## The 25th KIAS Combinatorics Workshop

Utop Ubless Hotel
Jeju, Korea
June 15-18, 2022

## Information

Title: The 25th KIAS Combinatorics Workshop
Date: June 15-18, 2022
Venue: Utop Ubless Hotel Jeju
Web: http://events.kias.re.kr/h/combinatorics/ or kcws.combinatorics.kr

## Invited Speakers

Debsoumya Chakraborti (IBS)
Ilkyoo Choi (Hankuk University of Foreign Studies)
Pascal Gollin (IBS)
Jaehoon Kim (KAIST)
Jang Soo Kim (Sungkyunkwan University)
SangWook Kim (Chonnam National University)
O-Joung Kwon (Hanyang University)
Sang June Lee (Kyung Hee University)
Ben Lund (IBS)
Boram Park (Ajou University)
Jaebum Sohn (Yonsei University)

## Organizers

Jang Soo Kim (Sungkyunkwan University)
Jeong Han Kim (KIAS)
Seog-Jin Kim (Konkuk University)
Young Soo Kwon (Yeungnam University)
Sang June Lee (Kyung Hee University)
Jongyook Park (Kyungpook National University)
Seunghyun Seo (Kangwon National University)

## Timetable

| 1st Day: June 15 (Wednesday) |  |
| :--- | :--- |
| $15: 30 \sim 16: 00$ |  |
| $16: 00 \sim 16: 00$ |  |
| $16: 00 \sim 17: 30$ | Registration |
| Chair: Young Soo Kwon | Opening address |
| $16: 00 \sim 16: 40$ | Jaehoon Kim |
| $16: 50 \sim 17: 30$ | Sang June Lee |
| $18: 00 \sim$ | Ramsey numbers of cycles versus general graphs <br> On the total variation distance between the binomial <br> random graph and the random intersection graph |


| 2nd Day: June 16 (Thursday) |  |  |
| :---: | :---: | :---: |
| 09:30 ~ 12:00 |  | Session (B) |
| Chair: Seog-Jin Kim |  |  |
| 09:30 ~ 10:10 | Ilkyoo Choi | Relaxations of coloring squares of graphs |
| 10:20 ~ 11:00 | O-Joung Kwon | A survey on the Erdős-Pósa property |
| 11:10 ~ 12:00 |  | Problem session and free discussion |
| 12:00 ~ 14:00 |  | Lunch |
| 14:00 ~ 17:30 |  | Session (C) |
| Chair: Sang-il Oum |  |  |
| 14:00 ~ 14:40 | Boram Park | Broadcast domination of cubic graphs |
| 14:50 ~ 15:30 | Debsoumya Chakraborti | Recent developments in graph saturation |
| 15:30 ~ 15:50 |  | Coffee Break |
| 15:50 ~ 17:30 |  | Problem session and free discussion |
| 18:00 ~ |  | Dinner |

## Timetable

| 3rd Day: June 17 (Friday) |  |  |
| :---: | :---: | :---: |
| 09:20 ~ 12:00 |  | Session (D) |
| Chair: Seunghyun Seo |  |  |
| 09:20 ~ 10:00 | Jaebum Sohn | A survey on (simultaneous) core partitions |
| 10:00 ~ 10:30 | Coffee Break |  |
| 10:30 ~ 11:10 | SangWook Kim | Enumeration of Fuss-Catalan paths by type and blocks |
| 11:20 ~ 12:00 | Jang Soo Kim | Affine Gordon-Bender-Knuth identities and cylindric Young tableaux |
| 12:00 ~ 14:00 |  | Lunch |
| 14:00 ~ 17:30 |  | Problem session and free discussion |
| 18:00 ~ |  | Banquet |


| 4th Day: June 18 (Saturday) |  |  |
| :---: | :---: | :---: |
| 09:30 ~ 10:00 |  | Coffee Break |
| 10:00 ~ 12:00 |  | Session (E) |
| Chair: Jongyook Park |  |  |
| 10:00 ~ 10:40 | Pascal Gollin | Product structure of graph classes with bounded treewidth |
| 10:50 ~ 11:30 | Ben Lund | Maximal 3-wise intersecting families |
| 11:30 ~ 12:00 |  | Free discussion |
| 12:00 ~ |  | Lunch |

