Discovery of a New Fundamental Dictating Galaxy Cluster Evolution from Gravitational Lensing

Keiichi Umetsu, ASIAA, Taipei, Taiwan

Galaxy clusters as a sensitive probe of nonlinear structure formation

- Standard paradigm for structure formation: $\Lambda$CDM
  - Collisionless, cold dark matter
- Clusters offer fundamental tests of assumed DM properties:
  - DM density, $\rho(r|\mathcal{M})$ Umetsu+11, 14, 16
  - Splashback density steepening Umetsu+17
  - Splashback in phase space Okumura+18
  - Halo shape and alignments Umetsu+18
  - Substructure distribution Okabe+14
  - DM/galaxy/gas offsets Clowe+14
End-product of collisionless collapse in an expanding universe, $\rho(r|M)$

Quasi-equilibrium halos with

$$\frac{1}{2}\langle \dot{I} \rangle = 2T - |W| - S \approx 0$$

Navarro-Frenk-White (NFW) profile

Quasi-equilibrium halos with Cuspy, steepening profile

$$d \ln \rho(r) \over d \ln r = -2 @ r = r_s$$

$$\max(V_{\text{circ}}) @ r \approx 2r_s$$
Cluster mass distribution—cuspy, NFW-like

~20 high-mass clusters with $<M_{200c}> = 1.4 \times 10^{15} M_{\text{sun}}$ (CLASH survey)

- Excellent agreement with CDM over 99% of radial range ($r_{\text{min}} = 40 \text{kpc}/h$)
- Innermost density slope: $\gamma = 0.9 \ (+0.2, -0.3)$ with $\rho(r) \sim r^{-\gamma} \ (\gamma_{\text{CDM}} = 1)$
- Concentration $c_{200c} = 3.8 \ +/- \ 0.3$ precisely matching the LCDM prediction
- Halo ellipticity $1^{-<b/a>} = 0.33 +/ - 0.07$, consistent with LCDM (Umetsu+18)
**Splashback steepening in cluster lensing data**

**$R_\Delta$-rescaled joint fit**

$R [\text{physical} \ h^{-1} \text{kpc}]$

$\Sigma/\Sigma(r_{200m})$

$\Delta/\sigma$

$R/r_{200m}$

$R_{sp}/r_{200m} > 0.89 \ (1\sigma)$

First lensing constraints on the splashback radius $R_{sp}$

How DM halos form and grow?

“Inside-out” growth scenario ($\Lambda$CDM)

- DM halos are assembled from the inside out (Wechsler+02, Zhao+03)
- Halo’s internal structure reflects their growth history (Ludlow+13)

1. **Fast-growth phase**
   Inner region ($<r_s$) grows rapidly via massive major mergers

2. **Slow-growth phase**
   Halo outskirts ($>r_s$) gradually grow via smooth matter accretion from surroundings, without changing the inner potential significantly

*Figure courtesy by Y. Fujita*
Key questions in this talk

How about real halos? \( \rightarrow \) high-mass clusters \((M \sim 10^{15} M_{\odot})\)

How about baryons? \( \rightarrow \) hot gas (>80% of the cluster baryons)

- **Lensing observables**: halo’s scale radius and mass \((r_s, M_s)\), with \(r_s \sim 0.2 R_{sp} \sim 500\text{kpc}\) for high-mass clusters

- **Baryonic observable**: X-ray gas temperature, \(T_X\)

Are the DM and baryon halo parameters tightly coupled?

- **If yes**: The hot gas was likely heated during the fast-growth phase (i.e., major mergers), and \(T_X\) was preserved in the subsequent slow-growth phase.

If so, how do \((M_s, r_s, T_X)\) correlate? What is the degree of scatter?

