Rényi Mutual Information in Holographic WCFT

Bin Chen

Peking University

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Based on:
BC and Jia-ju Zhang, 1309.5453
BC, Jiang Long and Jia-ju Zhang, 1312.5510
and the on-going work with Peng-xiang Hao and Wei Song

AdS_3/CFT_2 correspondence

A new window to study AdS/CFT without resorting to string theory

$$I = \frac{1}{16\pi G}\int d^3x\sqrt{-g}(R + \frac{2}{l^2})$$

3D AdS_3 Einstein gravity is special: No locally dynamical d.o.f

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More precisely they found that under appropriate boundary conditions the asymptotic symmetry group (ASG) of AdS_3 Einstein gravity is generated by two copies of Virasoro algebra with central charges

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In modern understanding: quantum gravity in AdS_3 is dual to a 2D CFT at AdS boundary

Asymptotic Symmetry

One important lesson is that the asymptotic boundary conditions play an essential role in defining a quantum gravity theory and its holographic dual.

One has to find the asymptotic boundary conditions carefully: not too restricted, which could kill interesting physics; but not too relaxed, which cannot induce nontrivial physics.

Given a set of consistent asymptotic boundary conditions, one may define the asymptotic symmetry group (ASG)

$$\text{ASG} = \frac{\text{Allowed asymptotic symmetry}}{\text{Trivial asymptotic symmetry}}$$

Brown-Henneaux boundary condition

For the AdS_3 gravity, the metric could be expanded in Fefferman-Graham coordinates (set $l = 1$)

$$ds^2 = \frac{dr^2}{r^2} + r^2 \left(g^{(0)}_{ab} + \frac{1}{r^2} g^{(2)}_{ab} + O(r^{-3}) \right) dx^a dx^b$$

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Brown and Heanneaux found a consistent set of parity-invariant boundary conditions:

$$g^{(0)}_{++} = g^{(0)}_{--} = 0, \quad g^{(0)}_{+-} = g^{(0)}_{-+} = -\frac{1}{2}.$$

where $t^{\pm} = t \pm \phi$, $\phi \sim \phi + 2\pi$. Under this condition, the ASG of the gravity is generated by two copies of Virasoro algebra with entral charges $c_L = c_R = \frac{3}{2G}$. The Virasoro symmetries act infinitesimally on the coordinates $t^{\pm}$ of the boundary cylinder as

$$\delta t^+ = \epsilon^+(t^+), \quad \delta t^- = \epsilon^-(t^-)$$

The semi-classical states, including BTZ BH, form representations of left and right Virasoro algebras.

Compere-Song-Strominger(CSS) boundary condition

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Compere-Song-Strominger(CSS) boundary conditions Compere et.al. 1303.2622

$$g^{(0)}_{--} = 0, \qquad g^{(0)}_{+-} = g^{(0)}_{-+} = -\frac{1}{2}.$$

$$g^{(0)}_{++} = \partial_+\bar{P}(t^+), \qquad g^{(2)}_{--} = 4G\Delta.$$

where Δ is a constant and $\partial_+\bar{P}$ is periodic.

1. The CSS B.C. are not parity invariant.
2. The corresponding ASG is generated by a right-moving noncompact $U(1)$ Kac-Moody-Virasoro algebra, which acts on the coordinates as

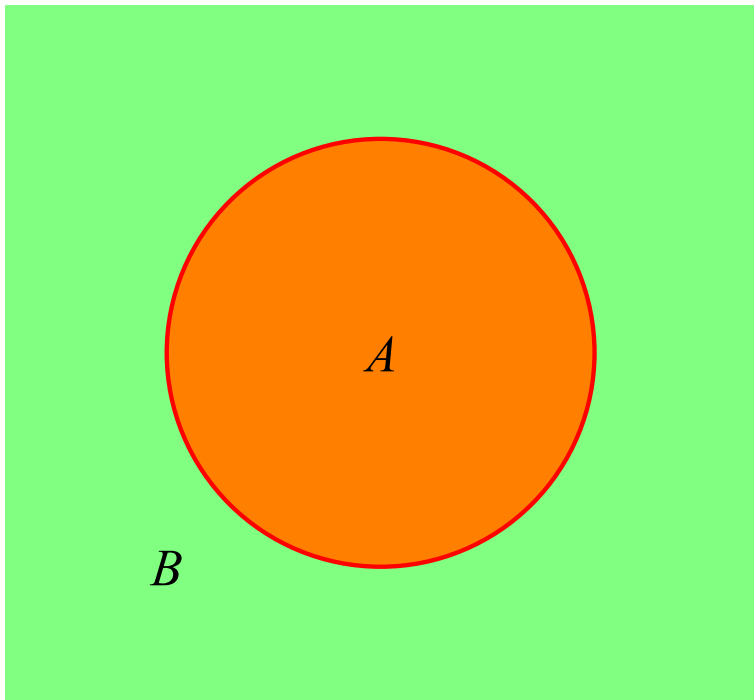
   $$\delta t^+ = \epsilon(t^+), \qquad \delta t^- = \sigma(t^+)$$

