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Polar Order in Active Ising Model

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Woo @ Univ. of Saarland

Motility-Induced Pinning in Flocking System with Discrete Symmetry

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We report a motility-induced pinning transition in the active Ising model for a self-propelled particle system with discrete symmetry. This model was known to exhibit a liquid-gas type flocking phase transition, but a recent study reveals that the polar order is metastable due to droplet excitation. Using extensive Monte Carlo simulations, we demonstrate that, for an intermediate alignment interaction strength, the steady state is characterized by traveling local domains, which renders the polar order short-ranged in both space and time. We further demonstrate that interfaces between colliding domains become pinned as the alignment interaction strength increases. A resonating back-and-forth motion of individual self-propelled particles across interfaces is identified as a mechanism for the pinning. We present a numerical phase diagram for the motility-induced pinning transition, and an approximate analytic theory for the growth and shrink dynamics of pinned interfaces. Our results show that pinned interfaces grow to a macroscopic size preventing the polar order in the regime where the particle diffusion rate is sufficiently smaller than the self-propulsion rate. The growth behavior in the opposite regime and its implications on the polar order remain unresolved and require further investigation.

DOI: 10.1103/PhysRevLett.133.188301

Active matter

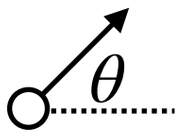
- Self-propelled particles can move by consuming energy.
  ex) cytoskeleton (actin, microtubule, …), unicellular organisms (bacteria), animals (fish schools, bird flocks, …), synthetic materials (Janus particles, robots, …)
- Motility is a relevant parameter

flying starlings ≃ flying XY spins

long range flocking order



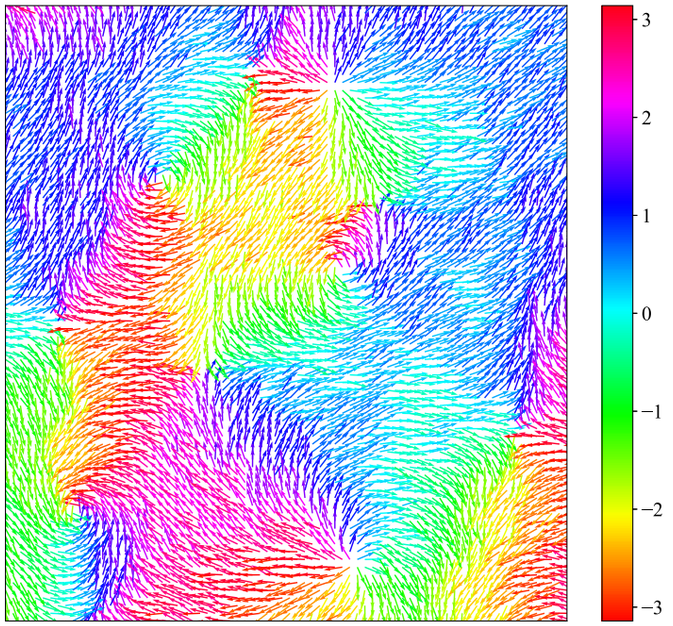
By John Holmes, CC BY-SA 2.0,
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≠

equilibrium XY spins

quasi long range order in 2D



By ChrisJLygouras - Own work, CC BY-SA 4.0,
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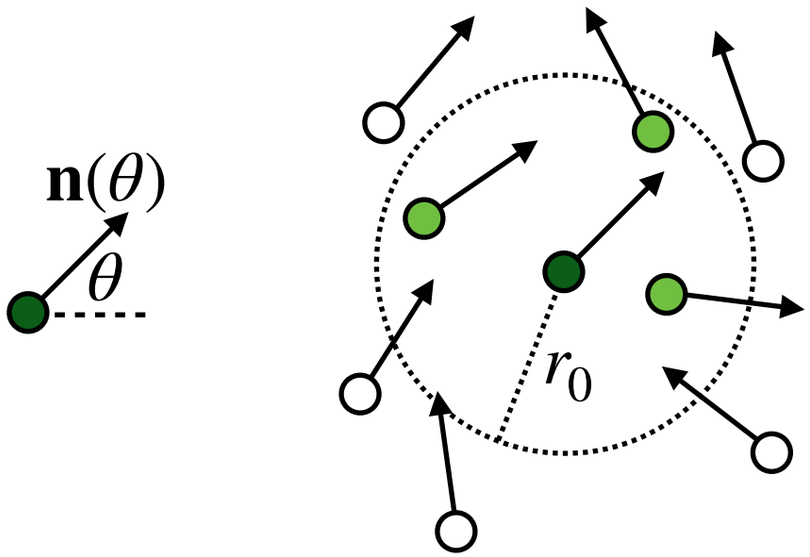
Continuous vs Discrete Symmetry

Vicsek model

VOLUME 75, NUMBER 6 PHYSICAL REVIEW LETTERS 7 AUGUST 1995

Novel Type of Phase Transition in a System of Self-Driven Particles

Tamás Vicsek,[1,2] András Czirók,[1] Eshel Ben-Jacob,[3] Inon Cohen,[3] and Ofer Shochet[3]



self-propulsion

$$\mathbf{r}_i(t+1) = \mathbf{r}_i(t) + v_0\mathbf{n}(\theta_i(t+1))$$

velocity alignment

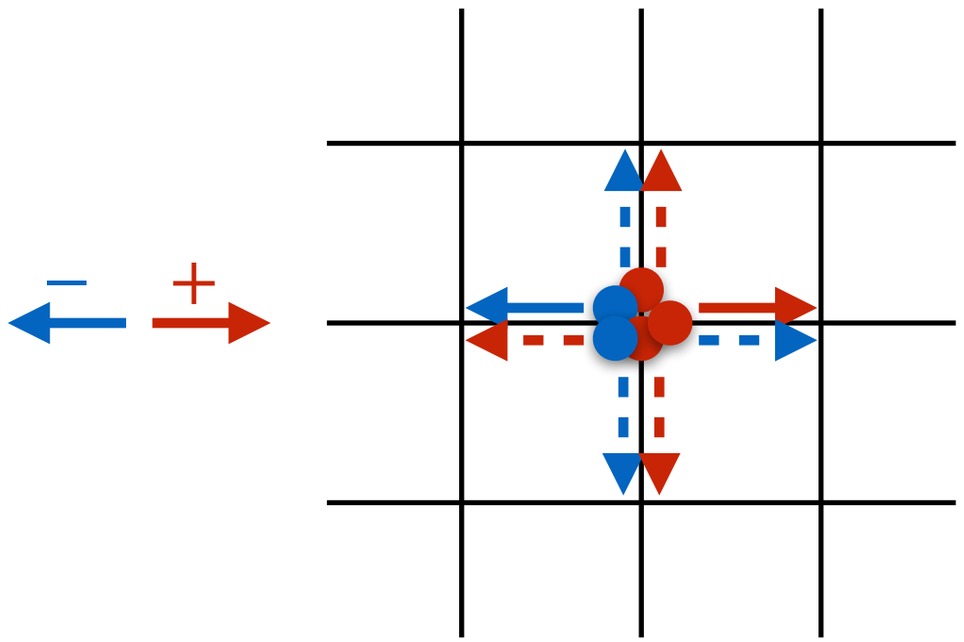
$$\theta_i(t+1) = \mathrm{Arg}\sum_{j\sim i} e^{i\theta_j(t)} + \pi\eta_i(t)$$

active Ising model (AIM)

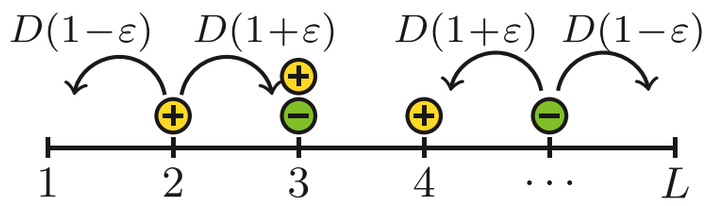
PRL 111, 078101 (2013) PHYSICAL REVIEW LETTERS week ending 16 AUGUST 2013

