Conformal matters on torus

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POSTECH

Based on

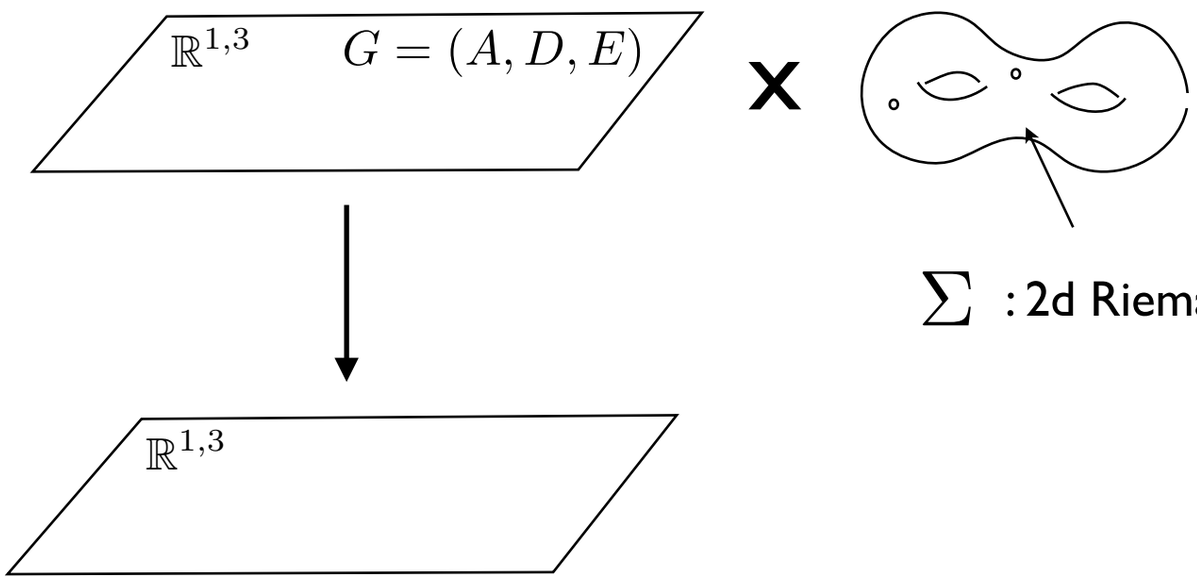
arXiv : 1709.02496, 1802.00620, 1806.07620 with Shlomo Razamat, Cumrun Vafa, Gabi Zafrir

arXiv : 1506.03871 with Davide Gaiotto

Compactification of 6d SCFTs

- 4d N=2 Class S theories [Gaiotto, Moore, Neitzke 08], [Gaiotto 09]

6d N=(2,0) ADE-type theories



Σ : 2d Riemann surface

4d N=2 theories $T_{4d}[G, \Sigma]$

- 6d N=(1,0) CFTs $\mathcal{T}_{6d}$ on Σ with flux F

⟶ 4d N=1 theories $T_{4d}^{N=1}[\mathcal{T}_{6d}, \Sigma, F]$

[Ohmori, Shimizu, Tachikawa, Yonekura 15], [Gaiotto, Razamat 15], [Del Zotto, Vafa, Xie 15], [Razamat, Vafa, Zafrir 16], [Bah, Hanany, Maruyoshi, Razamat, Tachikawa, Zafrir 17]…

6d conformal matter theories and compactifications

6d conformal matter theory

[Del Zotto, Heckman, Tomasiello, Vafa 14],
[Heckman, Morrison, Rudelius, Vafa 15],…

- $\mathcal{N} = (1, 0)$ theory on N M5-brane probing an ADE singularity.
- (G,G) conformal matter has G x G global symmetry (where G=A,D,E).

We shall discuss

- D=4,5 compactifications of 6d conformal matters on a torus with flux.
- Non-trivial flux of global symmetry, while preserving 4 real SUSYs.

In this talk, I will mostly focus on D-type conformal matter theory of a single M5-brane.

Compactification of E-string theory

E-string theory on a circle

6d SCFT on a M5-brane probing D4 singularity (or M9-brane E8-wall).

- Minimal D4 conformal matter theory. [Witten 95], [Ganor, Hanany 96]
- This theory has E8 global symmetry.

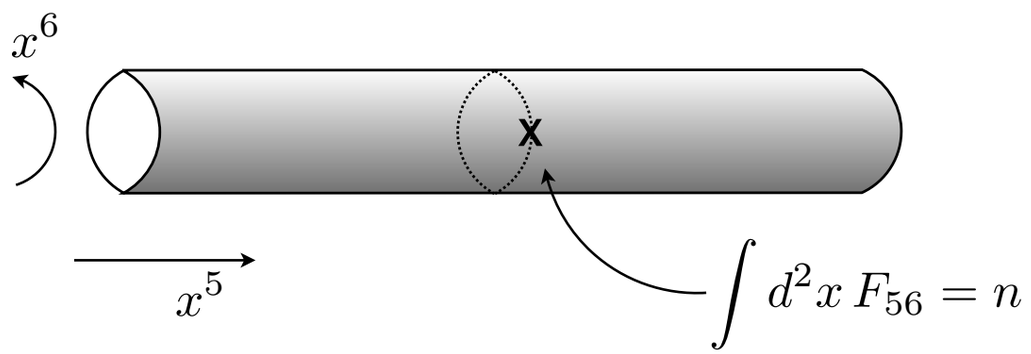
Circle reduction (without flux) leads to [Seiberg 96], [Ganor, Morrison, Seiberg 96]

- 5d $\mathcal{N}=1$ SU(2) gauge theory with 8 fundamental hypers.
- Vector multiplet $(A_\mu, \phi; \lambda)$
- Hypermultiplet $(q^A; \psi)$
- Classical symmetry is $SU(2)_R$ R-symmetry and $SO(16)$ flavor symmetry and $U(1)_I$ topological symmetry.
- UV completion is expected to be 6d E-string theory on a circle.
- Global symmetry enhances to $E_8 \times U(1)_{KK}$ at strong coupling.

