Thermodynamics of Precision in Open Quantum Systems

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East Asia Joint Seminars On Statistical Physics 2025

Oct 14-17 2025, KIAS

PRX QUANTUM **6,** 010343 (2025)

IYQ Collection

Fundamental Bounds on Precision and Response for Quantum Trajectory Observables

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(Received 10 December 2024; revised 4 February 2025; accepted 13 February 2025; published 6 March 2025)

arXiv > quant-ph > arXiv:2508.21567

Universal Precision Limits in General Open Quantum Systems

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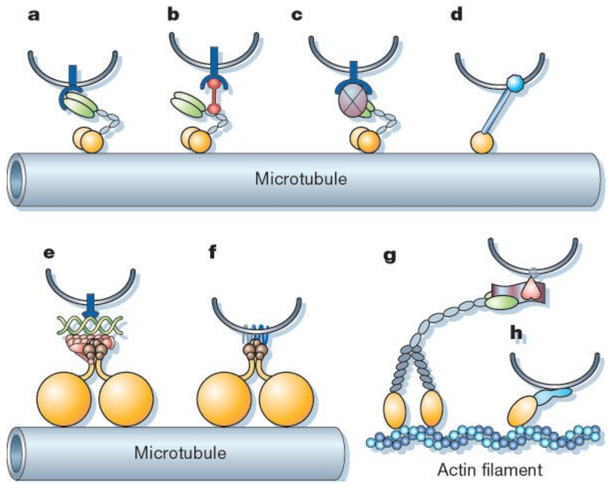
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Outline

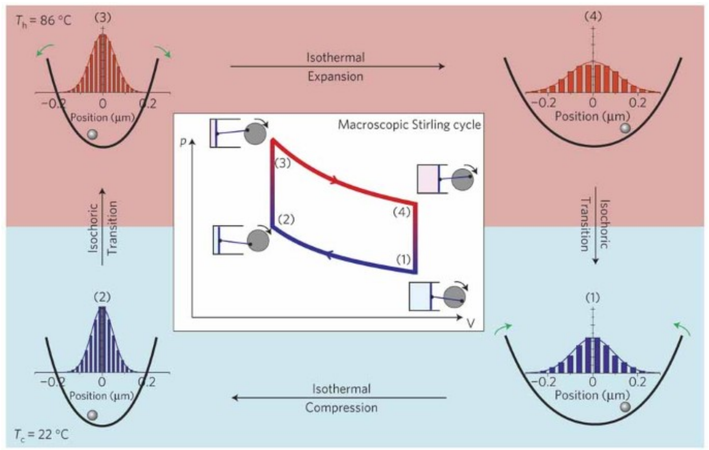
- Background & motivation
- Markovian dynamics [PRX Quantum **6**, 010343 (2025)]
- General open quantum dynamics [arXiv:2508.21567]
- Summary

Small systems



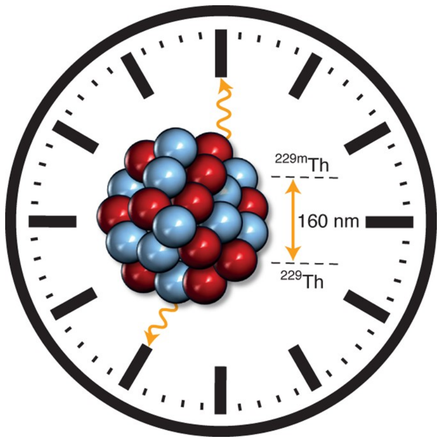
Molecular motor, Nature (2003)

traveled distance



Colloidal heat engine, Nat. Phys. (2012)

power current



Nuclear clock, Nat. Phys. (2018)

tick current

- How precise can these currents be?
- What is the cost of being precise?

Classical precision bounds

- Thermodynamic and kinetic uncertainty relations

$$F_\phi := \tau \frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2}$$

ϕ: time-integrated current

e.g., particle & heat current, motor's displacement

TUR: $F_\phi \geq \frac{2}{\sigma}$

Barato+, PRL (2015)

σ: entropy production rate

KUR: $F_\phi \geq \frac{1}{a}$

Garrahan, PRE (2017)

a: dynamical activity rate

TKUR: $F_\phi \geq \frac{4a}{\sigma^2} \Phi\left(\frac{\sigma}{2a}\right)^2 \geq \max\left(\frac{2}{\sigma}, \frac{1}{a}\right)$

Vo, TVV, Hasegawa, J. Phys. A (2022)

- Valid only for overdamped and Markov jump processes

Φ: inverse function of $x \tanh x$

Motivation

- TKUR can be violated for quantum systems due to quantum coherence

  consecutive occurrence of identical jumps

  x **classical**: $1 \to 0,\ 1 \to 0,\ \ldots$

  ✓ **quantum**: $|1\rangle \to |0\rangle, |1\rangle \to |0\rangle,\ \ldots$ ← precision can be enhanced

- How precision limits are modified in the presence of quantum effects?

