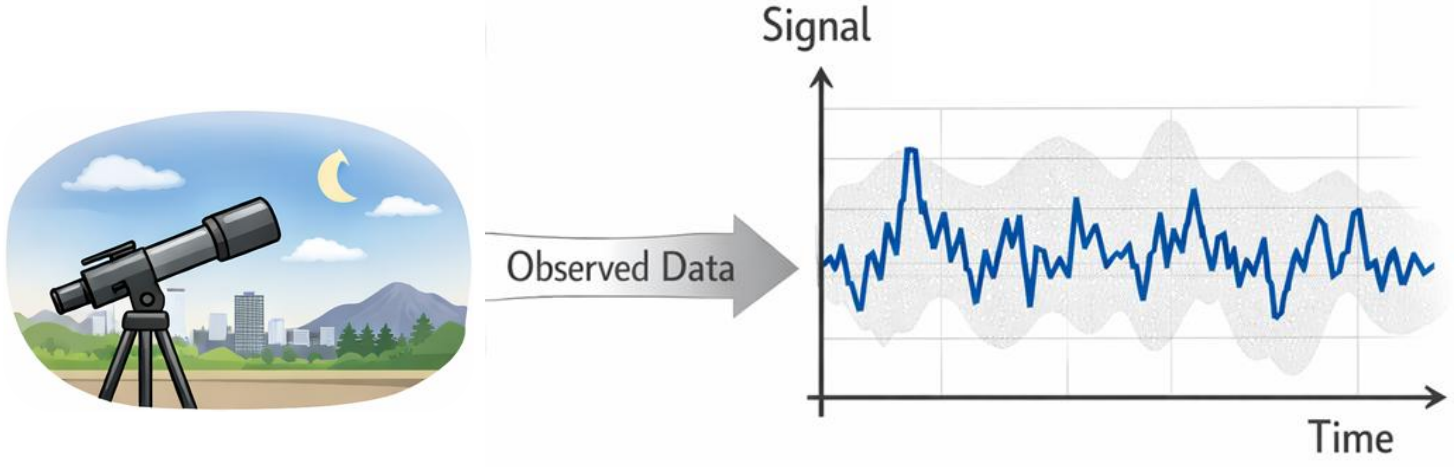
Distinguishing Chaos from Noise through Cross Prediction with Reservoir Computing

Jong-Min Park
Asia Pacific Center for Theoretical Physics

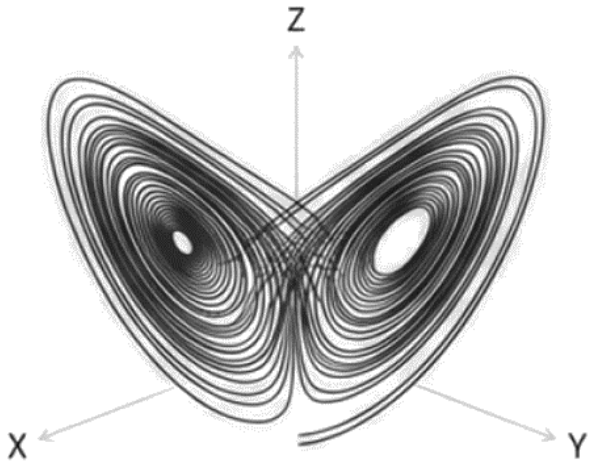
Jaesung Choi (CAINS, KIAS)
Athokpam Langlen Chanu (APCTP)

NSPCS2026 @ KIAS, 21 Jul. 2026



Where does **irregularity** in empirical time series come from?

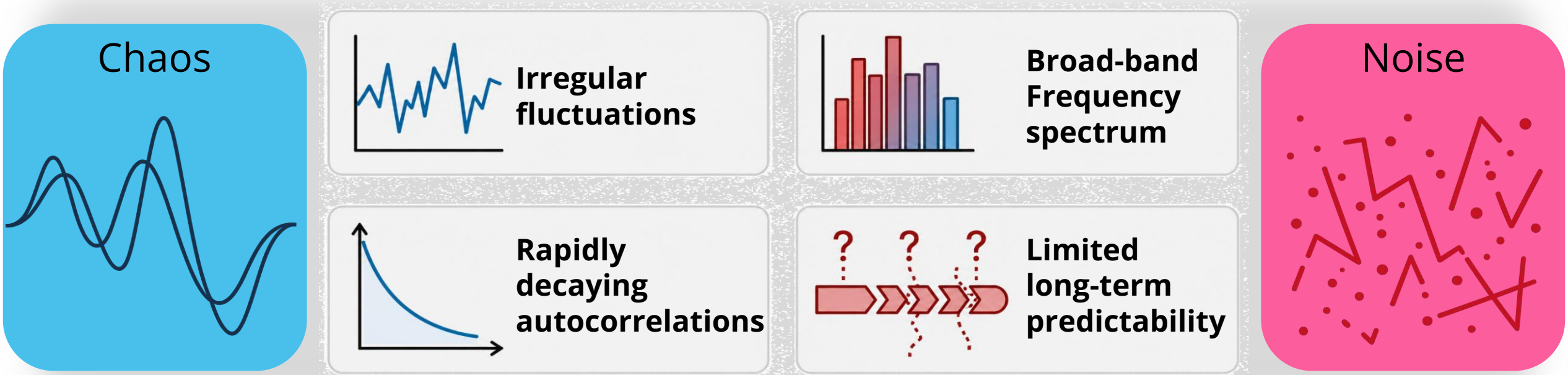
Chaos



Noise



Chaos and noise are often difficult to distinguish due to their **similar behavior**.



Chaos Signature Detection

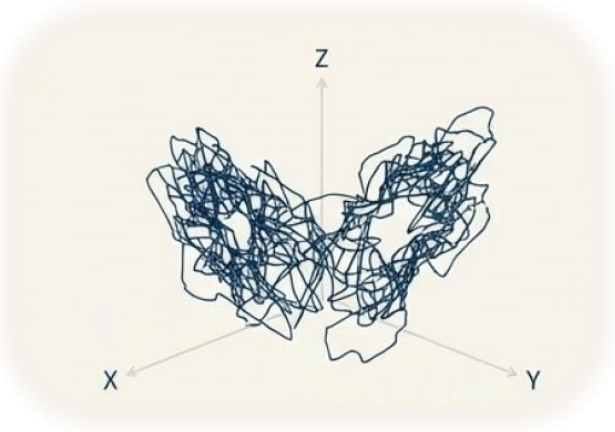
Lyapunov exponent

[Osborne *et al.*, Physica D (1989)]
[J. Theiler, Physics Letters A (1991)]

correlation dimension

[Tanaka *et al.*, Phys. Rev. E (1996)]
[Ikeguchi *et al.*, Int. J. Bifurc. Chaos (1997)]

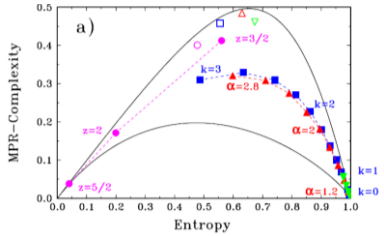
False positive



Ordinal pattern Analysis

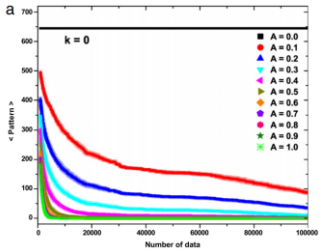
Entropy-complexity plain

[Rosso *et al.*, Phys. Rev. Lett. (2007)]



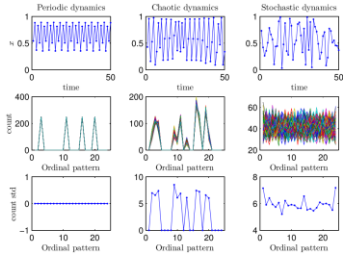
Forbidden pattern

[Rosso *et al.*, Physica (2012)]



Permutation spectrum

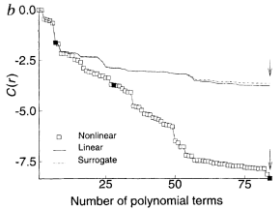
[Kulp *et al.*, Chaos (2014)]



Others

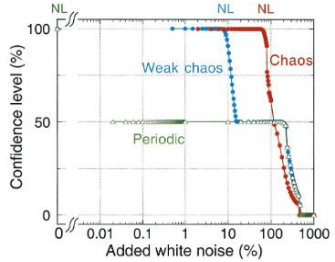
Nonlinear prediction test

[Barahona *et al.*, Nature (1996)]



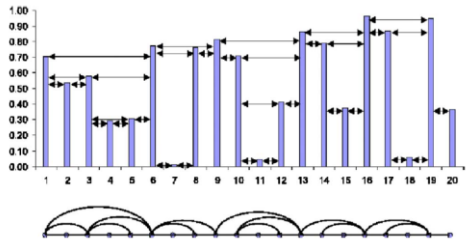
Noise titration

[Poon *et al.*, PNAS (2001)]



Visibility graph

[Lacasa *et al.*, Phys. Rev. E (2010)]

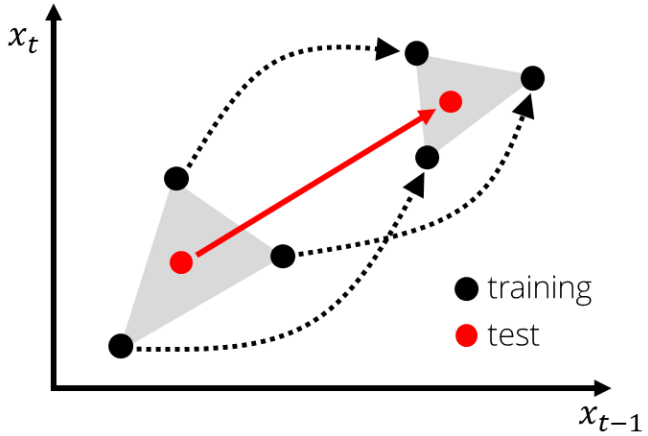


Short-term predictability

Nonlinear forecasting

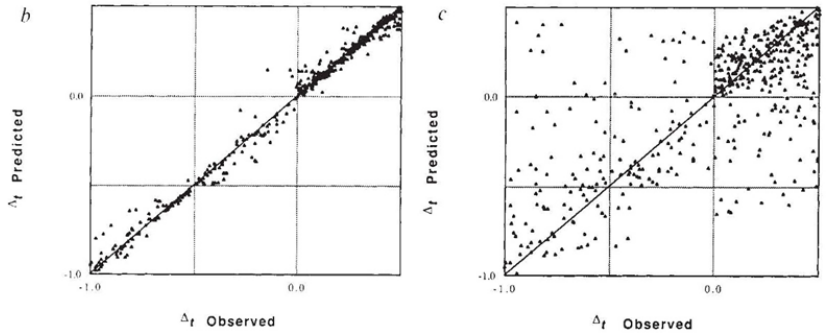
[Suglihara and May, Nature (1990)]

Simplex projection



Chaos

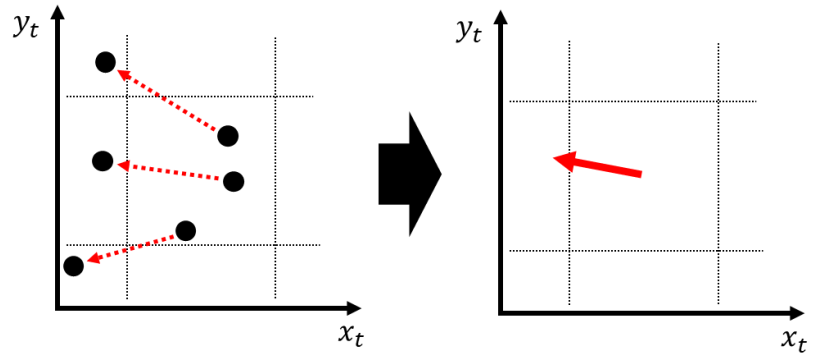
Noise



Deterministic dynamical flow

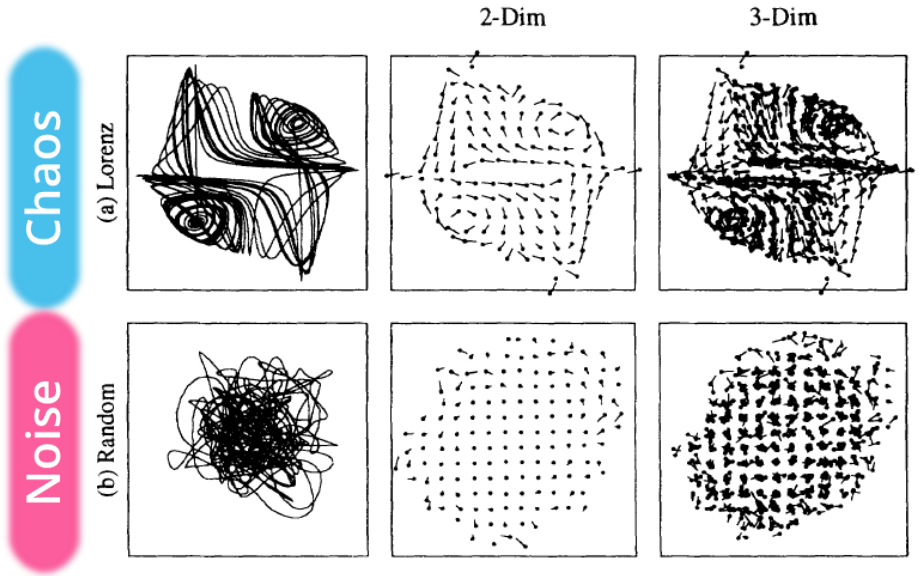
Text for determinism

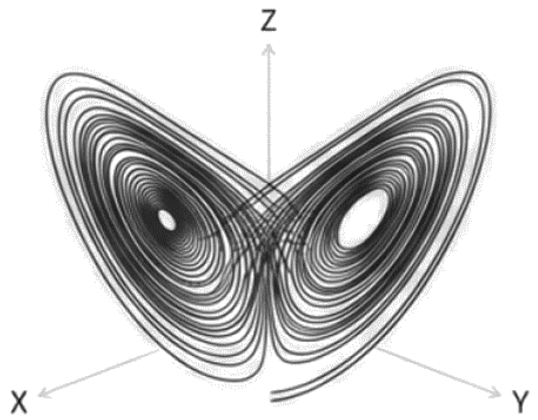
[Kaplan and Glass, Phys. Rev. Lett. (1992)]