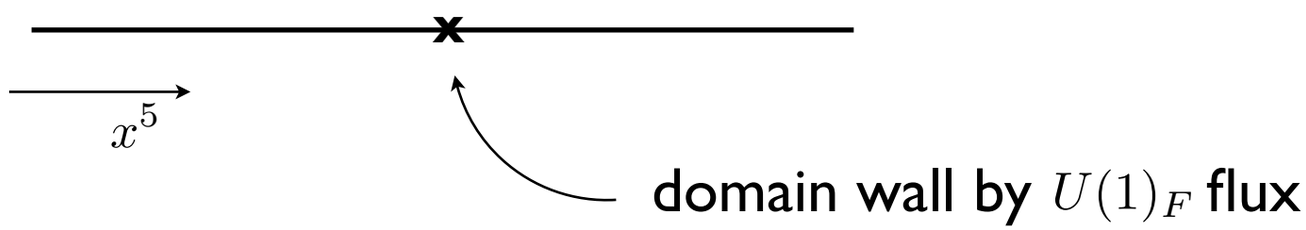
5d E-string theory and flux domain wall

5d reduction of 6d E-string theory $\longrightarrow$ 5d SU(2) with 8 fundamentals.

Let us now consider 6d E-string theory on a cylinder with $U(1)_F \subset E_8$ flux.



Then in 5d reduction, 6d flux gives rise to a domain wall



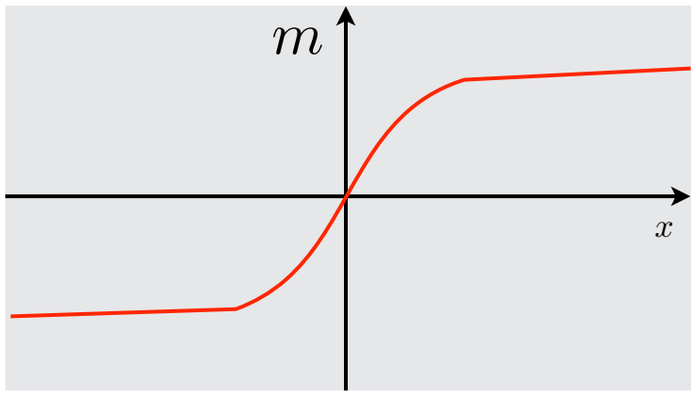
Domain walls in 5d E-string theory

6d flux leads to x^5 dependent holonomy in 5d. [Chan, Ganor, Krogh 00]

$$n = \int dx^5 d\theta F_{x^5\theta} = \int dx^5 d\theta \partial_{x^5} A_\theta = RA_\theta|_{x^5=\infty} - RA_\theta|_{x^5=-\infty}$$

Therefore, it introduces a domain wall changing $U(1)_F$ mass $m(x_5) \sim A_\theta(x_5)$.

It turns out that minimal flux (which is 1/2) for $U(1)_F$ flips mass sign $m \to -m$

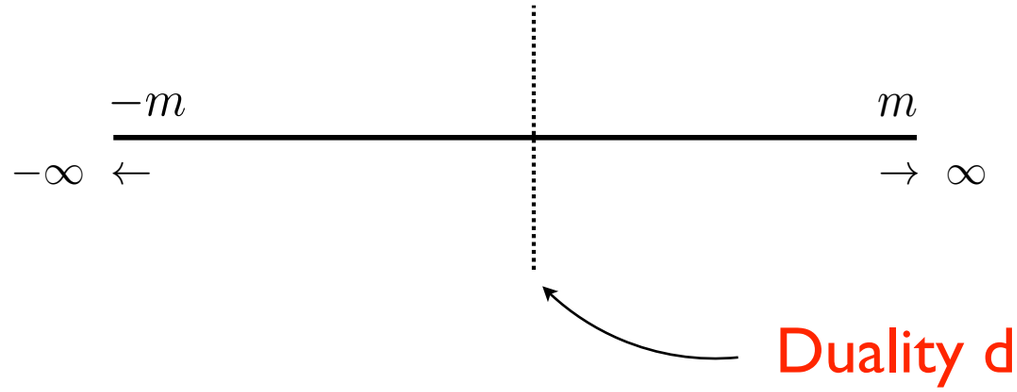


m : mass parameter

Domain walls in 5d E-string theory

6d minimal flux corresponds to a 5d domain wall flipping mass sign of $U(1)_F$.

Duality domain wall in [Gaiotto, H-C. Kim 15] is such domain wall!