   The 2D CFT with such a symmetry is called a warped CFT. Hofman and Strominger, 1107.2917, Detouney et.al. 1210.0539
3. AdS_3/WCFT correspondence: the AdS_3 gravity with CSS B.C. is holographically dual to a warped CFT (at least in the semiclassical sense).

In this talk, I would like to show how the study of Rényi mutual information shed light on semiclassical $\mathrm{AdS}_3/\mathrm{CFT}_2$ and $\mathrm{AdS}_3/\mathrm{WCFT}$ correspondences.

Entanglement entropy

Entanglement entropy is an important notion in quantum world.



For A and its complement B

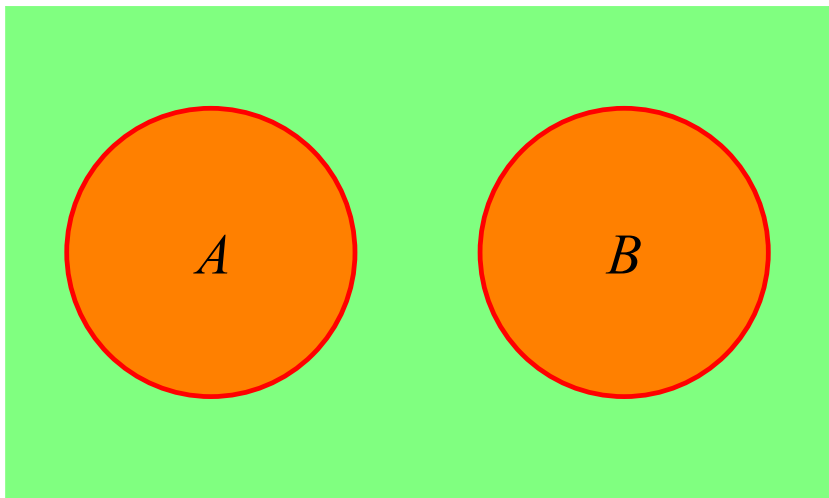
- $\mathcal{H}_{tot} = \mathcal{H}_A \otimes \mathcal{H}_B$ and $\rho_{tot} = |\Psi\rangle\langle\Psi|$
- Reduced density matrix: $\rho_A = \mathrm{tr}_B \rho_{tot}$
- Entanglement entropy $S_A = -\mathrm{tr}_A \rho_A \ln \rho_A$
- Rényi entropy $S_A^{(n)} = -\frac{\ln \mathrm{tr}_A \rho_A^n}{n-1}$
- $S_A = \lim_{n\to 1} S_A^{(n)}$

Rényi mutual information

- Choose two subsystems A and B which are not necessarily each other's complement
- Define the Rényi mutual information of A and B

