From Bricks to Mansions: Zero-Energy Loops, Entropic Basins, and the Organization of Ground States in the 2D Spin Glass

Lei-Han Tang
Center for Interdisciplinary Studies, Westlake University

How does the ground-state manifold of a disordered magnet organize itself—into a few broad valleys, or into many distinct basins separated by subtle, entropy-controlled bottlenecks? I will address this question in the 2D Edwards–Anderson (random-bond Ising) model using entropic-sampling Monte Carlo [1], which gives direct access to the low-energy, magnetization-resolved density of states and lets us see how ground states cluster.

The central "building blocks" are zero-energy loops: closed spin-flip contours that cost no energy but reshape the accessible configuration space. A useful first description of loop–loop interactions is captured by an effective hard-core lattice-gas picture (geometric exclusion / "entropy locking"). Turning this into a truly formal theory, however, is where difficulties begin: the mapping must confront systematic under-counting and over-counting of configurations when loops are not independent (overlaps, compatibility constraints, and composite moves), and this is precisely where deeper mathematical structure is hiding. A visual tour will compare representative ground states from different entropic basins across phases—ferromagnetic, the transition region, and the spin-glass regime—highlighting how basin structure connects to hierarchical organization and emergent ultrametricity *a la* Parisi, and how this viewpoint sharpens the intuition behind peculiar temperature chaos [2].

I will close by linking these ideas to an older theme from our 2007 PRL work on the Kuramoto model [3]: in both spin glasses and coupled oscillators, quenched disorder can produce "peculiar" mean-field behavior, while finite-dimensional systems reveal equally important, and sometimes counterintuitive, corrections driven by spatial structure. Hyunggyu has always insisted on rigor while I keep trying to dig underneath for the physics—so I like to think we have uncovered many good bricks in the 2D RBIM, and that after his formal retirement he will have the spare time to assemble them into proper mansions.

[1] Yi Liu, Ding Wang, Xin Wang, Dao-Xin Yao, and Lei-Han Tang, *Comm. Theor. Phys.* **77**, 125603 (2025).
[2] H Nishimori, M Ohzeki, and M Okuyama, *Phys. Rev. E* **112**, 044140 (2025).
[3] Hyunsuk Hong, Hugues Chaté, Hyunggyu Park, and Lei-Han Tang, *Phys. Rev. Lett.* **99**, 184101 (2007).