Construction of duality domain wall :

1. Boundary condition
   - Vector multiplet : $F_{5i}|_\partial = 0$
   - Hypermultiplet : $\Phi^{(1)} = (X, Y) \ : \ X|_\partial = 0, \ \partial_5 Y|_\partial = 0$

     $\Phi^{(2)} = (X', Y') \ : \ \partial_5 X'|_\partial = 0, \ Y'|_\partial = 0$

2. Domain wall degrees of freedom (q, b) and superpotential

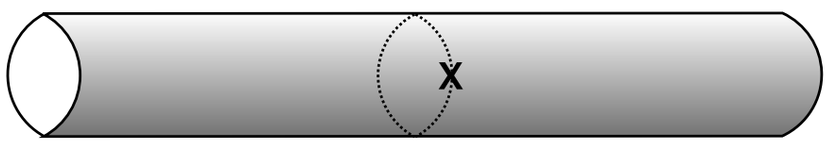
$$W = b\,\mathrm{det} q + Y q X'$$

Domain walls in 5d E-string theory

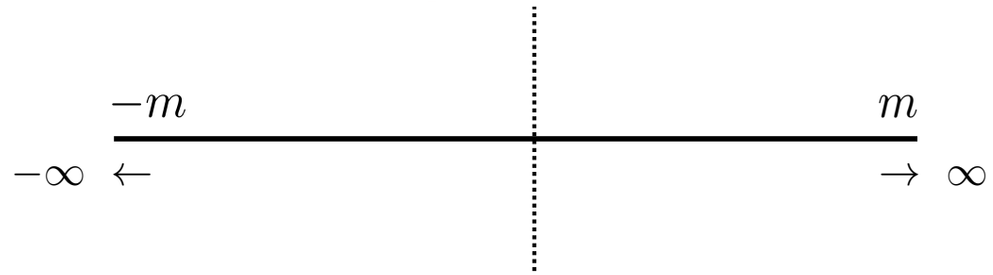
We claim

‘Duality’ domain wall realizes 1/2 unit $U(1)_F$ flux in 6d E-string theory

6d flux



5d duality domain wall



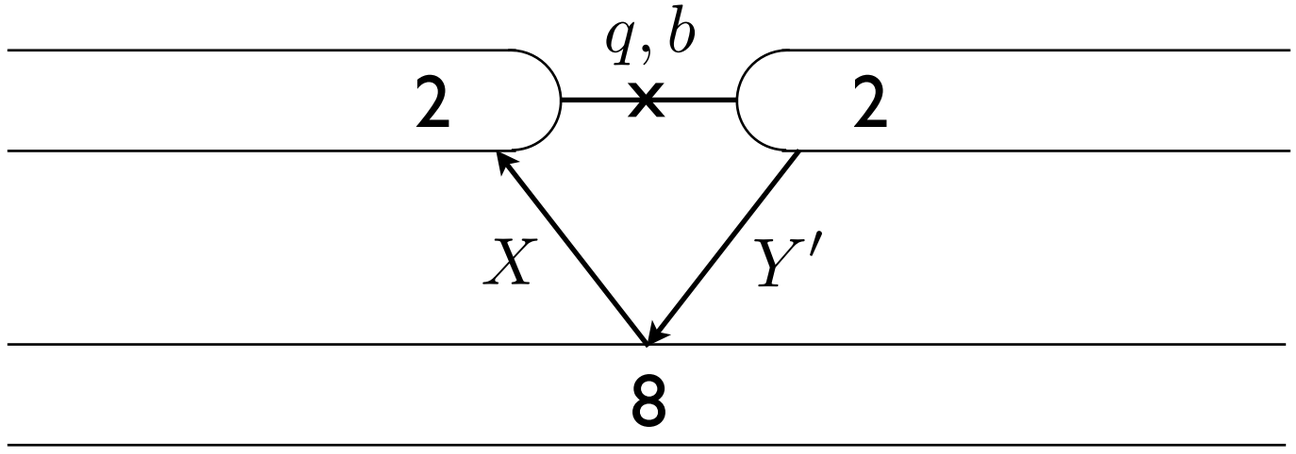
So, we will also call duality domain wall as ‘flux domain wall’.

Minimal flux domain wall

Choice of 1/2 BPS boundary condition at the domain wall :

$$\mathcal{B} = (s_1, s_2, \cdots, s_8) \quad \text{where} \quad \begin{cases} s_i = + & \text{if} \quad (\partial_5 X_i, Y_i) = 0 \\ s_i = - & \text{if} \quad (X_i, \partial_5 Y_i) = 0 \end{cases}$$

- For instance, we can define a duality domain wall with boundary conditions $\mathcal{B}^{(1)} = (+, +, \cdots, +)$, $\mathcal{B}^{(2)} = (-, -, \cdots, -)$

