Ordinal-Pattern-Based Analysis of Human Brain States from electroencephalography (EEG) phase dynamics

ATHOKPAM Langlen Chanu

Chanu, A. L., Park, Y., Choi, J., Cha, Y., Lee, U., Moon, J. Y.*, & Park, J. M.* (2026). *Chaos, Solitons & Fractals*, 209, 118416

Park, Y., Cha, Y., Kim, H., Kim, Y., Woo, J. H., Cho, H., Mashour, G.A., Ting., X., Lee, U., Hong, S.J., Honey, C.J., & Moon, J. Y.* (2026). *Current Biology*, 36, 1.

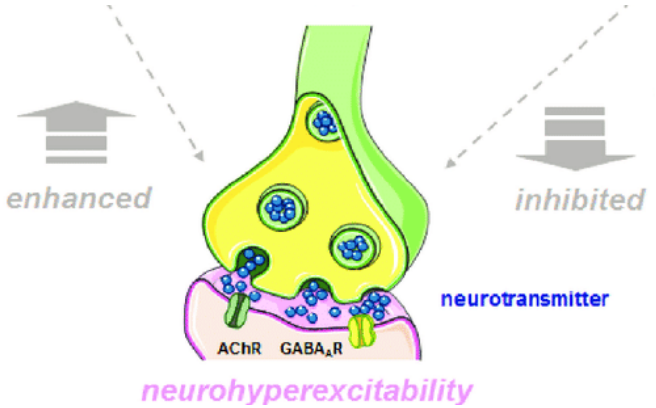
apctp
아시아태평양이론물리센터
asia pacific center for theoretical physics

21 July 2026

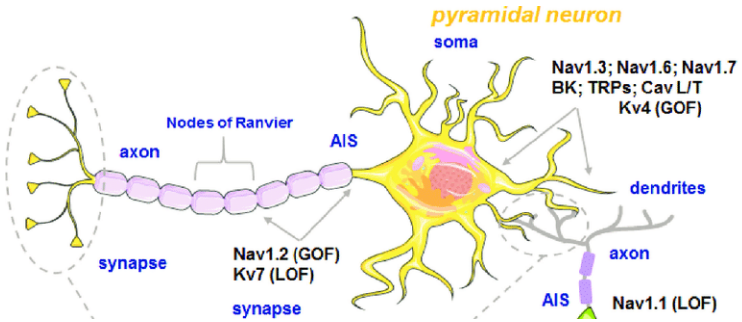
Nonequilibrium Statistical Physics of Complex Systems (NSPCS) 2026,
Korea Institute for Advanced Study, Seoul, South Korea

POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY
1986 포항공과대학교

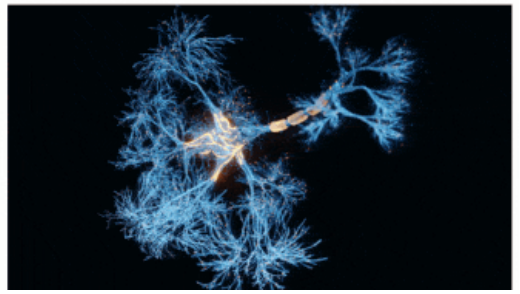
Brain as a Complex System

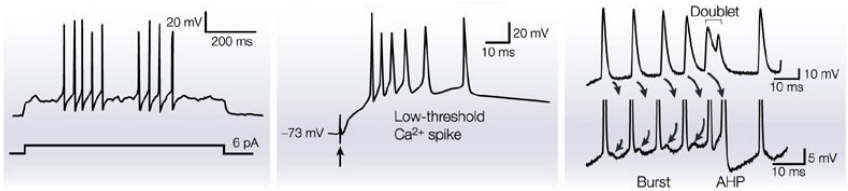


molecular-scale ions & ion channels



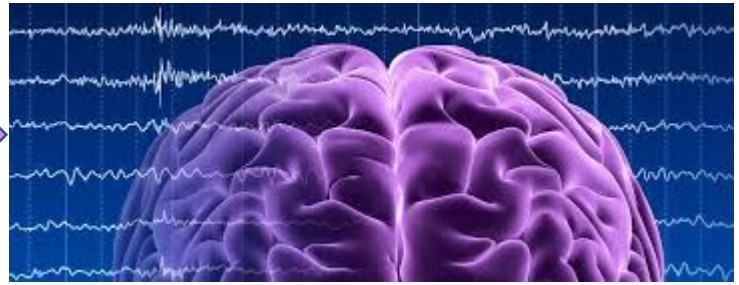
Single neuron



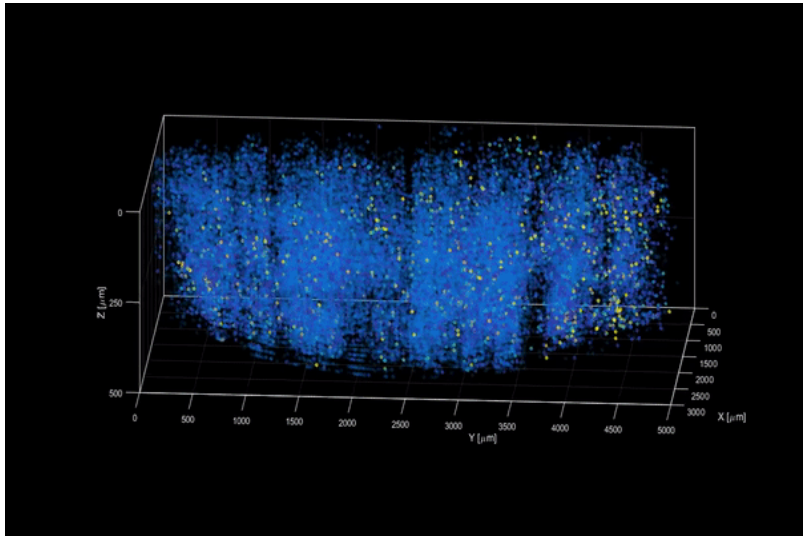




Whole brain dynamics



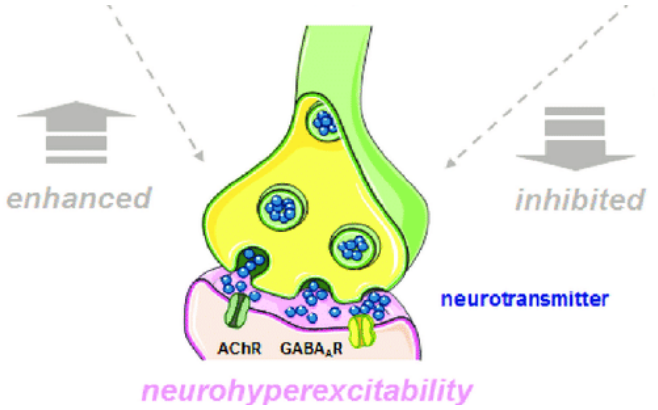
EEG signals



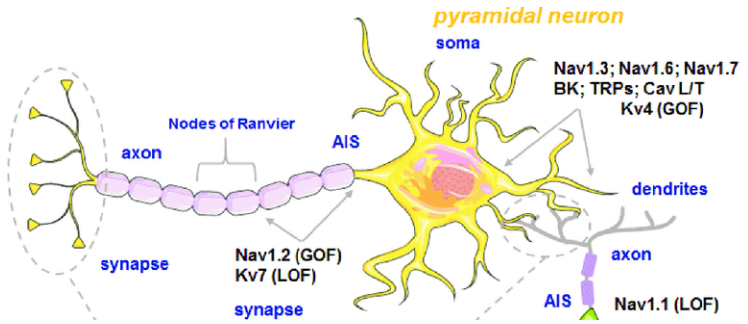
firing patterns of collective neurons

Simon, Herbert A., 1962, 'The Architecture of Complexity', reprinted in Simon, (1969), pp. 84-118
https://singularityhub.com/2021/09/27/this-amazing-gif-shows-a-million-neurons-firing-in-a-mouses-brain/
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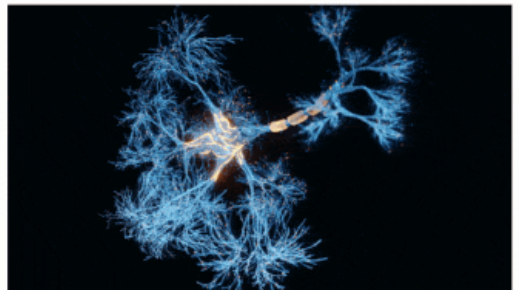
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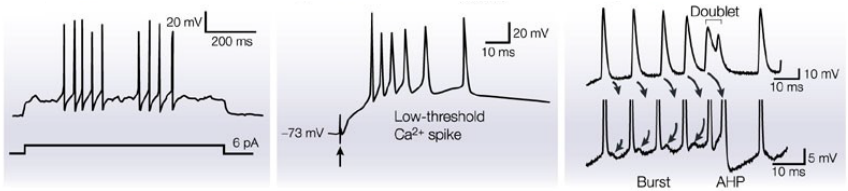


molecular-scale ions & ion channels



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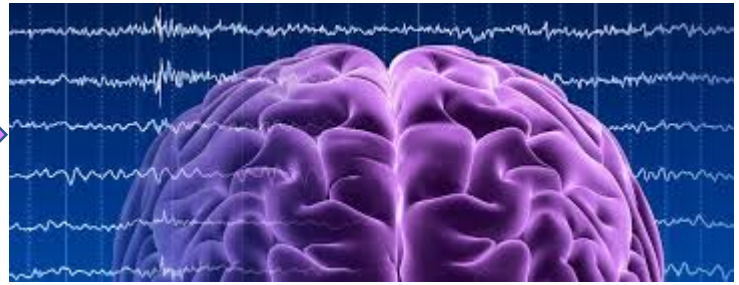


Hierarchy in the architecture of complexity. (H.A. Simon, 1962).

