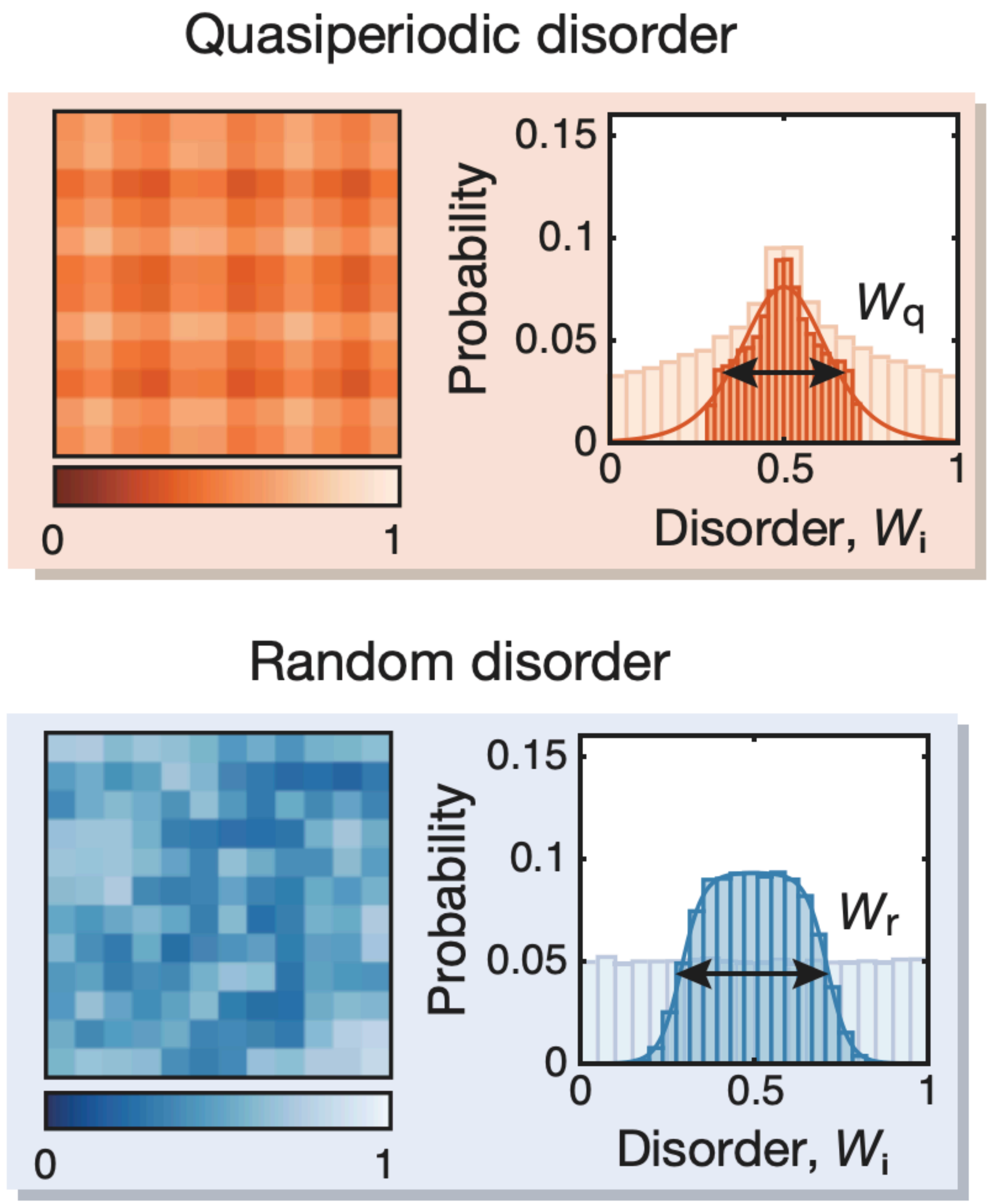
Many-body localization in the age of classical computing*