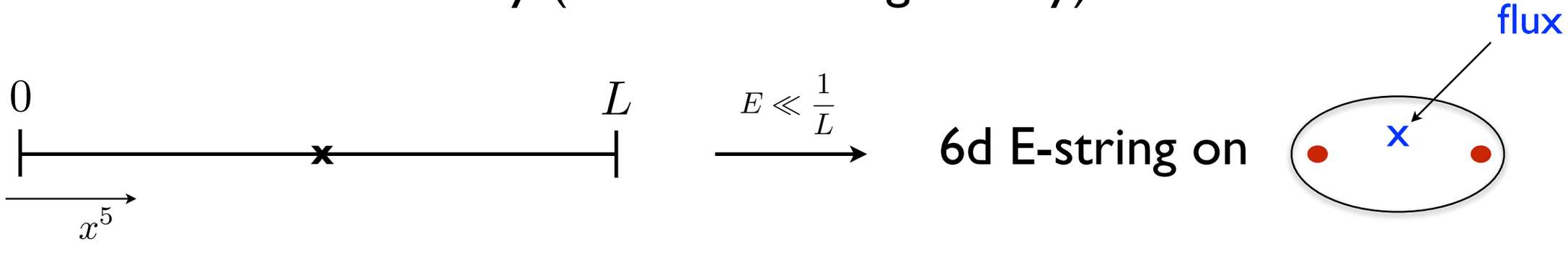


$$W = b\,\mathrm{det} q + X q Y'$$

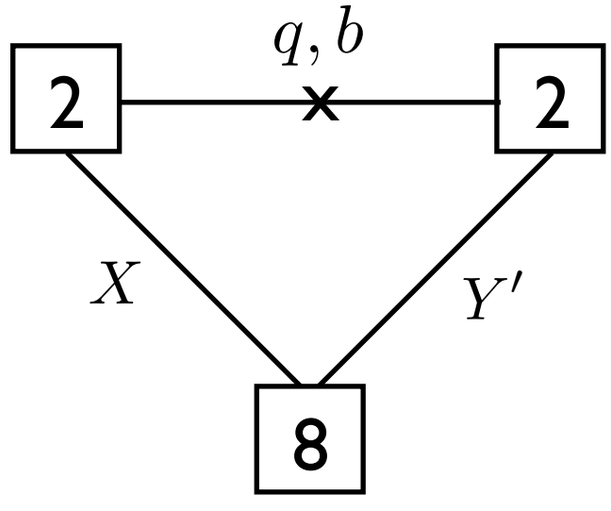
- This is minimal flux domain wall of flux $(\frac{s_1}{2}, \frac{s_2}{2}, \cdots, \frac{s_8}{2})$ along the flavor currents $J_i \subset E_8$ which breaks $E_8 \to SU(8) \times U(1)$. This domain wall flips sign of mass parameter for the flavor current $\sum_i s_i J_i$.

4d E-string theory with flux on tube

5d system with domain wall on an interval $\mathcal{I} : 0 \leq x_5 \leq L$ at low energy reduces to a 4d N=1 theory (or a 4d E-string theory).



For example, $(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$ flux in 6d E-string theory leads to 4d E-string theory of two punctured sphere (or tube).

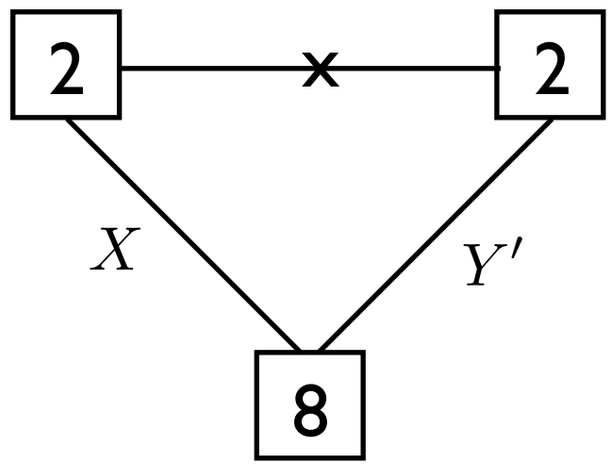


$$W = b\,\mathrm{det}q + XqY'$$

Note that $SU(2) \times SU(2)$ are symmetries coming from two punctures.