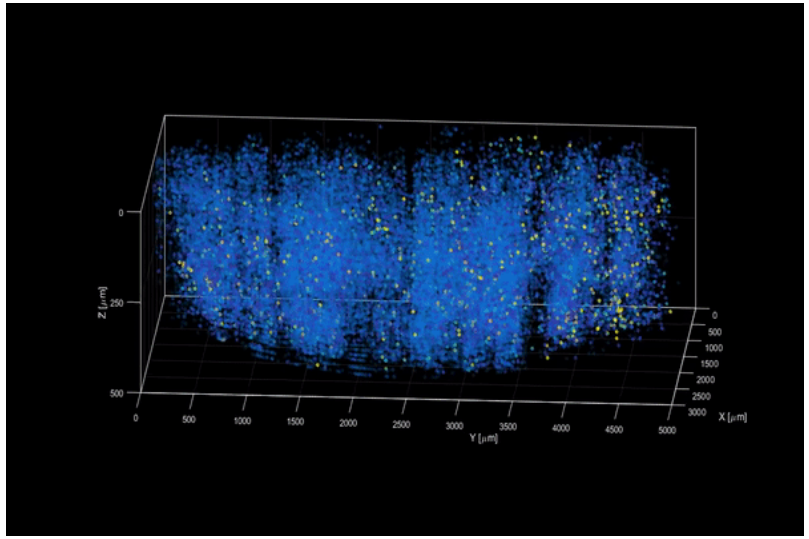
Consciousness, creativity, and intelligence are emergent properties.



Whole brain dynamics



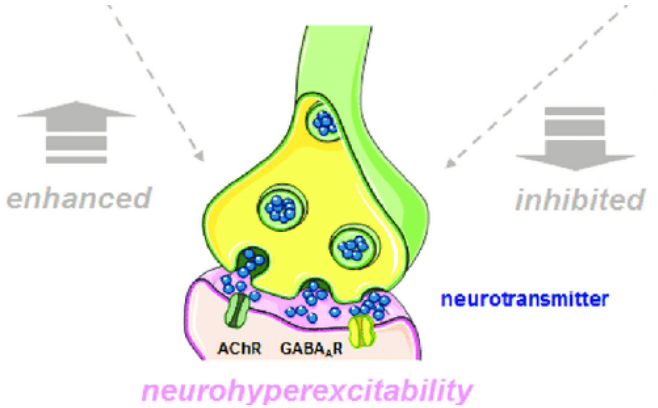
EEG signals



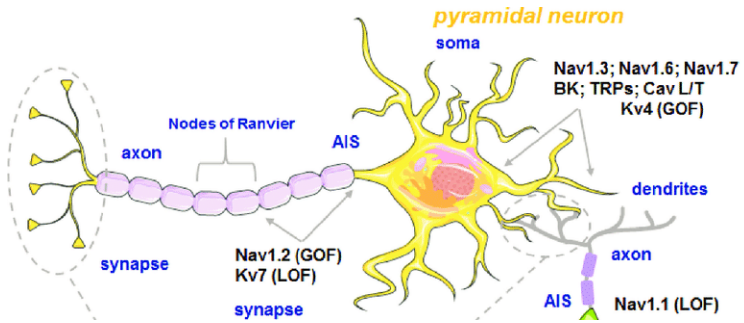
firing patterns of collective neurons

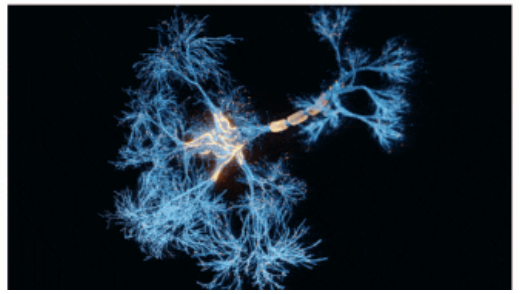
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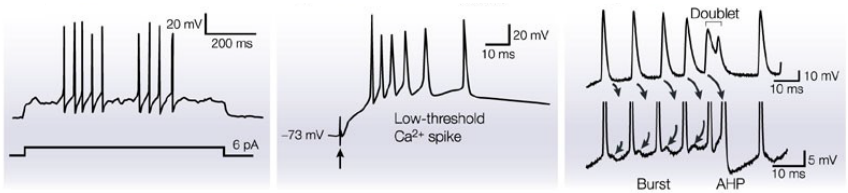
Brain as a Complex System



molecular-scale ions & ion channels





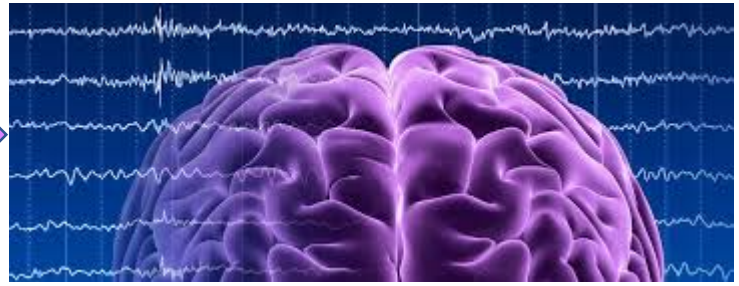


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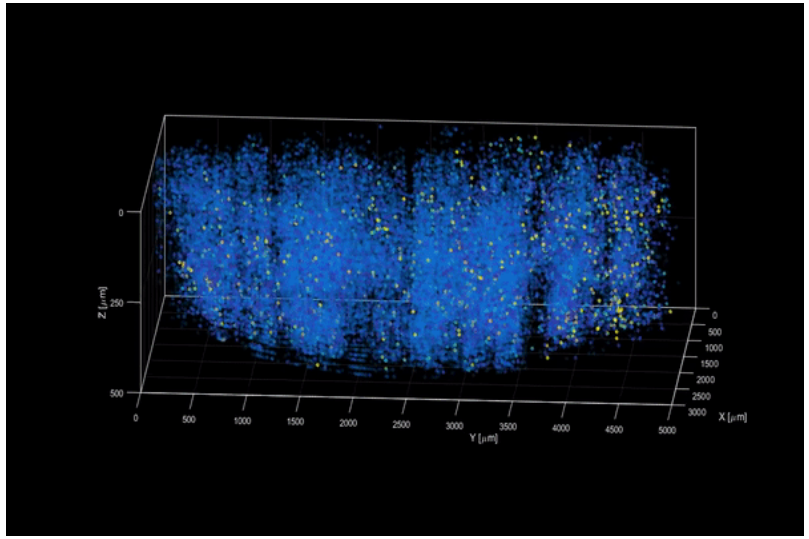




Whole brain dynamics

EEG signals

EEG allows us to characterize how the brain's collective organization changes across different states.



firing patterns of collective neurons

Simon, Herbert A., 1962, 'The Architecture of Complexity', reprinted in Simon, (1969), pp. 84-118
https://singularityhub.com/2021/09/27/this-amazing-gif-shows-a-million-neurons-firing-in-a-mouses-brain/
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Motivation

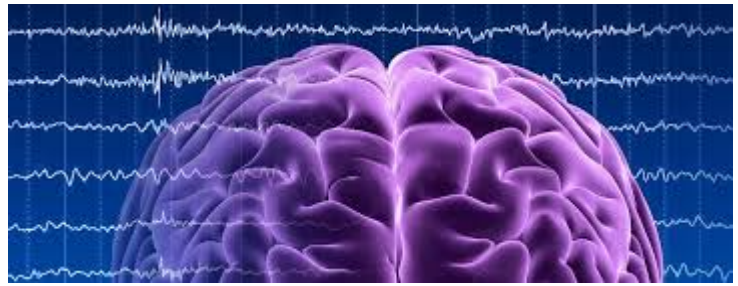
Cognitive functions



Human brain

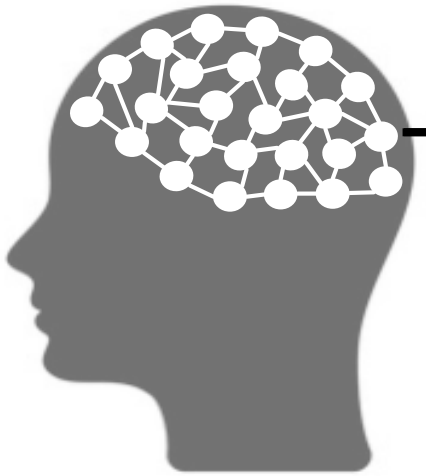
External factor (anesthesia)

Neurodevelopment disorder



Motivation

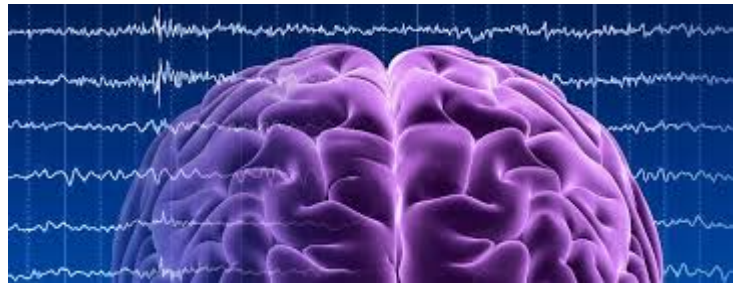
Cognitive functions



Human brain

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Neurodevelopment disorder



Understanding the relationship between brain states and statistical characteristics of brain signals is both scientifically and clinically important.

Motivation

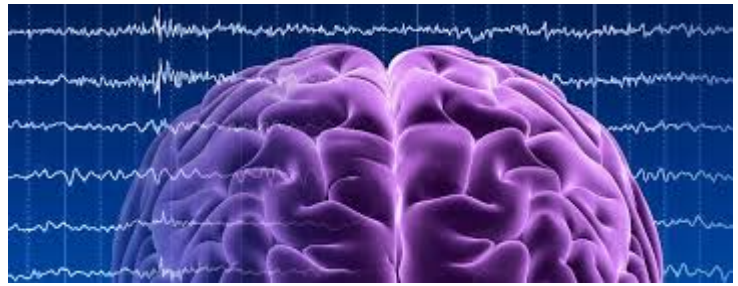
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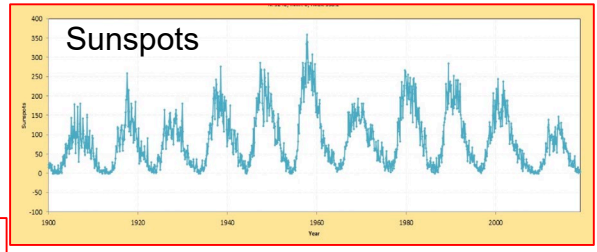
Understanding the relationship between brain states and statistical characteristics of brain signals is both scientifically and clinically important.

