Cost of Non-instantaneous Stochastic Resetting

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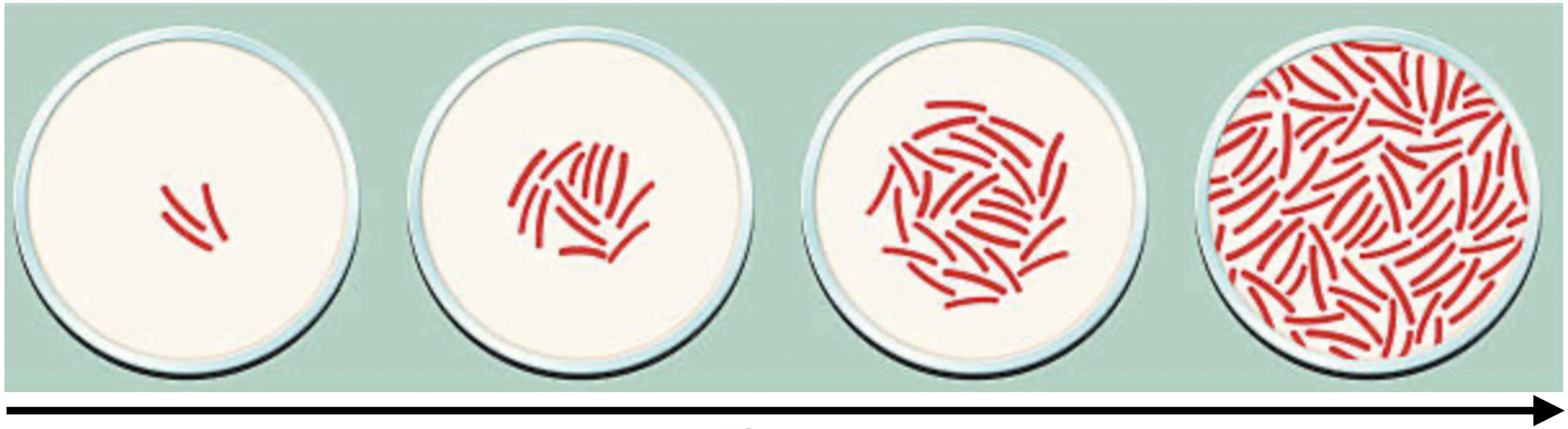
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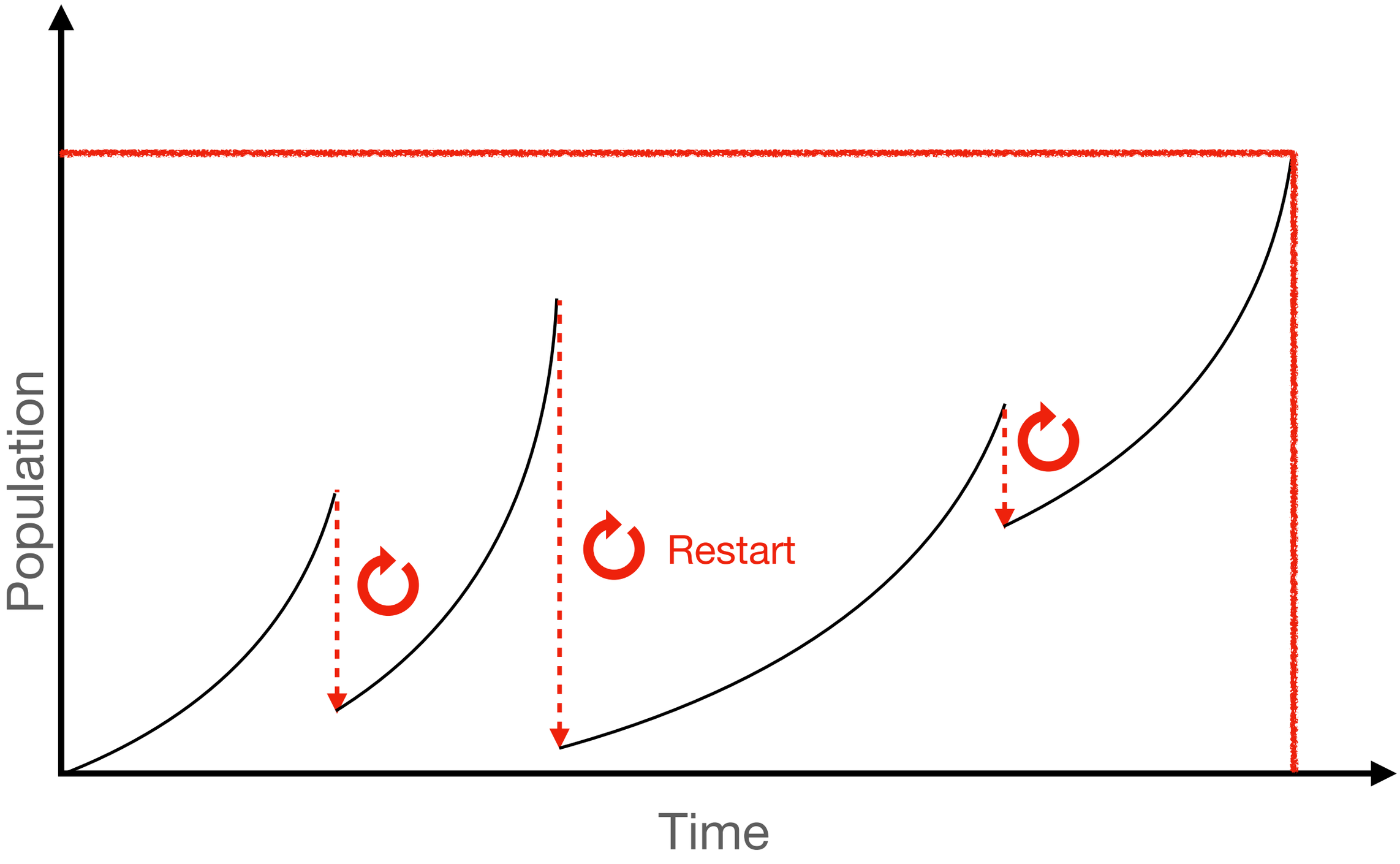

