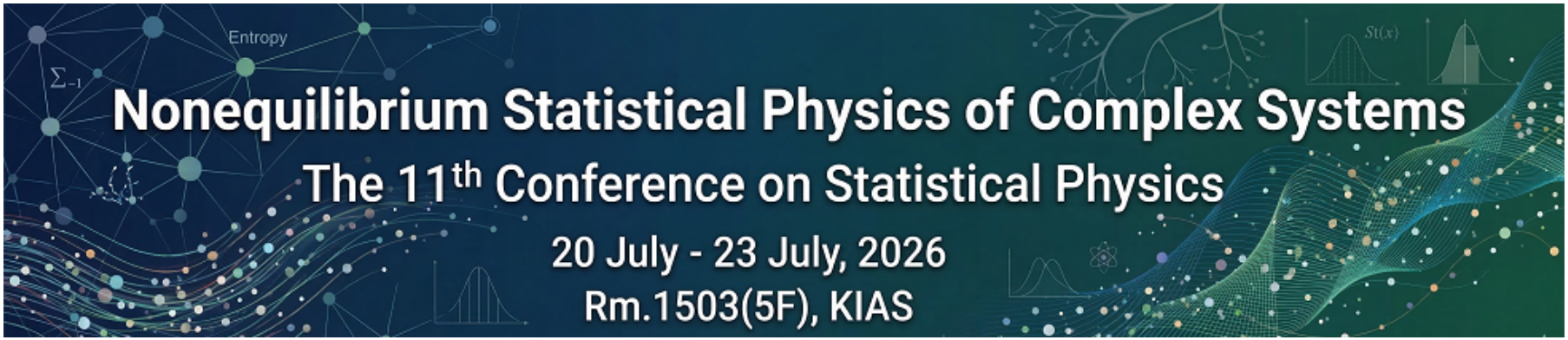
Thermodynamic cost of finite-time erasure

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Based on work[1] with



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Technische Universität
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Institut für Theoretische Physik II
Heinrich-Heine-Universität
Düsseldorf



Francesco Mori

Research Associate
Harvard Center of Mathematical
Sciences and Applications

[1] Mori, Olsen, and Krishnamurthy 2023; Olsen et al. 2023; Gupta, Olsen, and Krishnamurthy 2025.

Information Erasure $\Rightarrow$ Bit reset and Landauer principle

- A Bit has two states 0 and 1.

- An ”Erasure” involves resetting the bit to 1 by some means independent of the initial state.

- Such a procedure necessarily involves a ”Cost” if the operation is performed in an environment at temperature T. This cost can be termed as the *Work done* or the *Heat dissipated*.

- *Minimum* average work or *Minimum* average heat dissipated $\Rightarrow T\Delta S$

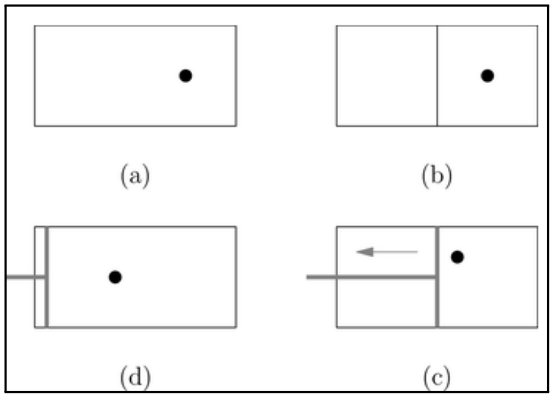
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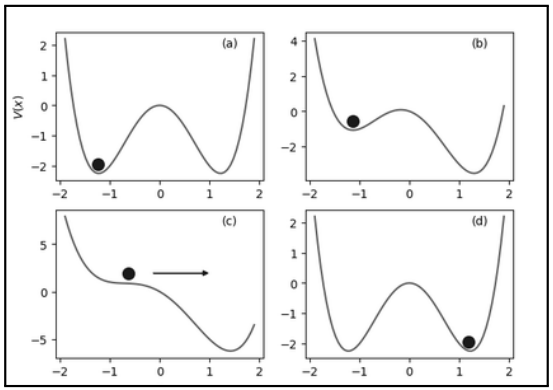
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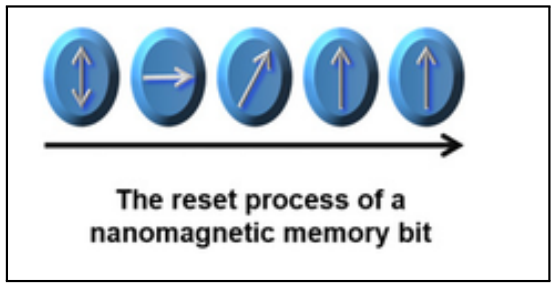
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A. Bérut *et al*, Nature 483, 187 (2012); Y. Jun, M. Gavrilov and J. Bechhoefer, PRL, 113 (2014); Jeongmin Hong *et al.* , Sci. Adv.2 (2016).

Information Erasure (Landauer Principle)

- Transforming an initial equilibrium distribution into a target (equilibrium) distribution

Equilibrium

- $\Delta U = Q + W$
- $\langle \Delta S_{\mathrm{Tot}} \rangle \geq 0$
- $\Delta F^{\mathrm{eq}} = \langle \Delta U \rangle - T \langle \Delta S_{sys} \rangle$
- $\langle W \rangle \geq \Delta F^{\mathrm{eq}}$

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Equilibrium

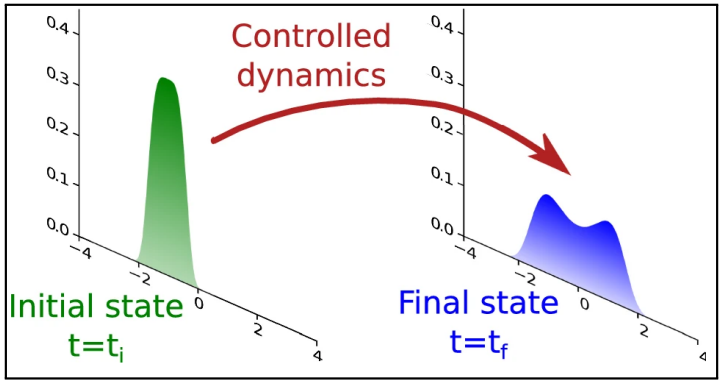
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- $\langle W \rangle \geq \Delta F^{\mathrm{eq}}$

Landauer bound for erasing one bit of information (any protocol in the *slow* limit):

$$\langle W \rangle_{\mathrm{min}} \geq \Delta F^{\mathrm{eq}} = -T \langle \Delta S_{\mathrm{sys}} \rangle = k_B T \log(2)$$

Information Erasure more generally

• Transforming an initial probability distribution into a target distribution in a given time τ using a *protocol* $\lambda(t)$. What is the minimum cost?

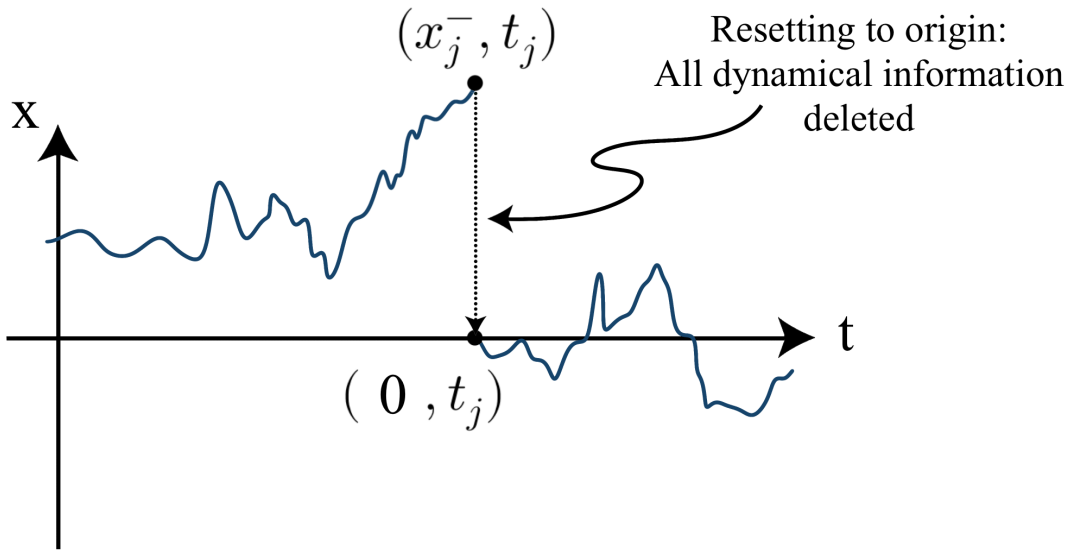


• Many results for classical and quantum systems in the form of *speed limits*.