How can we quantitatively measure the complexity of different brain states and use that information to distinguish them from EEG recordings?

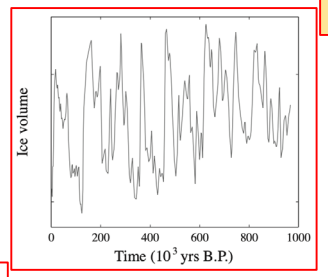
Introduction: Time series data & Ordinal patterns

$\{X_t\}_{t \in T}$

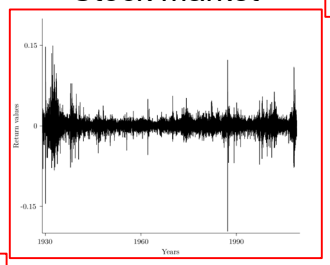
-astronomy
-geology
-meteorology
-finance
-birdsong
-biomedical
-biophysics



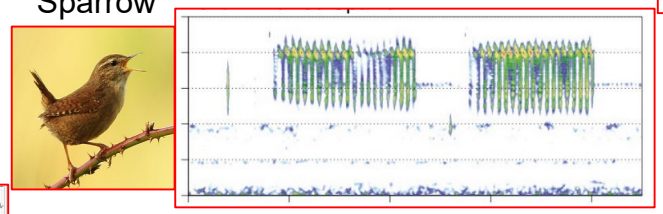
Volume of ice

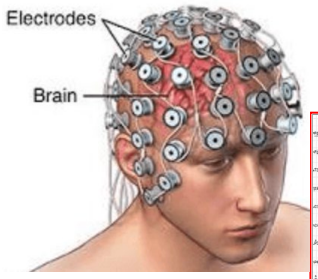


Stock market

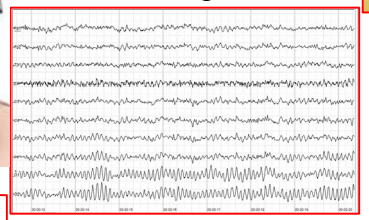


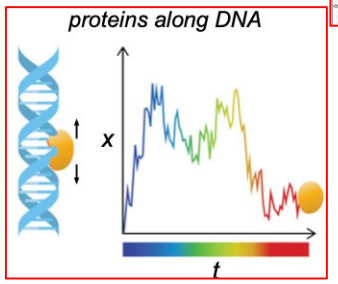
Sparrow



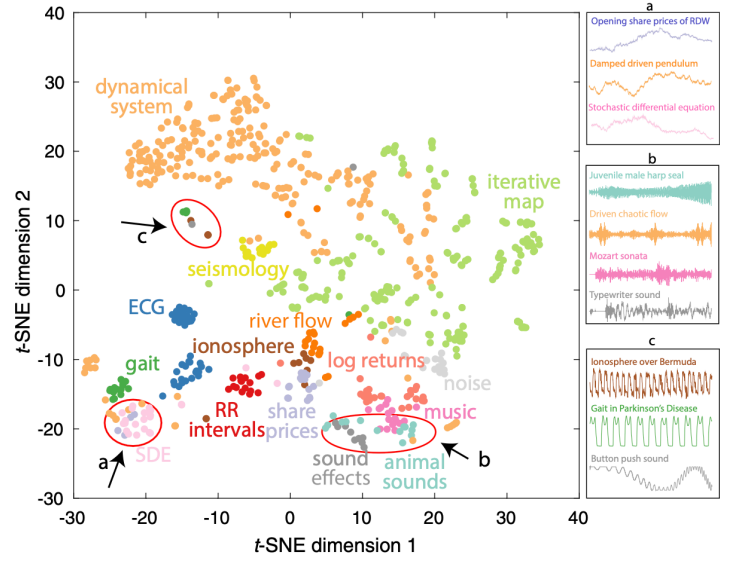


EEG signals





A self-organizing, living library of time-series data

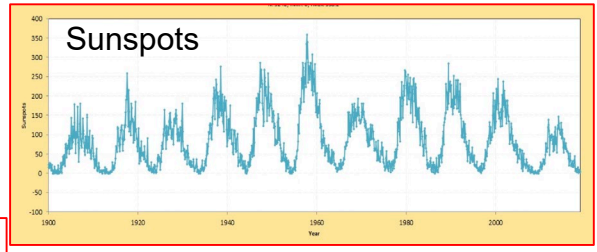


Fulcher, B. D., Lubba, C. H., Sethi, S. S., & Jones, N. S. (2020). *Scientific data*, *7*(1), 213.

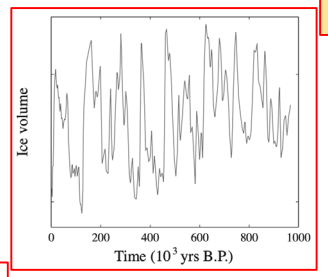
Introduction: Time series data & Ordinal patterns

$\{X_t\}_{t \in T}$

-astronomy
-geology
-meteorology
-finance
-birdsong
-biom
-bioph



Volume of ice



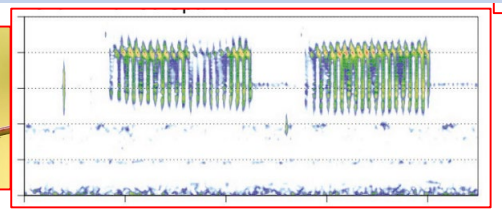
Stock market

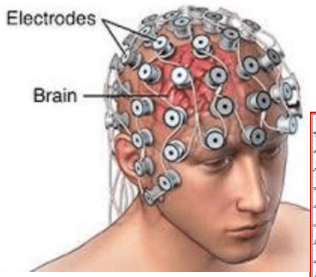


Patterns from Complex Systems emerge from the interplay between order and randomness, often governed by nonlinear interactions.

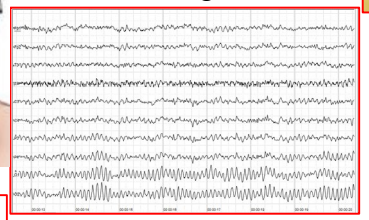
Sparrow

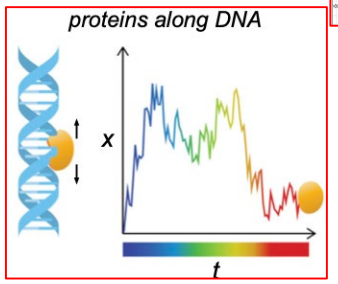




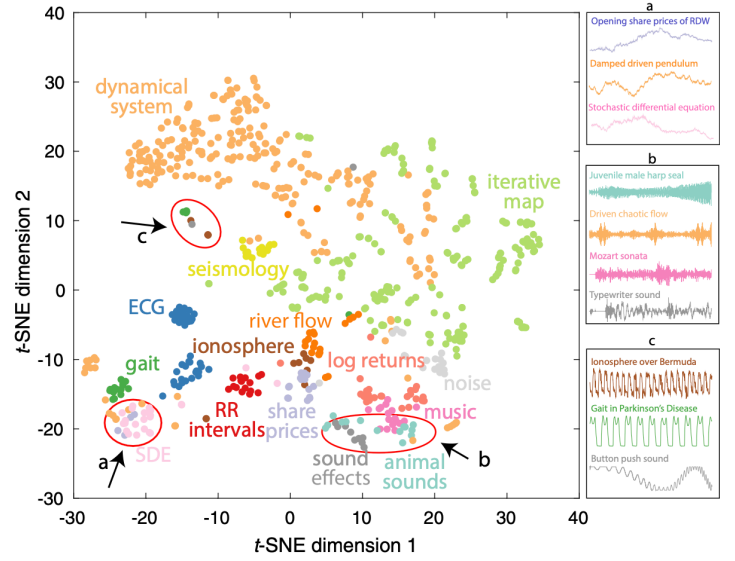


EEG signals





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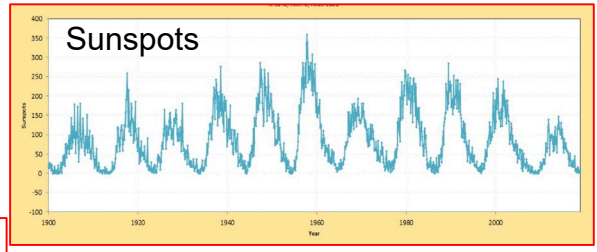


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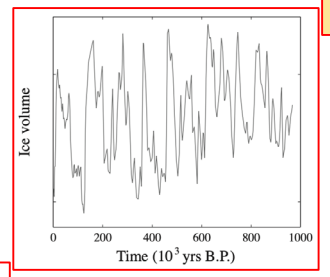
Introduction: Time series data & Ordinal patterns

$\{X_t\}_{t \in T}$

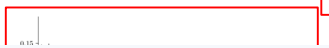
-astronomy
-geology
-meteorology
-finance
-birdsong
-biom
-biop



Volume of ice



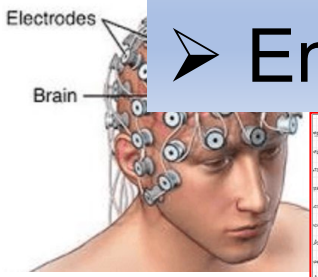
Stock market

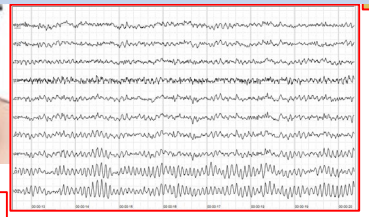


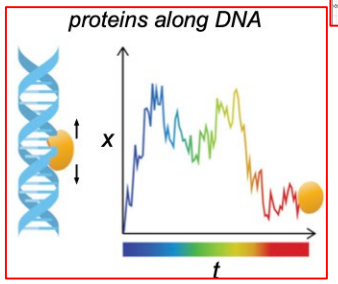
Some quantitative measures of complexity