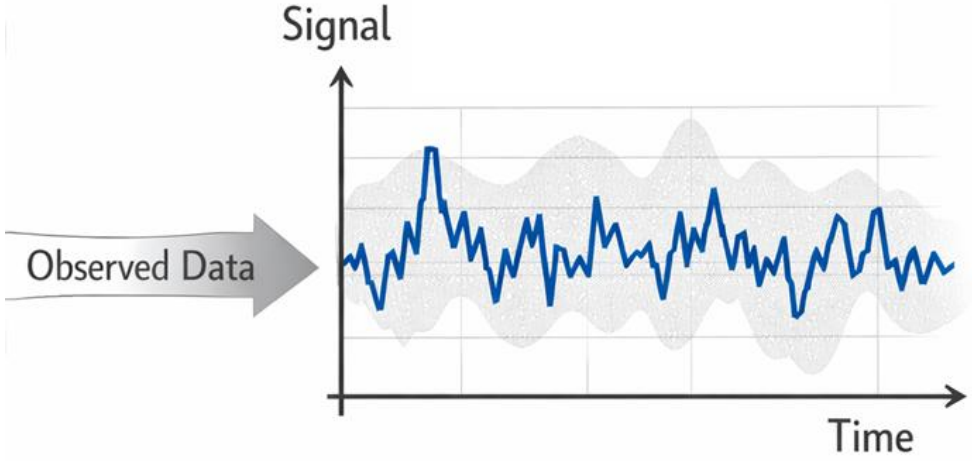


Chaos

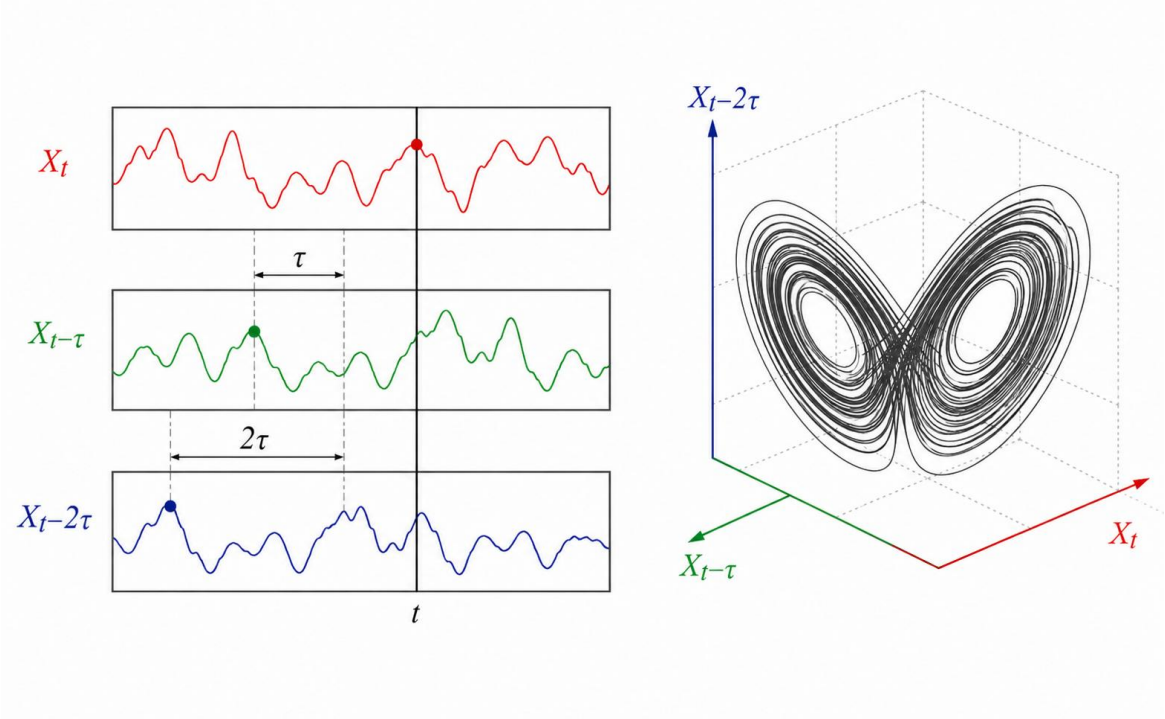
Noise

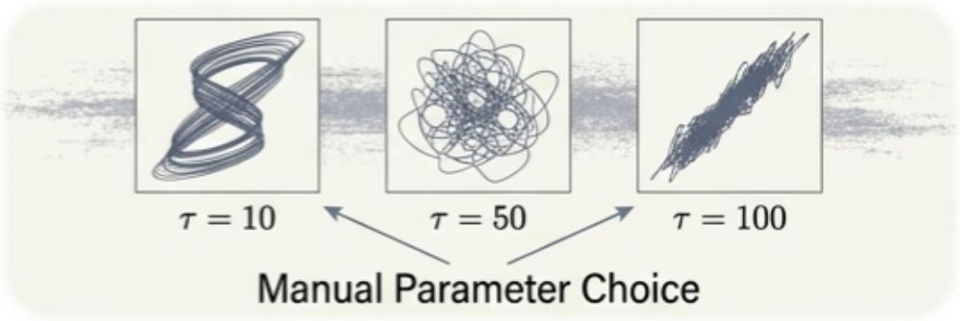






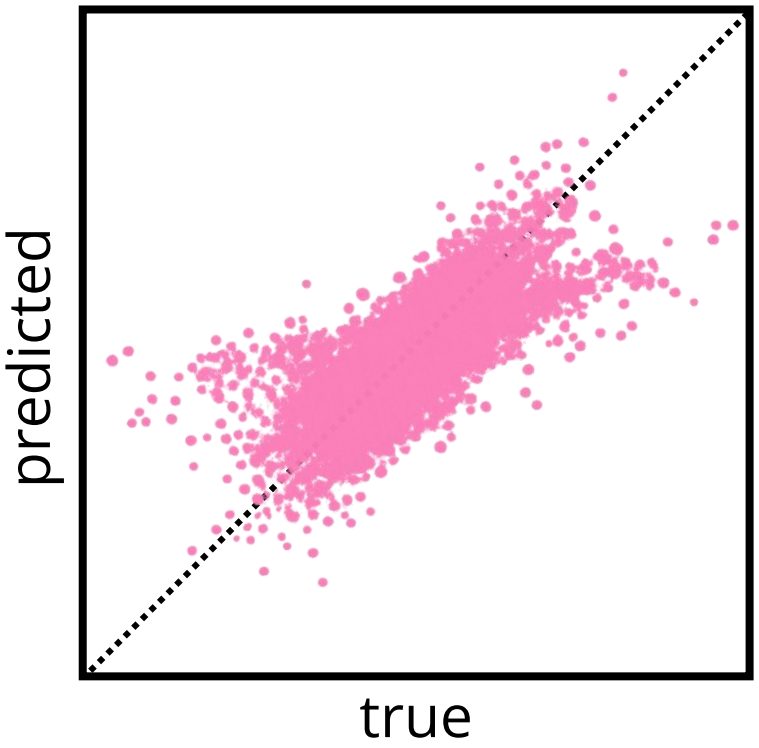
If only a scalar time series is available, we need to reconstruct an embedding space.



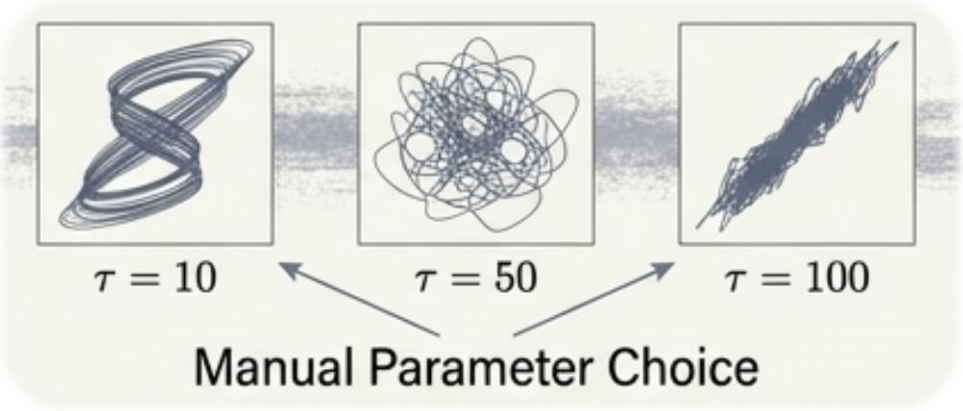




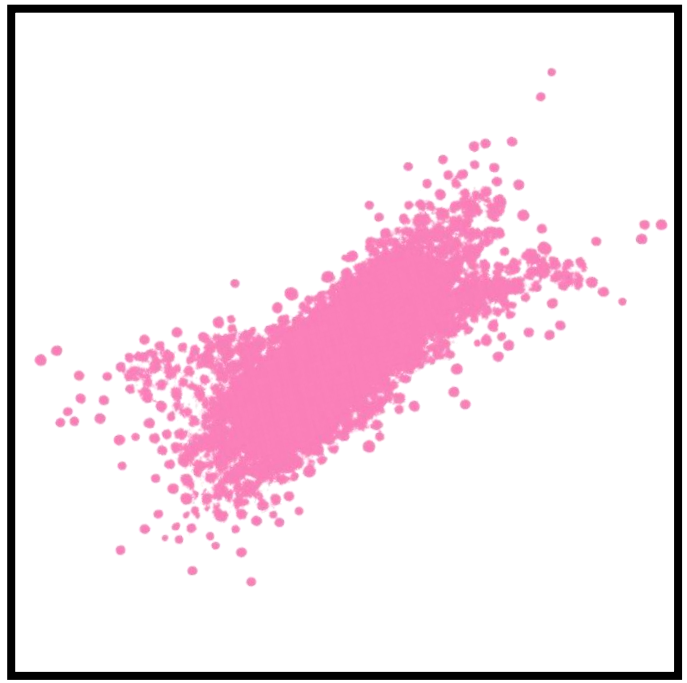
Strong temporal correlation in noise can arise a spurious predictability.