Speaker: Jaehoon Kim

## Affiliation: KAIST

Title: Ramsey numbers of cycles versus general graphs


#### Abstract

The Ramsey number $R(F, H)$ is the minimum number $N$ such that any $N$-vertex graph either contains a copy of $F$ or its complement contains $H$. Burr in 1981 proved a pleasingly general result that for any graph $H$, provided $n$ is sufficiently large, a natural lower bound construction gives the correct Ramsey number involving cycles: $R\left(C_{n}, H\right)=(n-1)(\chi(H)-1)+\sigma(H)$, where $\sigma(H)$ is the minimum possible size of a colour class in a $\chi(H)$-colouring of $H$. Allen, Brightwell and Skokan conjectured that the same should be true already when $n \geq|H| \chi(H)$. We improve this 40-year-old result of Burr by giving quantitative bounds of the form $n \geq C|H| \log ^{4} \chi(H)$, which is optimal up to the logarithmic factor. In particular, this proves a strengthening of the Allen-Brightwell-Skokan conjecture for all graphs $H$ with large chromatic number. This is joint work with John Haslegrave, Joseph Hyde and Hong Liu.


Speaker: Sang June Lee
Affiliation: Kyung Hee University
Title: On the total variation distance between the binomial random graph and the random intersection graph


#### Abstract

When each vertex is assigned a set, the intersection graph generated by the sets is the graph in which two distinct vertices are joined by an edge if and only if their assigned sets have a nonempty intersection. An interval graph is an intersection graph generated by intervals in the real line. A chordal graph can be considered as an intersection graph generated by subtrees of a tree. In 1999, Karoński, Scheinerman and Singer-Cohen [Combin Probab Comput 8 (1999), 131-159] introduced a random intersection graph by taking random assigned sets. The random intersection graph $G(n, m ; p)$ has $n$ vertices and their assigned sets are chosen to be i.i.d. random subsets of a fixed set $M$ of size $m$ where each element of $M$ belongs to each random subset with probability $p$, independently of all other elements in $M$. Fill, Scheinerman and Singer-Cohen [Random Struct Algorithms 16 (2000), 156-176] showed that the total variation between the random graph $G(n, m ; p)$ and the Erdös-Rényi graph $G(n, \hat{p})$ tends to 0 if $m=n^{\alpha}, \alpha>6$, where $\hat{p}$ is chosen so that the expected numbers of edges in the two graphs are the same. In this paper, it is proved that the total variation still tends to 0 whenever $m \gg n^{4}$. This is joint work with Jeong Han Kim and Joohan Na.


Speaker: Ilkyoo Choi

Affiliation: Hankuk University of Foreign Studies
Title: Relaxations of coloring squares of graphs


#### Abstract

Six months ago, the following two notions of relaxations of coloring squares of graphs were formally introduced by Petruševski and Škrekovski, and by Fabrici, Lužar, Rindošová, and Soták: An odd c-coloring (resp. proper conflict-free c-coloring) of a graph is a proper $c$-coloring such that each non-isolated vertex has a color appearing an odd number of times (resp. exactly once) on its neighborhood.

We show that for $c \geq 5$, every graph $G$ with $\operatorname{mad}(G) \leq \frac{4 c}{c+2}$ has a proper conflict-free $c$ coloring unless $G$ contains a 1-subdivision of the complete graph on $c+1$ vertices. We also provide results when $c=4$ and for planar graphs with girth restrictions. Our results completely resolve Cranston's conjecture in a much stronger form, and also improves upon results of Caro, Petruševski, and Škrekovski. This is joint work with Eun-Kyung Cho, Hyemin Kwon, and Boram Park.


Speaker: O-Joung Kwon

Affiliation: Hanyang University
Title: A survey on the Erdős-Pósa property


#### Abstract

Erdős and Pósa proved in 1965 that given a graph G and an integer k , either G contains k vertex-disjoint cycles or it contains a vertex set of size $O(k \log k)$ hitting all the cycles. Since then, lots of variations of this theorem have been considered. We may consider different objects, different containment relations, or restricted host graphs. I will present a survey on this topic.


Speaker: Boram Park

Affiliation: Ajou University
Title: Broadcast domination of cubic graphs


#### Abstract

For a graph $G$, a function $f$ from $V(G)$ to the set of nonnegative integers is called a dominating broadcast on $G$ if for every vertex $u$, there exists a vertex $v$ such that $d(v)>0$ and $d(u, v) \leq f(v)$. For a dominating broadcast $f$ on $G$, if $f(v) \leq 2$ for every $v \in V(G)$, then $f$ is called a 2limited dominating broadcast on $G$. The 2-limited broadcast domination number $\gamma_{b, 2}(G)$ of $G$ is the minimum cost $\sum_{v \in V(G)} f(v)$ of a 2-limited dominating broadcast $f$. Henning, MacGillivray and Yang showed that a cubic graph $G$ without $C_{4}$ nor $C_{6}$ satisfies $\gamma_{b, 2}(G) \leq \frac{|V(G)|}{3}$, and then they conjectured that $\gamma_{b, 2}(G) \leq \frac{|V(G)|}{3}$ for every cubic graph $G$. In this paper we show that the conjecture is true for cubic graphs without $C_{7}$.