  - Understand effects of quantum features (coherence & entanglement)
  - Utilize quantum features to enhance precision

Markovian dynamics - setup

- GKSL equation

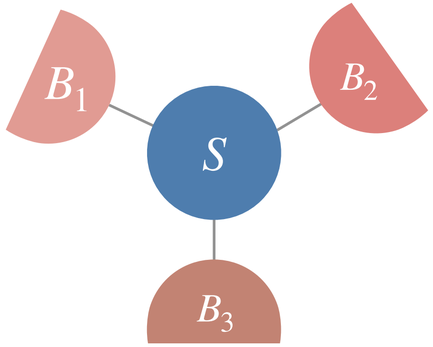
$$\dot{\varrho}_t = \mathscr{L}(\varrho_t),$$

$$\mathscr{L}(\circ) := -i[H, \circ] + \sum_{k\geq 1} (L_k \circ L_k^\dagger - \{L_k^\dagger L_k, \circ\}/2)$$

- Thermodynamic processes

  local detailed balance condition $L_k = e^{\Delta s_k/2} L_{k*}^\dagger$

  Δs_k: environmental entropy change due to jump L_k



- Entropy production rate & dynamical activity rate

$$\sigma = \sum_{k\geq 1} \mathrm{tr}(L_k \pi L_k^\dagger) \Delta s_k \qquad a = \sum_{k\geq 1} \mathrm{tr}(L_k \pi L_k^\dagger)$$

π: steady state

Unraveling & observables

- Quantum jump unraveling

$$\varrho_{t+dt} = \sum_{k\geq 0} M_k \varrho_t M_k^\dagger \qquad M_0 = \mathbf{1} - (iH + \sum_{k\geq 1} L_k^\dagger L_k/2)dt, \quad M_k = L_k\sqrt{dt}\ (k \geq 1)$$

no jump — kth jump

- Observables defined for stochastic trajectory $\Gamma = \{(t_1, k_1), \ldots, (t_J, k_J)\}$

$$\phi(\Gamma) := \sum_{i=1}^{J} c_{k_i}$$

c_k: coefficient associated with kth jump

e.g., entropy flux ($c_k = \Delta s_k$), number of jumps ($c_k = 1$)

- Currents: $c_k = -c_{k*}$ (time-antisymmetry)

Quantum thermokinetic uncertainty relation

- Quantum TKUR for currents

$$F_\phi \geq (1+\delta_\phi)^2 \frac{4a}{\sigma^2} \Phi\left(\frac{\sigma}{2a}\right)^2$$

$\delta_\phi = -\dfrac{(\mathbf{1}|\,\widehat{\mathscr{C}}\,\widehat{\mathscr{L}}^{+}\widehat{\mathscr{D}}_{\ell}\,|\pi)}{(\mathbf{1}|\,\widehat{\mathscr{C}}\,|\pi)}$: quantum correction term

$$\widehat{\mathscr{D}}_{\ell} = \sum_{k\geq 1} \ell_k \left[L_k \otimes L_k^* - \frac{1}{2}(L_k^\dagger L_k)\otimes \mathbf{1} - \frac{1}{2}\mathbf{1}\otimes (L_k^\dagger L_k)^\top \right]$$

$$\widehat{\mathscr{C}} = \sum_{k\geq 1} c_k L_k \otimes L_k^*$$

$\widehat{\mathscr{L}}^{+}$: pseudoinverse of $\widehat{\mathscr{L}}$

- Remarks
  - δ_ϕ vanishes in the classical limit
  - classical TKUR can be violated if $\delta_\phi \in (-2,0)$
  - generalizable to time-dependent driving
  - similar result can be obtained for quantum diffusion unraveling

<u>Derivation</u>:
quantum Cramer-Rao inequality
& virtual perturbation

TVV, PRX Quantum (2025)

Application to heat engines

- Power-efficiency trade-off for steady-state heat engines

$$P\frac{\eta}{\eta_C-\eta}\frac{T_c(1+\delta_P)^2}{D_P}\leq\frac{1}{2}$$

P: power, D_P: power fluctuation

η: efficiency, η_C: Carnot efficiency

cf. classical trade-off

$$P\frac{\eta}{\eta_C-\eta}\frac{T_c}{D_P}\leq\frac{1}{2}$$

Pietzonka+, PRL (2018)

$\mapsto D_P\to\infty$ as $\eta\to\eta_C$ at $P=O(1)$

$(1+\delta_P)^2=O(\eta_C-\eta)$ $\mapsto$ possible to achieve Carnot efficiency at finite power without divergent power fluctuation?

