Dynamically Emergent Correlations

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Plan

- **Dynamically emergent correlations** (**DEC**) between **independent** particles via **stochastic** fluctuation of the environment

  $\Longrightarrow$ **general setting**

- A simple model: **Switching** confining potential (box/trap)

- Exact **nonequilibrium stationary state (NESS)**

$\Longrightarrow$ **Conditionally independent and identically distributed (CIID)**

- Recent experiments using optical tweezers

- Generalisation to other models with **stochastically driven** environment

- Summary and Conclusion

Dynamically Emergent Correlations (DEC)

General idea: **Independent** units/particles with no **direct interaction** between them $\Longrightarrow$ get **strongly correlated** in time due to a shared **stochastically fluctuating** environment

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Real life examples:



Two people walking
(not known to each other)

Suddenly it starts raining

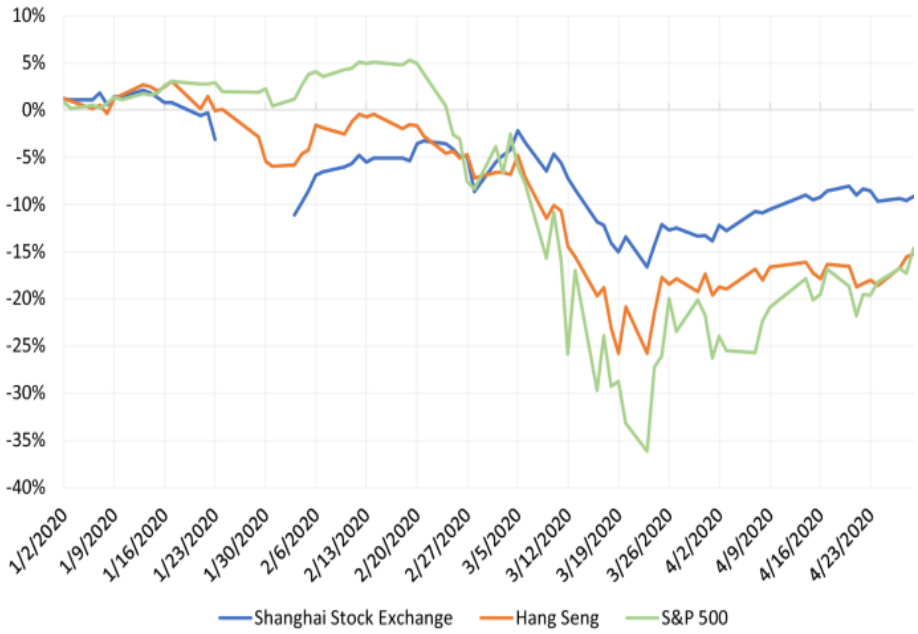
Both start running

An observer may think they influenced each other, but the **rain** influenced both

Dynamically Emergent Correlations (DEC)

General idea: **Independent** units/particles with no **direct interaction** between them $\Longrightarrow$ get **strongly correlated** in time due to a shared **stochastically fluctuating** environment

Real life examples:



Stock prices of unrelated companies may start evolving in a **correlated** way

due to sudden change of interest rates/oil prices/economic news

$\Longrightarrow$ **common fluctuating environment**

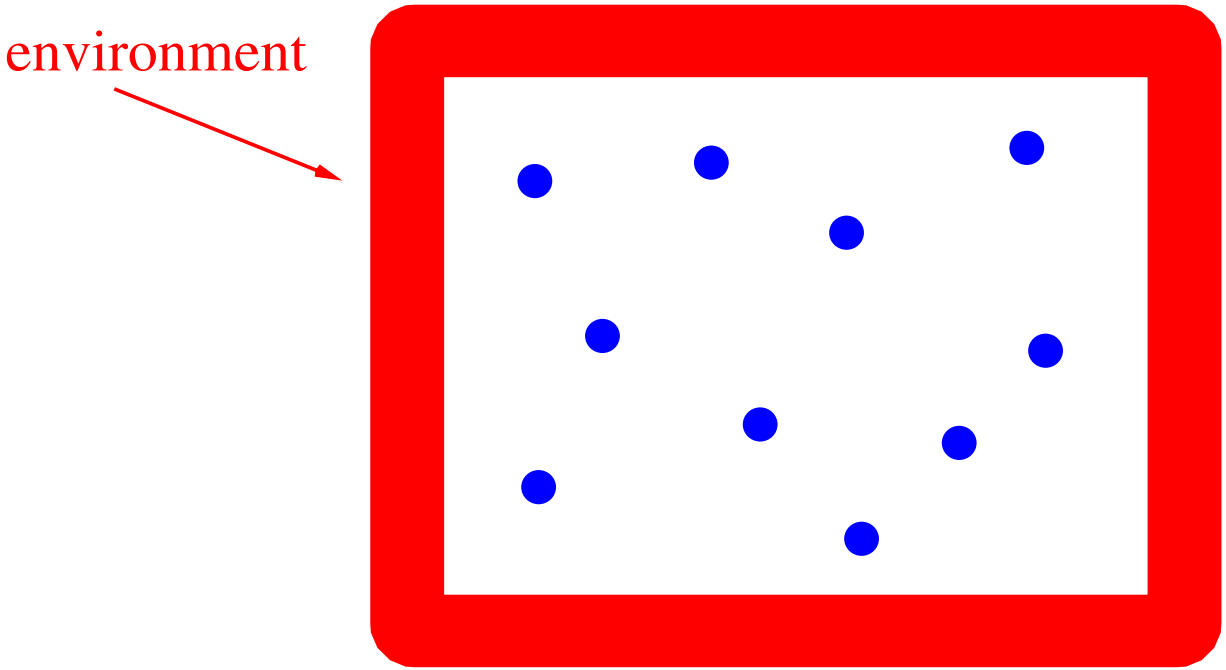
General Physical Setting

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S.M. & G. Schehr, *Dynamically Emergent Correlations*

$\longrightarrow$ a perspective article in EPL, 155, 11001 (2026)

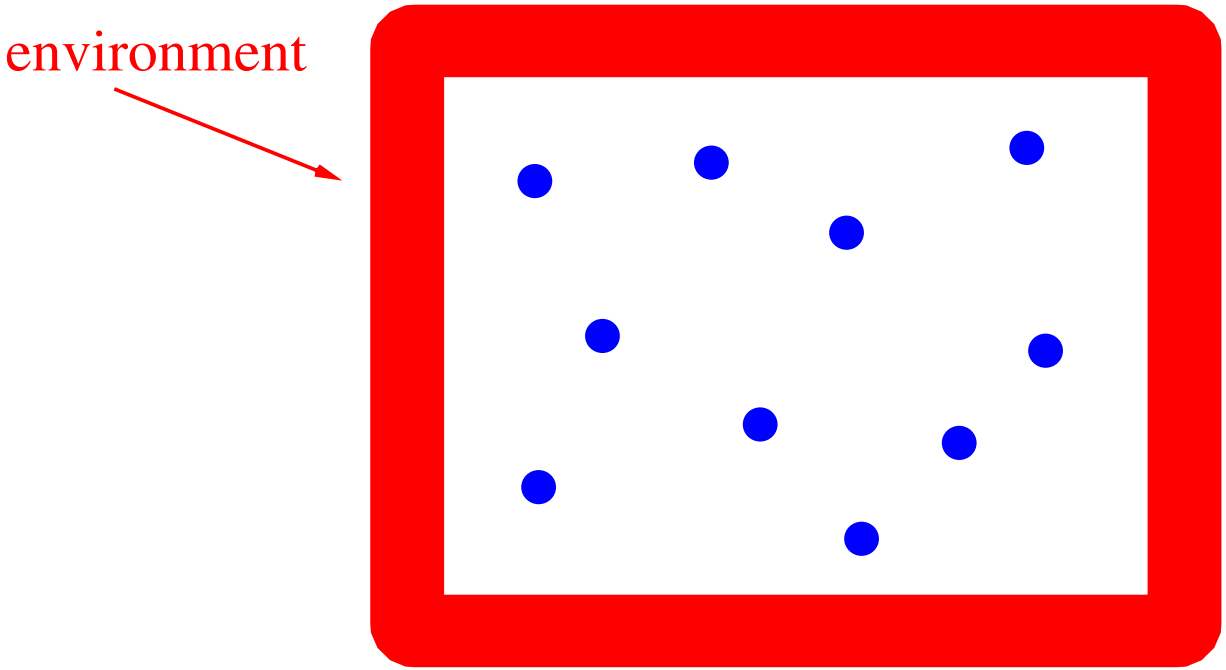
Dynamically emergent correlations : general setting



Particles: **non-interacting** (say Brownian)

Environment: **stochastic** (independent of particle motions)

Dynamically emergent correlations : general setting



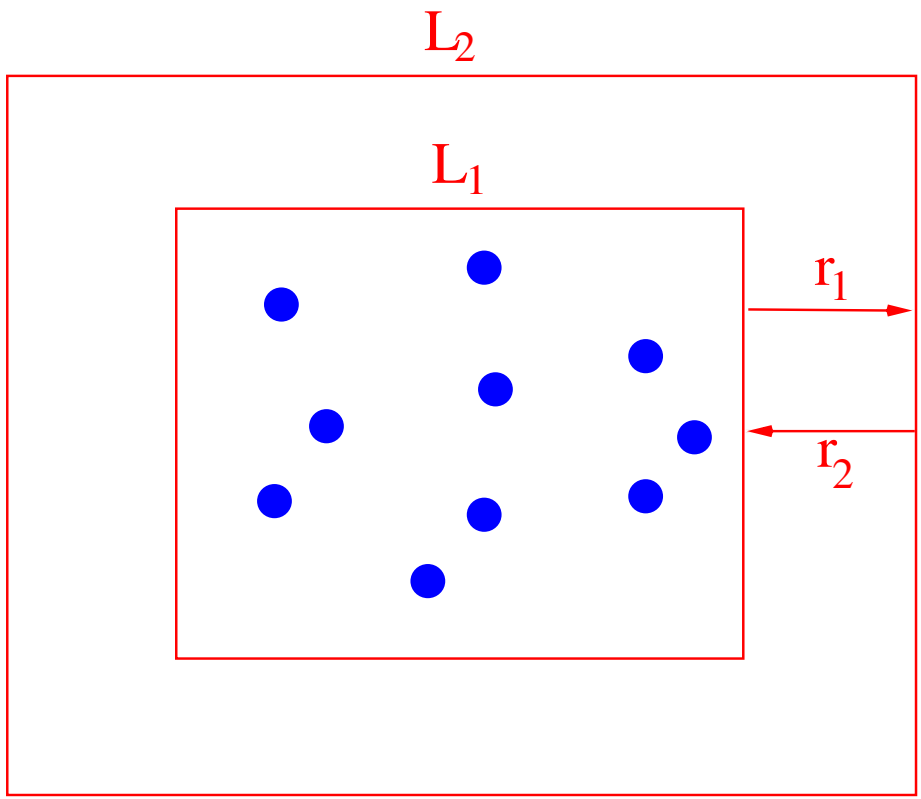
Particles: **non-interacting** (say Brownian)

Environment: **stochastic** (independent of particle motions)

Stochastic dynamics of the **shared environment** induces
strong correlations between particles

$\Longrightarrow$ **dynamically emergent correlations (DEC)**

A simple model : stochastically switching box size



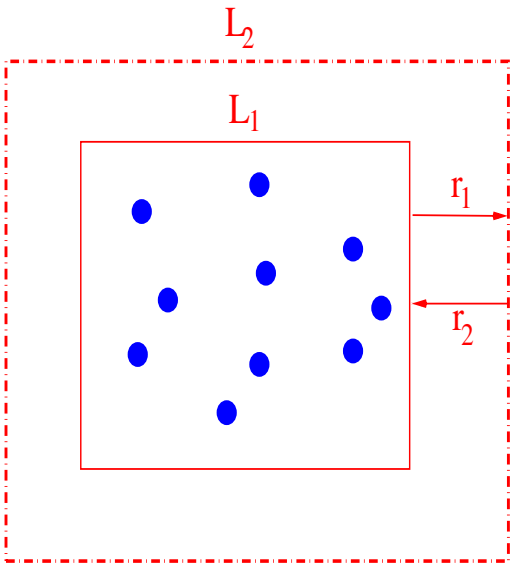
Particles $\Rightarrow$ **independent** Brownian with reflecting bc. at the walls

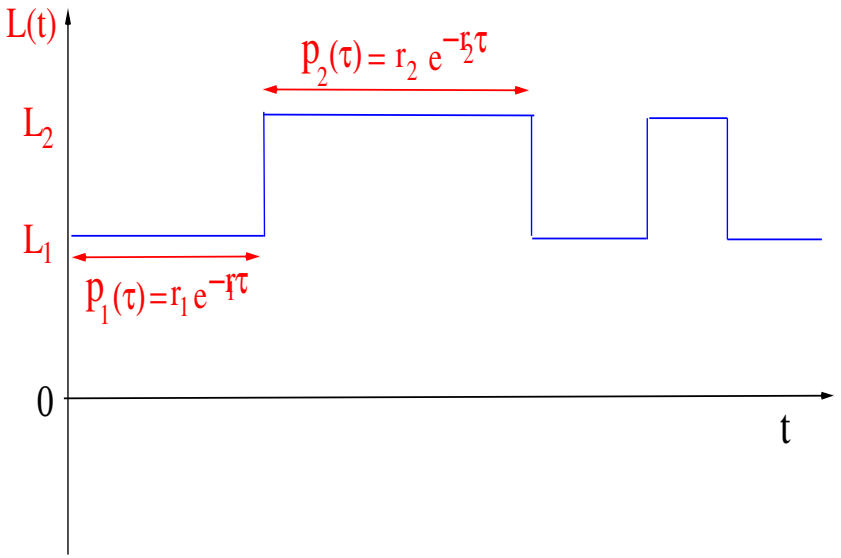
Box size $\Rightarrow$ **stochastically** switching between two values L_1 and L_2

$$L_1 \xrightarrow{r_1} L_2$$

$$L_1 \xleftarrow{r_2} L_2$$

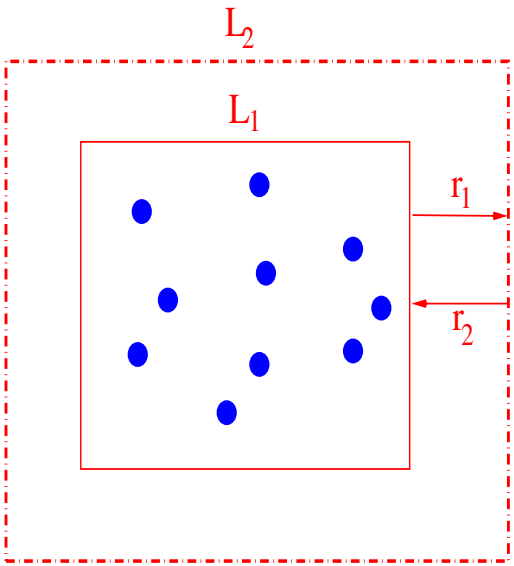
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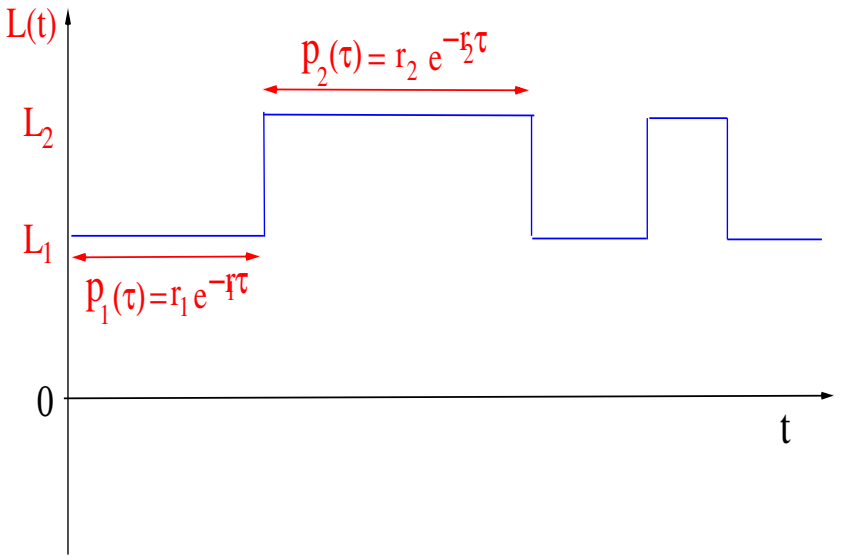




Box size $L(t) \Longrightarrow$ a **dichotomous telegraphic** process

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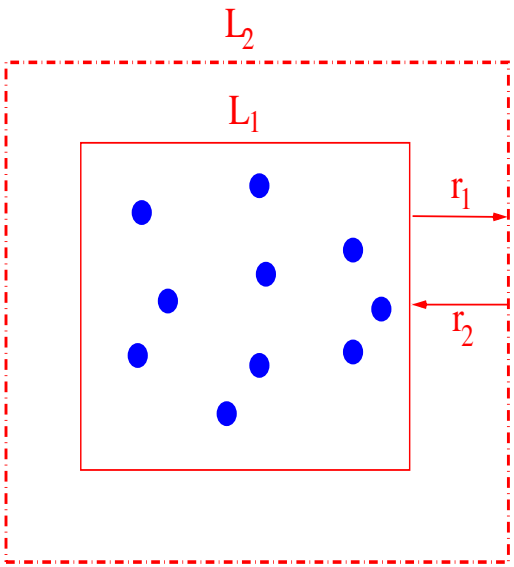
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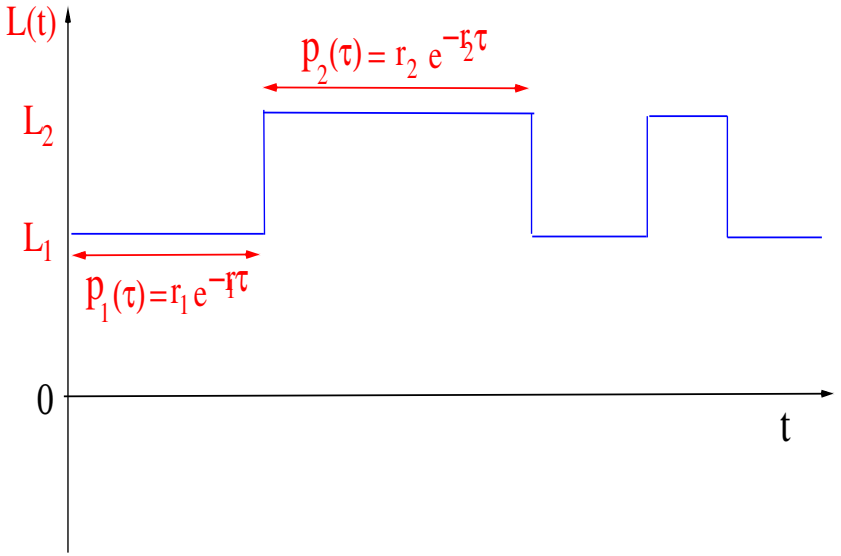
Goal: to compute the joint PDF $P(\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_N}|t)$ at time t