$$I_{A,B}^{(n)} = S_A^{(n)} + S_B^{(n)} - S_{A+B}^{(n)}.$$

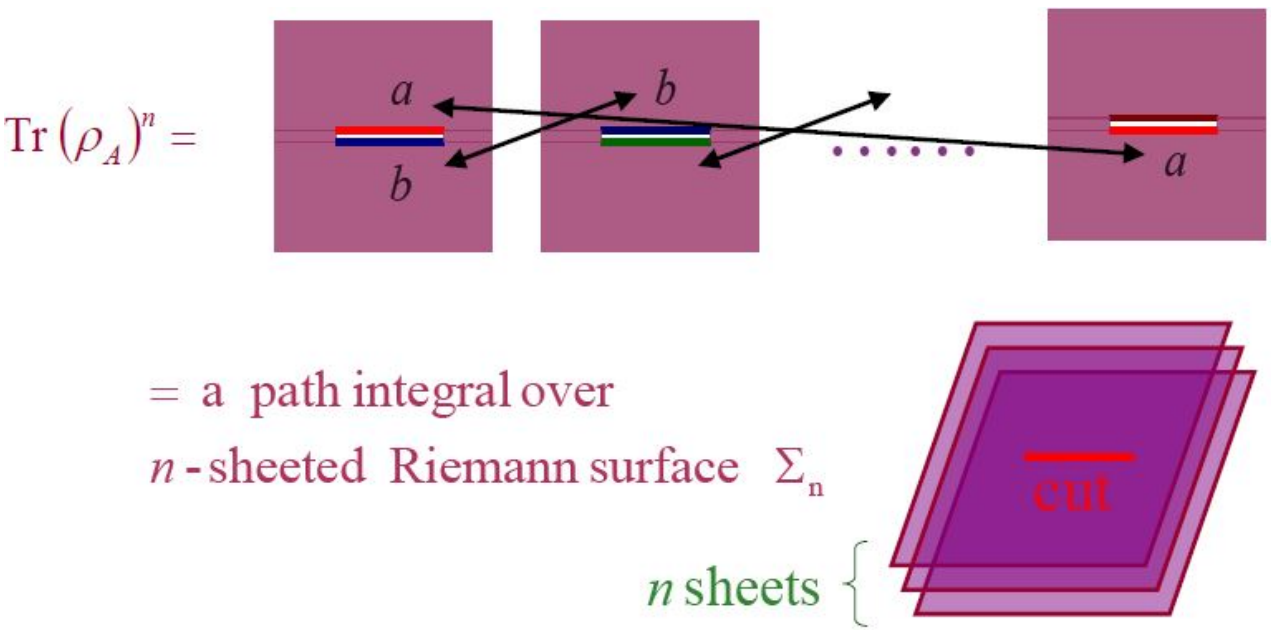
- For $n = 1$, it is called mutual information. From subadditivity, we know $I_{A,B} \geq 0$



Replica trick

In order to compute $\mathrm{tr}_A \rho_A^n$, one can use the replica trick J. Callan et.al. 9401072

For a 2D QFT, we have the picture Figures from T. Takayanagi's lecture



Replica trick II

- Picture 1: partition function on a n-sheeted Riemann surface Σ_n.
- Picture 2: multi-point function of the twist operators in a product orbifold $(\mathrm{CFT})^n/Z_n$.

The twist operators $\sigma, \tilde{\sigma}$ at the branch points are primary and have conformal dimensions

$$h = \bar{h} = \frac{c}{24}\left(n - \frac{1}{n}\right).$$

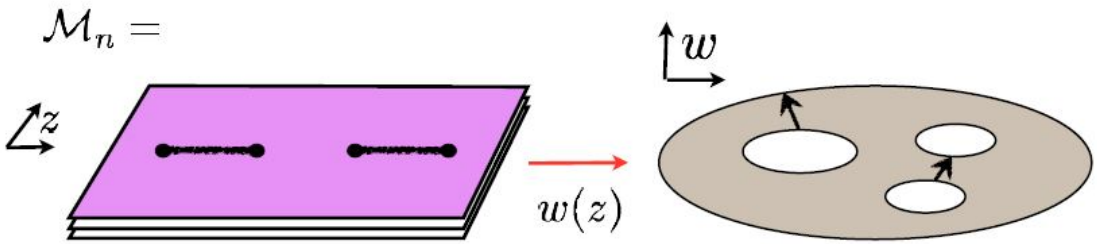
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In the two-interval case, Σ_n is of genus $(n-1)$. The partition function on Σ_n is very difficult to compute.
Nevertheless, in the case that the intervals are short, we may use operator product expansion(OPE) of the twist operators to compute.