Revisiting the Flocking Transition Using Active Spins

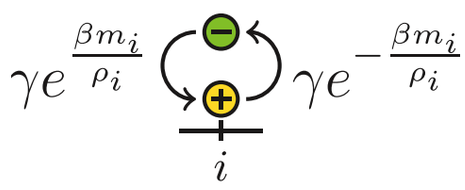
A. P. Solon and J. Tailleur



Biased Diffusion

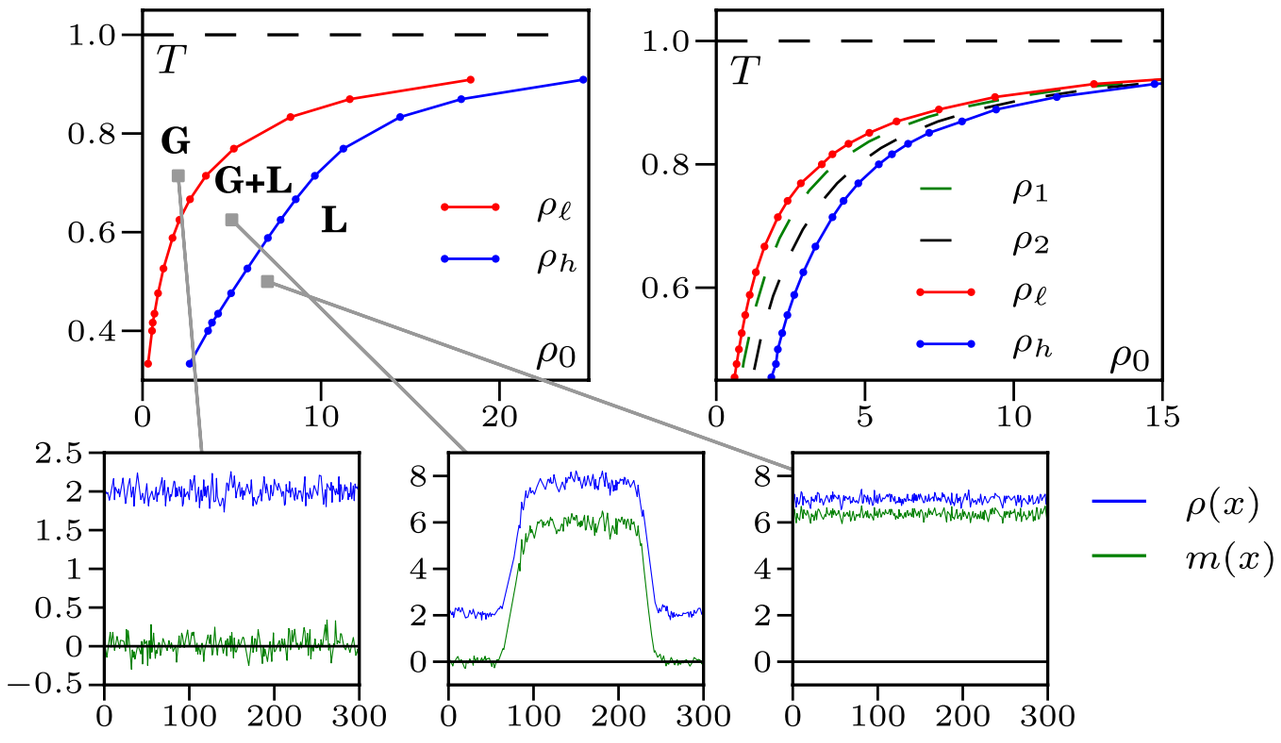


Spin Flips

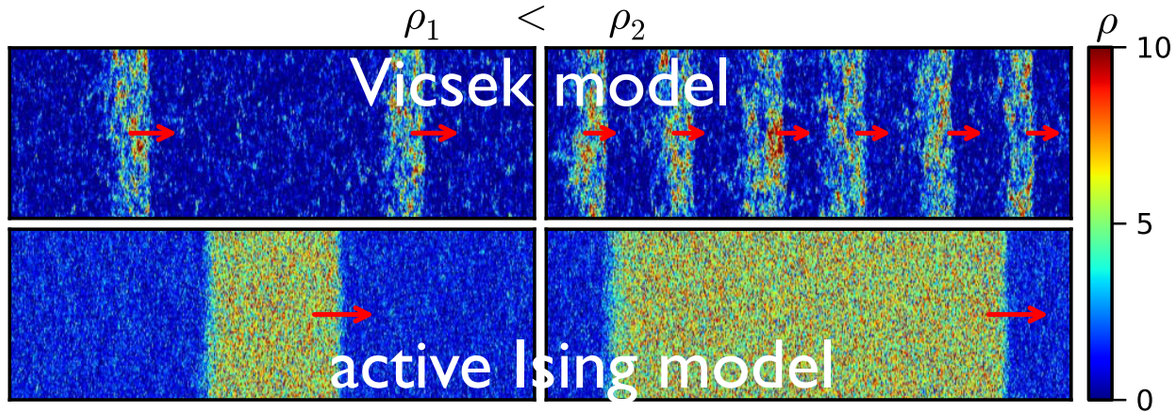


Liquid-Gas Transition

Solon and Tailleur, PRL (2013)



Solon, Chate, and Tailleur, PRL (2015)



coexistence of a disordered gas state ($m = 0$) and an ordered liquid state ($m \neq 0$)

microphase vs macrophase separation
giant vs normal density fluctuations

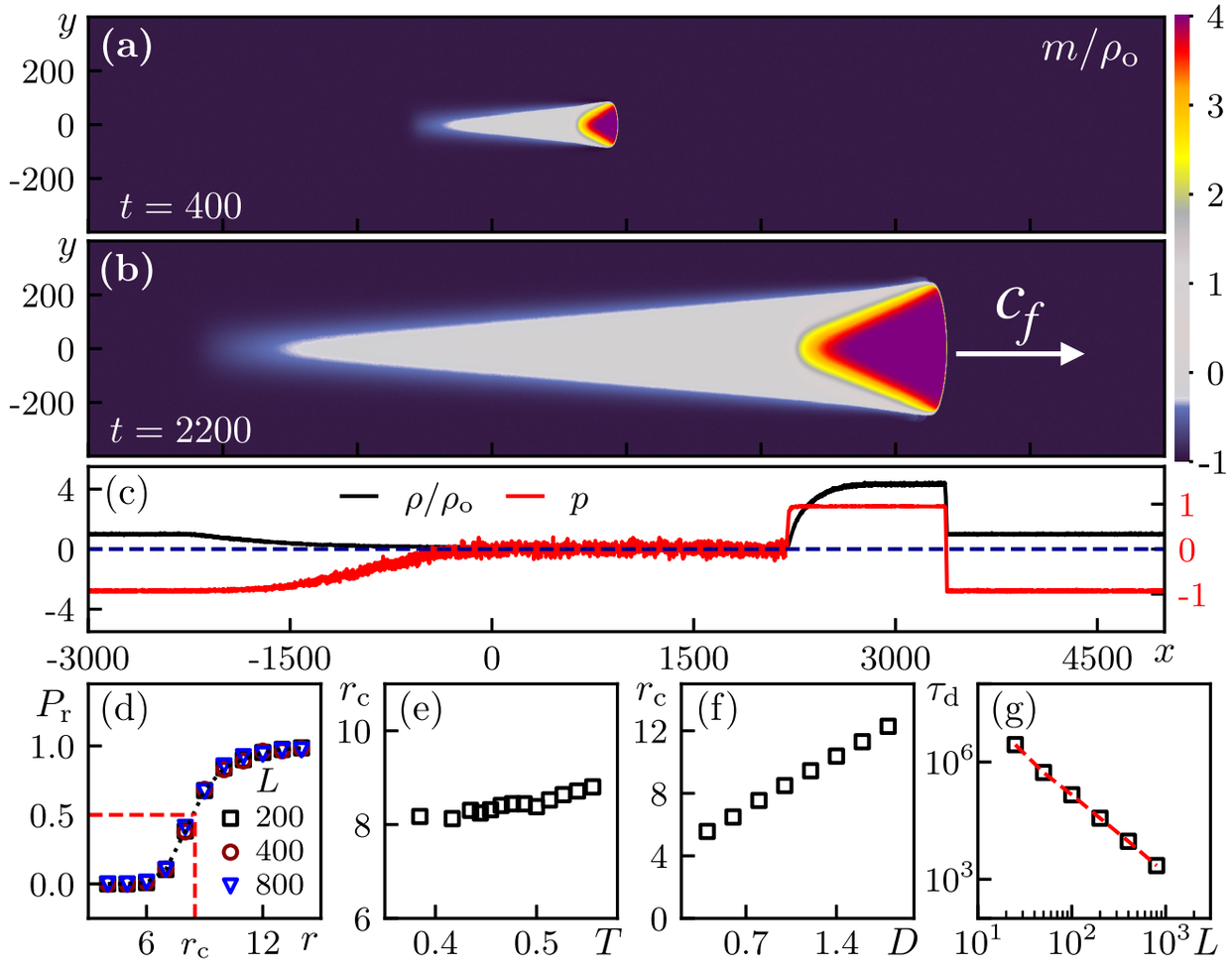
AIM Ordered State is Metastable

PHYSICAL REVIEW LETTERS **131,** 218301 (2023)

Editors' Suggestion

Metastability of Discrete-Symmetry Flocks

Brieuc Benvegnen,[1] Omer Granek,[2] Sunghan Ro,[3] Ran Yaacoby,[2] Hugues Chaté,[4,5,1] Yariv Kafri,[2] David Mukamel,[6] Alexandre Solon,[1] and Julien Tailleur[3]

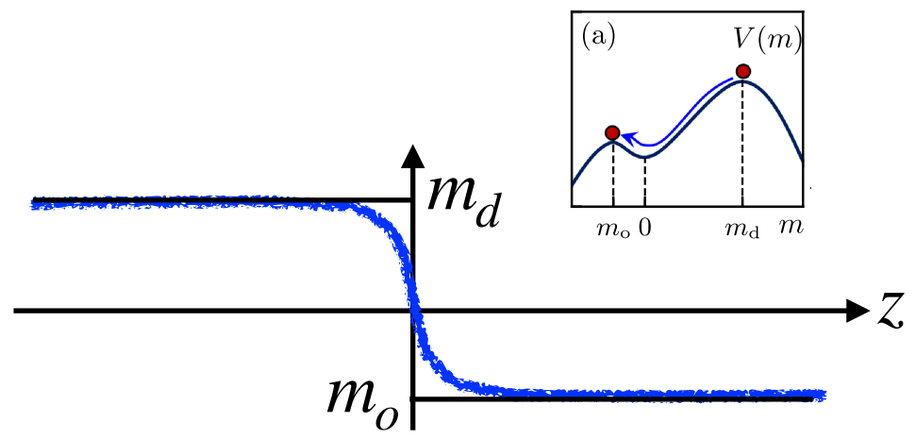


survival prob. characteristic droplet size nucleation time

$\tau_d \sim L^{-2}$

hydrodynamic equation for density field $\rho(\mathbf{r}, t)$ and magnetization field $m(\mathbf{r}, t)$