An application in the field of Stochastic resetting

Resetting

At either random times or periodically, the system is instantaneously reset to its initial state.



[2]

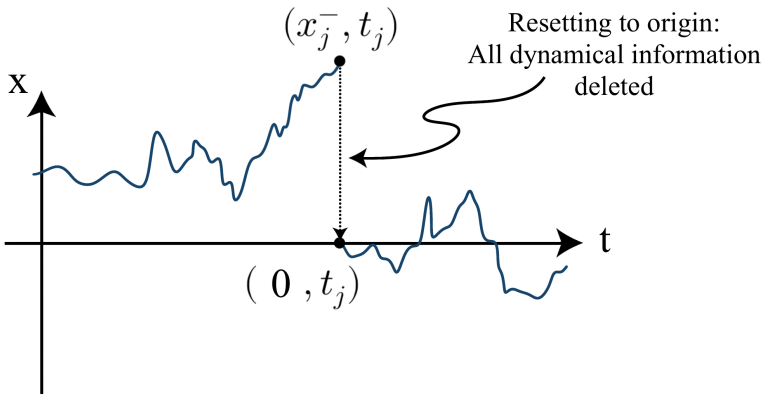
$$x(\tau + dt) = \begin{cases} x(\tau) - U'(x)dt + \sqrt{2D}\eta(\tau)dt & \text{with probability } 1 - rdt\,, \\ x_{\rm res} & \text{with probability } rdt\,, \end{cases}$$

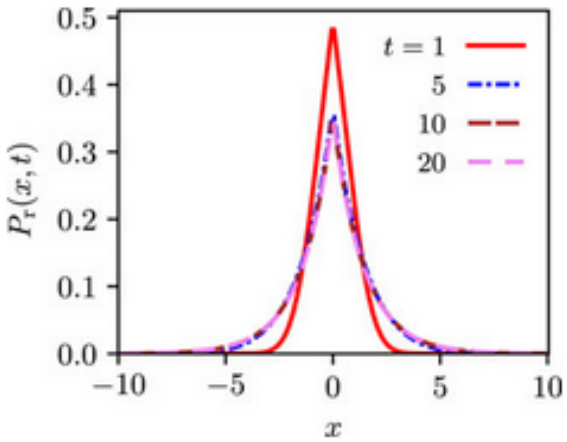
In steady state

$$j_{\rm st}(x) = -D\partial_x p_{\rm st}(x) - V'(x)p_{\rm st}(x)\,.$$

[2] Evans and Majumdar 2011; Evans, Majumdar, and Schehr 2020.

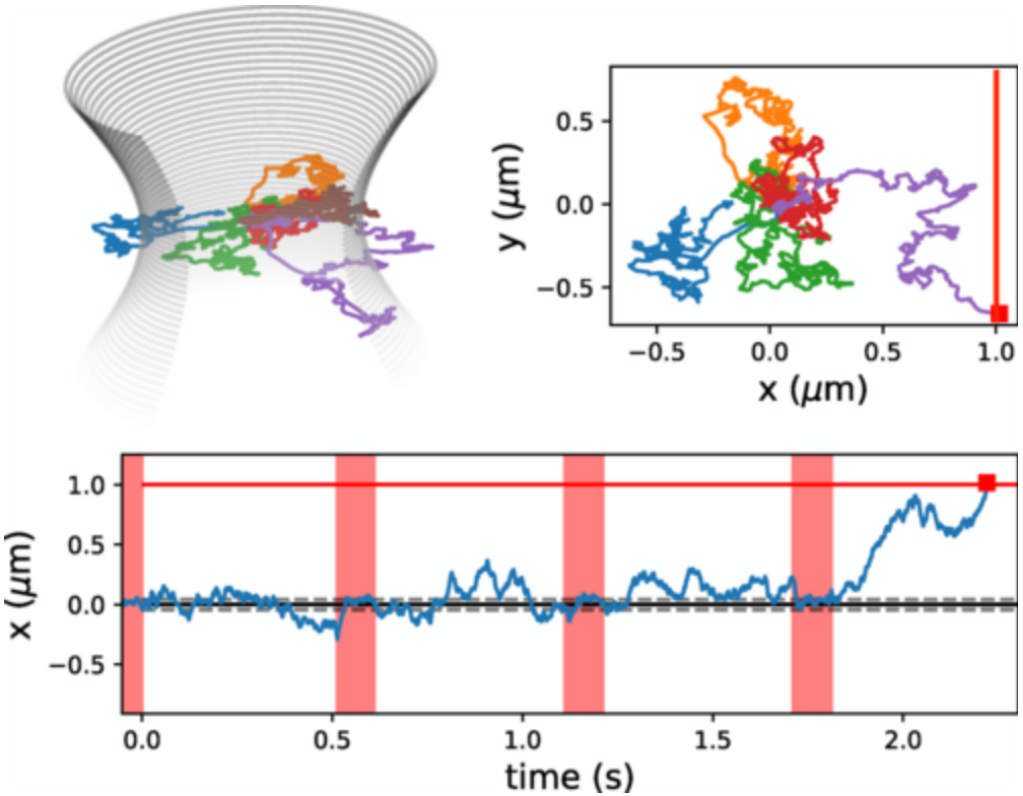
An application in the field of Stochastic resetting