Speaker: Debsoumya Chakraborti

## Affiliation: IBS

Title: Recent developments in graph saturation


#### Abstract

Graph saturation is one of the oldest areas of investigation in extremal combinatorics. A graph $G$ is called $F$-saturated if $G$ does not contain a subgraph isomorphic to $F$, but the addition of any edge creates a copy of $F$. The function sat ( $\mathrm{n}, \mathrm{F}$ ) is defined to be the minimum number of edges in an $n$-vertex $F$-saturated graph. One of the central conjectures in graph saturation made by Tuza (1986) states that for every graph $F$, the limit $\lim _{n \rightarrow \infty} \frac{\operatorname{sat}(\mathrm{n}, \mathrm{F})}{n}$ exists. We make progress in the negative direction of this conjecture. We resolve one of the most fundamental questions of minimizing the number of cliques of size $r$ in a $K_{s}$-saturated graph with sufficiently large numbers of vertices, confirming a conjecture of Kritschgau, Methuku, Tait, and Timmons. We will also discuss the above problems in the context of other classical variants of saturation. This talk will be based on joint work with Po-Shen Loh.


Speaker: Jaebum Sohn

Affiliation: Yonsei University

Title: A survey on (simultaneous) core partitions


#### Abstract

t -core partitions have played important roles in the theory of partitions and related areas. In this survey, we briefly summarize interesting and important results on t-cores from classical results like how to obtain a generating function to recent results like simultaneous cores. Since there have been numerous studies on $t$-cores, it is infeasible to survey all the interesting results. Thus, we mainly focus on the roles of $t$-cores in number theoretic aspects of partition theory. This includes the modularity of $t$-core partition generating functions, the existence of $t$-core partitions, asymptotic formulas and arithmetic properties of $t$-core partitions, and combinatorial and number theoretic aspects of simultaneous core partitions. We also explain some applications of t-core partitions, which include relations between core partitions and self-conjugate core partitions, a t -core crank explaining Ramanujan's partition congruences, and relations with class numbers. At the end of the talk, we will focus on some new results about the number of (self-conjugate) t-core partitions with given number of corners. (This talk is based on joint work with Hyunsoo Cho, Byungchan Kim, and Hayan Nam and with Hyunsoo Cho, JiSun Huh, and Hayan Nam.)


Speaker: SangWook Kim

Affiliation: Chonnam National University
Title: Enumeration of Fuss-Catalan paths by type and blocks


#### Abstract

Armstrong enumerated the number of Fuss-Catalan paths with a given type and Rhoades provided the number of Dyck paths with a given type and a given number of blocks. In this talk we generalize those results to enumerate the number of Fuss-Catalan paths with a fixed type and a fixed number of blocks. We also discuss two possible generalizations for Fuss-Schroder paths.


Speaker: Jang Soo Kim

Affiliation: Sungkyunkwan University
Title: Affine Gordon-Bender-Knuth identities and cylindric Young tableaux


#### Abstract

The Gordon-Bender-Knuth identities are determinant formulas for the sum of Schur functions of partitions with bounded length. There are interesting combinatorial consequences of the Gordon-Bender-Knuth identities, for instance, connections between standard Young tableaux of bounded height, lattice walks in a Weyl chamber, and noncrossing matchings. In this talk we prove an affine analog of the Gordon-Bender-Knuth identities and study their combinatorial properties. As a consequence we obtain an unexpected connection between cylindric standard Young tableaux and r-noncrossing and s-nonnesting matchings. This is joint work with JiSun Huh, Christian Krattenthaler, and Soichi Okada.


Speaker: Pascal Gollin
Affiliation: IBS
Title: Product structure of graph classes with bounded treewidth


#### Abstract


TBA

Speaker: Ben Lund

## Affiliation: IBS

Title: Maximal 3-wise intersecting families


#### Abstract

A family F of subsets of $\{1,2, \cdots, \mathrm{n}\}$ is called maximal k -wise intersecting if every collection of at most k members from F has a common element, and moreover, no set can be added to F while preserving this property. In 1974, Erdős and Kleitman asked for the smallest possible size of a maximal $k$-wise intersecting family, for $\mathrm{k} \geq 3$. We resolve this problem for $\mathrm{k}=3$ and n even and sufficiently large. This is joint work with Kevin Hendrey, Casey Tompkins, and Tuan Tran.


