

Exact Results in Stochastic Processes with Division, Death, and Diffusion

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We consider a generic class of stochastic particle-based models whose state at an instant in time is described by a set of continuous degrees of freedom (e.g. positions), and the length of this set changes stochastically in time due to birth-death processes. Using a master equation formalism, we write down the dynamics of the corresponding (infinite) set of probability distributions: this takes the form of coupled Fokker-Planck equations with model-dependent source and sink terms. We derive the general expression of entropy production rate for this class of models in terms of path irreversibility. To demonstrate the practical use of this framework, we analyze a biologically motivated model incorporating division, death, and diffusion, where spatial correlations arise through the division process. By systematically integrating out excess degrees of freedom, we obtain the marginal probability distribution, enabling exact calculations of key statistical properties such as average density and correlation functions. We validate our analytical results through numerical Brownian dynamics simulations, finding excellent agreement between theory and simulation. Our method thus provides a powerful tool for tackling previously unsolved problems in stochastic birth-death dynamics.

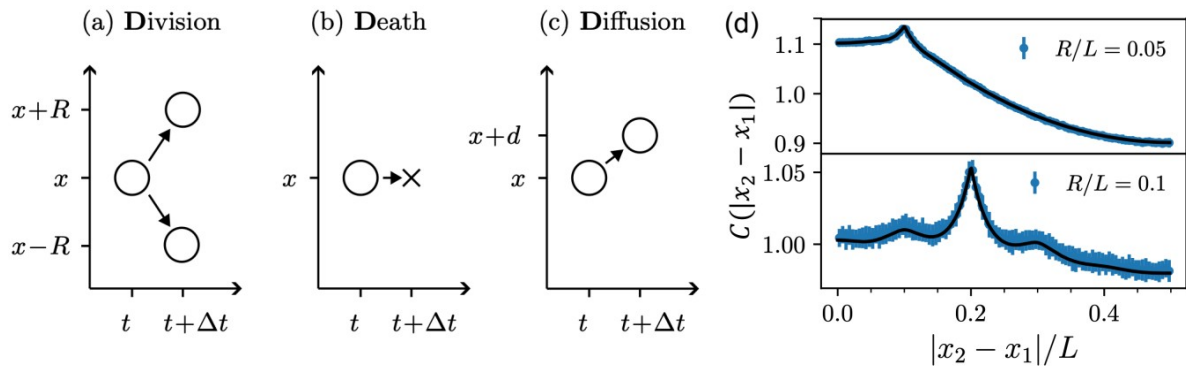


Figure: Schematic of (a) division, (b) death, and (c) diffusion processes in our model. (d) Two-body correlation function in the steady state. Markers are numerical results and solid lines are analytic results.

[1] S. Cameron, and E. Tjhung, Phys. Rev. E. 112, 034109 (2025).