Time







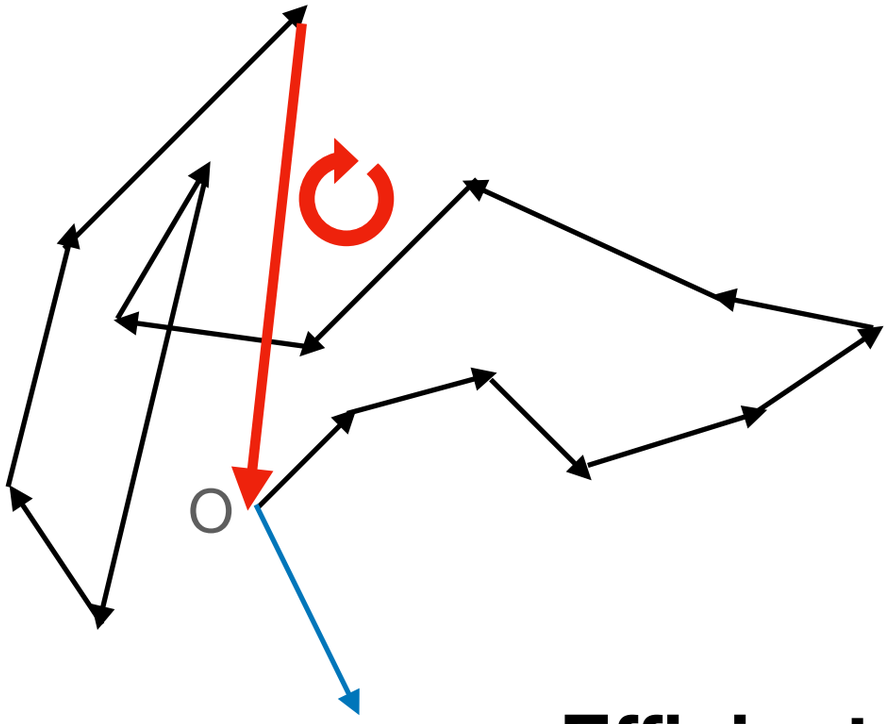


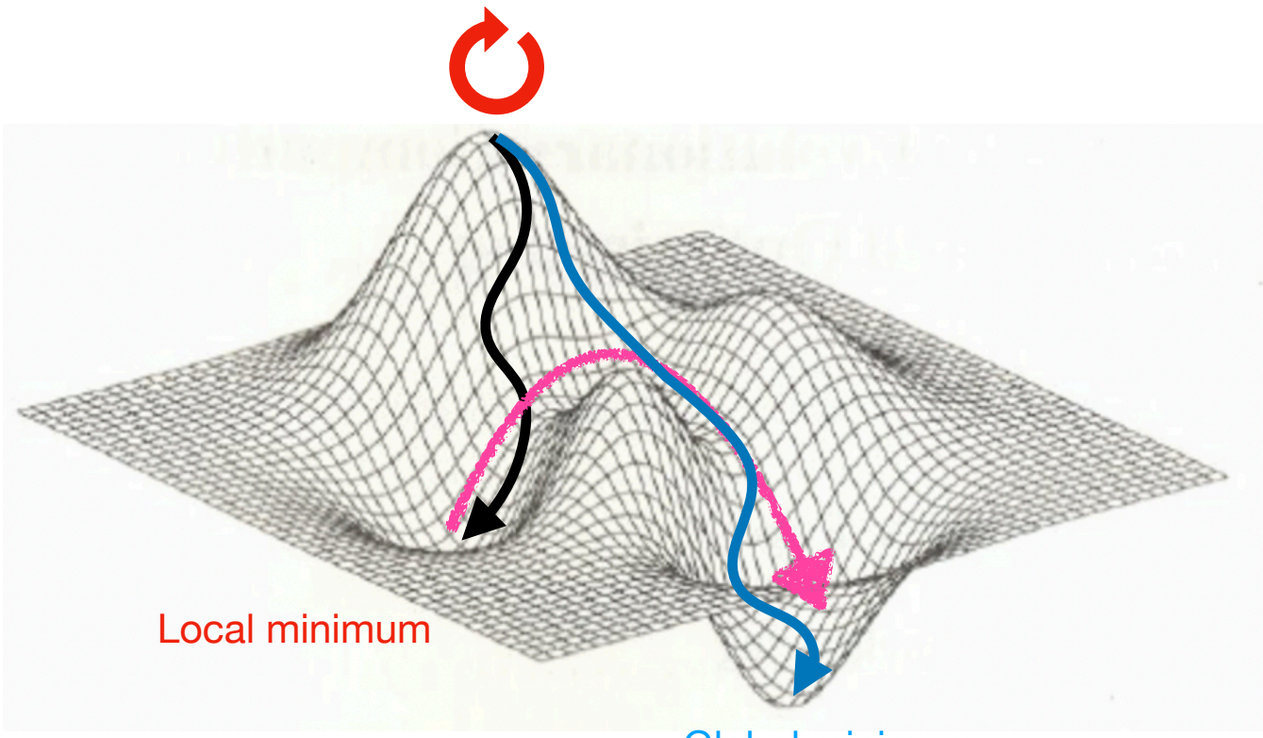

Population
Restart
Restart
Restart
Time

Stochastic Resetting

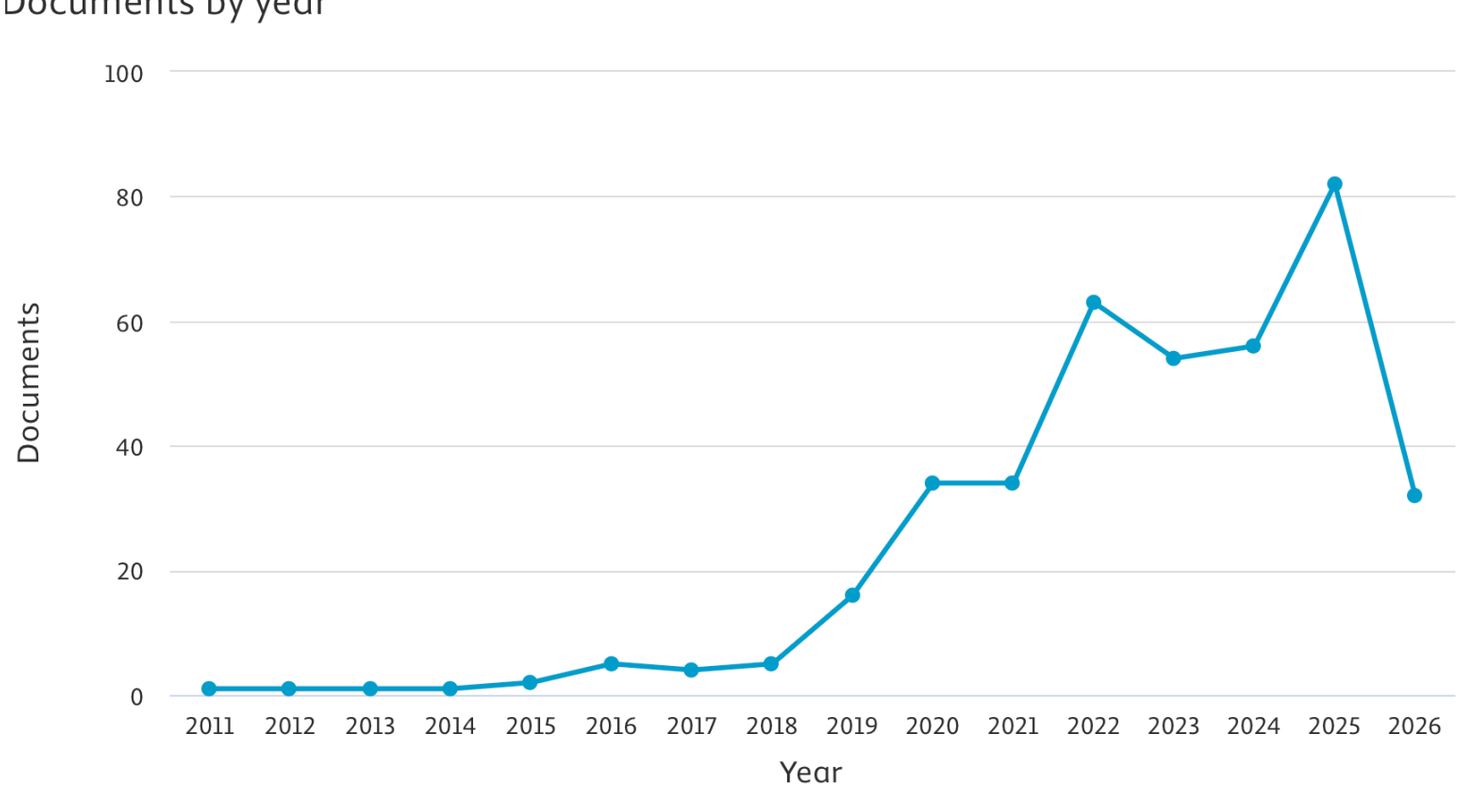


M.R. Evans, S.N. Majumdar. PRL 106, 160601 (2011)



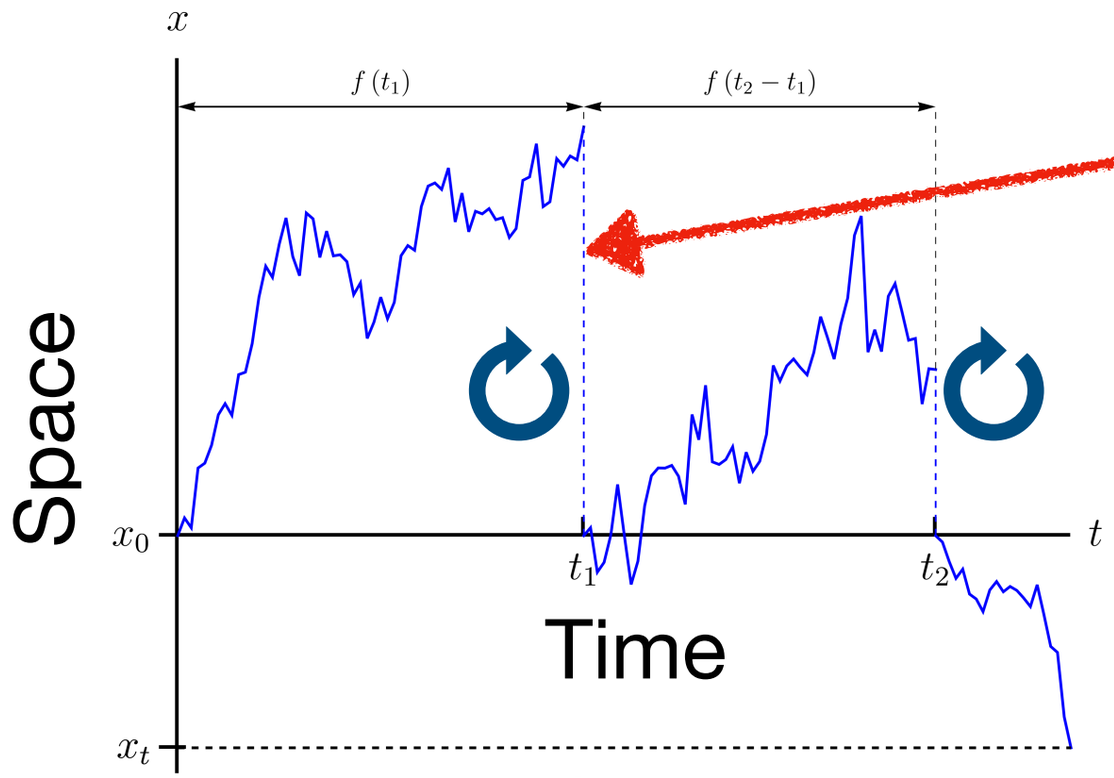


Efficient search strategies….



M.R. Evans, S.N. Majumdar, G. Schehr. J. Phys. A. Math. Theor. 53, 193001 (2020)

Stochastic Resetting?



Instantaneous resetting

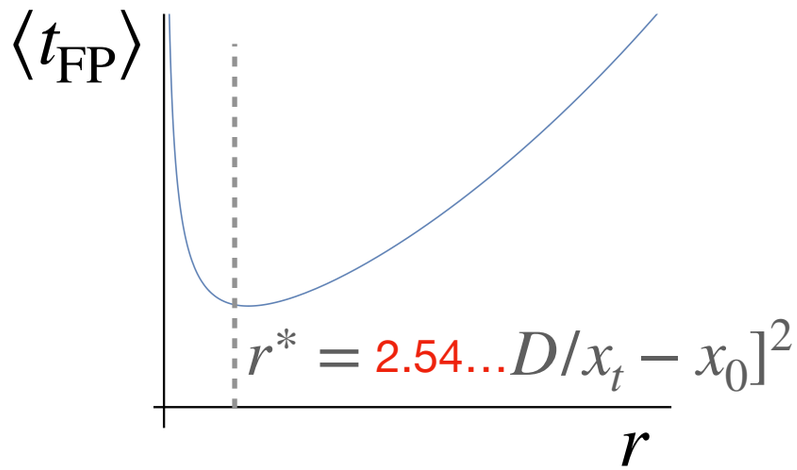
No resetting

$$p(x, t \,|\, x_0) \propto \exp[-(x - x_0)^2/(4Dt)]$$

$$\langle t_{\mathrm{FP}} \rangle \equiv \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} t_i = \infty$$

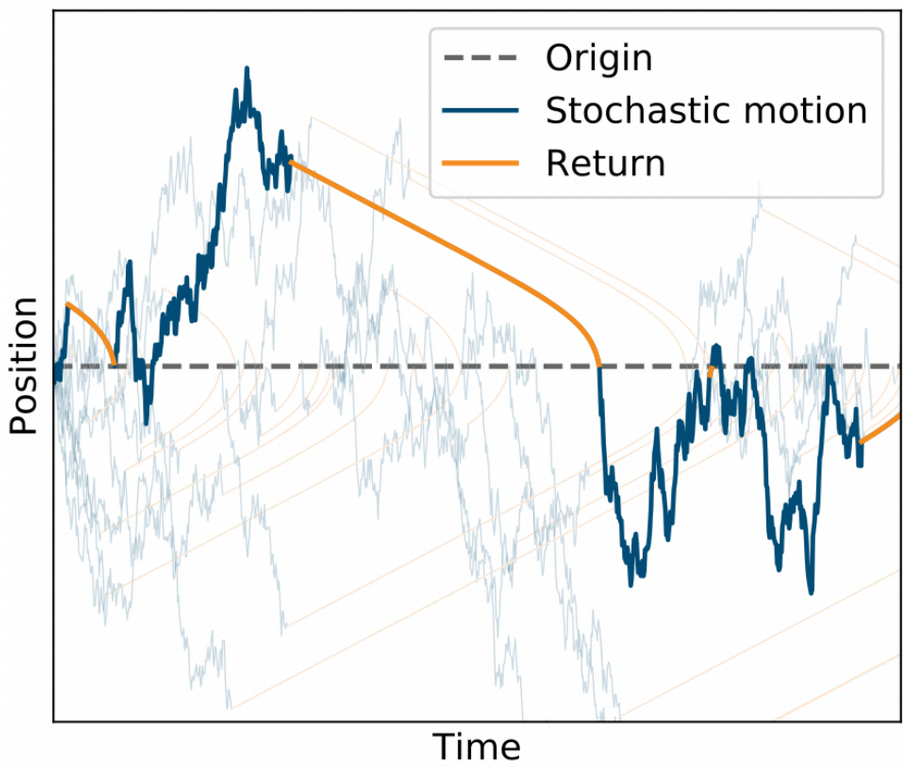
With resetting: $f(t) = r \exp(-rt)$

$$p_{\mathrm{ss}}(x) \propto \exp[-\sqrt{r/D}\,|x - x_0|\,]$$

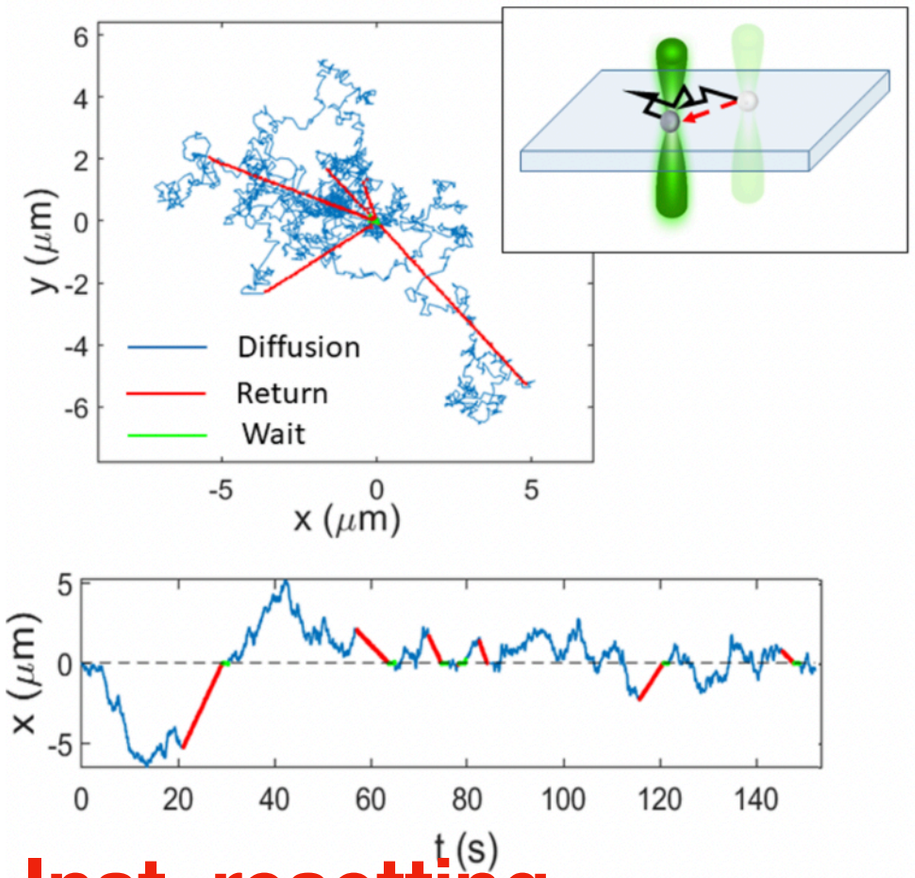


Evans and Majumdar. PRL 106, 160601 (2011)