In particular, the stationary state (if it exists)

$$P^{\rm st}(\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_N}) = \mathbf{P}(\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_N}|\mathbf{t} \to \infty)$$

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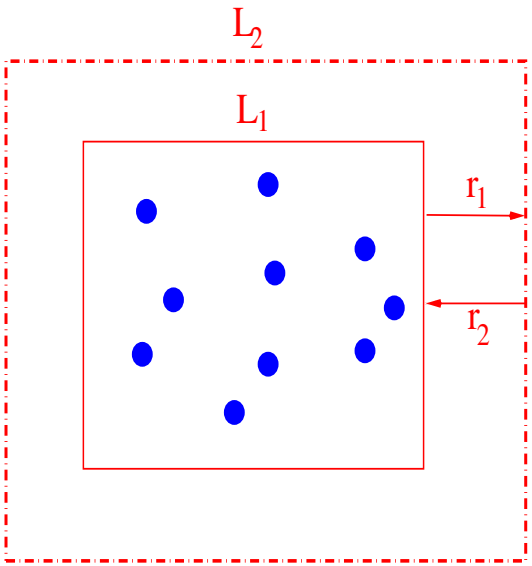
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Stationary joint PDF does not **factorize** $\Longrightarrow$ **emergent correlations**

An interesting limiting case

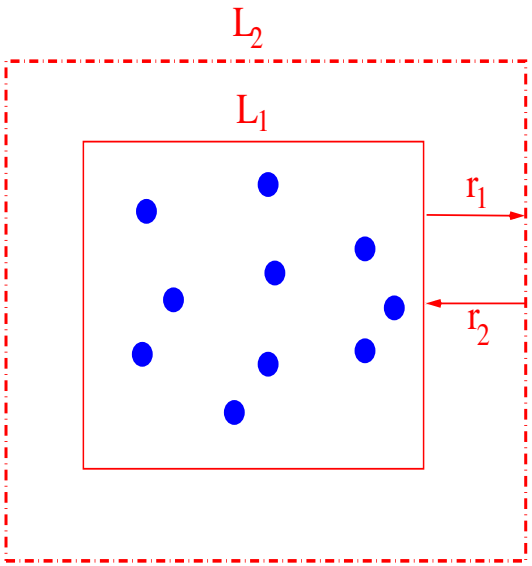


A limiting case:

$L_1 = 0$ and $L_2 \to \infty$

$r_1 \to \infty$ and $r_2 = \mathbf{r}$

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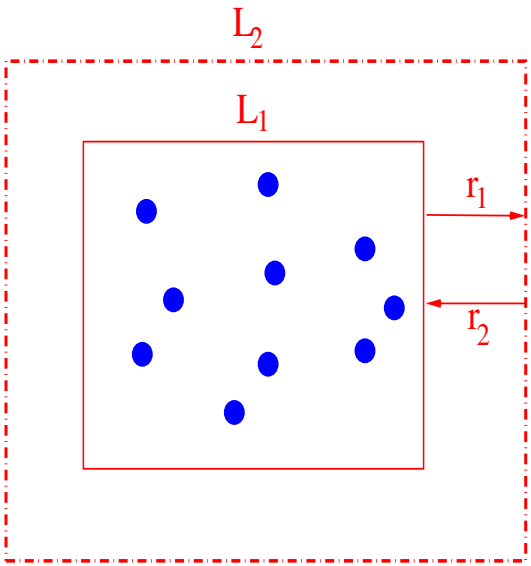


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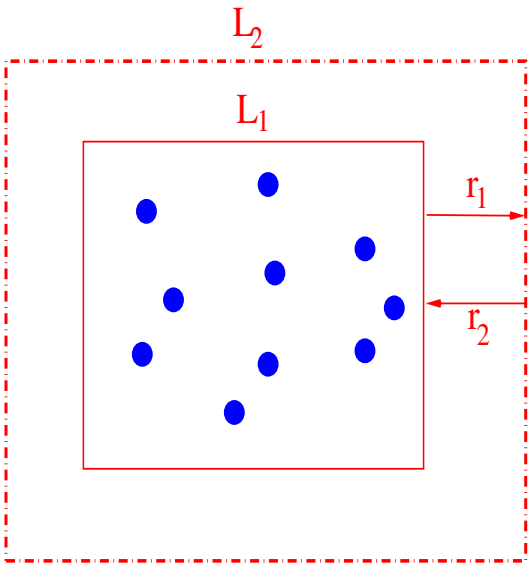
$L_1 = 0 \longrightarrow$ all particles **reset** to origin

$L_2 \to \infty \longrightarrow$ **free** diffusion

$r_1 \to \infty \longrightarrow$ following every reset, free diffusion starts instantaneously

$r_2 = r \longrightarrow$ free independent diffusion continues up to a random time drawn from $p(\tau) = r\, e^{-r\tau}$

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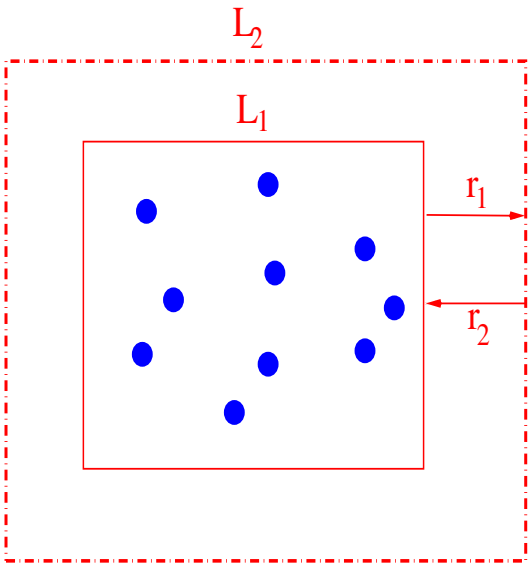
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Reset to the origin and **free diffusion** phases alternate stochastically

An interesting limiting case

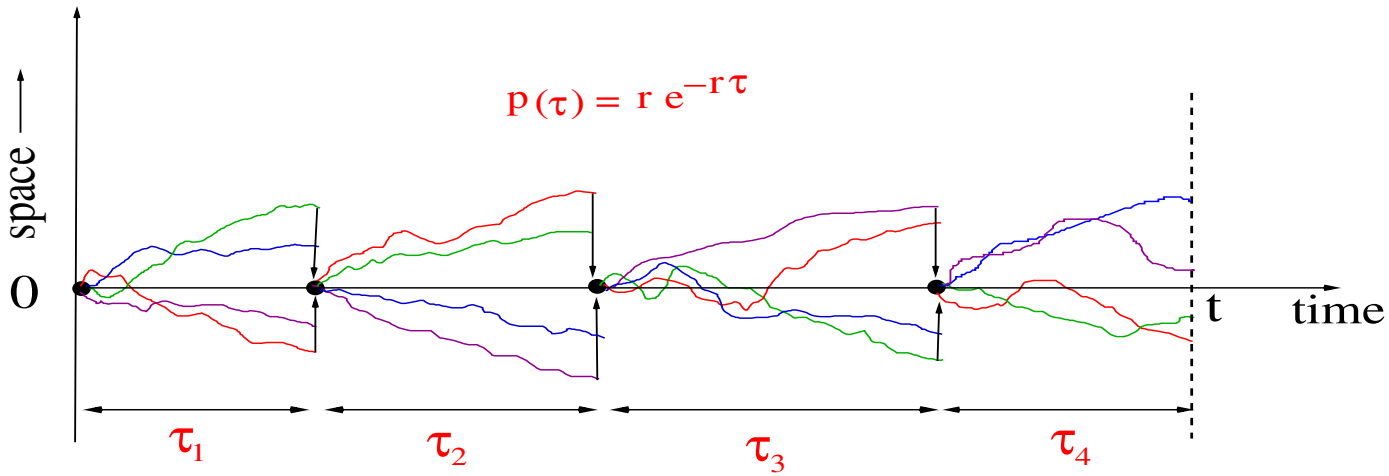


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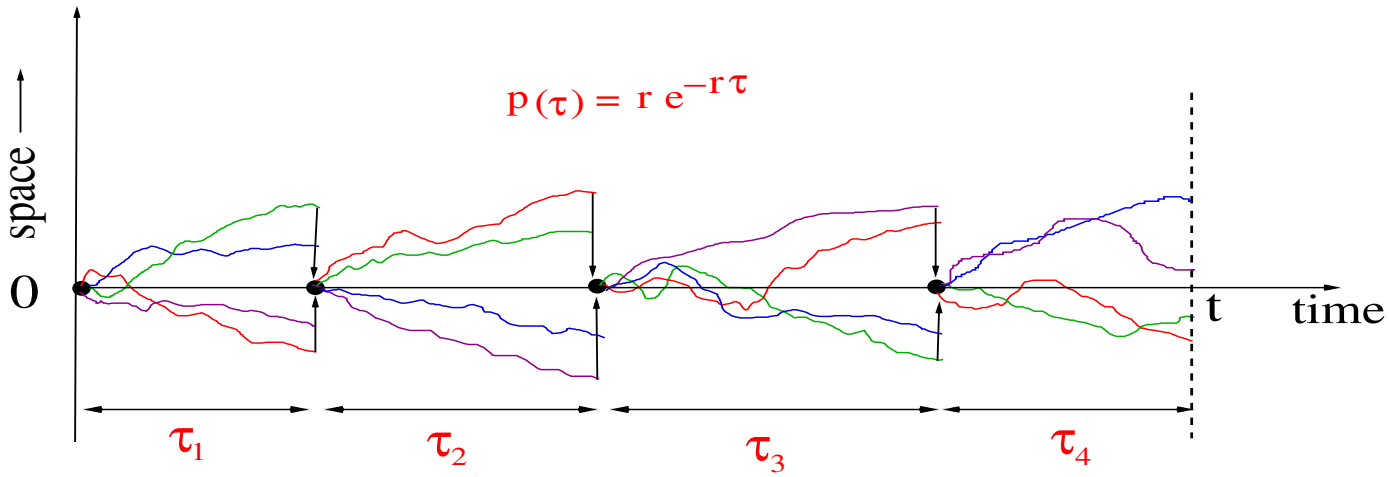
$r_1 \to \infty$ and $r_2 = \mathbf{r}$

N independent Brownian motions $\Longrightarrow$ **simultaneously reset** (**instantaneously**) to the origin with rate **r** (e.g. in $d = 1$)



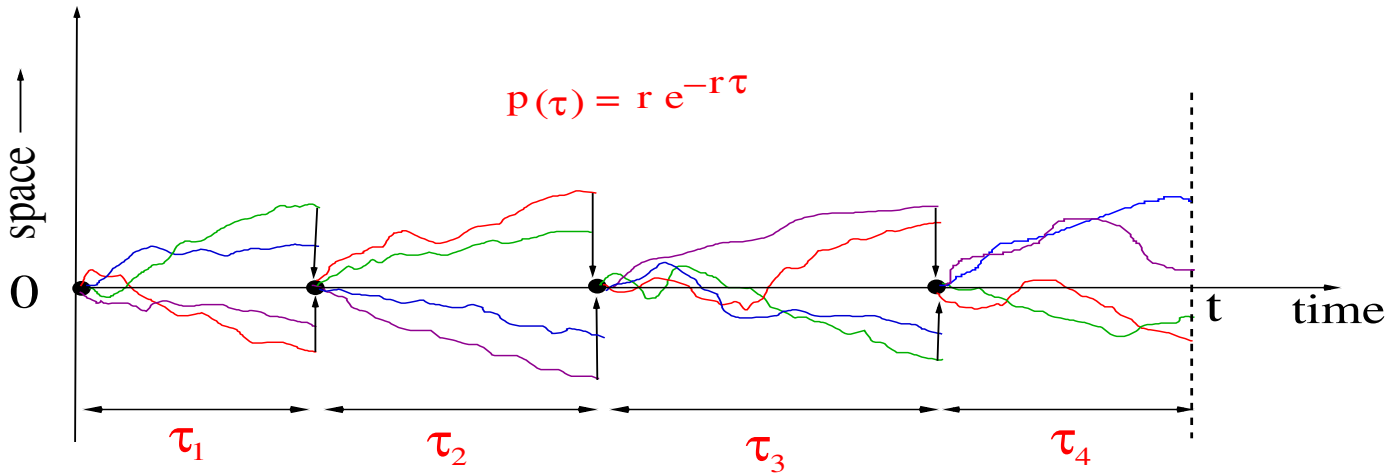
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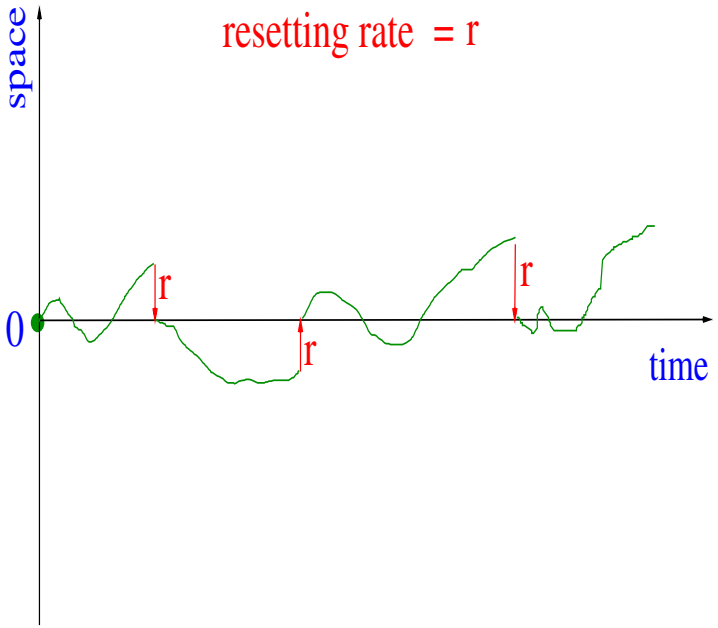


$\Longrightarrow$ A **multiparticle** generalization of **stochastic resetting** of a **single N = 1** diffusing particle

[M.R. Evans & S.M., PRL, 106, 160601 (2011)]

Single particle (N = 1)

$N = 1 \Rightarrow$ a single Brownian particle in $d = 1$

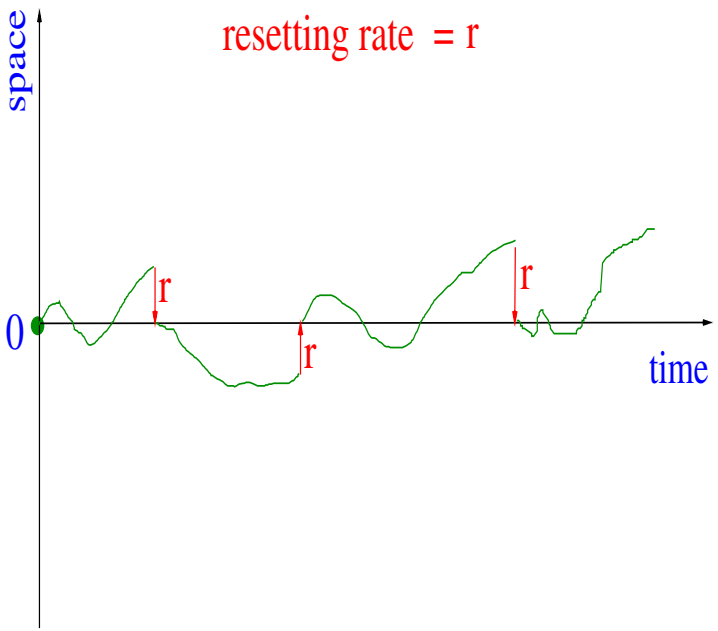


Poissonian resetting

Time intervals between successive resettings distributed as:

$$p(\tau) = r\, e^{-r\tau}$$

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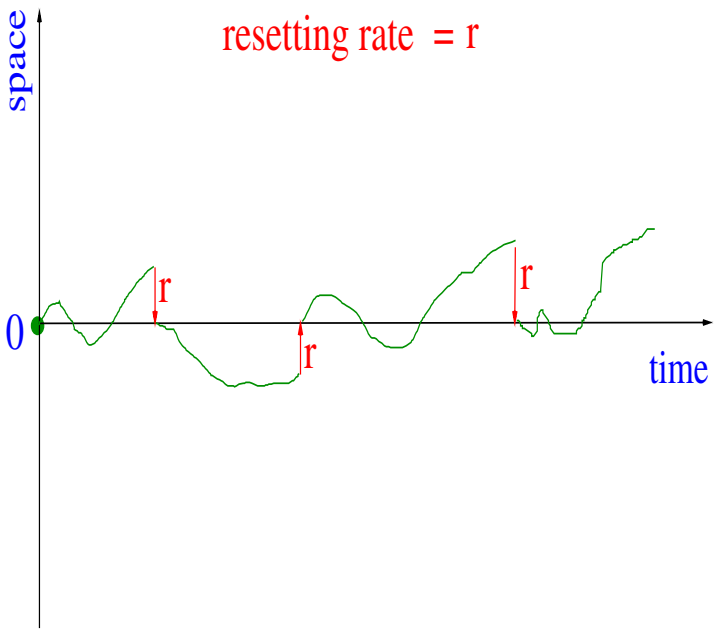
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Dynamics: In a small time interval Δt

$$x(t + \Delta t) = 0 \qquad \text{with prob. } r\Delta t \qquad \text{(resetting)}$$

$$= x(t) + \eta(t)\, \Delta t \qquad \text{with prob. } 1 - r\Delta t \qquad \text{(diffusion)}$$

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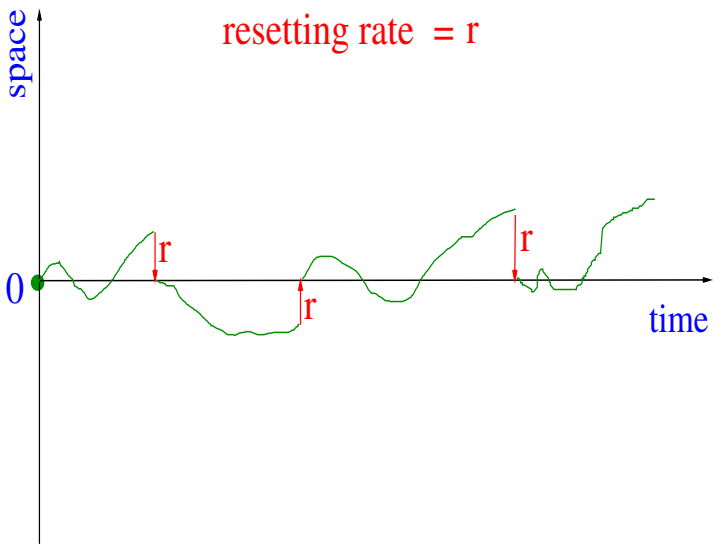
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$\eta(t) \rightarrow$ Gaussian white noise: $\langle \eta(t) \rangle = 0$ and $\langle \eta(t)\eta(t') \rangle = 2\, D\, \delta(t - t')$

[M.R. Evans & S.M., PRL, 106, 160601 (2011)]