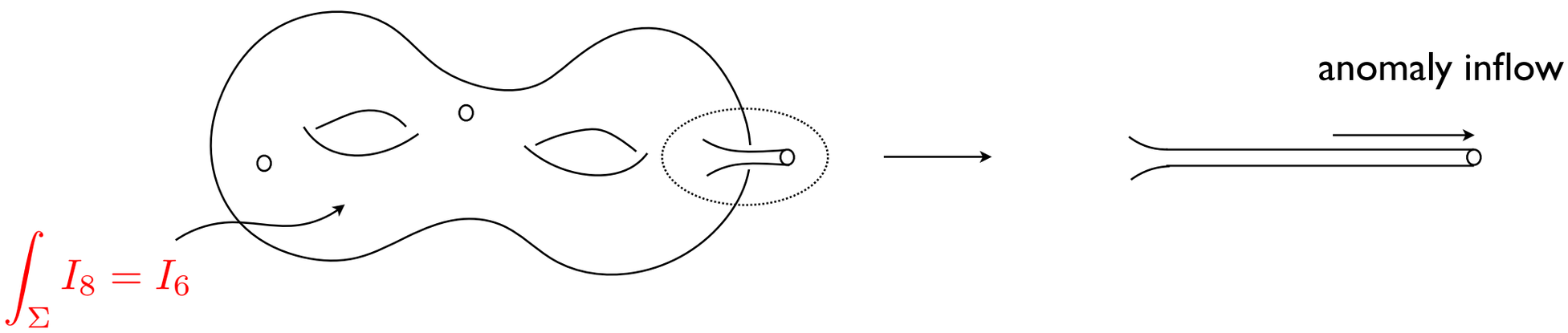
4d E-string theory with flux on tube

4d E-string theory of a tube with 1/2 flux



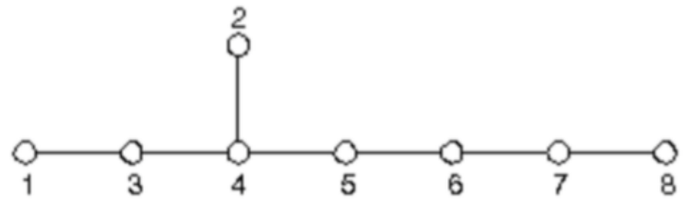
$$W = b\,\mathrm{det}q + XqY'$$

Central charges and 't Hooft anomalies match $\int_\Sigma I_8 = I_6$ from 6d anomaly polynomial I_8 of E-string theory [Ohmori, Shimizu, Tachikawa 14] together with anomaly inflow contribution from each puncture (which can be computed by analyzing the boundary condition of the 5d E-string theory).



E-string theory with general flux

In 6d E-string theory , we have many different flux choices preserving different subgroups of E_8 .



Unit fluxes in orthogonal basis (corresponding to nodes in E8 Dynkin diagram) are

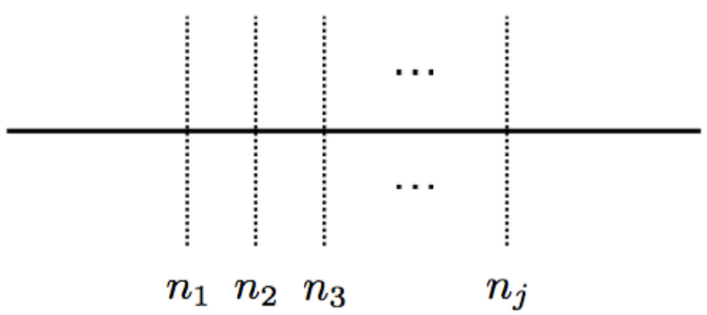
Node	Associated vectors	Commutant in E_8
8	$(2,2,0,0,0,0,0,0),(1,1,1,1,1,1,1,1)$	$U(1)\times E_7$
1	$(4,0,0,0,0,0,0,0),(2,2,2,2,0,0,0,0)$	$U(1)\times SO(14)$
7	$(2,2,2,2,2,2,0,0),(3,3,1,1,1,1,1,1)$	$U(1)\times SU(2)\times E_6$
2	$(4,2,2,2,2,0,0,0),(5,1,1,1,1,1,1,1)$	$U(1)\times SU(8)$
6	$(4,4,2,2,2,2,0,0),(4,4,4,0,0,0,0,0)$	$U(1)\times SU(3)\times SO(10)$
3	$(6,2,2,2,2,2,0,0)$	$U(1)\times SU(2)\times SU(7)$
5	$(4,4,4,4,4,0,0,0),(5,5,5,1,1,1,1,1)$	$U(1)\times SU(4)\times SU(5)$
4	$(9,3,3,3,3,1,1,1)$	$U(1)\times SU(2)\times SU(3)\times SU(5)$

Ex) $(1,1,1,1,1,1,1,1)=2\times\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$

$(2,2,0,0,0,0,0,0)=2\times\left((\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})+(\frac{1}{2},\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2})\right)$

General fluxes can be obtained by gluing multiple minimal flux domain walls with different boundary conditions!