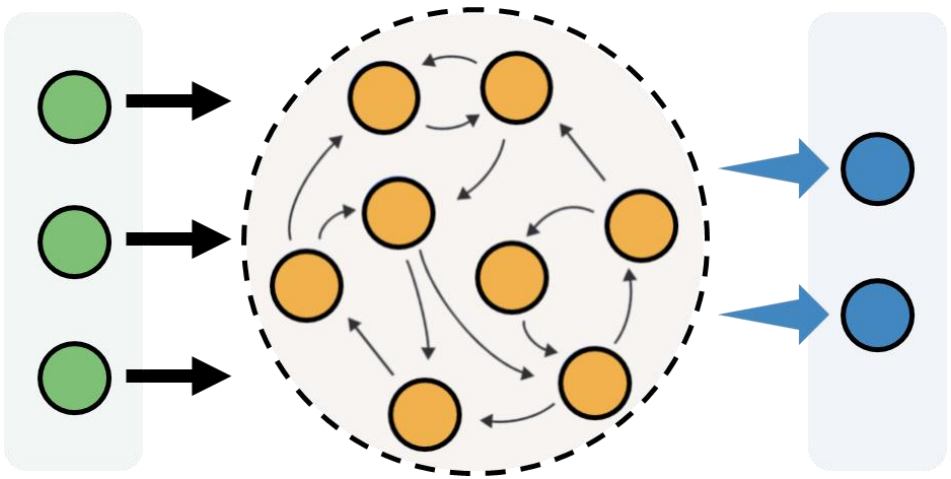
⚠ Necessity of embedding parameters



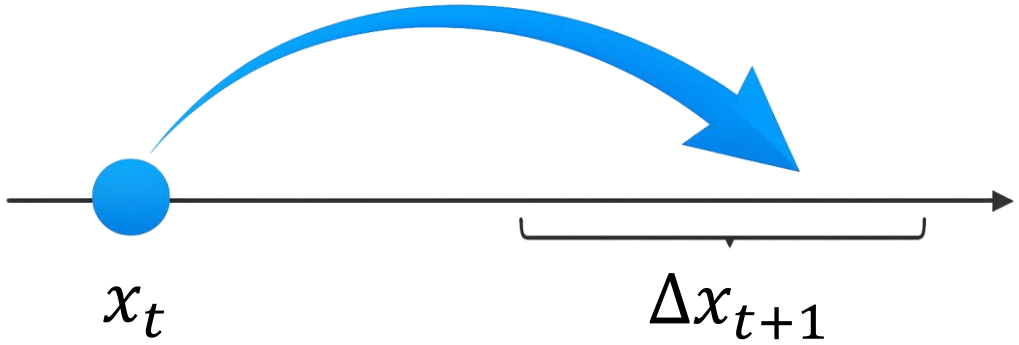
⚠ Correlation induced predictability



Reservoir computing



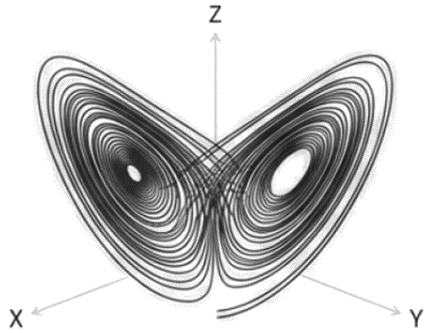
Cross prediction

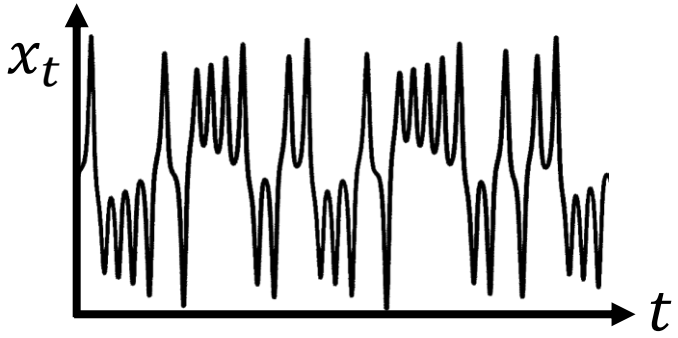


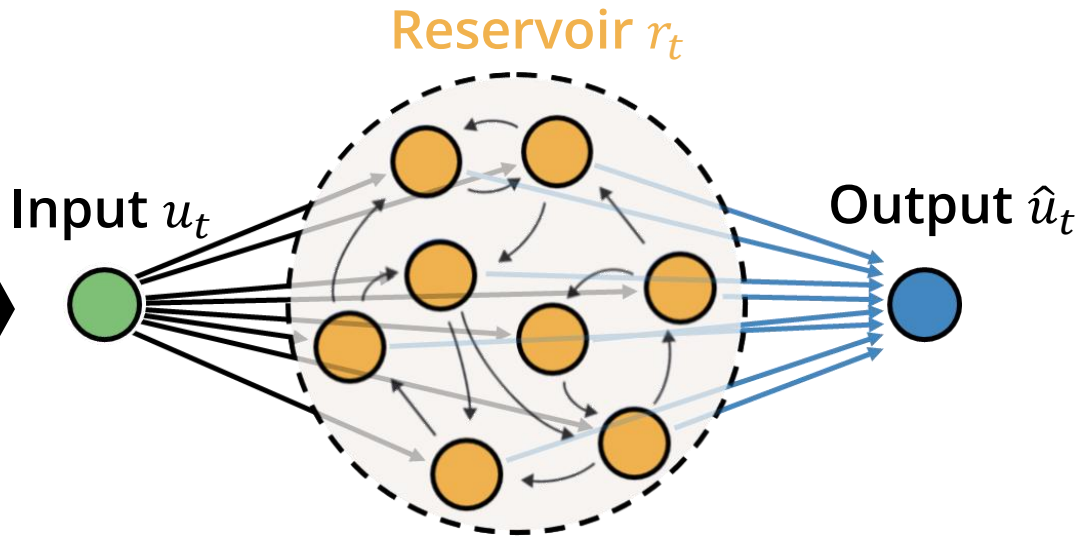
⚠ Necessity of embedding parameters

⚠ Correlation induced predictability

Reservoir computing





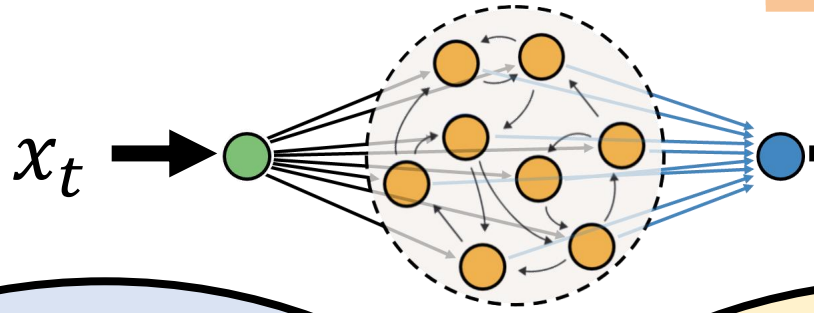


Update rule $\boldsymbol{r}_{t+1} = (1 - a_{\text{leak}})\boldsymbol{r}_t + a_{\text{leak}} \tanh(\mathbf{W}_{\text{res}}\boldsymbol{r}_t + \mathbf{W}_{\text{in}}u_t)$

Loss function $L \equiv \sum_{t=0}^{L-1} (u_{t+1} - \hat{u}_{t+1})^2 , \qquad \hat{u}_{t+1} = \mathbf{W}_{\text{out}}\boldsymbol{r}_{t+1}$

Preprocessing $x_t \mapsto x_t - \langle x_t \rangle \quad x_t \mapsto x_t / \max_t\{|x_t|\}$