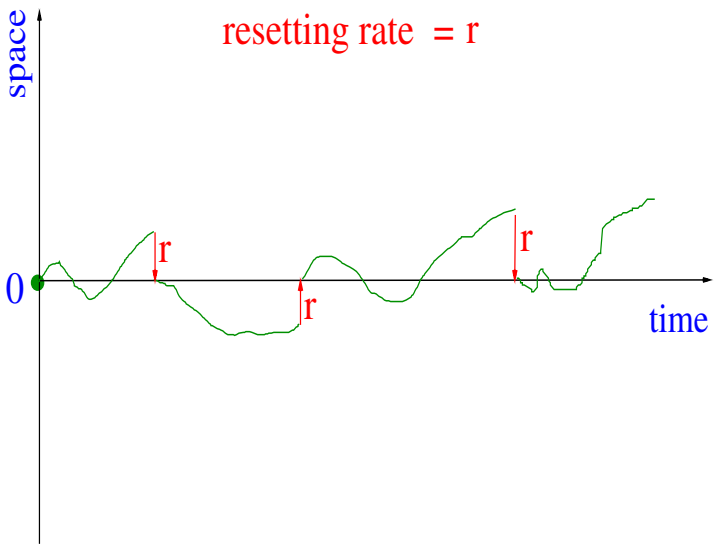
Prob. density $p_r(x,t)$ with resetting rate $r > 0$



$p_r(x,t) \rightarrow$ prob. density at time t,

given $p_r(x,0) = \delta(x)$

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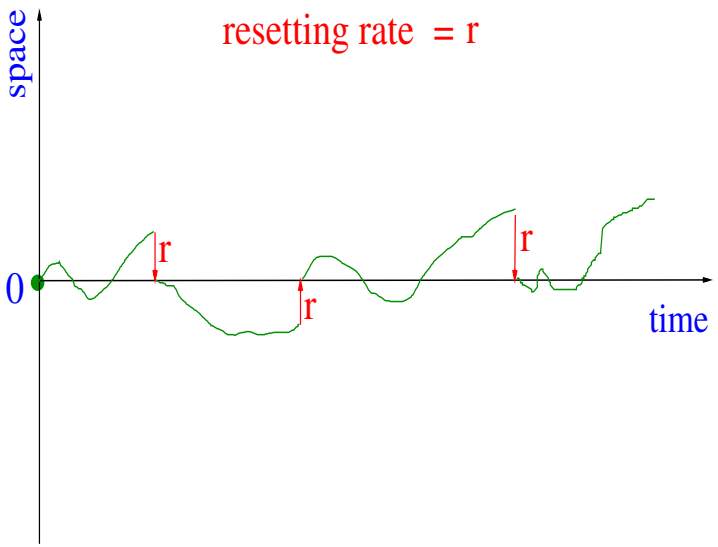


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$$p_0(x,t) = \frac{1}{\sqrt{4\pi\, D\, t}}\ \exp[-x^2/4Dt]$$

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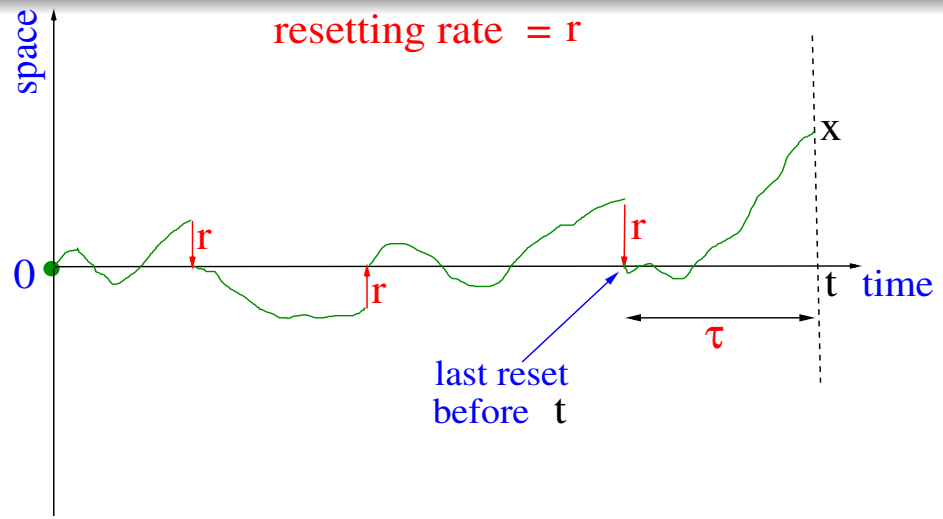
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Exact solution valid at all times t



- Exact solution at all times t:

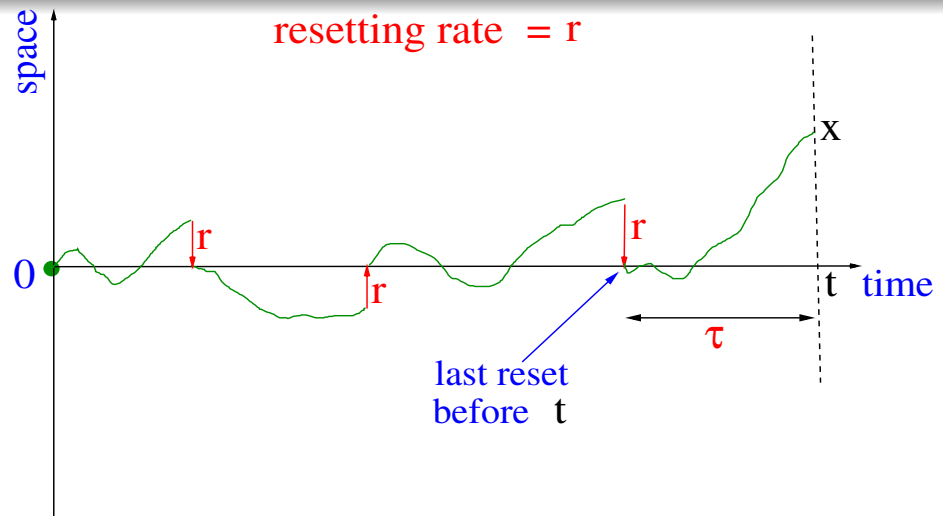
$$p_r(x|t) = e^{-r\,t}\, p_0(x|t) + \int_0^t d\tau\, (r\, e^{-r\,\tau})\, p_0(x|\tau)$$

where $p_0(x|\tau) = \text{diffusion propagator} = \frac{1}{\sqrt{4\pi\, D\, \tau}}\, \exp[-x^2/4D\tau]$

Renewal interpretation: $\tau \rightarrow$ time since the last resetting during which

$\Longrightarrow$ free diffusion

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- As $t \rightarrow \infty$, $p_r^{\rm st}(x) = r\int_0^\infty p_0(x|\tau)\,e^{-r\,\tau}\,d\tau = \frac{\alpha_0}{2}\,\exp[-\alpha_0\,|x|]$

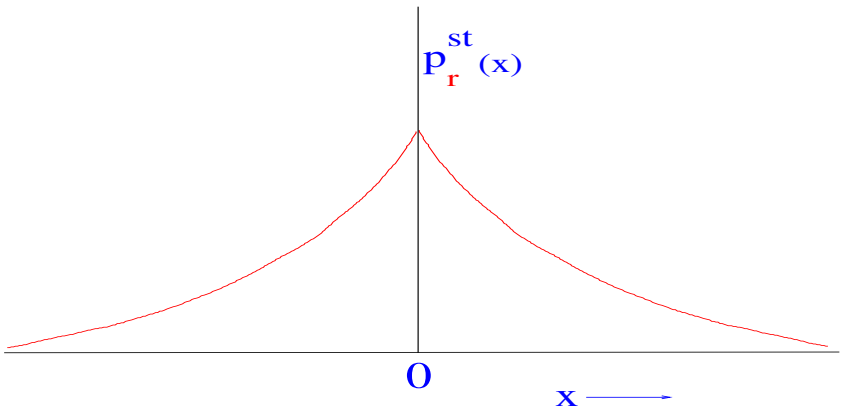
where $\alpha_0 = \sqrt{r/D}$

Stationary State

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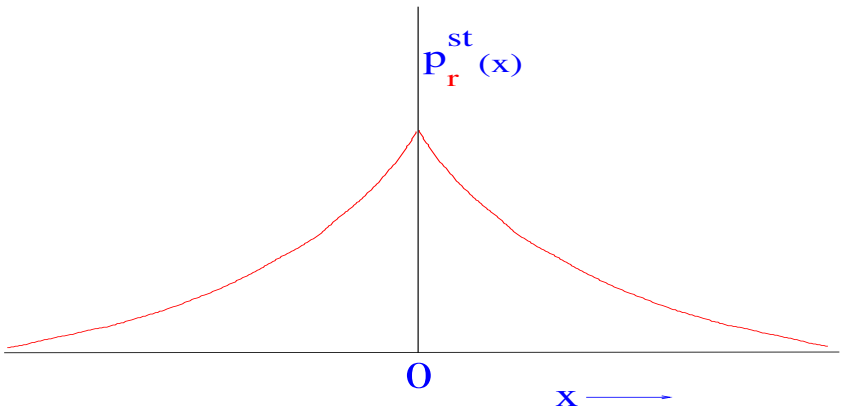
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$\Rightarrow$ current carrying with detailed balance $\rightarrow$ violated

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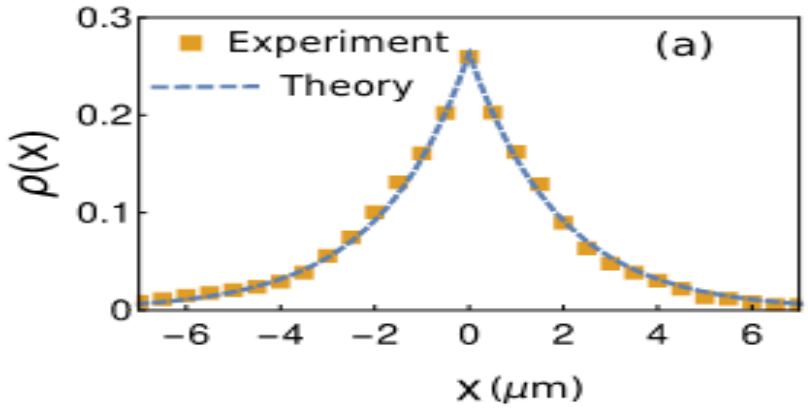
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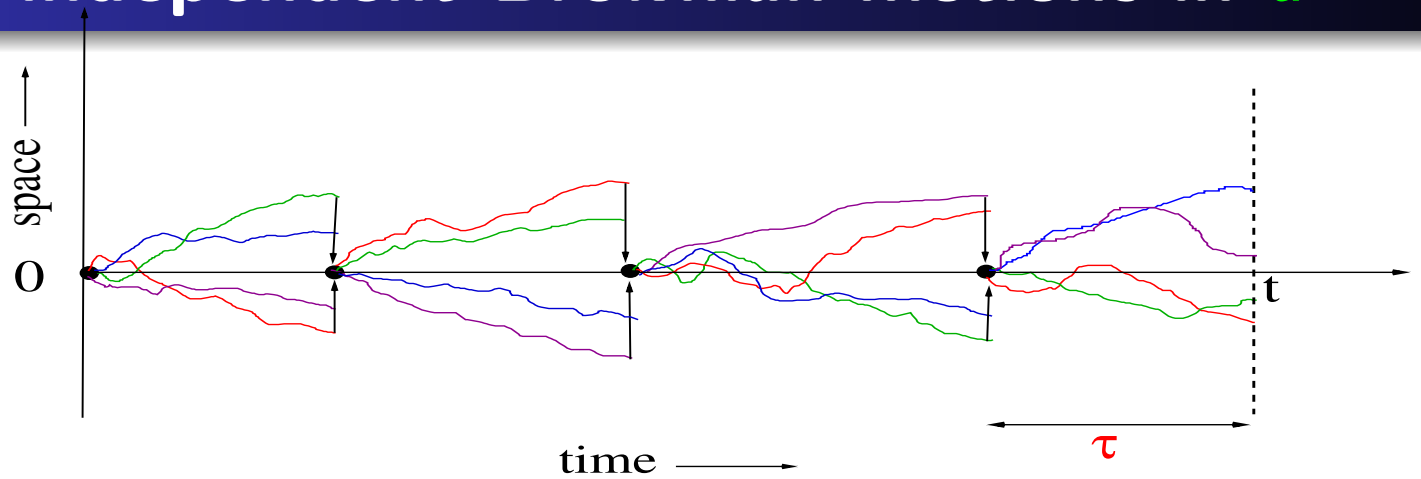
Experimental verificaion using holographic optical tweezers

Tal-Friedman, Pal, Sekhon, Reuveni, & Roichman

J. Phys. Chem. Lett. 11, 7350 (2020)

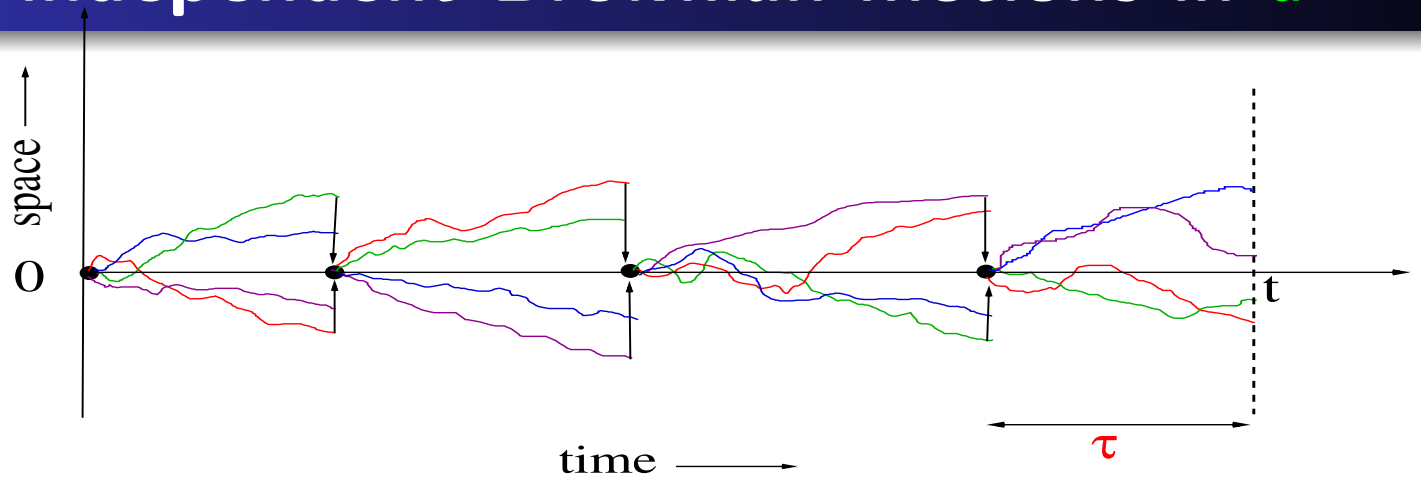
Multiparticle Generalization (N $>$ 1)

$N > 1$ independent Brownian motions in $d = 1$



Consider N Brownian motions (independent) that are **simultaneously** reset with rate r to the origin

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Joint distribution at any time t:

$$P_r\left(\{x_i\}|t\right) = e^{-r\,t}\prod_{i=1}^{N} p_0(x_i|t) + r\int_0^t d\tau\, e^{-r\,\tau}\prod_{i=1}^{N} p_0(x_i|\tau)$$

where $p_0(x|\tau) = \frac{1}{\sqrt{4\pi D\tau}}\, e^{-x_i^2/4D\tau}$

M. Biroli, H. Larralde, S. M., G. Schehr, PRL, 130, 207101 (2023)

Dynamically emergent correlations

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The joint distribution does not **factorize** $\Longrightarrow$ **correlated** resetting gas

In this model, interactions between particles are **not built-in**, but the correlations are generated by the dynamics (**simultaneous resetting**), that persist all the way to the stationary state

$\longrightarrow$ **dynamically emergent correlations (DEC)**

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Correlation function at time t for any pair $i \neq j$:

By symmetry, $\langle x_i x_j\rangle - \langle x_i\rangle\langle x_j\rangle = 0$ at all times t

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$\longrightarrow$ **dynamically emergent correlations (DEC)**

Correlation function at time t for any pair $i \neq j$:

By symmetry, $\langle x_i x_j\rangle - \langle x_i\rangle\langle x_j\rangle = 0$ at all times t

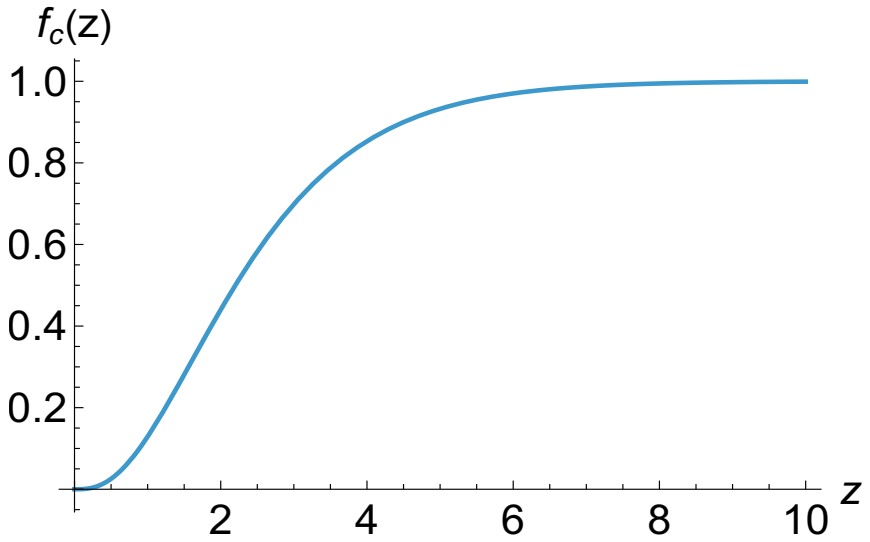
One needs to go to higher order to detect the correlations

Dynamically emergent correlations

Simplest higher order correlation function for $i \neq j$

$$C_{i,j}(t) = \langle x_i^2\, x_j^2 \rangle - \langle x_i^2 \rangle \langle x_j^2 \rangle = \frac{4D^2}{r^2} f_c(r\, t)$$

