Dualities and 5-brane webs for 5d rank 2 SCFTs

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arXiv: 1801.03916, 1806.10569

- In this talk, we focus on 5d N=1 supersymmetric gauge theories (with 8 supercharges).
  - superconformal theories (SCFTs) at UV fixed point

Seiberg 97

- 5-brane web diagram in Type IIB string theory has provided a powerful tool to study these 5d N=1 gauge theories.

Aharony, Hanany, Kol 97

- Possibly, all 5d SCFTs could be obtained from a circle compactification with some twist from 6d theories, full classification is still not complete
  - dualities
  - symmetries

- Recently there has been progress on classification of 5d SCFTs:

- From the field theory:
  milder constraint on the theory in the Coulomb branch
  with **the positivity of the monopole string tension**
  —> a classification of 5d SCFTs of a single gauge group.

  [17 Jefferson- HC Kim-Vafa-Zafrir]

- geometric construction
  M-theory on local shrinkable Calabi-Yau threefold
  Rank 2 theories are classified (with duality).

  [18 Jefferson-Katz-HC Kim-Vafa]

- In this talk, we propose **all the 5-brane webs for rank 2 SCFTs**
  e.g., marginal theories (UV completion as 6d theory)

  G_2 + 6F | $SU(3)_9$ | $SU(3)_{3/2}$ + 1 Sym

- Employ them to **test duality** between different gauge descriptions
  e.g.,

  Pure G_2 $\longleftrightarrow$ $SU(3)_7$

  G_2 + 2F $\longleftrightarrow$ $SU(3)_6$ + 2F $\longleftrightarrow$ $Sp(2)_\pi$ + 2AS

  Sp(2) + 1AS + 8F $\longleftrightarrow$ $SU(3)_{3/2}$ + 9F

Prepotential

- On Coulomb branch (G -> $U(1)^{\text{rank}(G)}$),

  low energy physics is characterized by prepotential.

- Prepotential is at most cubic and 1-loop exact:

$$\mathcal{F} = \frac{1}{2g_0^2} h_{ij}\phi^i\phi^j + \frac{\kappa}{6} d_{ijk}\phi^i\phi^j\phi^k + \frac{1}{12}\Big(\sum_{\text{Roots}} |R\cdot\phi|^3 - \sum_f \sum_{w\in W_f} |w\cdot\phi + m_f|^3 \Big)$$

$$h_{ij} = \text{Tr}(T_i T_j),\ d_{abc} = \frac{1}{2}\text{Tr}T_a(T_bT_c + T_cT_b), \quad W_f = \text{Weight of } G \text{ in the rep. } r_f$$

[96 Morrison-Seiberg]

[97 Intriligator-Morrison-Seiberg]

Magnetic monopole string tension: $\phi_{D_i} = \partial_i \mathcal{F}$

Effective coupling: $\tau_{ij} = \partial_i\partial_j \mathcal{F}$

An example: SU(2) Prepotential

- With N_f massless hypermultiplet in the fundamental representation,

$$\mathcal{F}_{SU(2)} = \frac{1}{g_0^2}a^2 + \frac{1}{6}(8 - N_f)a^3$$

Magnetic monopole string tension: $\phi_{D_i} = \partial_i \mathcal{F}$ $\frac{a}{2}\left(\frac{4}{g_0^2} + (8 - N_f)a\right)$

Effective coupling: $\tau_{ij} = \partial_i \partial_j \mathcal{F}$ $\frac{2}{g_0^2} + (8 - N_f)a$

Type IIB 5-brane

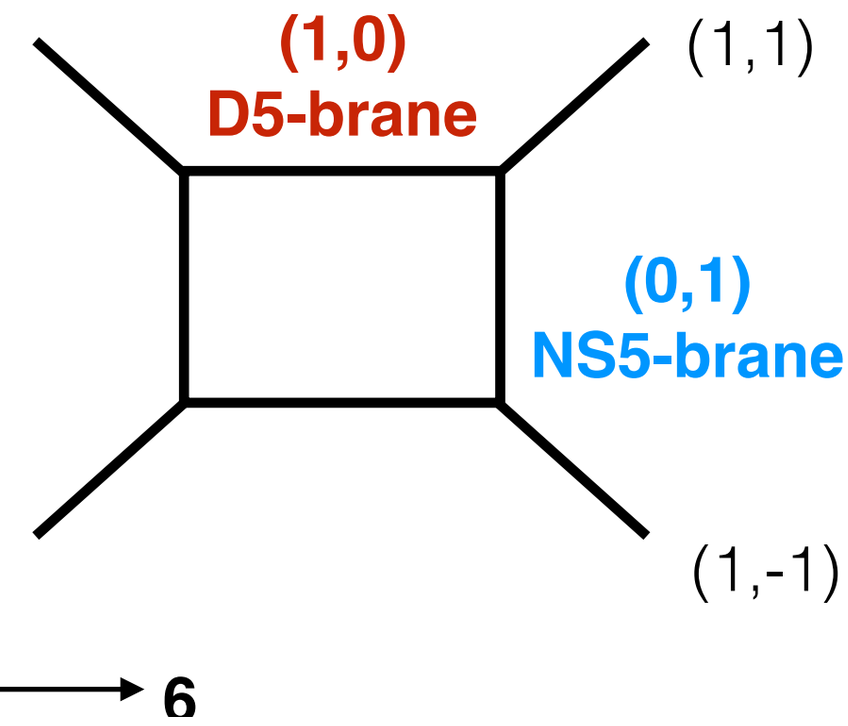
Such prepotential information is encoded in 5-brane webs.

	0	1	2	3	4	5	6	7	8	9
NS5	–	–	–	–	–	–				
D5	–	–	–	–	–		–			
(p,q)	–	–	–	–	–	-	-			

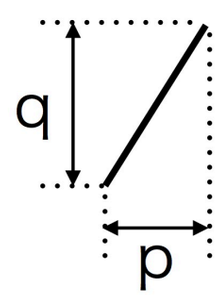
5d world vol.