Pal et al. New J. Phys. 21 (2019) 113024



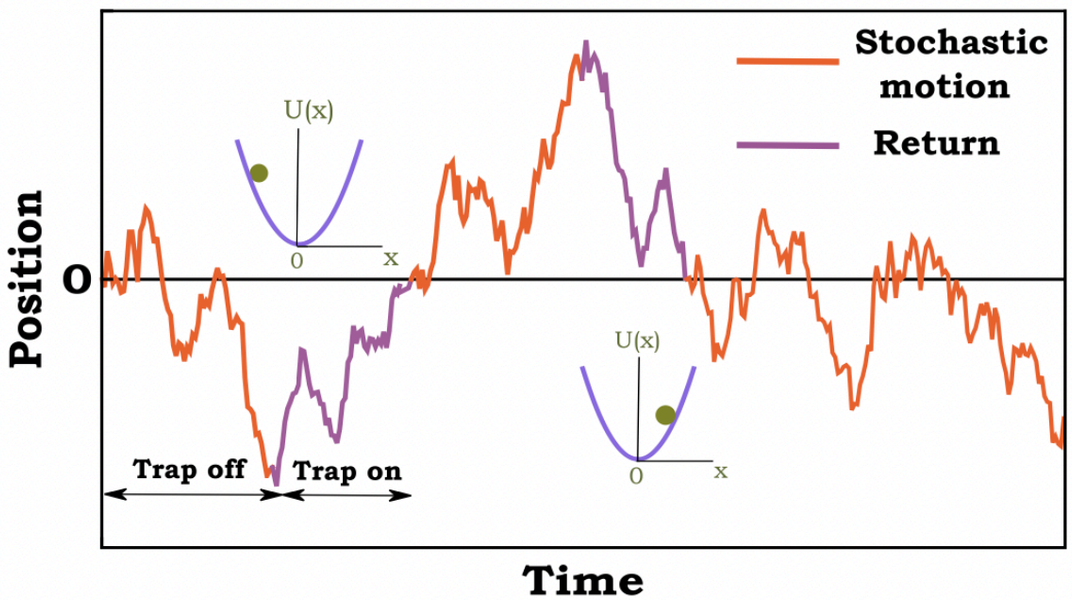
Tal-Friedman et al. J. Phys. Chem. Lett. 2020, 11, 17, 7350–7355



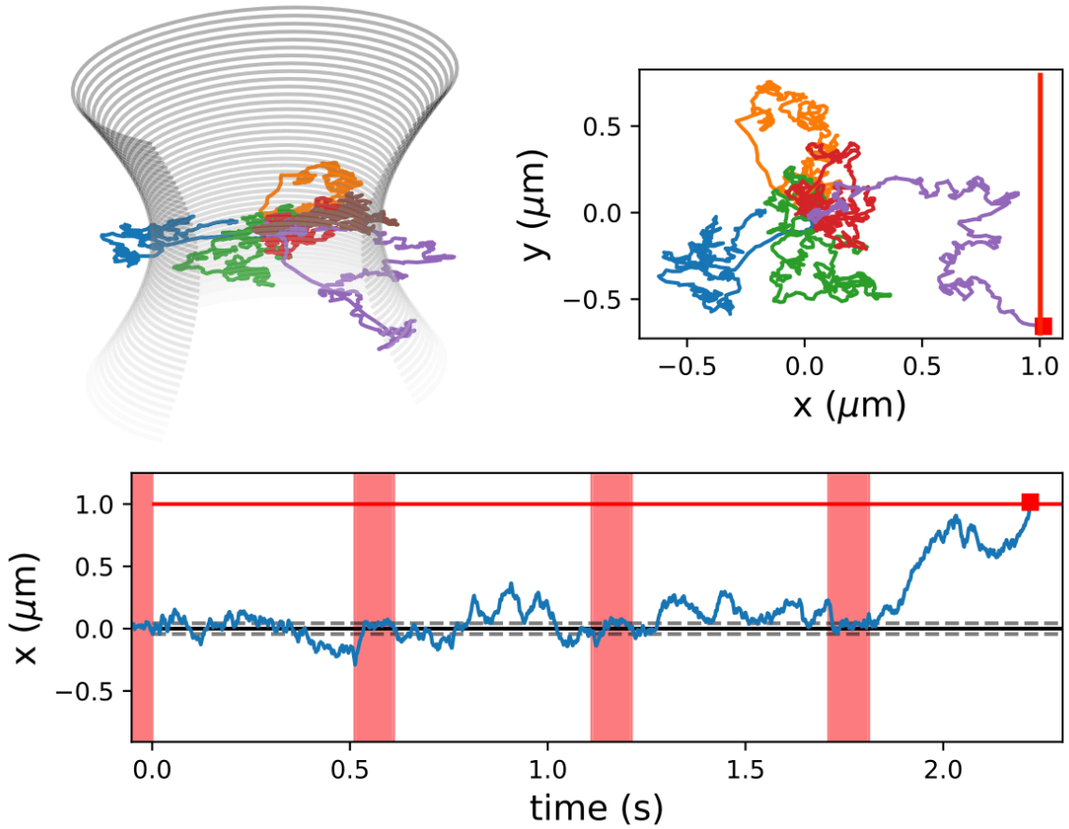
Instantaneous → Non Inst. resetting

Gupta et al. J. Phys. A: Math. Theor. 54 (2021) 025003

Gupta, Pal, and Kundu. JSTAT 2021 (4), 043202

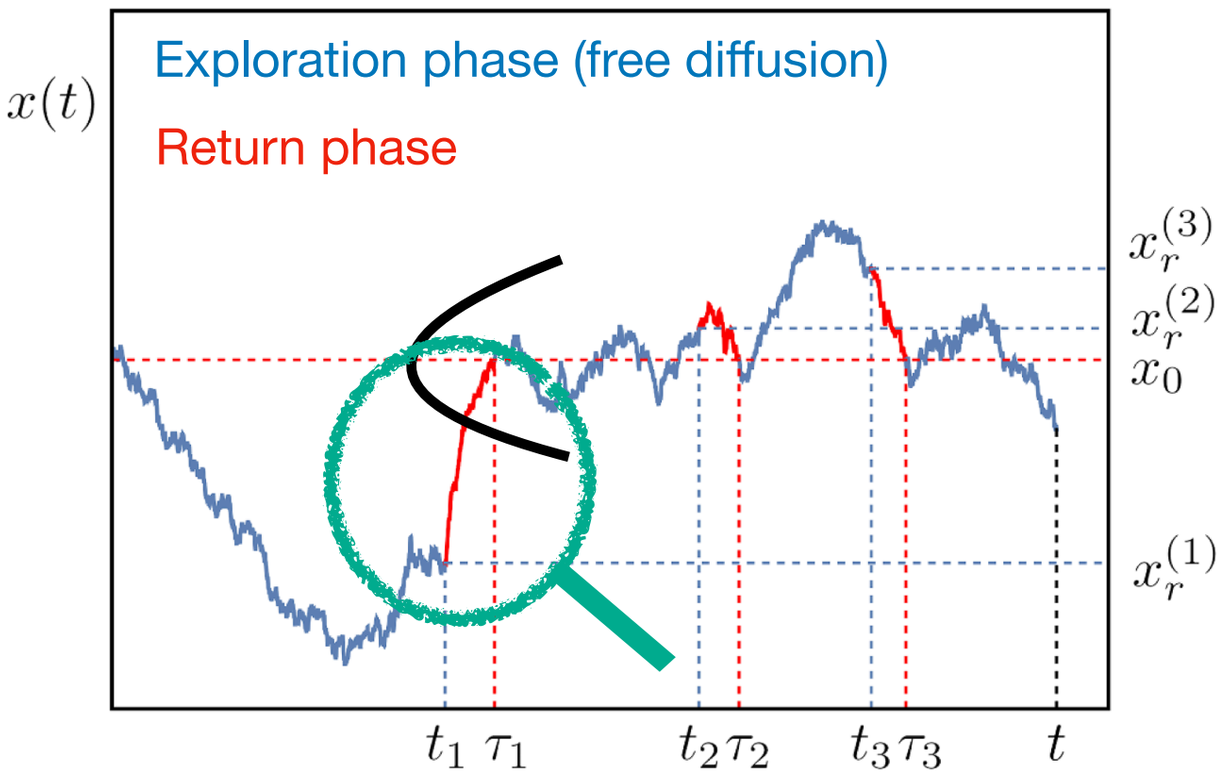


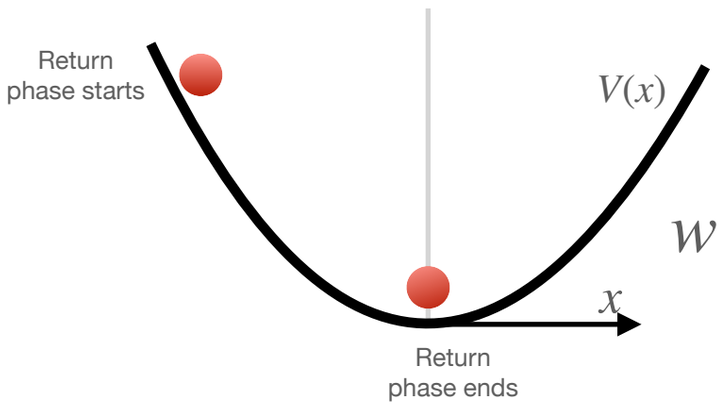
Besga et al. Phys. Rev. Research **2**, 032029(R) (2020)