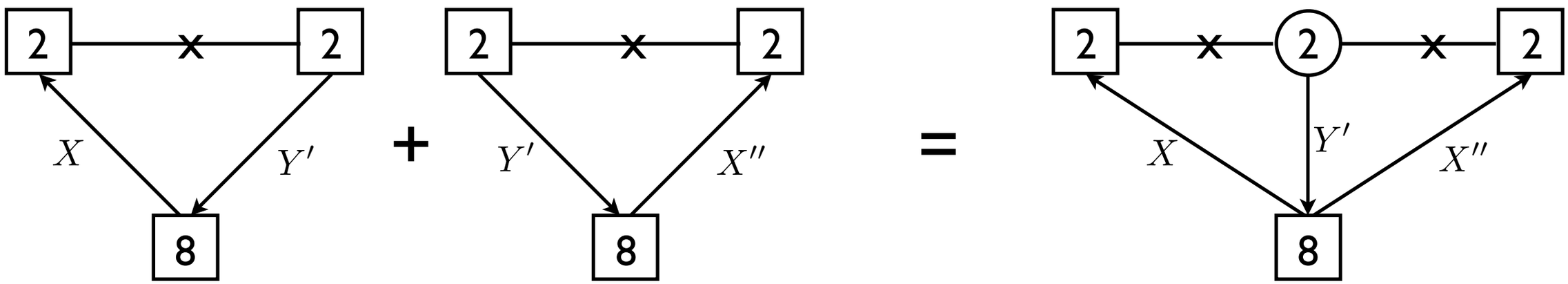
$$n_{\rm tot}=\sum_i n_i$$ from



Gluing flux domain walls

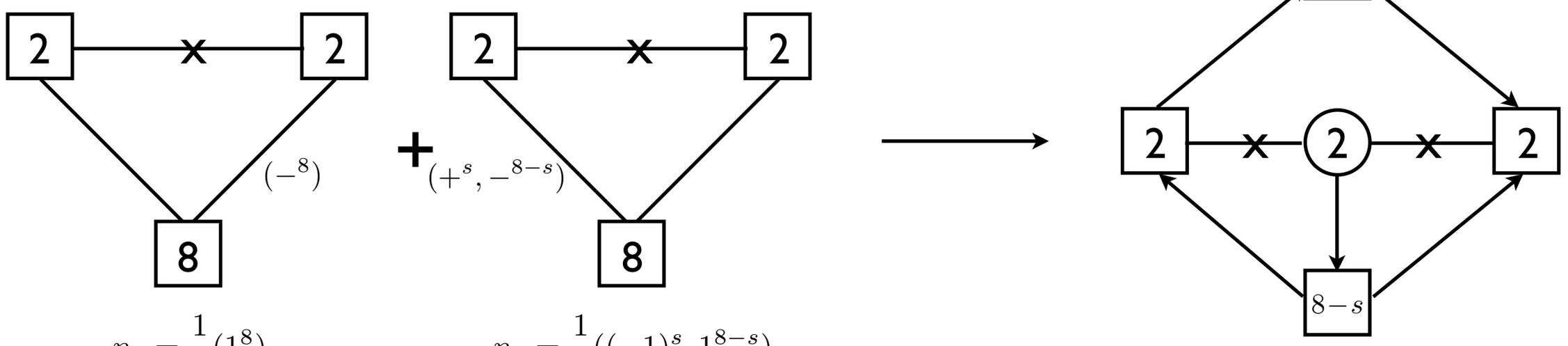
We can glue two or more flux domain walls for general fluxes.

ex 1) two domain walls with 1/2 fluxes on the same U(1)



$n = \frac{1}{2}$ $\quad$ $n = \frac{1}{2}$ $\quad$ $n = \frac{1}{2} + \frac{1}{2} = 1$

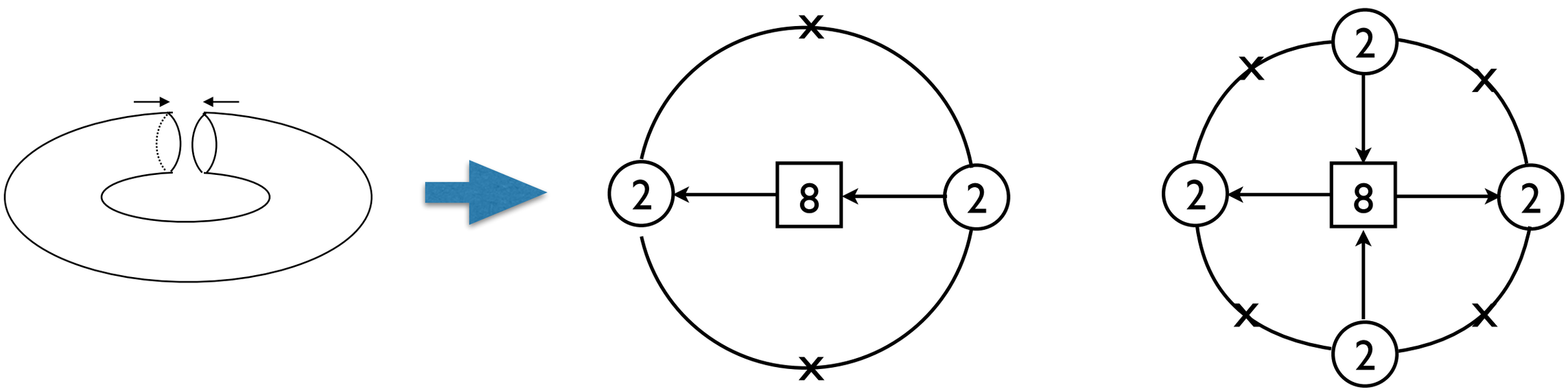
ex 2) two domain walls with 1/2 fluxes along different U(1)'s



$n_1 = \frac{1}{2}(1^8)$ $\quad$ $n_2 = \frac{1}{2}((-1)^s, 1^{8-s})$ $\quad$ $n = n_1 + n_2 = (0^s, 1^{8-s})$ $\quad$ s : even

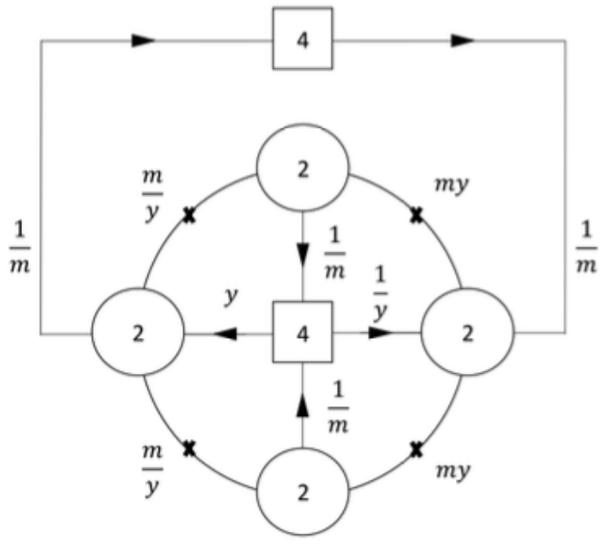
4d theories from 6d E-string on a torus

By gluing two ends, we can obtain 4d theories of torus with general fluxes.