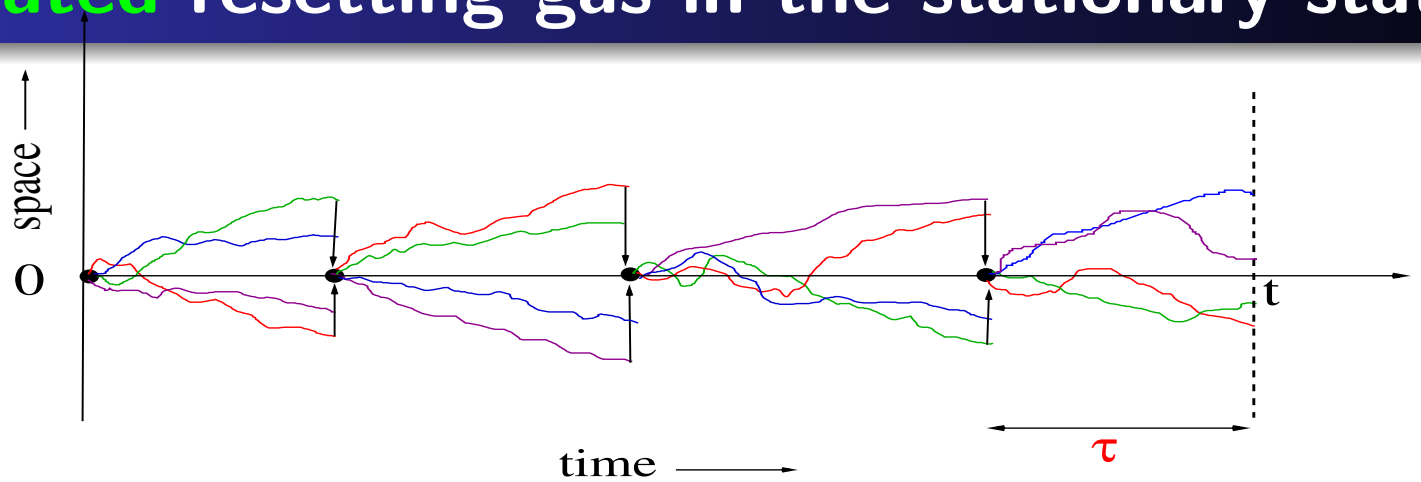
where $$f_c(z) = 1 - 2\, z\, e^{-z} - e^{-2\, z}$$



$$f_c(z) \approx \begin{cases} \frac{1}{3} z^3 & z \to 0 \\ 1 - 2\, z\, e^{-z} & z \to \infty \end{cases}$$

correlations **grow with time**

Correlated resetting gas in the stationary state



N Brownian motions (independent) that are **simultaneously** reset with rate r to the origin

The joint position distribution approaches a nonequilibrium stationary state (NESS) at long times $t \to \infty$

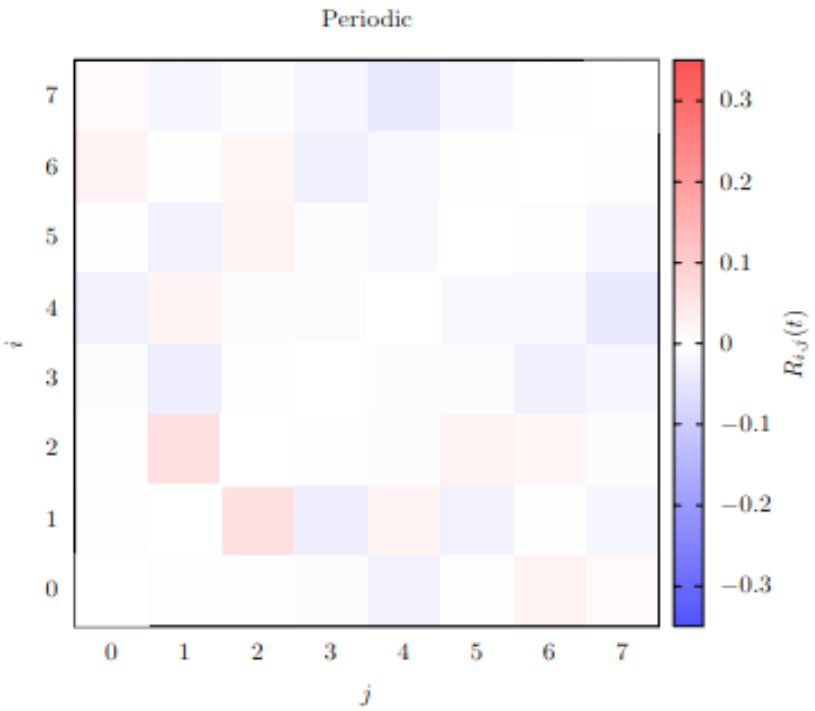
$$P_r^{\rm st}(\{x_i\}) = r \int_0^\infty d\tau\, e^{-r\tau} \prod_{i=1}^N \frac{1}{\sqrt{4\pi D\tau}}\, e^{-x_i^2/4D\tau}$$

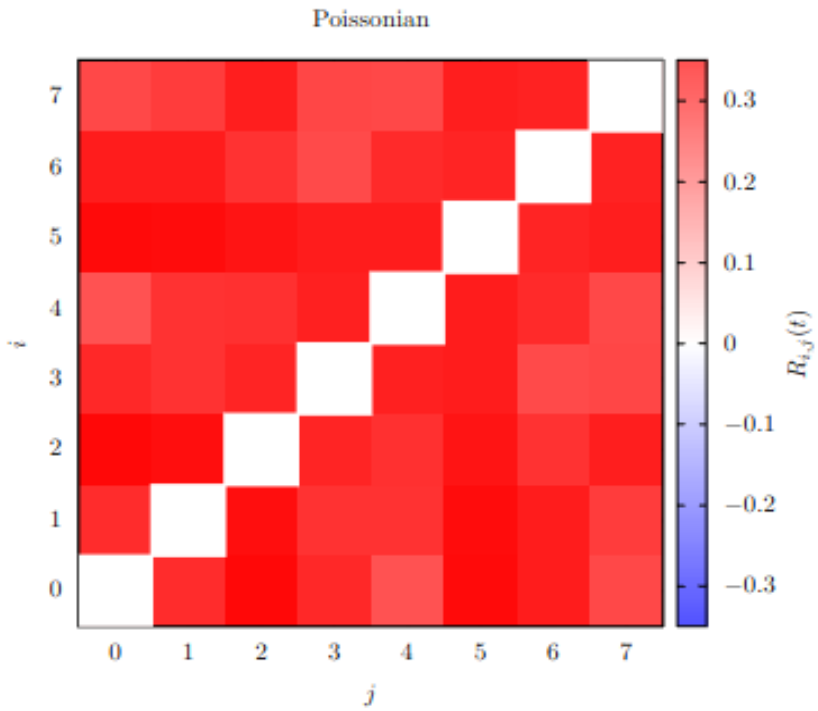
The joint distribution does not **factorize** even in the NESS

$$\langle x_i^2 x_j^2 \rangle - \langle x_i^2 \rangle \langle x_j^2 \rangle = 4\frac{D^2}{r^2} \Longrightarrow \textbf{attractive all-to-all} \text{ interaction}$$

$$\Longrightarrow \textbf{strongly correlated} \text{ resetting gas}$$

Periodic vs. Stochastic resetting

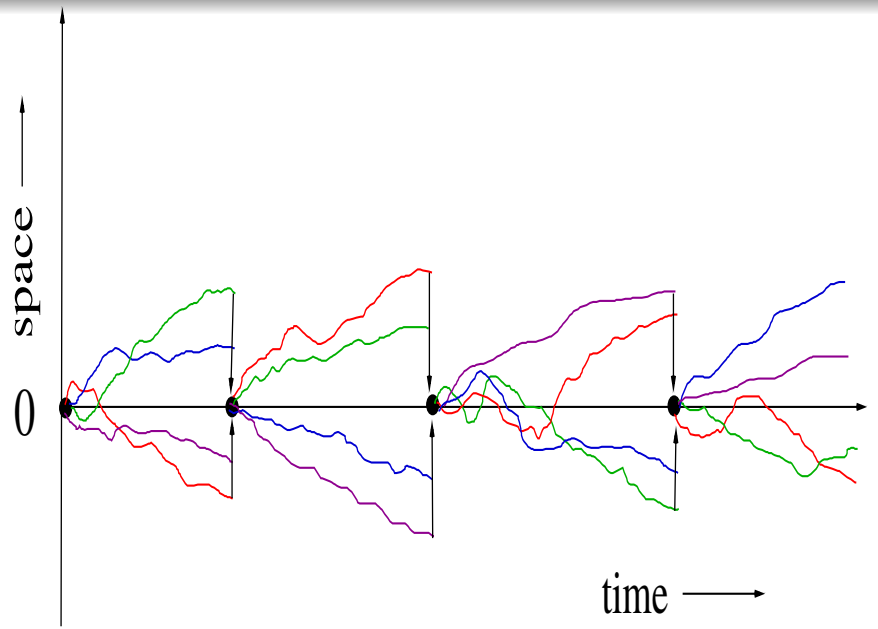




Periodic Stochastic

Normalized correlation matrix: $R_{i,j}(t) = \frac{\langle x_i^2\, x_j^2 \rangle}{\langle x_i^2(t) \rangle \langle x_j^2(t) \rangle} - 1$ (for $i \neq j$)

Solvable Correlated Gas



Joint distribution:

$$P_r^{\rm st}(\{x_i\}) = r \int_0^\infty d\tau\, e^{-r\tau} \prod_{i=1}^N p_0(x_i|\tau)$$

$$p_0(x|\tau) = \frac{1}{\sqrt{4\pi D\tau}}\, e^{-x_i^2/4D\tau}$$

The stationary joint distribution has a **CIID** structure $\Longrightarrow$ Solvable

$$\boxed{P_r^{\rm st}(x_1, x_2, \ldots, x_N) = \int_{-\infty}^{\infty} du\, h(u) \prod_{i=1}^N p_0(x_i|u)}$$

Here $u \equiv \tau$ and $h(\tau) \equiv r\, e^{-r\tau}$

CIID $\Longrightarrow$ Conditionally Independent and Identically Distributed

Solvable Correlated Gas

Joint distribution : $$P_r^{\rm st}(\{x_i\}) = r \int_0^\infty d\tau\, e^{-r\,\tau} \prod_{i=1}^{N} p_0(x_i|\tau)$$

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Despite the presence of **strong correlations**, several physical observables can be computed **exactly** in the NESS due to the **CIID** structure

(1) Compute any observable for the ideal gas $\Rightarrow$ I.I.D variables with distribution $p_0(x|\tau)$ parametrized by $\tau \Longrightarrow$ easy

(2) Average over the **random** parameter τ using $h(\tau) = r\, e^{-r\,\tau}$

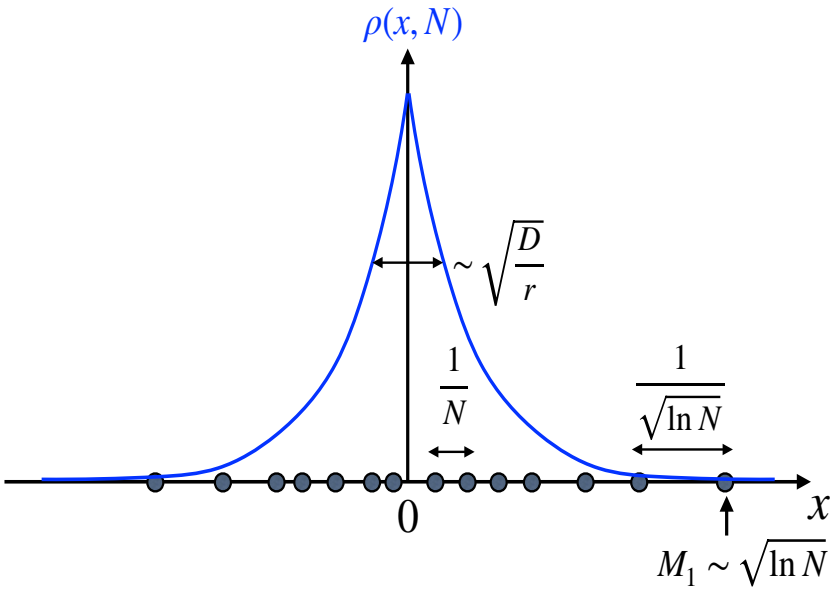
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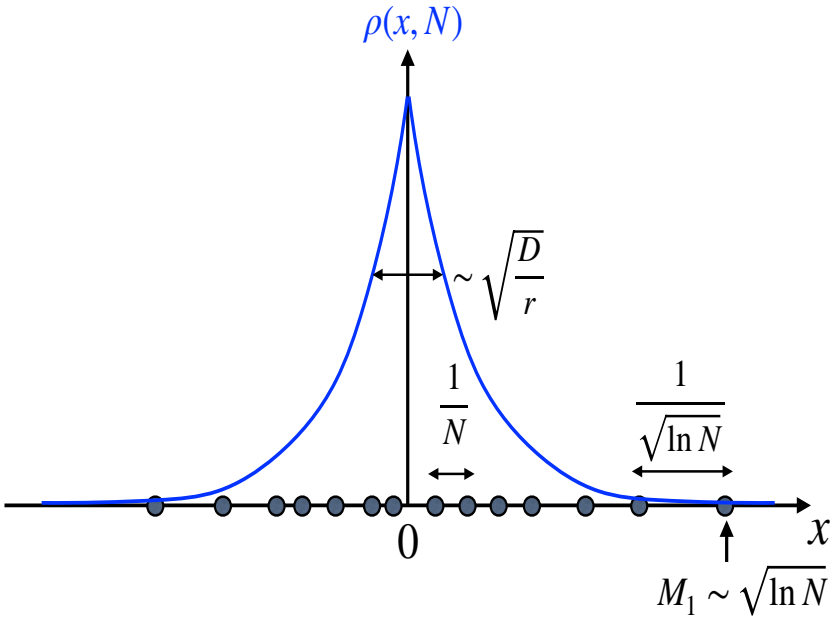
Examples:

- Average density
- Distribution of the k-th maximum: Order statistics
- Spacing distribution
- Full Counting Statistics in $[-L, L]$

Observables $\Longrightarrow$ natural in **random matrix theory**

Explicit Results

Average Density



Joint distribution:

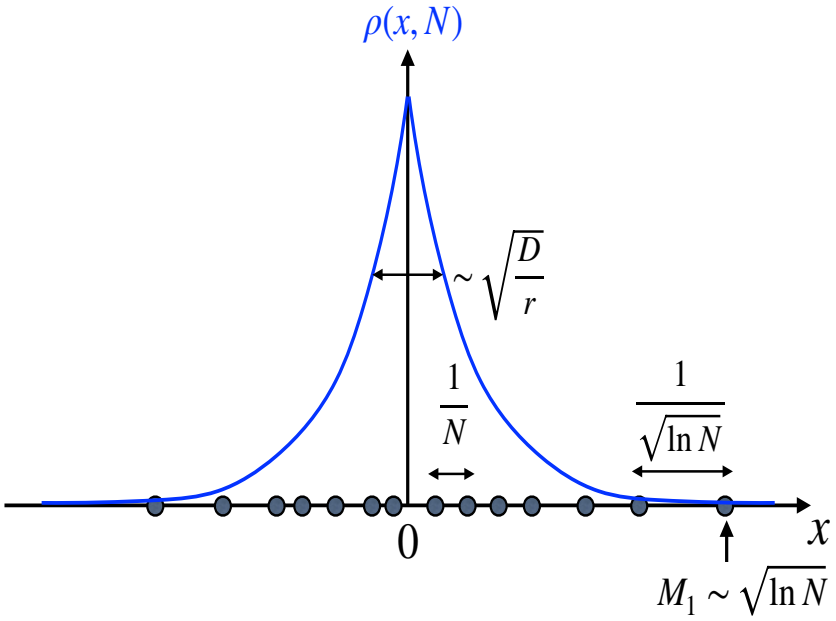
$$P_r^{\rm st}\left(\{x_i\}\right) = r\int_0^\infty d\tau\, e^{-r\tau}\prod_{i=1}^{N} p_0(x_i|\tau)$$

$$p_0(x|\tau) = \frac{1}{\sqrt{4\pi D\tau}}\, e^{-x_i^2/4D\tau}$$

Average density:

$$\rho(x,N) = \frac{1}{N}\sum_{i=1}^{N}\langle\delta(x_i - x)\rangle = \int P_r^{\rm st}\left(x,\, x_2, \ldots, x_N\right)\, dx_2\, dx_3 \ldots dx_N$$

Average Density



Joint distribution:

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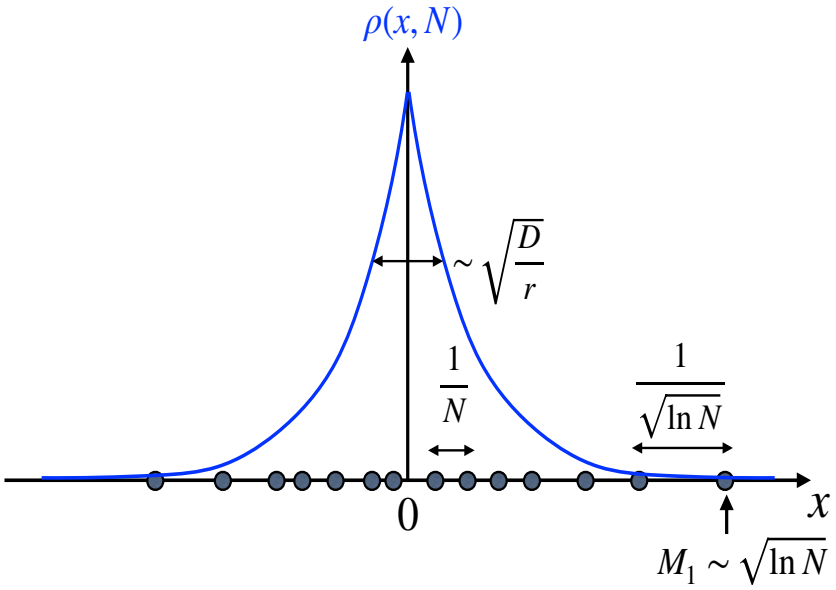
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$$= r\int_0^\infty d\tau\, e^{-r\,\tau}\, p_0(x|\tau) = \frac{\alpha_0}{2}\, \exp[-\alpha_0\,|x|]$$

where $\alpha_0 = \sqrt{r/D}$

$\Longrightarrow$ same as the single particle position distribution

Order Statistics



$M_k \Longrightarrow k$-th maximum

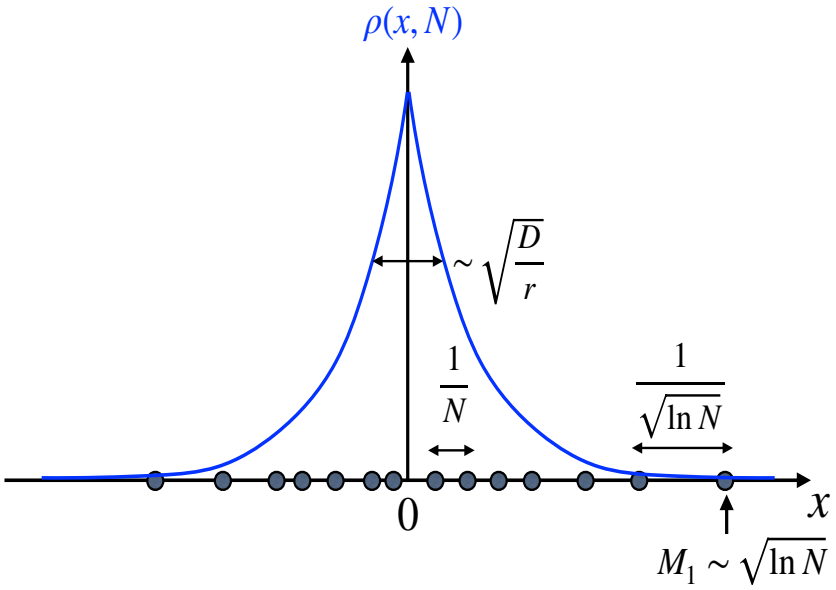
Set $k = \alpha N$

$\alpha \sim O(1) \Longrightarrow$ **bulk**

$\alpha \sim O(1/N) \Longrightarrow$ **edge**

We set $\alpha < \mathbf{1/2}$

Order Statistics



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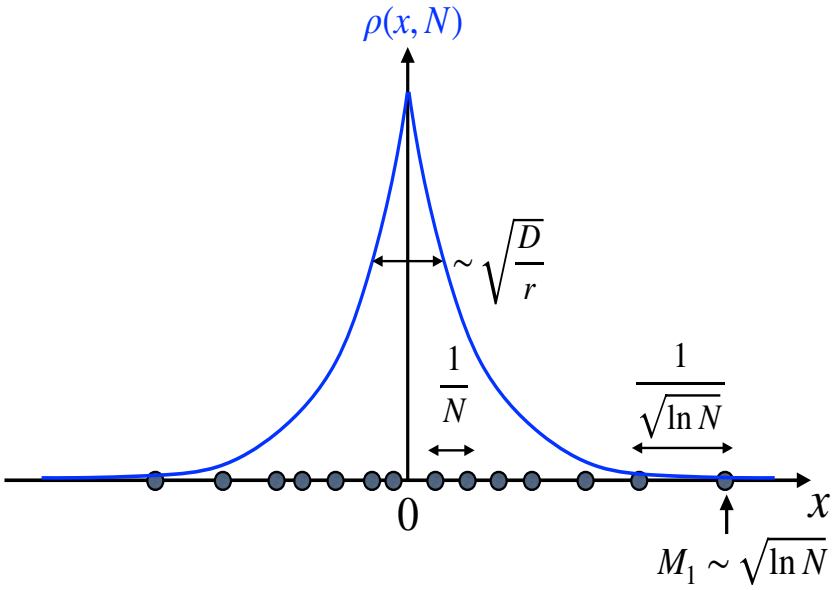
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Order Statistics



