Recent theoretical advances on integrability and non-integrability in quantum many-body systems

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We consider integrability and non-integrability of quantum spin chains. Here, the word integrable/non-integrable stands for the presence/absence of nontrivial local conserved quantities.

In the first part, we investigate a striking dichotomy. All quantum spin chains studied so far fall into two extremes: either the system possesses k-local conserved quantities for all k (complete integrability), or it possesses none at all (non-integrability). However, this dichotomy has remained entirely unexplained from a mathematical point of view. In this work, we rigorously prove the dichotomy for several classes of quantum spin chains. For nearest-neighbor, inversion-symmetric spin-1/2 chains, we prove this dichotomy by giving a complete classification into completely integrable and non-integrable models [1,2]. In particular, we show that all models other than the known integrable models are non-integrable. We also establish the dichotomy for SU(2)-symmetric spin-S chains with arbitrary S [3]. In this setting, we further show that the two cases are distinguished by an extremely simple computable quantity D^3: the model is completely integrable if (D^3=0), and non-integrable otherwise.

In the second part, we shed new light on quantum integrability. In the standard approach based on the Yang--Baxter equation, one starts from an R-matrix, and the Hamiltonian is obtained from logarithmic derivatives of the transfer matrix. Therefore, to solve a given Hamiltonian, one has to find a suitable R-matrix by a kind of craftsmanship, and thus it is far from obvious whether such an R-matrix exists. In contrast, we prove that Yang-Baxter solvability can be determined simply by examining a nested commutator of the local Hamiltonian [4]. This criterion, which is equivalent to the Reshetikhin condition, is also the lowest-order nontrivial consequence of the Yang-Baxter equation. Our result therefore reveals a surprising structure: the lowest-order relation already contains the full information of the Yang-Baxter equation.

[1] Mizuki Yamaguchi, Yuuya Chiba, Naoto Shiraishi, arXiv:2411.02162

[2] Mizuki Yamaguchi, Yuuya Chiba, Naoto Shiraishi, arXiv:2411.02163

[3] Naoto Shiraishi and Mizuki Yamaguchi, Phys. Rev. B 113, L241111 (2026)

[4] Naoto Shiraishi, Mizuki Yamaguchi, and Fuga Ishii, in preparation