(p,q) plane

5-brane web for SU(2)

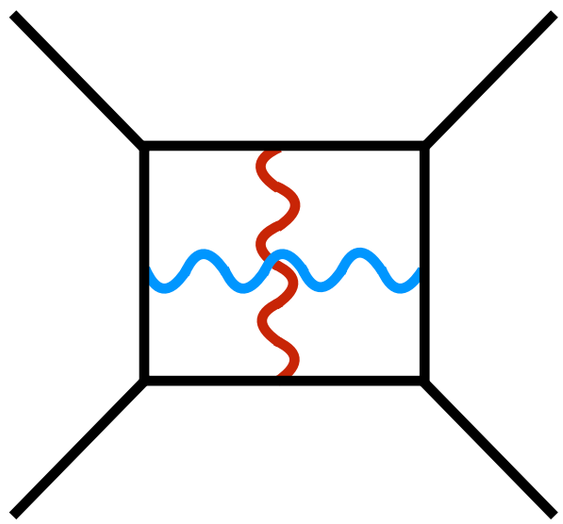


- (p,q) charges: p D5 charge + q NS5 charge,
  (p,q) 5-brane has a slope q/p



pure SU(2) theory and 5-brane web

- Coulomb branch : a = $< \phi >/2$

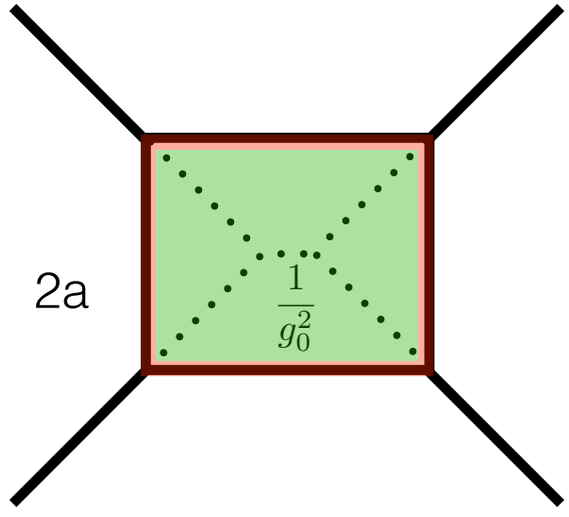


F1 SU(2) **W-bosons**

D1 SU(2) **Instantons**

- **Monopole string tension**

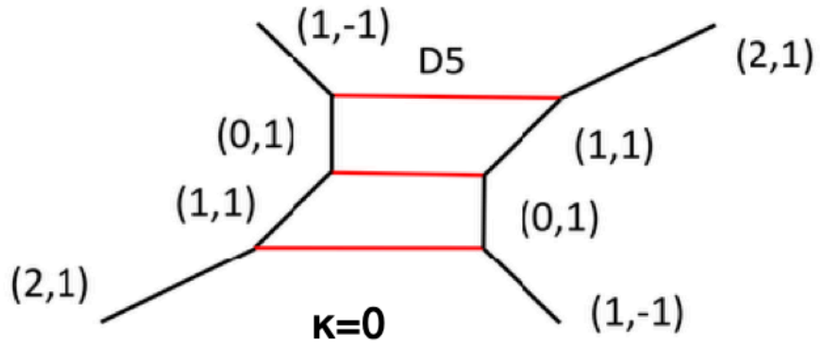
Area $2a\left(\frac{1}{g_0^2}+2a\right)$

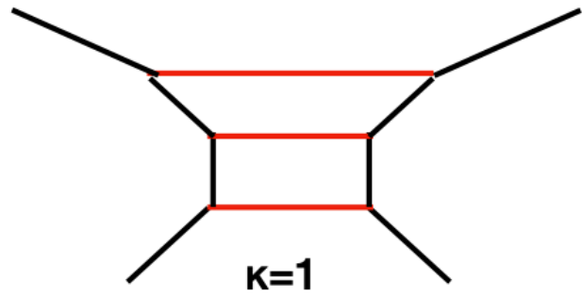


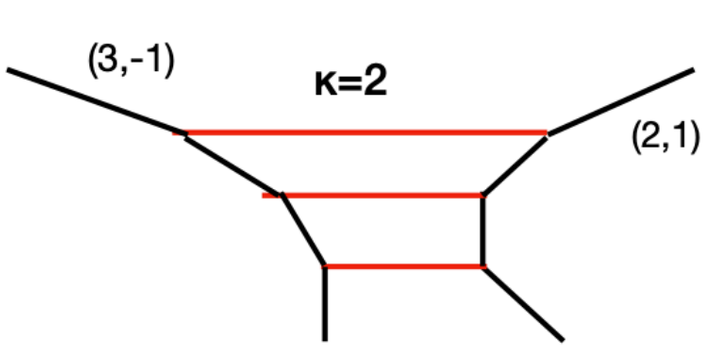
- **Effective couplings**

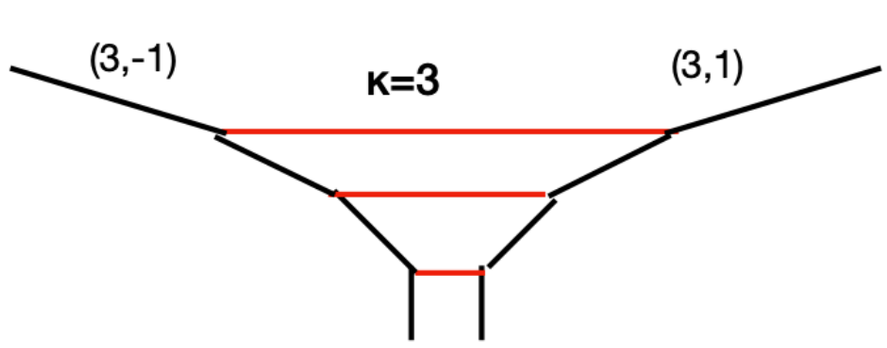
Circumference $\frac{2}{g_0^2}+8a$

- A 5-brane web for **$SU(N)_k$** gauge theories is obtained from N parallel D5 branes between 2 NS5 branes.
  - for small Chern-Simons level k









- With the introduction of an **orientifold 5-plane (or O7)**, one may realize an **Sp(N)** or an **SO(N)** gauge group.