Thermodynamics of resetting

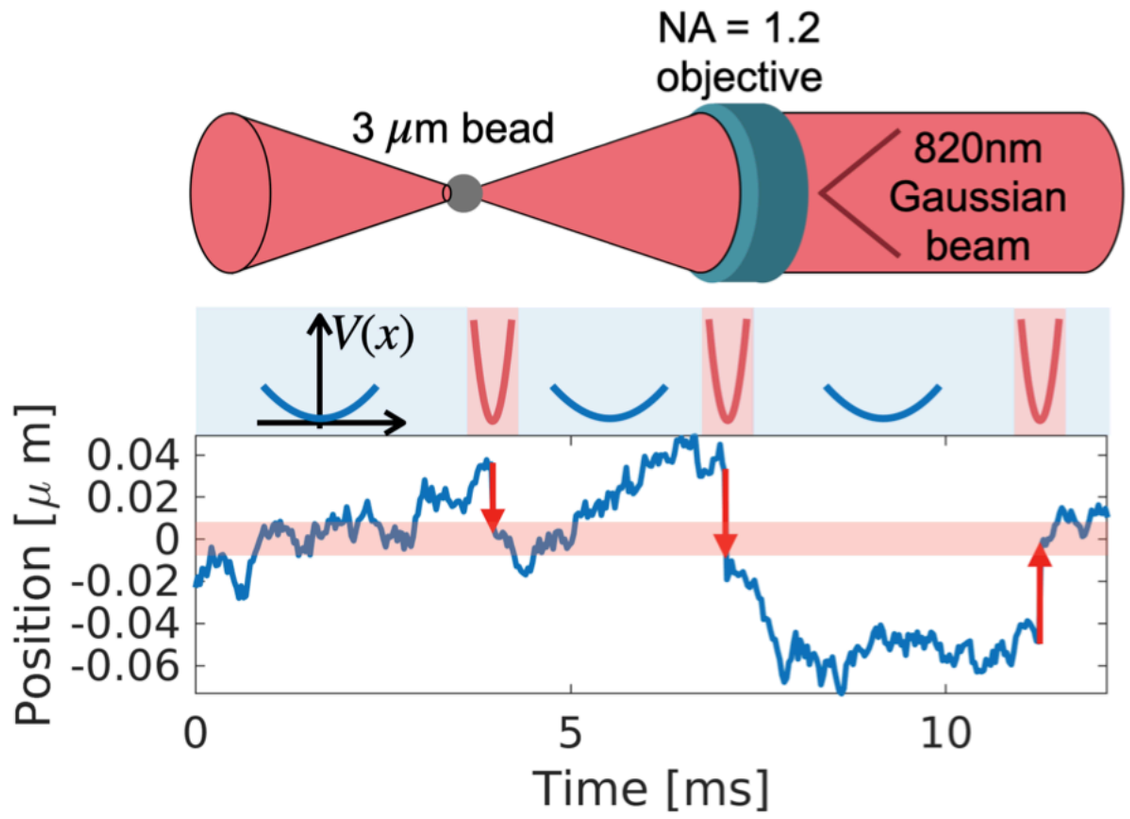
Gupta and Plata, New J. Phys. 24 (2022) 113034





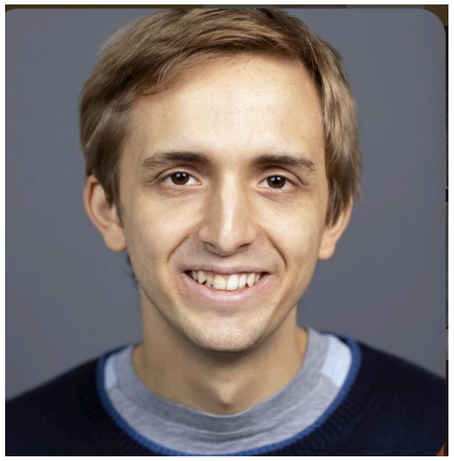
$$w = \sum_{x_{\mathrm{RP,start}}} V(x_{\mathrm{RP,start}})$$

Goerlich et al. Phys. Rev. E 112, 064116 (2025)



1. Gupta, Plata, and Pal. Work Fluctuations and Jarzynski Equality in Stochastic Resetting. PRL **124**, 110608 (2020).
2. Busiello, Gupta, and Maritan. Entropy production in systems with unidirectional transitions. PRR **2,** 023011 (2020).
3. Mori, Olsen, and Krishnamurthy. Entropy production of resetting processes. PRR **5**, 023103 (2023).
4. Fuchs, Goldt and Seifert. Stochastic thermodynamics of resetting. EPL **113,** 60009 (2016).
5. ……

Thermodynamic cost of finite-time stochastic resetting



Dr. F. Mori
Harvard University



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HHU, Düsseldorf

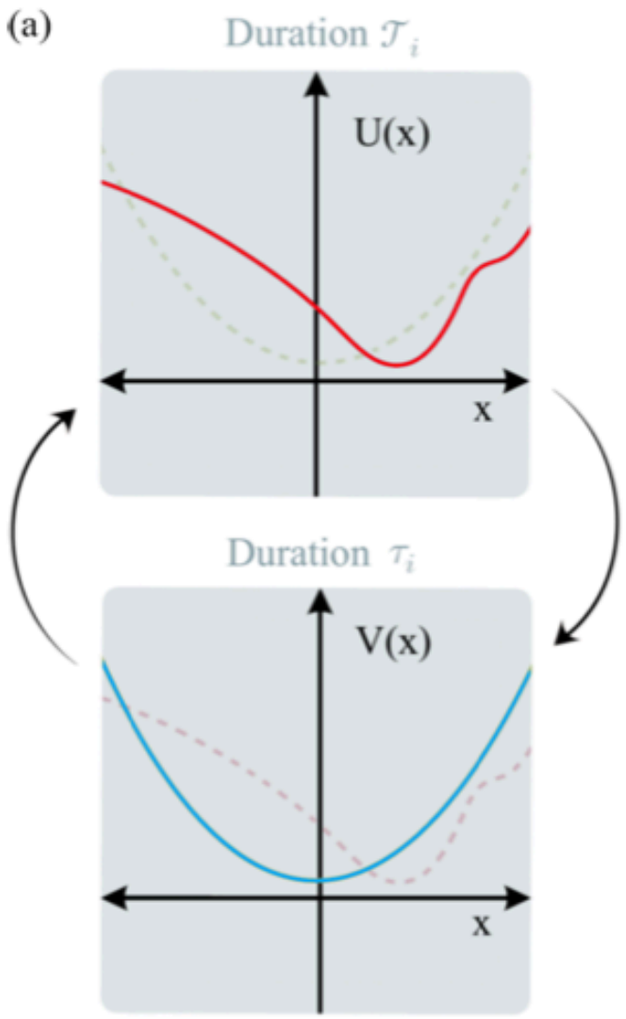


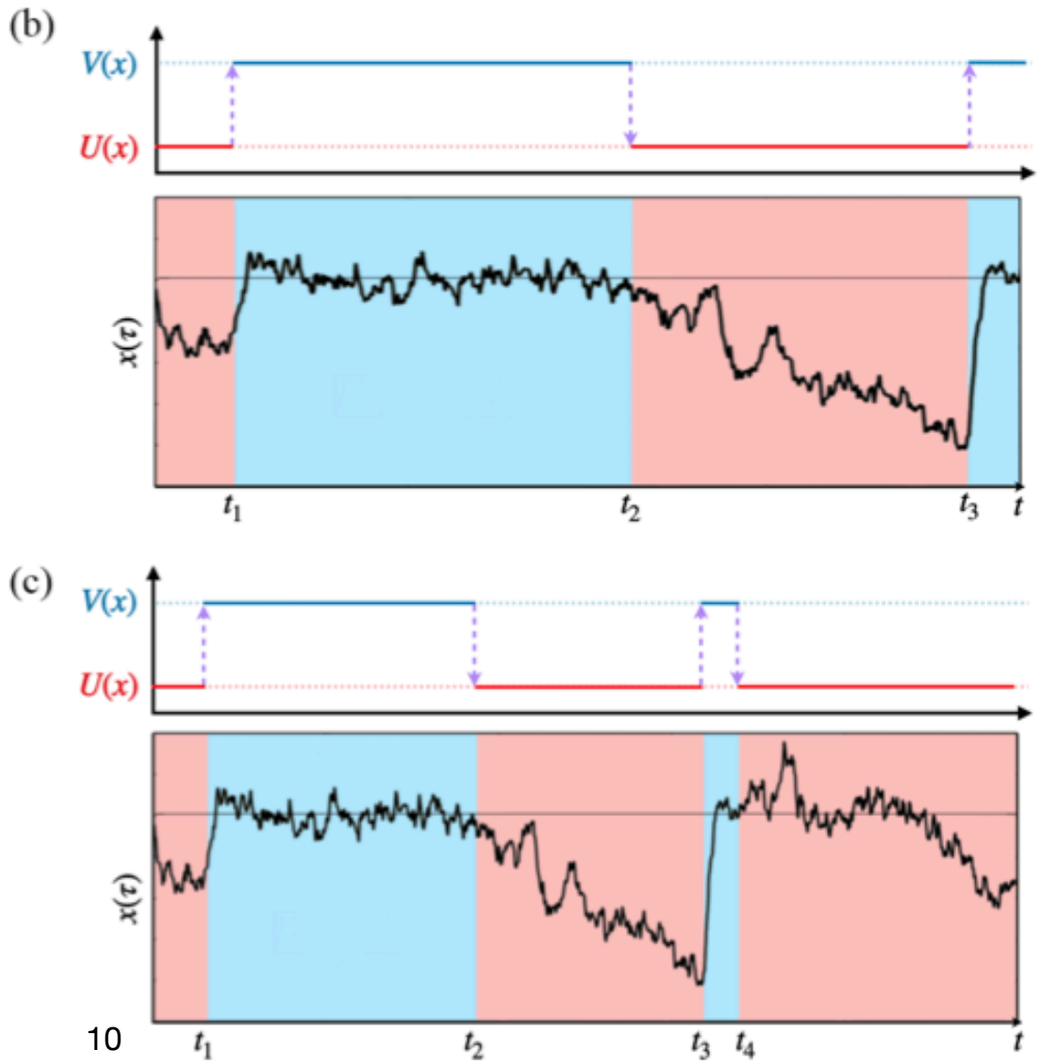
Prof. S. Krishnamurthy
Stockholm University

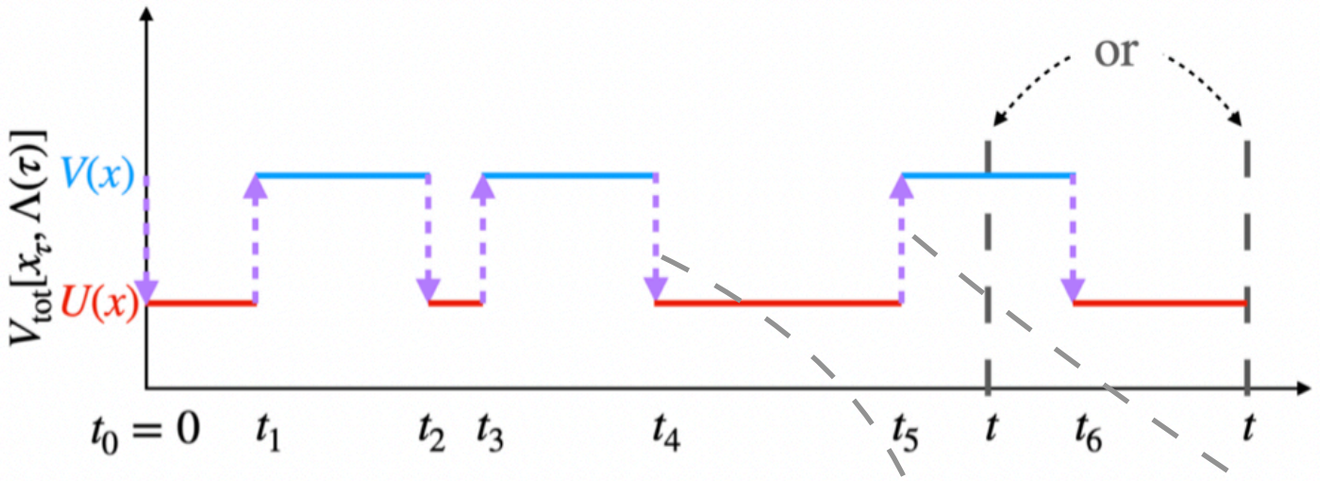
Phys. Rev. Res. 6, 033343 (2024)

