Dynamics of a Local Defect in 1D Driven Flow:

Interplay of Jamming, Condensation, and Tracer Diffusion

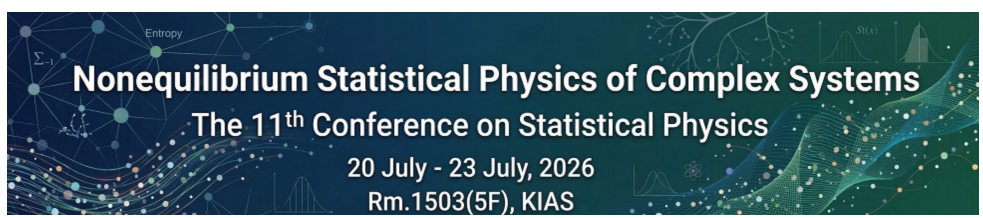
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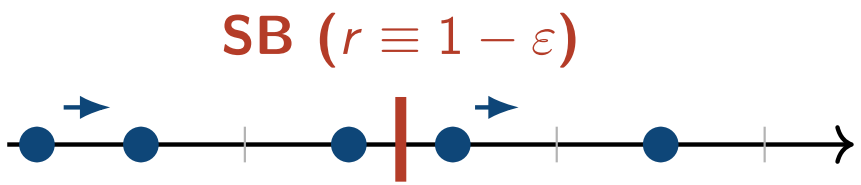
Key question for this talk

Localized defect in a 1D Kardar-Parisi-Zhang (KPZ)-type flow

If a single bond is slowed with the hopping probability from 1 to $r \equiv 1 - \varepsilon$, is a weak defect still visible on macroscopic scales?

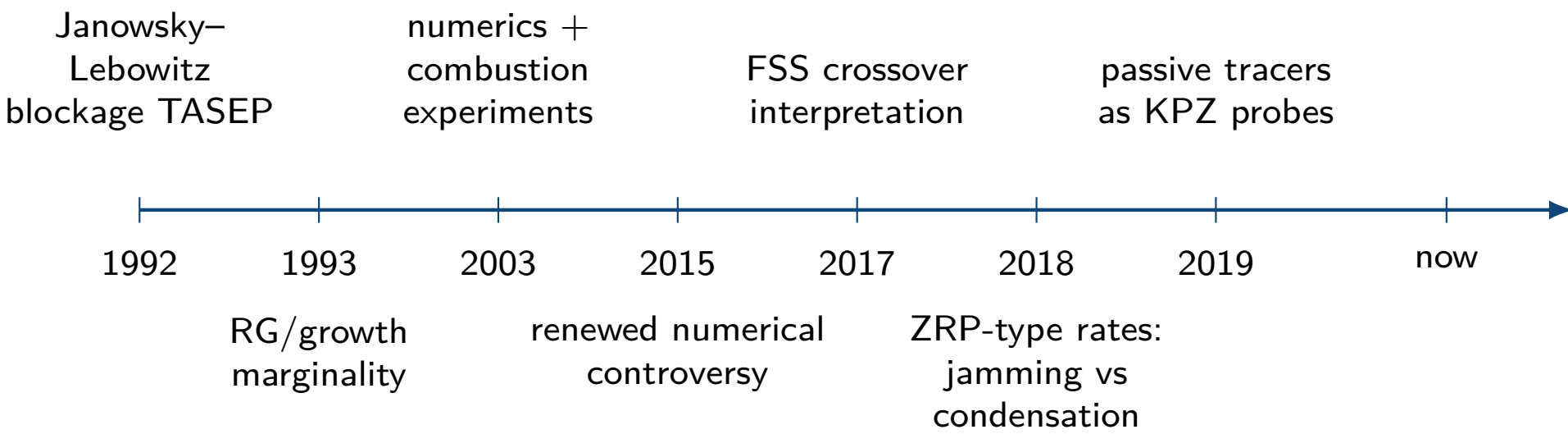
- **1D TASEP**[a]: hard-core exclusion + drive $\Rightarrow$ 1D KPZ scaling.
- **Slow-Bond problem**: local in space, but can create a global queue.
- With interactions, **Zero-Range Process (ZRP)-type hopping** can induce **condensation and particle-hole segregation**. For passive (positive and negative) tracers, non-invasive probes of deformed KPZ fluctuations.

[a] TASEP=Totally Asymmetric Simple Exclusion Process



current J and density $\rho(x)$

Historical progress I: **Physics route (1992–2019, now)**



Can SB change macroscopic current and KPZ fluctuation structure?

Lesson for this talk

The local defect became a controlled probe of marginal relevance, finite-size crossover, and interaction-induced screening.

Janowsky–Lebowitz; Tang–Lyuksyutov; Balents–Kardar; MH–Timonen–den Nijs; Myllys *et al.*; Schmidt–Popkov–Schadschneider; Soh–Baek–MH–Jeong; Soh–MH–Jeong; Soh–MH; MH.

Recap of Physics Debate (1992–200X)

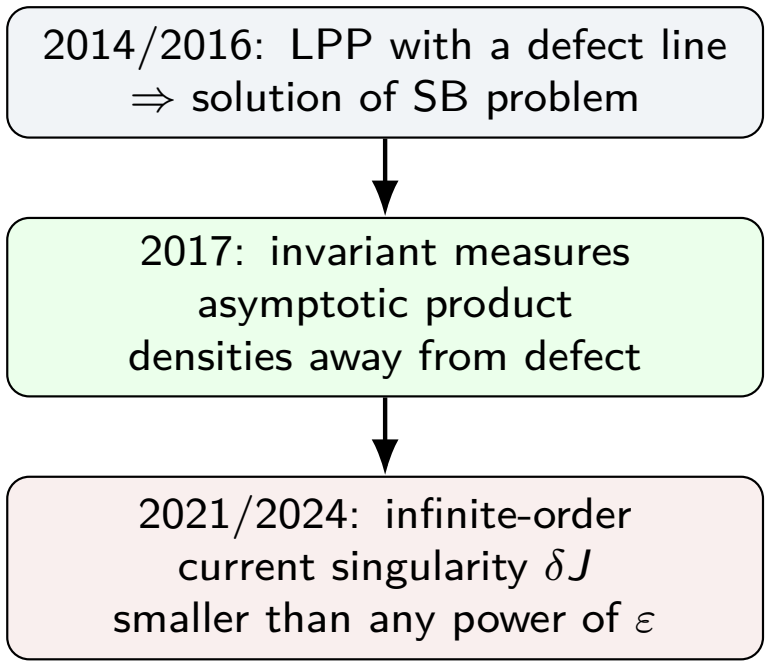
- **1992: Genesis of the Problem**
  Janowsky and Lebowitz first posed the SB problem, arguing via MF theory that any defect strength ($\epsilon > 0$) induces a macroscopic queue, implying a critical threshold of $\epsilon_c = 0$.
- **1993–1998: Analytical Consensus**
  RG arguments and series expansions (Tang & Lyuksyutov; Kolomeisky) supported the existence of an essential singularity at $\epsilon_c = 0$.
- **2003: Empirical & Numerical Challenge**
  MH, Timonen, and den Nijs proposed the existence of a non-queued SB phase via finite-size scaling (estimating $\epsilon_c \approx 0.20$). Concurrently, Myllys et al. observed similar thresholds in slow paper combustion experiments, sparking a decades-long debate.