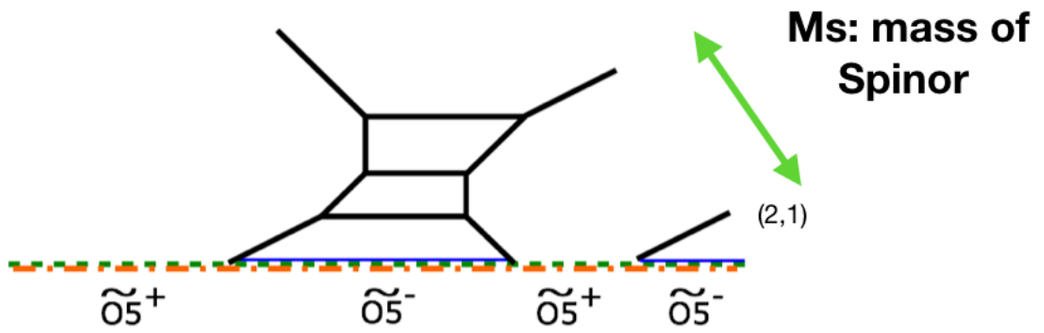
Brunner, Karch 97,
Brandhuber, Itzhaki, Sonnenschein, Theisen, Yankielowizc 97
Hanany, Zaffaroni 99
Zafrir 15

G_2 gauge theory and 5-brane webs

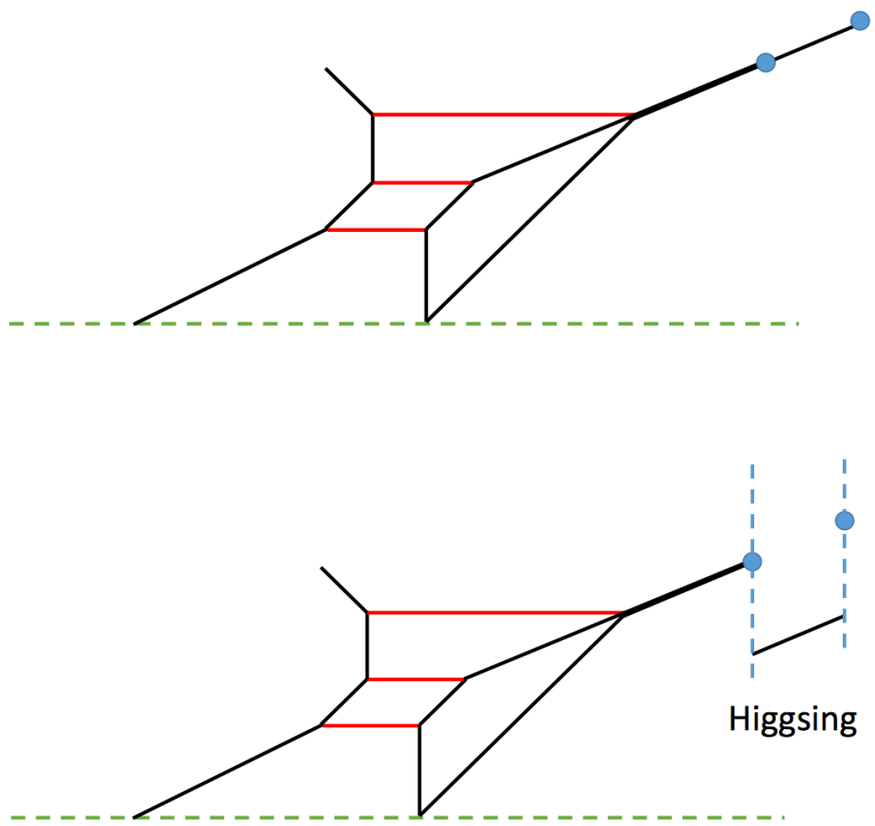
G2 = Higgsing of SO(7) with spinor

SO(7) + **spinor** matter

[Zafrir'15]



SO(7) with a **massless** spinor



Higgsing gives **pure G2**

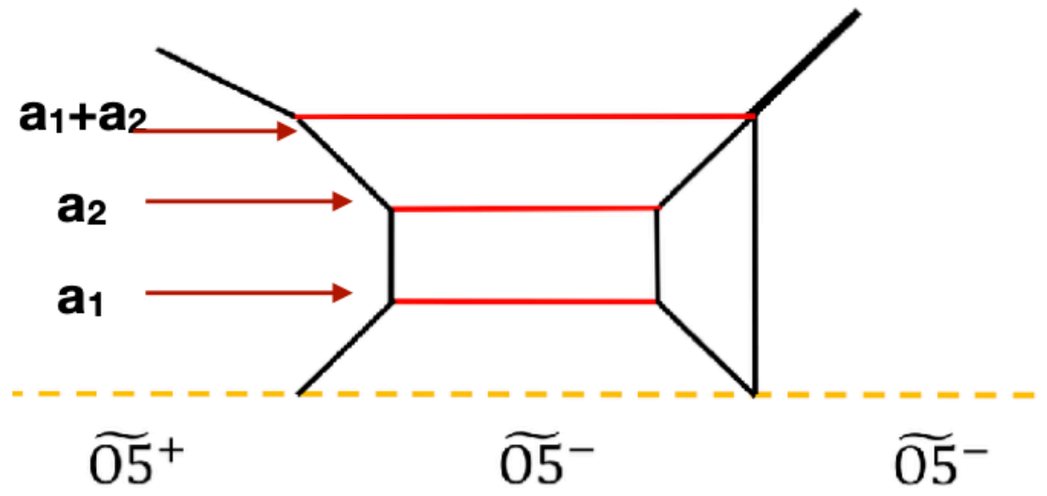
[Hayashi, SSK, Lee, Yagi 1801.03916]

Using a topological vertex method with an O5-plane, **[SSK, Yagi 1709.01928]**
we confirmed that it reproduces the known G2 partition function.

['12 Hanany, Mekareeya, Razama] ['12 Keller, J Song] ['18 H-C Kim, J Kim, S Kim, K-H Lee, J Park]

This 5-brane web also reproduces the correct **monopole string tension**, consistent with the prepotential