Cross prediction

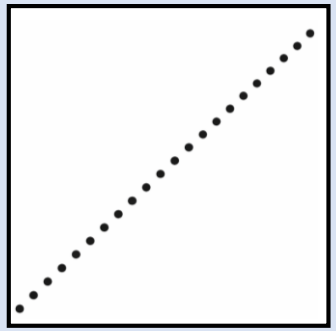


x_t → $\Delta\hat{x}_{t+1} = v(x_{t+1})$

Short-term predictability

$x_t \rightarrow x_{t+1}$

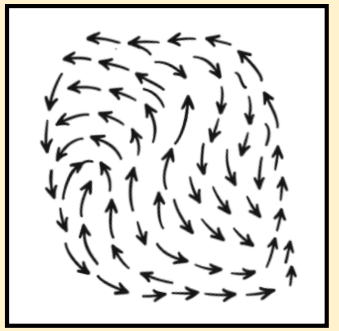
Predict next value



Determinism test

$v(x_t) = \Delta x_t$

Infer deterministic flow



cross-prediction scheme

Lorenz system

$$\dot{x} = \sigma(y - x)$$

$$\dot{y} = x(\rho - z) - y$$

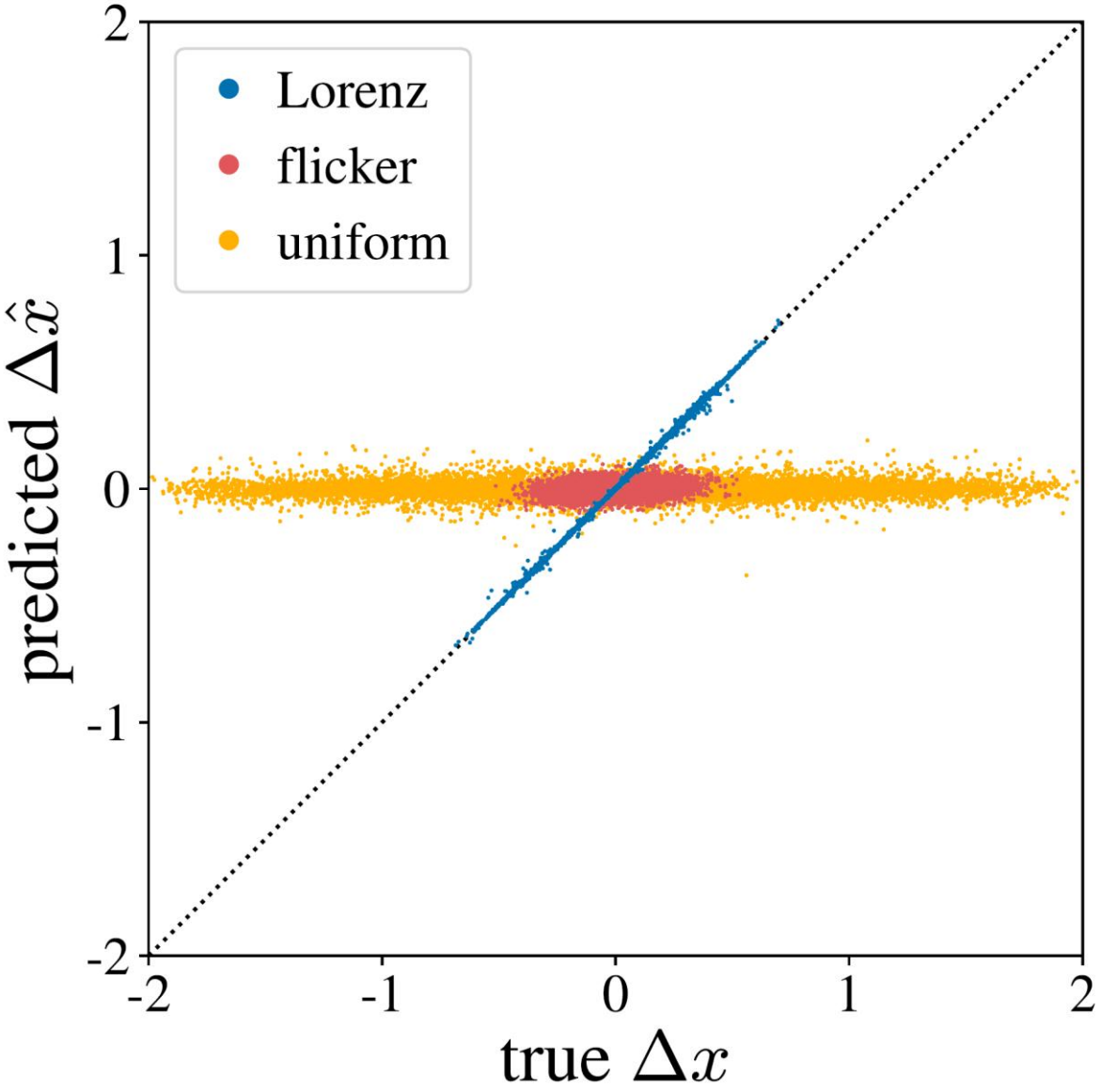
$$\dot{z} = xy - \beta z$$

Stochastic noise

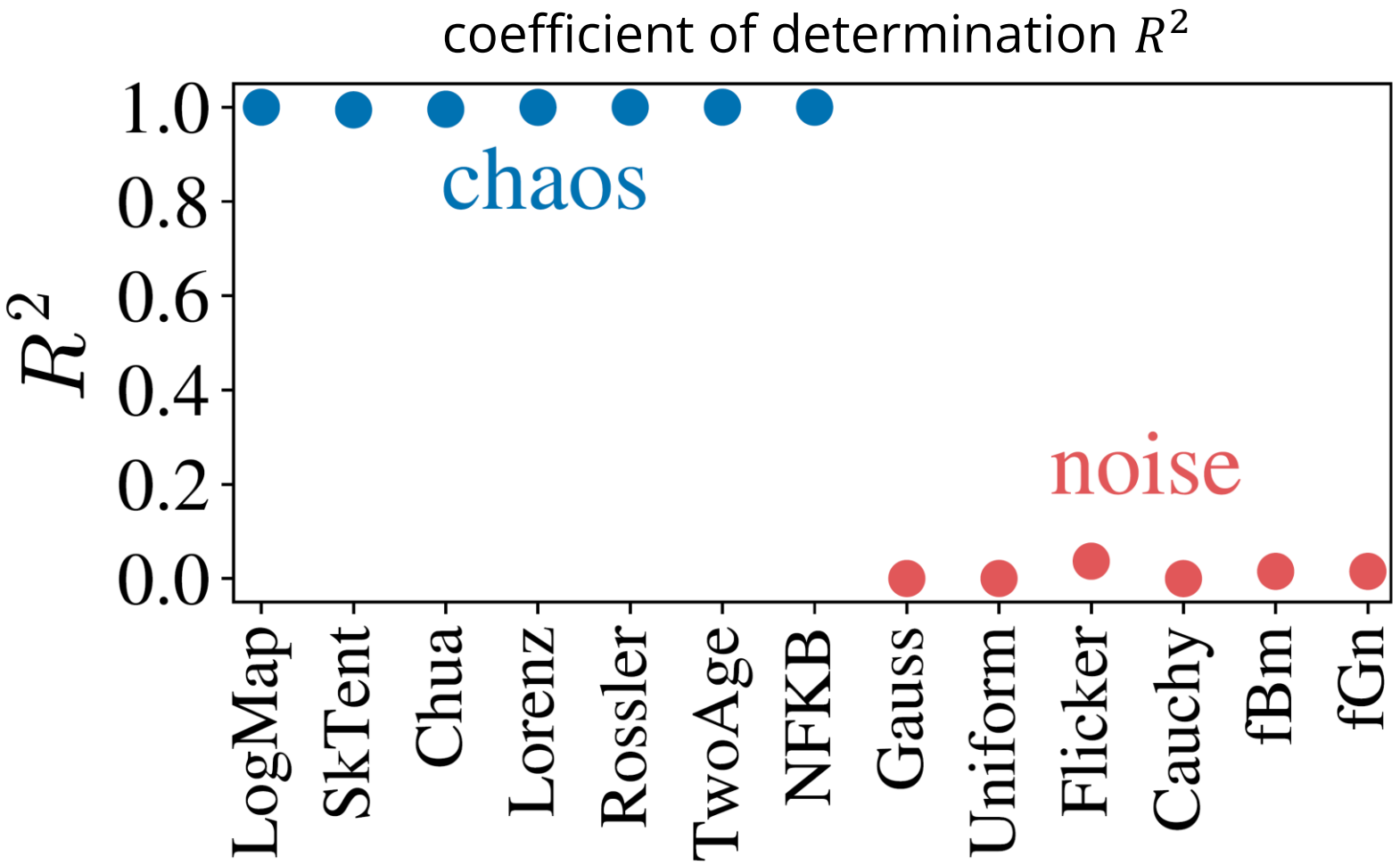
$$x(t) \sim \mathcal{N}$$

flicker noise $S(f) \sim 1/f$

uniform noise $x(t) \in (-1,1)$



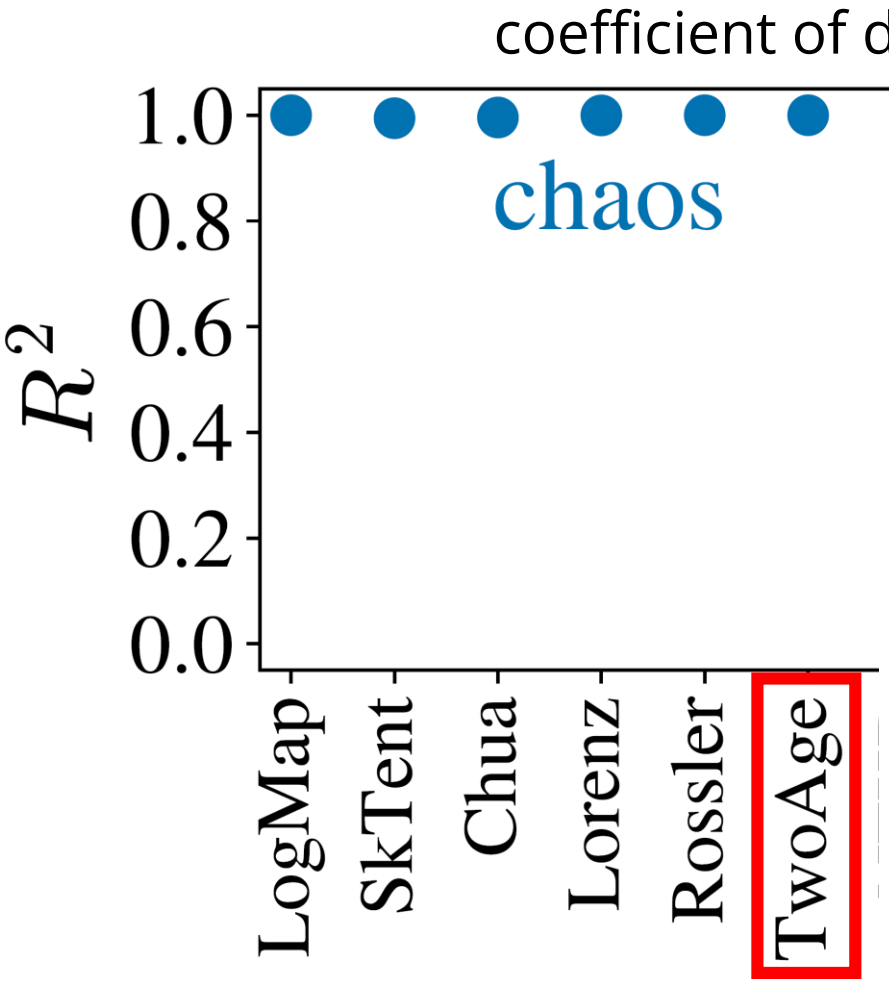
Results for simulated synthetic dynamics

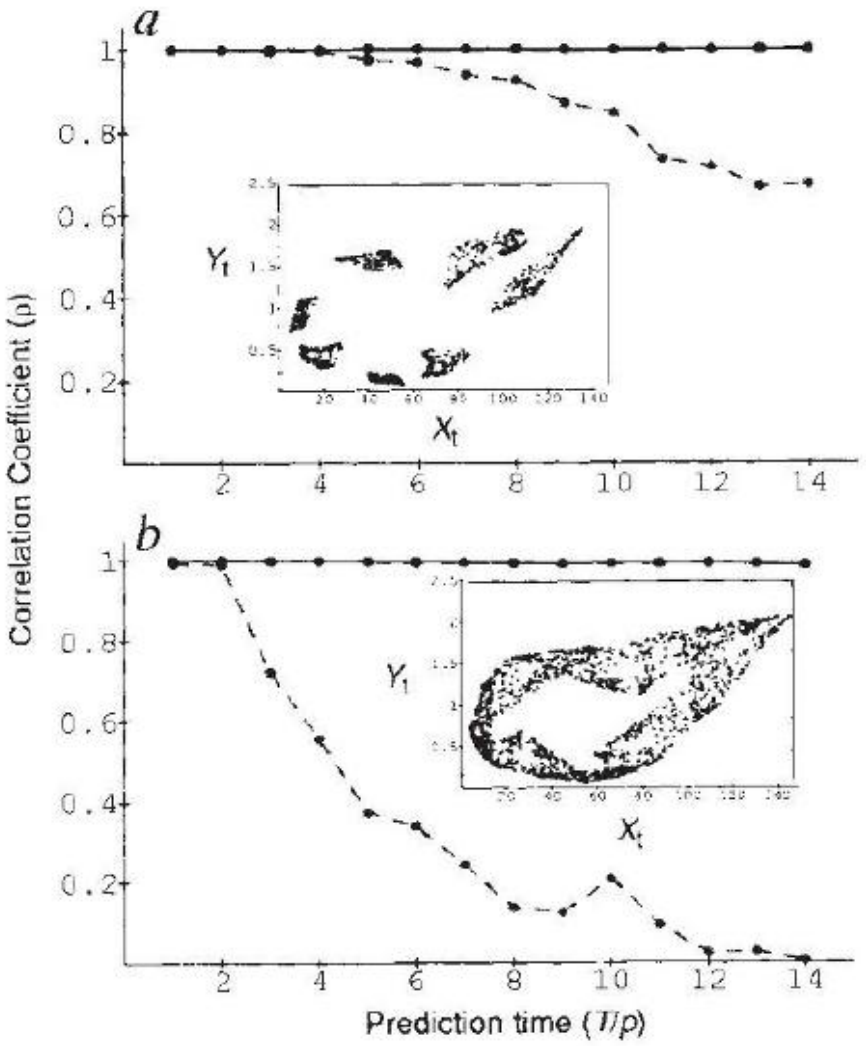


LogMap – logistic map
SkTent – skew tent map
Chua – Chua's circuit
Lorenz – Lorenz system
Rossler – Rossler system
TwoAge – two-age-class model
NFKB – NF-κB model ($d = 6$)

Gauss – white Gaussian noise
Uniform – white uniform noise
Flicker – flicker noise
Cauchy – Cauchy noise
fBm – fractional Brownian motion
fGn – fractional Gaussan noise

Results for simulated synthetic dynamics

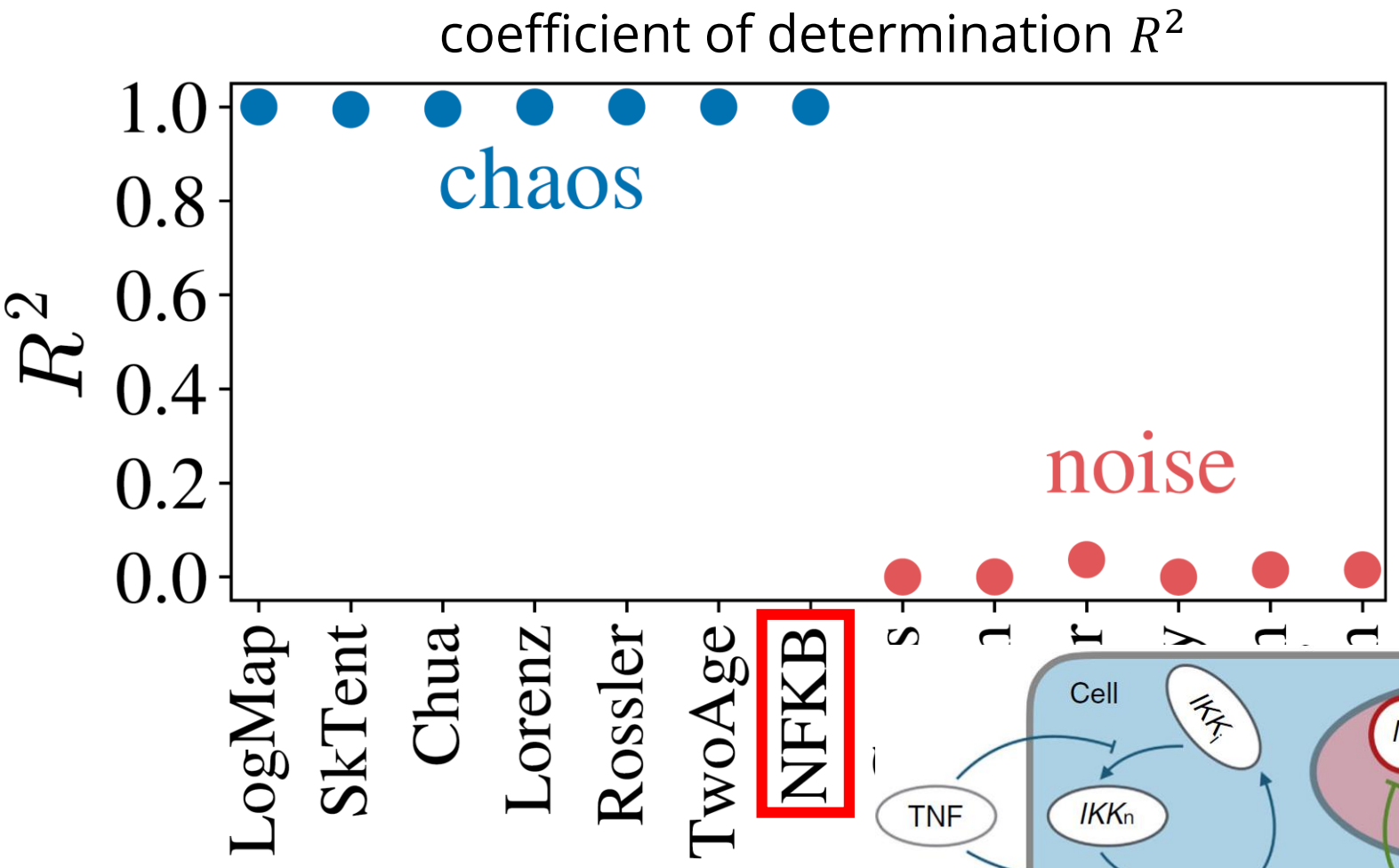




LogMap – logistic map
nt – skew tent map
– Chua's circuit
nz – Lorenz system
'er – Rossler system
lge – two-age-class model
– NF-κB model ($d = 6$)

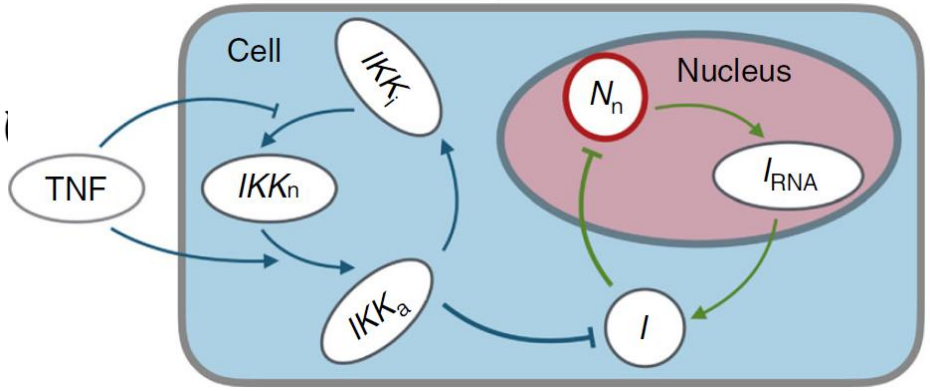
s – white Gaussian noise
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er – flicker noise
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- fractional Brownian motion
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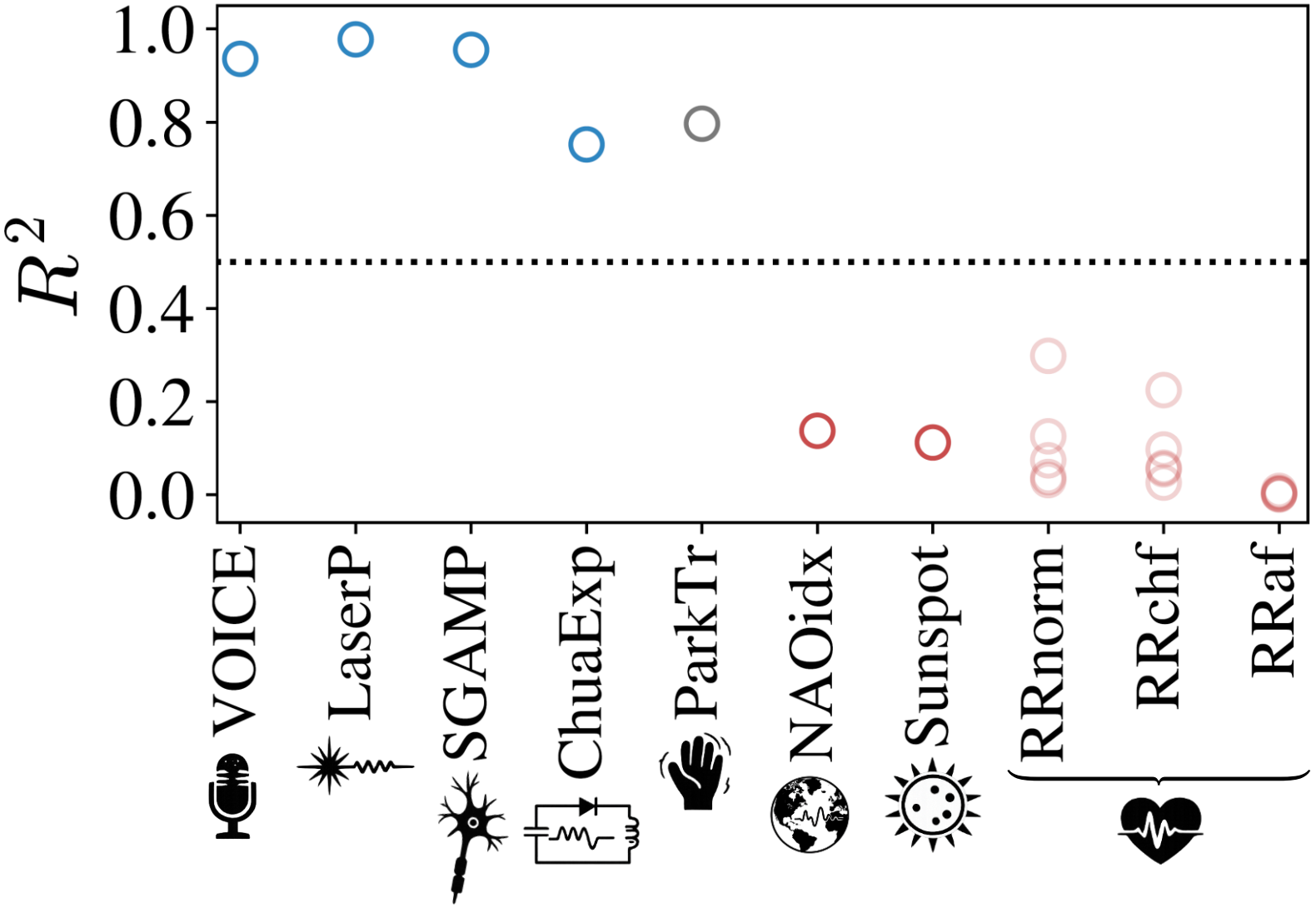