Historical progress II: **Mathematics route (up to now)**

From TASEP to **LPP**[a]

[a]LPP = Last-Passage Percolation

- TASEP current maps to passage time in exponential LPP.
- SB maps to a reinforced defect line.
- SB problem: question about optimal geodesics and diagonal local time.



Rigorous resolution (Basu–Sidoravicius–Sly, 2014/2016)[a]

[a]Basu, R., Sidoravicius, V., and Sly, A. (2016). Last passage percolation with a defect line and the solution of the slow bond problem. Ann. Prob., **44**, 2918-2987.

Every $\varepsilon > 0$ reduces the asymptotic current: $\varepsilon_c = 0$.

Key Mathematical Refs.: Basu–Sidoravicius–Sly; Basu–Sarkar–Sly; Sarkar–Sly–Zhang.

Recap of Rigorous Proofs & Reconciliation (up to now)

- **2013: Return to Essential Singularity**
  Costin et al. provided strong analytical limits that support the reduced current scaling $\delta J \sim \exp(-b/\varepsilon)$, reinvigorating $\varepsilon_c = 0$.
- **2014: Mathematical Resolution**
  Basu, Sidoravicius, and Sly delivered a rigorous mathematical proof that $\epsilon_c = 0$ for the 1D TASEP using LLP models.
- **2015–2017: Physics Reconciliation**
  High-precision simulations by Schmidt et al. (2015) and comprehensive finite-size scaling by Soh, Baek, MH, and Jeong (2017) demonstrated that the apparent $\epsilon_c > 0$ phase is actually a **finite-size crossover artifact**.

Current Paradigm

Our focus has shifted from searching for a phase transition to **mapping the anomalous dynamic scaling, tracer diffusion, and condensation behaviors induced by the defect with crossover scaling**.

Defining Last Passage Percolation (LPP) for TASEP

The dynamics of 1D TASEP can be exactly mapped to a directed LPP model on a 2D grid ($\mathbb{Z}^2_{\geq 0}$).

Canonical Mapping

- **Vertices & Weights:**
  Each vertex (i, j) in the grid is assigned an independent random waiting time $\omega_{i,j}$, drawn from an exponential distribution with rate 1.
- **Directed Paths:**
  A directed path π from $(0, 0)$ to (M, N) consists of steps moving only right $(+1, 0)$ or up $(0, +1)$.
- **Passage Time:**
  The last passage time $G(M, N)$ is the maximum accumulated weight over all valid paths: $G(M, N) = \max_{\pi:(0,0)\to(M,N)} \sum_{(i,j)\in\pi} \omega_{i,j}$.

Physical Interpretation: The macroscopic current J in the TASEP directly maps to the asymptotic growth rate of $G(M, N)$ in the exponential LPP geometry.

SB as a reinforced defect line in LPP

Introducing a localized quenched SB of strength ε at the origin in 1D TASEP fundamentally alters the 2D LPP landscape.

- **Defect Line:**
  SB strictly maps to the main diagonal ($i = j$) of the LPP grid.
- **Modified Weights:**
  Vertices on this diagonal are assigned exponentially distributed weights with a reduced rate $r \equiv 1 - \varepsilon$.
- **Heavier Rewards:**
  A lower rate means a higher mean waiting time $\langle \omega_{i,i} \rangle = \frac{1}{1-\varepsilon}$.

Geodesic Competition

SB problem is a competition of two optimal geodesics

1. **Bulk Wandering:**
   Paths that wander into the bulk, driven by KPZ transverse fluctuations ($\sim t^{2/3}$).
2. **Diagonal Pinning:**
   Paths that stick to the diagonal to accumulate "heavier" rewards by discrete geodesic local time.

Resolution: The rigorous proof that $\epsilon_c = 0$ hinges on showing that for any $\epsilon > 0$, the optimal geodesic spends macroscopic time pinned to this reinforced diagonal, strictly reducing the asymptotic current.

Most recent progress

1. SB: rigorous resolution and infinite-order singularity

Sarkar, Sly, and Zhang (2024)[a] proved that the current perturbation has an **infinite-order phase transition at $\varepsilon = 0$, helping to explain why finite simulations could appear contradictory**.

[a] Sarkar, S., Sly, A. and Zhang, L. (2024), Infinite order phase transition in the slow bond TASEP. Comm. Pure Appl. Math., **77**: 3107-3140. https://doi.org/10.1002/cpa.22185

2. Inhomogeneity and boundaries in KPZ fixed-point theory

Recent TASEP/KPZ work[a] extends **exact and fixed-point tools to inhomogeneous rates and half-space geometries, making local/boundary perturbations a central modern theme**.

[a] Bisi, E., Liao, Y., Saenz, A. et al. Non-intersecting Path Constructions for TASEP with Inhomogeneous Rates and the KPZ Fixed Point. Commun. Math. Phys. **402**, 285–333 (2023). https://doi.org/10.1007/s00220-023-04723-8

Most recent progress cont'd

3. **Finite-volume KPZ universality fluctuations**[a]

[a] Prolhac, S. KPZ fluctuations in finite volume. SciPost Phys. Lect. Notes 81 (2024). https://scipost.org/SciPostPhysLectNotes.81

Finite-volume relaxation at $t \sim L^{3/2}$ is now a mature perspective; it **directly informs how to interpret simulation windows for defects**.

$+$ ... more

Why we should still care?

Although the theorem says the defect is relevant, the infinite-order scale explains why simulations see long-lived "almost irrelevant" behavior.

Then how does it show in finite systems?