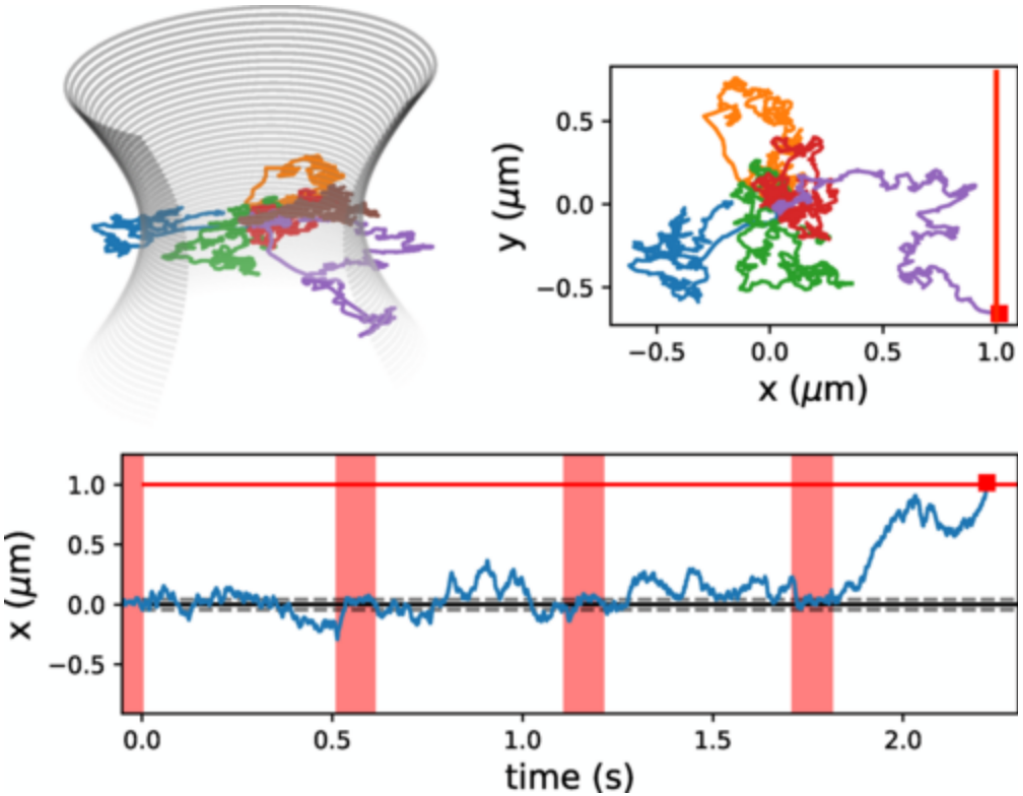


$$\partial_t p(x,t) = -\partial_x j(x,t) - rp(x,t) + r\delta(x)$$

A recent experimental setup for implementing stochastic resetting

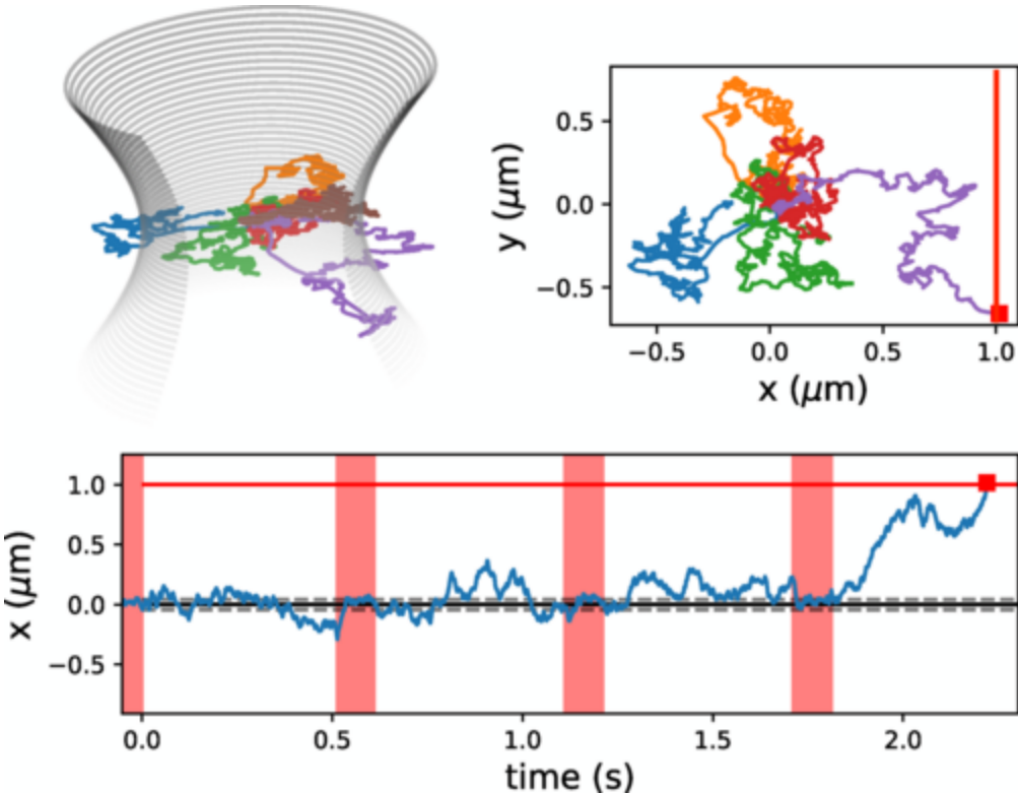


A recent experimental setup for implementing stochastic resetting



- What is the thermodynamic cost of implementing such a process?

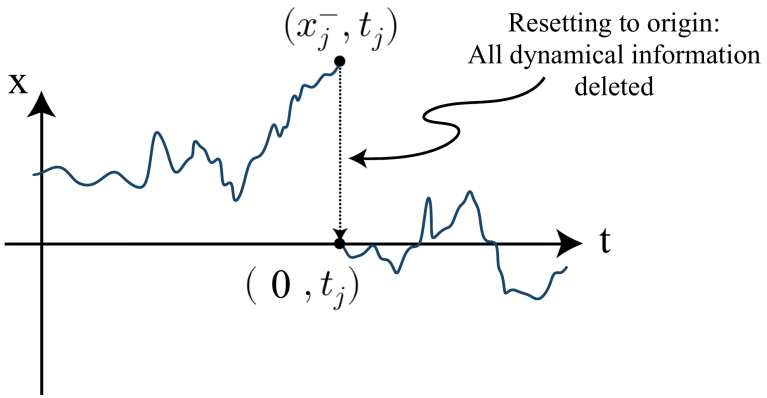
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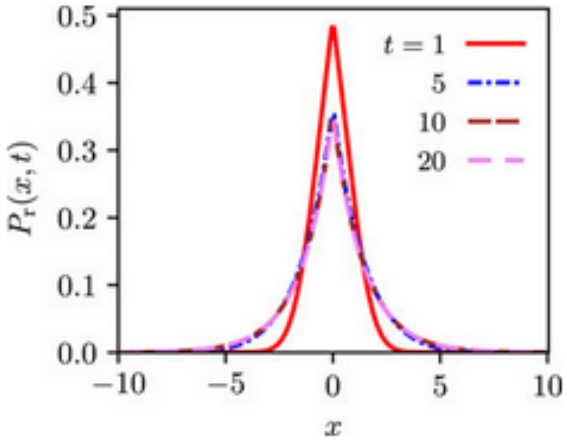


- What is the thermodynamic cost of implementing such a process?
- What is the *minimal* thermodynamic cost of implementing such a process?

B. Besga, A. Bovon, A. Petrosyan, S. N. Majumdar and S. Ciliberto, Phys. Rev. Res. **2** 032029 (2020)

An application in the field of Stochastic resetting





$$\partial_t p(x,t) = -\partial_x j(x,t) - rp(x,t) + r\delta(x)$$

- A slightly modified version $\delta(x) \to p_R(x)$[3] for which thermodynamic costs can be assigned consistently.

- The "thermodynamic" (Entropic) cost of $p_{\rm st}(x) \to p_R(x)$[4]

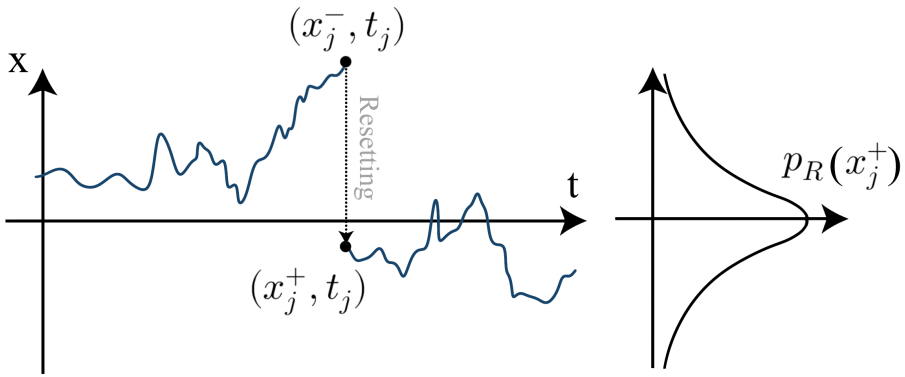
- *Other* costs have also been considered[5]

[3] Evans, Majumdar, and Schehr 2020.

[4] Mori, Olsen, and Krishnamurthy 2023.

[5] Fuchs, Goldt, and Seifert 2016; Sunil et al. 2023.