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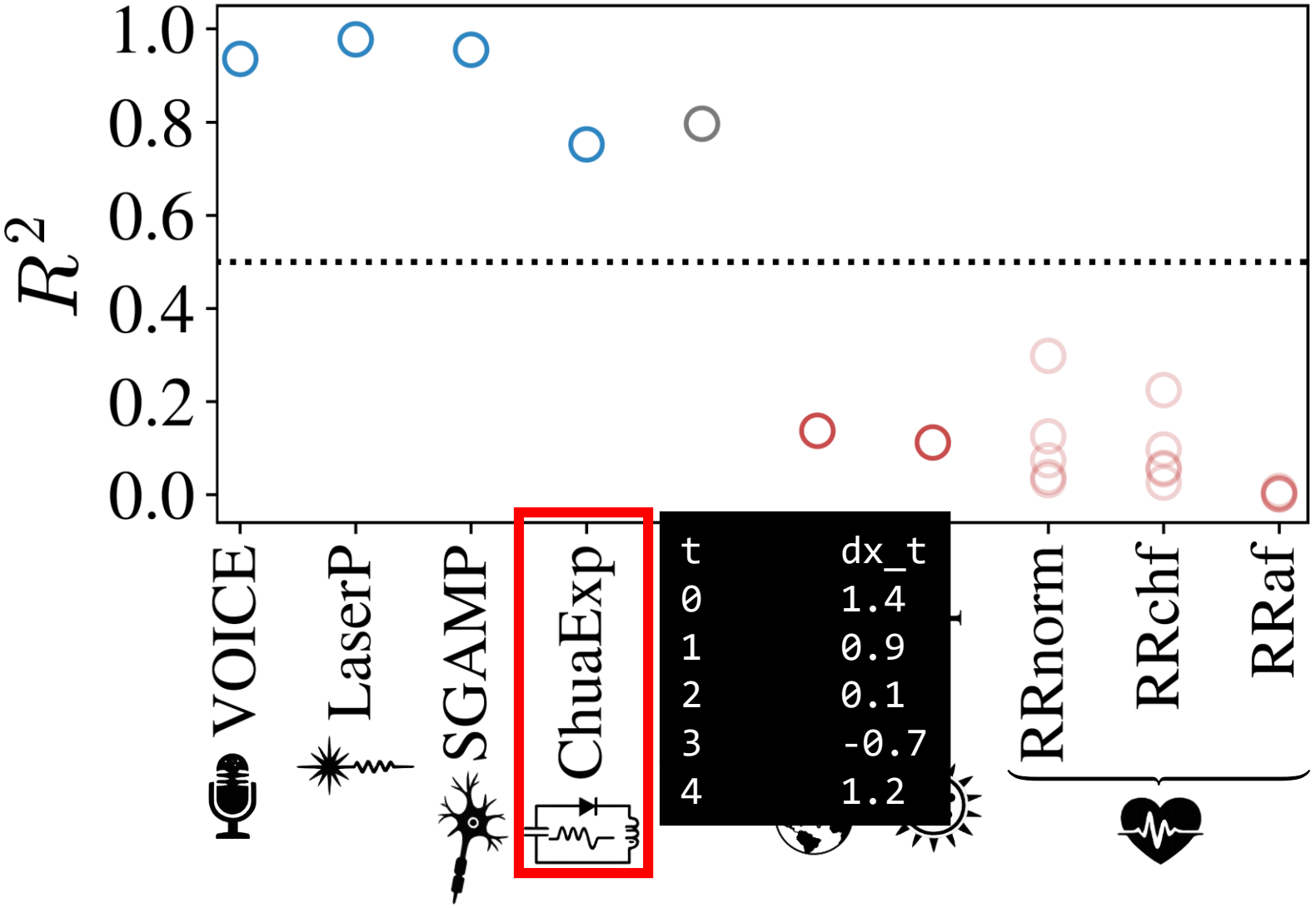
Results for empirical data



VOICE – voice recoding
LaserP – laser intensity pulsing
SGAMP – squid giant axon membrane potential
ChuaExp – Chua's circuit experiment
ParkTr – Parkinsonian tremor
NAOidx – mean North Atlantic Oscillation index
Sunspot – sunspot number

RR-interval data from…
RRnorm – healthy subjects
RRchf – congestive heart failure
RRaf – atrial fibrillation

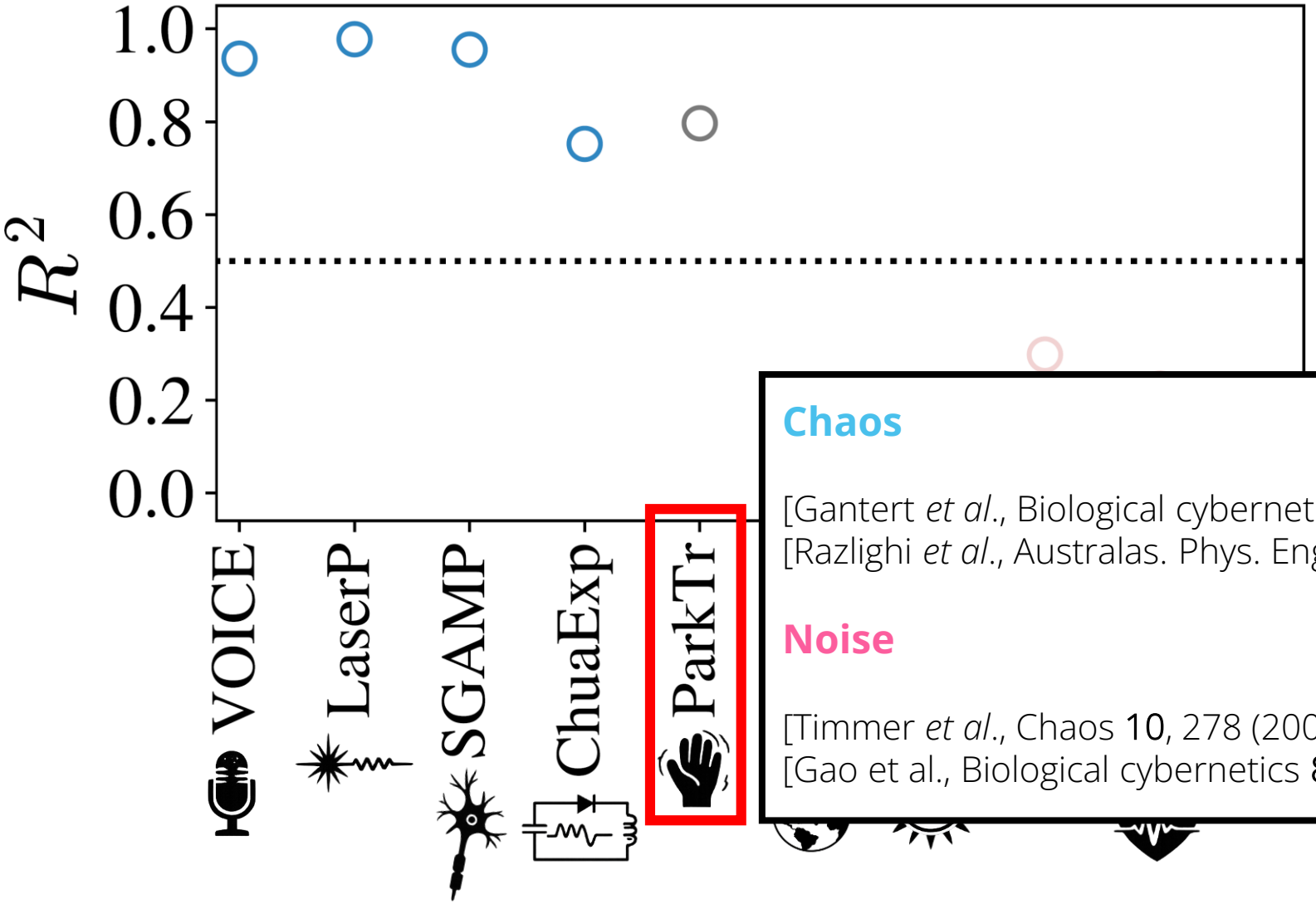
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Results for empirical data



Chaos

[Gantert *et al*., Biological cybernetics **66**, 479 (1992)]
[Razlighi *et al*., Australas. Phys. Eng. Sci. Med. **35**, 25 (2012)]

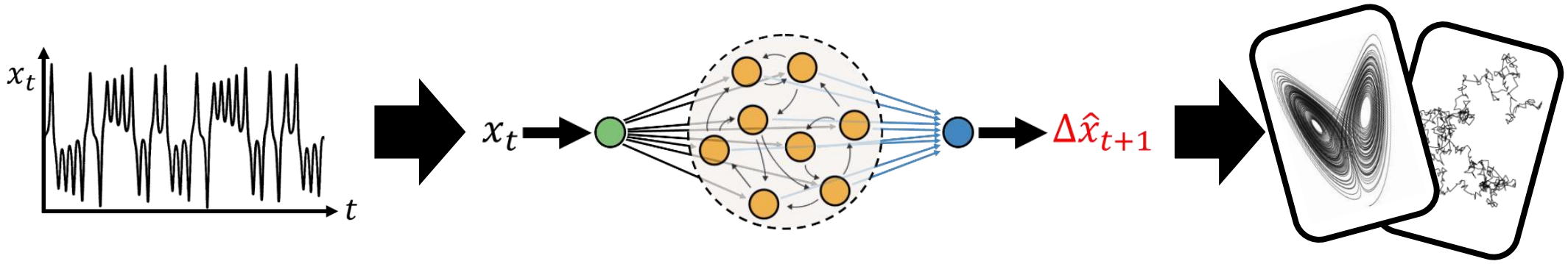
Noise

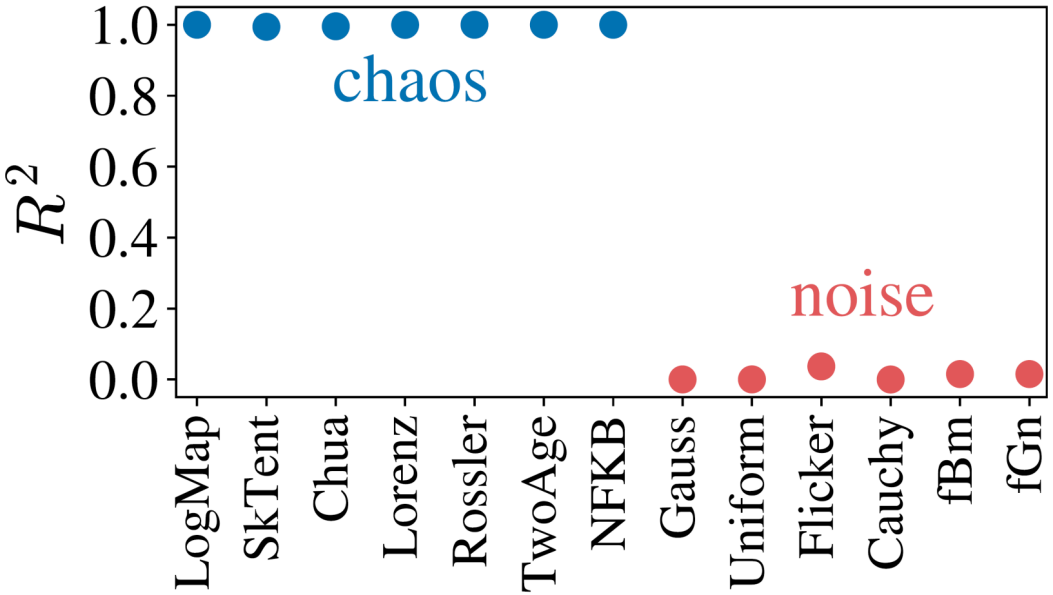
[Timmer *et al*., Chaos **10**, 278 (2000)]
[Gao et al., Biological cybernetics **86**, 263 (2002)]

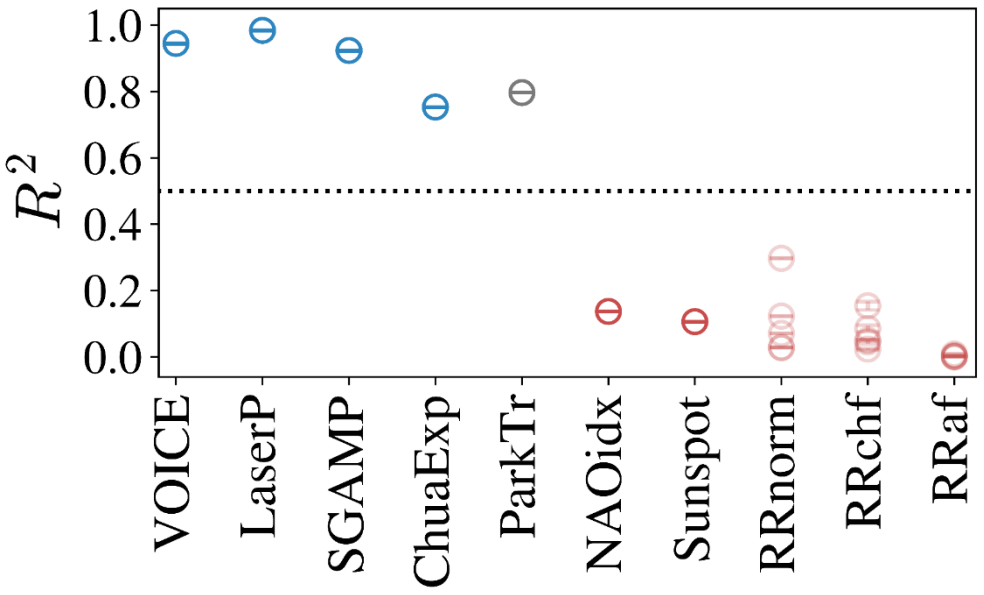
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Gait – ...rom...
Epil – ... subjects
CHF – ...e heart failure
RRaf – atrial fibrillation

Summary

Pure data-driven method to distinguish chaos and noise
using cross-prediction with reservoir computing







Thank you for your kind attention!