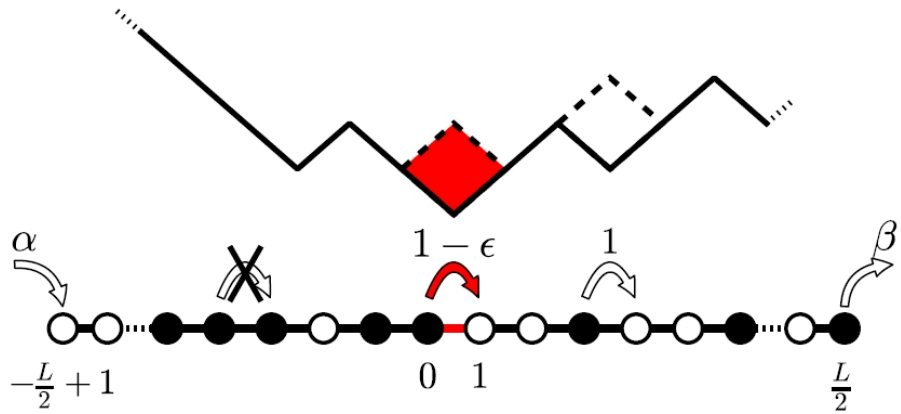
Model: TASEP $\leftrightarrow$ KPZ surface growth

Driven lattice gas

$$\bullet\circ \rightarrow \circ\bullet, \qquad r \equiv 1-\varepsilon.$$

Main Quantities

- Density: $\rho(x) = \langle n_x \rangle$.
- Current $J = \langle n_x(1-n_{x+1}) \rangle$.
- Maximal-Current point: $\rho_{\rm bulk} = 1/2,\ J_{\rm MC} = 1/4$.



Body-Centered Solid on Solid (BCSOS) representation of TASEP with a SB[a] that is located between $x = 0$ and $x = 1$ for $x \in [-\frac{L}{2}+1, +\frac{L}{2}]$.

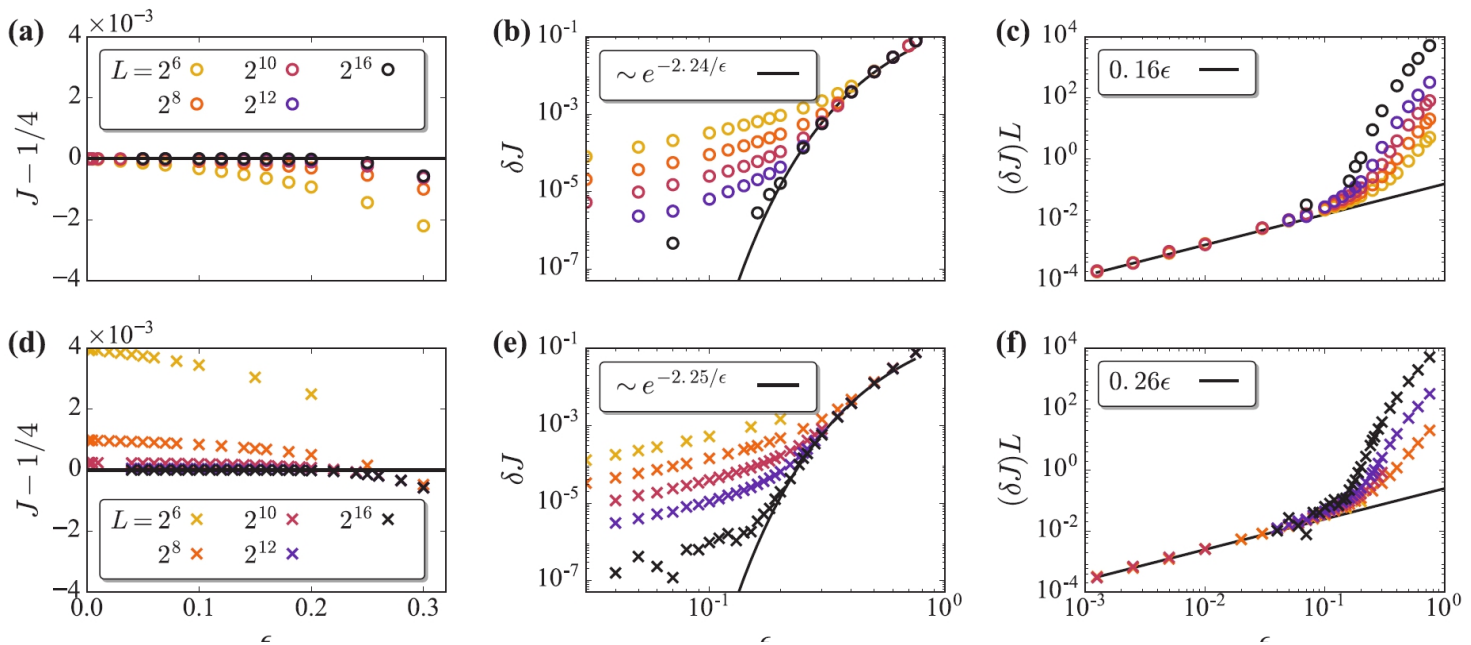
[a] Soh, H., Baek, B., MH, and Jeong, H., Phys. Rev. E **95**, 042123 (2017).

Baseline of SB problem: **Why it was hard**

Thermodynamic prediction

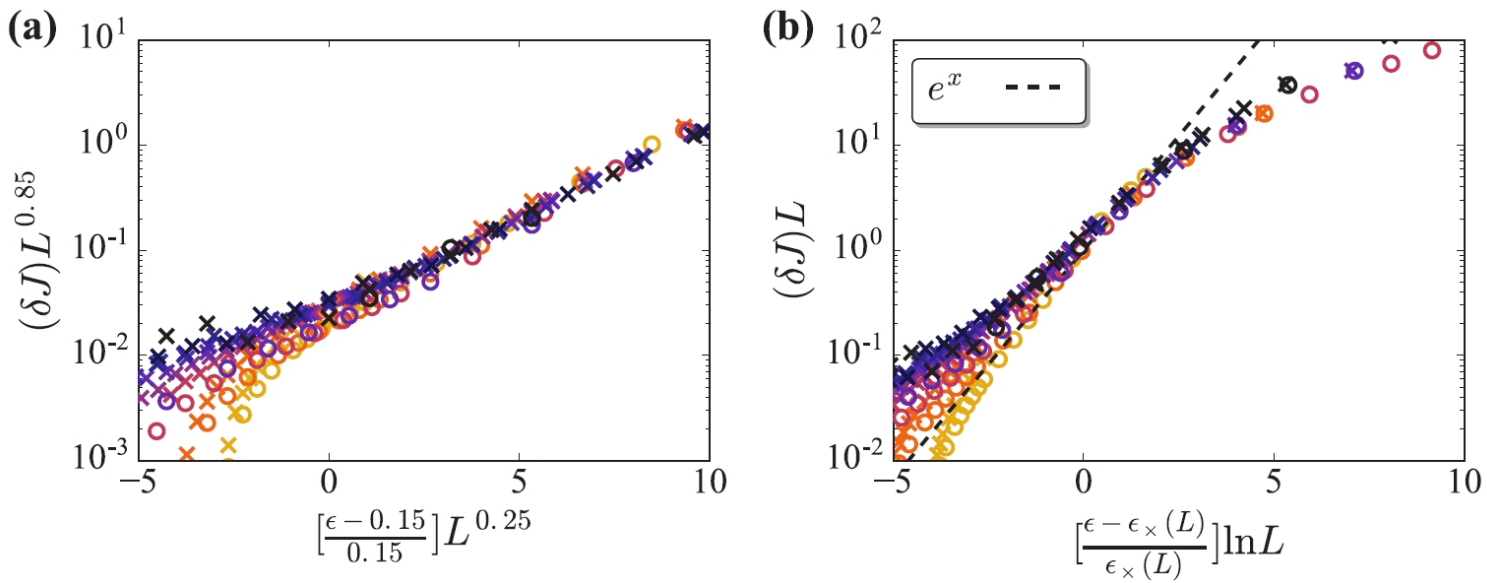
$$\delta J(\varepsilon; L) \equiv J(0; L) - J(\varepsilon; L) \sim e^{-b/\varepsilon} \text{ vs. } \varepsilon/L.$$

Since this essential singularity is numerically subtle, finite systems see an apparent threshold, finite $\varepsilon_\times(L) \simeq \frac{b}{\ln L}$. Weak SB regime for $\varepsilon < \varepsilon_\times$ can look like a stable phase, but shrinks slowly with L.