Entropy production of resetting processes[6]



$$\partial_t p(x,t) = -\partial_x j(x,t) - r p(x,t) + r p_R(x)$$

$$p_R(x) = \frac{1}{2a} e^{-|x|/a} \rightarrow \text{V-shaped confining potential}$$

$$p_R(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/(2\sigma^2)} \rightarrow \text{Harmonic confining potential}$$

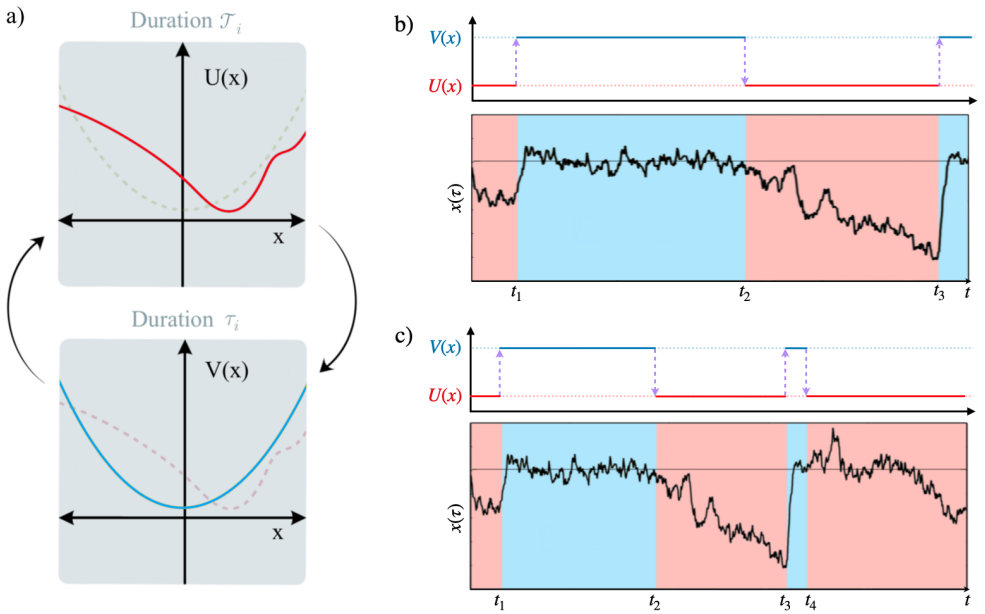
If resetting is implemented by allowing equilibriation in the resetting potential.

[6] Mori, Olsen, and Krishnamurthy 2023.

Work done for implementing resetting

$$\frac{dx_t}{dt} = -\gamma^{-1}\nabla\mathcal{U}[x_t, \lambda(t)] + \eta(t)\,,$$

$$W = \int_0^\tau \frac{\partial\mathcal{U}}{\partial\lambda}\frac{d\lambda}{dt}dt$$

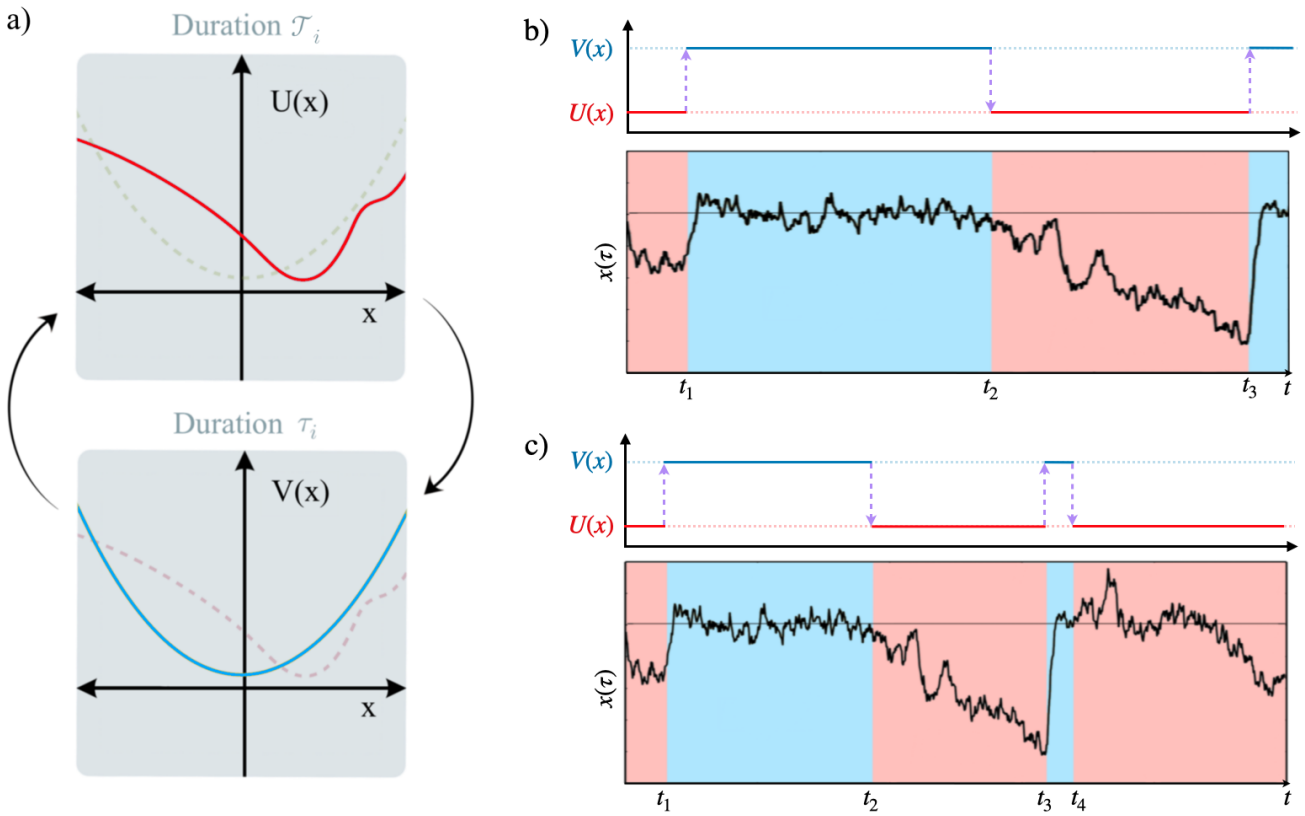


[7] Olsen et al. 2023.

Work done for implementing resetting[8]

The work done to reset the system is the net energy change at the instances $U(x) \leftrightarrow V(x)$

$$w[\{x_t\}_0^t] \equiv \sum_{j=1}[U(x^+_{2j-2}) - V(x^-_{2j-2})] + \sum_{j=1}[V(x^+_{2j-1}) - U(x^-_{2j-1})]$$



[8] Olsen et al. 2023.

Interesting Open questions

MGF For Heat dissipated while resetting?

$$q[\{x_\tau\}_0^t] \equiv \sum_{j=1}[U(x_{2j-2}^+) - U(x_{2j-1}^-)] + \sum_{j=1}[V(x_{2j-1}^+) - V(x_{2j}^-)]$$

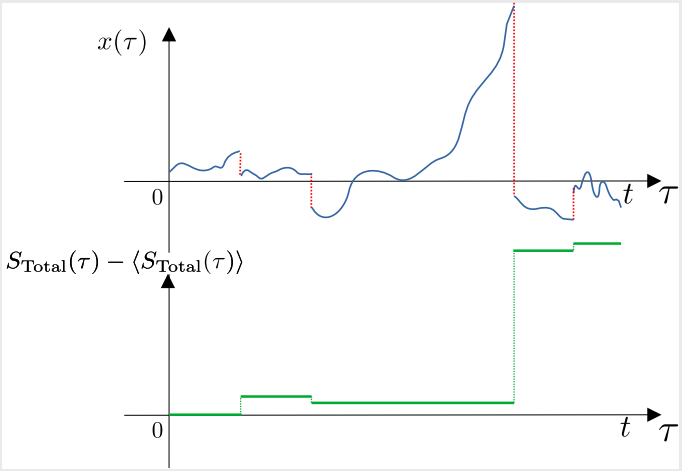
$$q[\{x_\tau\}_0^t] \equiv \sum_{j=1}[U(x_{2j-2}^+) - U(x_{2j-1}^-)] + \sum_{j=1}\log\frac{p_R(x_{2j}^+)}{p_R(x_{2j-1}^+)}$$

Gaussian resetting distribution : Condensation transition

$$p_R(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/(2\sigma^2)}$$

$$p(s) \approx \sqrt{\frac{e}{2s}} e^{-\sqrt{2s}}$$

$\Rightarrow$ S_{tot} rate function shows a condensation transition



- Does the MGF for the work distribution show such a transition?

- What is the **minimal** thermodynamic cost for performing stochastic resetting by switching potentials?

- What is the **minimal** thermodynamic cost for performing stochastic resetting by switching potentials?

- Initial distribution is not an equilibrium distribution!

- The **time** needed for the transformation needs to be taken into account.