✉ jongmin.park@apctp.org



KIAS KOREA INSTITUTE FOR ADVANCED STUDY

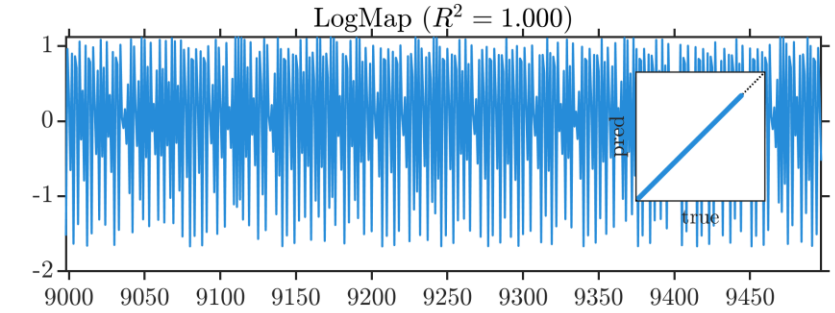
Jaesung Choi

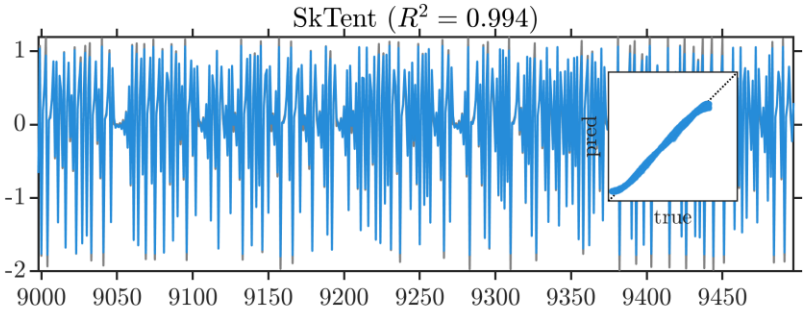


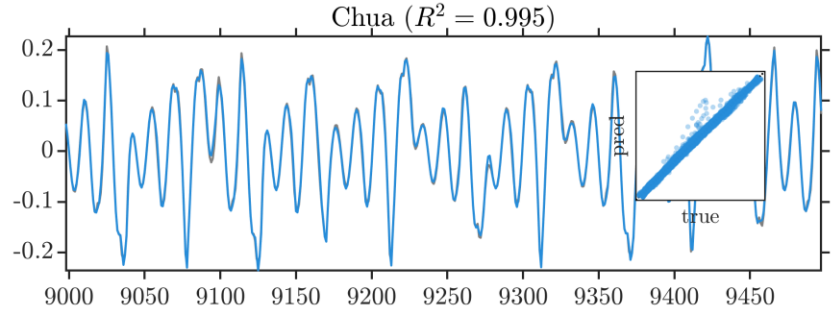
apctp asia pacific center for theoretical physics

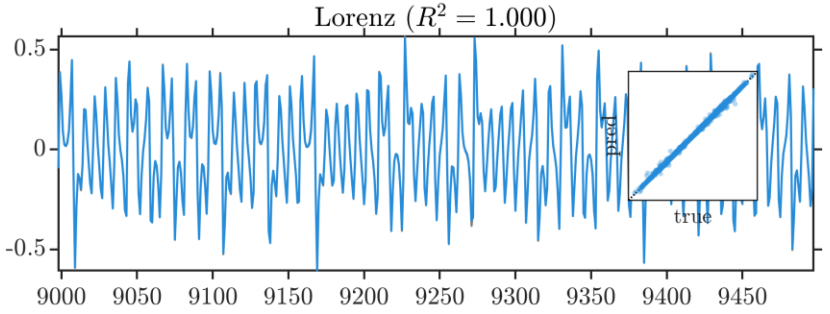
Athokpam Langlen Chanu

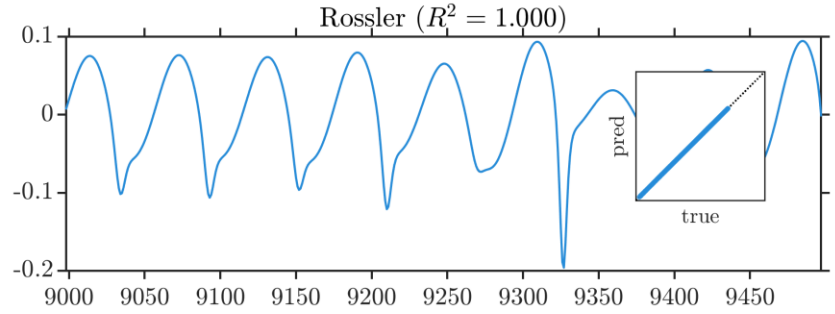
Appendix 1 – Results for synthetic data

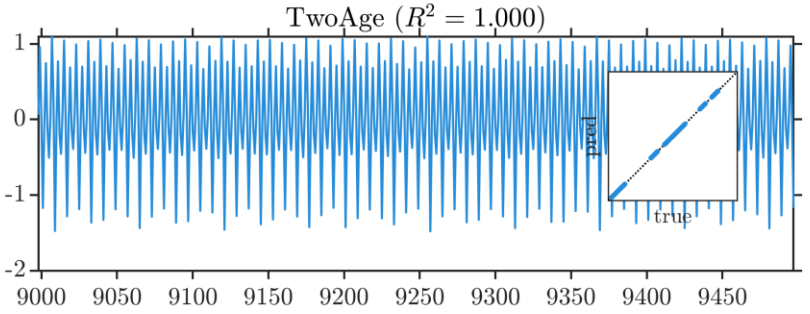


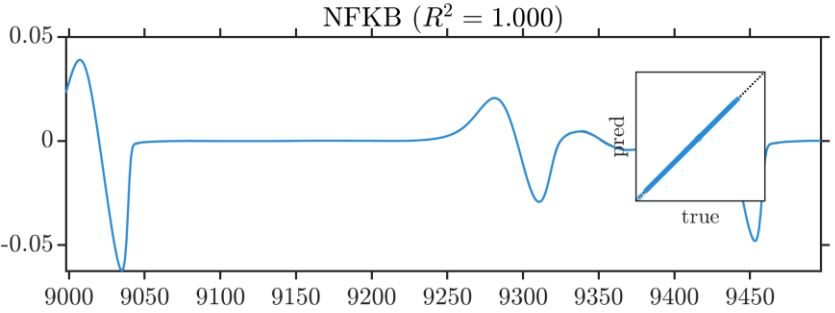




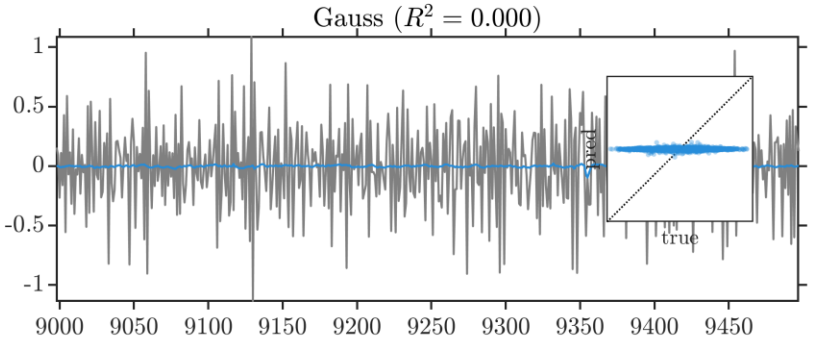


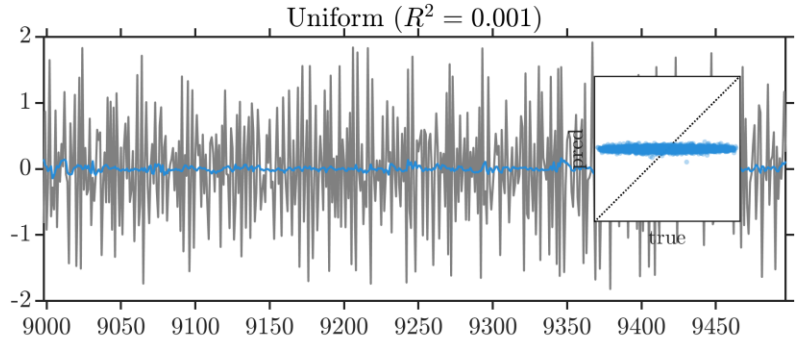


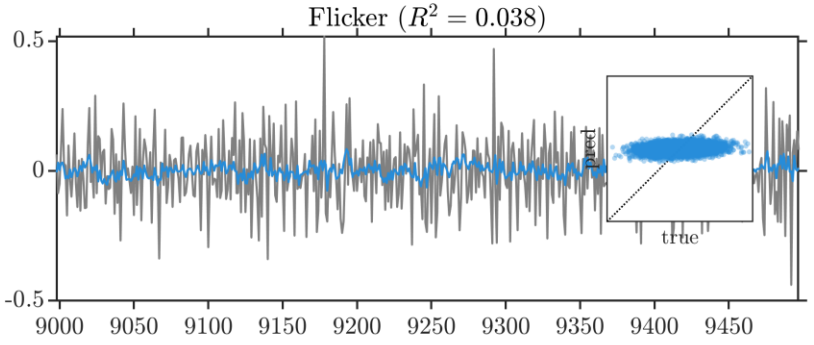


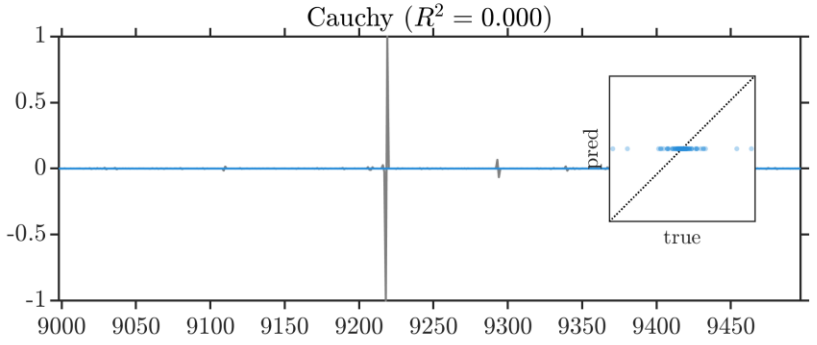


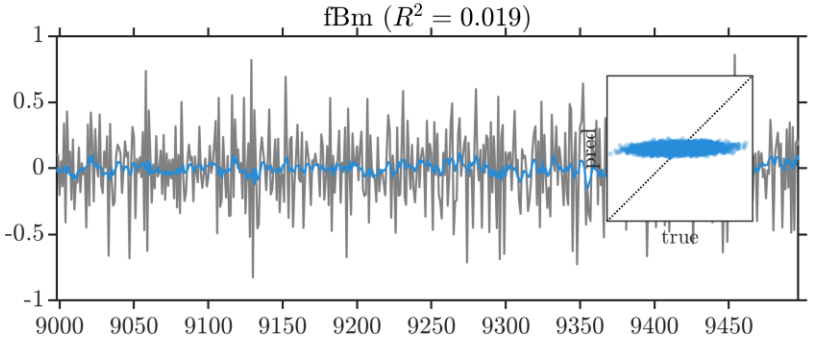
LogMap – logistic map
SkTent – skew tent map
Chua – Chua's circuit
Lorenz – Lorenz system
Rossler – Rossler system
TwoAge – two-age-class model
NFKB – NF-κB model ($d = 6$)