small density gradient approx. $\partial^2\rho/\partial x^2 \simeq 0$

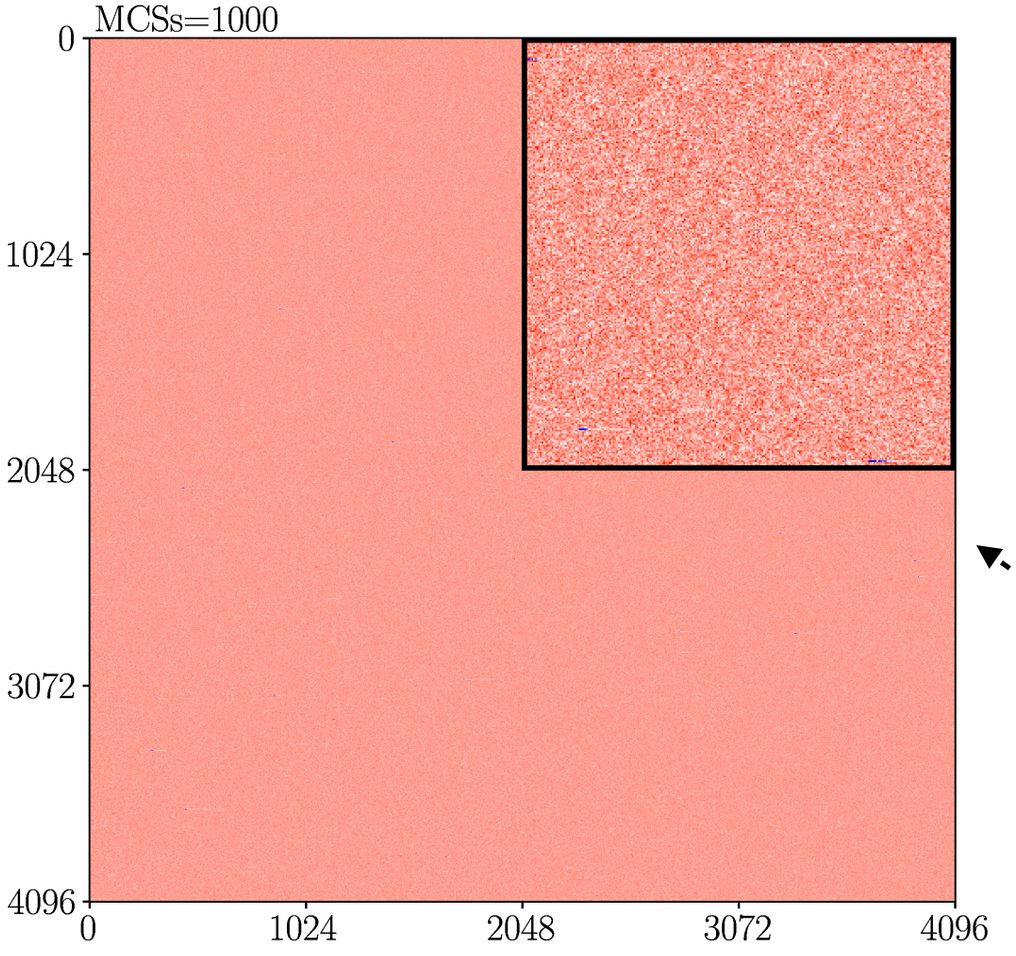


⇒ propagating droplet solution for sufficiently large β

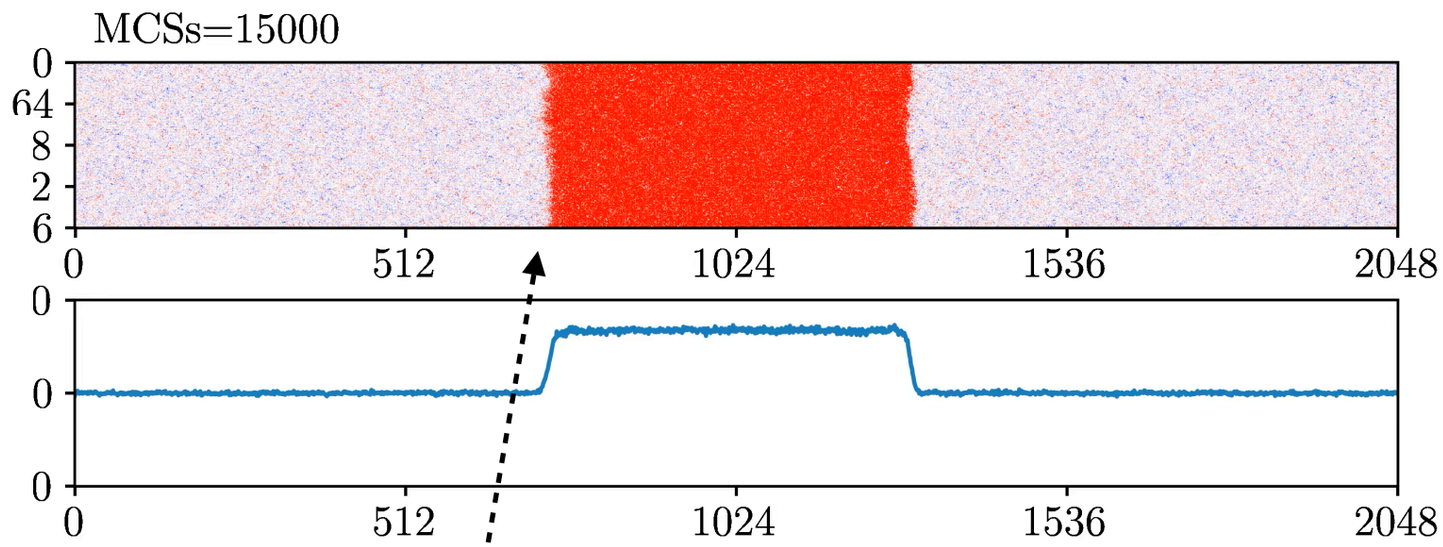
Propagating droplets destroy long range polar order.

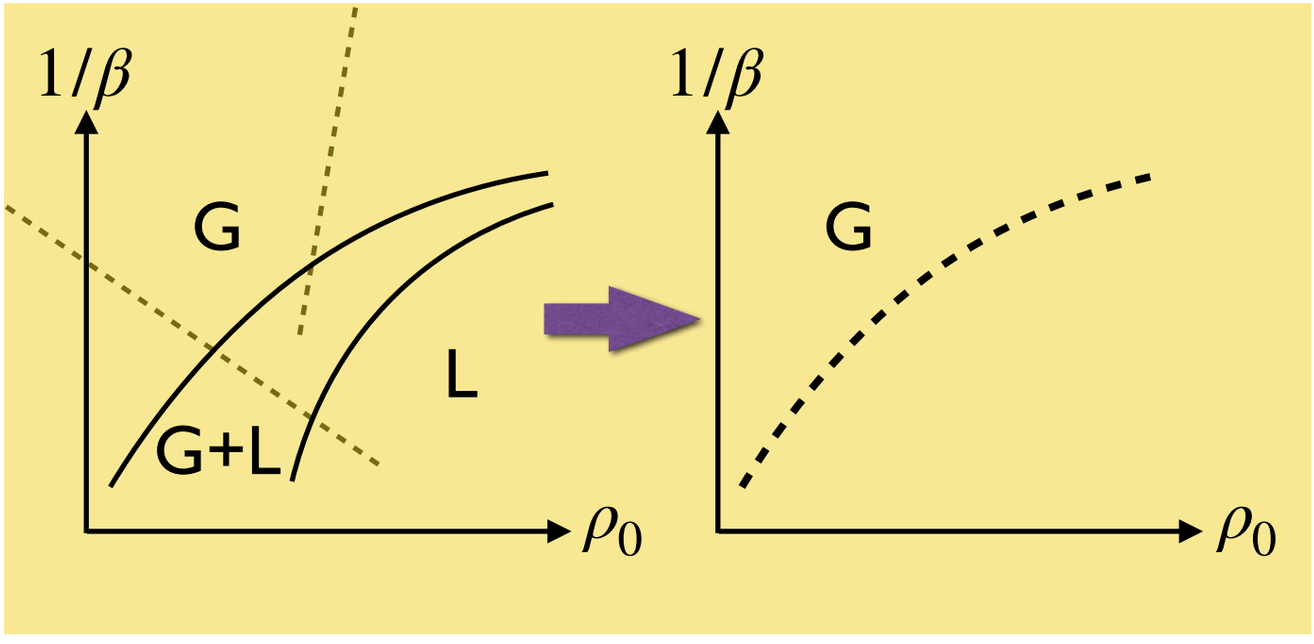
Disordered droplet phase

$L_x = 4096, L_y = 4096, \rho_0 = 8, v = 1, D = 0.05, \beta = 2$



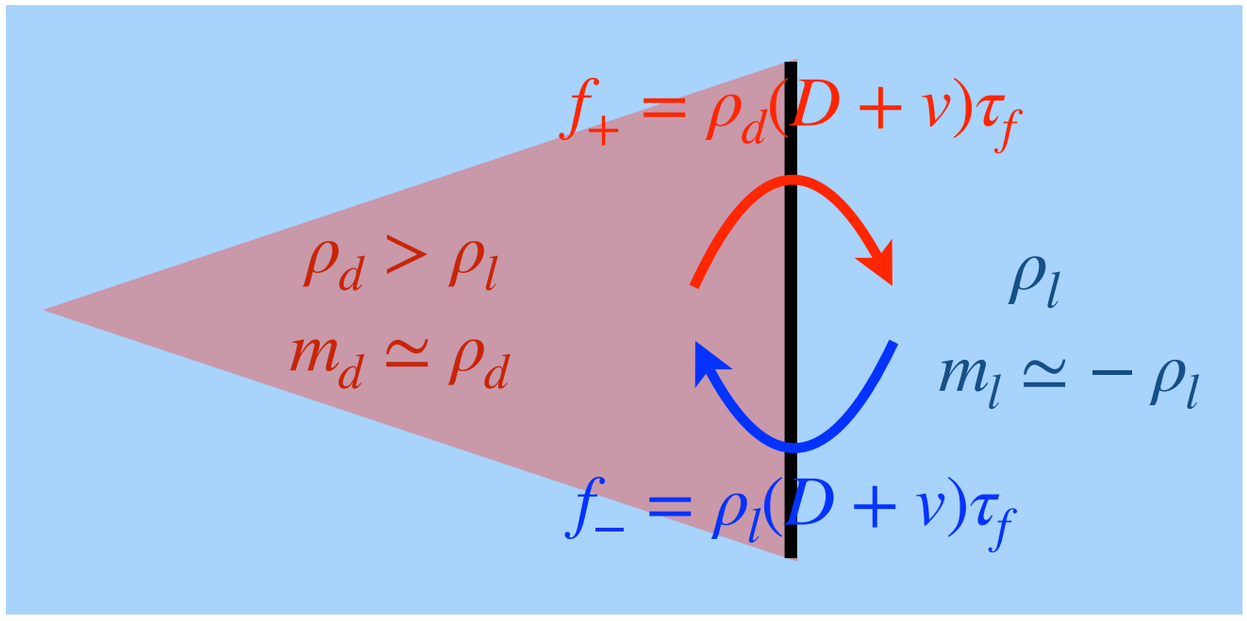
$L_x = 2048, L_y = 256, \rho_0 = 3.00, v = 1.00, D = 0.10, \beta = 2.00$