Application to quantum clocks

- Precision of quantum clocks is limited by both dissipation and coherence

ϕ: number of clock cycles

$\mathcal{N} := \dfrac{\langle \phi \rangle}{\mathrm{Var}[\phi]}$: clock precision

$\Sigma_{\mathrm{tick}} := \dfrac{\tau\sigma}{\langle \phi \rangle}$: entropy production per tick

$$\frac{F_\phi}{(1+\delta_\phi)^2} \geq \frac{2}{\sigma} \quad \Rightarrow \quad \mathcal{N} \leq \frac{\Sigma_{\mathrm{tick}}}{2(1+\delta_\phi)^2}$$

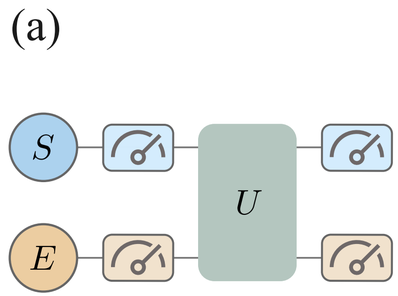
$\mapsto$ precision can be arbitrarily large as $1+\delta_\phi \to 0$

cf. $\mathcal{N} = O(e^{\Sigma_{\mathrm{tick}}})$ Meier+, Nat. Phys. (2025)

General open quantum dynamics - setup

- General unitary dynamics

$$\dot{\varrho}_t = -[H, \varrho_t]$$
$$H = H_S + H_E + H_I$$



- Projective measurements on both system and environments

$$\varrho_S = \sum_n p_n |n\rangle\langle n|, \quad \varrho_E = \sum_\mu p_\mu |\mu\rangle\langle\mu| \ (= e^{-\beta H_E}/Z)$$

- Stochastic trajectory $\gamma = \{n, (\nu_1, \mu_1), \ldots, (\nu_N, \mu_N), m\}$

Symmetry-breaking quantities

- Entropy production

$$\Sigma := \Delta S + \beta Q = \left\langle \ln \frac{\mathsf{P}(\gamma)}{\widetilde{\mathsf{P}}(\widetilde{\gamma})} \right\rangle$$

ΔS: von Neumann entropy change

Q: heat dissipation

$\widetilde{\gamma} = \{m, (\mu_N, \nu_N), \ldots, (\mu_1, \nu_1), n\}$: time-reversed trajectory

$\mathsf{P}, \widetilde{\mathsf{P}}$: path probabilities in forward and backward processes

- Forward-backward asymmetry

$$\Sigma_* := \left\langle \ln \frac{\mathsf{P}(\gamma)}{\widetilde{\mathsf{P}}(\gamma)} \right\rangle$$

- difference between forward and backward processes
- vanishes when $\mathsf{P} \equiv \widetilde{\mathsf{P}}$
- arises due to dynamical factors

Generalized TUR

- Time-antisymmetric currents $\phi(\gamma) = -\phi(\widetilde{\gamma})$

- Generalized TUR

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq f(\Sigma + \Sigma_*)$$

$$f(x) = 4[\Phi(x/2)/x]^2 - 1 \in (0, +\infty)$$

Φ: inverse function of $x\tanh(x)$

$\mapsto$ high precision of currents requires large dissipation or strong asymmetry

✓ tight and saturable

✓ fundamental limit on current precision

TVV, R. Honma, K. Saito, arXiv:2508.21567

Generalized KUR

- Generic observables $\phi(\gamma) = 0 \ \forall \gamma \in I, \quad I = \{\gamma \,|\, \mu_i = \nu_i \, \forall i\}$

  trajectories that have no jump

- Generalized KUR

$$\frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2} \geq \frac{1}{\mathscr{P}^{-1} - 1}$$

$\mathscr{P} := \sum_{\gamma \in I} \mathrm{P}(\gamma)$: probability of inactivity