Current reduction and finite-size scaling (FSS) in the original SB problem: (a)-(c) for OBCs and (d)-(f) for PBCs.

Two FSS narratives can fit the same finite data



Competing finite-size scaling forms: nonzero critical defect vs. logarithmic crossover.

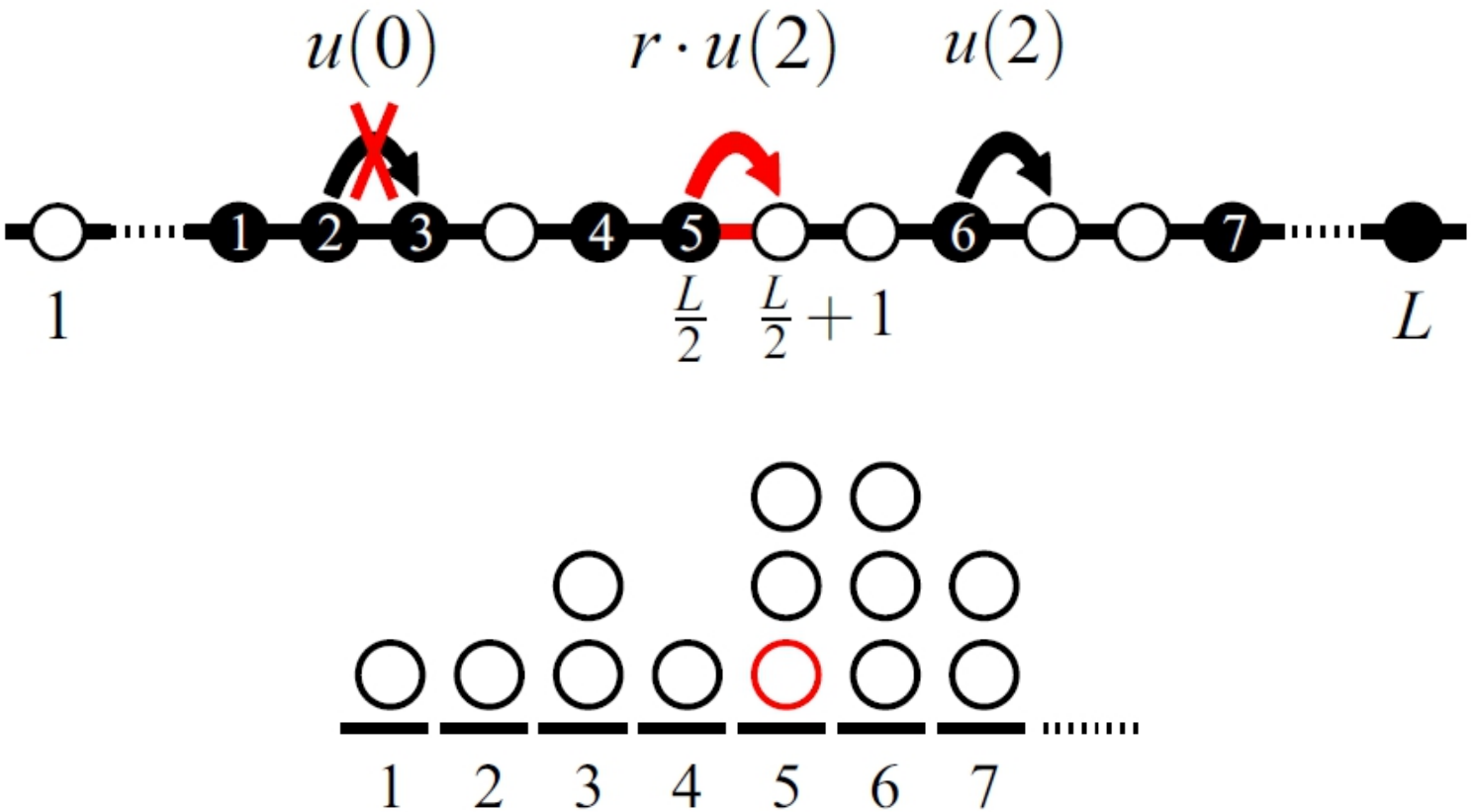
Key caution for numerical simulations: $\delta J = L^{-\beta/\nu} f(\tilde{\epsilon} L^{1/\nu})$

To distinguish $L^{1/\nu}$ from $\ln L$ requires an enormous dynamic range.

Working in progress with physical interpretation

The localized defect (SB) is marginally relevant in the ordinary TASEP, but its observable footprint is exponentially weak as $\varepsilon \to 0$.

Model: TASEP $\leftrightarrow$ ZRP



Modified TASEP with a SB (red)[1] and its ZRP-type mapping with a slow (red) particle.

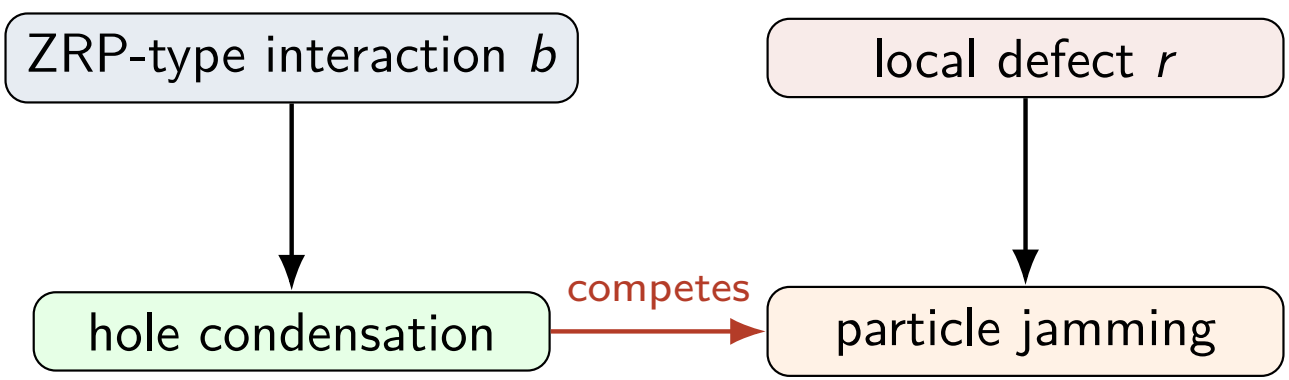
[1] Soh, H., MH, and Jeong, H., Phys. Rev. E **97**, 032120 (2018).