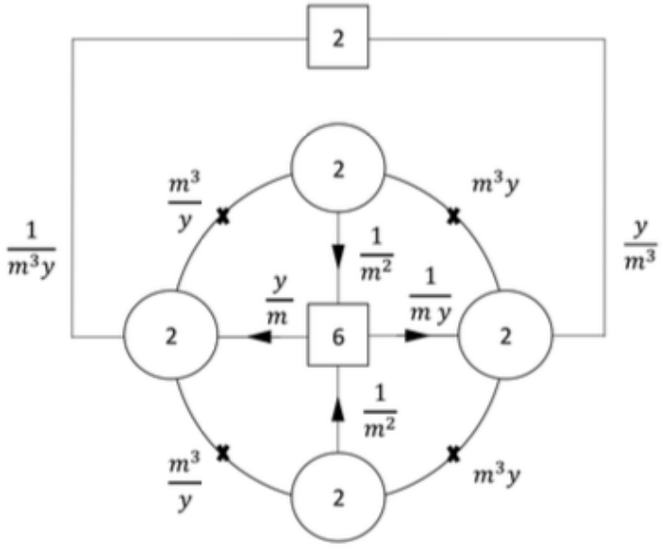


$$n = \frac{1}{2} + \frac{1}{2} = 1 \qquad n = 4 \times \frac{1}{2} = 2$$

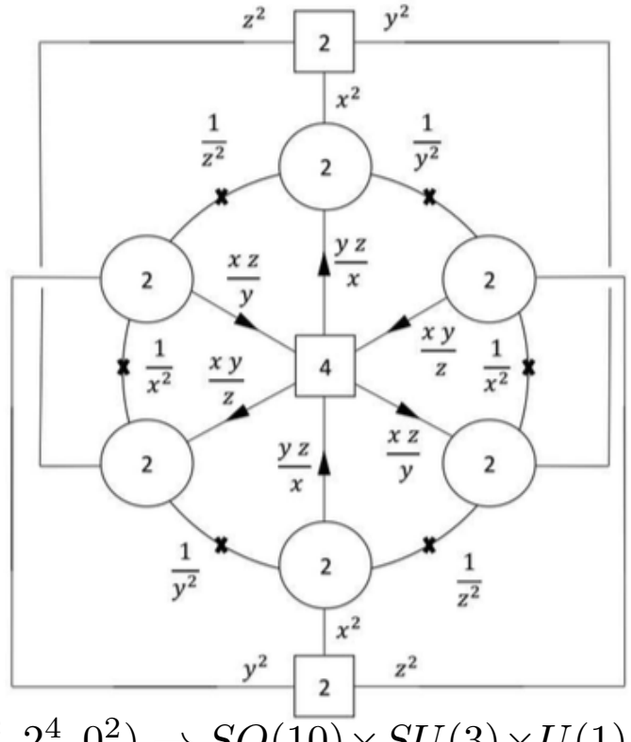
More examples :



$n = (2^4, 0^4) \to SO(14) \times U(1)$



$n = (2^6, 0^2) \to E_6 \times SU(2) \times U(1)$



$(4^2, 2^4, 0^2) \to SO(10) \times SU(3) \times U(1)$

Compactification of D-type conformal matters

D-type conformal matter on a circle

6d SCFT on a M5-brane probing D_{N+3} singularity. [Del Zotto, Heckman, Tomasiello, Vafa 14]

- $Sp(N-1)$ gauge theory with 2N+6 hypers.
- N=1 theory is the E-string theory.

Circle reduction (without flux) leads to 5d $\mathcal{N}=1$

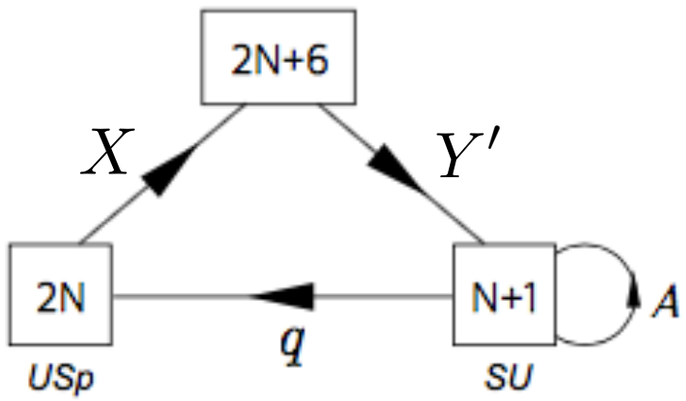
- Affine D_{N+3} quiver theory
- $SU(N+1)$ gauge theory with 2N+6 hypers. [Hayashi, S-S. Kim, K. Lee, Taki, Yagi 15]
- $USp(2N)$ gauge theory with 2N+6 hypers. [Gaiotto, H-C. Kim 15]

Different 5d descriptions come from different holonomies of 6d symmetries.
We will focus on the 2nd and 3rd descriptions in 5d.

Flux domain wall in 5d D-type conformal matter

Duality domain wall connecting $USp(2N)$ and $SU(N+1)$ gauge theories.