Motility-Induced Interface Pinning

Dynamics near Droplet Front



diffusion rate $\propto D$: slow

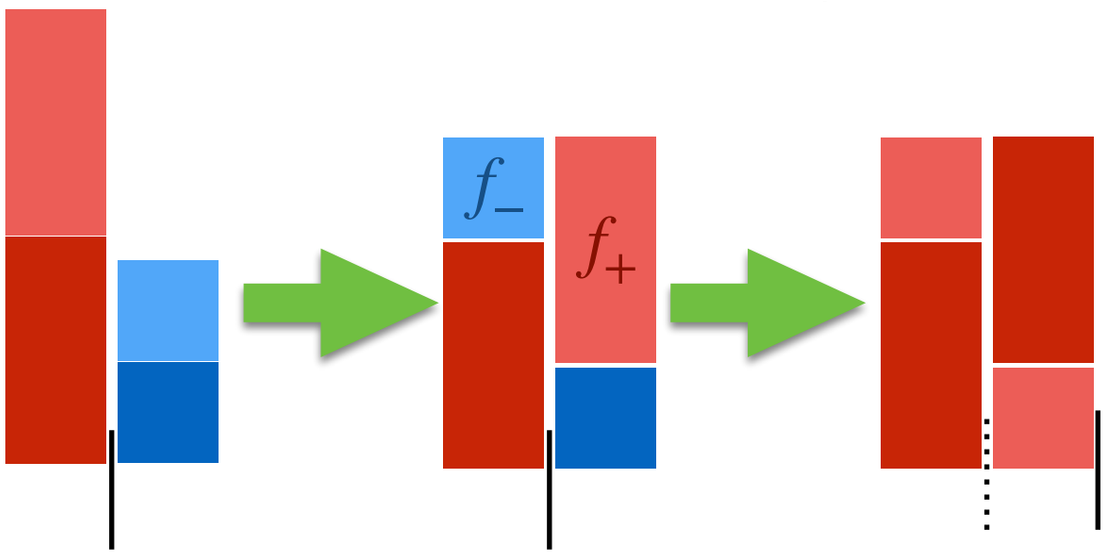
self-propulsion rate $\propto v$: slow

spin flip rate $\propto e^{\beta}$: fast

particle flux per site during $\Delta t = e^{-\beta}$

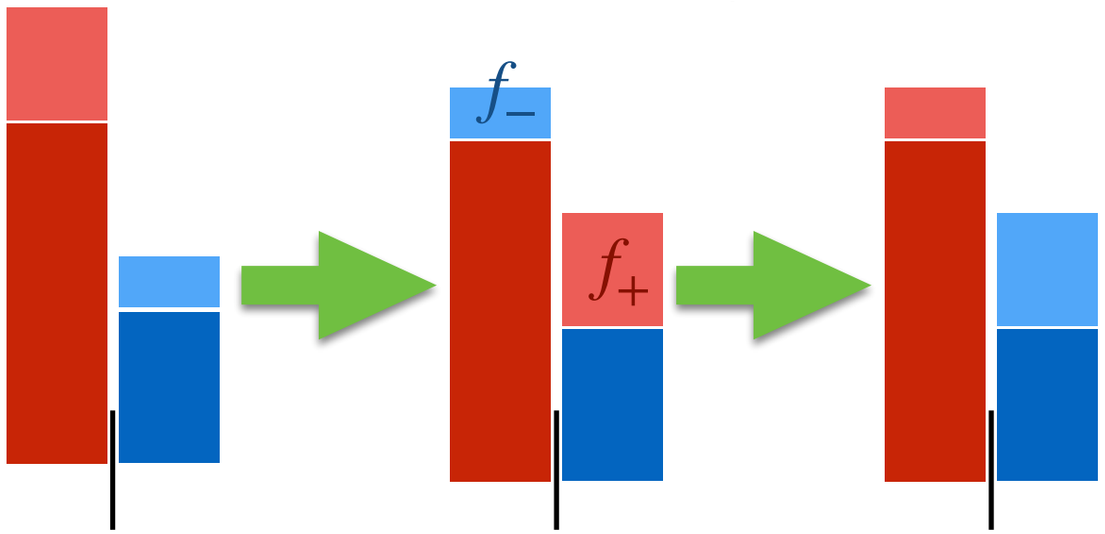
$f_+ = \rho_d(D+v)\tau_f$ and $f_- = \rho_l(D+v)\tau_f$

the droplet front advances if $f_+ \gg \rho_l - f_-$



$\beta_1 \ll \beta < \beta_p$

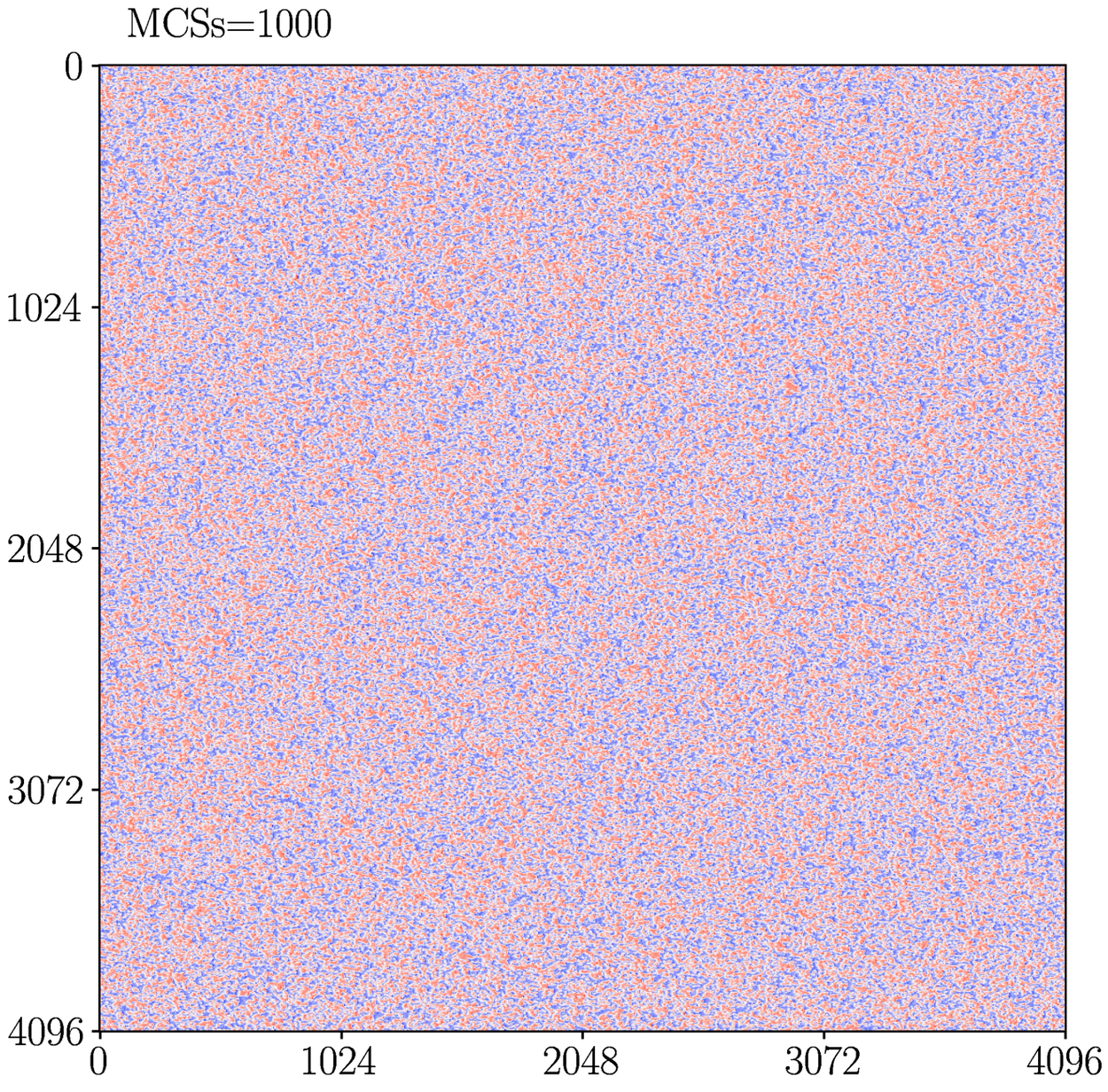
the interface gets pinned if $f_+ \ll \rho_l - f_-$.