After an SL(2,Z),



- Prepotential $2\phi_1 - 3\phi_2 = a_2 - a_1,\ -\phi_1 + 2\phi_2 = a_1$

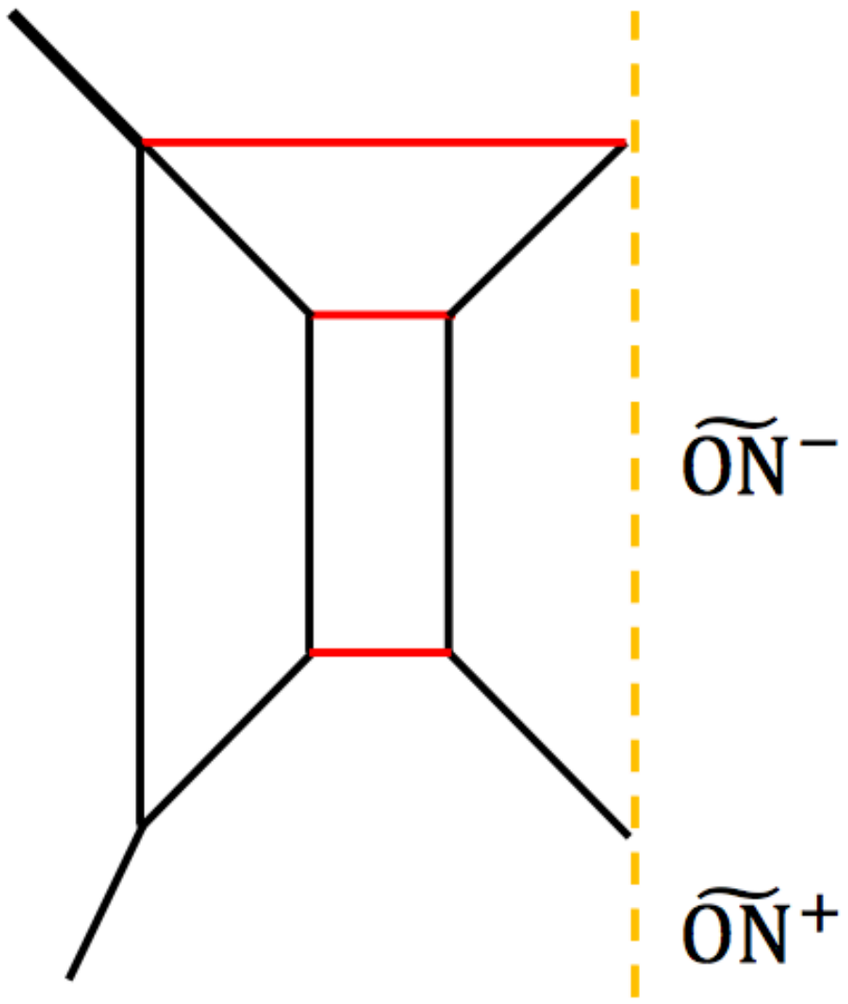
$$\mathcal{F}_{G_2} = m_0(\phi_1^2 - 3\phi_1\phi_2 + 3\phi_2^2) + \frac{4}{3}\phi_1^3 - 4\phi_1^2\phi_2 + 3\phi_1\phi_2^2 + \frac{4}{3}\phi_2^3$$

- Monopole tension

$$T_1 = \partial\mathcal{F}/\partial\phi_1,\ T_2 = \partial\mathcal{F}/\partial\phi_2$$

Duality

S-dual to the 5-brane web for the pure G_2 gauge theory:



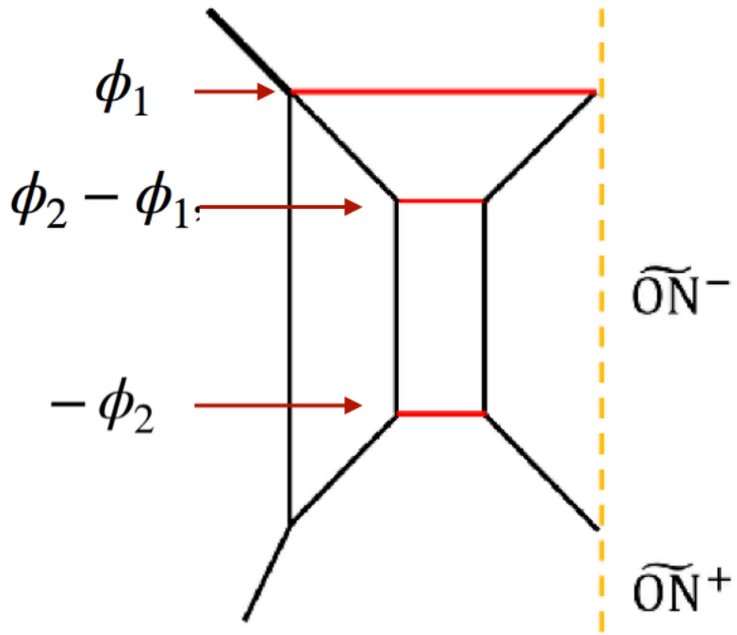
3 color D5 branes

SU(3) gauge theory

CS level?

The monopole string tension corresponds to the area of compact face.

We confirmed that this SU(3) gauge theory has $\kappa = 7$



- Prepotential

$$\mathcal{F}_{SU(3)_7} = m_0(\phi_1^2 - \phi_1\phi_2 + \phi_2^2) + \frac{4}{3}\phi_1^3 + 3\phi_1^2\phi_2 - 4\phi_1\phi_2^2 + \frac{4}{3}\phi_2^3$$

- Monopole tension $a_1 = \phi_1, a_2 = \phi_2 - \phi_1, a_3 = \phi_3 - \phi_2,$

$$T_1 = \partial\mathcal{F}/\partial\phi_1,\ T_2 = \partial\mathcal{F}/\partial\phi_2$$