Exploration phase

Return phase







$V(x)$: Return potential

$U(x)$: Exploration potential

Initial Condition : $\rho(x_0) \propto e^{-\beta V(x)}$

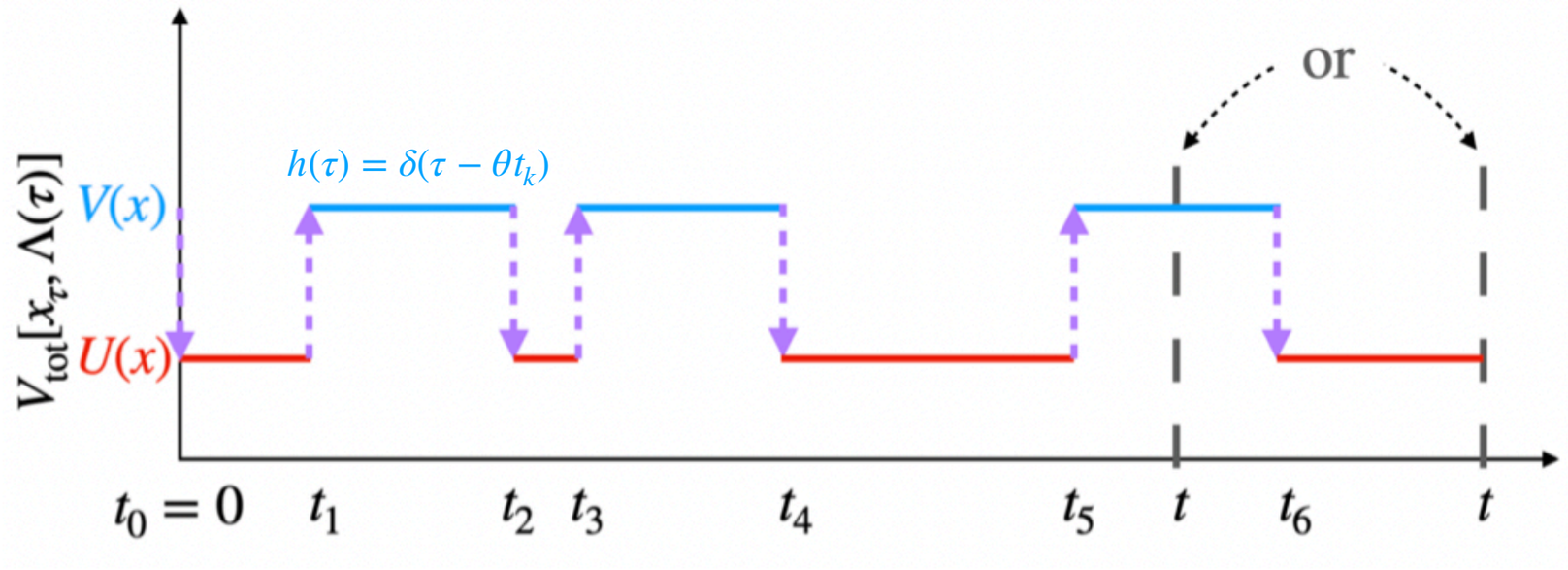
Thermodynamic cost :

$$w[\{x_\tau\}_0^t] \equiv \sum_{j=1} [U(x_{2j-2}^+) - V(x_{2j-2}^-)] + \sum_{j=1} [V(x_{2j-1}^+) - U(x_{2j-1}^-)]$$

$$x(t_{\cdots}^{\pm})$$

Stochastic quantity (initial conditions, thermal fluctuations, stochastic switching times)

Prob . Density Function $p(w, t) = ?$



$V(x)$: Return potential

$U(x)$: Exploration potential

Assumption :

$$C(k,t) \equiv \langle e^{kw} \rangle_{\{\text{initial conditions+thermal fluctuations+stochastic switching times}\}}$$

The particle relaxes to its steady state at the end of each return phase .

How to implement this in finite time?

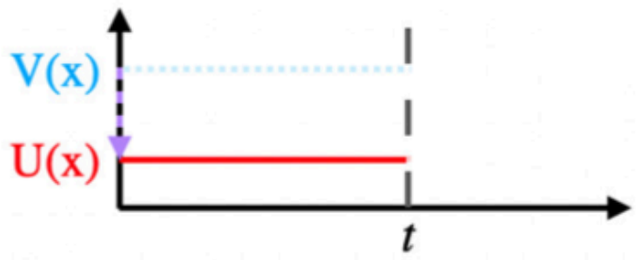
Besga et al. Phys. Rev. Research **2**, 032029(R) (2020)

Goerlich et al. Phys. Rev. E 112, 064116 (2025)

$$C(k,t) = \sum_{n=0}^{\infty} \psi_n(k,t)$$

(a)

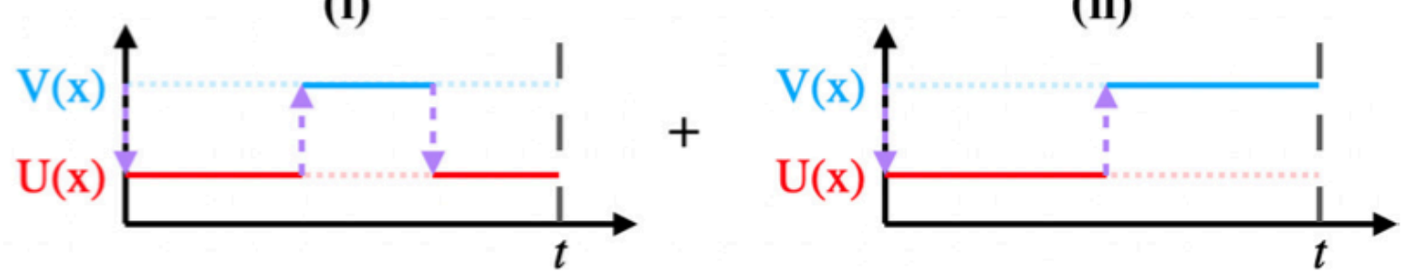
$\psi_0(k) =$



$$\psi_0(\mathbf{k},\mathbf{t}) = \mathbf{g(t)} \int_{-\infty}^{+\infty} \mathbf{dx}\ \mathbf{p}_{\mathrm{eq}}(\mathbf{x}) \mathbf{e}^{\mathbf{k[U(x)-V(x)]}}$$

(b)

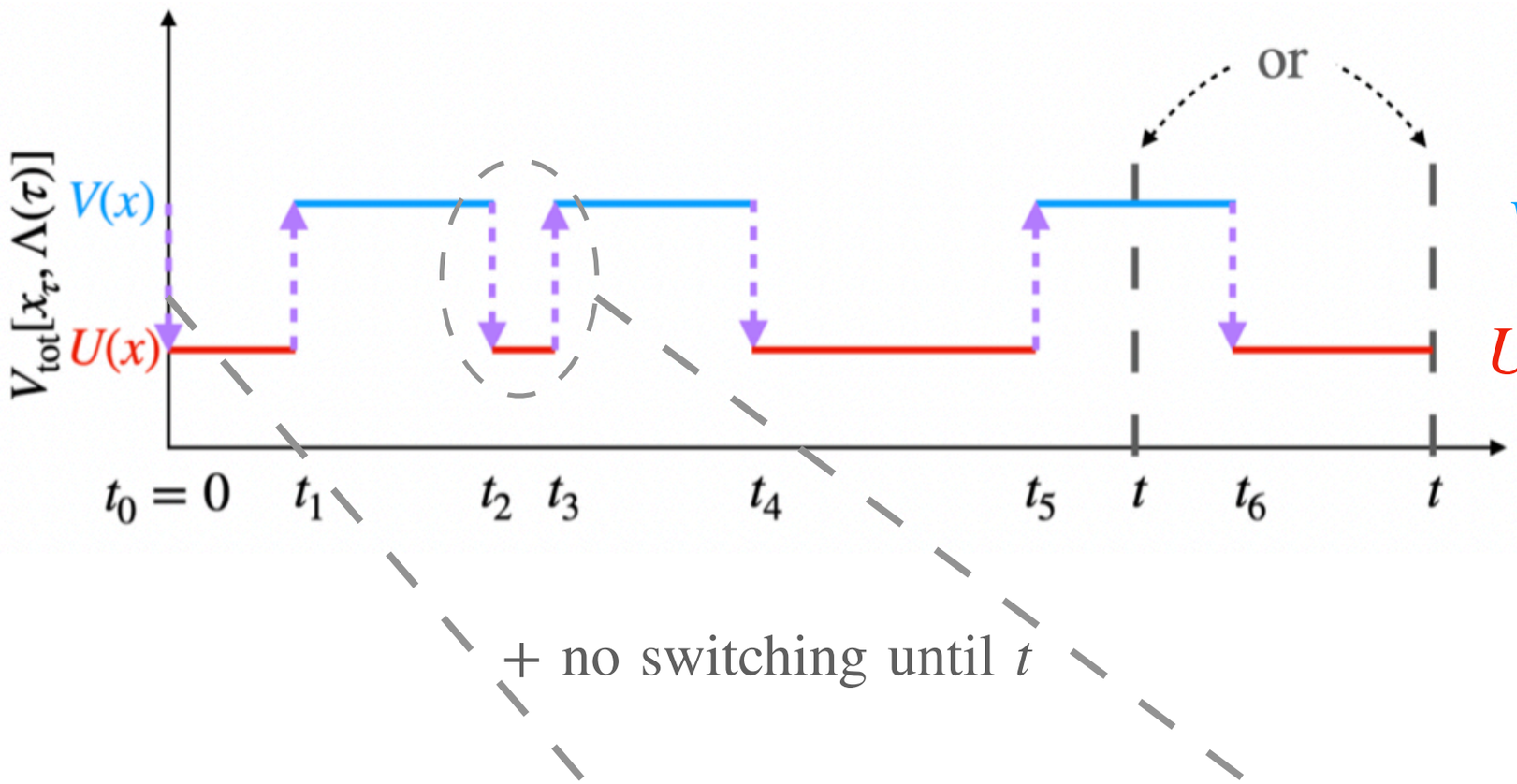
$\psi_1(k) =$



$$\psi_1^{(\mathbf{i})} \equiv \int_0^{\mathbf{t}} \mathbf{d}\mathscr{T}_1\ \mathbf{I}(\mathbf{k},\mathscr{T}_1) \int_0^{\mathbf{t}-\mathscr{T}_1} \mathbf{d}\tau_1\ \mathbf{h}(\tau_1)\ \psi_0(\mathbf{k},\mathbf{t}-\mathscr{T}_1-\tau_1)$$

$$\psi_1^{(\mathbf{i})} \equiv \int_0^{\mathbf{t}} \mathbf{d}\mathscr{T}_1\ \mathbf{I}(\mathbf{k},\mathscr{T}_1)\ \mathbf{H}(\mathbf{k},\mathbf{t}-\mathscr{T}_1)$$

$$\mathbf{I}(\mathbf{k},\mathbf{t}) = \mathbf{f(t)} \int_{-\infty}^{+\infty} \mathbf{dx}\ \mathbf{p}_{\mathrm{eq}}(\mathbf{x}) \mathbf{e}^{\mathbf{k[U(x)-V(x)]}} \int_{-\infty}^{+\infty} \mathbf{dx'}\ \mathbf{p}(\mathbf{x'},\mathbf{t}\,|\,\mathbf{x}) \mathbf{e}^{\mathbf{k[V(x')-U(x')]}}$$