- Lyapunov exponents
- Fractal dimension
- Entropy

data





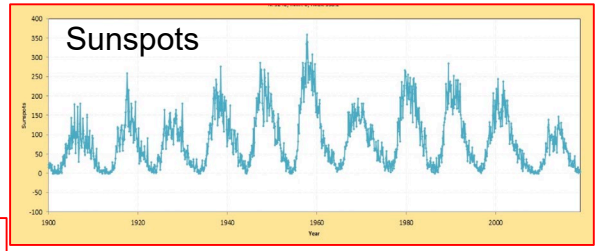


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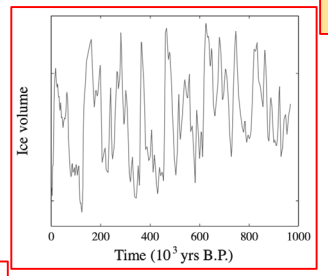
Introduction: Time series data & Ordinal patterns

$\{X_t\}_{t \in T}$

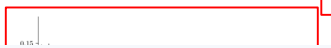
-astronomy
-geology
-meteorology
-finance
-birdsong
-biom
-biophysics



Volume of ice



Stock market

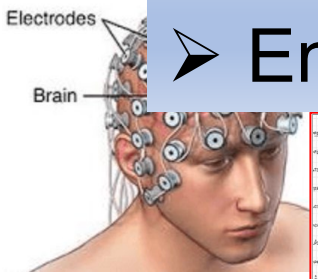


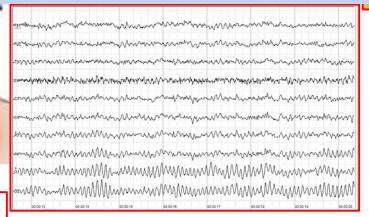
Some quantitative measures of complexity

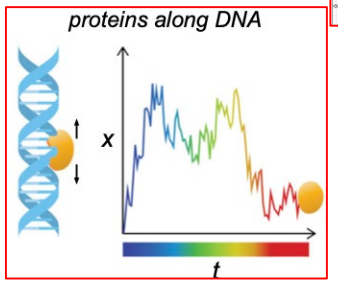
- Lyapunov exponents
- Fractal dimension
- Entropy ⟶ Permutation Entropy (Bandt and Pompe)

(calculated from ordinal patterns)

data







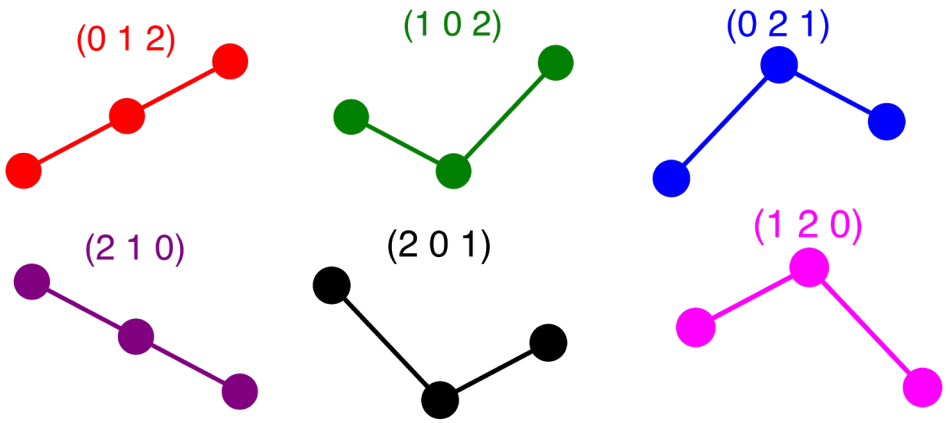
Introduction: Time series data & Ordinal patterns

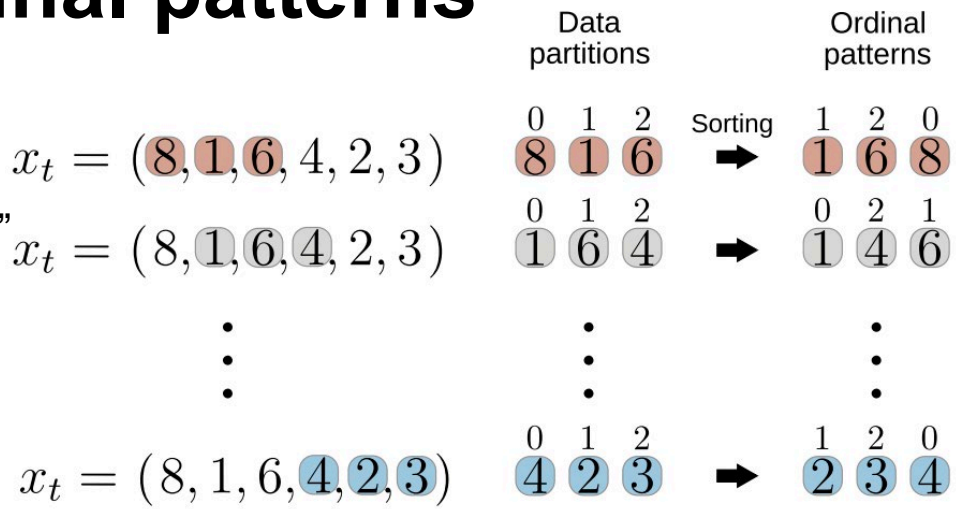
Illustration with 1D-time series data $x_t = (8, 1, 6, 4, 2, 3)$

Define an embedding dimension or permutation order $dx = 3$

So, we have 3! = 6 possible "ordinal patterns" or "permutation states"

$$\pi_1 = (0,1,2), \pi_2 = (1,0,2), \pi_3 = (0,2,1),$$
$$\pi_4 = (2,1,0), \pi_5 = (2,0,1), \pi_6 = (1,2,0)$$





$$\pi_p = ((1,2,0), (0,2,1), (2,1,0), (1,2,0))$$

$$\{x_t\} \longrightarrow \{\pi\}$$

Time series

symbolic sequence of ordinal patterns or permutation symbols

Ordinal probability distribution $P = \{\rho_j(\pi_j)\}\ ;\ j = 1, \ldots, r.$
The relative frequency of all possible patterns in the symbol sequences

C. Bandt and B. Pompe, *Phys. Rev. Lett.* **88**, 174102 (2002).
Rosso, O. A., Larrondo, H. A., Martin, M. T., Plastino, A., & Fuentes, M. A. (2007). *Phys. Rev. Lett. 99*(15), 154102.

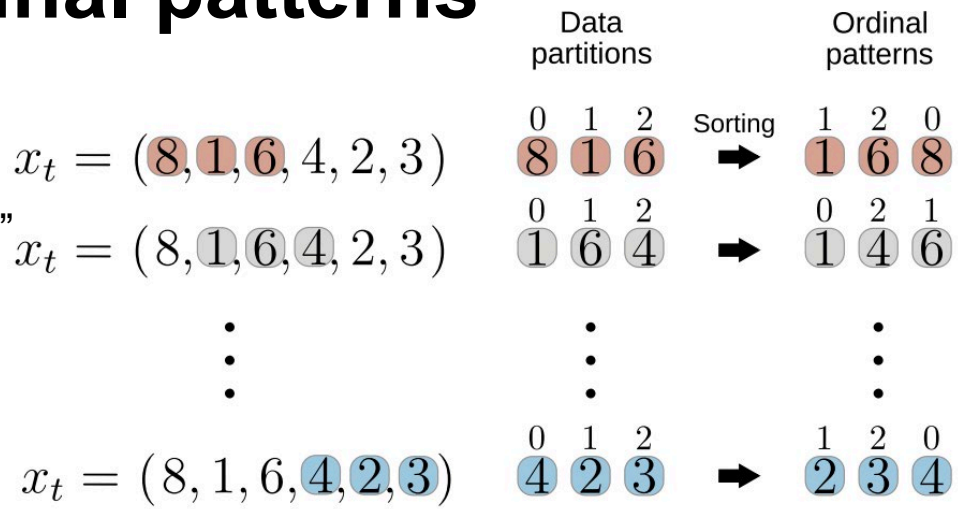
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$$\pi_p = ((1,2,0), (0,2,1), (2,1,0), (1,2,0))$$

Permutation entropy is the Shannon entropy of ordinal patterns. (Bandt and Pompe)

$$S(P) = -\sum_{j=1}^{dx!} \rho_j(\pi_j) \log \rho_j(\pi_j)$$

$$H(P) = \frac{S(P)}{\log dx!}$$

$\{x_t\} \longrightarrow \{\pi\}$

Time series

symbolic sequence of ordinal patterns or permutation symbols

Permutation entropy reveals temporal structure through local ordering patterns in time series.

Ordinal Patterns

Model-free data driven approach

Ordinal probability distribution $P = \{\rho_j(\pi_j)\}$; $j = 1, \ldots, r$.
The relative frequency of all possible patterns in the symbol sequences

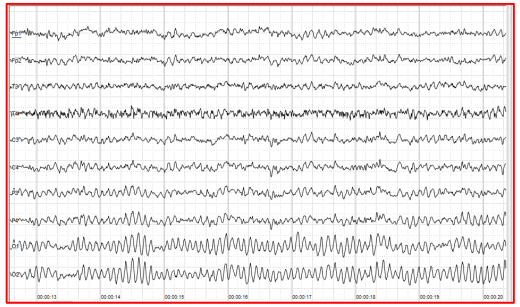
C. Bandt and B. Pompe, *Phys. Rev. Lett.* **88**, 174102 (2002).

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Human brain



A 128-channel system



EEG data

Two independent datasets of EEG recordings:

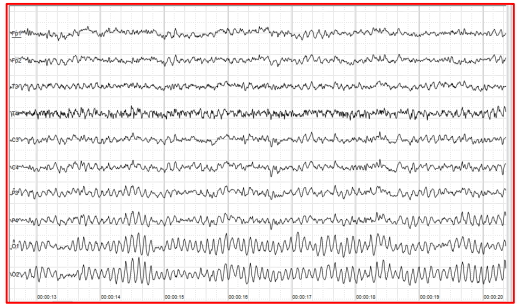
(i) Obtained from subjects undergoing a general anesthesia protocol,

(ii) Acquired in the resting state from healthy control subjects and individuals with inattentive-type attention deficit hyperactivity disorder (ADHD), at eyes-open and eyes-closed conditions.