Adding interactions: **Modified TASEP w/ ZRP-type jumps**

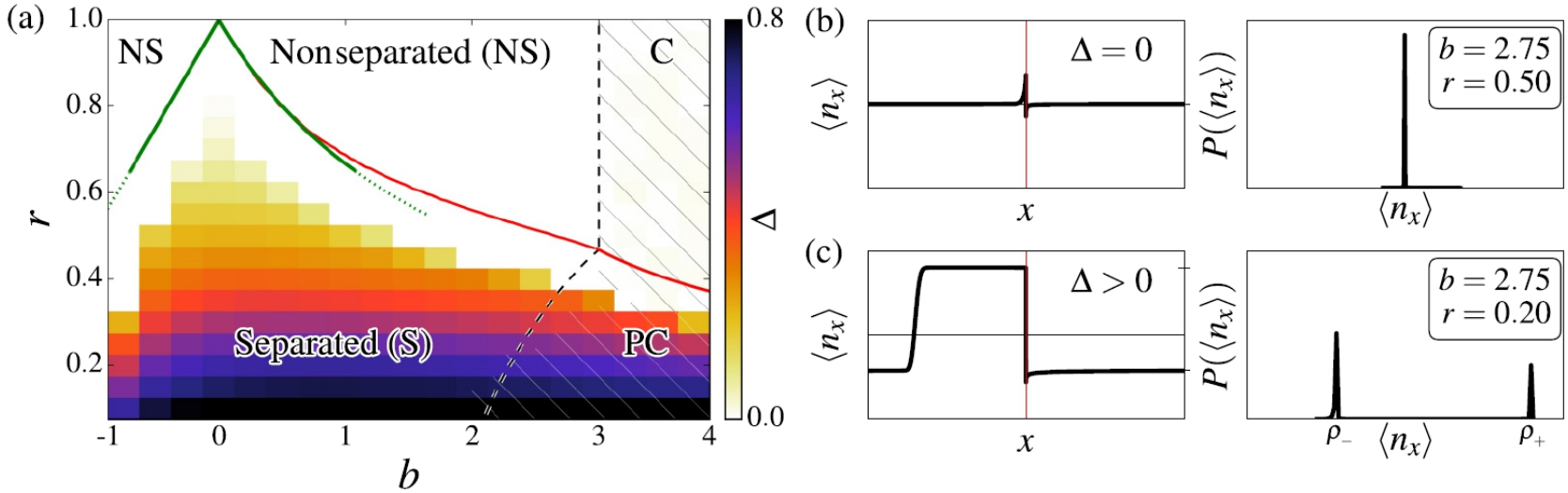
Hopping probability depends on the gap ℓ ahead

$$q(\ell) = \frac{u(\ell)}{u_{\max}}, \qquad u(\ell) = 1 + \frac{b}{\ell}, \qquad q_{\mathrm{SB}} = r\, q(\ell).$$

- $b < 0$: effective repulsion, more homogeneous spacing.
- $b = 0$: ordinary TASEP with slow bond.
- $b > 0$: attraction/hole condensation tendency.
- $r < 1$: defect-induced jamming tendency.



Phase diagram: **Jamming can be suppressed**



Phase diagram in (b, r) plane, Density profiles, and Probabilities, where $\Delta \equiv \rho_+ - \rho_-$: **S**eparated (S), **N**on-**S**eparated (NS), **C**ondensed (C), **P**artially **C**ondensed (PC).

Four phases/regimes

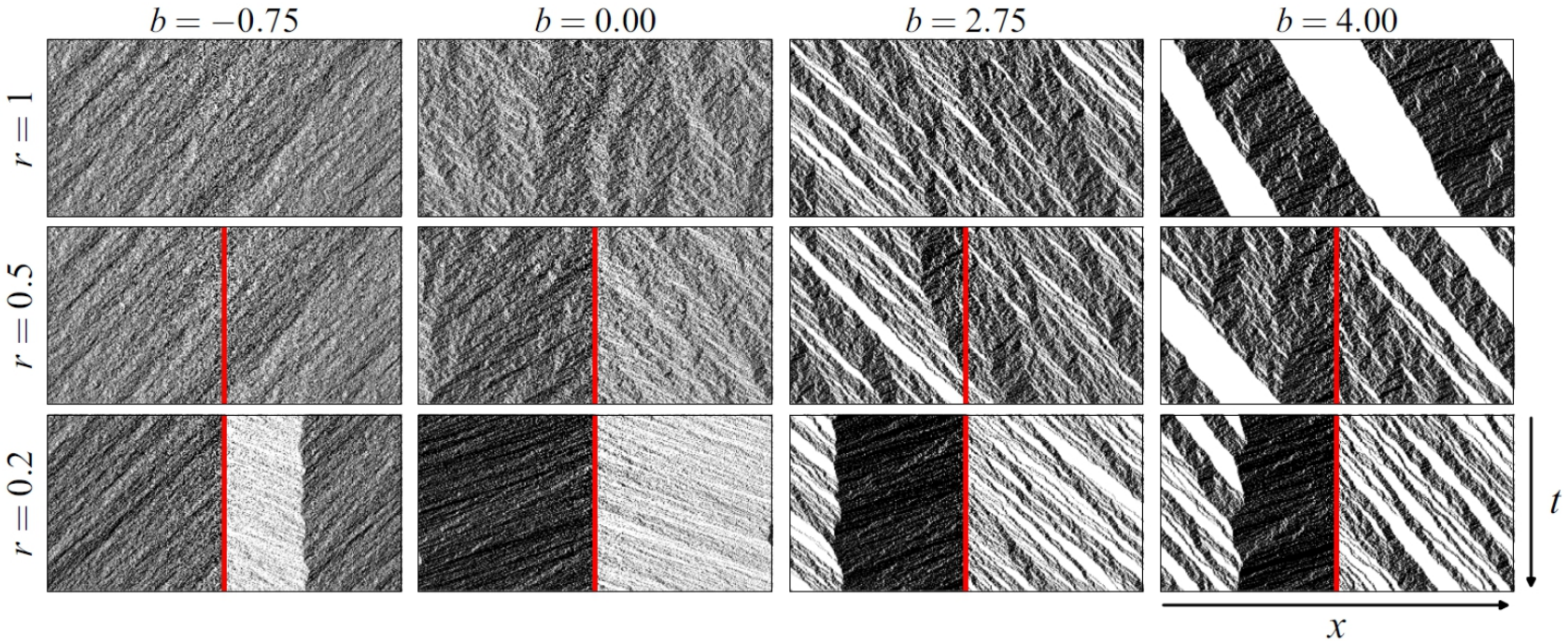
NS: localized SB effect, single density, **S**: macroscopic density separation,

C: full hole condensation, SB irrelevant, **PC**: jamming & PC coexist.