$\beta_p < \beta$

Motility-Induced Interface Pinning at large $\beta < \infty$

$L_x = 4096, L_y = 4096, \rho_0 = 4, v = 1, D = 1.00, \beta = 5$



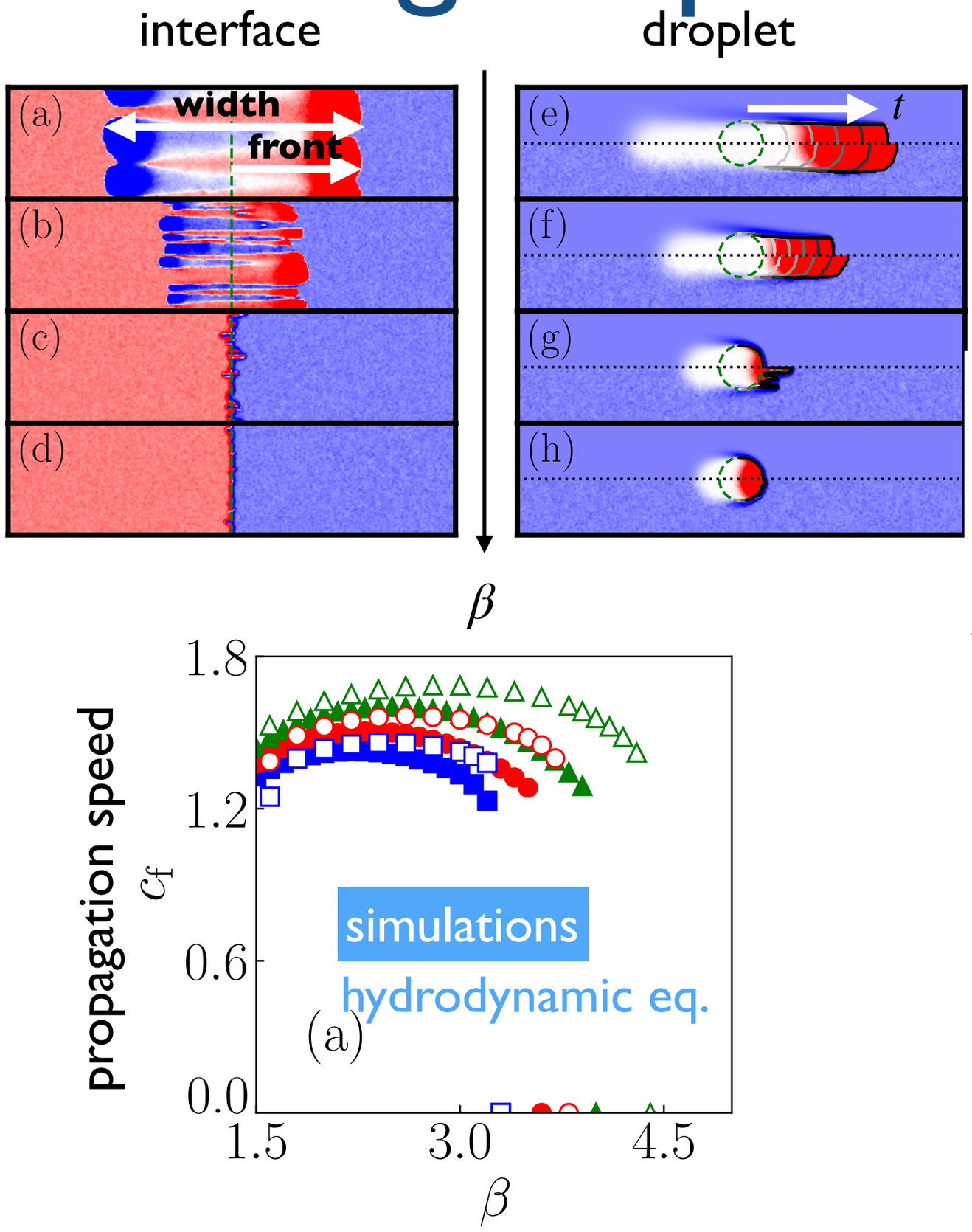
random initial configuration

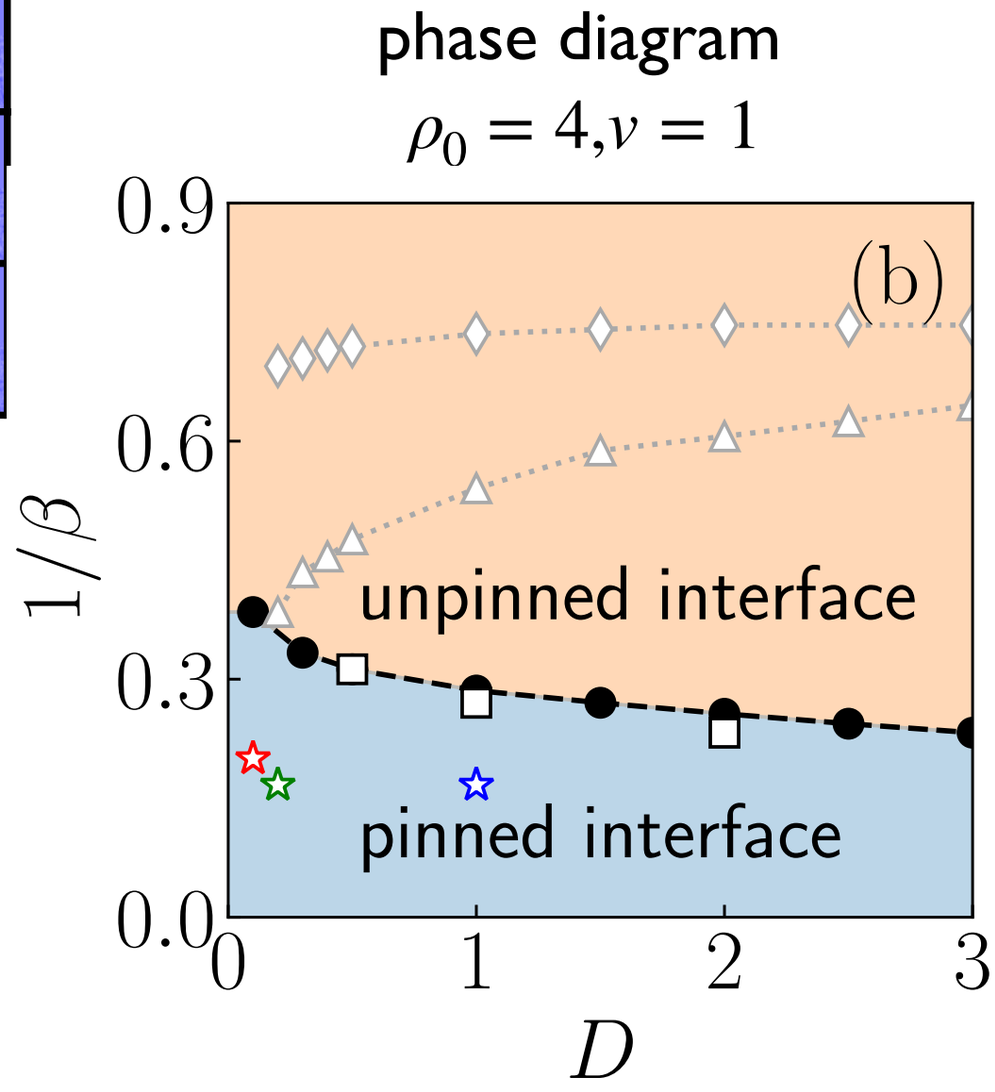
small droplets

pinned interfaces

coarsening dynamics
(growth and shrink)