**Canonical predictions** (e.g., virial equilibrium, Komatsu-Seljak pressure model):

\[
T \propto f(c) \frac{M_s}{r_s} \sim \frac{M_s}{r_s}
\]
Deep multi-wavelength data sets from the CLASH survey


X-ray analysis of all 25 CLASH clusters with deep Chandra and XMM X-ray imaging and spectroscopy (Donahue+14, ApJ, 797, 34)
High-resolution space imaging with *HST* (ACS/WFC3) for strong lensing

Subaru/Suprime-Cam multi-color imaging for weak lensing shear & magnification

Cluster sample and data

- **20 CLASH clusters**
  - $0.18 < z < 0.69$
  - $5 < M_{200c}/10^{14} M_{\odot} < 30$

- **$M_s$, $r_s$**
  - Marginalized posteriors of NFW ($M_{200c}$, $c_{200c}$) from weak+strong lensing (Umetsu+16)
  - $<r_s> \sim 500\text{kpc}$

- **$T_X$**
  - Core-excised *Chandra* temperature $T(50-500\text{kpc})$ (Donahue+14)
  - $<T_X>\sim 8\text{keV}$

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Results: Principal Component Analysis

A fundamental plane (FP) exists in DM-baryon parameter space with 0.045dex (10%) scatter!

→ $T_X$ is tightly coupled with $(r_s, M_s)$

$$a \log(r_s) + b \log(M_s) + c \log(T_X) = \text{const.}$$
Plane Angle

The observed plane is significantly tilted from the canonical “virial” expectation, \( T_X \propto M_s/r_s \)

Direction of the plane normal: \( P_3 \)

\[
\begin{align*}
\log T_X & \\
\log r_s & \\
\log M_s & \\
\theta & \\
\Phi & \\
\end{align*}
\]

Figure courtesy by Y. Fujita

Simulations

- MUSIC
- NF0
- FB0
- FB1
- SSol

Contours: CLASH

Fujita, Umetsu+18a
**N-body + hydro Simulations**

- Our sample is relatively small ($N = 20$)
  - Checks for selection bias (e.g., relaxation state) needed
  - Interpret data within the framework of $\Lambda$CDM
- **Adiabatic simulations**
  - 402 mass-selected halos with $M > 2 \times 10^{14} M_{\text{sun}}/h$ from Meneghetti et al. (2014)
    - MUSIC: $z=0.25$
- **Radiative cooling + feedback (AGN/SNe) simulations**
  - 29 cluster halos with $M = (1-30) \times 10^{14} M_{\text{sun}}/h$ from Rasia et al. (2015)
    - FB0: $z=0$
    - NF0: $z=0$, adiabatic for comparison
    - FB1: $z=1$

Fujita, Umetsu+18a
(1) Adiabatic simulations

• Simulated cluster halos form a tight plane (0.025dex)!
  – The angle is consistent with the data
• The plane formed by most unrelaxed halos is almost the same as that of most relaxed halos, but with an increased scatter
• Selection bias due to the degree of relaxation is not significant
(2) Radiative cooling + feedback simulations

- The angle is consistent with the data
- FB0 plane is almost the same as NF0 (adiabatic) plane
  - The effects of cooling and feedback are not important at $r_s \sim 500$ kpc
- FB1($z=1$) and FB0($z=0$) halos lie on the same plane (0.037 dex)
  - Clusters evolve within the plane along the direction of $P_1$

![Cross Section](image)

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<th>P_3 (orthogonal to FP)</th>
<th>Core-excised T used</th>
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What’s the physics governing FP?

A possible explanation: Bertschinger (1985) secondary-infall similarity solution for accretion of gas in a matter-dominant universe

- Shock-heated ideal gas, \( P \propto \rho^{5/3} \)
- Accretion shock radius (shock jump conditions satisfied)
- Cold accretion \((P=0, V_{\text{infall}})\)

Cluster outskirts

Progenitor halo \((t_s, M_s, r_s)\)

\[ \bar{\rho} \propto a^{-3} \propto t^{-2} \]
What’s the physics governing FP?

A possible explanation: Bertschinger (1985) secondary-infall similarity solution for accretion of gas in a matter-dominant universe

Cold accretion \((P=0, V_{\text{infall}})\)

Cluster outskirts

Progenitor halo \((t_s, M_s, r_s)\)

Shock-heated ideal gas, \(P \propto \rho^{5/3}\)

Accretion shock radius (shock jump conditions satisfied)

(1) Entropy integral of B85
(2) \(P(k) \propto k^{-2}\) at cluster scales

\(T \propto \frac{M^{1.5}}{r^2}, \quad r \propto M^{1/2}\)