$\rightarrow \mathscr{P}^{-1} - 1$ can be interpreted as an activity term

$\mapsto$ high precision of observables requires high activity

✓ saturable for $\phi(\gamma) = 1 \ \forall \gamma \notin I$

✓ tightens and generalizes previous results

Hasegawa, PRL 126, 010602 (2021)
Hasegawa, PRL 127, 240602 (2021)

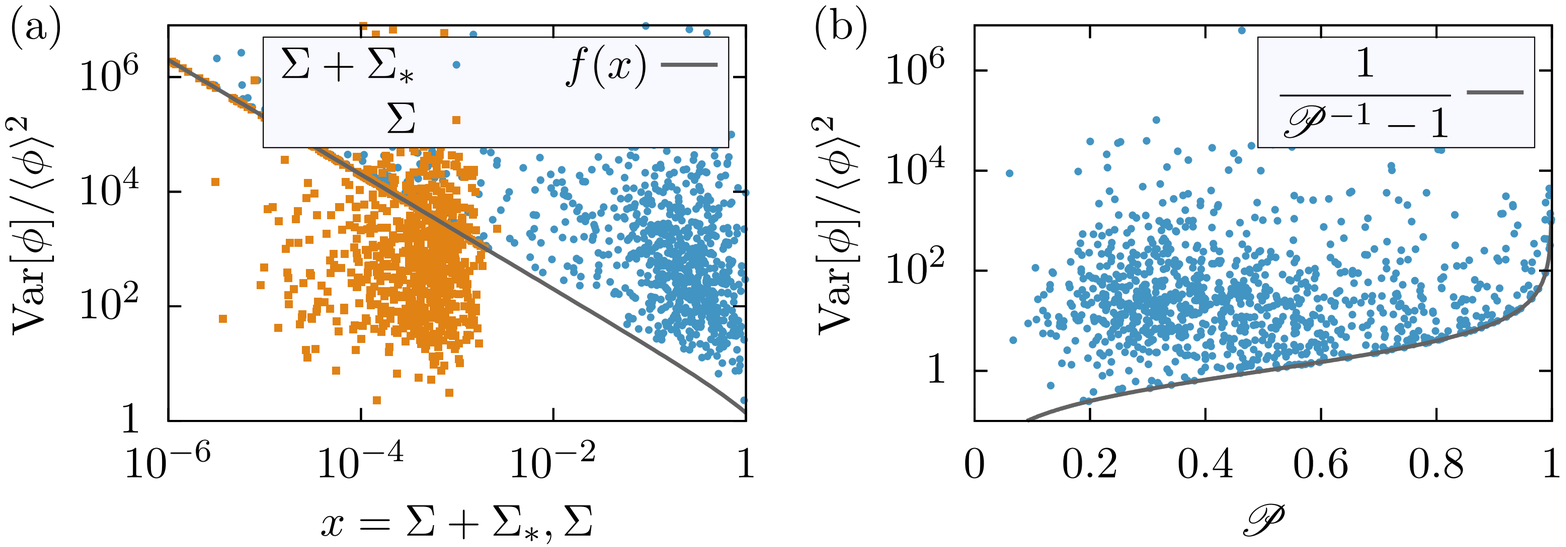
TVV, R. Honma, K. Saito, arXiv:2508.21567

Demonstration - Qubit

$$H = \frac{1}{2}(\omega_z \sigma_z + \omega_x \sigma_x) \otimes \mathbf{1}_E + \mathbf{1}_S \otimes H_E + \lambda V_S \otimes V_E$$

$$\overline{S}_{\mathrm{EE}} := \sum_{n,\nu} p_n p_\nu S_{\mathrm{EE}}(\varrho'_{n,\nu})$$: average entanglement entropy

$$\mathcal{Q} := (\mathrm{Var}[\phi]/\langle\phi\rangle^2)/f(\Sigma)$$: quality factor



Summary

Thank you for your attention!

- Quantum TKUR: quantum coherent dynamics constrains current precision along with dissipation and activity

$$F_\phi \geq (1+\delta_\phi)^2 \frac{4a}{\sigma^2} \Phi\left(\frac{\sigma}{2a}\right)^2$$

- Generalized TUR: current precision is limited by both dissipation and asymmetry

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq f(\Sigma + \Sigma_*)$$

- Generalized KUR: observable precision is limited by a generalized activity

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq \frac{1}{\mathscr{P}^{-1} - 1}$$