Main physical message

Interactions can render SB effectively irrelevant in the thermodynamic limit.

Spatiotemporal mechanism: **Queue** vs. **Condensate**

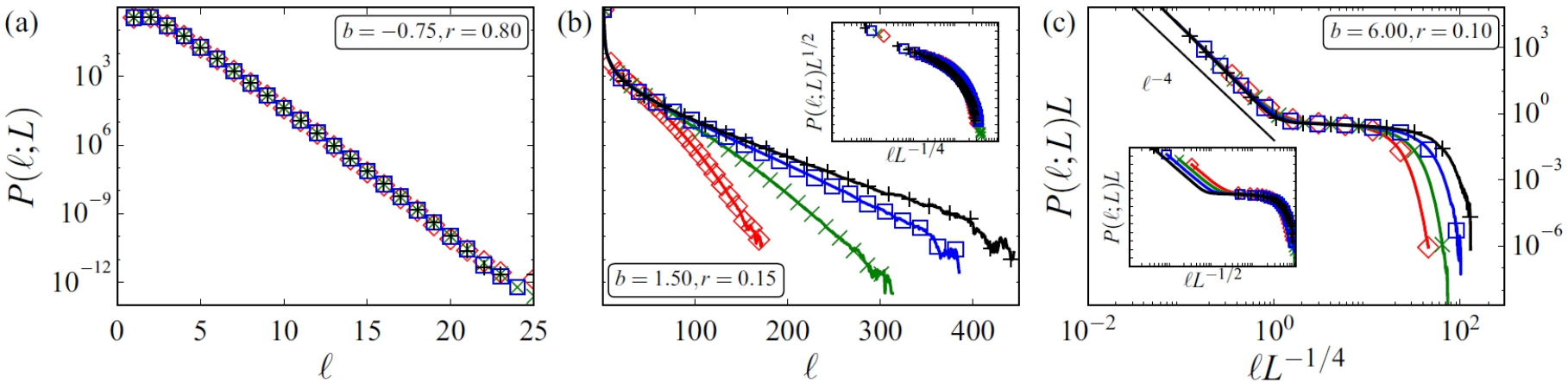


Spatiotemporal patterns: particles, SB (red) line, and interaction-controlled clustering.
bright (white)/dark (black) domains: density-separated flow;
attractive interaction: promotes long gaps/hole clusters;
strong SB: stabilizes a jam behind the defect.

Competition of two organizing fields

Density separation & inter-particle distance (condensate) statistics.

Scaling of inter-particle distances (condensates)

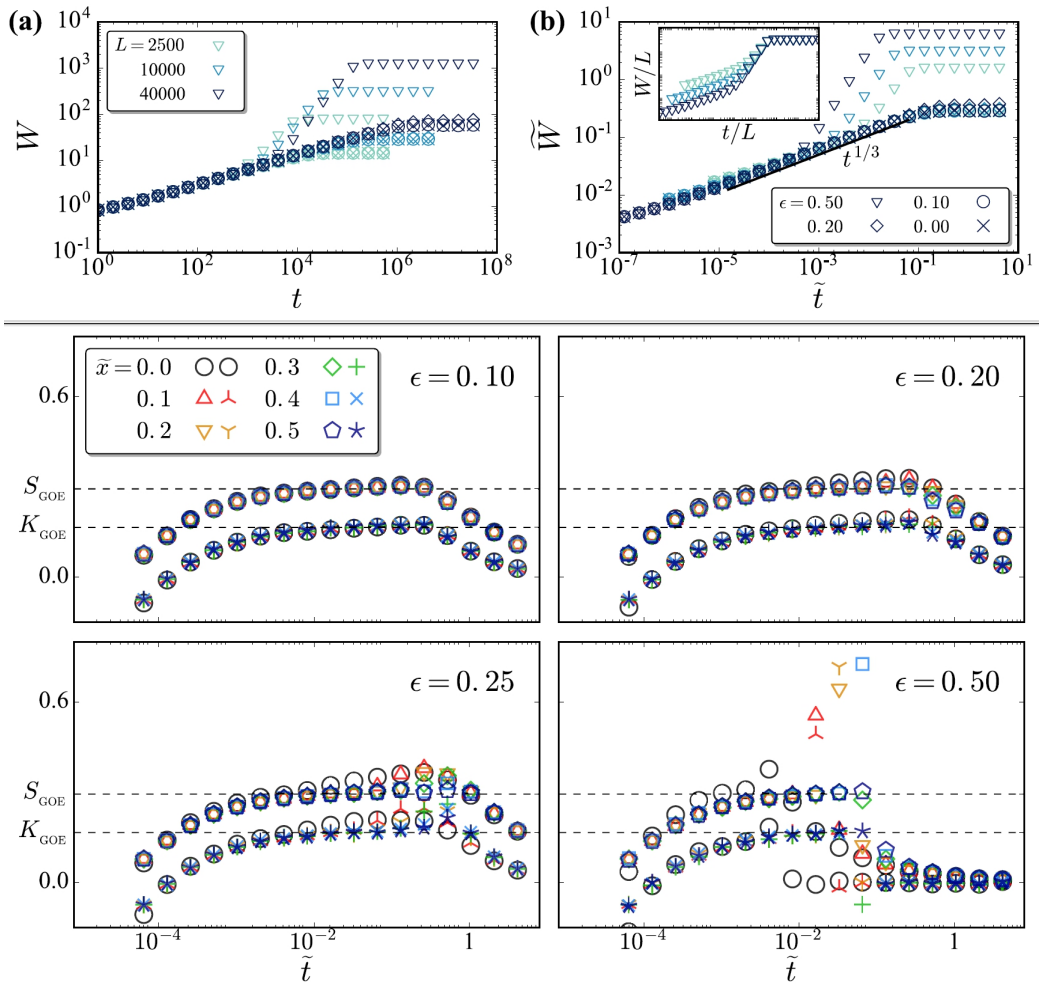


Inter-particle distance distributions in **NS**, **S**, and **PC** phases.

Representative scaling forms

- **S** phase: finite-size-controlled gap scale as $P(\ell; L)L^{1/2} = f_{\mathrm{S}}(\ell/L^{1/4})$.
- **PC** phase: additional condensate scale $\ell_{\mathrm{PC}} \sim L^{1/2}$ as $P(\ell; L)L = f_{\mathrm{PC}}(\ell/L^{1/4}, \ell/L^{1/2})$.