@ \(r = r_s\)
Fundamental Plane vs. Similarity Solution

**Similarity solution (SSol)**

\[
\frac{r_s^2 M_s}{(n+11)/6} T = \text{const.}
\]

\[
n \equiv \frac{d \ln P}{d \ln k} \approx -2 \text{ @ cluster scales}
\]

- The predicted angle (SSol) is consistent with observations and simulations.
- In this picture, clusters are continuously evolving and growing with time.
  - Momentum flux from matter accretion
  - Not in “virial” equilibrium \( \frac{1}{2} \langle \tilde{t} \rangle = 2T - |W| - S \approx 0 \)
  - Gas in pressure equilibrium

Fujita, Umetsu+18a
Stability of FP against mergers

Evolutionary track of a typical halo in the FB0+FB1 sample

- Clusters evolve along $P_1$
- $T$ and $(M_s, r_s)$ are tightly coupled and co-evolve
- Even during major mergers (A, B, E), the halo stays in the plane
  - $T$ and $(M_s, r_s)$ are anti-correlated
  - Mergers contributing to the thickness of the FP
Application: cluster mass calibration

Calibrating X-ray hydrostatic mass estimates using the “shift” of the FP (Fujita, Umetsu et al. 2018b)

Red: 20 CLASH clusters
Black: 44 clusters from Ettori+10 (XMM-Newton)

Mass calibration weakly depends on concentration $c_\Delta$ of the target sample

$$f_M=M_{X\text{-FP}}/M_{FP} = 0.85+/-0.20$$, consistent with the expected level of HS bias (10-15%)
Summary

1. Observed clusters form a tight plane in DM-baryon parameter space \((r_s, M_s, T)\).

2. The observed plane is significantly tilted from the virial expectation, \(T \propto M_s/r_s\), and can be explained by a similarity solution (Bertschinger 1985):
   - Cluster outskirts are evolving with time through continuous mass accretion.
   - The Bertschinger 85 picture works for adiabatic gas + collisionless DM (e.g., Shi 2016).

3. Numerical simulations reproduce the observed plane, independently of the gas physics implemented in the code.

4. Cluster halos are predicted to evolve within the plane along the direction of P1:
   - The plane is stable even against major mergers!!!

5. See Paper II (Fujita, Umetsu+18b) for further applications: e.g., cluster mass calibration, origin of the \(M-T\) relation
Supplemental slides
A close look at simulated clusters

Similarity solution: \( r_s \propto M_s^{1/2} \) \( (r_s \propto M_s^{1/1.65} \text{ by Zhao+09}) \)

Formation epoch of halos \( \rho_{\text{crit}}(t_f) \sim \frac{M_s}{r_s^3} \equiv \rho_s \)

The self-similar solution predicts \( r_s \propto M_s^{1/2} \) for cluster-scale halos \( (P(k) \sim k^{-2}) \), which is consistent with the direction of cluster evolution \( (P_1) \) found from our observations and simulations.

Cluster evolution depends on the initial matter power spectrum \( P(k) \sim k^n \)

FP projected on \( r_s - M_s \)

Cluster evolution depends on the initial matter power spectrum \( P(k) \sim k^n \)

Fujita, Umetsu+18
Weak lensing: shear & magnification

- **Shear** (Kaiser 92)
  - Shape distortion: $\delta e \sim \gamma$

- **Magnification** (Broadhurst+95)
  - Flux amplification: $\mu F$
  - Area distortion: $\mu \Delta \Omega$

Sensitive to “differential” matter density:

$$\Sigma_c \gamma_+ = \Delta \Sigma(R) \equiv \Sigma(<R) - \Sigma(R)$$

Sensitive to “total” matter density:

$$\mu \approx 1 + 2\kappa; \quad \Sigma_c \kappa = \Sigma(R) = \int (\rho - \bar{\rho}_m) dl$$
Cluster sample and data

- 20 CLASH clusters
  - $0.18 < z < 0.69$
  - $4 < \frac{M_{200c}}{10^{14} M_{\odot}/h} < 20$
  - 16 X-ray regular clusters
  - 4 high-magnification lenses

- $M_s, r_s$
  - Marginalized posteriors of NFW ($M_{200c}, c_{200c}$) from weak+strong lensing (Umetsu+16)