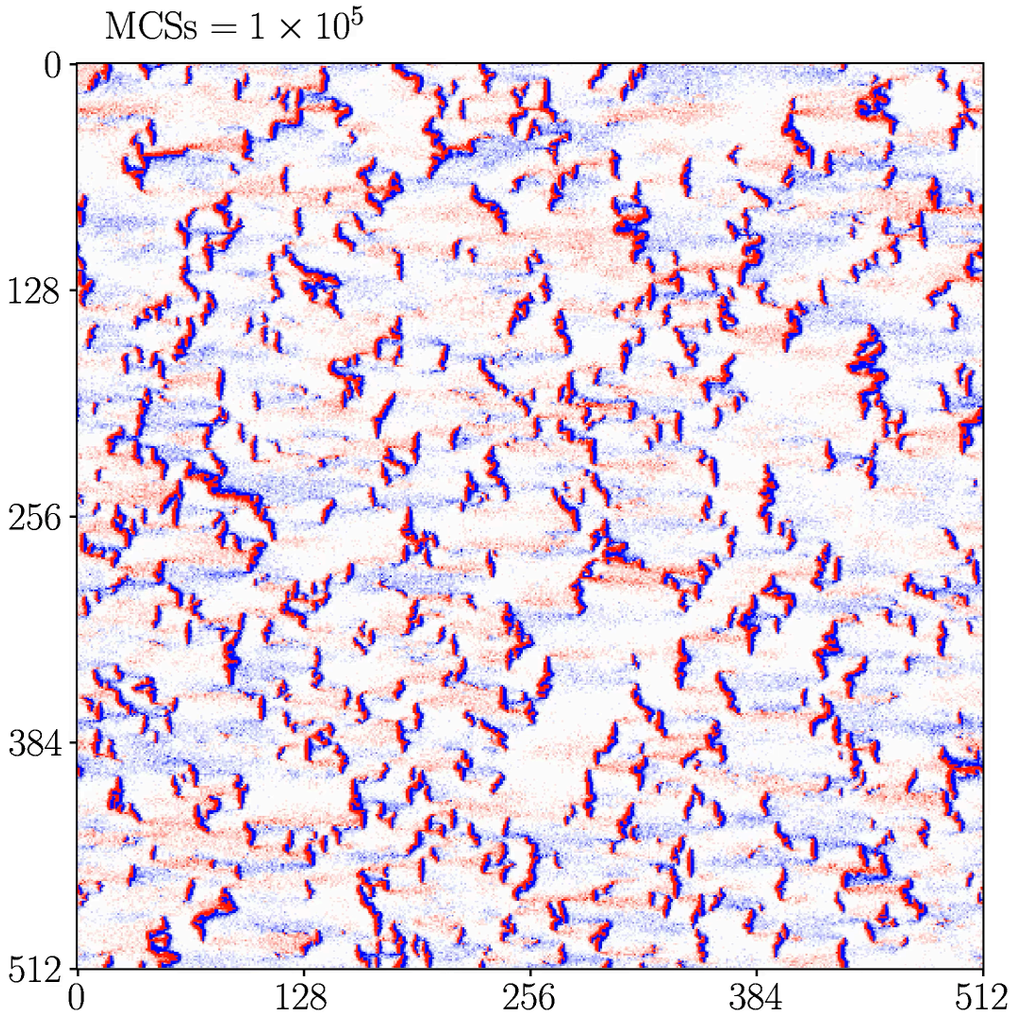
Pinning-Depinning Transition





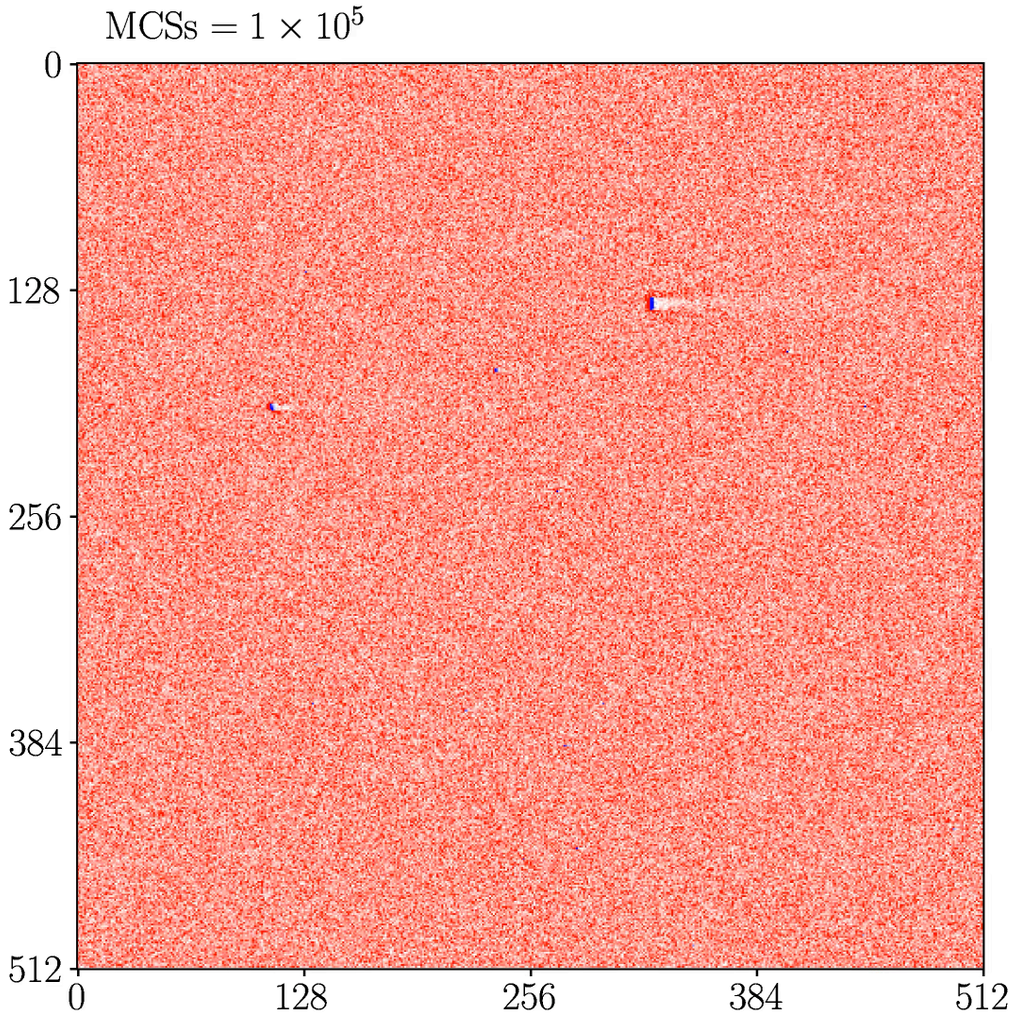
Macroscopic Pinned Interfaces for Small D