Two-interval case: large distance expansion

M. Headrick 1006.0047, P. Calabrese et.al. 1011.5482, BC and J-j Zhang 1309.5453

$$\sigma(z,\bar{z})\tilde{\sigma}(0,0) = c_n \sum_K d_K \sum_{m,r\geq 0} \frac{a_K^m}{m!}\frac{\bar{a}_K^r}{r!} \frac{1}{z^{2h-h_K-m}\bar{z}^{2\bar{h}-\bar{h}_K-r}} \partial^m\bar{\partial}^r\Phi_K(0,0),$$

with the summation K being over all the independent quasiprimary operators of CFT_n.
We are interested in the two-interval case, then

$$\begin{aligned} \mathrm{Tr}\rho_A^n &= \langle\sigma(1+y,1+y)\tilde{\sigma}(1,1)\sigma(y,y)\tilde{\sigma}(0,0)\rangle_C \\ &= c_n^2 x^{-\frac{c}{6}\left(n-\frac{1}{n}\right)} \left(\sum_K \alpha_K d_K^2 x^{h_K} F(h_K,h_K;2h_K;x)\right)^2 \end{aligned}$$

where x is the conformal-invariant cross ratio.
In the large distance limit, x is small such that to each order of x only finite number of the quasi-primary operators contribute.
In general, the result depends on the OPE coefficients α_K. However, the things get simplified in the holographic CFT.

Large c CFT

The holographic CFT in the large c limit is dual to the semiclassical AdS_3 gravity under Brown-Heanneaux B.C. . One attractive feature of the CFT is that in the large c limit

$$c \to \infty, \qquad h/c \quad \text{kept finite}$$

the vacuum module dominates the 4-point function

$$\langle O_1(x_1)O_2(x_2)|\mathcal{O}\rangle\langle\mathcal{O}|O_3(x_4)O_4(x_4)\rangle = \mathcal{F}_{\mathcal{O}}(x_i) \simeq e^{-\frac{c}{6}f(h_{\mathcal{O}}/c, h_i/c, x_i)}$$

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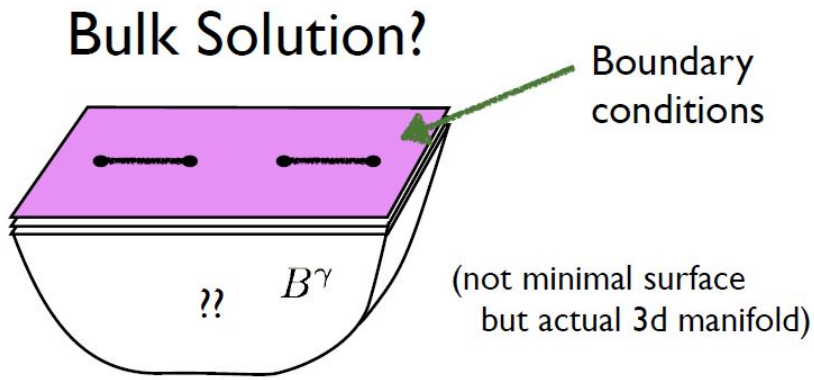
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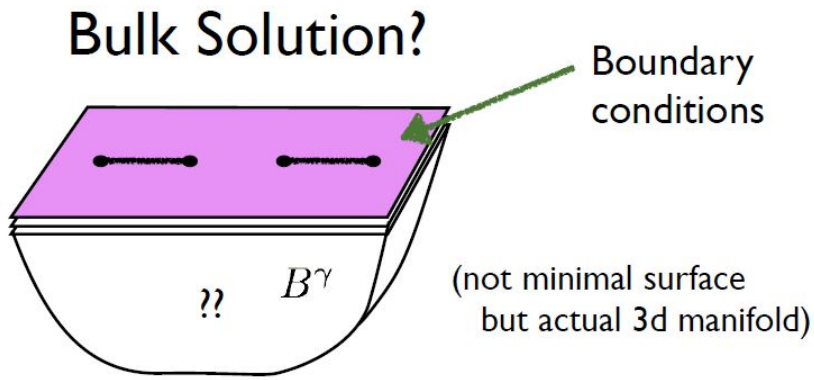
This allows us to focus on the Virasoro vacuum module in the CFT study. T. Hartman 1303.6955,...

Holographic picture T. Faulkner 1303.7221, T. Barrella et.al. 1306.4682



Find the bulk gravitational configurations B^γ such that $\partial B^\gamma = \Sigma_n$.
It turns out that $B^\gamma = \text{AdS}_3/\Gamma_\gamma$, where Γ_γ is the Schottky group.
From $\text{AdS}_3/\text{CFT}_2$, the classical regularized bulk action should reproduces the leading order $\mathcal{O}(c)$ partition function on Σ_n.