- $T_X$
  - Core-excised Chandra temperature $T(50-500\text{kpc})$ (Donahue+14)

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<td>18.1$^{+4.4}_{-4.1}$</td>
<td>10.8$^{±0.60}$</td>
</tr>
<tr>
<td>MACS J0329.7-0211</td>
<td>0.450</td>
<td>254$^{+95}_{-63}$</td>
<td>1697$^{+129}_{-129}$</td>
<td>1.4$^{+0.6}_{-0.4}$</td>
<td>8.6$^{+2.1}_{-1.8}$</td>
<td>8.0$^{±0.60}$</td>
</tr>
<tr>
<td>RX J1347.5-1145</td>
<td>0.451</td>
<td>840$^{+339}_{-299}$</td>
<td>2084$^{+290}_{-290}$</td>
<td>9.8$^{+5.6}_{-3.6}$</td>
<td>34.2$^{+8.1}_{-8.3}$</td>
<td>15.5$^{±0.60}$</td>
</tr>
<tr>
<td>MACS J1149.5+2223</td>
<td>0.544</td>
<td>1108$^{+404}_{-291}$</td>
<td>2334$^{+169}_{-178}$</td>
<td>10.8$^{+5.4}_{-3.7}$</td>
<td>25.0$^{+5.8}_{-5.3}$</td>
<td>8.7$^{±0.90}$</td>
</tr>
<tr>
<td>MACS J0717.5+3745</td>
<td>0.548</td>
<td>1300$^{+347}_{-271}$</td>
<td>2387$^{+154}_{-169}$</td>
<td>13.2$^{+5.3}_{-3.0}$</td>
<td>26.8$^{+5.6}_{-5.3}$</td>
<td>12.5$^{±0.70}$</td>
</tr>
<tr>
<td>MACS J0647.7+7015</td>
<td>0.584</td>
<td>468$^{+254}_{-160}$</td>
<td>1884$^{+189}_{-192}$</td>
<td>3.3$^{+2.3}_{-1.3}$</td>
<td>13.7$^{+3.8}_{-3.6}$</td>
<td>13.3$^{±1.80}$</td>
</tr>
<tr>
<td>MACS J0744.9+3027</td>
<td>0.686</td>
<td>574$^{+200}_{-192}$</td>
<td>1982$^{+185}_{-185}$</td>
<td>4.9$^{+3.1}_{-2.0}$</td>
<td>17.9$^{+5.3}_{-4.6}$</td>
<td>8.9$^{±0.80}$</td>
</tr>
</tbody>
</table>
A close look at simulated clusters

Similarity solution: \( r_s \propto M_s^{1/2} \) (by Zhao+09)

Formation epoch of halos \( \rho_{\text{crit}}(t_f) \sim \frac{M_s}{r_s^3} \equiv \rho_s \)

The self-similar solution predicts \( r_s \propto M_s^{1/2} \) for cluster-scale halos \( (P(k) \sim k^{-2}) \), which is consistent with the direction of cluster evolution \( (P_1) \) found from our observations and simulations.

Cluster evolution depends on the initial matter power spectrum \( P(k) \sim k^n \)
NFW vs. Similarity solution

NFW profile (DM)

Slow growth
Fast growth

$M_s$
$r \sim r_s$
$r = R_{sp}$

Similarity solution (ICM)

Secondary infall
Overdense perturbation

$m_{ita}$
$r \propto r_{ita}$
$r = R_{ac}$
Projections of simulated clusters

Halos evolve with \( r_s \propto M_s^{1/2} \) \( (r_s \propto M_s^{1/1.65} \) by Zhao+09)

Formation epoch of halos \( \rho_{\text{crit}}(t_f) \sim \frac{M_s}{r_s^3} \)

MUSIC cosmological simulations (DM + adiabatic gas)

Fujita, Umetsu+18
Gravitational Shear

\[ \gamma = \partial \partial \Psi / 2 \]

\[ \partial := e^{i\phi} \partial_r \]
Gravitational Magnification

\[ \kappa = \partial \partial^* \Psi / 2 = \Delta \Psi / 2 \]

\[ \partial := e^{i\phi} \partial_r \]

MACSJ1149 (z=0.54)