$V(x)$: Return potential

$U(x)$: Exploration potential

Main result:

$$\tilde{C}(k,s) = \frac{\tilde{\psi}_0(k,s) + \tilde{H}(s)\ \tilde{I}(k,s)}{1 - \tilde{h}(s)\ \tilde{I}(k,s)}$$

$$p(w,t) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dk\ e^{-kW}\ \ \mathscr{L}^{-1}\left[\tilde{C}(k,s)\right]$$

$$\langle w^n \rangle = \mathscr{L}^{-1}\left[\left.\frac{\partial^n \tilde{C}(k,s)}{\partial k}\right|_{k=0}\right]$$

Example

1D systems

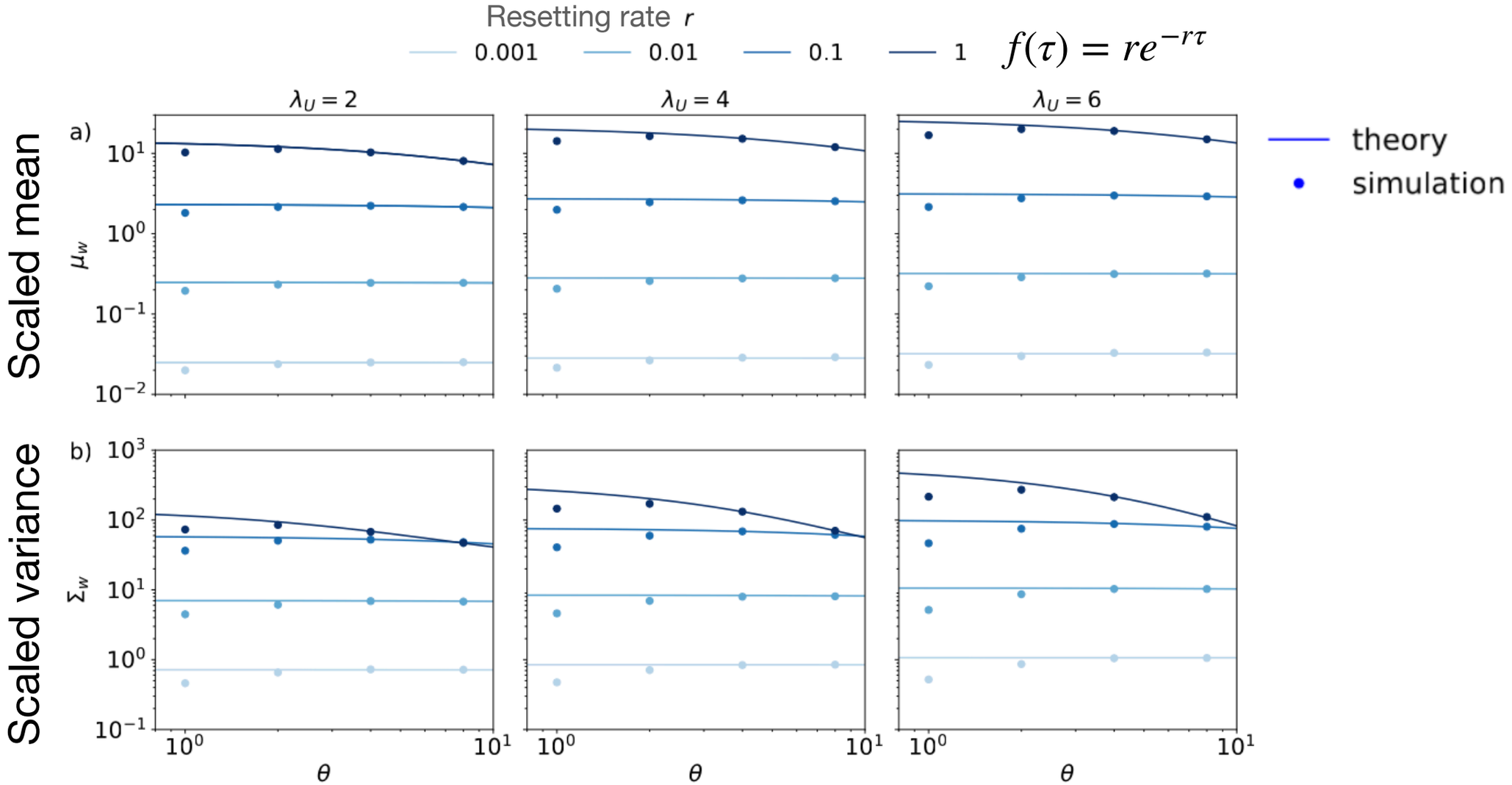
$$U(x) = \lambda_U x^2/2,\ V(x) = \lambda_V x^2/2$$

Goerlich et al. Phys. Rev. E 112, 064116 (2025)

Return phase, *V*: $h(\tau) = \delta(\tau - \theta t_k)$

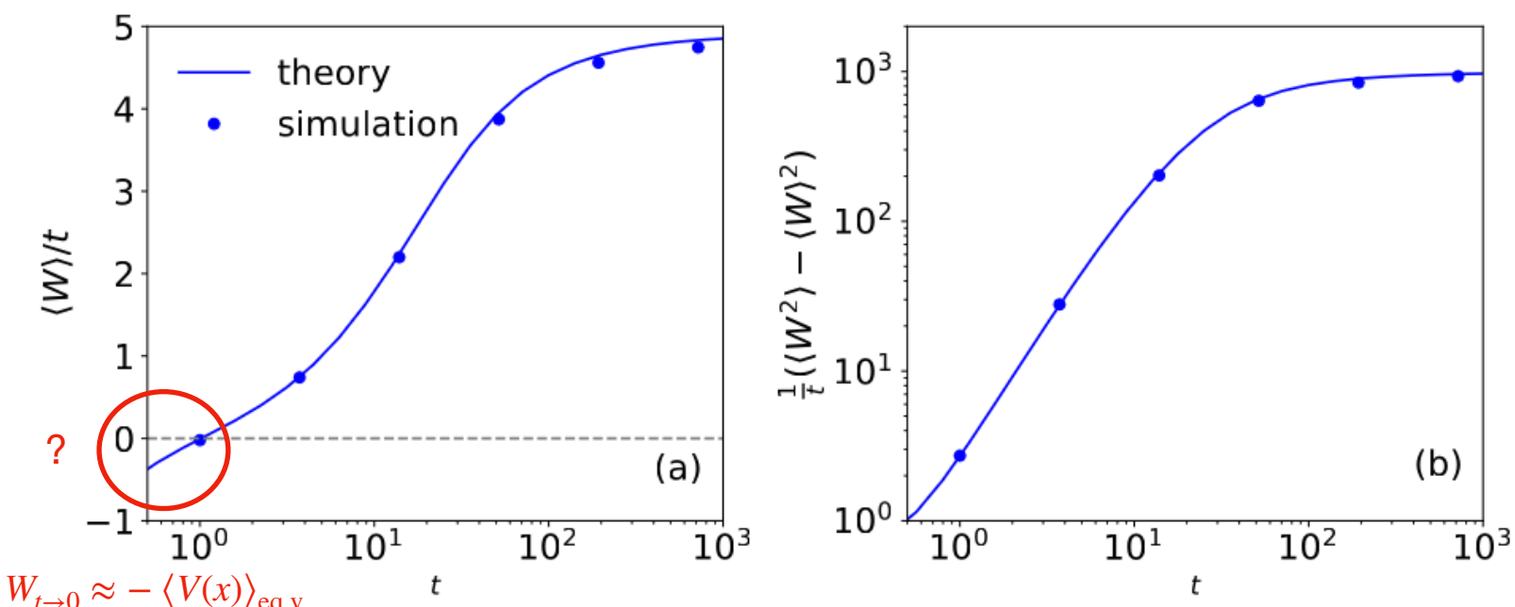
Exploration phase, *U*: $f(\tau) = re^{-r\tau}$

Results:



$t \to \infty$

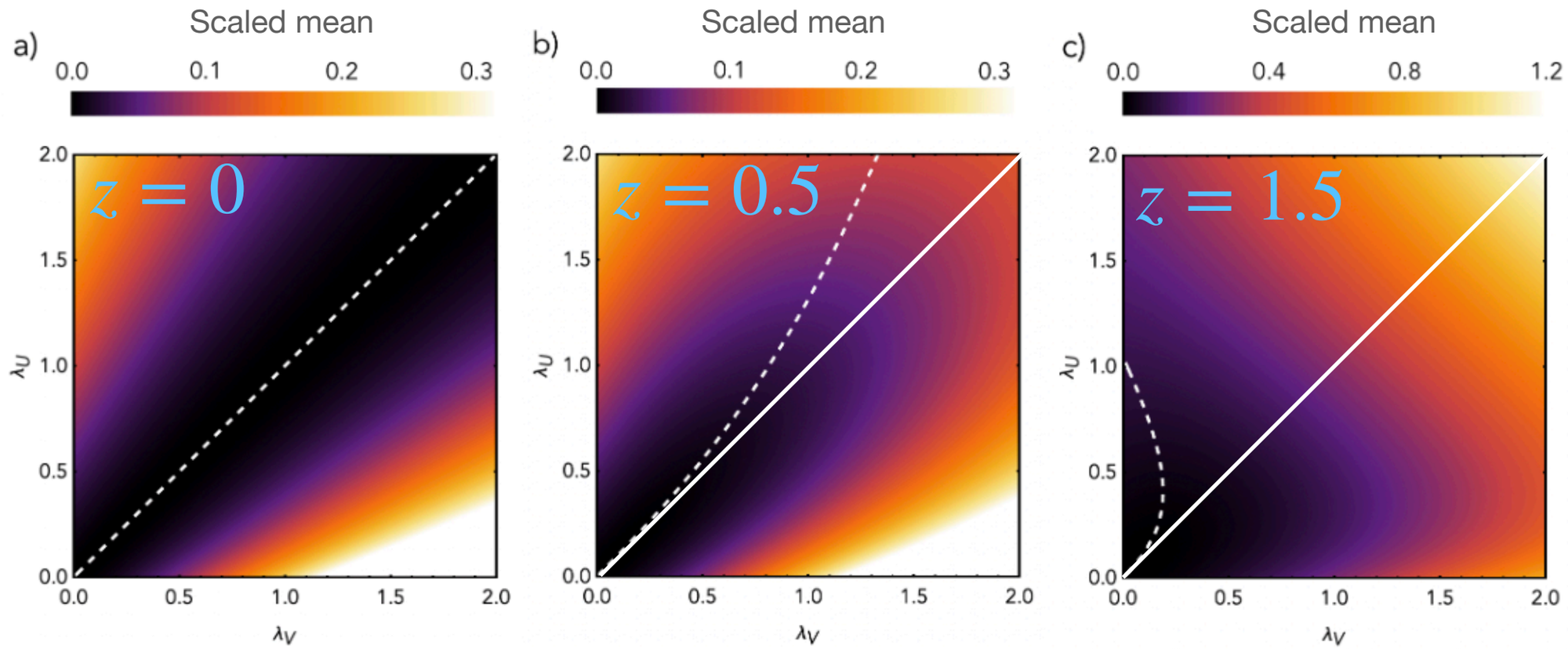
Number of relaxation times of the return trap $h(\tau) = \delta(\tau - \theta t_k)$