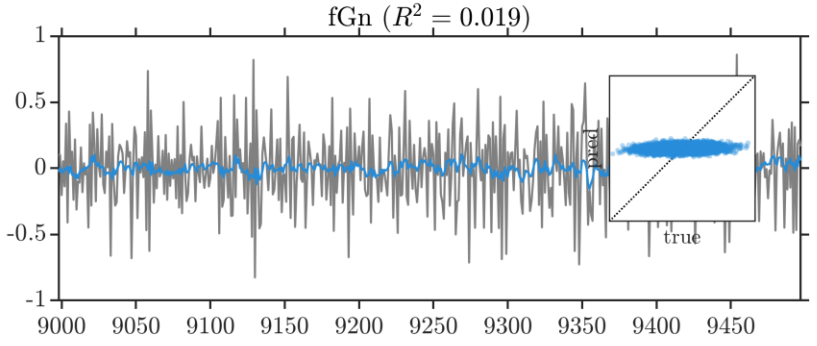






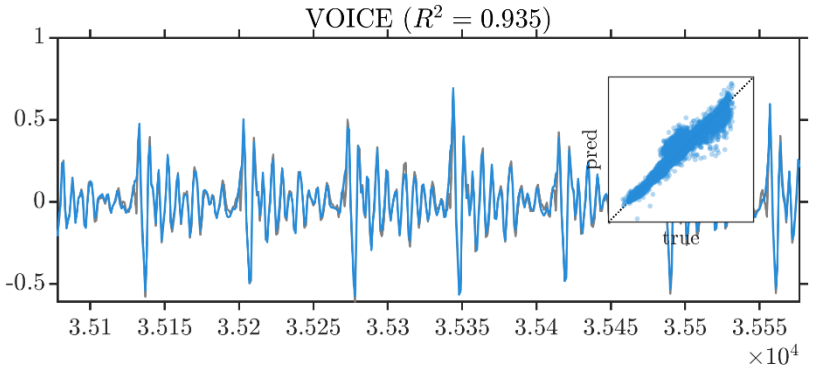


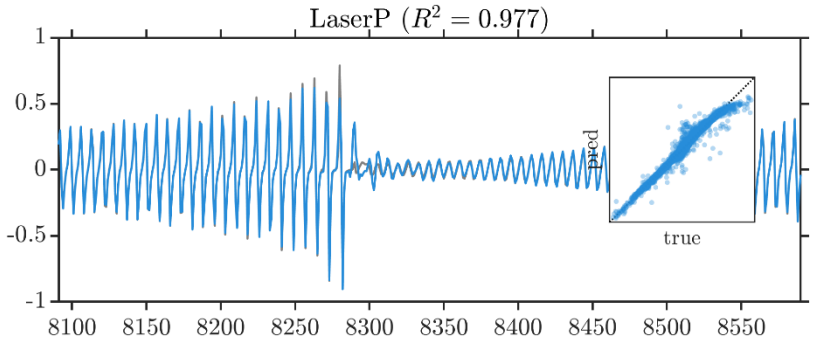


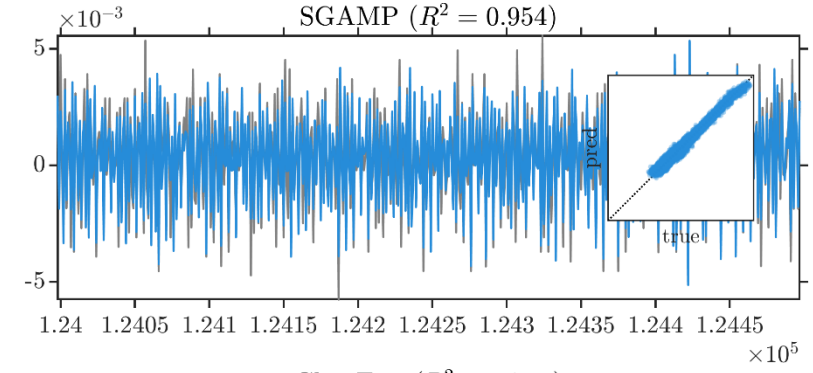


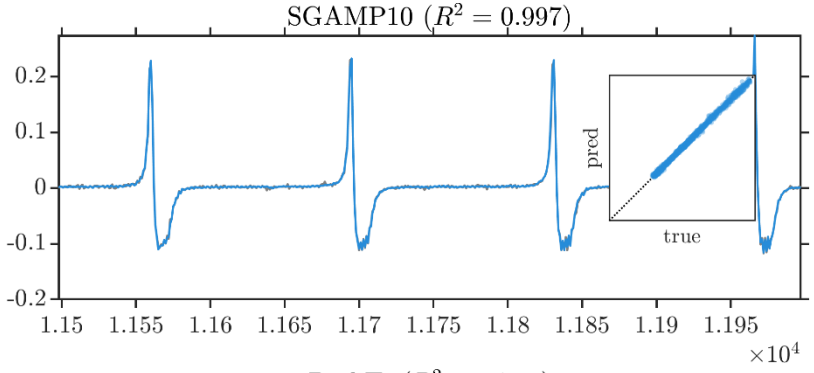
Gauss – white Gaussian noise
Uniform – white uniform noise
Flicker – flicker noise
Cauchy – Cauchy noise
fBm – fractional Brownian motion
fGn – fractional Gaussan noise

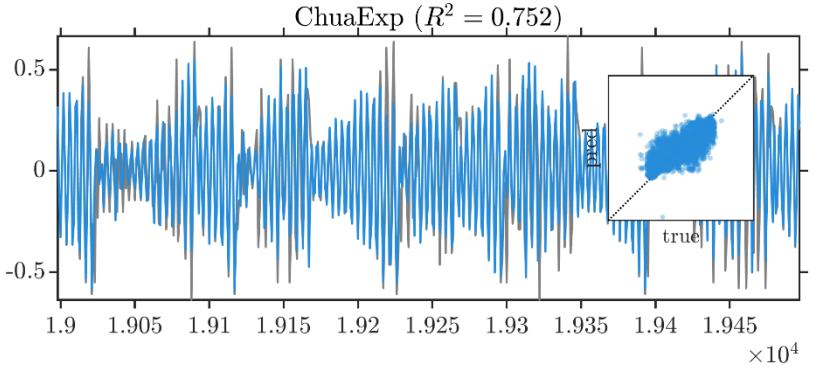
Appendix 2 – Results for empirical data

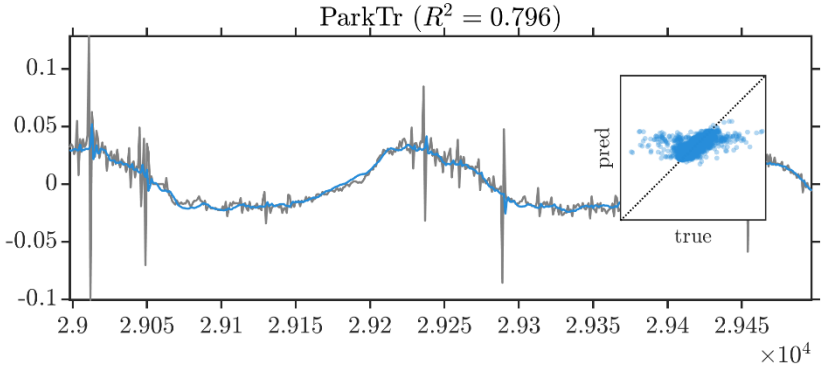




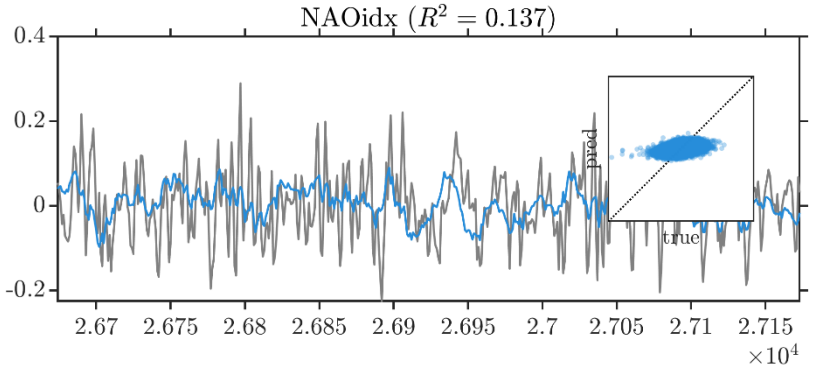


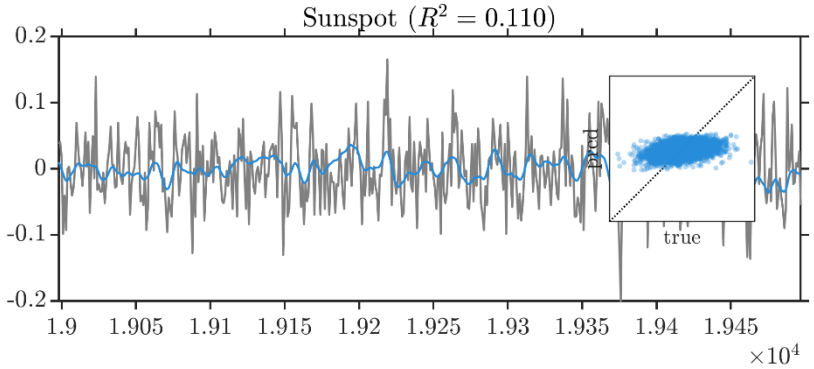


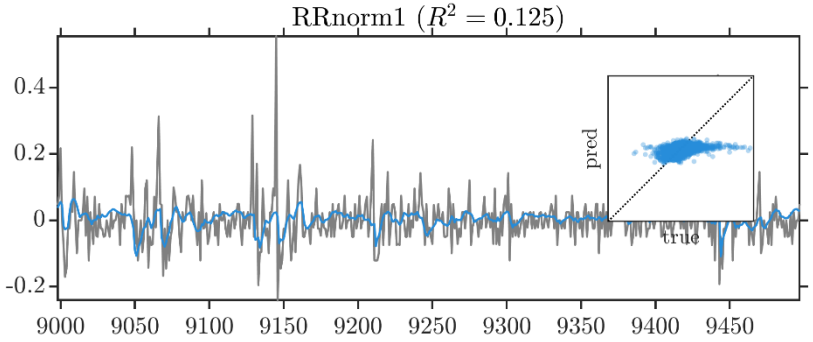


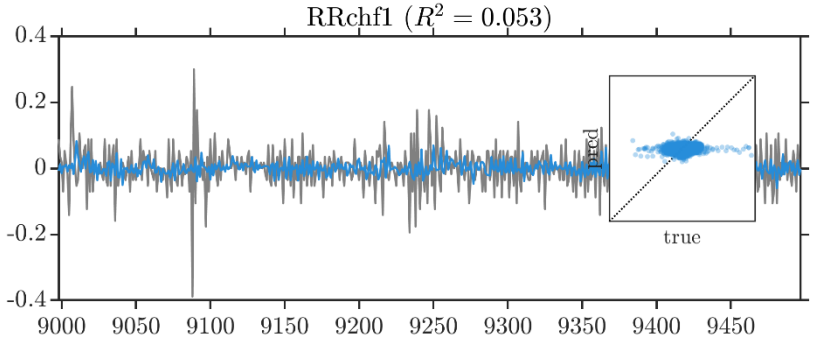


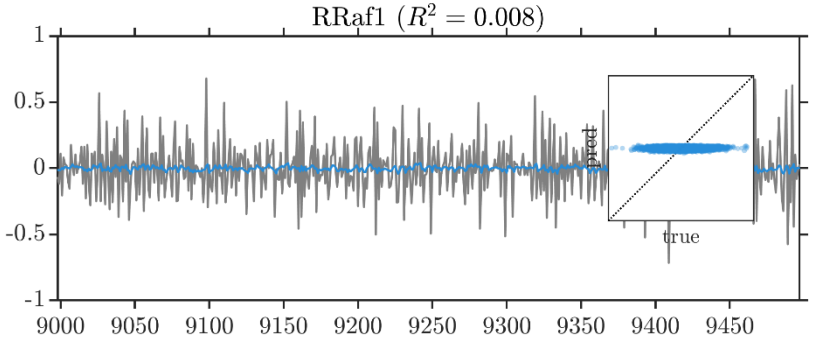
VOICE – voice recoding
LaserP – laser intensity pulsing
SGAMP – squid giant axon membrane potential
ChuaExp – Chua's circuit experiment
ParkTr – Parkinsonian tremor