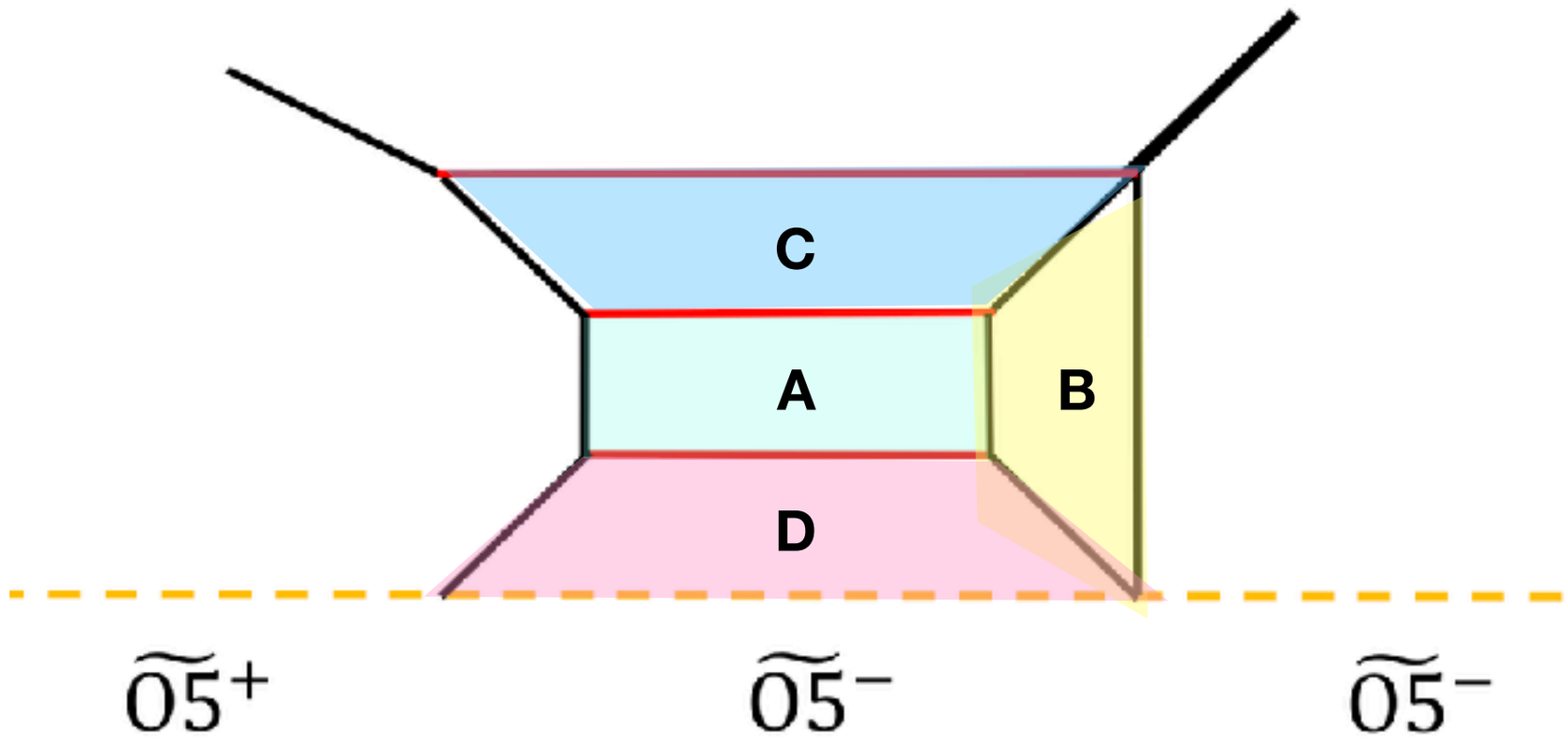
- Duality between G_2 & $SU(3)_7$

Parameter Map

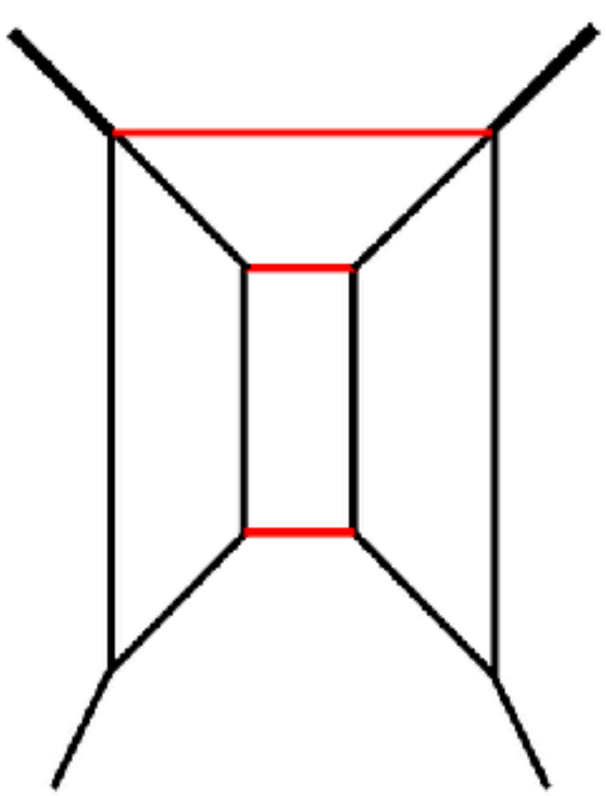
$$m_0^{SU(3)} = -\frac{m_0^{G_2}}{3},$$
$$\phi_1^{SU(3)} = \phi_2^{G_2} + \frac{1}{3} m_0^{G_2},$$
$$\phi_2^{SU(3)} = \phi_1^{G_2} + \frac{2}{3} m_0^{G_2},$$

Monopole tension

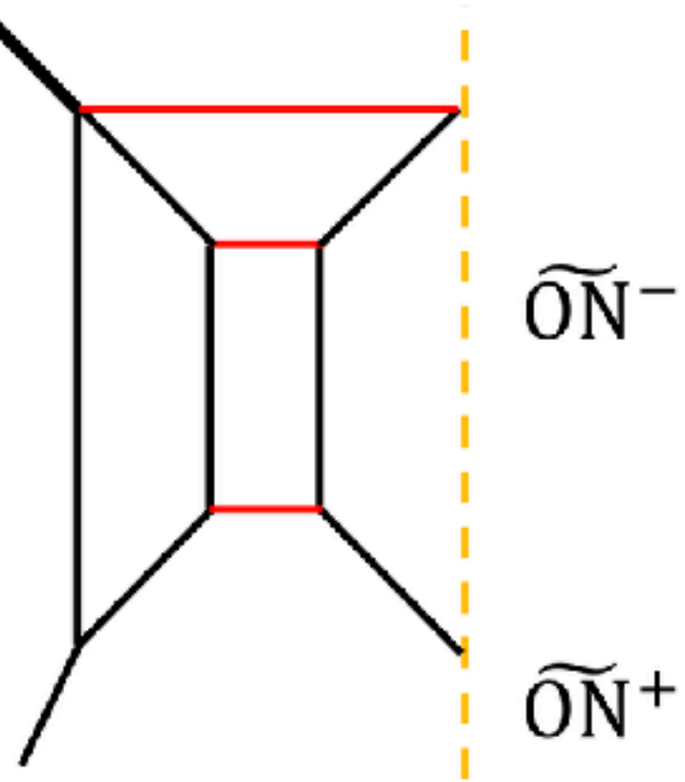
$$T_1^{SU(3)_7} = T_2^{G_2} = \mathbf{B} + \mathbf{C} + \mathbf{2D}$$
$$T_2^{SU(3)_7} = T_1^{G_2} = \mathbf{A}$$



- In fact, we can further increase the Chern-Simons levels.