[Gaiotto, H-C. Kim 15]

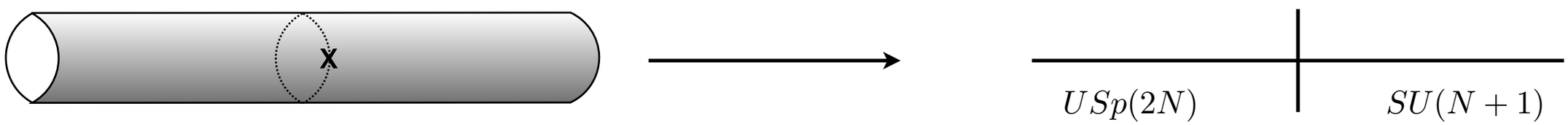


$$\mathcal{W} = X \cdot q \cdot Y' + q \cdot q \cdot A$$

A : antisymmetric in $SU(N+1)$

We claim

'Duality domain wall' implements 1/2 unit $U(1)_F$ flux in D-type Conformal matter theory

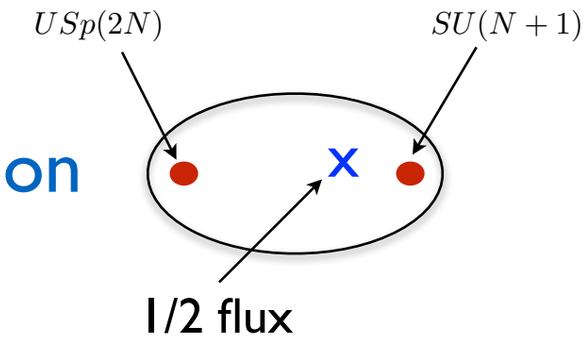


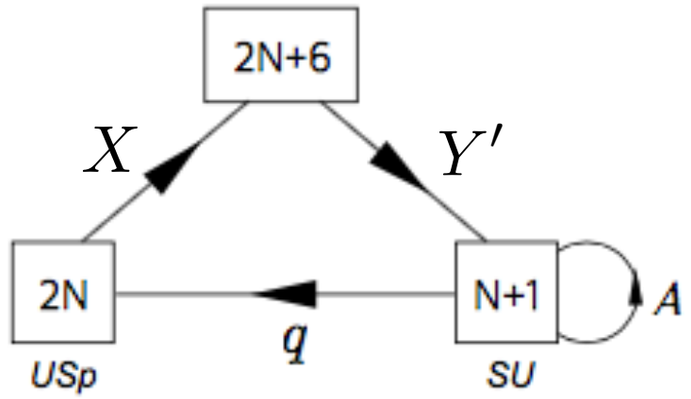
6d flux

5d domain wall

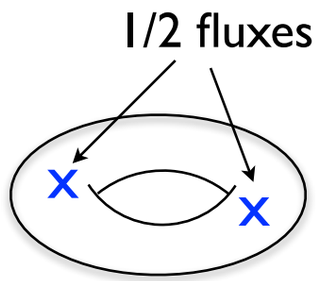
4d D-type conformal matters

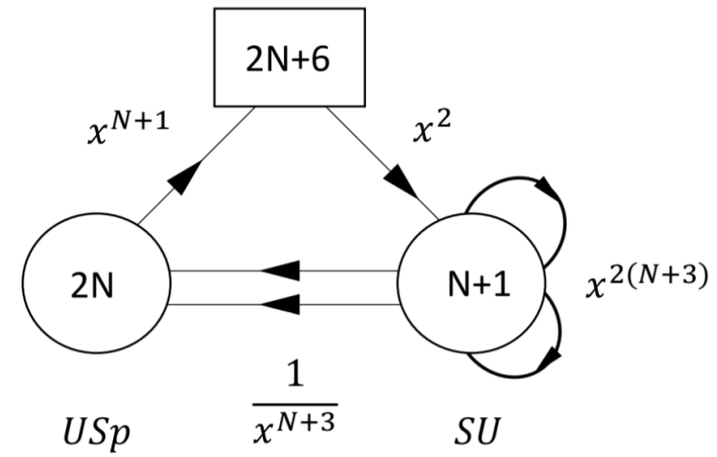
6d theory on





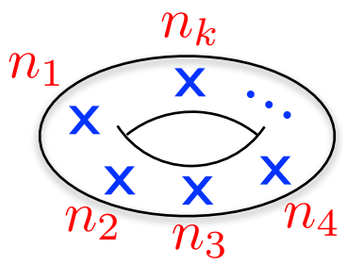
6d theory on

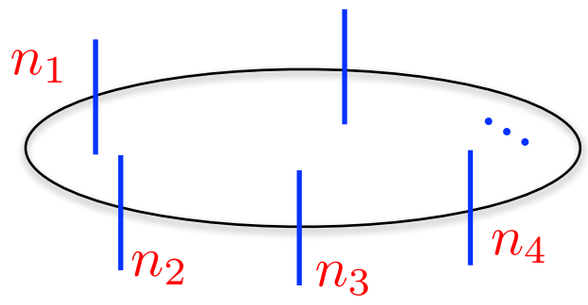




General fluxes $n_{\text{tot}} = \sum_i n_i$ can be obtained by gluing multiple domain walls with corresponding boundary conditions for n_i .

6d theory on





Summary

- We proposed a systematic construction of domain wall configurations for general 6d fluxes in 5d D-type conformal matters.

- 4d theories of a torus with fluxes in 6d theory can be obtained by 4d reduction of 5d domain wall configurations.

- 4d theories of other conformal matter theories. [H-C. Kim, Razamat, Vafa, Zafrir 18]
- Compactification on other Riemann surfaces? [H-C. Kim, Razamat, Vafa, Zafrir 17]
- More studies on 5d domain walls?

Thank you very much !