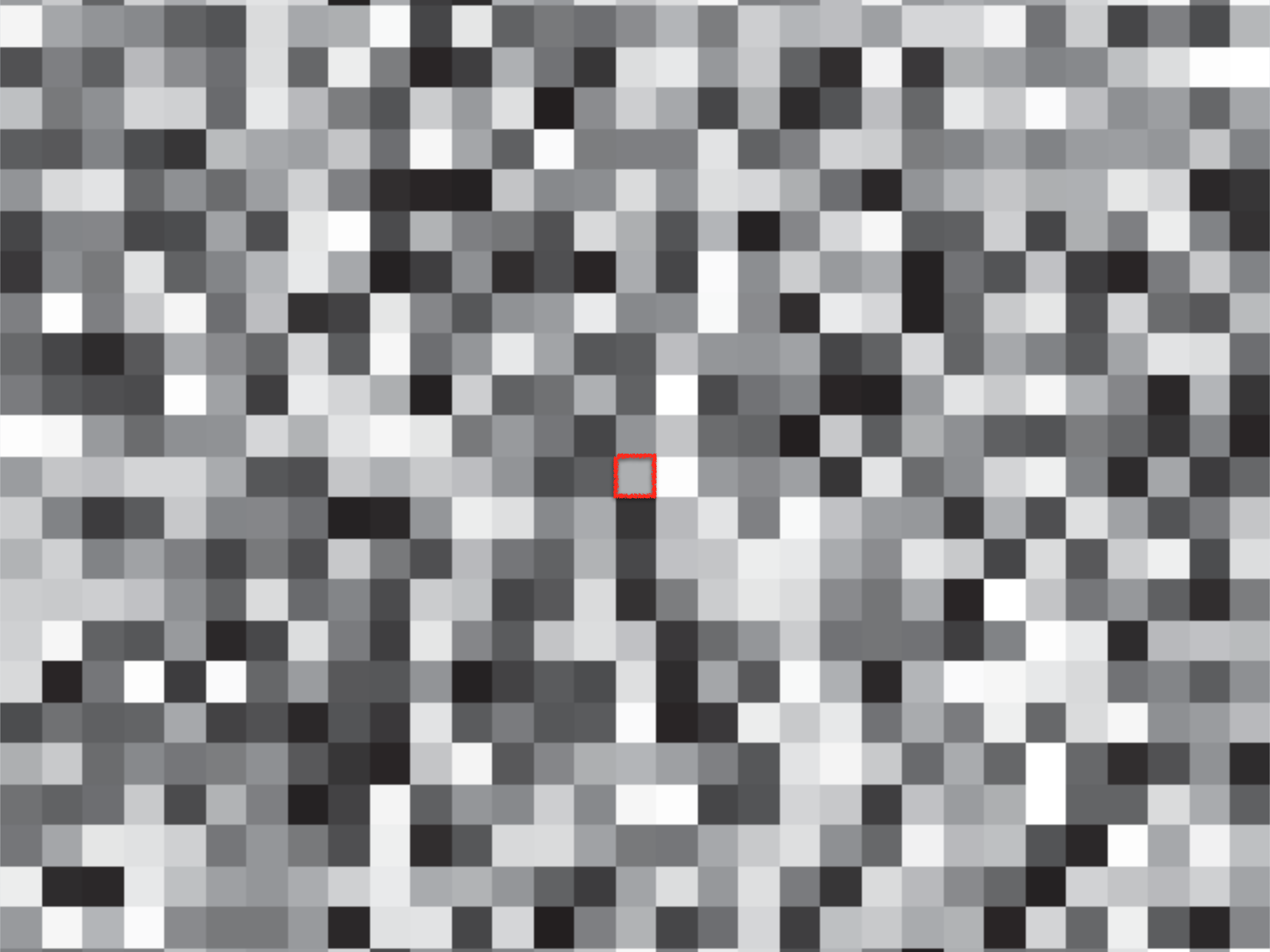
Piotr Sierant[1,], Maciej Lewenstein[1,2], Antonello Scardicchio[3], Lev Vidmar[4,5] and Jakub Zakrzewski[6,7]**