Information Erasure: Initial State

- If the initial state is not an equilibrium state $\rightarrow \Delta F^{\mathrm{neq}}$

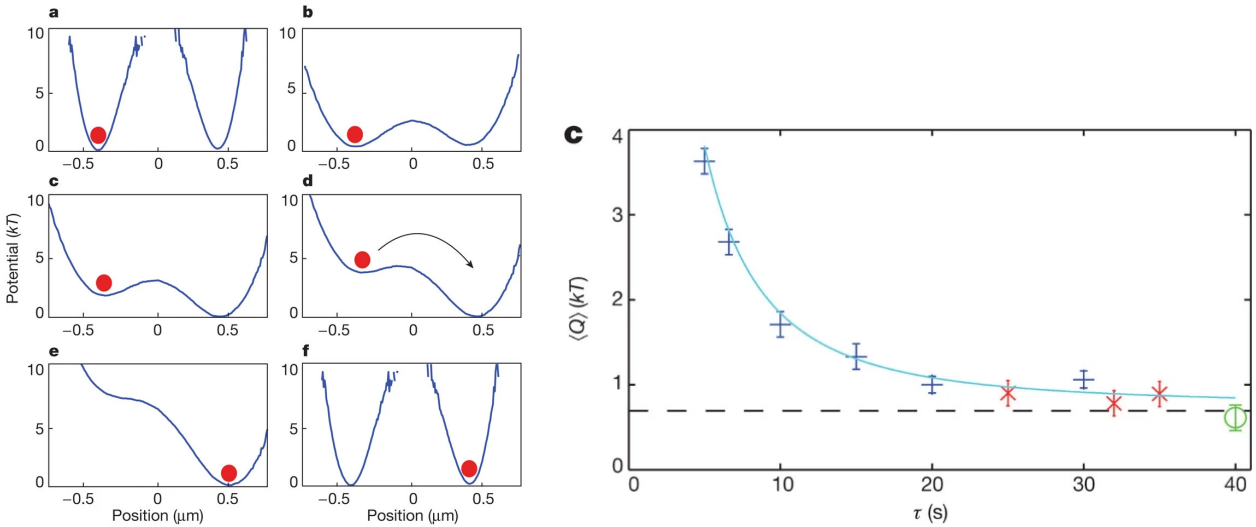
$$F^{\mathrm{neq}} = F^{\mathrm{eq}} + k_B T D_{\mathrm{KL}}\left[p||p^{\mathrm{eq}}\right]$$

$$\langle W \rangle - \Delta F^{\mathrm{neq}} \geq 0$$

B. Gaveau and L. Schulman (1997), M. Esposito and C. Van den Broeck (2011), J.M. R Parrondo, J. M. Horowitz and T. Sagawa (2015)

Finite-time Information Erasure

- **Experiments on the work cost of Erasure: Overdamped systems**

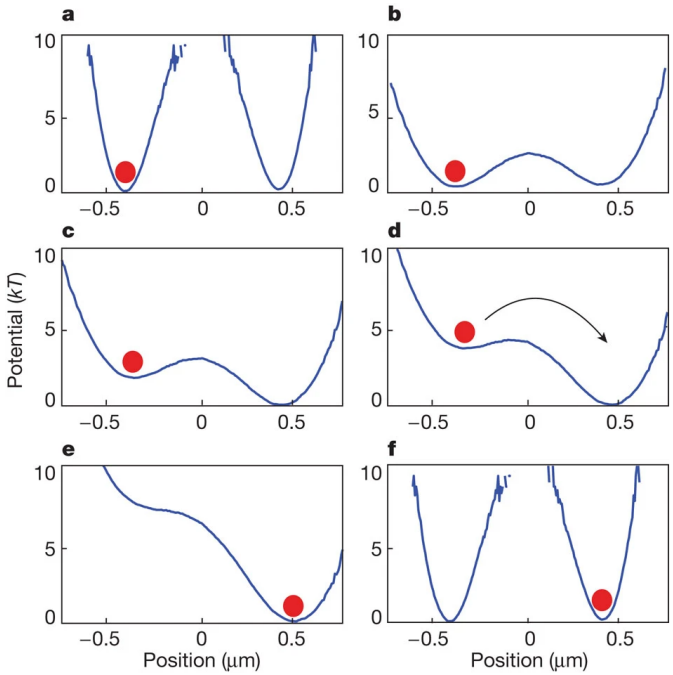


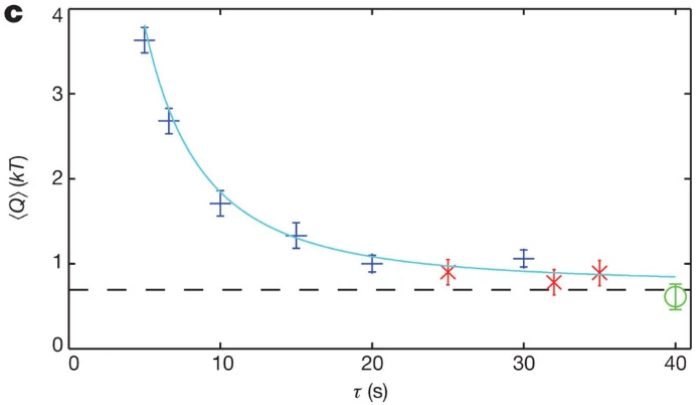
$$\langle W \rangle_{\min} \geq \Delta F^{\mathrm{eq}} = -T \langle \Delta S_{\mathrm{sys}} \rangle = k_B T \log(2)$$

- $\langle W \rangle_{\min}$ achieved for the slowest protocol (when $\langle \Delta S_{tot} \rangle = 0$).
- $\langle W \rangle \sim \Delta F^{\mathrm{eq}} + \frac{B(\tau)}{\tau}$

A. Bérut, A. Arakelyan, A. Petrosyan, S. Ciliberto, R. Dillen- schneider, and E. Lutz, **Experimental verification of landauer's principle linking information and thermodynamics**, Nature 483, 187 (2012); Y. Jun, M. Gavrilov and J. Bechhoefer, **High-Precision Test of Landauer's Principle in a Feedback Trap**, PRL, 113 (2014)

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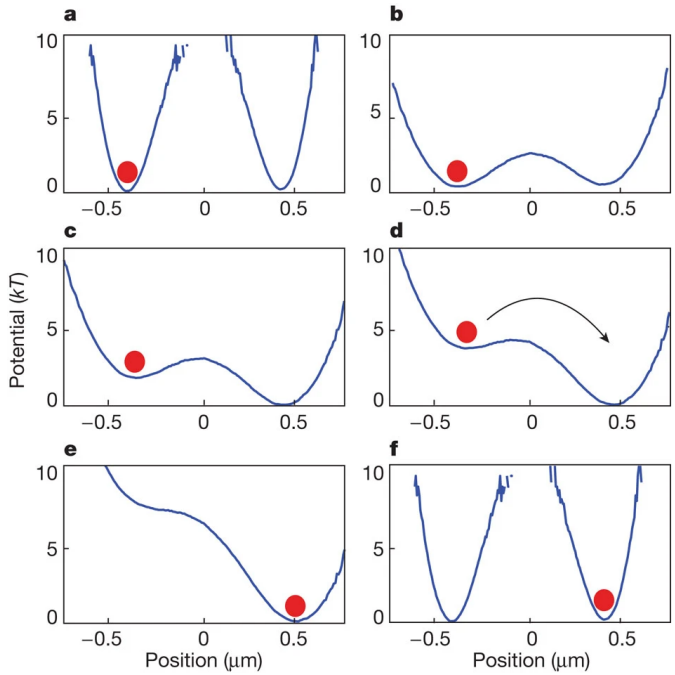


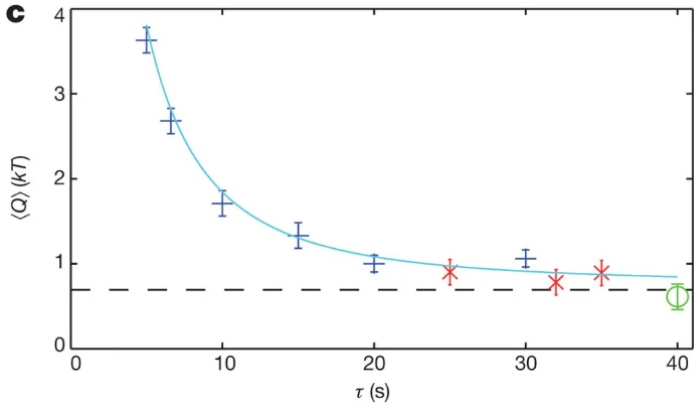


$$\langle W \rangle \geq \Delta F^{\mathrm{neq}} + \frac{B_{\mathrm{opt}}(\tau)}{\tau}$$

E. Aurell, C. Mejía-Monasterio and P. Muratore-Ginanneschi, Phys. Rev. Lett. 106 (2011); K. Prosemans, J. Ehrich and J. Bechhoefer, PRL 125, 100602 (2020); M. Nakazato and S. Ito, PRR, 3 (2021); A. Dechant and Y. Sakurai, arXiv:1912.08405; S. Oikawa *et al* Nat Commun 16, 10424 (2025).