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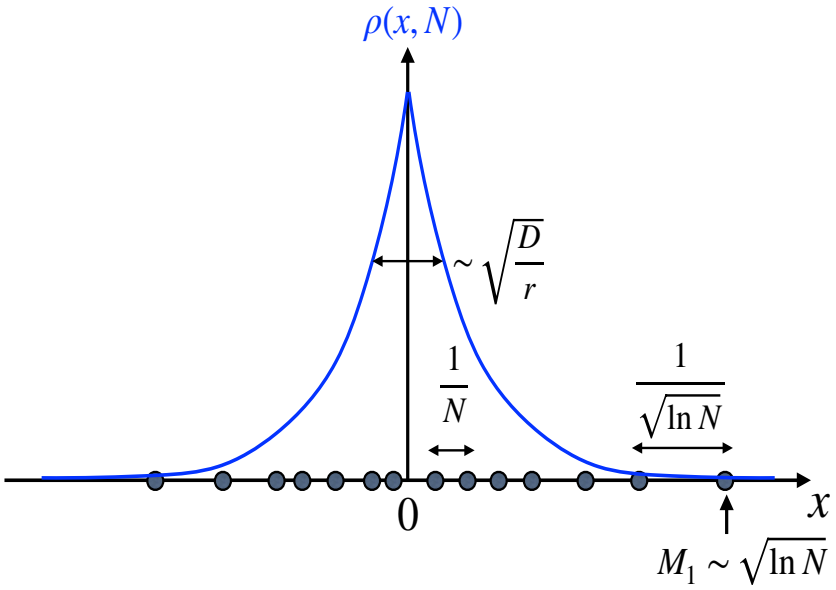
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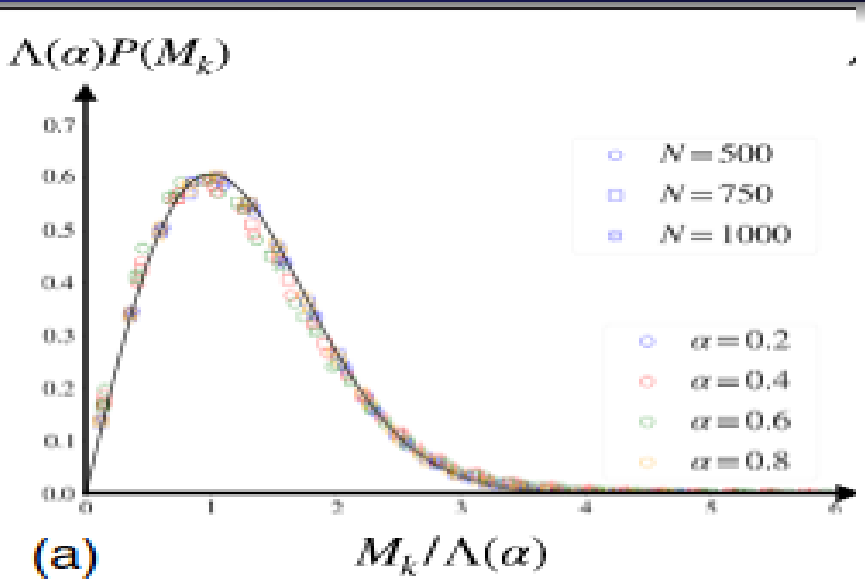
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The scaling function $\boxed{\mathbf{f(z) = 2\,z\,e^{-z^2}\,\theta(z)}}$

$\Longrightarrow$ **universal** (indep. of $\alpha < 1/2$)

M. Biroli, H. Larralde, S. M., G. Schehr, PRL, 130, 207101 (2023)

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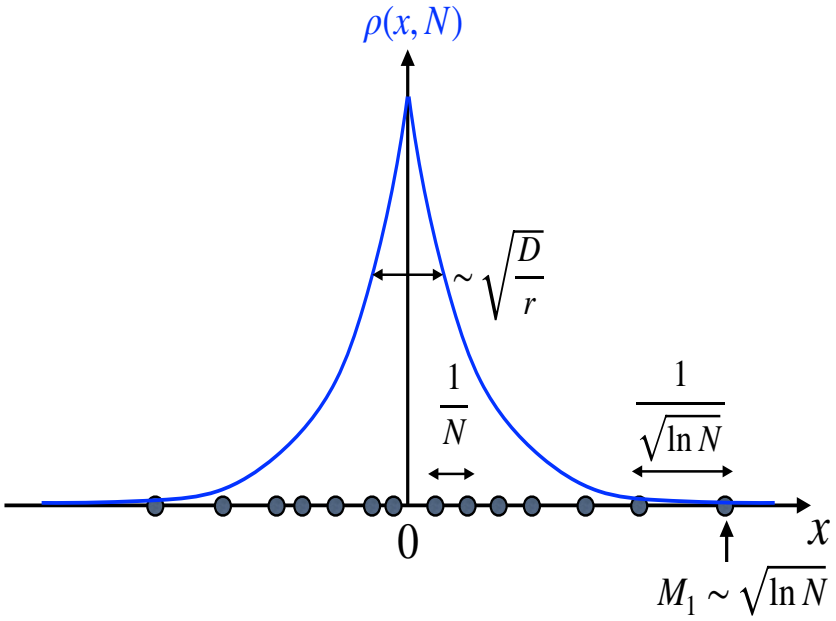
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Gap/Spacing Statistics



$M_k \Longrightarrow k$-th maximum

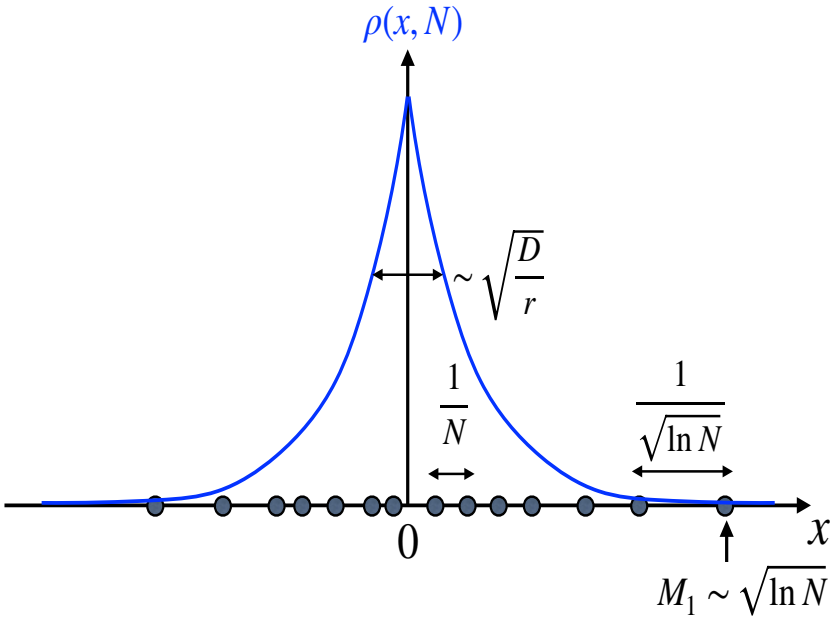
k-th gap: $d_k = M_k - M_{k+1}$

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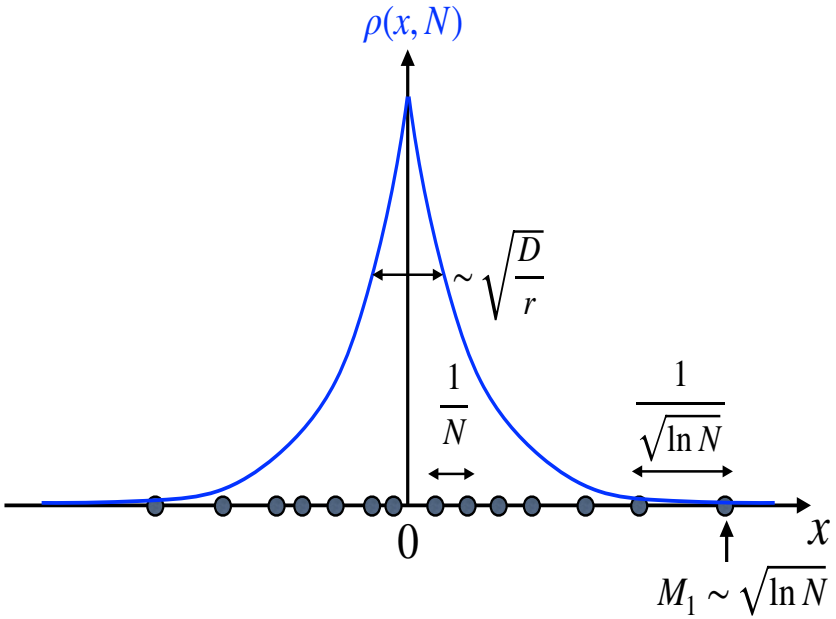
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Gap/Spacing Statistics



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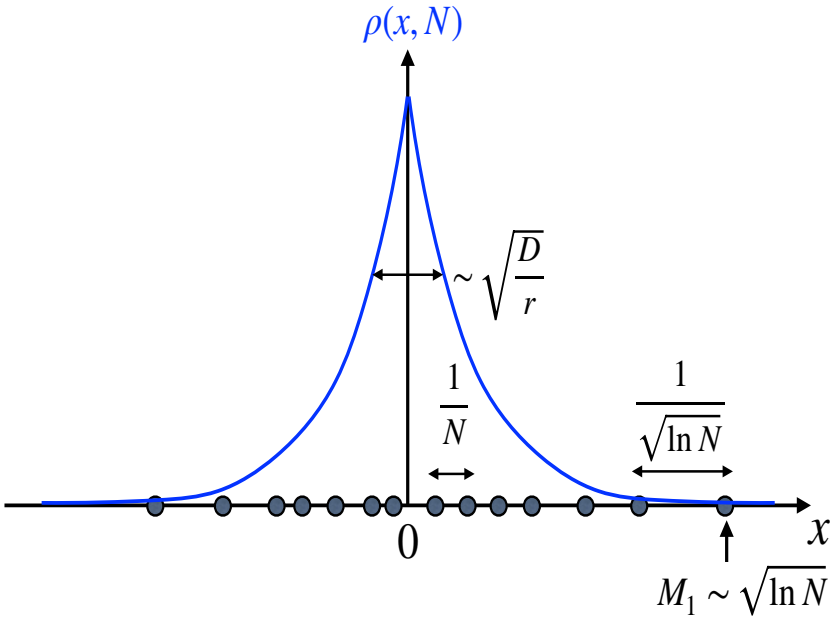
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Gap/Spacing Statistics



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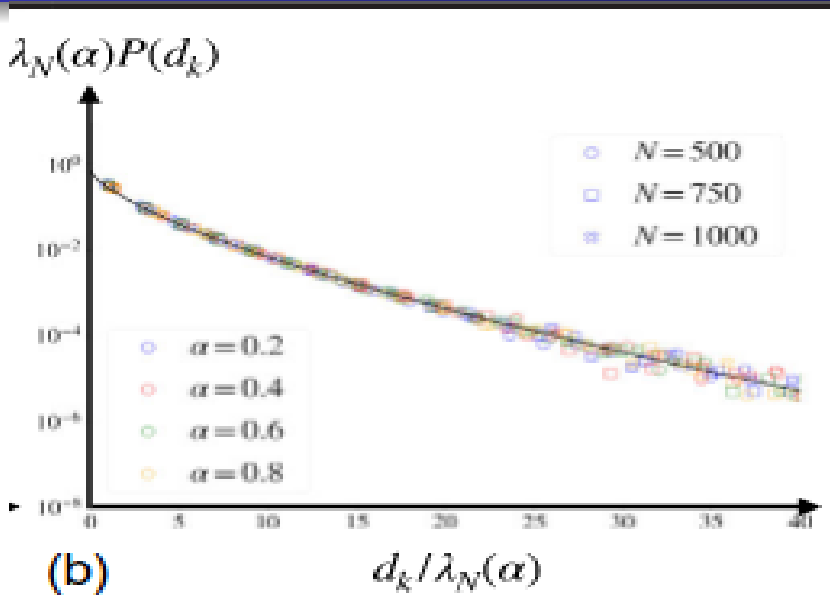
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The scaling function $\boxed{h_{\text{gap}}(z) = 2\int_0^\infty du\, e^{-u^2 - z/u}}$ $(z \geq 0)$

$\Longrightarrow$ **universal** (indep. of α)

Gap/Spacing Statistics



(b)

The gap scaling function:

$$h_{\rm gap}(z) = 2\int_0^\infty du\, e^{-u^2 - z/u}$$

$\to \sqrt{\pi}$ as $z \to 0$

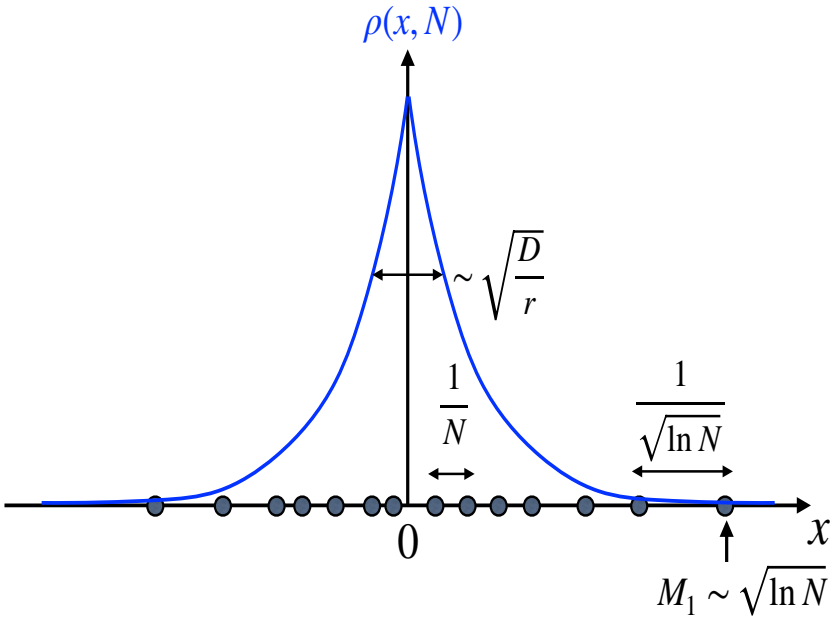
$\sim e^{-3(z/2)^{2/3}}$ as $z \to \infty$