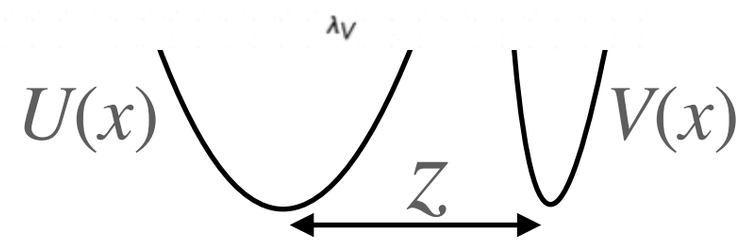


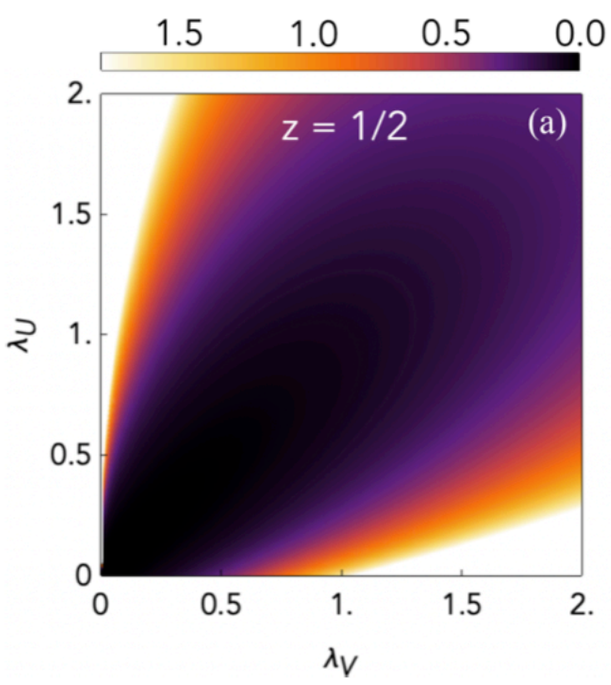
$U(x) = 0; \theta = 2$

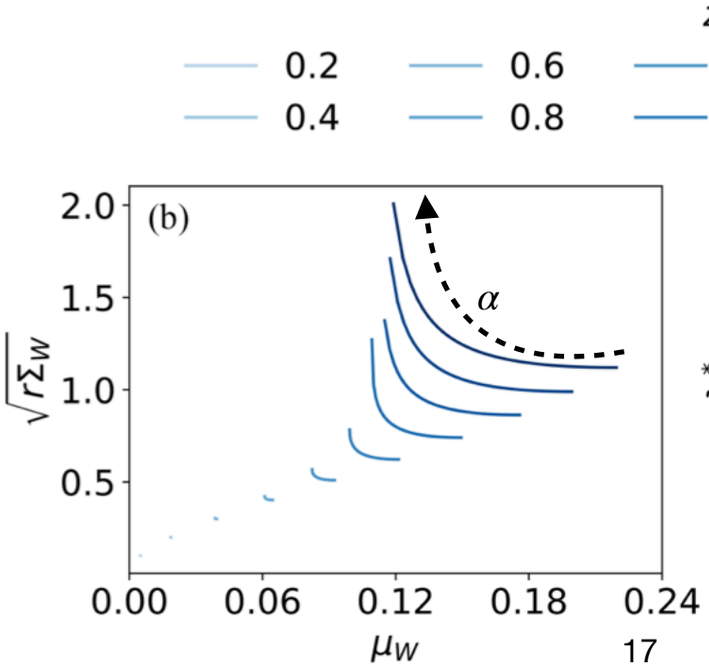
Time

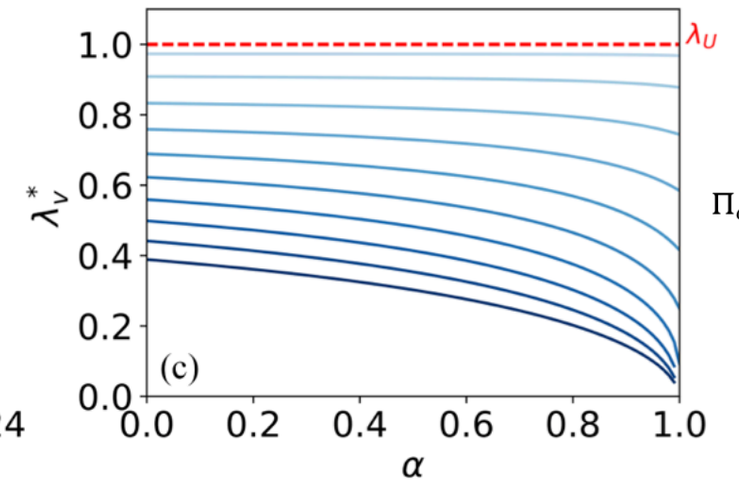
Optimal λ_V







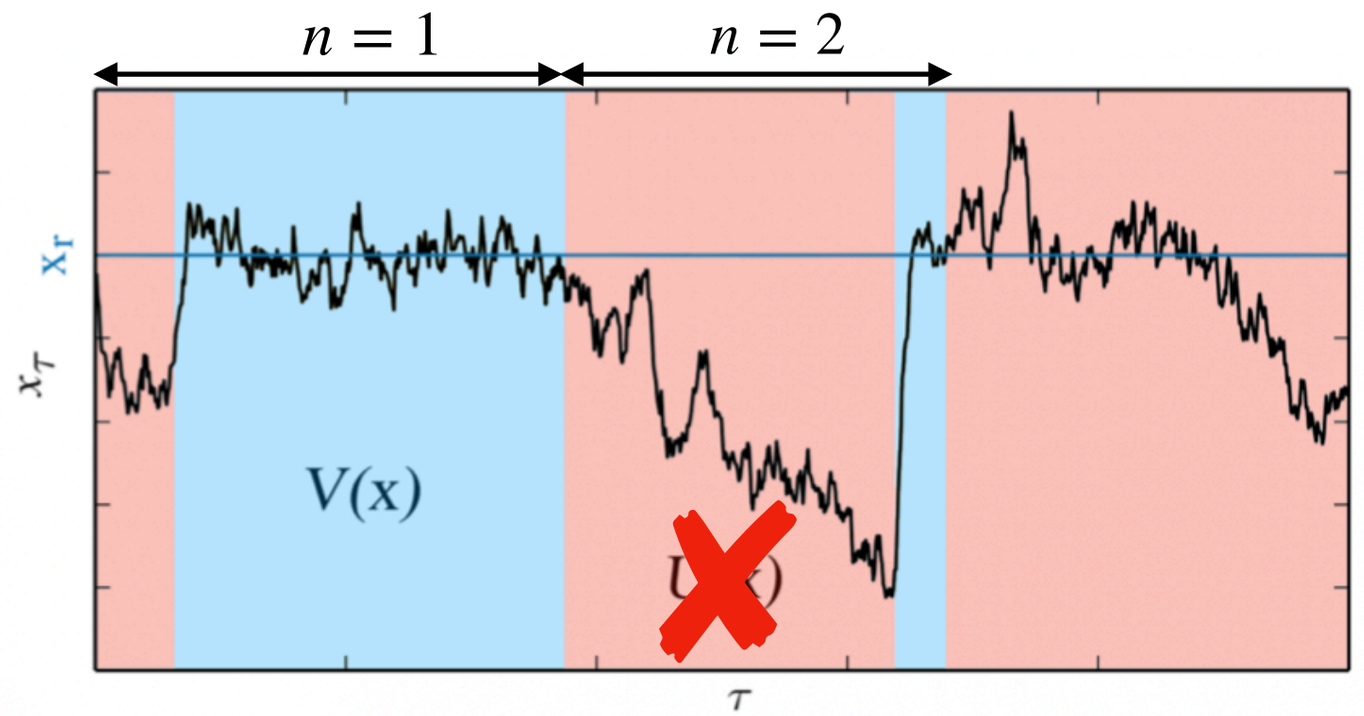




$$\Pi_\alpha(\lambda_V) \equiv \alpha\mu_W(\lambda_V) + (1-\alpha)\sqrt{r\Sigma_W(\lambda_V)}$$

Previous assumption :

The particle relaxes to its steady state at the end of each return phase.



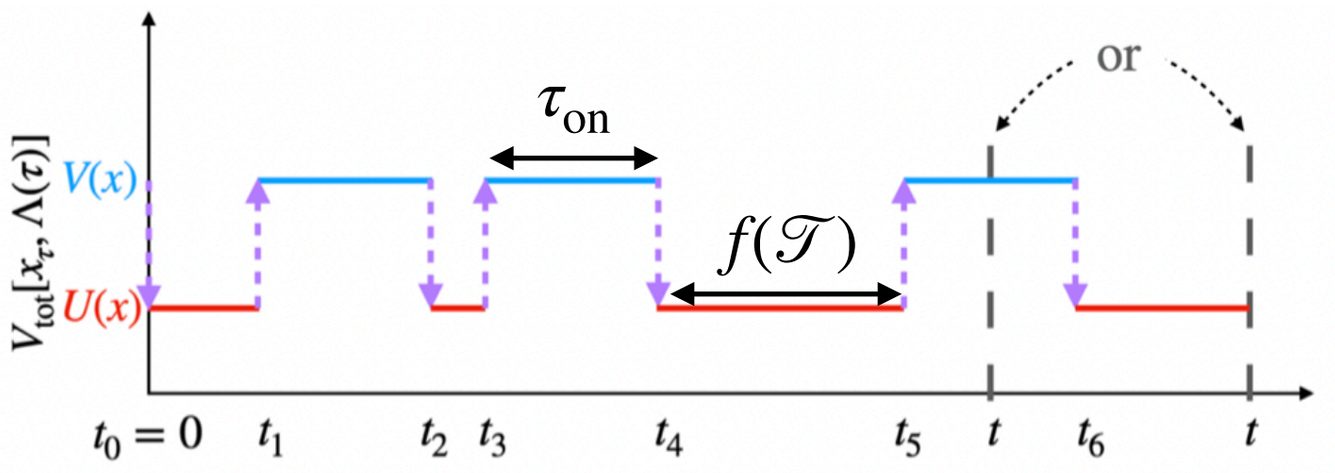
$V(x) \propto x^2$

$$\langle W_n \rangle = \lambda_V \left[D\langle \mathcal{T} \rangle \left\{ n - \tilde{h}(2\tau_{\text{rel}}^{-1}) \frac{1-\tilde{h}^n(2\tau_{\text{rel}}^{-1})}{1-\tilde{h}(2\tau_{\text{rel}}^{-1})} \right\} + \frac{1-\tilde{h}^n(2\tau_{\text{rel}}^{-1})}{2} \left(\langle x_0^2 \rangle - D\tau_{\text{rel}} \right) \right]$$

Large n : $\langle W_n \rangle \to nD\lambda_V \langle \mathcal{T} \rangle$ $\quad \langle t_n \rangle = n[\langle \mathcal{T} \rangle + \langle \tau \rangle]$ $\quad \frac{\langle W_n \rangle}{\langle t_n \rangle} \approx \frac{D\lambda_V}{1 + \langle \tau \rangle / \langle \mathcal{T} \rangle}$