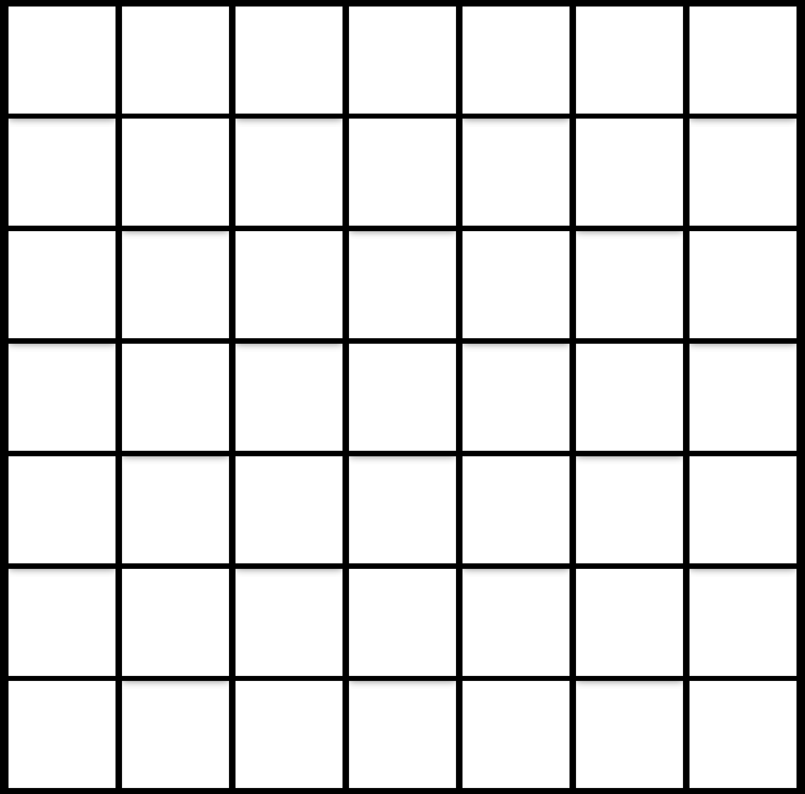


Stability of 2D MBL experiment

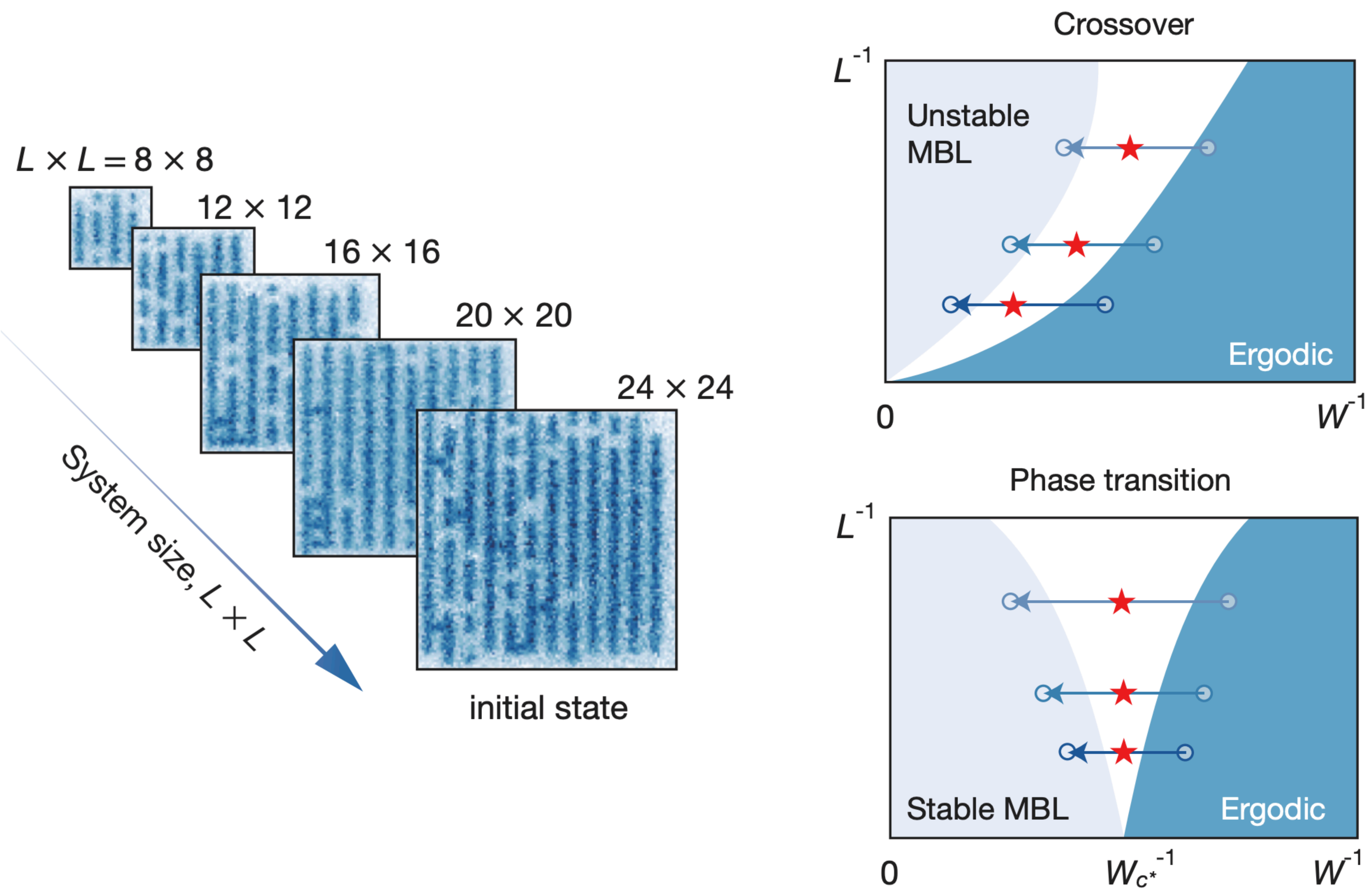




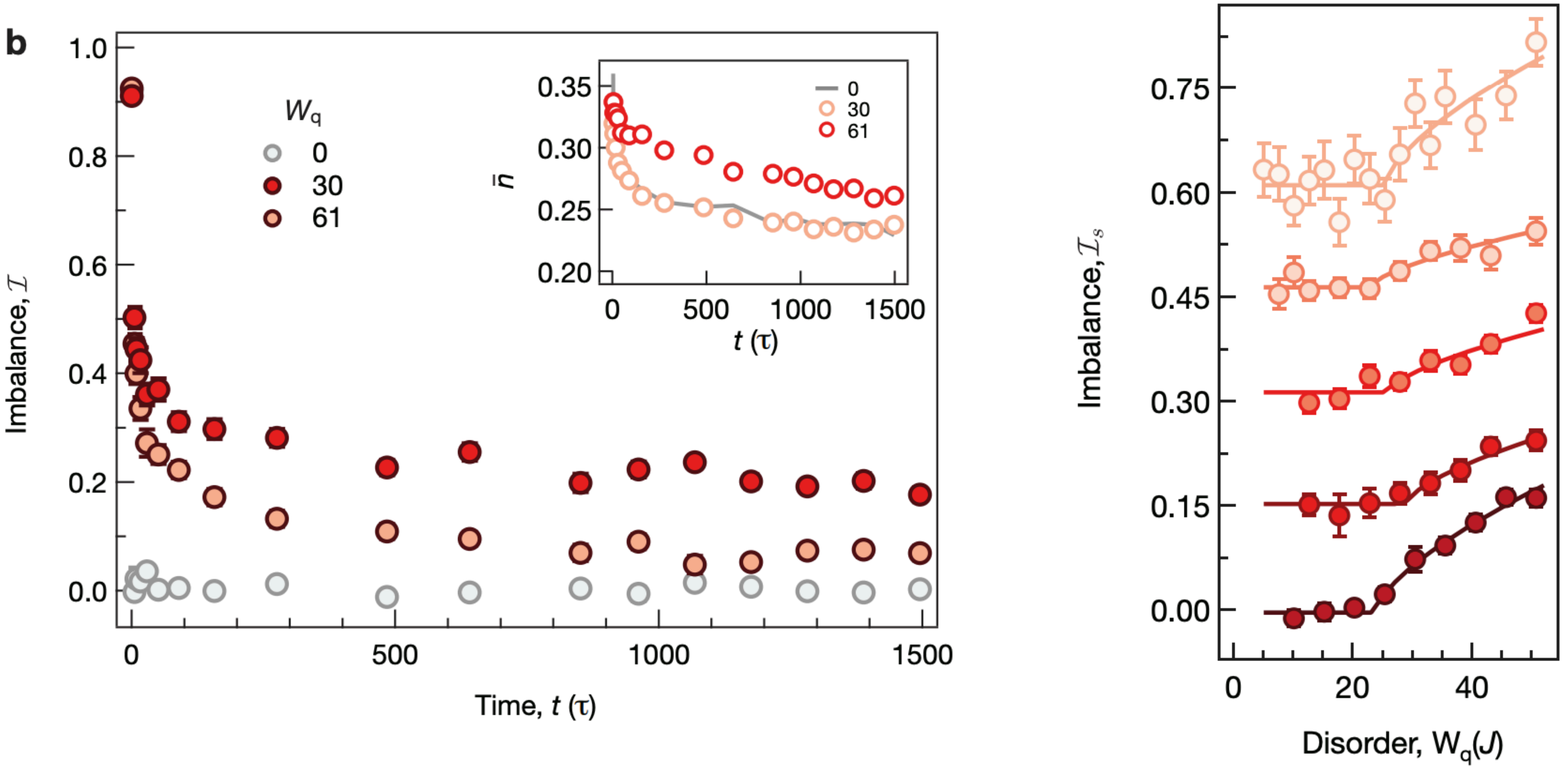