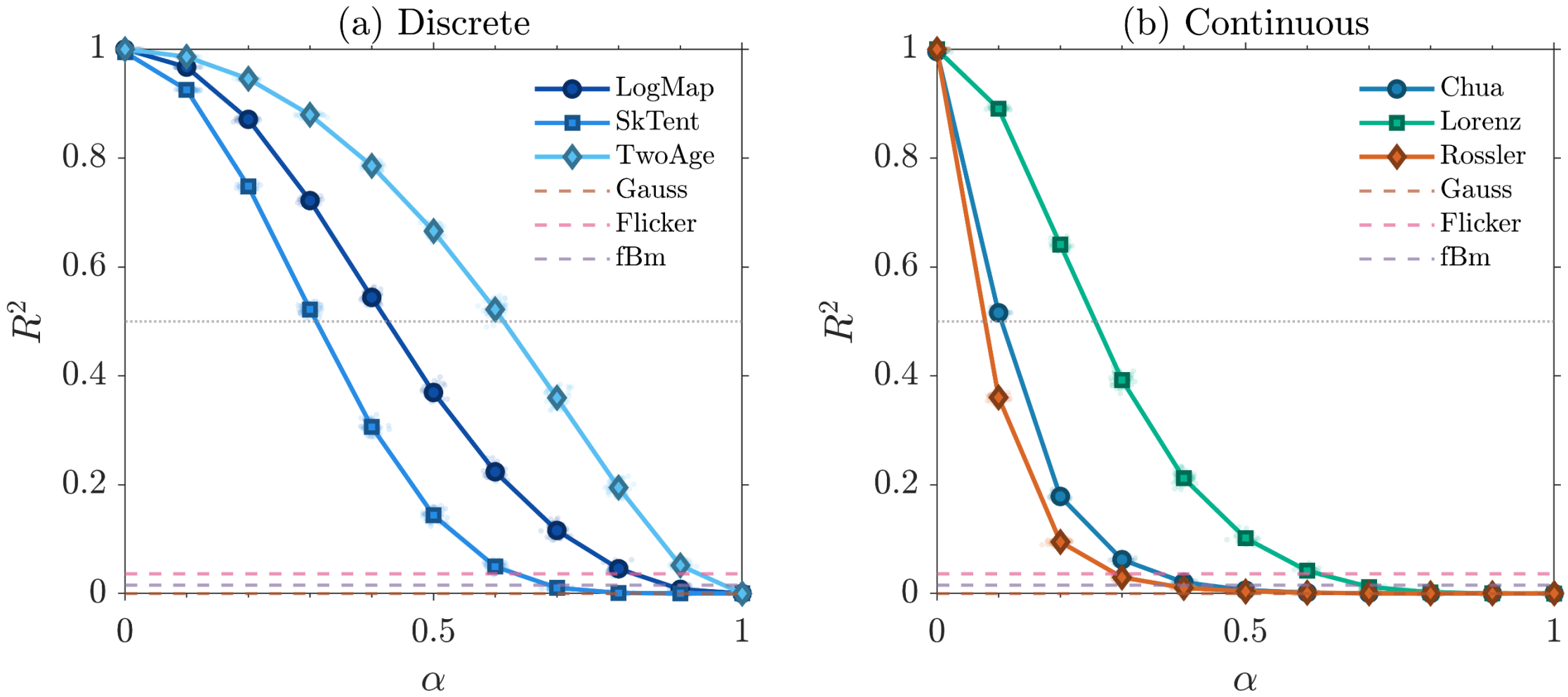


NAOidx – mean North Atlantic Oscillation index
Sunspot – sunspot number

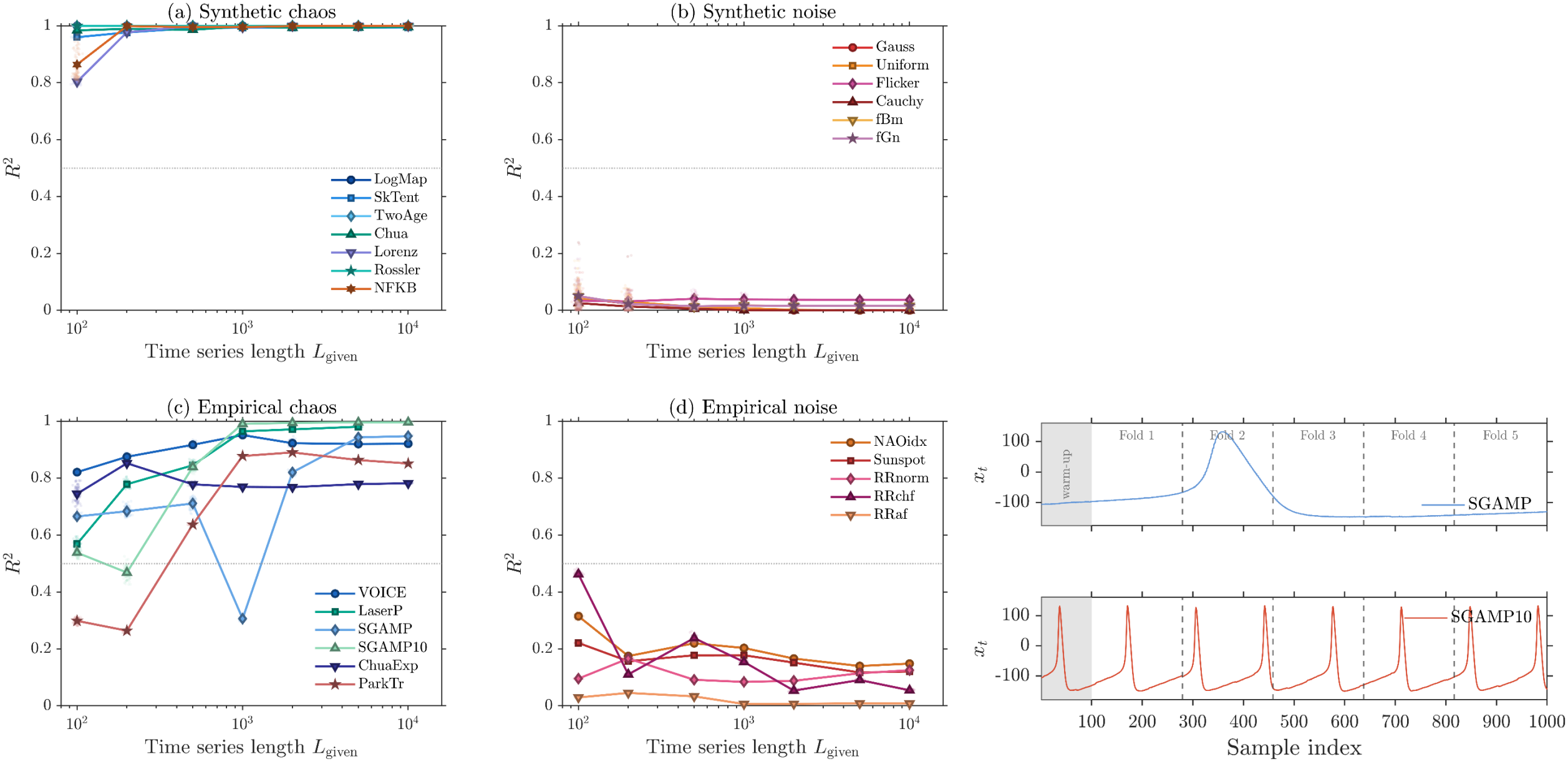
RR-interval data from…
RRnorm – healthy subjects
RRchf – congestive heart failure
RRaf – atrial fibrillation

Appendix 3 – Noise mixing

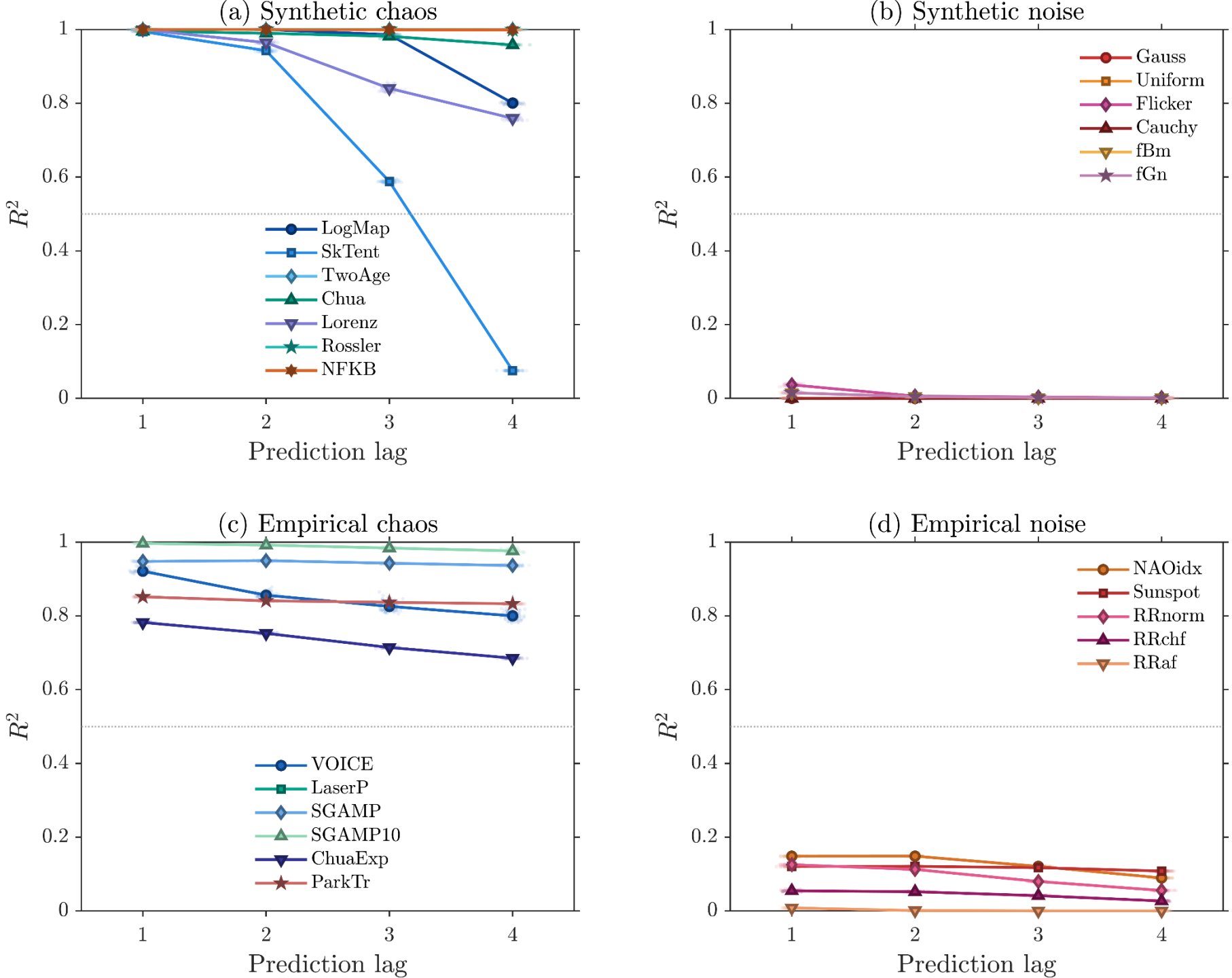
$$z_t = \sqrt{1-\alpha}x_t, +\sqrt{\alpha}\xi_t$$



Appendix 4 – Data length


(a) Synthetic chaos
(b) Synthetic noise
(c) Empirical chaos
(d) Empirical noise
R^2
Time series length L_{given}
LogMap
SkTent
TwoAge
Chua
Lorenz
Rossler
NFKB
Gauss
Uniform
Flicker
Cauchy
fBm
fGn
VOICE
LaserP
SGAMP
SGAMP10
ChuaExp
ParkTr
NAOidx
Sunspot
RRnorm
RRchf
RRaf
warm-up
Fold 1
Fold 2
Fold 3
Fold 4
Fold 5
x_t
SGAMP
SGAMP10
Sample index

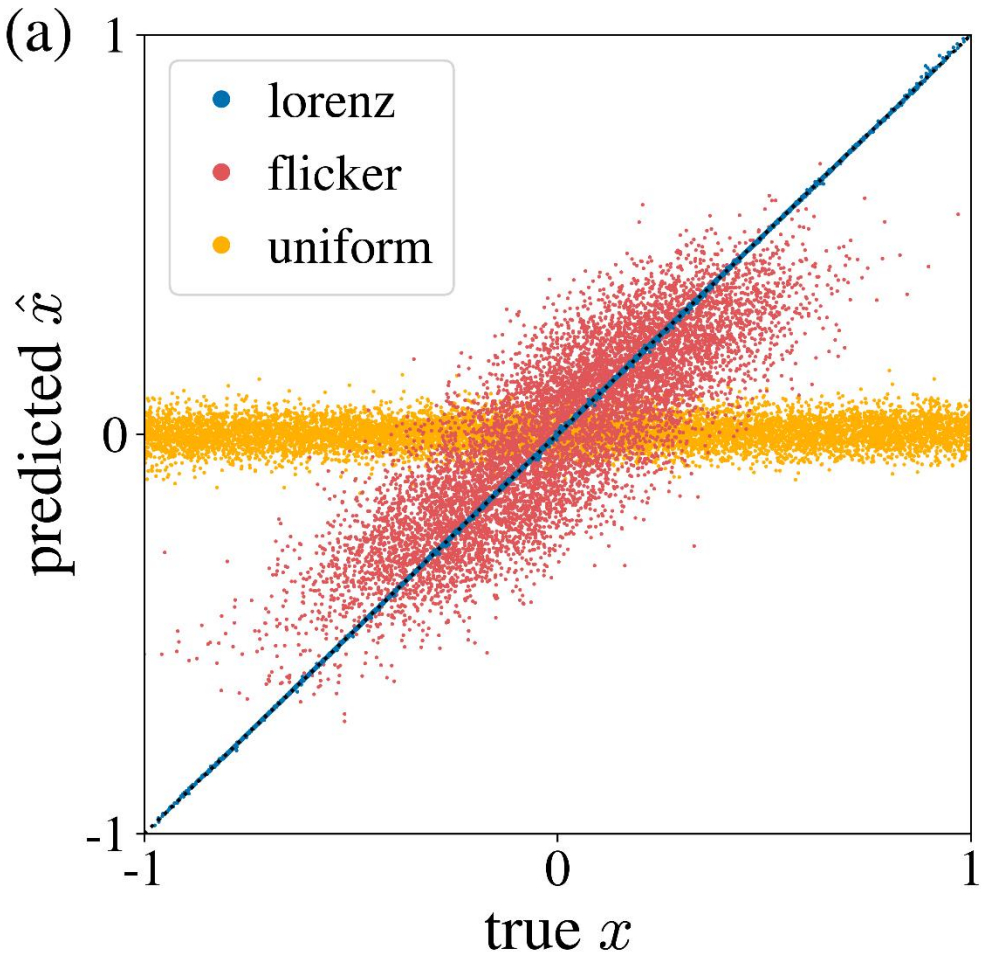
Appendix 5 – Prediction horizon


(a) Synthetic chaos
(b) Synthetic noise
(c) Empirical chaos
(d) Empirical noise
R^2
Prediction lag
LogMap
SkTent
TwoAge
Chua
Lorenz
Rossler
NFKB
Gauss
Uniform
Flicker
Cauchy
fBm
fGn
VOICE
LaserP
SGAMP
SGAMP10
ChuaExp
ParkTr
NAOidx
Sunspot
RRnorm
RRchf
RRaf

Appendix 6 – Conventional schemes

Testing on synthetic dynamics

Scheme 1
Value-to-value



Scheme 2
Difference-to-difference