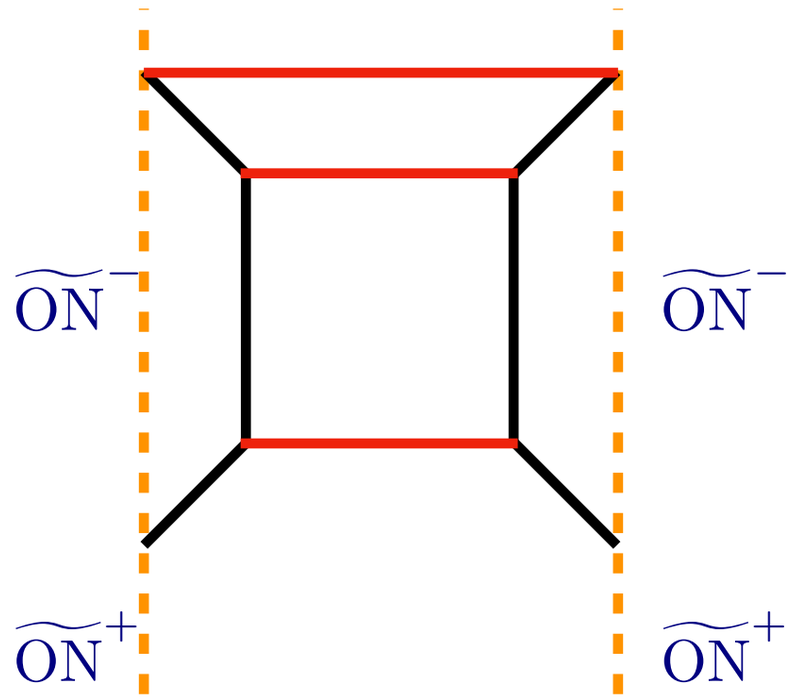


$\kappa = 5$



$\kappa = 7$

- A 5-brane web diagram for SU(3) with Chern-Simons level κ=9

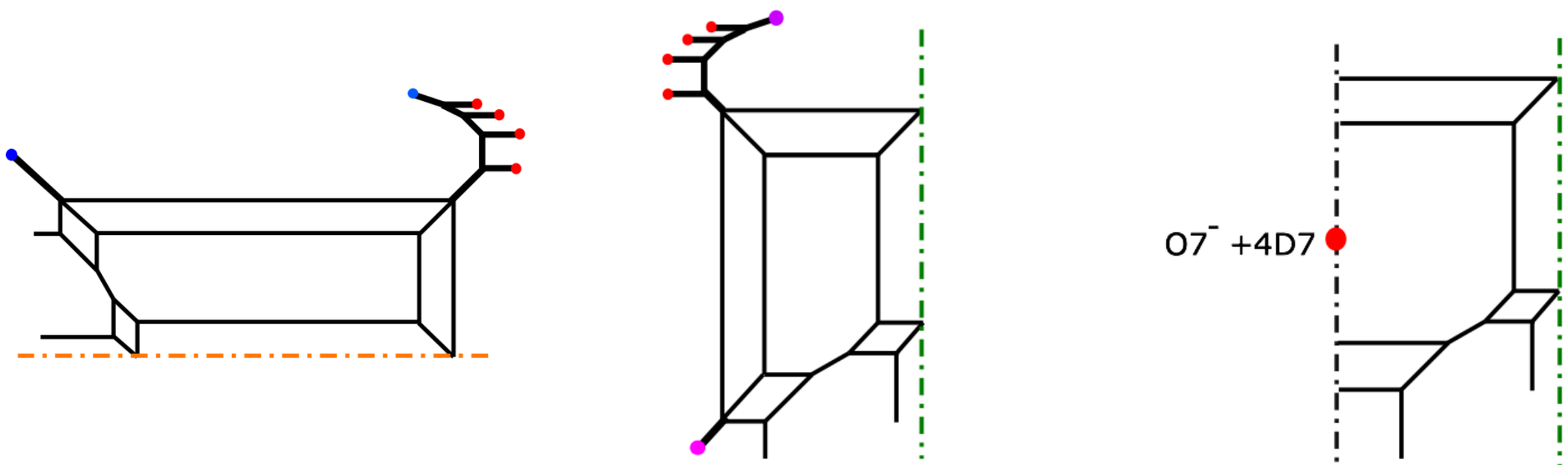


- The $SU(3)_9$ theory has been found and constructed from geometry.

Jefferson, Kim, Vafa, Zafrir 17
Jefferson, Katz, Kim, Vafa 18

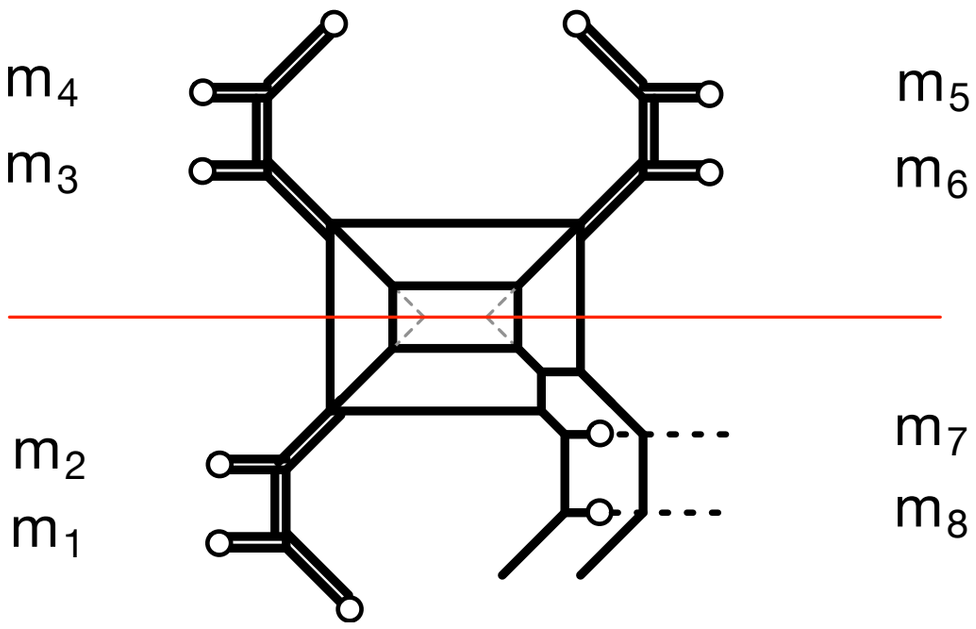
Adding more Fundamental hypers yields **marginal** theories of G_2-SU(3)-Sp(2) sequences