KPZ statistics with a defect: **Transient surface probe**



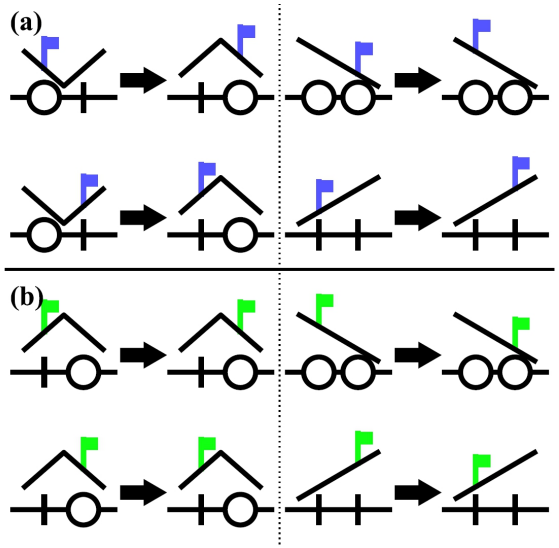
Surface width $W(L,t) \sim L^{1/2}\mathcal{W}(t/L^{3/2})$ and cumulant diagnostics (Skewness S and Kurtosis K) in BCSOS/KPZ representation:
$S_{\mathrm{GOE}} = 0.29346452408$ & $K_{\mathrm{GOE}} = 0.1652429384$.

What the transient tells us

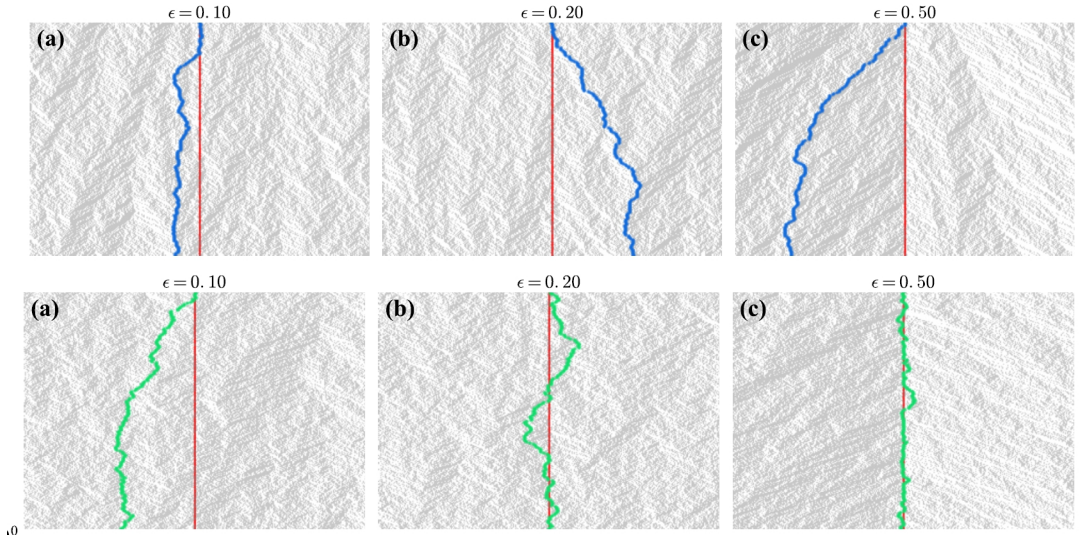
- Without a strong SB, fluctuations approach flat-KPZ/GOE[a] Tracy-Widom statistics as $\tilde{W} \equiv WL^{-1/2}$ against $\tilde{t} \equiv tL^{-3/2}$.
- **A strong SB produces a propagating faceting shock.**
- $S(x,t)$ and $K(x,t)$ locate where local KPZ statistics are already distorted.

[a] GOE = Gaussian Orthogonal Ensemble

Passive tracer[2] as probe of deformed KPZ dynamics



Possible dynamics of tracers:
(a) positive & (b) negative



Advection (blue, upper) vs. Anti-Advection (green, lower) away from SB (red) in strong SB regime.

Positive tracer: $t_c \sim v_g^{-1} \sim \delta J^{-1/2} \sim e^{b/2\varepsilon}$.

- essentially a second-class particle-like probe;
- super-diffusive without a strong SB:
  $\sigma_p(t) \sim t^{1/z_p}$, where $z_p = z_{\mathrm{KPZ}} = 3/2$ (weak SB) [cf. 1 (strong SB)]
- finite drift emerges from the SB-induced density slope.

[2] H. Soh and M. Ha, J. Stat. Mech. (2019) 094009.

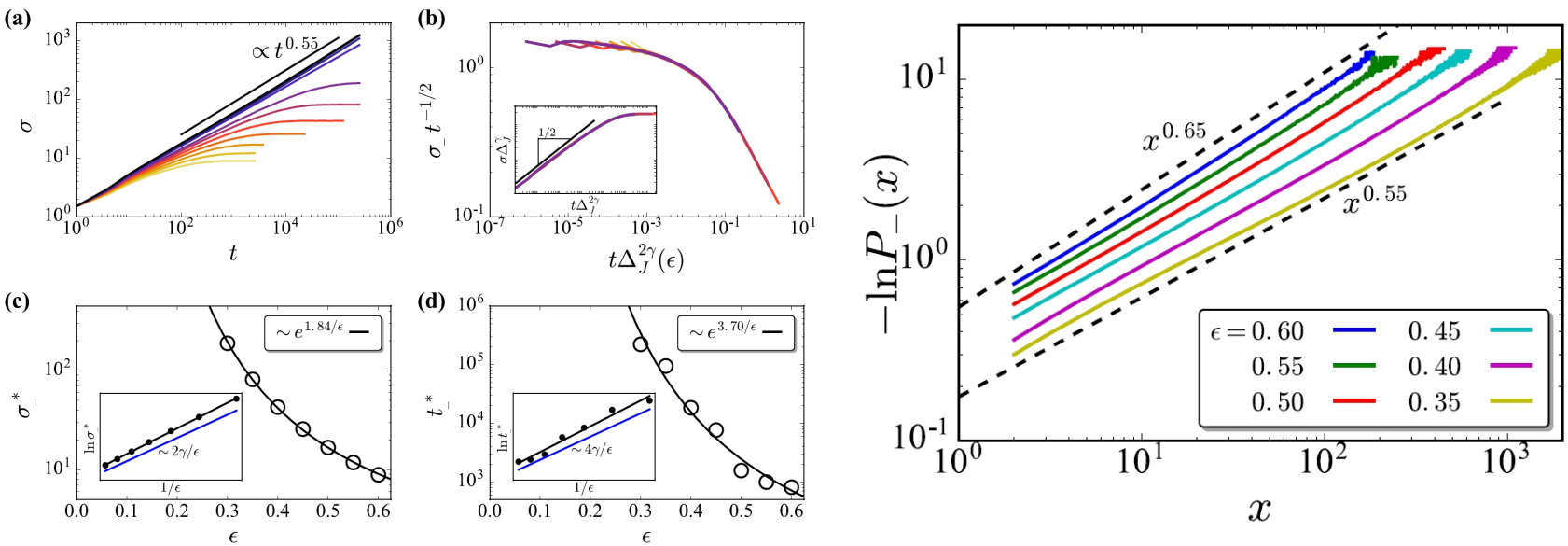
Negative tracer:

$$\frac{dx(t)}{dt} = -[v(x;\varepsilon)]_x + \zeta(t),\ \partial_t P_-(x,t) = K\partial_x^2 P_-(x,t)(x,t) + v_g \mathrm{sgn}(x)\partial_x P_-(x,t)$$