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Full Counting Statistics

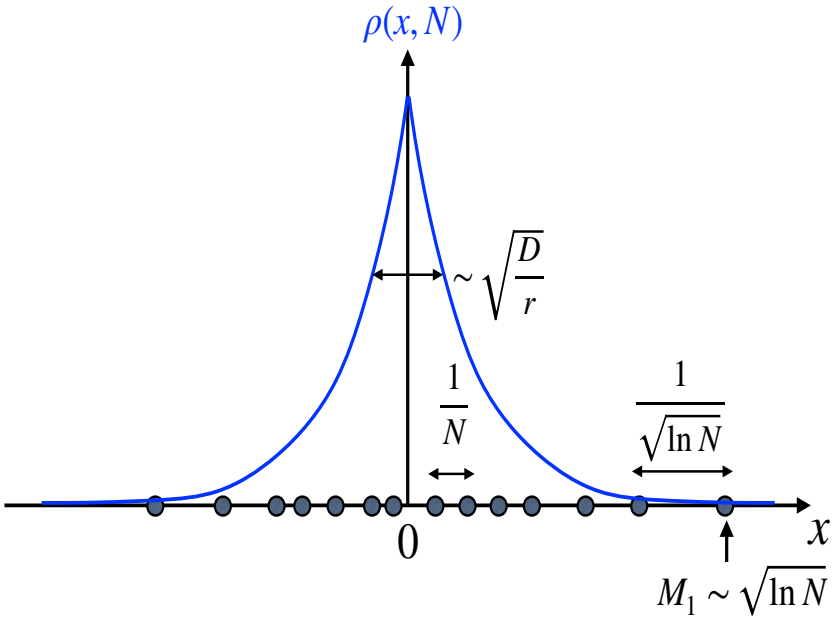


$N_L \Longrightarrow$ number of particles in $[-L, L]$

Clearly, $0 \leq N_L \leq N$

$P(N_L, N) = ?$

Full Counting Statistics



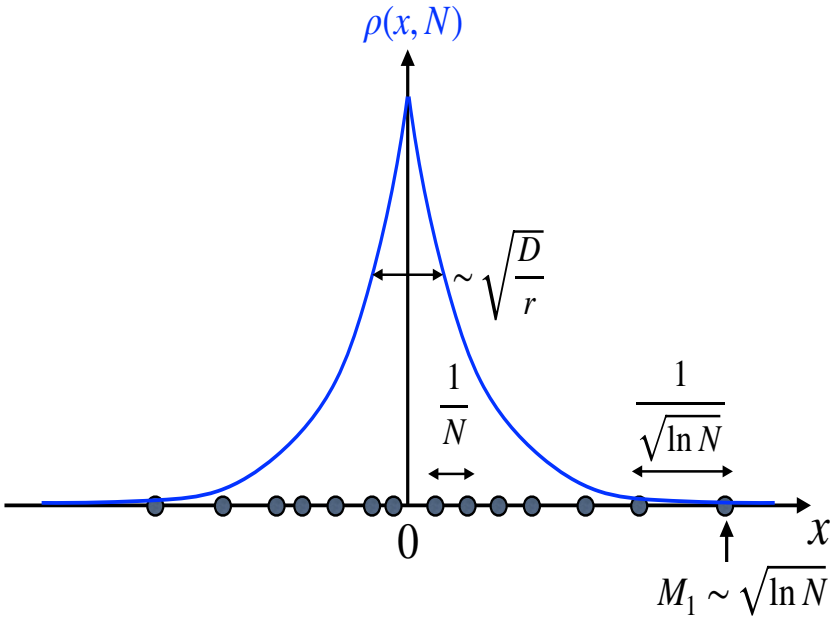
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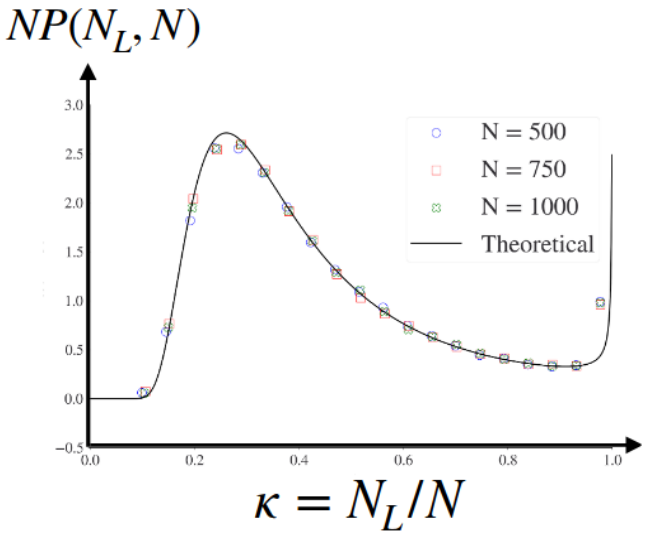
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where the scaling function :

$$\boxed{H(\kappa) = \gamma\sqrt{\pi}\,[u(\kappa)]^{-3}\,\exp\left[-\gamma\,u^{-2}(\kappa) + u^2(\kappa)\right]}$$

with $\gamma = r\,L^2/(4D)$ and $u(\kappa) = \mathrm{erf}^{-1}(\kappa)$

Full Counting Statistics



The scaling function $H(\kappa)$

$$H(\kappa) \to \frac{8\gamma}{\pi\,\kappa^3}\,\exp\left[-\frac{4\gamma}{\pi\,\kappa^2}\right] \text{ as } \kappa \to 0$$

$$H(\kappa) \to \frac{\gamma}{(1-\kappa)\,[\ln(1-\kappa)]^2} \quad \text{as } \kappa \to 1$$

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M. Biroli, H. Larralde, S. M., G. Schehr, PRL, 130, 207101 (2023)

Generalisations

The structure of the joint distribution in the **NESS** for N independent particles driven by simultaneous resetting is very general:

$$P_r^{\rm st}(\{x_i\}) = r \int_0^\infty d\tau\, e^{-r\tau} \prod_{i=1}^{N} p_0(x_i|\tau)$$

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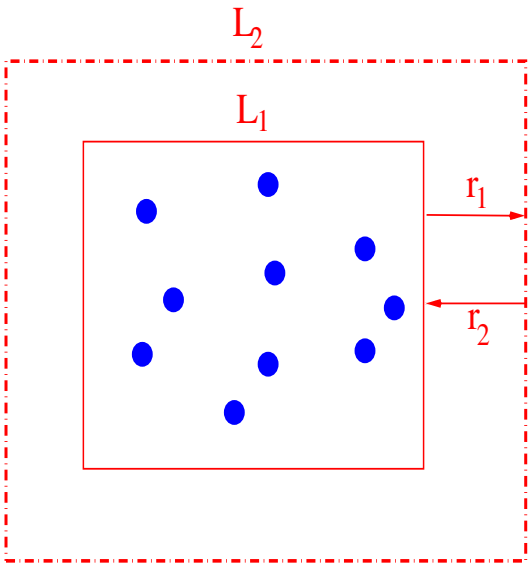
Ex: ballistic motion, Lévy flights etc.

$\Longrightarrow$ a whole class of **solvable** correlated gases in their **NESS**

$\Longrightarrow$ a new application of stochastic resetting

M. Biroli, H. Larralde, S. M., G. Schehr, Phys. Rev. E **109**, 014101 (2024)

Summary so far . . .

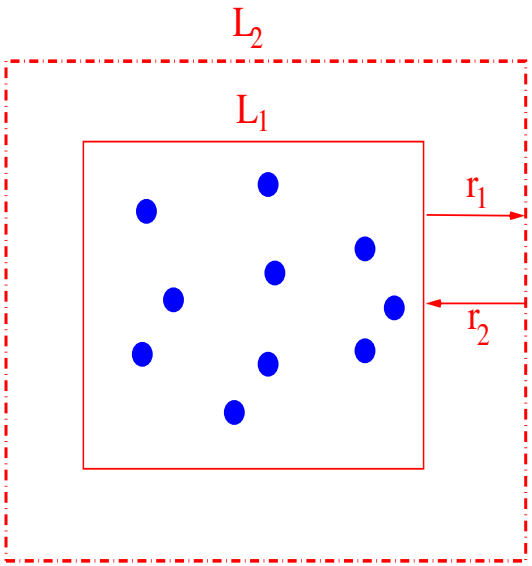


A limiting case:

$L_1 = 0$ and $L_2 \to \infty$

$r_1 \to \infty$ and $r_2 = \mathbf{r}$

Summary so far . . .



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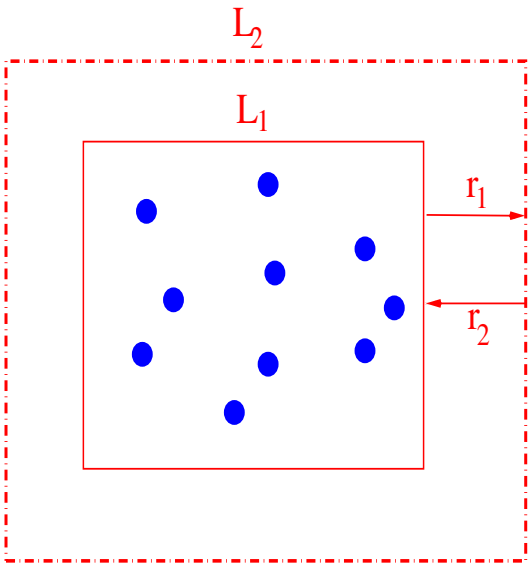
$L_1 = 0$ and $L_2 \to \infty$

$r_1 \to \infty$ and $r_2 = \mathbf{r}$

- This limiting case $\Longrightarrow$ **Exactly solvable** with a **strongly correlated NESS**
- The stationary joint distribution in the **NESS** $\Longrightarrow$ **CIID** structure

$\Longrightarrow$ allows us to compute physical observables expliciitly

Summary so far . . .



A limiting case:

$L_1 = 0$ and $L_2 \to \infty$

$r_1 \to \infty$ and $r_2 = \mathbf{r}$

- This limiting case $\Longrightarrow$ **Exactly solvable** with a **strongly correlated NESS**
- The stationary joint distribution in the **NESS** $\Longrightarrow$ **CIID** structure

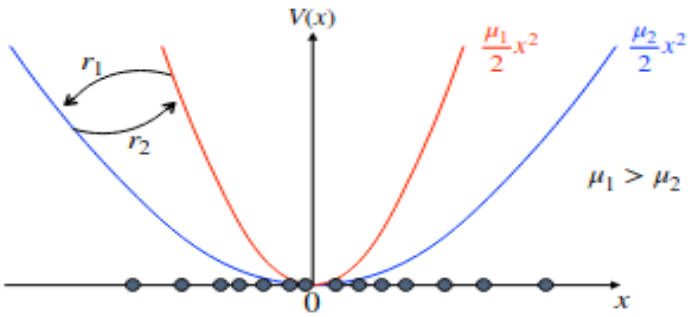
  $\Longrightarrow$ allows us to compute physical observables expliciitly
- How generic is the **CIID** structure $\Longrightarrow$ going beyond this limiting case ?

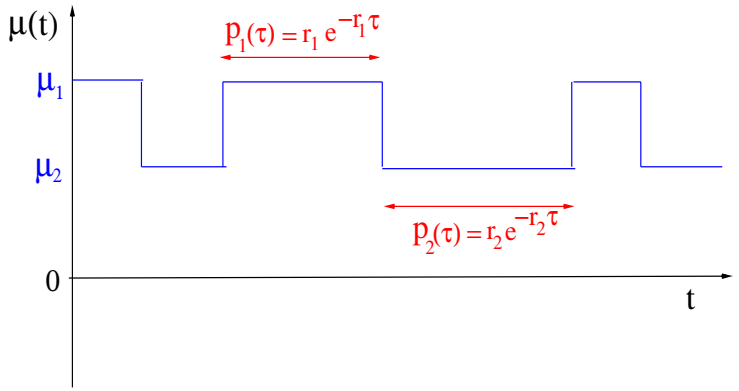
Switching Harmonic Trap

Switching Harmonic Trap

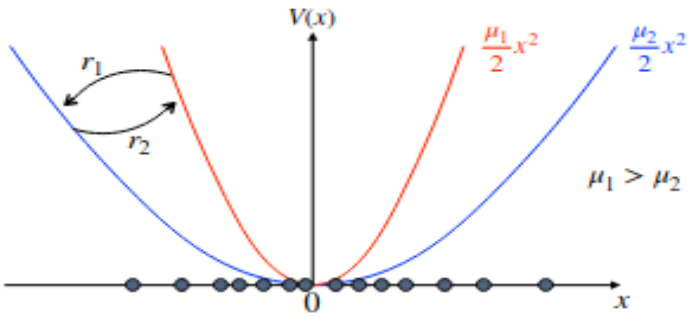
(a cousin of the box problem)

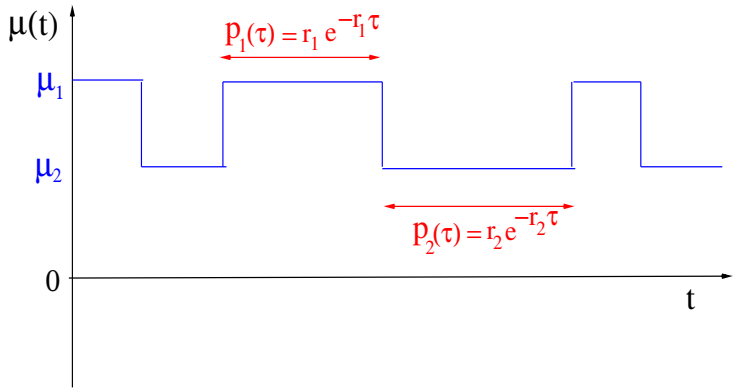
N Brownian particles in a switching harmonic trap





N Brownian particles in a switching harmonic trap



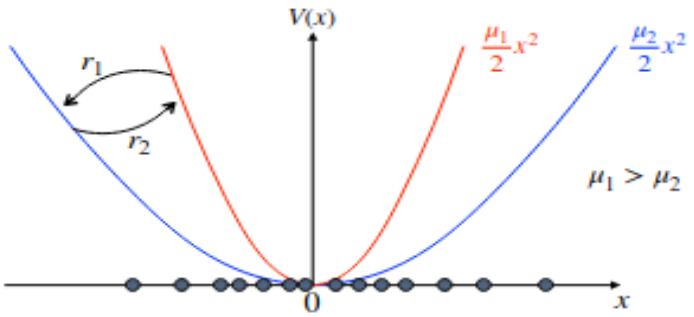


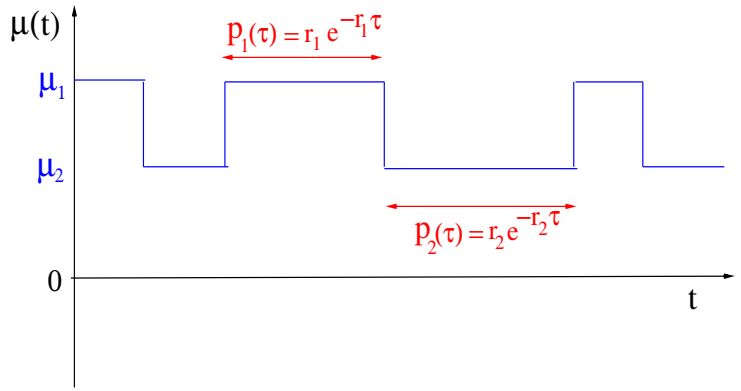
$$\frac{dx_i}{dt} = -\mu(t)\, x_i + \sqrt{2D}\, \eta_i(t)$$

$\eta_i(t) \longrightarrow$ Gaussian white noise with zero mean
and correlator $\langle \eta_i(t)\eta_j(t')\rangle = \delta_{i,j}\,\delta(t-t')$

The stiffness $\mu(t)$ of the harmonic trap changes from $\mu_1 \to \mu_2 < \mu_1$ with rate r_1 and $\mu_2 \to \mu_1$ with rate $r_2 \Longrightarrow$ dichotomous telegraphic noise

N Brownian particles in a switching harmonic trap





$$\frac{dx_i}{dt} = -\mu(t)\, x_i + \sqrt{2D}\, \eta_i(t)$$

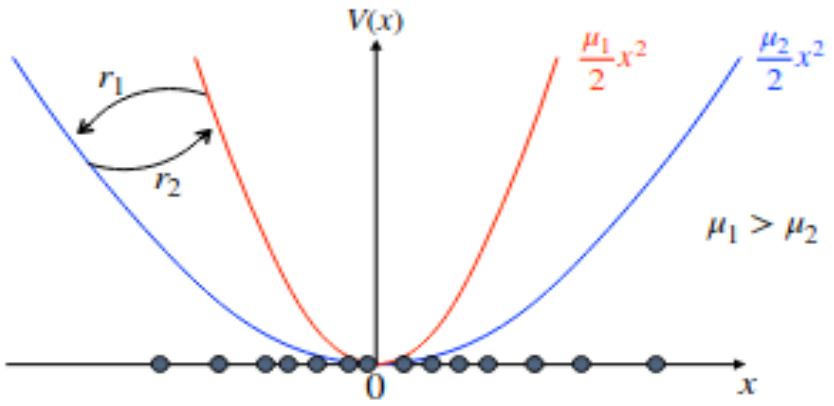
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$\Longrightarrow$ drives the system into a correlated NESS with a stationary joint distribution $P(x_1, x_2, \ldots, x_N | t \to \infty) = P^{\rm st}(\vec{x}|t \to \infty) = ?$

Biroli, Kulkarni, S.M., Schehr, PRE, 109, L032106 (2024)

Switching harmonic trap $\longrightarrow$ Exact Solution



Generic case:

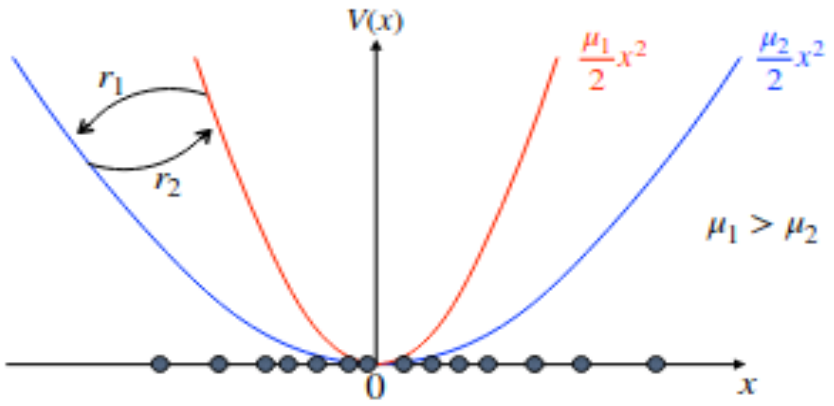
Arbitrary $\mu_1 > 0$, $\mu_2 > 0$,

$r_1 > 0$ and $r_2 > 0$

$\Longrightarrow$ renewal approach **no longer valid**

$\Longrightarrow$ yet **fully solvable**

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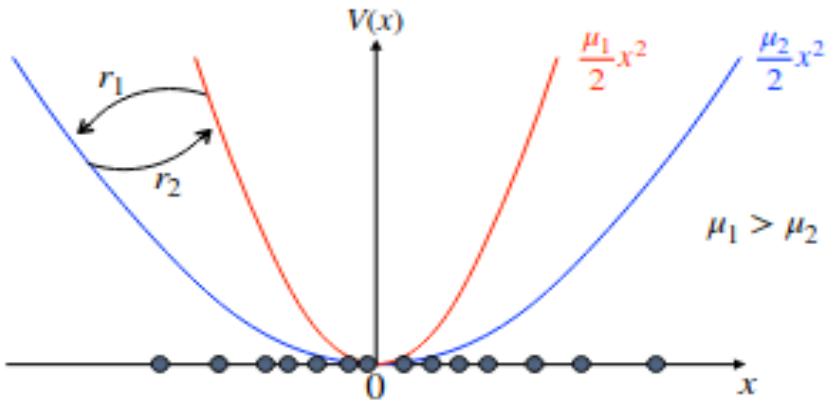
$r_1 > 0$ and $r_2 > 0$

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$P_{1,2}(\vec{x}|t) \longrightarrow$ Prob. that the position is $\vec{x}$ and the stiffness is μ_1 (or μ_2) at time t

Switching harmonic trap $\longrightarrow$ Exact Solution



Generic case:

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$P_{1,2}(\vec{x}|t) \longrightarrow$ Prob. that the position is $\vec{x}$ and the stiffness is μ_1 (or μ_2) at time t

They satisfy a pair of coupled Fokker-Planck equations:

$$\partial_t P_1 = D \sum_{i=1}^{N} \partial_{x_i}^2 P_1 + \mu_1 \sum_{i=1}^{N} \partial_{x_i} (x_i P_1) - r_1 P_1 + r_2 P_2$$

$$\partial_t P_2 = D \sum_{i=1}^{N} \partial_{x_i}^2 P_2 + \mu_2 \sum_{i=1}^{N} \partial_{x_i} (x_i P_2) - r_2 P_2 + r_1 P_1$$

M. Biroli, M. Kulkarni, S.M., G. Schehr, PRE, 109, L032106 (2024)

'Hidden' CIID structure of the stationary state

Exact stationary solution $\longrightarrow$ a **CIID** representation

$$P^{\rm st}(\vec{x}) = P_1^{\rm st}(\vec{x}) + P_2^{\rm st}(\vec{x}) = \int_0^1 du\, h(u) \prod_{i=1}^{N} \frac{1}{\sqrt{2\,V(u)}}\, e^{-x_i^2/(2\,V(u))}$$