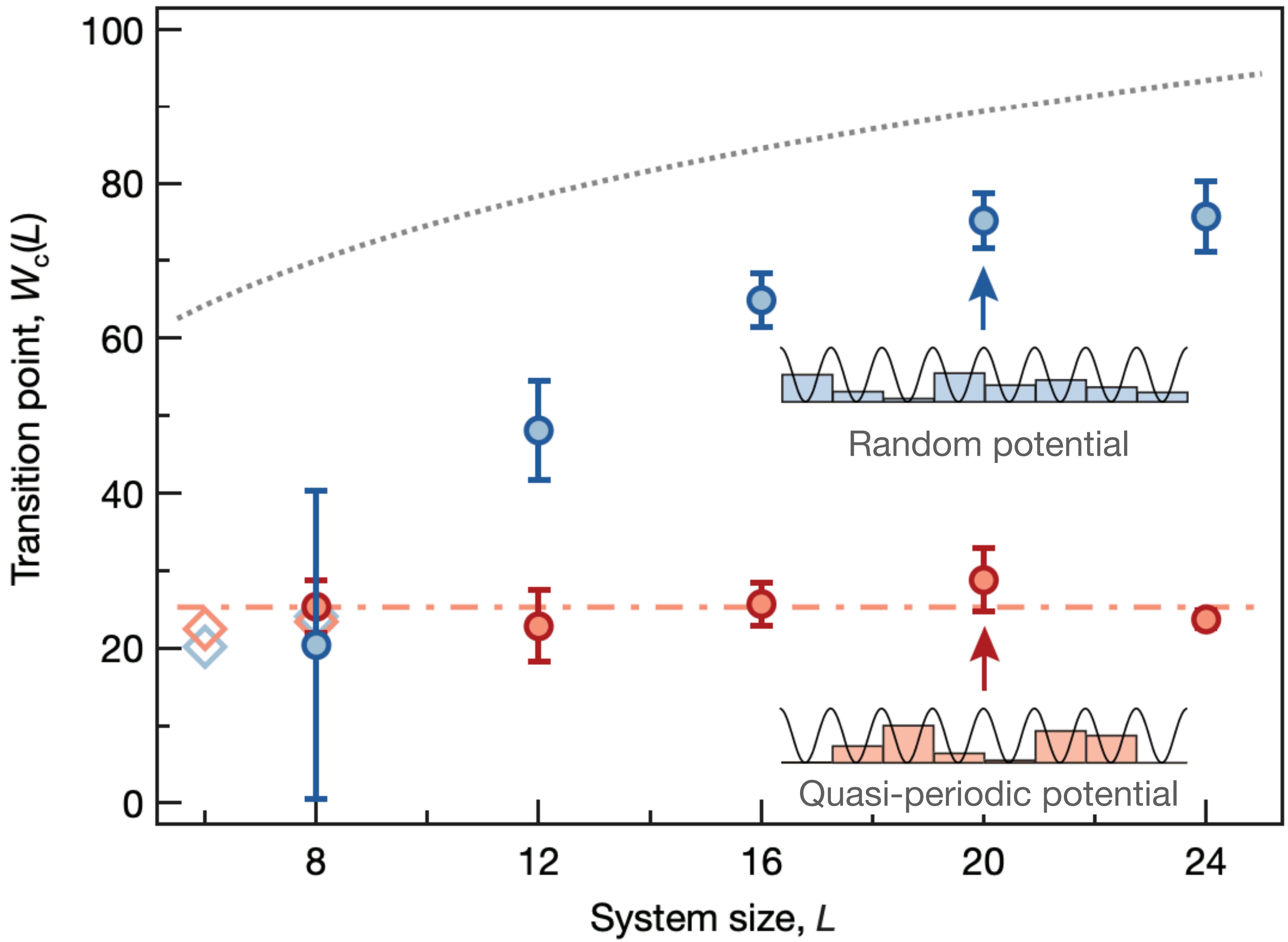
Stability of 2D MBL experiment

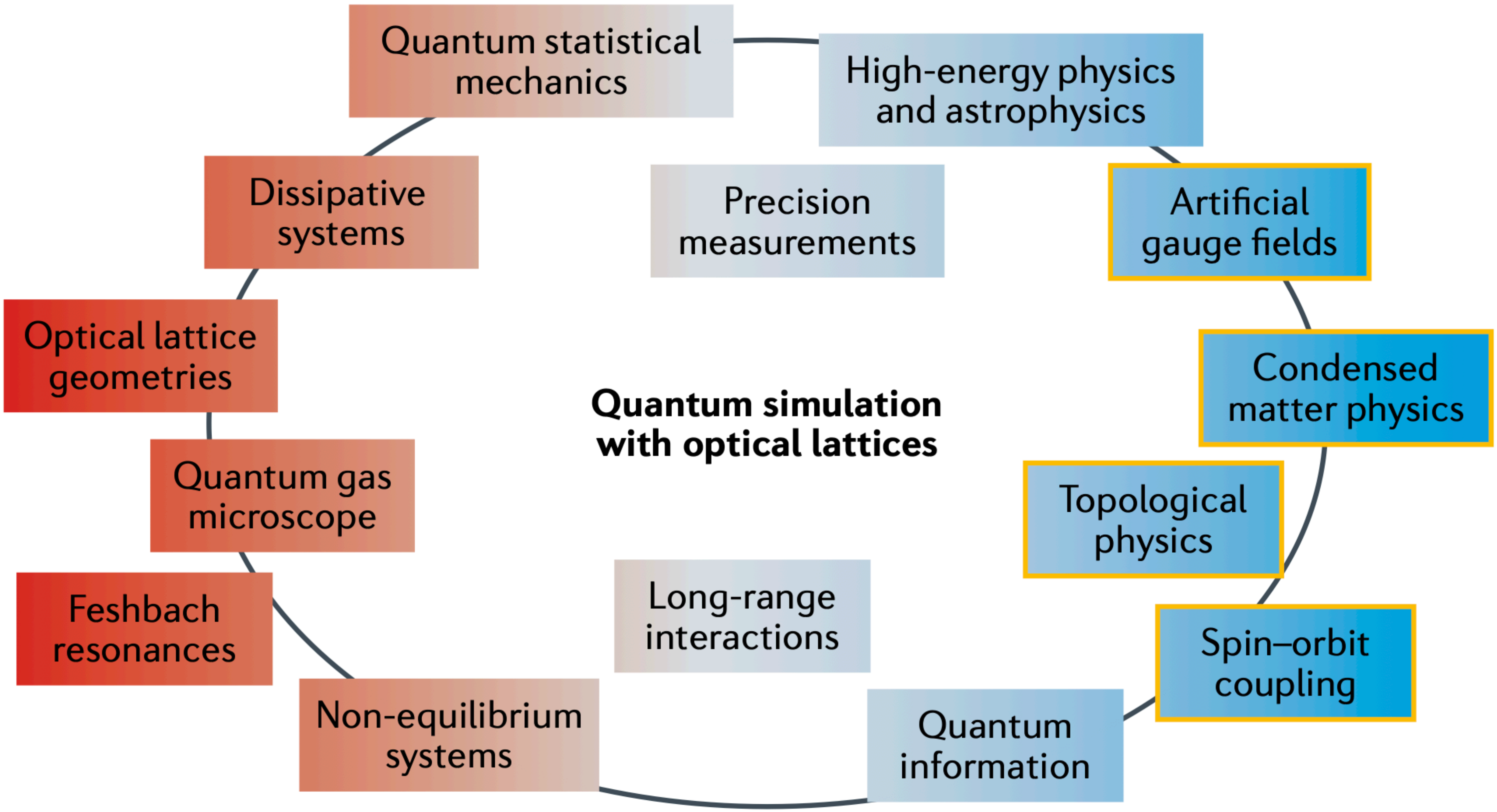


Stability of 2D MBL experiment