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For a fixed handle-body solution obtained from the Schottky group, its 1-loop partition function Giombi et.al. 0804.1773, X.Yin 0710.2129

$$Z^{1-loop} = \prod_{\gamma\in\mathcal{P}} \prod_{m=2}^{\infty} \frac{1}{|1-q_\gamma^m|},$$

should correspond to the order $\mathcal{O}(c^0)$ partition function on Σ_n.

Rényi mutual information: leading order

We may organize the result in orders of $1/c$ in the large c limit and in orders of x in the large distance limit.

The leading part, being proportional to the central charge c,

$$
\begin{aligned}
I_n^{LO} = & \frac{c(n-1)(n+1)^2x^2}{144n^3} + \frac{c(n-1)(n+1)^2x^3}{144n^3} \\
& + \frac{c(n-1)(n+1)^2\left(1309n^4-2n^2-11\right)x^4}{207360n^7} \\
& + \frac{c(n-1)(n+1)^2\left(589n^4-2n^2-11\right)x^5}{103680n^7} \\
& + \frac{c(n-1)(n+1)^2(805139n^8-4244n^6-23397n^4-86n^2+188)x^6}{156764160n^{11}} \\
& + (\text{the terms proportional to } x^7 \text{ and } x^8) + \mathcal{O}\left(x^9\right)
\end{aligned}
$$

It matches exactly with the holographic result up to order x^8.M. Headrick 1006.0047, BC et.al. 1309.5453, 1312.5510

The classical mutual information $(n = 1)$ is vanishing when the two intervals are far apart.

Rényi mutual information: next-to-leading order

The NLO part from the vacuum module, being proportional to c^0, is

$$I_n^{NLO} = \frac{(n+1)\left(n^2+11\right)\left(3n^4+10n^2+227\right)x^4}{3628800n^7}$$
$$+\frac{(n+1)\left(109n^8+1495n^6+11307n^4+81905n^2-8416\right)x^5}{59875200n^9}$$
$$+\frac{(n+1)\left(1444050n^{10}+19112974n^8+140565305n^6+1000527837n^4-167731255n^2-14142911\right)x}{523069747200n^{11}}$$
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The mutual information is not really vanishing due to the quantum correction



Rényi mutual information: NNLO

Remarkably there is also the NNLO contribution, being proportional to $1/c$ BC et.al. 1309.5453,1312.5510

$$I_n^{NNLO} = \frac{(n+1)\left(n^2-4\right)(19n^8+875n^6+22317n^4+505625n^2+5691964)x^6}{70053984000n^{11}c} - \frac{(n+1)\left(n^2-4\right)\left(276n^{10}+12571n^8+317643n^6+7151253n^4+79361381n^2-9428724\right)x^7}{326918592000n^{13}c} + (\text{the terms proportional to } x^8) + \mathcal{O}\left(x^9\right),$$

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Actually there is nonvanishing quantum 3-loop contribution, being proportional to $1/c^2$, for $S^{(n)}, n > 3$.

We have shown the semiclassical picture on Rényi mutual information between two disjoint intervals.

How about $AdS_3/WCFT$?

Basics of WCFT

Consider a two-dimensional local field theory on the (x, y) plane, with the following local symmetry.

$$x \to w = f(x), \qquad y \to y + g(x)$$

where $f(x)$ and $g(x)$ are arbitrary smooth functions depending only on x. This defines the warped CFTs, which are non-relativistic. The two directions are not on the equal footing in the sense that x has the scaling symmetry while y does not.
The global symmetry is $SL(2, R) \times U(1)$, while the local symmetry is enhanced to the Virasoro-Kac-Moody symmetry.

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c}{12} n(n^2 - 1)\delta_{n+m,0}$$

$$[L_n, P_m] = -mP_{m+n}$$

$$[P_n, P_m] = \frac{k}{2} n\delta_{n+m,0}$$

OPE in WCFT

The primary operator in a warped CFT is characterized by the conformal weight h and the $U(1)$ charge Q. The two- and three-point functions of the primary operators in the $SL(2,R) \times U(1)$ invariant vacuum can be fixed by the global symmetry[W. Song and J.f. Xu, 1706.07621]

$$\langle \phi(x_1, y_1)\phi^\dagger(x_2, y_2)\rangle = d e^{iQy_{12}} \frac{1}{x_{12}^{2h}}$$

$$\langle \phi(x_1, y_1)\phi^\dagger(x_2, y_2)\phi'(x_3, y_3)\rangle = C' e^{iQy_{12}} \frac{1}{x_{12}^{2h-h_k}} \frac{1}{x_{31}^{h_k}} \frac{1}{x_{23}^{h_k}}$$

where Q is the U(1) charge of ϕ, while $\phi^\dagger$ has the opposite charge to ϕ. The charge of ϕ' equals to zero due to the charge conservation condition. C' is the three-point coefficient, characterizing the dynamics of the theory.