- **Experiments on the work cost of Erasure: Overdamped systems**





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$B_{\mathrm{opt}} \rightarrow$ Wasserstein distance square

L^2- Wasserstein Distance

Example: Wasserstein distance between two Gaussians

$$p(x) = \frac{1}{\sqrt{2\pi\sigma_a}} e^{\frac{(x-\mu_a)^2}{2\sigma_a}}$$

$$q(x) = \frac{1}{\sqrt{2\pi\sigma_b}} e^{\frac{(x-\mu_b)^2}{2\sigma_b}}$$

$$\mathcal{W}(p,q)^2 = (\mu_a - \mu_b)^2 + (\sqrt{\sigma_a} - \sqrt{\sigma_b})^2$$

(Recurrent) Erasure using a time-varying potential

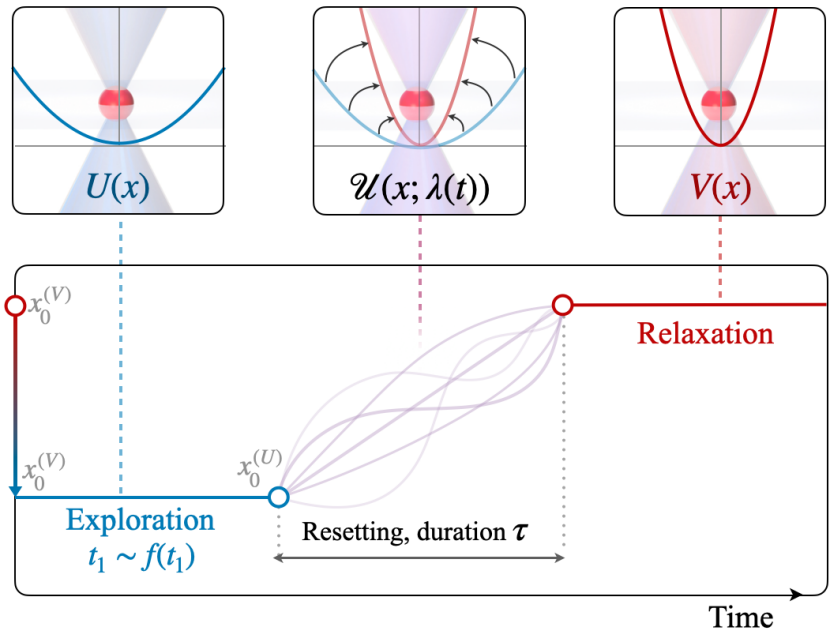
Erasure Scheme

i) A particle moves in a potential $U(x)$ for a duration $t \in f(t)$.

ii) $U(x)$ is transformed into $V(x)$ **over time** τ.

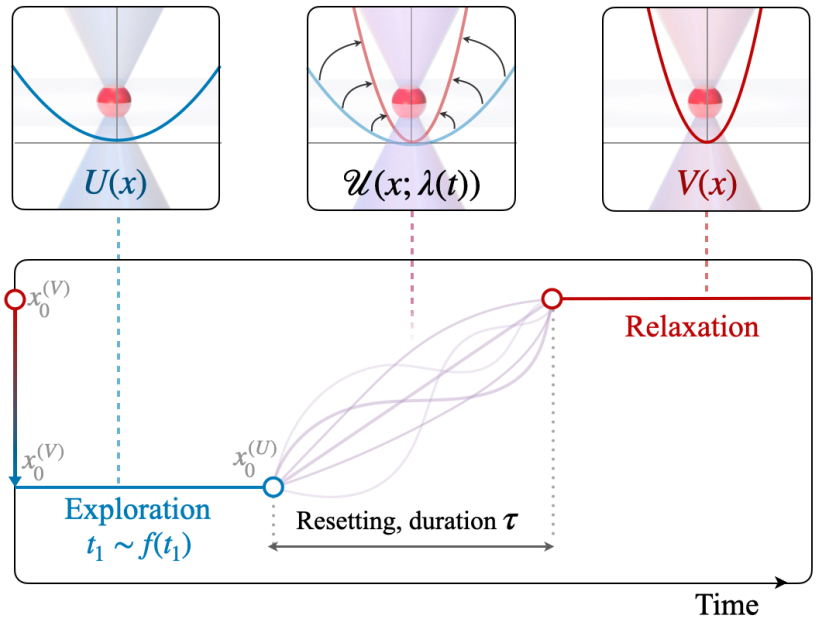
iii) The particle relaxes in $V(x)$ to reach equilibrium over a duration $t \in \psi_{\rm on}(t)$.

iv) The potential V is switched abruptly to U and system begins again at step i)



D. Gupta, K.S.Olsen and S.Krishnamurthy, Comm.Phys. **8**,355 (2025), K.S.Olsen, D. Gupta *et al*, PRR 6, 033343 (2023)
Many earlier works studying work cost beginning from equilibrium : T. Schmiedl and U. Seifert PRL (2007), T. Speck, J. Phys A (2011), C. Kwon *et al* (PRE 2013), A. Ryabov *et al* J. Phys. A Math. Theor. 46, 075002 (2013).
Studying intermittent potentials: Mercado-Vásquez *et al* (2020), I. Santra *et al* (2021); Alston et al (2022).

Recurrent erasure: Overdamped description

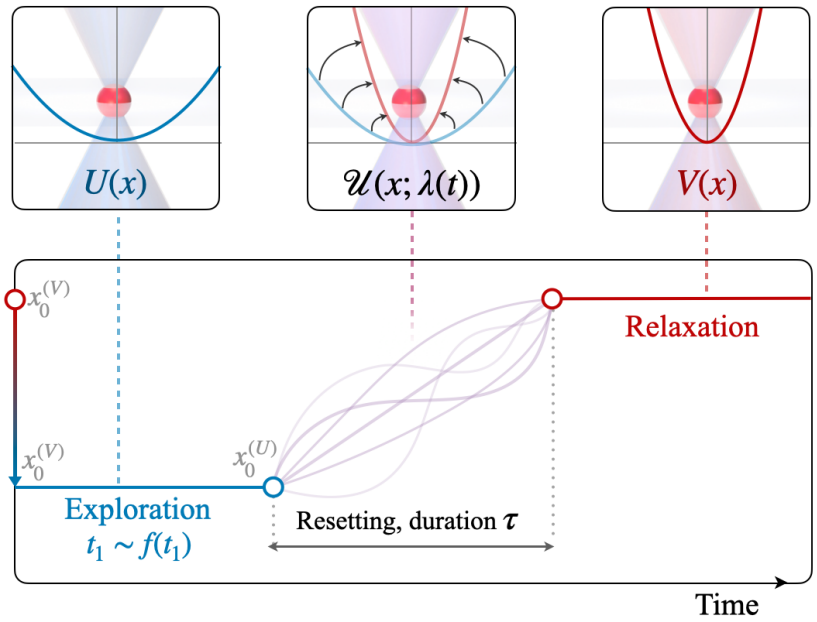


$$\frac{dx}{dt} = -\beta D \frac{\partial \mathcal{U}(x;\lambda)}{\partial x} + \sqrt{2D}\,\eta(t)\,,$$

$$w_1(x) \equiv U(x) - V(x)\,,$$

$$w_2[x(\cdot)] \equiv \int_0^\tau dt\,\dot{\lambda}\frac{\partial \mathcal{U}(x;\lambda)}{\partial \lambda} \quad ; \quad \mathcal{U} = \frac{\lambda(t)}{2}x^2$$