Stability of 2D MBL experiment

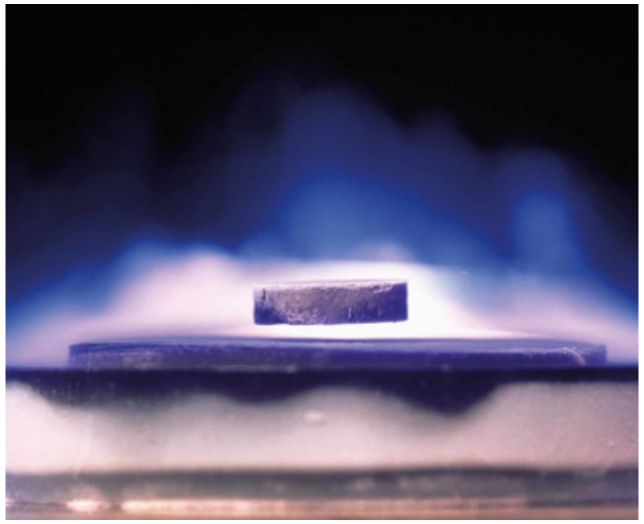




Nature Reviews Physics **2**, 411 (2020).
See also Science **357**, 995 (2017).

Neutral atom quantum simulator

Real material

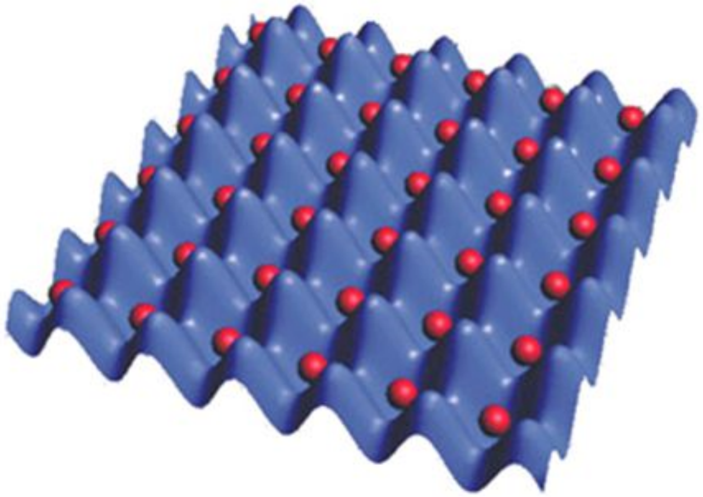


Model Hamiltonian

$$H = -t \sum_{(i,j),\sigma} \hat{c}^\dagger_{i\sigma}\hat{c}_{j\sigma} + V \sum_{i,\sigma,\sigma'} \hat{c}^\dagger_{i\sigma}\hat{c}^\dagger_{i\sigma'}\hat{c}_{i\sigma'}\hat{c}_{i\sigma}$$