Human brain



A 128-channel system



EEG data

Two independent datasets of EEG recordings:

(i) Obtained from subjects undergoing a general anesthesia protocol,

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Ordinal-pattern analysis faces limitations when applied to high-dimensional time series.

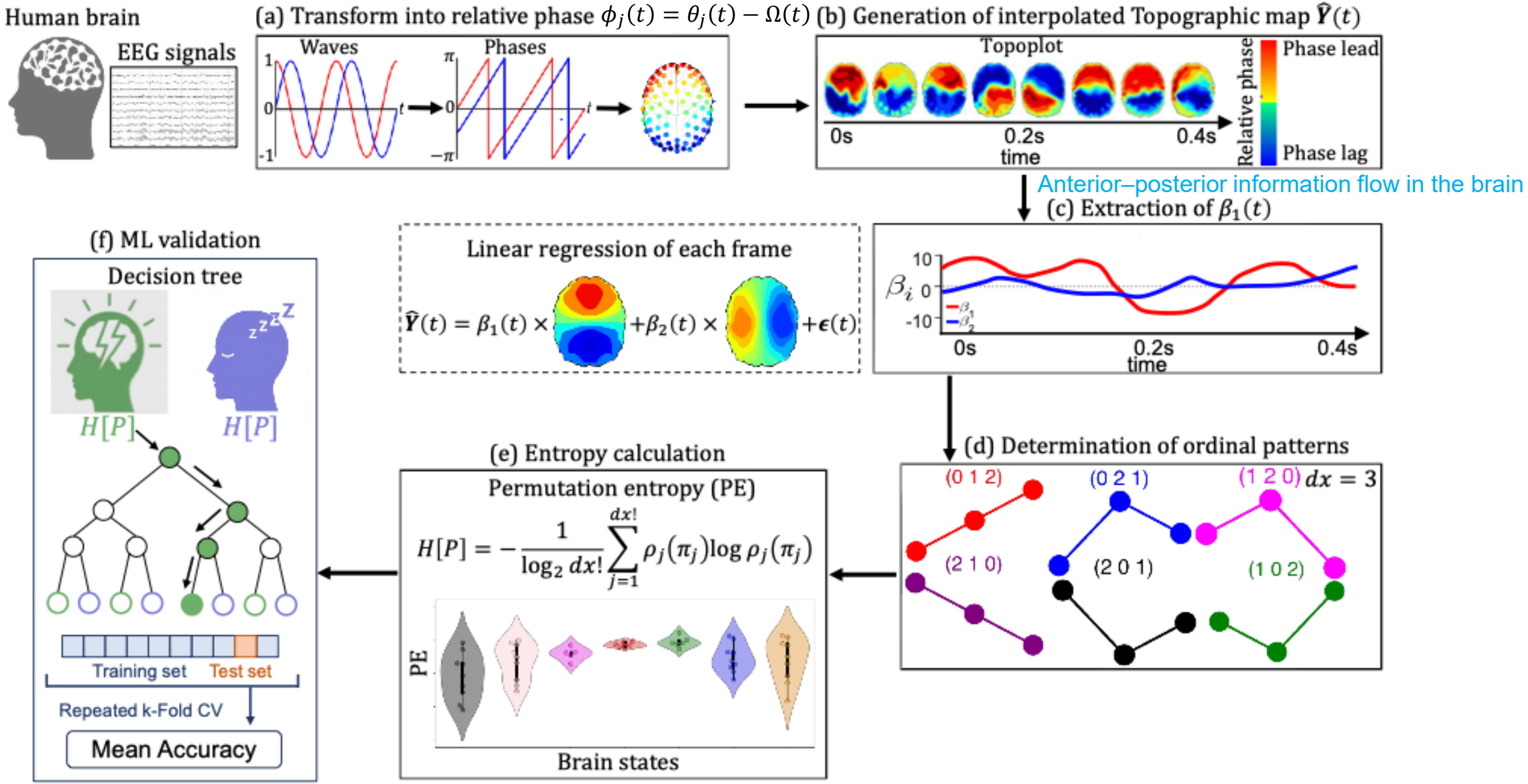
It is important to identify a representative low-dimensional time series that sufficiently captures the essential information embedded in high-dimensional EEG recordings.

Recent research* has identified two principal modes of information flow from EEG phase dynamics: top-down flow mode and bottom-up flow mode.

EEG phase dynamics allows high-dimensional multichannel EEG data to be effectively represented by the time series of a dominant principal mode.

Park, Y., Cha, Y., Kim, H., Kim, Y., Woo, J. H., Cho, H., ... & Moon, J. Y. (2026). *Current Biology*, 36, 1.

Workflow



Park, Y., Cha, Y., Kim, H., Kim, Y., Woo, J. H., Cho, H., ... & Moon, J. Y. (2026). *Current Biology*, 36, 1.

I. General Anesthesia Data

Seven distinct brain states

- Eyes Closed (EC): 5-minute eyes-closed resting state before anesthesia
- Propofol injection period (P): 5-minute post-propofol injection
- Loss of Consciousness (LOC): 3-minute post-LOC marker
- Burst period (B): 5-minute extracted burst period
- Suppression period (S): 5-minute extracted suppression period
- Post-Burst/Suppression anesthesia (PBS): 5-minute pre-recovery final state
- Recovery of Consciousness (ROC): 5-minute post-ROC marker, return of consciousness

(data recorded at)

UNIVERSITY OF MICHIGAN MEDICAL SCHOOL
MICHIGAN MEDICINE
Ann Arbor, Michigan, USA



(Prof. UnCheol Lee)

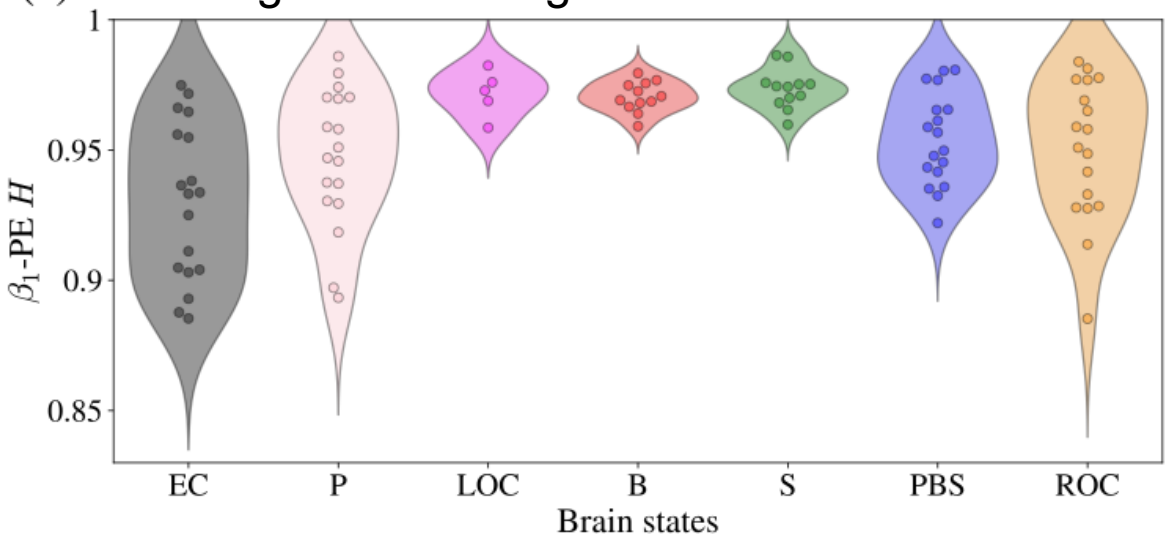
II. Healthy controls and Inattentive-type ADHD

Healthy Brain Network (HBN) dataset
(released by the Child Mind Institute)
L.M. Alexander, *et al.*, Scientific Data 4 (2017) 1.

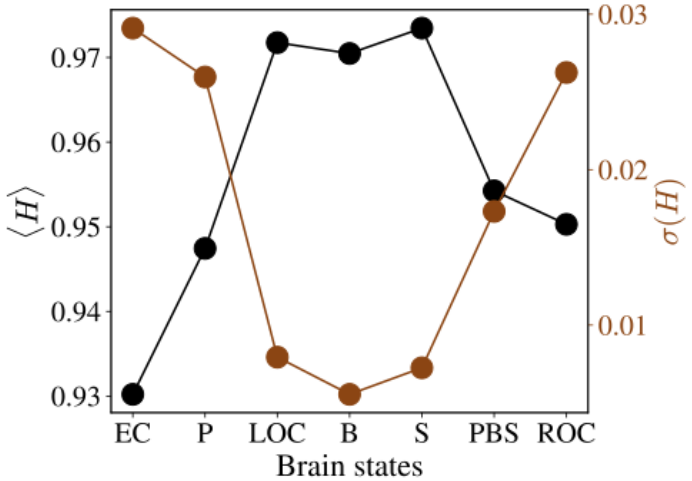
The resting-state protocol consists of five blocks, each comprising 20 seconds of eyes-open followed by 40 seconds of eyes-closed.