Recurrent erasure: MGF of total work



$$C(k,\tau) \equiv \langle e^{kw} \rangle \,,$$

$$\langle (w_1+w_2)^n \rangle = \int_0^\infty dt\; f(t) \int_{-\infty}^{+\infty} dx_0^{(V)}\; P_{\mathrm{eq},V}(x_0^{(V)})$$
$$\times \int_{-\infty}^{+\infty} dx_0^{(U)}\; P_U(x_0^{(U)}, t | x_0^{(V)})$$
$$\times \sum_{m=0}^{n} \binom{n}{m} w_1^m(x_0^{(V)}) \langle w_2^{n-m} \rangle_{x_0^{(U)}}$$

Out-of-equilibrium initial state

$$P_{\mathrm{ta,U}}(x_0^{(U)}) = \int_0^\infty dt\, f(t) \int_{-\infty}^{+\infty} dx_0^{(V)}\, P_{\mathrm{eq},V}(x_0^{(V)})\, P_U(x_0^{(U)}, t|x_0^{(V)})$$

A dimensionless parameter quantifying "non-equilibrium":

$$\zeta \equiv \int_{-\infty}^{+\infty} dx\, P_{\mathrm{ta,U}}(x)\, x^2 = \sqrt{1-(1-t_V/t_U)\tilde{f}(2/t_U)}$$

where $t_{V,U} \equiv (\beta D \lambda_{V,U})^{-1}$

$$W_2 = \frac{D}{2}\int_0^\tau dt\, \dot{\lambda}\left[t_U \zeta^2 G(t,0) + 4\int_0^t ds\, G(t,s)\right],$$

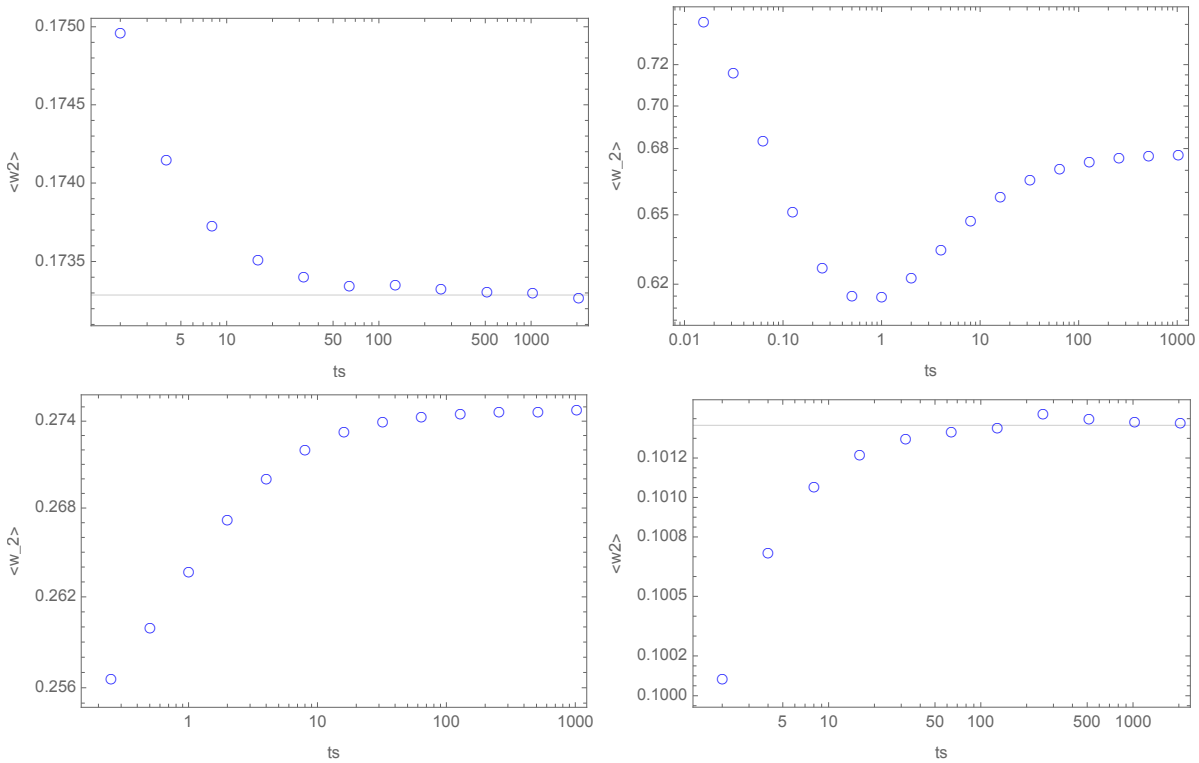
for

$$G(t,s) \equiv e^{-2\beta D \int_s^t dt' \lambda(t')}.$$

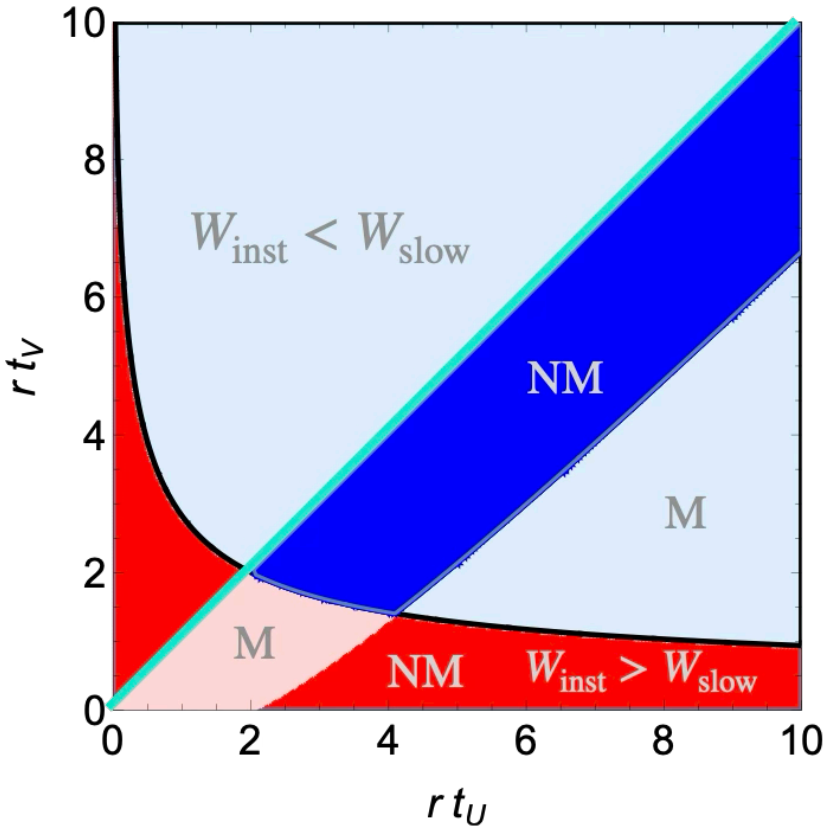
Parameters and protocols

Protocols $\lambda(t)$

(i) $\lambda(t) = \lambda_U + (\lambda_V - \lambda_U)\tanh[t/t^*]$; $f(t) = r\exp(-rt)$



Non-Monotonic average work



$$W_2 - \Delta F^{\rm eq}_{U\to V} = \alpha \frac{k_{\rm B}T}{4} t_U \left[\zeta^2 - \frac{1}{2}\left(1 + \frac{t_V^2}{t_U^2}\right)\right] + \mathcal{O}(\alpha^2)$$

where $\alpha \propto 1/t^*$.

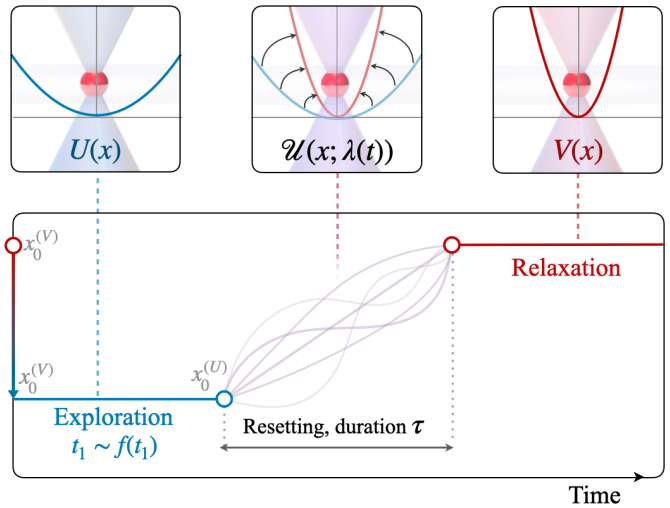
∗ Work fluctuations in a time-dependent harmonic potential: Rigorous results beyond the overdamped limit, Chulon Kwon, Jae Dong Noh and Hyunggyu Park, Phys. Rev. E, **88** (2013).