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where

$$V(u) = D\left(\frac{u}{\mu_2} + \frac{1-u}{\mu_1}\right)$$

and

$$h(u) = A\, u^{R_1-1}\,(1-u)^{R_2-1}\left[\frac{u}{\mu_2} + \frac{1-u}{\mu_1}\right]$$

with $R_1 = r_1/2\mu_1$ and $R_2 = r_2/2\mu_2$

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Since $\int_0^1 h(u)du = 1$, the function $h(u)$ can be interpreted as the PDF of the random variable $u \in [0,1] \longrightarrow$ the fraction of time each particle spends in μ_2 phase

Biroli, Kulkarni, S.M., Schehr, PRE, 109, L032106 (2024)

All observables $\longrightarrow$ exactly solvable

Using the explicit **CIID** structure of the stationary joint PDF

$$P^{\rm st}(\vec{x}) = \int_0^1 du\, h(u) \prod_{i=1}^{N} p_0(x_i|V(u))$$

all observables in the correlated **NESS** can be computed explicitly and they exhibit rich and interesting behaviors

Biroli, Kulkarni, S.M., Schehr, PRE, 109, L032106 (2024)

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Biroli, Kulkarni, S.M., Schehr, PRE, 109, L032106 (2024)

For example, the extreme value statistics (**EVS**) (setting $r_1 = r_2 = r$):

$$P(M_1 = w, N) \to \frac{1}{\sqrt{\ln N}}\, f\left(\frac{w\sqrt{r}}{\sqrt{4D \ln N}}\right)$$

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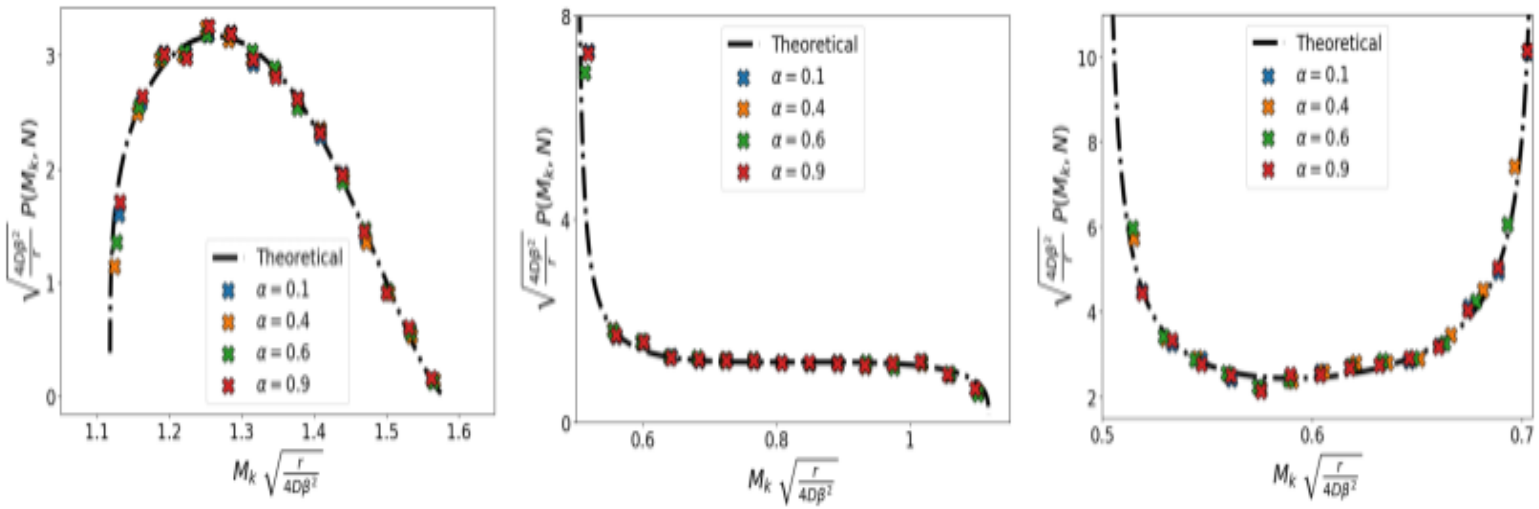
$$P(M_1 = w, N) \to \frac{1}{\sqrt{\ln N}}\, f\left(\frac{w\sqrt{r}}{\sqrt{4D \ln N}}\right)$$

where the exact scaling function (with $R_1 = \frac{r}{2\mu_1}$ and $R_2 = \frac{r}{2\mu_2}$):

$$f(z) = B\, z^3 \left(1 - \frac{z^2}{R_2}\right)^{R_2 - 1} \left(\frac{z^2}{R_1} - 1\right)^{R_1 - 1} \text{ with } \sqrt{R_1} \le z \le \sqrt{R_2}$$

$\longrightarrow$ a new extreme value distribution of **strongly** correlated random variables with a **finite** support

EVS with a finite support: Universality



The exact scaling function for the distribution of the scaled k-th maximum M_k

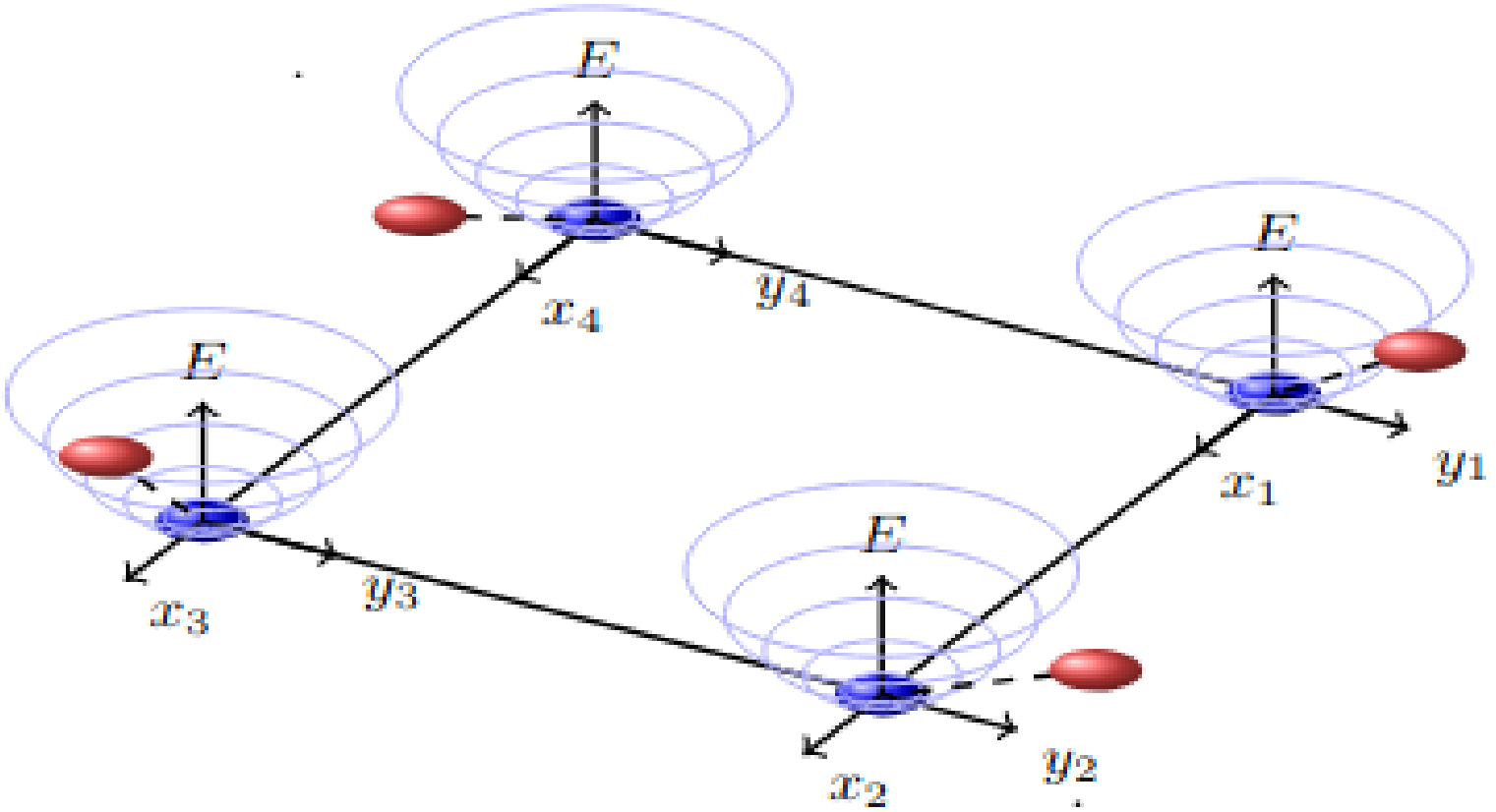
$$f(z) = B\, z^3 \left(1 - \frac{z^2}{R_2}\right)^{R_2 - 1} \left(\frac{z^2}{R_1} - 1\right)^{R_1 - 1} \text{ with } \sqrt{R_1} \leq z \leq \sqrt{R_2}$$

The scaling function $f(z) \longrightarrow$ **universal**, i.e,., same for all M_k's in $d = 1$ and also for all $\mathbf{d \geq 1}$

Biroli, Kulkarni, S.M., Schehr, PRE, 109, L032106 (2024)

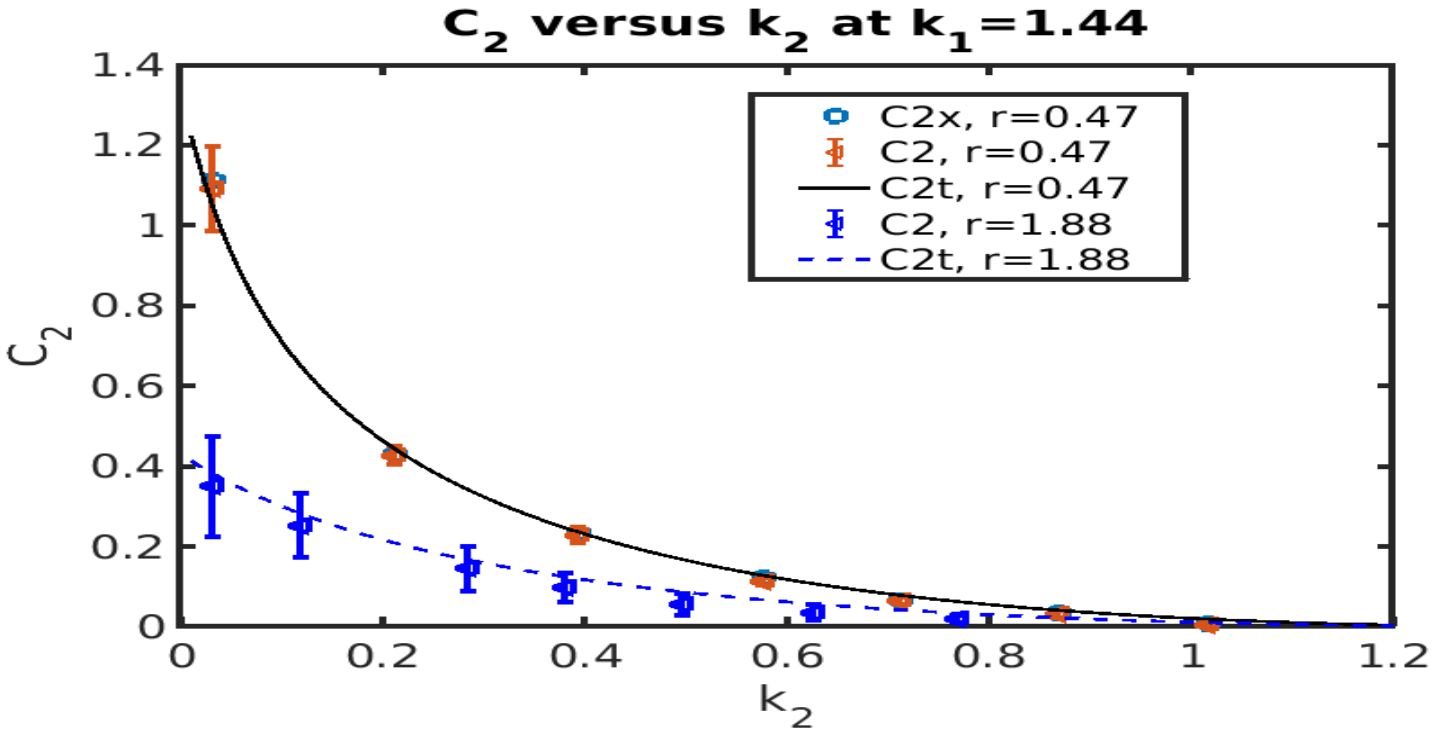
Experimental results :

[set-up: S. Ciliberto (ENS Lyon)]



- colloidal particles in synchronized harmonic traps
- particles are immersed in fluid $\Longrightarrow$ long-range hydrodynamic interaction (**neglected** in the theoretical model)

Experimental results on C_2



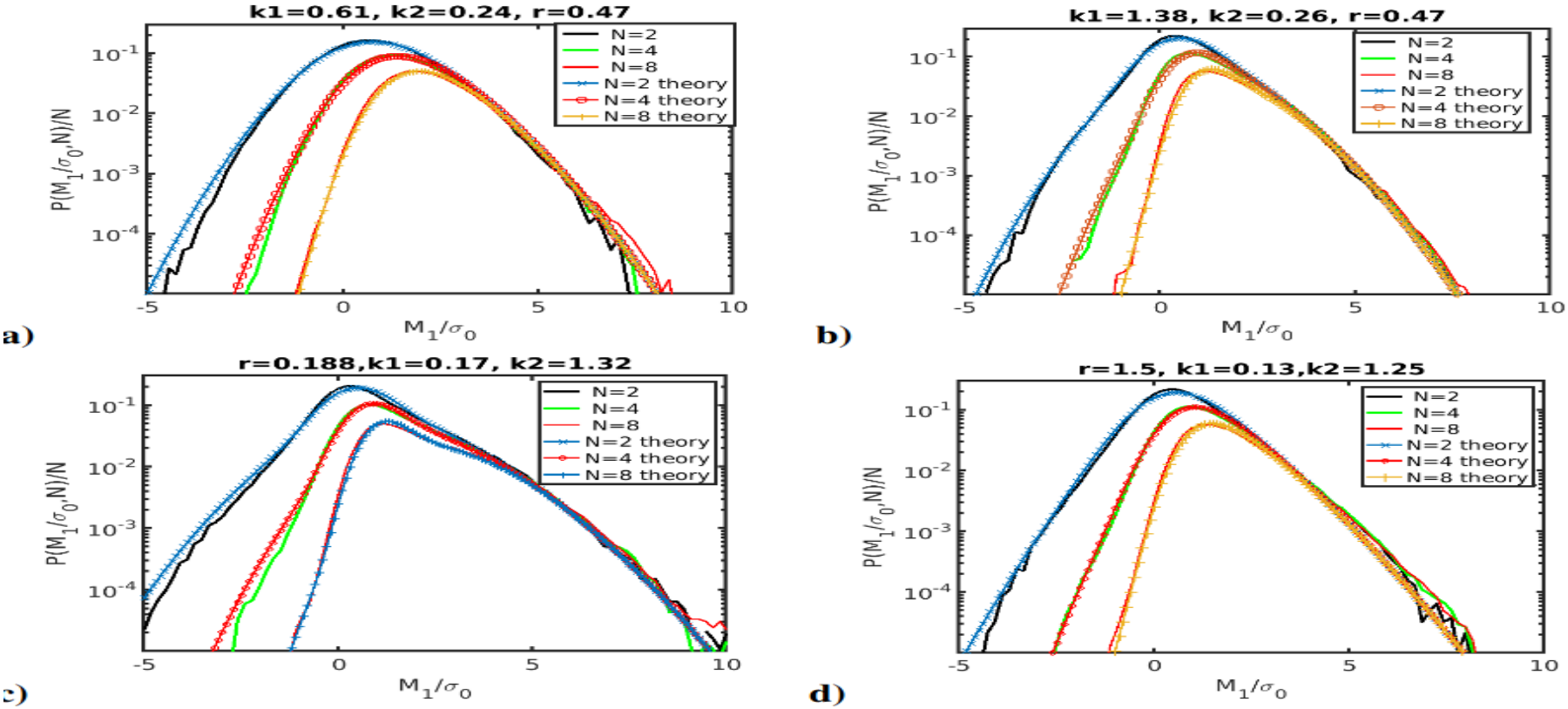
$$C_2 = \frac{\langle x_i^2\, x_j^2\rangle}{\langle x_i^2\rangle\langle x_j^2\rangle} - 1 \qquad \mu_1 = 1.44 \text{ and } \mu_2 = k_2$$

Biroli, Ciliberto, Kulkarni, **S.M.**, Petrosyan, Schehr, PRL, 137, 037102 (2026)

Theoretical prediction [for $r_1 = r_2 = r$ and with $R_1 = \frac{r}{2\mu_1}$ and $R_2 = \frac{r}{2\mu_2}$]:

$$\boxed{C_2 = \frac{(\mu_2 - \mu_1)^2(2 + 3R_1 + 3R_2 + 4R_1R_2)}{(2 + R_1 + R_2)(2r + \mu_1 + \mu_2)^2}}$$

Experimental results on M_1:

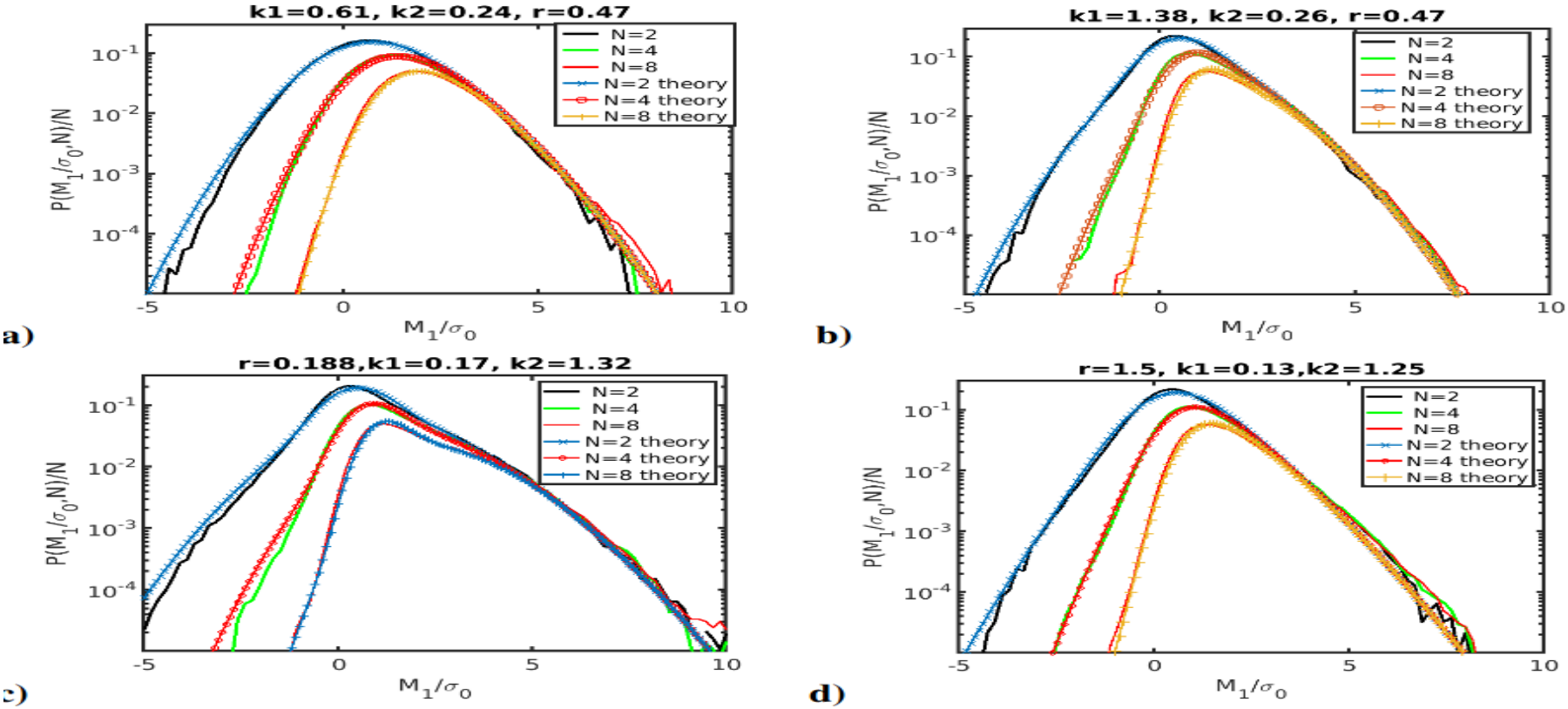


Up to $N = 8$ optical traps/particles

Biroli, Ciliberto, Kulkarni, **S.M.**, Petrosyan, Schehr, PRL, 137, 037102 (2026)

Emergent correlations dominate over hydrodynamic interactions

Experimental results on M_1:



Up to $N = 8$ optical traps/particles

Biroli, Ciliberto, Kulkarni, **S.M.**, Petrosyan, Schehr, PRL, 137, 037102 (2026)

Emergent correlations dominate over hydrodynamic interactions

Similar conclusion in a different setup $\longrightarrow$ Vatash & Roichmann (2025)

Other models with CIID structure

Classical models:

- N noninteracting particles in a harmonic trap whose center fluctuates **stochastically** S. Sabhapandit & S.M. J. Phys. A.: Math. Theor. **57**, 335003 (2024)
- N noninteracting Brownian motions with a common **diffusing diffusivity** N. Mesquita, S.M., & S. Sabhapandit, J. Stat. Mech. 103207 (2025)
- N noninteracting Brownian motions under simultaneous **non-Poissonian** resetting
  G. de Mauro, M. Biroli, S.M. & G. Schehr, Phys. Rev. E **113**, 014120 (2026)
- N noninteracting Brownian motions with **batch** resetting
  G. de Mauro, S.M., G. Schehr, arXiv: 2601.20077
- N noninteracting Brownian motions $\longrightarrow$ resetting with **memory**
  D. Boyer & S.M., J. Phys. A: Math. Theor. **59**,125001 (2026)
- Models of **catastrophes** under diffusion approximation
  T. Galla, arXiv:2602.21635
- **Linear statistics** for systems with DEC K. S. Olsen, arXiv:2603.01852

Other models with CIID structure

Quantum models:

- N noninteracting Bosons evolving **unitarily** in a harmonic trap and subject to quantum resetting

  M. Kulkarni, S.M. & S. Sabhapandit, J. Phys. A: Math. Theor. **58**, 105003 (2025)

  N. Mesquita, M. Kulkarni, S.M., & S. Sabhapandit, arXiv: 2509.00487

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  N. Mesquita, M. Kulkarni, S.M., & S. Sabhapandit, arXiv: 2509.00487

All these models (both classical and quantum) exhibit dynamically emergent correlations (DEC) and the NESS has the CIID (conditionally independent and identically distributed) structure

$\Longrightarrow$ **solvable** family of **strongly correlated many-body systems**

Summary and Conclusion

- **Stochastic fluctuation** of the **common** environment

  $\Longrightarrow$ **strong emerging** correlations between particles

- Exactly solvable example: **switching** harmonic trap

- The NESS has a **CIID** structure

  $\Longrightarrow$ Several physical observables are exactly computable and have rich interesting behaviors, despite being a **strongly correlated** system

- Comparison with experiments on colloidal particles in an optical trap

- Easily generalisable to a whole new class of solvable correlated gases in their nonequilibrium stationary state $\longrightarrow$ ballistic particles, Lévy flights, harmonic potential with a stochastic center, noninteracting bosons, ..

  $\Longrightarrow$ all have this **CIID** structure $\Rightarrow$ **Exactly solvable**

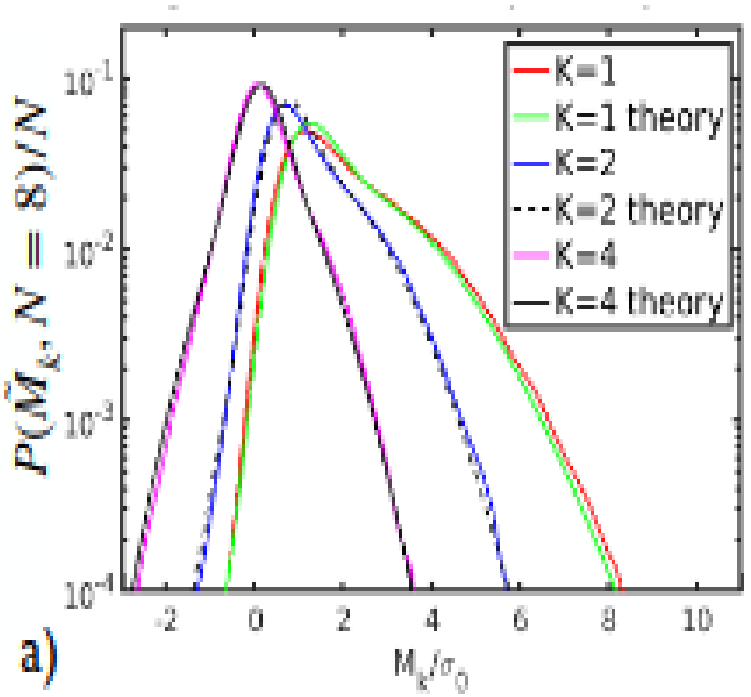
Collaborators:

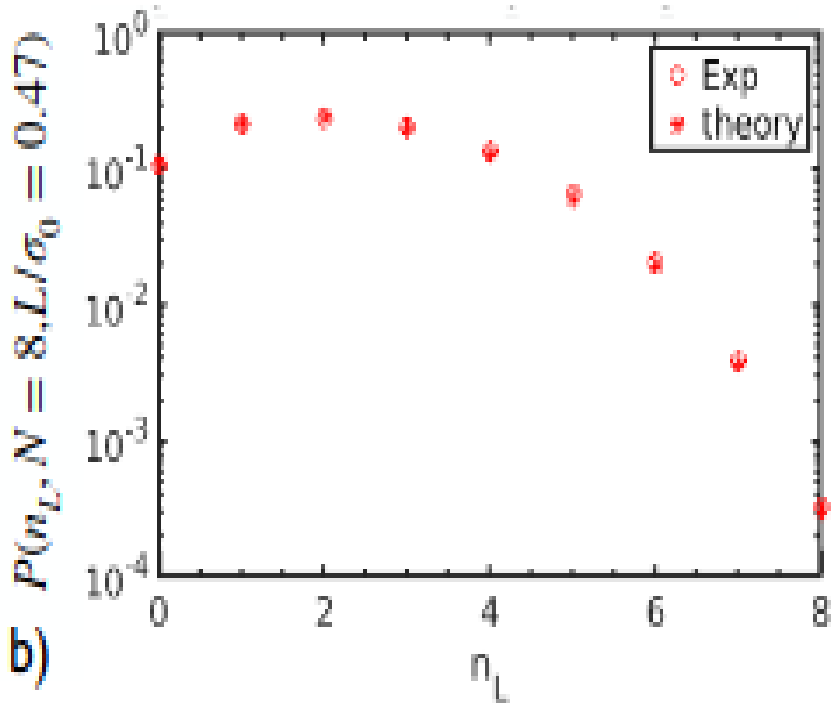
- **Marco Biroli (LPTMS, Univ. Paris Saclay $\longrightarrow$ Univ. of Chicago)**
- Arup Biswas, Arnab Pal (IMSC, Chennai)
- Denis Boyer (UNAM, Mexico)
- Sergio Ciliberto, A. Petrosyan (ENS, Lyon)
- Manas Kulkarni (ICTS, Bangalore)
- Hernan Larralde (UNAM, Mexico)
- **Gabriele de Mauro (LPTMS, Orsay)**
- Nikhil Mesquita (RRI, Bangalore)
- Sanjib Sabhapandit (RRI, Bangalore)
- Gregory Schehr (LPTHE, Univ. Sorbonne)

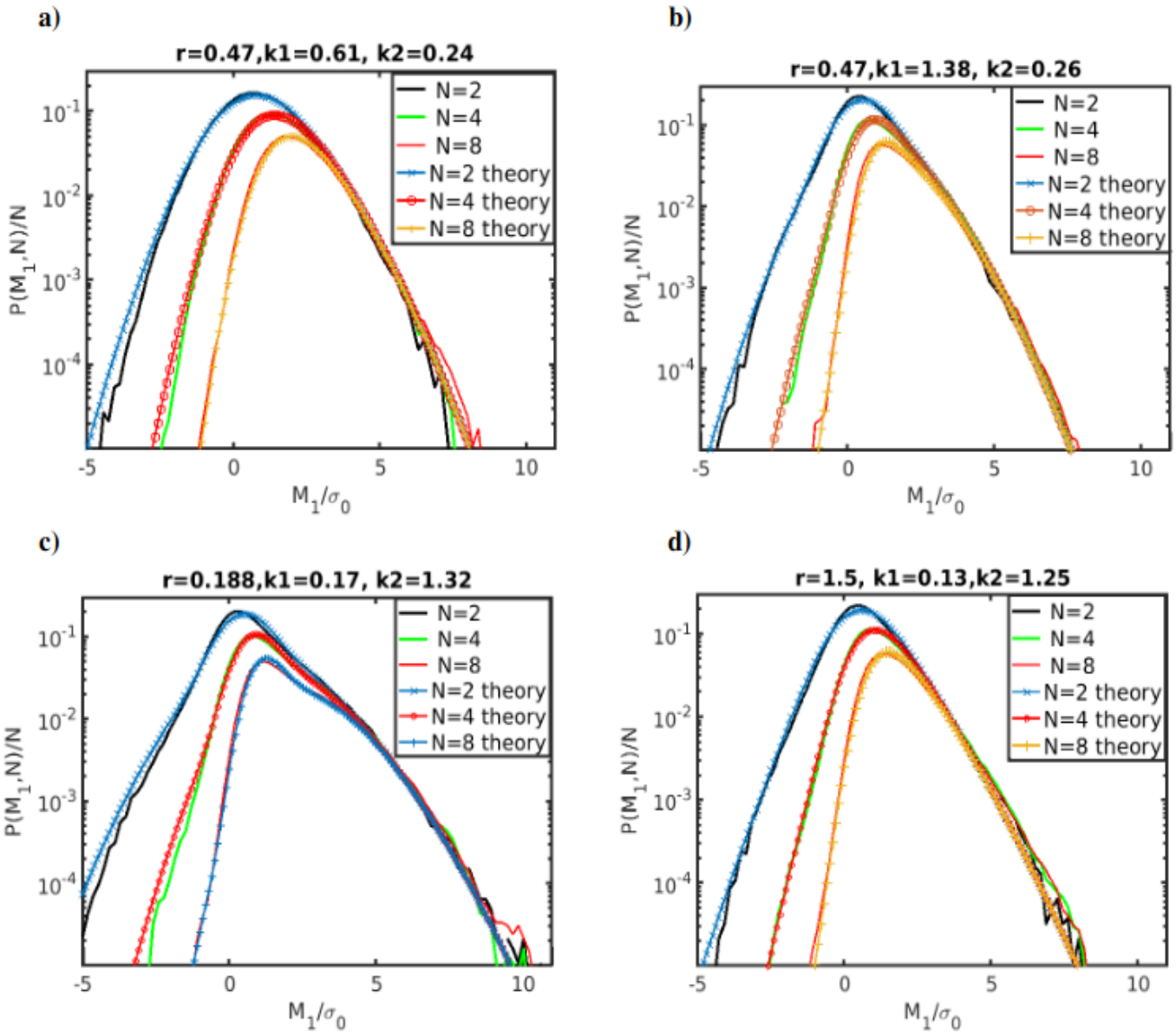
Earlier work on **Stochastic Resetting** with Martin Evans (Edinburgh)

Ongoing work with Urna Basu (S.N. Bose Center, Kolkata)

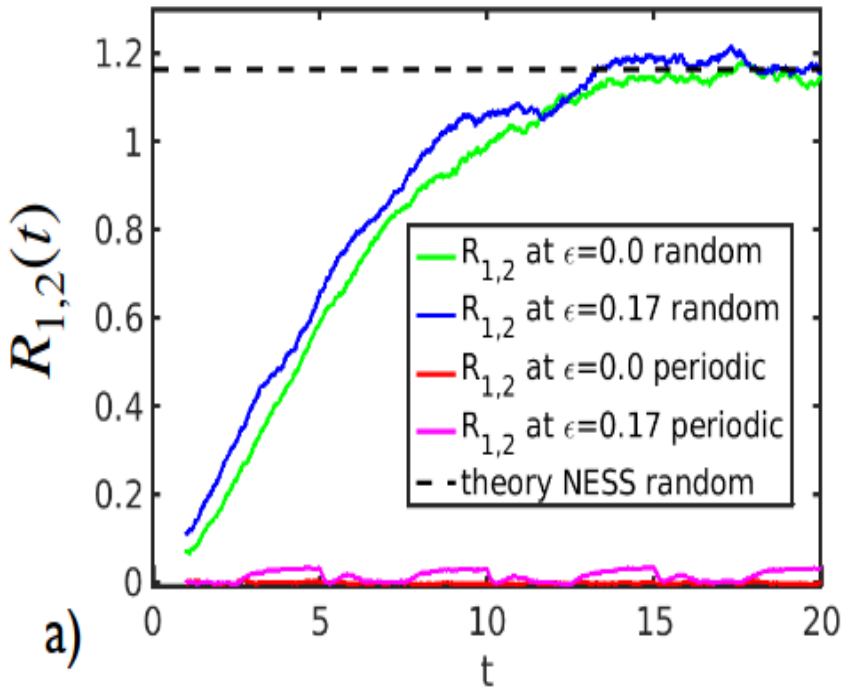
EVS and FCS

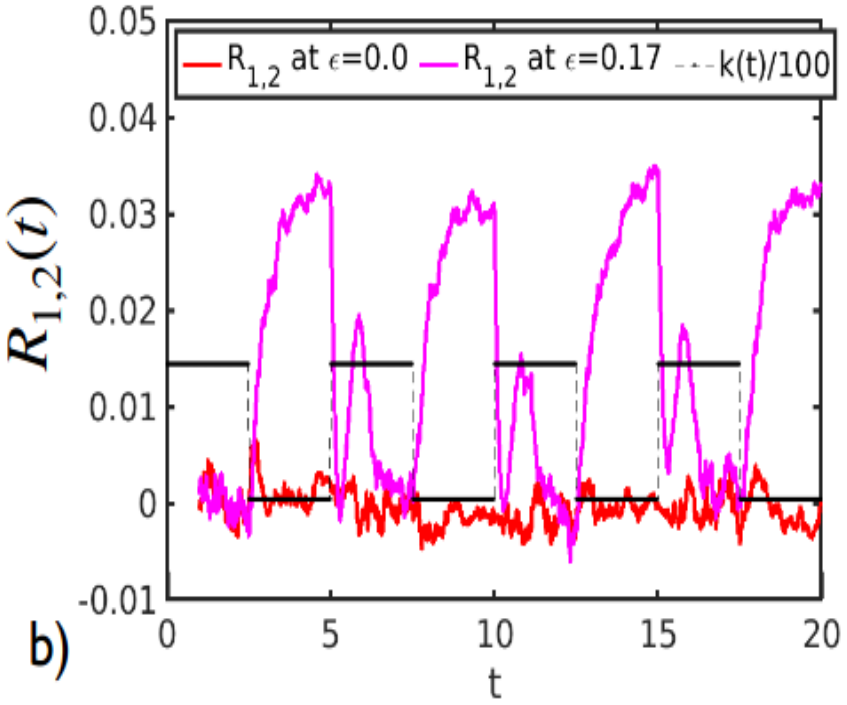

K=1
K=1 theory
K=2
K=2 theory
K=4
K=4 theory
$P(\tilde{M}_k, N = 8)/N$
M_k/σ_0
a)


Exp
theory
$P(n_L, N = 8, L/\sigma_0 = 0.47)$
n_L
b)


a)
r=0.47,k1=0.61, k2=0.24
b)
r=0.47,k1=1.38, k2=0.26
c)
r=0.188,k1=0.17, k2=1.32
d)
r=1.5, k1=0.13,k2=1.25
N=2
N=4
N=8
N=2 theory
N=4 theory
N=8 theory
P(M1,N)/N
M1/σ0

Simulations with/without hydrodynamic interactions





References:

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- M. Biroli, H. Larralde, S. N. Majumdar, G. Schehr, "Exact extreme, order and sum statistics in a class of strongly correlated system", Phys. Rev. E **109**, 014101 (2024).

- M. Biroli, M. Kulkarni, S. N. Majumdar, G. Schehr, "Dynamically emergent correlations between particles in a switching harmonic trap ", Phys. Rev. E **109**, L032106 (2024).

- S. Sabhapandit, S. N. Majumdar, "Noninteracting particles in a harmonic trap with a stochastically driven center ", J. Phys. A: Math. Theor. **57**, 335003 (2024).

- M. Kulkarni, S. N. Majumdar, S. Sabhapandit, "Dynamically emergent correlations in bosons via quantum resetting ", J. Phys. A: Math. Theor. **58**, 105003 (2025).

- M. Biroli, S. N. Majumdar, G. Schehr, "Resetting Dyson Brownian motion ", Phys. Rev. E **112**, 014101 (2025)).

- N. Mesquita, S. N. Majujmdar, S, Sabhapandit, "Dynamically emergent correlations in a Brownian gas with diffusing diffusivity", arXiv: 2506.20859

- M. Biroli, S. Ciliberto, M. Kulkarni, S. N. Majumdar, A. Petrosyan, G. Schehr, "Experimental evidence for strong emergent correlations between particles in a switching trap", arXiv: 2508.07199

- G. de Mauro, M. Biroli, S. N. Majumdar, G. Schehr, "Dynamically emergent correlations in Brownian particles subject to simultaneous non-Poissonian resetting protocols", arXiv: 2509.06658