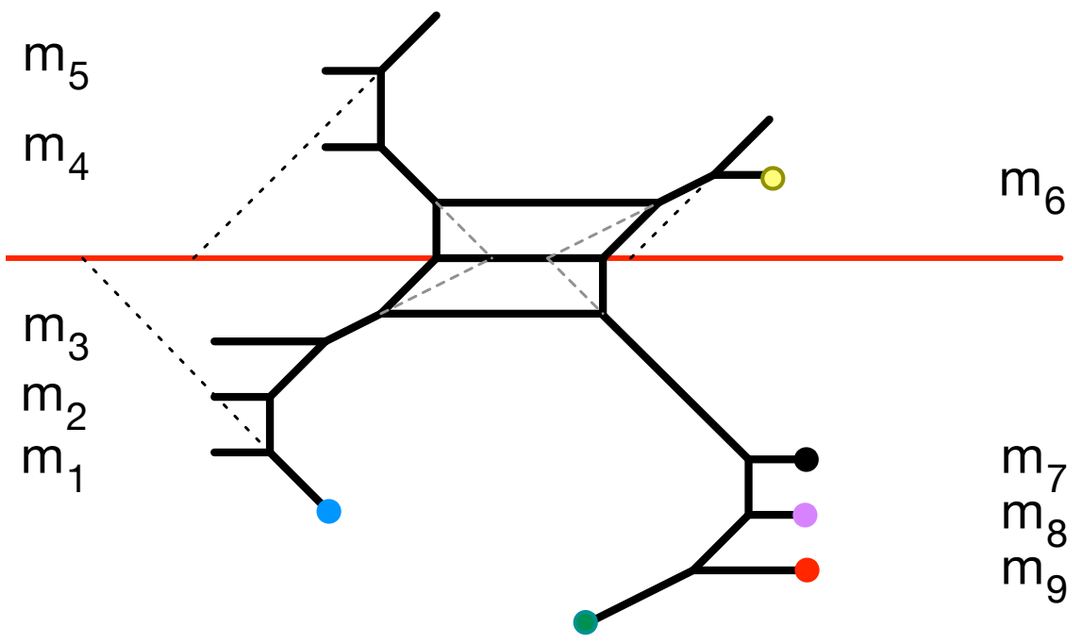
G_2 + 6F ⟷ **$SU(3)_4$ + 6F** ⟷ **Sp(2) + 2AS+ 4F**

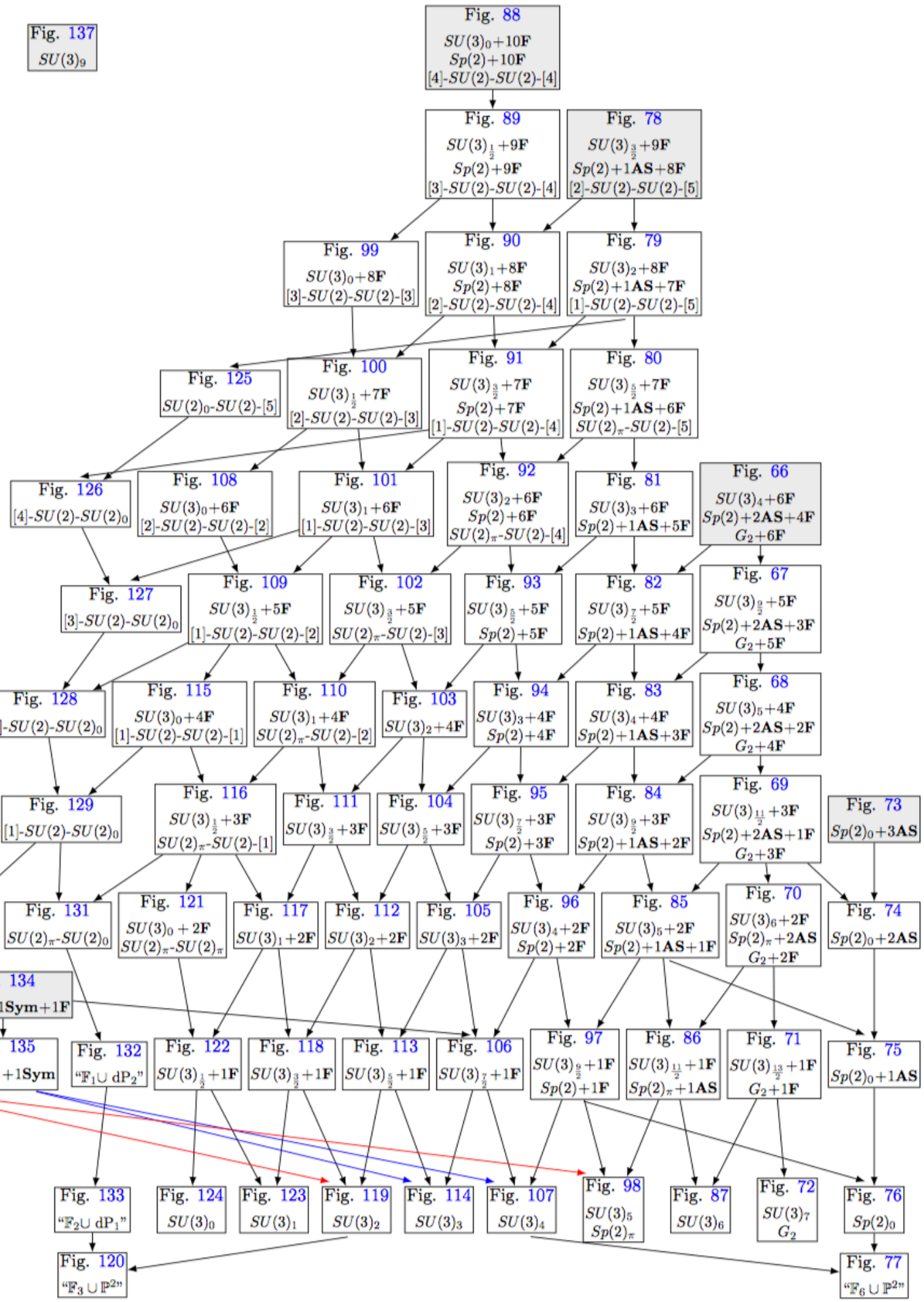


The **duality maps** can be read off from the 5-brane webs.