$L_x = 512, L_y = 512, \rho_0 = 4, v = 1, D = 0.20, \beta = 6$



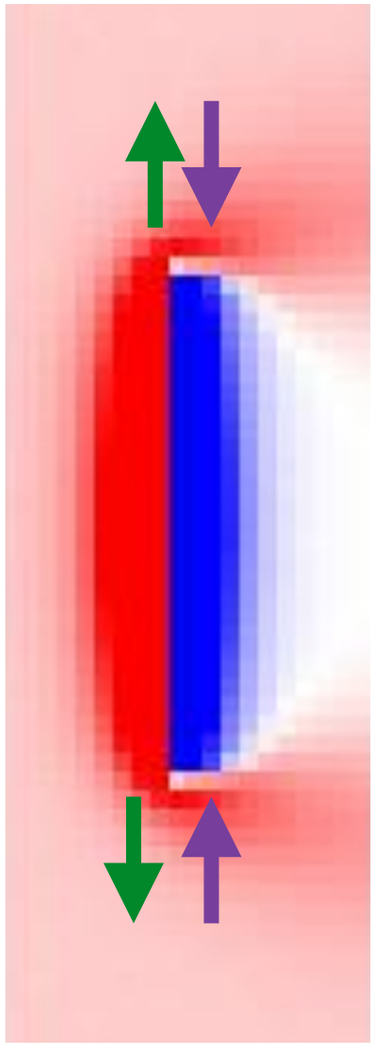
from a random initial state

$L_x = 512, L_y = 512, \rho_0 = 4, v = 1, D = 0.20, \beta = 6$



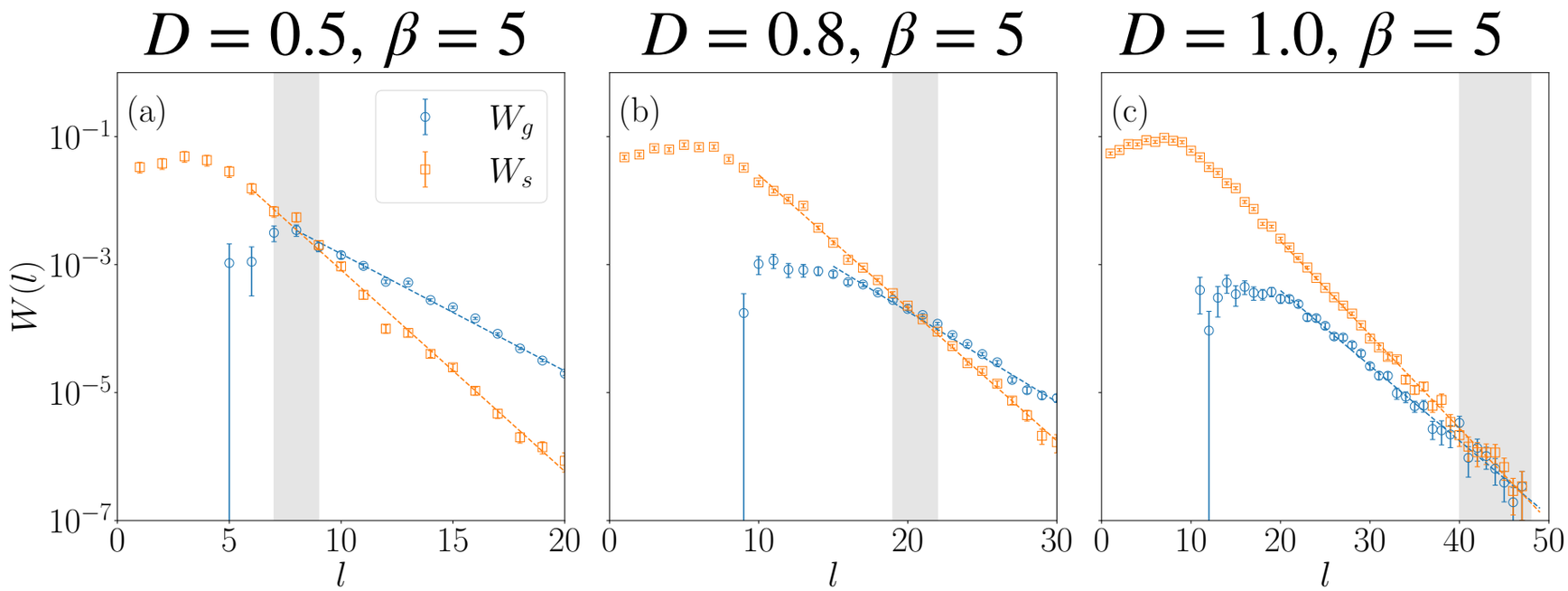
from an ordered initial state

Dynamics of Pinned Interfaces



pinned interface

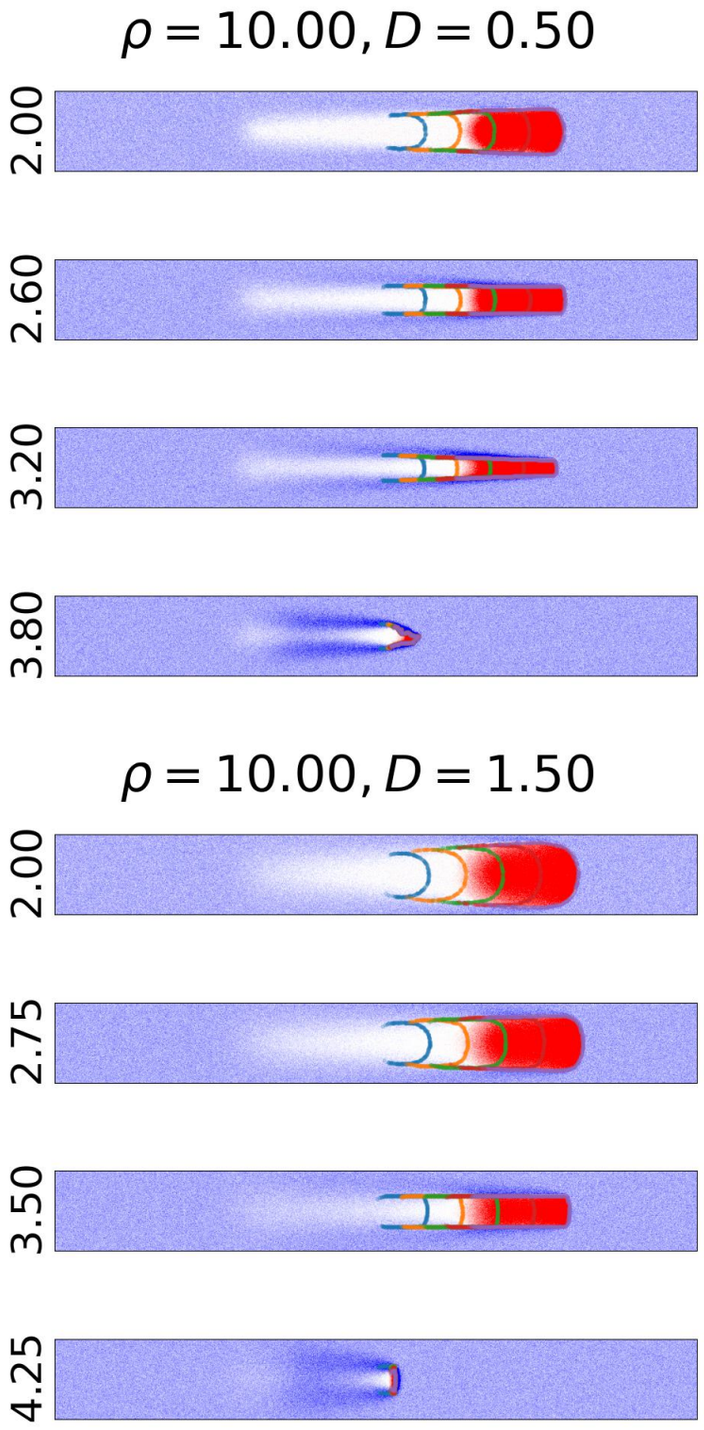
growth or shrink

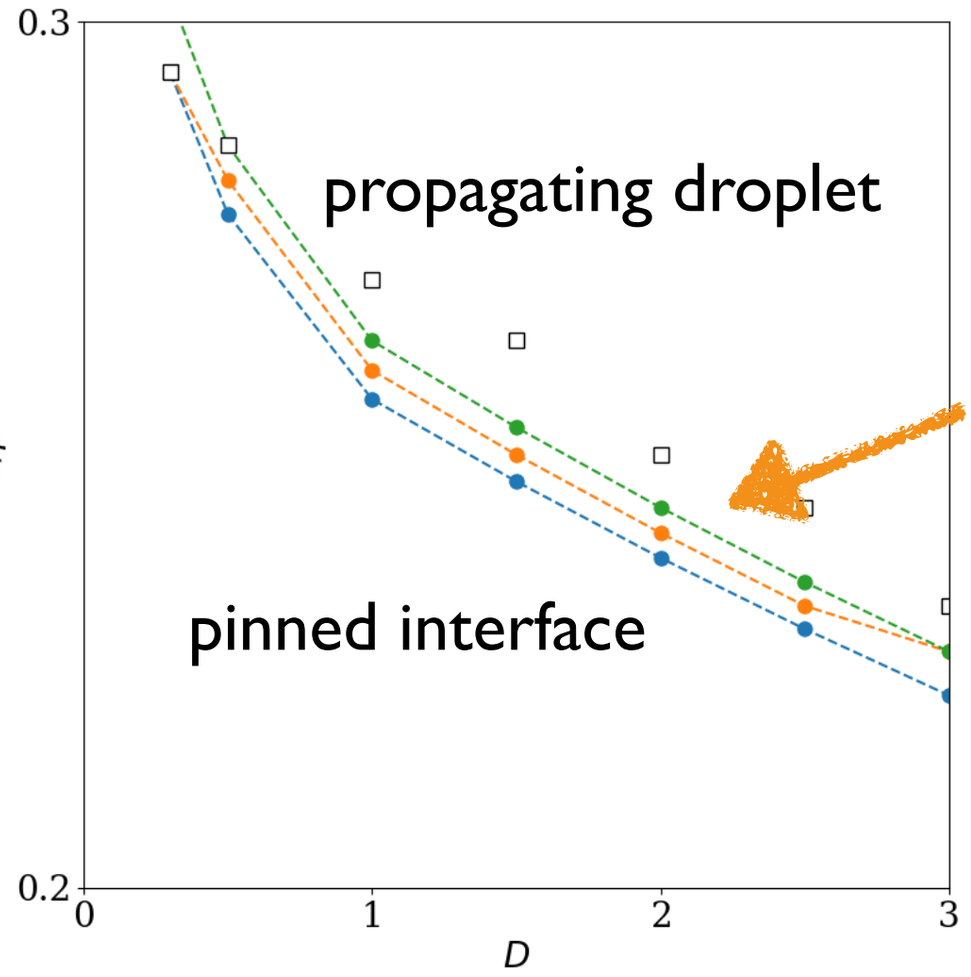


- length dependent growth rate $W_g(l) \sim e^{-l\,/\,l_g}$
- length dependent shrink rate $W_s(l) \sim e^{-l\,/\,l_s}$
- growth dominates ($l_g > l_s$) for small D $\Rightarrow$ no polar order
- IF shrink dominates ($l_g < l_s$) $\Rightarrow$restoring polar order?