Localization & fractional Fokker-Planck picture

Phenomenology

$$\partial_t P_- = \partial_x \left[K D_t^{1-2/z_m} \partial_x + v_g \,\mathrm{sgn}(x) \right] P_- .$$



Anti-advection: Anomalous diffusion in weak SB regime;

Saturation trapped by a convergent density field in strong SB regime; Crossover scaling close to stretched exponential as $P_-(x) \sim \exp[-\Delta_b |x|^\gamma]$, where $\gamma = z_m - 1$ (cf. $z_{\mathrm{KPZ}} = 1.5$).

SB (localized quenched defect) is a diagnostic, not just a perturbation

SB in TASEP

- marginally relevant;
- current suppression is extremely weak for small ε;
- finite-size crossover is unavoidable.

Interacting/tracer-augmented flow

- interaction competes with jamming;
- condensation creates new length scales;
- tracer dynamics reveals deformation of KPZ fluctuations.

Central message from passive tracer dynamics

The local defect acts as a controlled probe of how KPZ universality is reshaped by conservation, interaction, and emergent inhomogeneity.

Mathematical Derivation: Current (J) to Shape Constant (f)

In TASEP, Current J measures particle flux per unit time. In LPP, the hydrodynamic limit establishes a deterministic **Shape Constant** (f) via the Law of Large Numbers.

Step-by-Step Derivation

Let $T(L, L)$ be the last passage time from $(0, 0)$ to (L, L) on the $\mathbb{Z}^2$ grid.

$$\text{Shape Constant Limit: } \lim_{L\to\infty} \frac{T(L,L)}{L} = f(\chi)$$

$$\text{Physical Time Mapping: Total Time } t \approx L \cdot f(\chi)$$

$$\text{Current : } J \equiv \frac{\text{of successful jumps } (L)}{\text{Total time } (t)} = \frac{L}{L \cdot f(\chi)} = \frac{1}{f(\chi)}$$

Pure TASEP Duality

For a homogeneous system with density ρ; $J = \rho(1-\rho)$. Thus, the LPP time constant directly evaluates to the inverse of the particle flux: $f = \frac{1}{\rho(1-\rho)}$.

Mathematical Derivation cont'd: Density (ρ) to Geodesic Slope (j/i)

The spatial coordinate x and local particle density $\rho(x)$ map directly onto the **geometric slope** of the optimal path (Geodesic) in the LPP lattice plane.

Slope Formula

Let i be the particle index (horizontal) and j be the jump counter (vertical).

Spatial growth slope:

$$\tan\theta = \frac{j}{i} = \left(\frac{\rho}{1-\rho}\right)^2$$

Solving directly for the particle density ρ:

$$\rho = \frac{\sqrt{j/i}}{1+\sqrt{j/i}}$$

Physical Interpretations

- **Maximal-Current Fluctuation ($\rho = 0.5$):** Slope $j/i = 1$ (MC)
  Geodesic traces cleanly along the main diagonal.
- **High-Density Jam ($\rho > 0.5$):**
  Slope $j/i > 1$ (HD)
  Path bends vertically, mapping onto particle congestion.
- **Low-Density Void ($\rho < 0.5$):**
  Slope $j/i < 1$ (LD)
  Path shifts horizontally, indicating empty gaps (hole condensation).

Finite-Size Crossover Scaling Derivation: $L_\times \sim \varepsilon^{-3/2}$

Weak SB regime: persists as long as the deterministic energy gain (time) from the defect line is masked by the intrinsic background KPZ noise.

Energy Balance Principle

Let L be the system size (acting as the time horizon in LPP models) and evaluate two competing scales:

$$\text{Background Noise Fluctuation: } \Delta F_{\text{pure}} \sim L^{\beta_{\text{KPZ}}} = L^{1/3}$$

$$\text{Defect Perturbation Energy: } \Delta F_{\text{defect}} \sim \varepsilon \cdot L \quad \text{(linear accumulation)}$$

The critical crossover scale $L_\times$ marks the boundary, where these competing forces balance out $\Delta F_{\text{pure}} \sim \Delta F_{\text{defect}} \implies L_\times^{1/3} \sim \varepsilon \cdot L_\times$.

Scaling Connection

Solving for the crossover length $L_\times$ explicitly yields: $\varepsilon \sim L_\times^{-2/3}$.
$\implies L_\times \sim \varepsilon^{-3/2}$, where the exponent $-2/3$ is exactly $-1/z_{\text{KPZ}}$.

Take-home messages of this talk

- **Debate Settled:**
  SB critical threshold $\epsilon_c = 0$ as $L \to \infty$; perceived stability of weak SB regime is entirely a finite-size crossover artifact. SB relevance in TASEP is masked by a logarithmic; infinite-order crossover scaling:

$$\varepsilon_\times = L_\times^{-1/z_{\mathrm{KPZ}}} \implies (\delta J) L^{1/3} \sim \varepsilon L^{2/3}.$$

- **Interplay Mechanisms:**
  Particle-hole interaction parameter b breaks the traditional particle-hole symmetry, allowing condensation to insulate systems against localized jamming. ZRP-type interactions with a slow particle produce a genuine competition between jamming and condensation.

- **Anomalous Profiling:**
  Saturated passive tracer measures provide direct, sensitive mathematical fingerprints of localized defect perturbations to deform KPZ dynamics.



Thank you for listening to my talk.

Remaining issues & questions



- **Scaling theory for the interaction-induced SB-free regime**:
  Can the NS–S boundary be described beyond mean-field and finite-size collapse?
- **Universality of Partial Condensation**:
  Is $\ell_{\mathrm{PC}} \sim L^{1/2}$ robust under open boundaries, stochastic rates, or multiple defects?
- **Tracer statistics as a fixed-point observable**:
  Can passive-tracer distributions be derived from KPZ fixed point, or directed landscape objects with localized perturbations?