$$H = J \sum_i \hat{S}_i \cdot \hat{S}_{i+1}$$

Quantum simulator

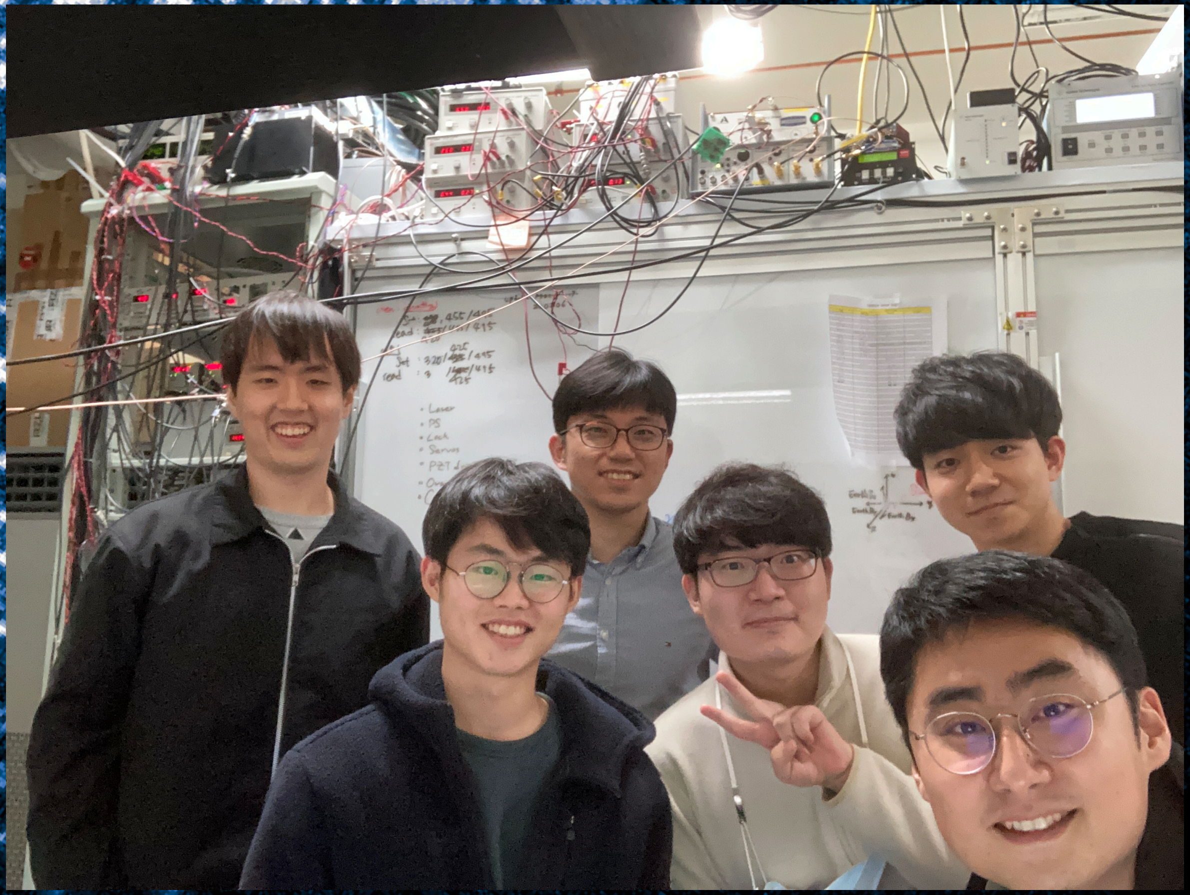


Measurement outcome

$$|\psi(t)\rangle = e^{-\frac{i}{\hbar}Ht}|\psi(0)\rangle$$

$$\langle s_i s_{i+1} \rangle$$

Thank you



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