Optimal Protocols ⇔ Bounds

•$W_2[\lambda(t)]$ is a functional of $\lambda(t)$

• Minimization of $W_2 \rightarrow$ optimal protocol λ^*

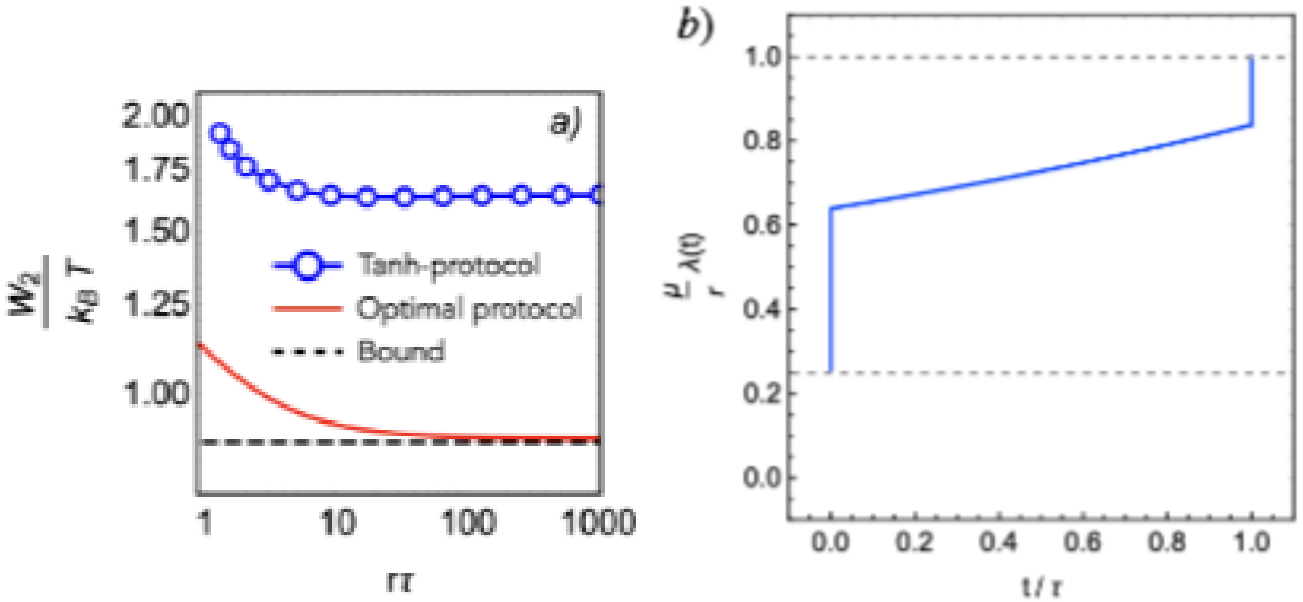
• The optimal protocol $\lambda_t^{\rm opt}$ is the one that minimizes the work functional, $\frac{\delta W_2[\{\lambda_t^{\rm opt}\}]}{\delta_{\lambda_t^{\rm opt}}} = 0.$



$$\lambda_t^{\rm opt} = \gamma \frac{1 - t_U \zeta^2 c_2 (1 + c_2 t)}{t_U \zeta^2 (1 + c_2 t)^2}$$

T. Schmiedl and U. Seifert PRL (2007)

Optimal Protocols ⇔ Bounds



D. Gupta, K. S. Olsen and S. Krishnamurthy, Comm.Phys. **8**,355 (2025)

Optimal Protocols $\Leftrightarrow$ full time dependence

$$
\begin{aligned}
W_2 =& \Delta F^{\text{eq}}_{U\to V} + \frac{k_{\text{B}}T}{2}\left[\ln\zeta^2 + (1-\zeta^2)\right] \\
& -k_BT\log\left[\frac{\sqrt{1+\frac{2t_V}{\tau}+\frac{\zeta^2 t_U t_V}{\tau^2}}}{(1+\frac{2t_V}{\tau})} + \frac{\sqrt{\zeta^2 t_U t_V}}{\tau(1+\frac{2t_V}{\tau})}\right] \\
& -k_BT\frac{t_U\zeta^2}{\tau}\left[\frac{1}{\zeta}\sqrt{\frac{t_V}{t_U}}\frac{\sqrt{1+\frac{2t_V}{\tau}+\frac{\zeta^2 t_U t_V}{\tau^2}}}{(1+\frac{2t_V}{\tau})} - \frac{(1+\frac{t_V}{\tau})}{(1+\frac{2t_V}{\tau})}\right]
\end{aligned}
$$

D. Gupta, K. S. Olsen and S. Krishnamurthy,Comm.Phys. **8**,355 (2025)

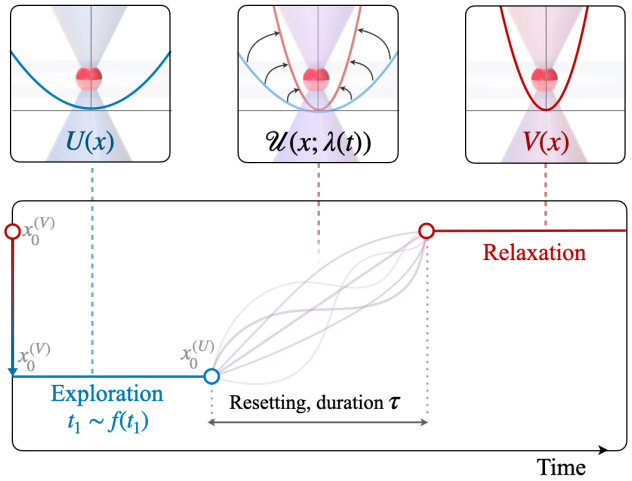
Optimal Protocols $\Leftrightarrow$ full time dependence of Mean work

$$\text{At large times: } W_2^* = \Delta F^{\text{eq}}_{U\to V} + \frac{k_{\text{B}}T}{2}\left[\ln\zeta^2 + (1-\zeta^2)\right] + \frac{k_B T}{2\tau}\left[\sqrt{2t_V} - \zeta\sqrt{2t_U}\right]^2$$

- $\frac{k_{\text{B}}T}{2}\left[\ln\zeta^2 + (1-\zeta^2)\right] \to -k_{\text{B}}T\, D_{\text{KL}}(\pi[P_{\text{ta},U}] \parallel P_U^{\text{eq}})$.
  - $\frac{k_B T}{2\tau}\left[\sqrt{2t_V} - \zeta\sqrt{2t_U}\right]^2 \to$ Wasserstein distance.

Outlook

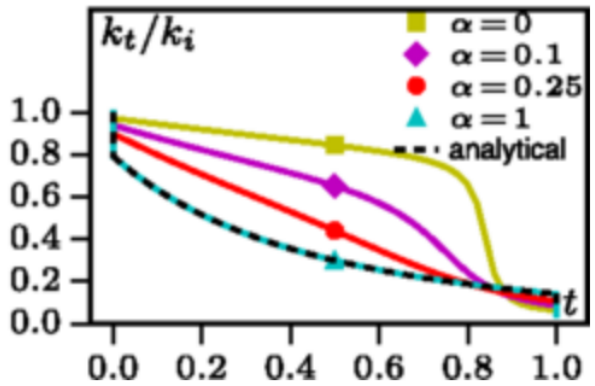
- Many particle systems: cost of dynamically emergent correlations [9]



[9] work with Deepak Gupta

Outlook

- Protocols to optimise work fluctuations? [10]



A.P. Solon and J. M. Horowitz, Phys. Rev. Lett. 120, 180605; S. Blaber and D. A. Sivak, J. Chem. Phys. 153, 244119 (2020).

[10] work with Felix Herber and Deepak Gupta

Thank you!



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