Assumption : $U(x), V(x) \propto x^2$

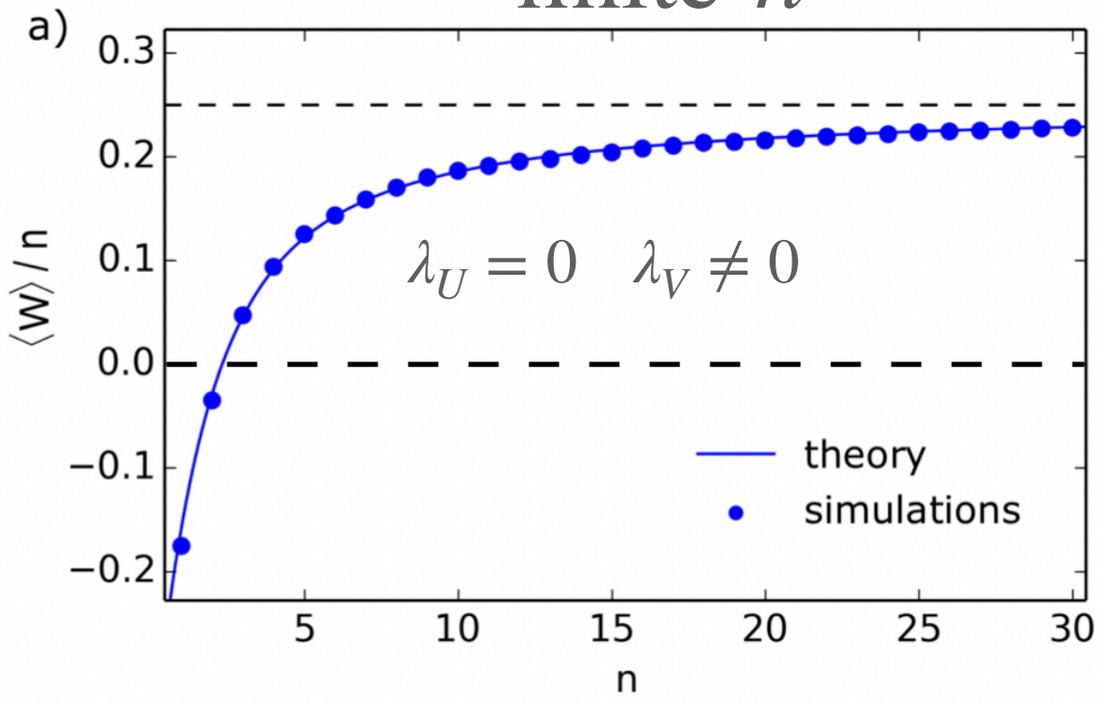
System in the steady state(long – time or large – n limit)



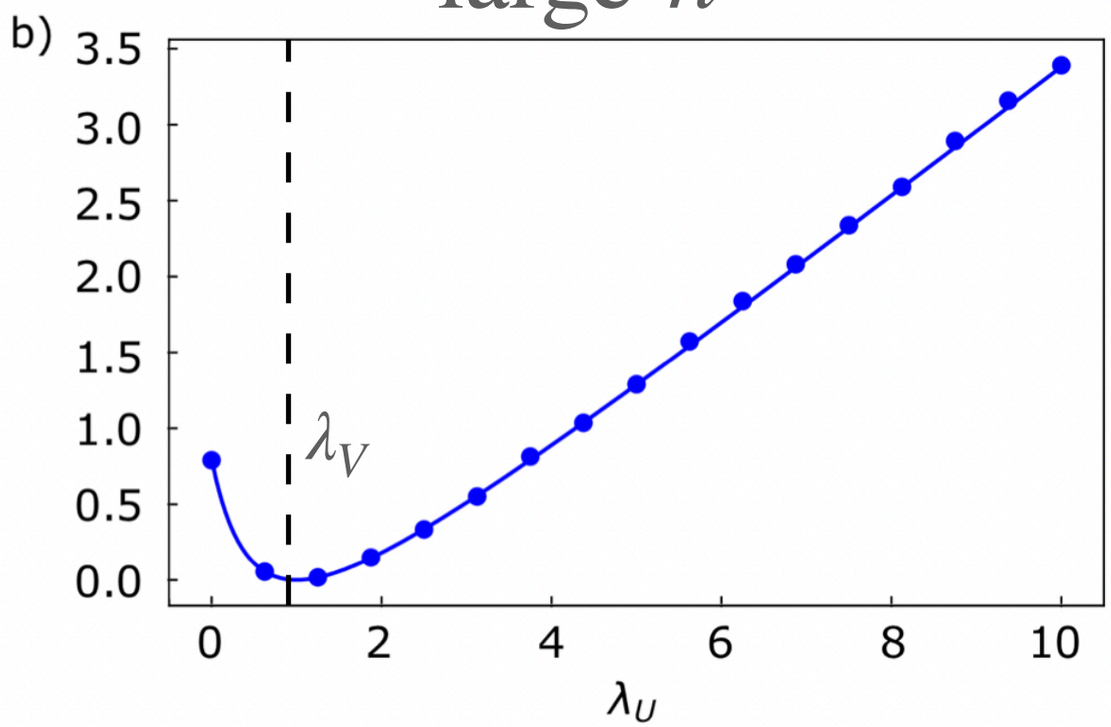
$$f(\mathcal{T}) \propto e^{-\mathcal{T}^2/(2\sigma^2)},\ \mathcal{T} > 0$$

$$\lim_{n\to\infty} \frac{\langle W_n \rangle}{n} = \frac{D}{2}(\lambda_V - \lambda_U)^2 \frac{(e^{2\lambda_V \tau_{\text{on}}} - 1)[e^{2\lambda_U^2\sigma^2}\,\text{Erfc}(\sqrt{2}\lambda_U\sigma) - 1]}{\lambda_U\lambda_V[e^{2\lambda_U^2\sigma^2}\,\text{Erfc}(\sqrt{2}\lambda_U\sigma) - e^{2\lambda_V\tau_{\text{on}}}]}$$

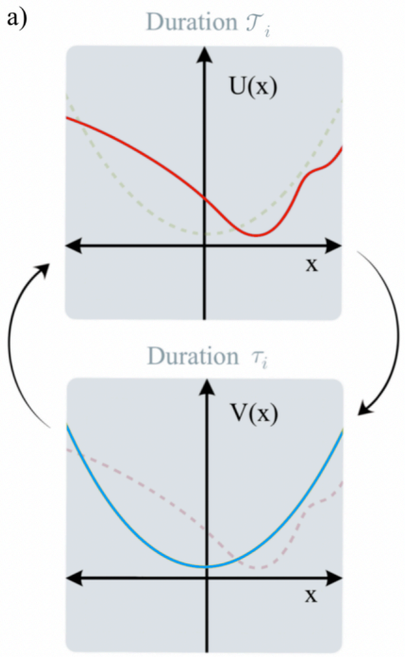
finite n

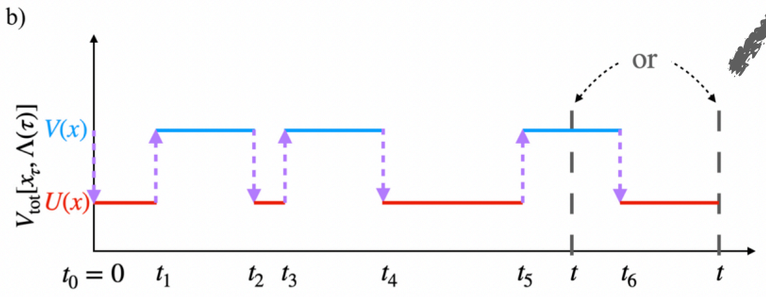


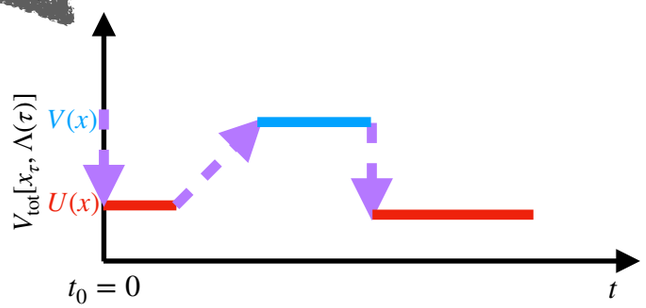
large n

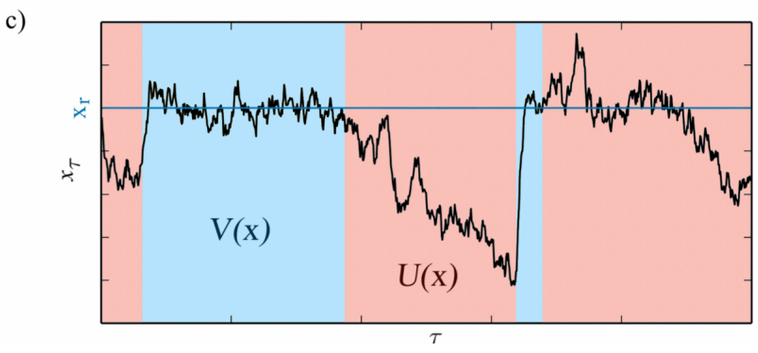


Summary

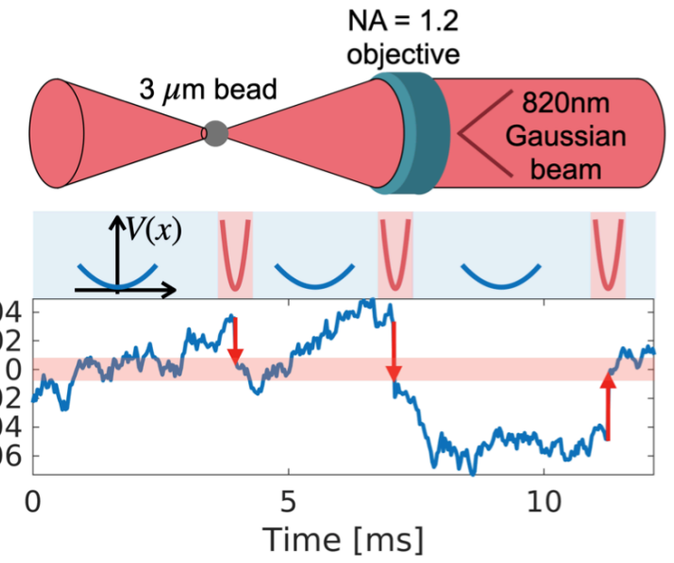


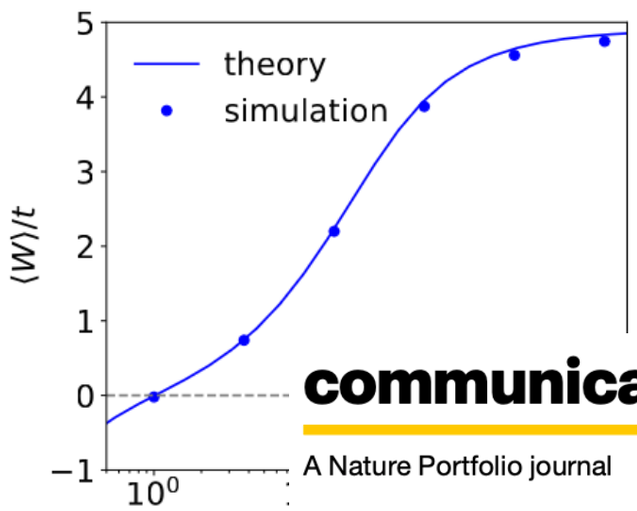


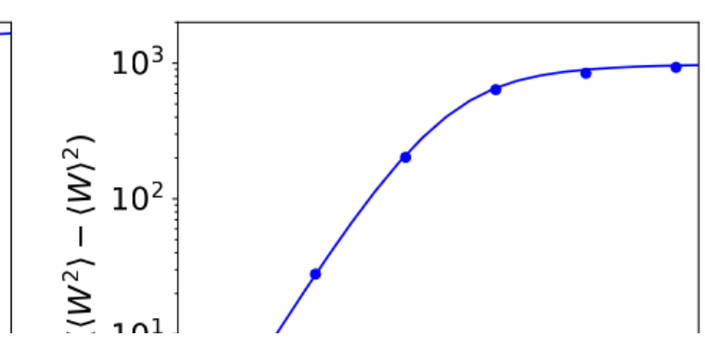




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Article

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Thermodynamic cost of recurrent erasure

Check for updates

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Thanks!