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where Q is the U(1) charge of ϕ, while $\phi^\dagger$ has the opposite charge to ϕ. The charge of ϕ' equals to zero due to the charge conservation condition. C' is the three-point coefficient, characterizing the dynamics of the theory.
Considering the scaling behaviour, we have the OPE,

$$\phi(x_1, y_1)\phi^\dagger(x_2, y_2) = \sum_{k,n,m} C_k a_{k,n,m} \frac{1}{x_{12}^{2h-h_k-n}} \partial_{x_2}^n \partial_{y_2}^m e^{iQy_{12}} \phi_k(x_2, y_2)$$

where $C_k = \frac{C'}{d}$,and ϕ_k runs over every quasi-primary operator in the theory with zero $U(1)$ charge.

RMI in holographic WCFT

For the holographic warped CFT, it has a large central charge and a sparse light spectrum. In the large c limit, the vacuum module dominates the contribution in the large c expansion. We can organize the RMI in orders of $1/c$

$$I_{n,WCFT} = I^{(LO)}_{n,WCFT} + I^{(NLO)}_{n,WCFT} + ...$$

The leading order result up to x^3 is

$$\begin{aligned} I^{(LO)}_{n,WCFT} &= \frac{(n-1)l_{AdS}}{8nG_N}x + \frac{(n-1)l_{AdS}}{16nG_N}x^2 + \frac{(n-1)l_{AdS}}{24nG_N}x^3 \\ &+ \frac{c(n-1)(n+1)^2}{288n^3}x^2 + \frac{c(n-1)(n+1)^2}{288n^3}x^3 + \cdots \end{aligned}$$

while the next-to-leading order result up to x^3 is

$$I^{(NLO)}_{n,WCFT} = \frac{(n+1)\left(n^2+11\right)}{1440n^3}x^2 + \frac{(n+1)\left(23n^4+233n^2-40\right)}{30240n^5}x^3 + \cdots$$

LO result from conformal block

Recall that

$$I_n(x) = \frac{1}{n-1} \log \frac{F_n(x)}{F_1(x)^n}$$

where

$$F_n(x) = \frac{\langle \sigma_n(\infty)\tilde{\sigma}_n(1)\sigma_n(x)\tilde{\sigma}_n(0)\rangle}{\langle \sigma_n(\infty)\tilde{\sigma}_n(1)\rangle\langle \sigma_n(x)\tilde{\sigma}_n(0)\rangle}$$

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The warped conformal block could be factorized

$$V_{T+P}(c, h_i, k, q_i, h_p, x) = V_T(c-1, h_i - \frac{q_i^2}{k}) V_P(k, q_i, x),$$

where h_i, q_i are the conformal weights and $U(1)$ charges of the primary operators. The Kac-Moody block has a closed form when the exchanged operator is neutral.

$$V_P(k, q_i, x) = x^{\frac{-2q_1^2}{k}} (1-x)^{\frac{2q_1 q_2}{k}}.$$

The Virasoro block can be computed using the recursive relation.

LO result: comparision

The one from the conformal block

$$
\begin{aligned}
I^{(LO)}_{n,WCFT}(x) \quad = \quad & -\frac{n-1}{8n}\frac{l_{AdS}}{G_N}\log(1-x) \\
& + \frac{c(n-1)(n+1)^2x^2}{288n^3} + \frac{c(n-1)(n+1)^2x^3}{288n^3} \\
& + \frac{c(n-1)(n+1)^2\left(1309n^4-2n^2-11\right)x^4}{414720n^7} + \cdots
\end{aligned}
$$

The one from the OPE

$$
\begin{aligned}
I^{(LO)}_{n,WCFT} \quad = \quad & \frac{(n-1)l_{AdS}}{8nG_N}x + \frac{(n-1)l_{AdS}}{16nG_N}x^2 + \frac{(n-1)l_{AdS}}{24nG_N}x^3 \\
+ \quad & \frac{c(n-1)(n+1)^2}{288n^3}x^2 + \frac{c(n-1)(n+1)^2}{288n^3}x^3 + \cdots
\end{aligned}
$$