Droplets and Pinned Interfaces in 3D AIM

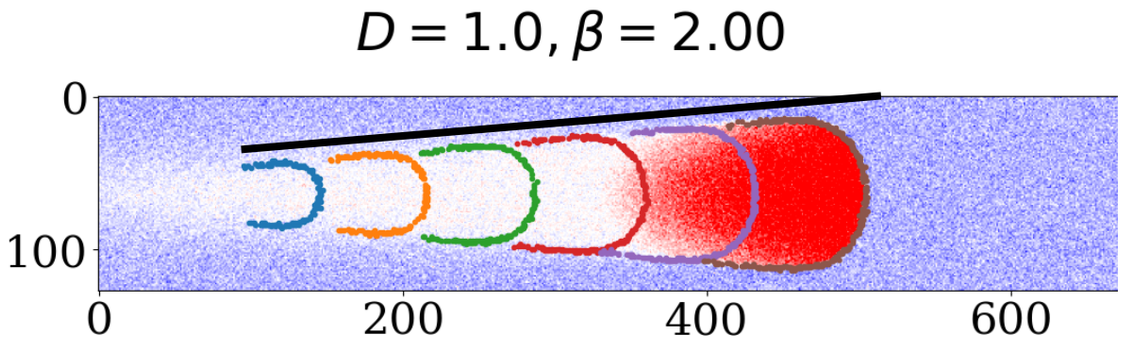
MIIP in 3D AIM



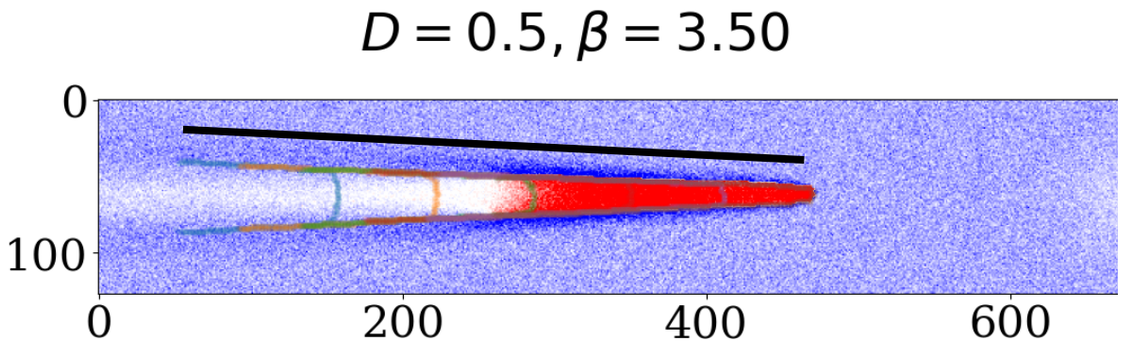


pinning-depinning

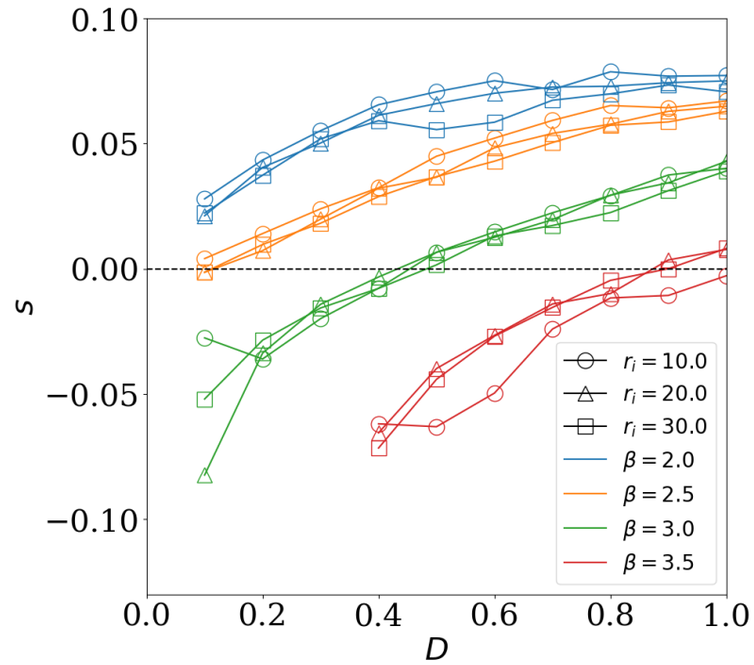
Dynamics of Droplets

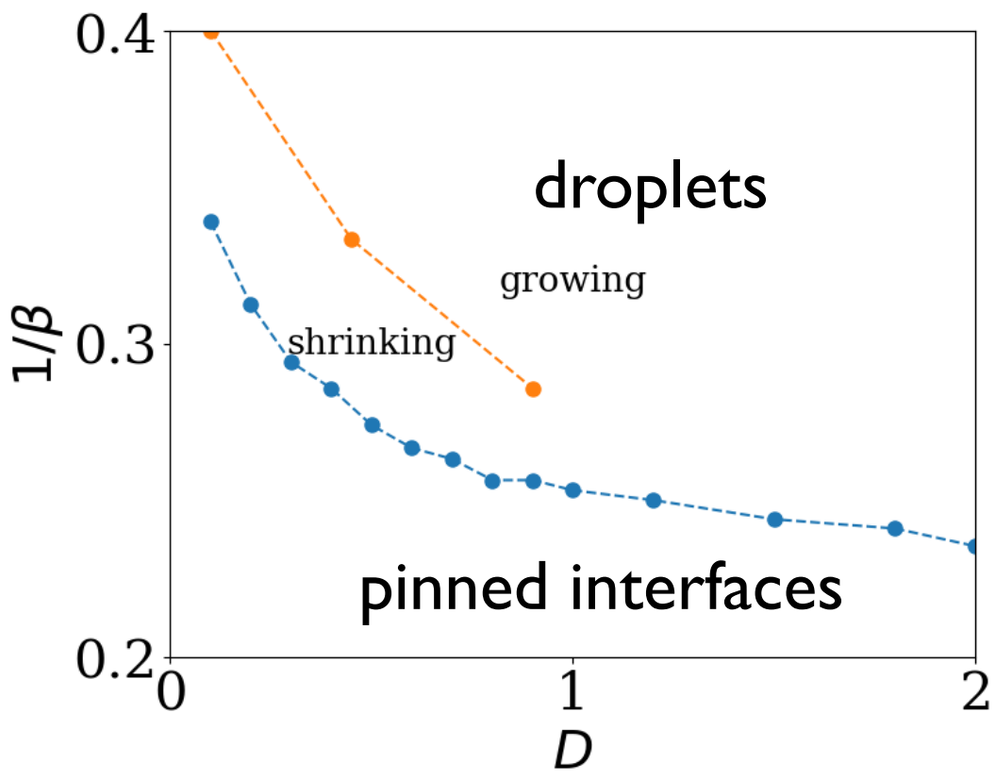


propagating and growing droplets ($s > 0$)

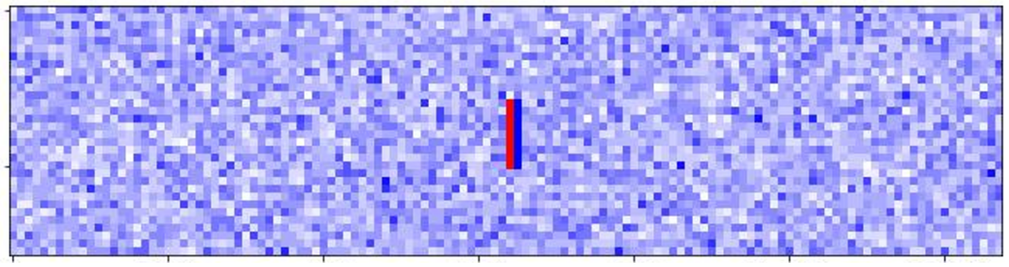


propagating and shrinking droplets ($s < 0$)



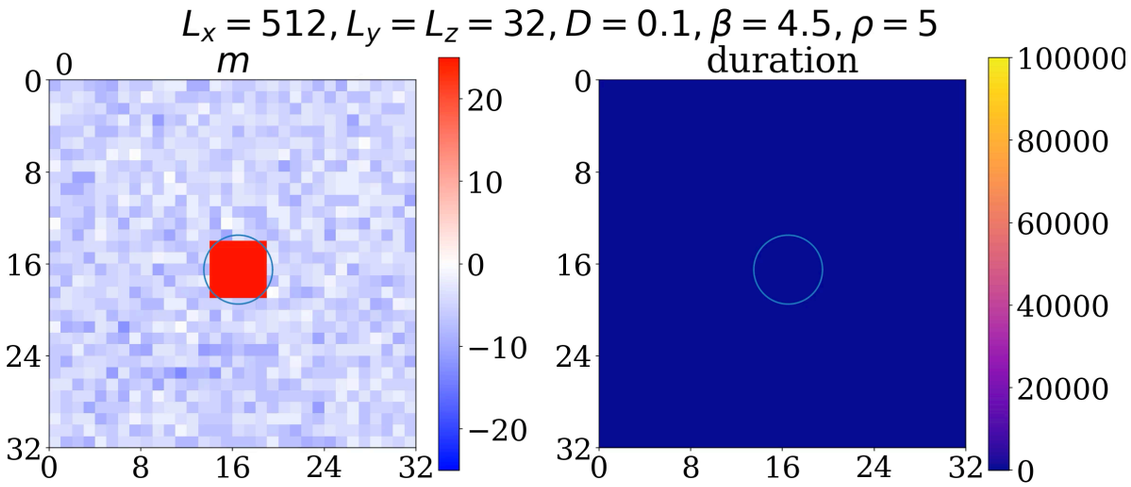


Dynamics of Pinned Interfaces



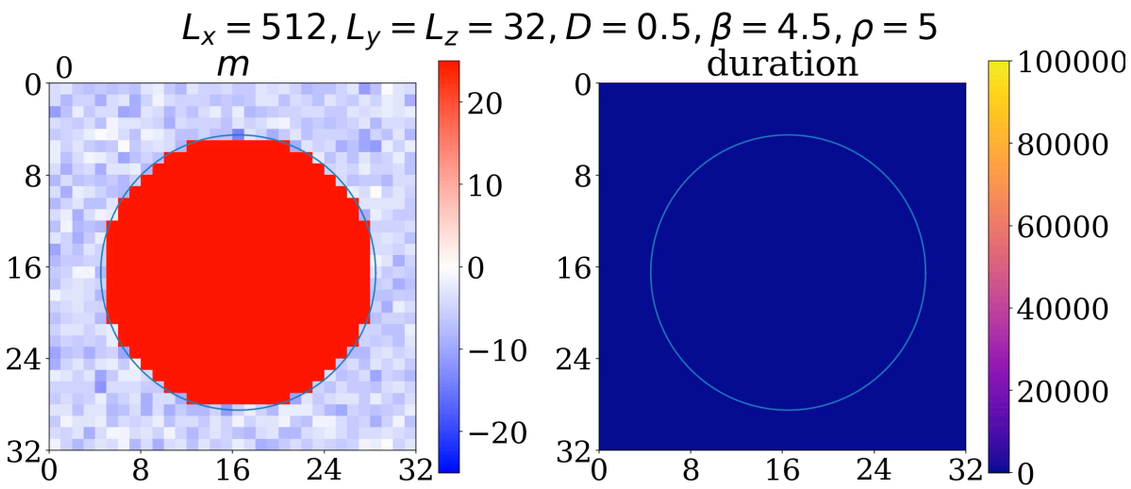
small D

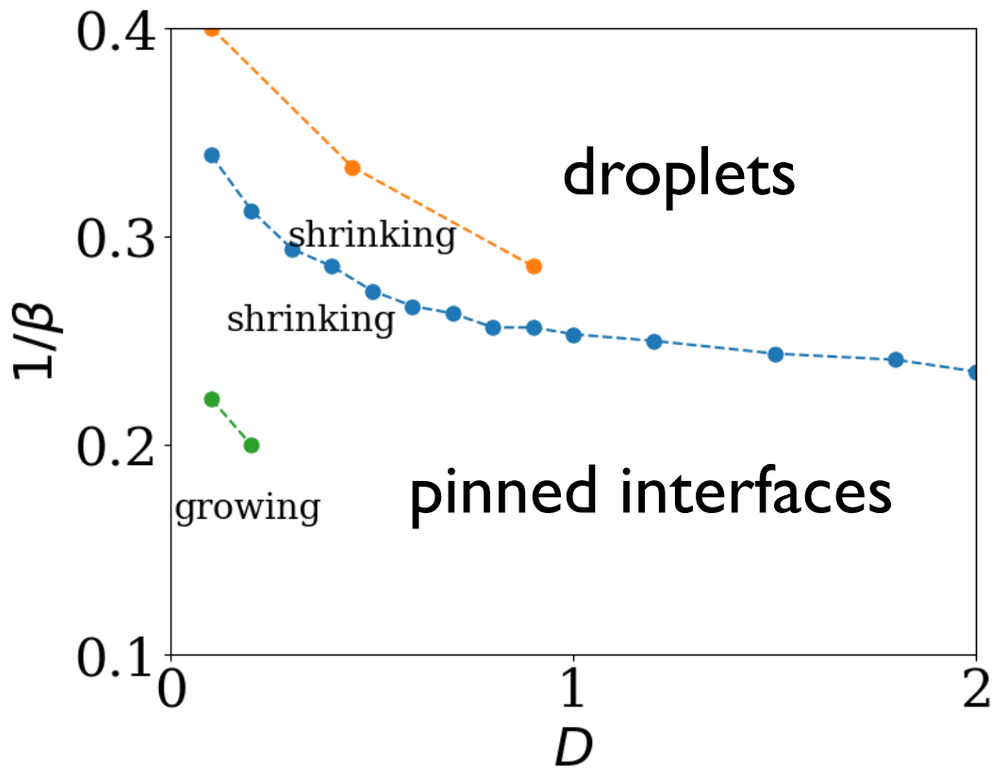
$r_0 = 3$



large D

$r_0 = 12$





Phase Diagram