I. Results

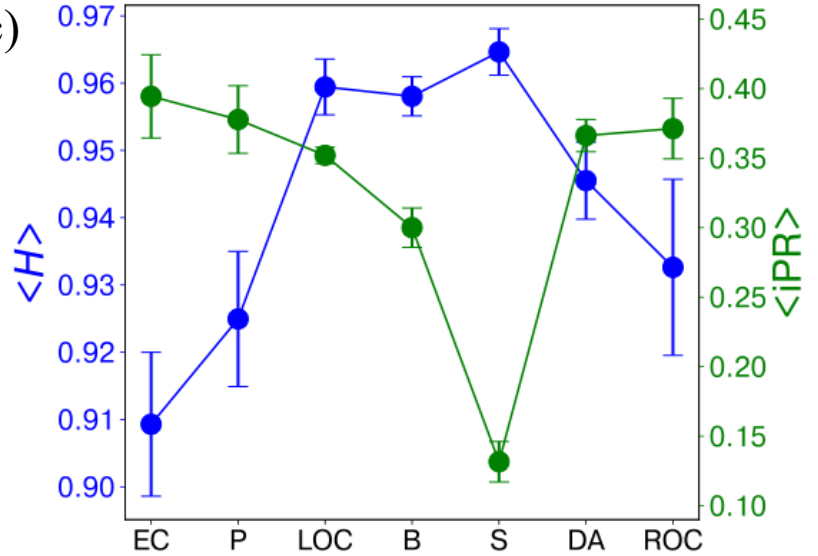
(a) Time segments of length 1500 with dx = 4 across subjects



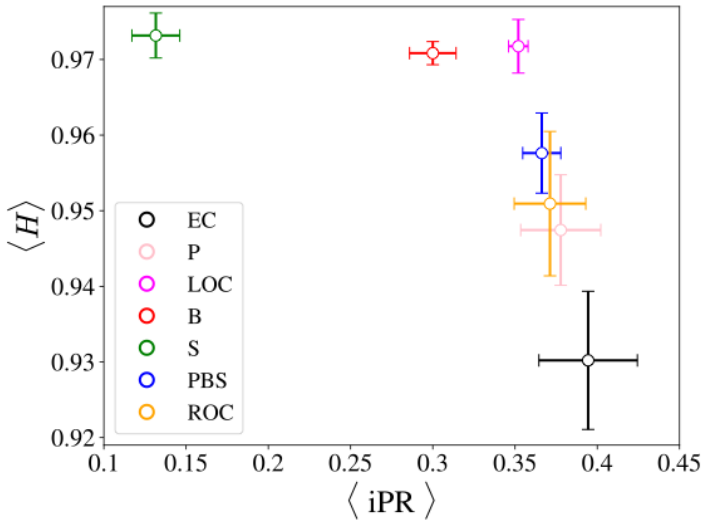
(b)



(c)



(d)



The higher the value of iPR,
the lower the depth of anesthesia*.

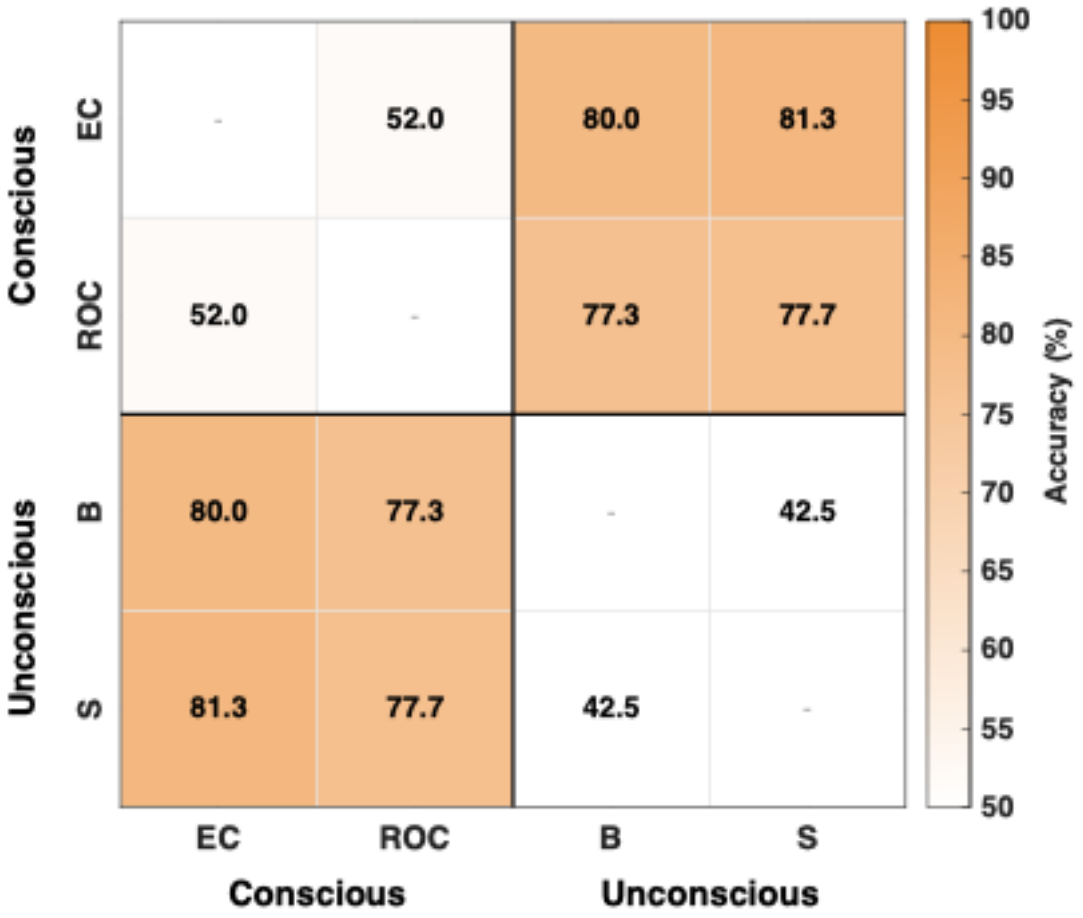
The permutation entropy derived from EEG phase dynamics can serve as an effective indicator of brain states in relation to consciousness levels.

Park, Y., Cha, Y., Kim, H., Kim, Y., Woo, J. H., Cho, H., ... & Moon, J. Y. (2026). *Current Biology*, 36, 1.

I. Results

Classification accuracy of a decision tree for distinguishing conscious and unconscious states in the general anesthesia dataset

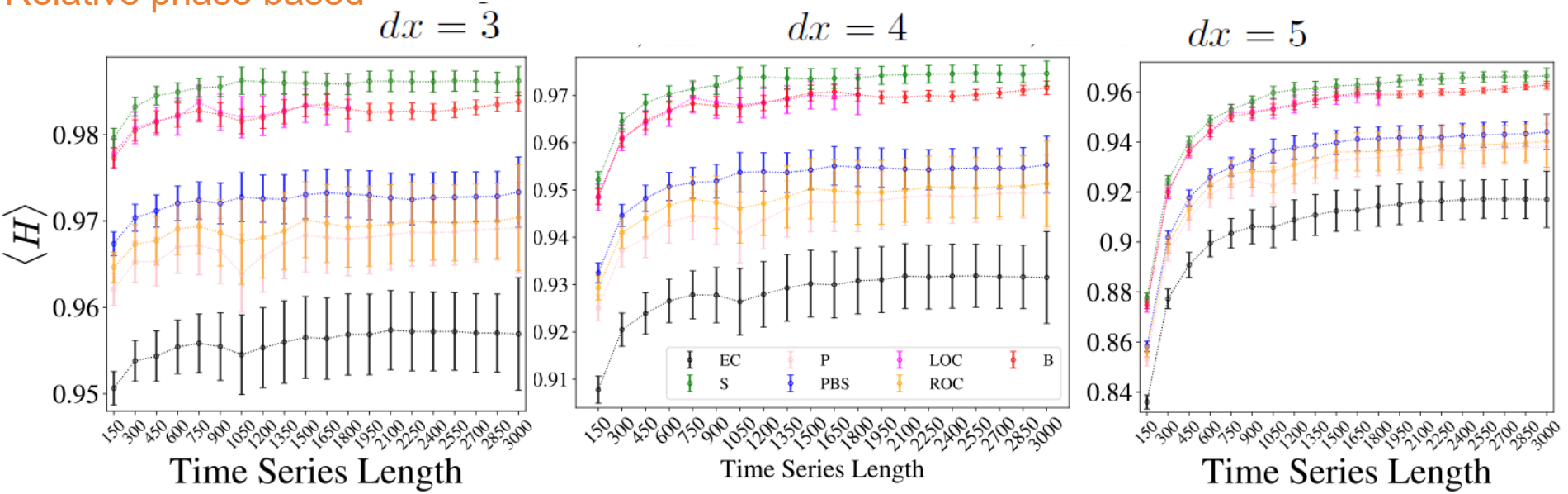
Permutation entropy as a classification feature



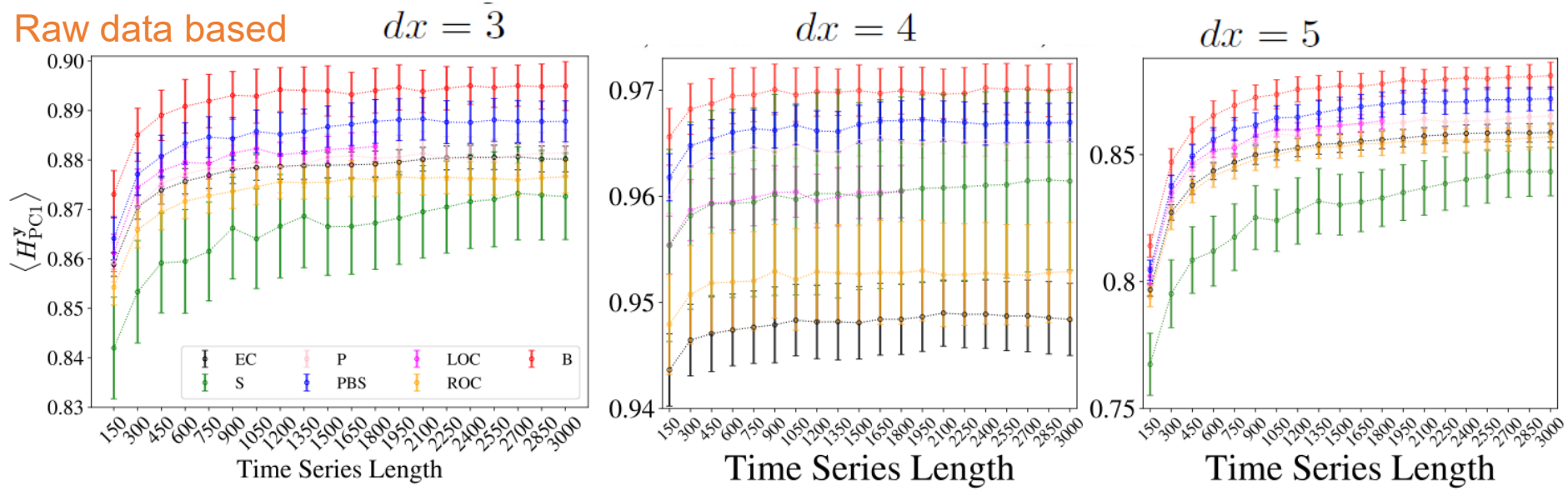
- Shows a clear distinction between conscious (EC and ROC) and unconscious (B,S) states.
- Brain states with different levels of consciousness are readily distinguishable.
- Permutation entropy can serve as an effective feature for distinguishing between conscious and unconscious states.