Comparision II

The LO result is actually

$$
\begin{aligned}
I^{(LO)}_{n,WCFT}(x) \quad = \quad & -\frac{n-1}{8n}\frac{l_{AdS}}{G_N}\log(1-x) \\
& + \frac{c(n-1)(n+1)^2x^2}{288n^3} + \frac{c(n-1)(n+1)^2x^3}{288n^3} \\
& + \frac{c(n-1)(n+1)^2\left(1309n^4-2n^2-11\right)x^4}{414720n^7} + \cdots
\end{aligned}
$$

The one in usual holographic CFT

$$
\begin{aligned}
I^{LO}_{n,CFT} = & \frac{c(n-1)(n+1)^2x^2}{144n^3} + \frac{c(n-1)(n+1)^2x^3}{144n^3} \\
& + \frac{c(n-1)(n+1)^2\left(1309n^4-2n^2-11\right)x^4}{207360n^7} \\
& + \frac{c(n-1)(n+1)^2\left(589n^4-2n^2-11\right)x^5}{103680n^7} \\
& + \frac{c(n-1)(n+1)^2(805139n^8-4244n^6-23397n^4-86n^2+188)x^6}{156764160n^{11}} \\
& + (\text{the terms proportional to } x^7 \text{ and } x^8) + \mathcal{O}\left(x^9\right)
\end{aligned}
$$

Final picture on LO result

$$
\begin{aligned}
I^{(LO)}_{n,WCFT}(x) &= -\frac{n-1}{8n}\frac{l_{AdS}}{G_N}\log(1-x) \\
&\quad +\frac{1}{2}I^{(LO)}_{n,CFT}(x).
\end{aligned}
$$

This should be reproduced from classical action of the bulk configuration.

Final picture on LO result

$$I^{(LO)}_{n,WCFT}(x) = -\frac{n-1}{8n}\frac{I_{AdS}}{G_N}\log(1-x) + \frac{1}{2}I^{(LO)}_{n,CFT}(x).$$

This should be reproduced from classical action of the bulk configuration.

The mutual information is simply vanishing in the leading order.

Fluctuations in the AdS_3 gravity with CSS B.C.

For the AdS_3 gravity with CSS boundary condition, the 1-loop determinant for the BTZ black hole takes the form A. Castro et.al. 1707.06245

$$Z_{BTZ}^{1-loop} = \prod_{l'=1}^{\infty} \frac{1}{1-q^{l'}} \prod_{l=2}^{\infty} \frac{1}{1-q^{l}}.$$

1. The fluctuations include a (holomorphic-) vector and a (holomorphic-) graviton
2. Only the holomorphic sector survives the CSS boundary condition

1-loop correction in AdS_3 gravity with CSS B.C.

The 1-loop determinant for the handlebody solution could be

$$Z^{1-loop} = \prod_{\gamma\in\mathcal{P}}\prod_{l'=1}^{\infty}\frac{1}{1-q_\gamma^{l'}}\prod_{l=2}^{\infty}\frac{1}{1-q_\gamma^{l}}.$$

The NLO result from OPE

$$I^{(NLO)}_{n,WCFT} = \frac{(n+1)\left(n^2+11\right)}{1440n^3}x^2+\frac{(n+1)\left(23n^4+233n^2-40\right)}{30240n^5}x^3+\cdots$$

Actually they match with each other up to order x^3.

Conclusions and Discussions

$$I_n^{(LO)}(x) = -\frac{n-1}{8n}\frac{l_{AdS}}{G_N}\log(1-x) + \frac{1}{2}(\text{Rényi mutual information of usual holographic CFT}).$$

How to understand this relation from bulk point of view?

Can we derive it from the chiral Liouville action? G. Compere et.al. 1303.2660

$$Z^{1-loop} = \prod_{\gamma\in\mathcal{P}} \prod_{l'=1}^{\infty} \frac{1}{1-q_\gamma^{l'}} \prod_{l=2}^{\infty} \frac{1}{1-q_\gamma^{l}}.$$

Can we classify all the gravitational solutions in AdS_3 gravity w.r.t. CSS B.C.?

Thanks for your attention!