Sp(2) + 1AS + 8F ⟷ **SU(3)$_{3/2}$ + 9F**






Fig. 137
$SU(3)_9$
Fig. 88
$SU(3)_0$+10F
$Sp(2)$+10F
[4]-$SU(2)$-$SU(2)$-[4]
Fig. 89
$SU(3)_{\frac{1}{2}}$+9F
$Sp(2)$+9F
[3]-$SU(2)$-$SU(2)$-[4]
Fig. 78
$SU(3)_{\frac{3}{2}}$+9F
$Sp(2)$+1AS+8F
[2]-$SU(2)$-$SU(2)$-[5]
Fig. 99
$SU(3)_0$+8F
[3]-$SU(2)$-$SU(2)$-[3]
Fig. 90
$SU(3)_1$+8F
$Sp(2)$+8F
[2]-$SU(2)$-$SU(2)$-[4]
Fig. 79
$SU(3)_2$+8F
$Sp(2)$+1AS+7F
[1]-$SU(2)$-$SU(2)$-[5]
Fig. 125
$SU(2)_0$-$SU(2)$-[5]
Fig. 100
$SU(3)_{\frac{1}{2}}$+7F
[2]-$SU(2)$-$SU(2)$-[3]
Fig. 91
$SU(3)_{\frac{3}{2}}$+7F
$Sp(2)$+7F
[1]-$SU(2)$-$SU(2)$-[4]
Fig. 80
$SU(3)_{\frac{5}{2}}$+7F
$Sp(2)$+1AS+6F
$SU(2)_\pi$-$SU(2)$-[5]
Fig. 126
[4]-$SU(2)$-$SU(2)_0$
Fig. 108
$SU(3)_0$+6F
[2]-$SU(2)$-$SU(2)$-[2]
Fig. 101
$SU(3)_1$+6F
[1]-$SU(2)$-$SU(2)$-[3]
Fig. 92
$SU(3)_2$+6F
$Sp(2)$+6F
$SU(2)_\pi$-$SU(2)$-[4]
Fig. 81
$SU(3)_3$+6F
$Sp(2)$+1AS+5F
Fig. 66
$SU(3)_4$+6F
$Sp(2)$+2AS+4F
G_2+6F
Fig. 127
[3]-$SU(2)$-$SU(2)_0$
Fig. 109
$SU(3)_{\frac{1}{2}}$+5F
[1]-$SU(2)$-$SU(2)$-[2]
Fig. 102
$SU(3)_{\frac{3}{2}}$+5F
$SU(2)_\pi$-$SU(2)$-[3]
Fig. 93
$SU(3)_{\frac{5}{2}}$+5F
$Sp(2)$+5F
Fig. 82
$SU(3)_{\frac{7}{2}}$+5F
$Sp(2)$+1AS+4F
Fig. 67
$SU(3)_{\frac{9}{2}}$+5F
$Sp(2)$+2AS+3F
G_2+5F
Fig. 128
[2]-$SU(2)$-$SU(2)_0$
Fig. 115
$SU(3)_0$+4F
[1]-$SU(2)$-$SU(2)$-[1]
Fig. 110
$SU(3)_1$+4F
$SU(2)_\pi$-$SU(2)$-[2]
Fig. 103
$SU(3)_2$+4F
Fig. 94
$SU(3)_3$+4F
$Sp(2)$+4F
Fig. 83
$SU(3)_4$+4F
$Sp(2)$+1AS+3F
Fig. 68
$SU(3)_5$+4F
$Sp(2)$+2AS+2F
G_2+4F
Fig. 129
[1]-$SU(2)$-$SU(2)_0$
Fig. 116
$SU(3)_{\frac{1}{2}}$+3F
$SU(2)_\pi$-$SU(2)$-[1]
Fig. 111
$SU(3)_{\frac{3}{2}}$+3F
Fig. 104
$SU(3)_{\frac{5}{2}}$+3F
Fig. 95
$SU(3)_{\frac{7}{2}}$+3F
$Sp(2)$+3F
Fig. 84
$SU(3)_{\frac{9}{2}}$+3F
$Sp(2)$+1AS+2F
Fig. 69
$SU(3)_{\frac{11}{2}}$+3F
$Sp(2)$+2AS+1F
G_2+3F
Fig. 73
$Sp(2)_0$+3AS
Fig. 130
$SU(2)_0$-$SU(2)_0$
Fig. 131
$SU(2)_\pi$-$SU(2)_0$
Fig. 121
$SU(3)_0$ + 2F
$SU(2)_\pi$-$SU(2)_\pi$
Fig. 117
$SU(3)_1$+2F
Fig. 112
$SU(3)_2$+2F
Fig. 105
$SU(3)_3$+2F
Fig. 96
$SU(3)_4$+2F
$Sp(2)$+2F
Fig. 85
$SU(3)_5$+2F
$Sp(2)$+1AS+1F
Fig. 70
$SU(3)_6$+2F
$Sp(2)_\pi$+2AS
G_2+2F
Fig. 74
$Sp(2)_0$+2AS
Fig. 134
$SU(3)_0$+1Sym+1F
Fig. 136
$SU(3)_{\frac{3}{2}}$+1Sym
Fig. 135
$SU(3)_{\frac{1}{2}}$+1Sym
Fig. 132
"$\mathbb{F}_1 \cup$ dP$_2$"
Fig. 122
$SU(3)_{\frac{1}{2}}$+1F
Fig. 118
$SU(3)_{\frac{3}{2}}$+1F
Fig. 113
$SU(3)_{\frac{5}{2}}$+1F
Fig. 106
$SU(3)_{\frac{7}{2}}$+1F
Fig. 97
$SU(3)_{\frac{9}{2}}$+1F
$Sp(2)$+1F
Fig. 86
$SU(3)_{\frac{11}{2}}$+1F
$Sp(2)_\pi$+1AS
Fig. 71
$SU(3)_{\frac{13}{2}}$+1F
G_2+1F
Fig. 75
$Sp(2)_0$+1AS
Fig. 133
"$\mathbb{F}_2 \cup$ dP$_1$"
Fig. 124
$SU(3)_0$
Fig. 123
$SU(3)_1$
Fig. 119
$SU(3)_2$
Fig. 114
$SU(3)_3$
Fig. 107
$SU(3)_4$
Fig. 98
$SU(3)_5$
$Sp(2)_\pi$
Fig. 87
$SU(3)_6$
Fig. 72
$SU(3)_7$
G_2
Fig. 76
$Sp(2)_0$
Fig. 120
"$\mathbb{F}_3 \cup \mathbb{P}^2$"
Fig. 77
"$\mathbb{F}_6 \cup \mathbb{P}^2$"

136 figures!

One more thing !

Thanks for organizing such a wonderful workshop!

Organizing Committee

Chong-Sun Chu (NCTS / NTHU)
Yoshihisa Kitazawa (KEK)
Kimyeong Lee (KIAS) - Chair
Jian-Xin Lu (USTC)

Local Organizing Committee

Nakwoo Kim (Kyung Hee U. / KIAS)
Sungjay Lee (KIAS)
Kimyeong Lee (KIAS)
Piljin Yi (KIAS)

Secretaries

And **all the participants**

Thanks for organizing such a wonderful workshop!

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Sungjay Lee (KIAS)
Kimyeong Lee (KIAS)
Piljin Yi (KIAS)

Secretaries

And **all the participants**

See you again in Taiwan in 2019

2019年在台湾再见　　2019年在臺灣再見

2019年に台湾でまた会おう

2019년 대만에서 다시뵙겠습니다.