I. Results: Comparison with raw data

Relative phase based



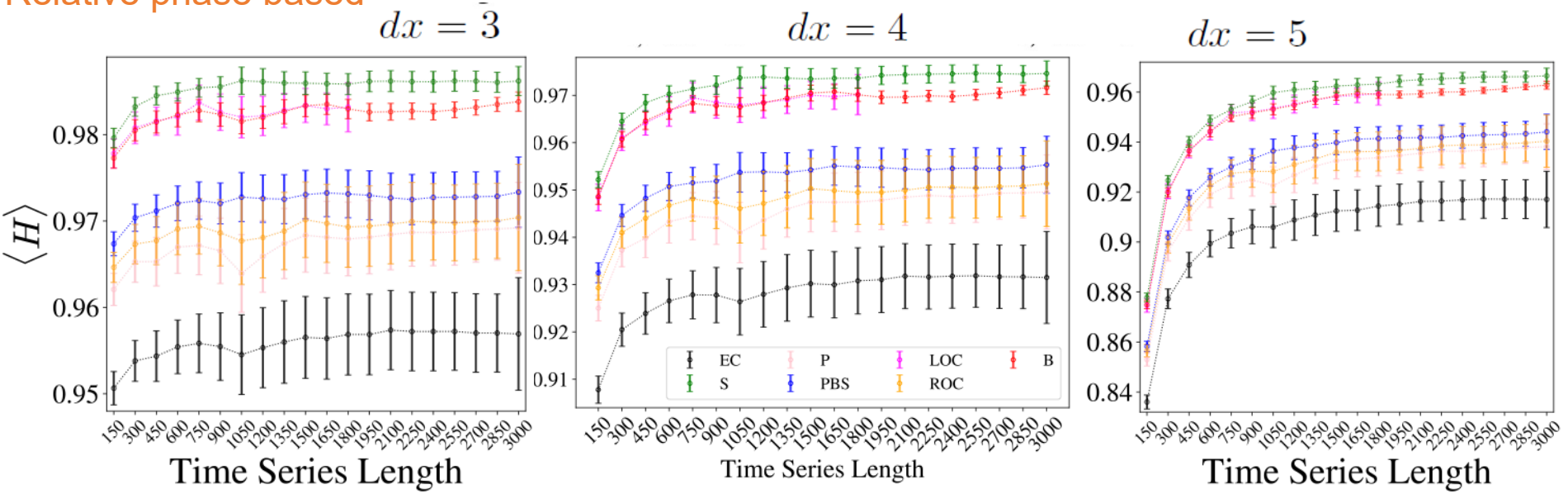
Raw data based

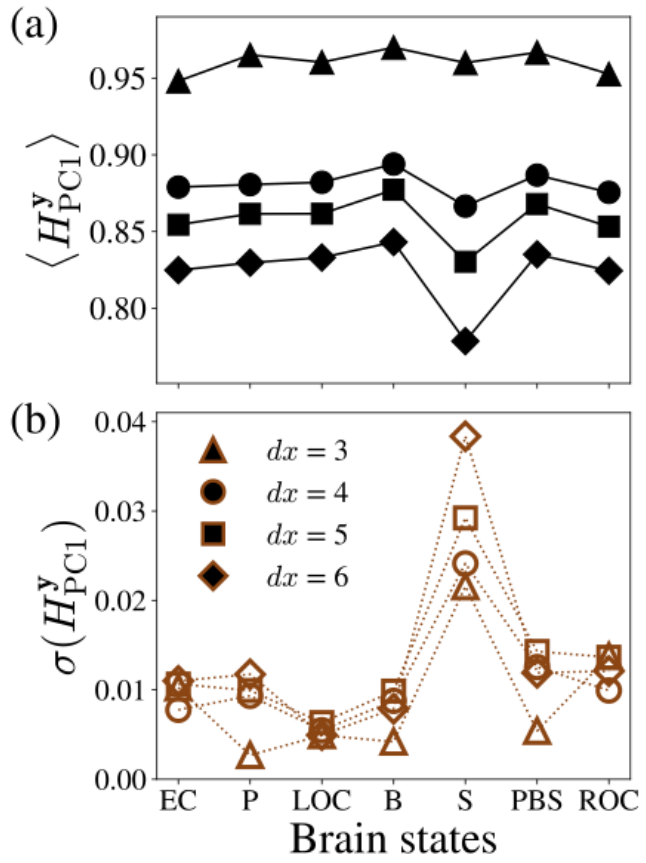


Raw-data-based permutation entropy (1) requires a larger number of data points to reliably estimate the ordinal pattern statistics, (2) exhibits a strong dependence on the embedding dimension.

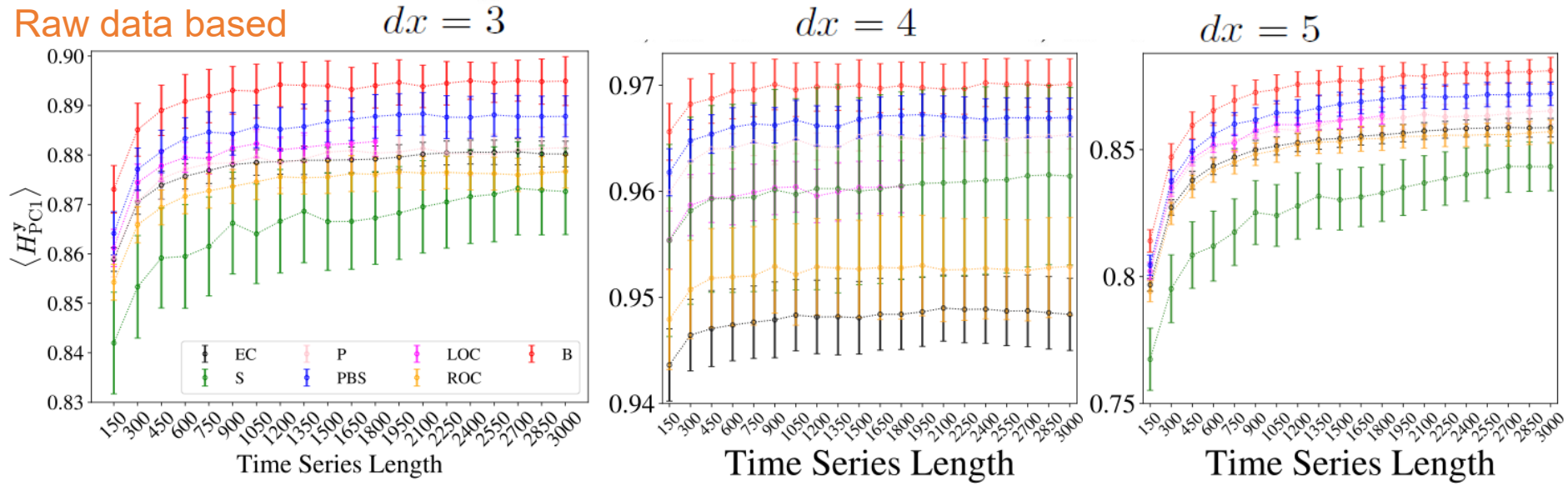
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Relative phase based





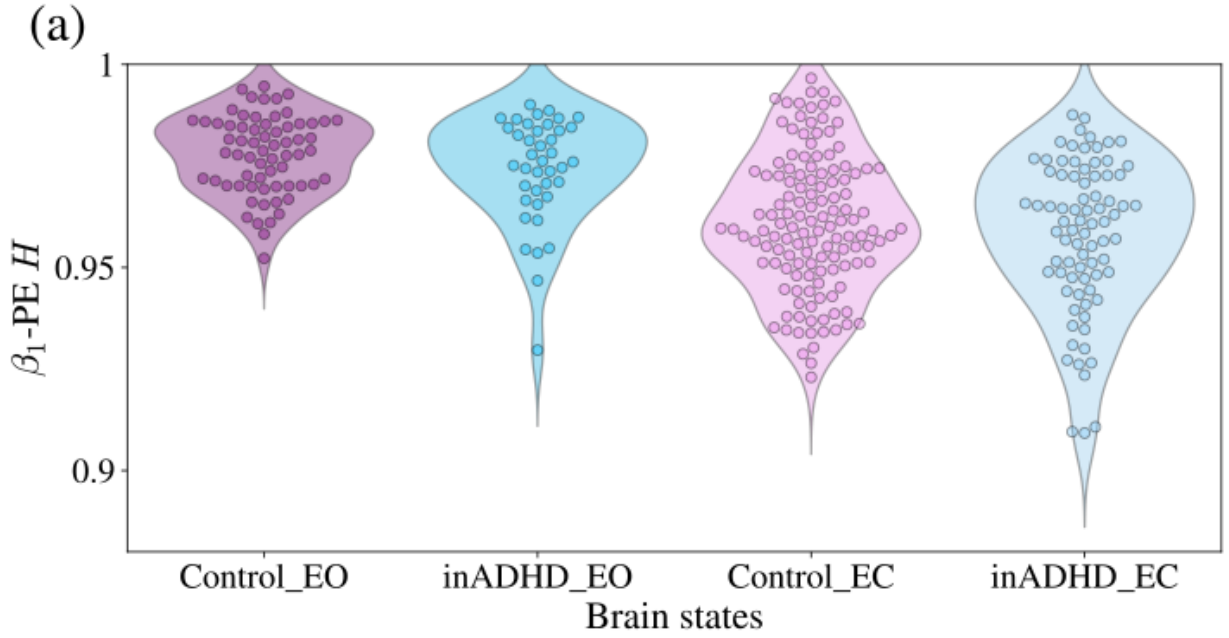
Raw data based

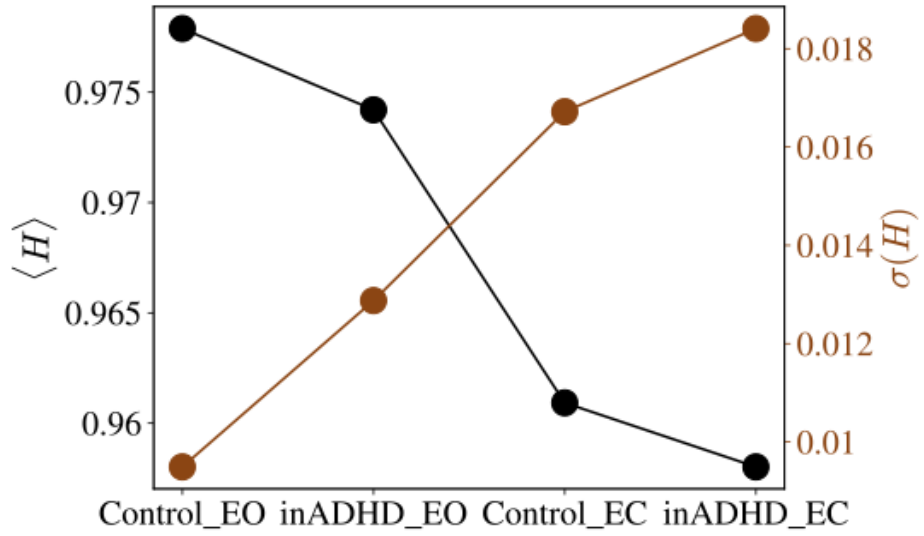


(3) does not show a clear correlation with brain states.

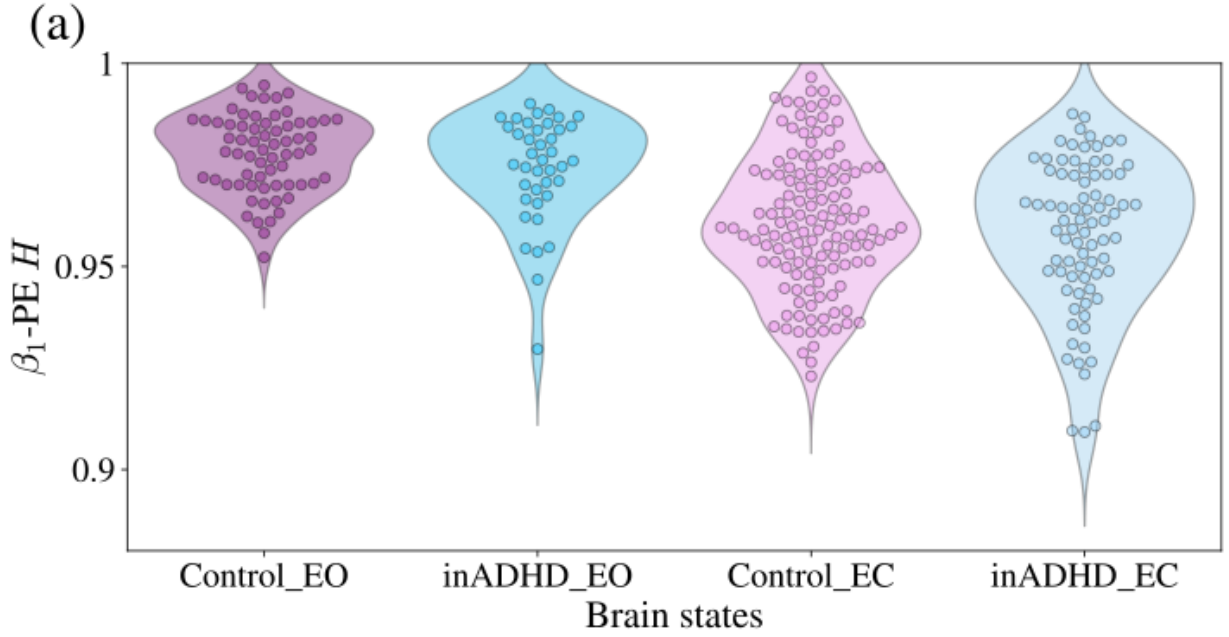
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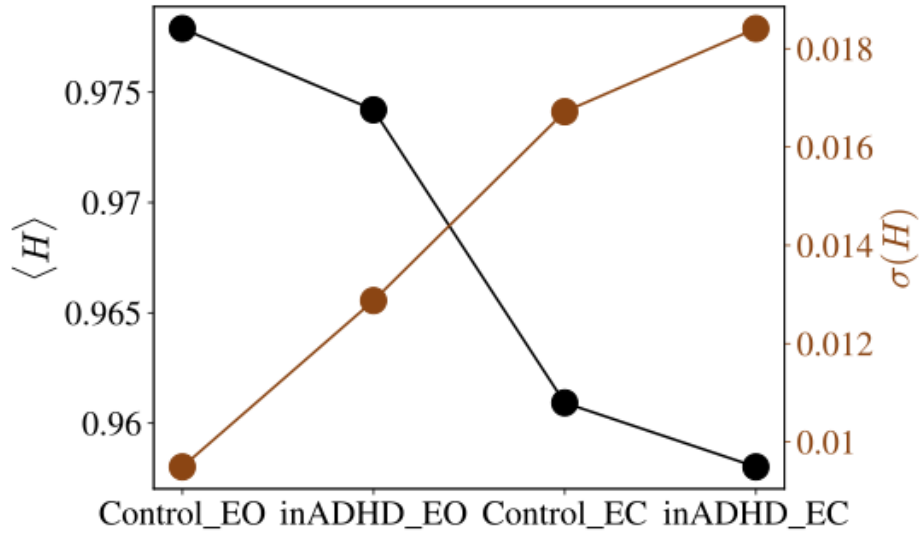
II. Results: Healthy controls and Inattentive-type ADHD across eyes-open and eyes-closed resting states



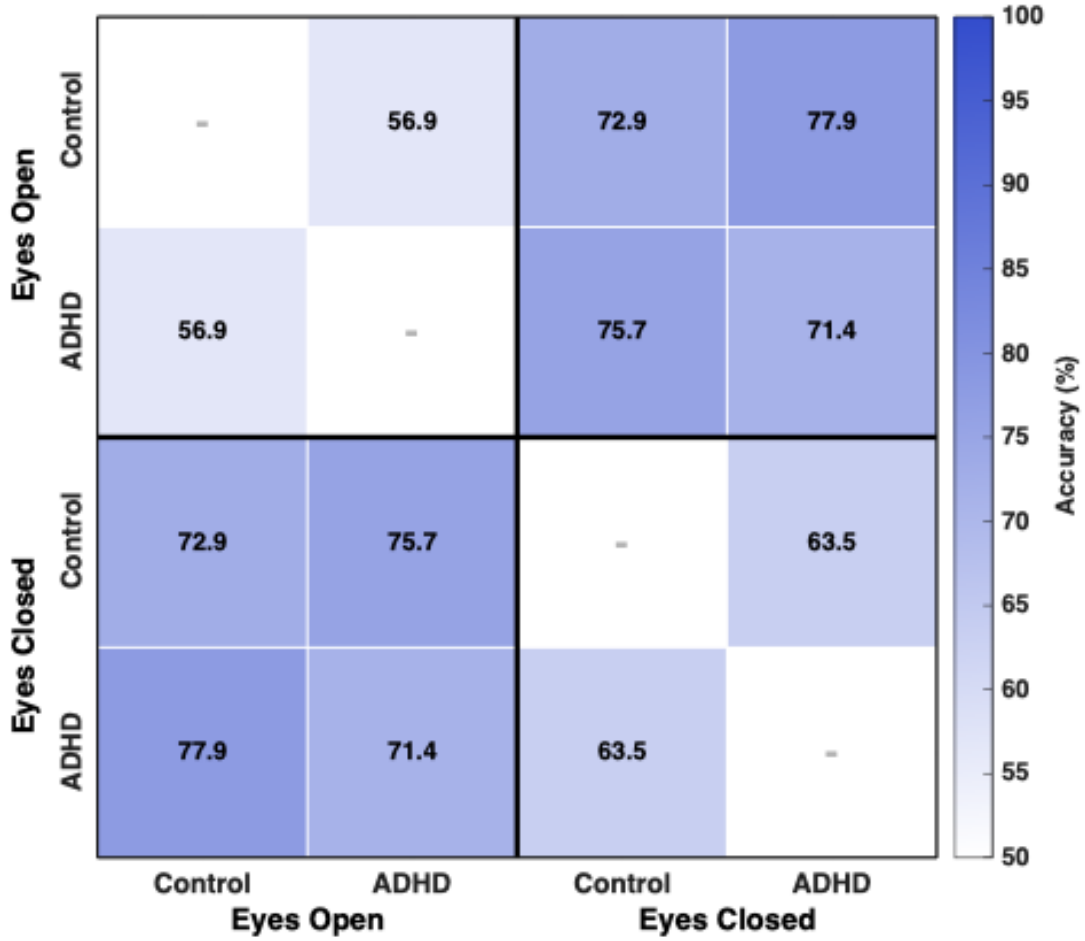


II. Results: Healthy controls and Inattentive-type ADHD across eyes-open and eyes-closed resting states





Classification accuracy of a random forest for distinguishing resting states and inattentive type ADHD condition



Concluding remarks

- Permutation entropy distributions of EEG phase dynamics exhibit a clear dependence on brain state.
- In particular, conscious, inattentive-type ADHD, and eyes-closed conditions show lower mean values and larger standard deviations of permutation entropy.
- Permutation entropy computed directly from raw EEG data is highly sensitive to the choice of embedding parameter and does not provide a consistent characterization of brain states. In contrast, the phase-based representation yields a low-dimensional signal that preserves state-dependent structure and enables robust estimation of ordinal pattern statistics.
- The distinction between conscious and unconscious states can be reliably captured in the general-anesthesia dataset.
- In the resting-state dataset, eyes-open and eyes-closed conditions are distinguishable.
- Classification between control and inattentive-type ADHD groups does not show clear separability.
- Overall, permutation entropy derived from EEG phase dynamics provides an effective indicator of brain states, particularly in relation to consciousness, while also highlighting its limitations for identifying individuals with inattentive-type ADHD.

Thank you all
for your kind attention!