Quantum Teleportation and Traversable Wormholes

Dongsu Bak

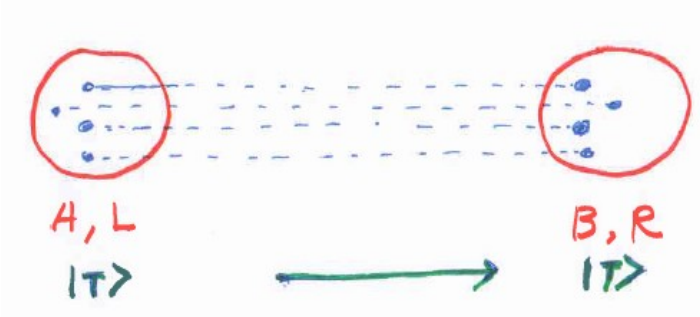
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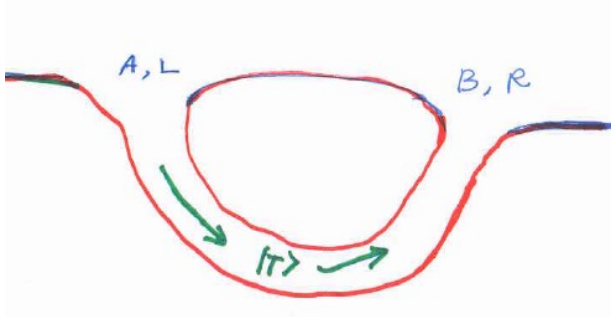
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Quantum teleportation

- Alice located in L sends a quantum state to Bob in a remote place R.

- The main holographic picture is that the qubit is sent through bulk traversable wormhole.



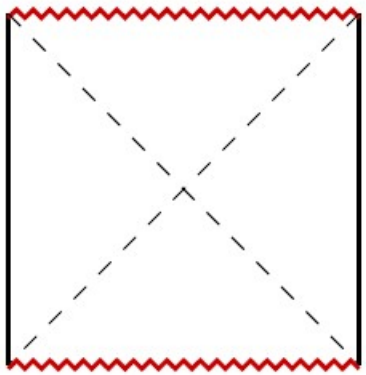


Holographic Decoding

- It has an important implication in holographic decoding problems in AdS/CFT.

- We are interested in the decoding of the behind horizon degrees and in the wormhole spacetime the horizon can disappear.

- One can in principle understand what happens to behind horizon degrees and the fate of future spacelike singularities.



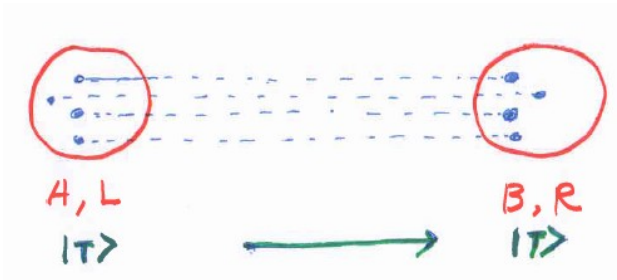
Quantum teleportation

- Alice on the left side would like to teleport a qubit state

$$|T\rangle = c_0|0\rangle_T + c_1|1\rangle_T = \sum_{k=0}^{1} c_k|k\rangle_T$$

- To send the qubit from L to R, L and R should be entangled as a required setup. We model the L-R entanglement by an EPR pair

$$|\Psi\rangle_{LR} = \frac{1}{\sqrt{2}}\sum_{i=0}^{1}|i\rangle_L|i\rangle_R$$



- The qubit $|T\rangle$ is added to the left side at some point. One then represents the left side system in a new basis spanned by

$$|\theta_1\theta_2\rangle_M = \frac{1}{\sqrt{2}}\left[|\theta_1 0\rangle_{TL} + (-)^{\theta_2}|1-\theta_1 \quad 1\rangle_{TL}\right]$$

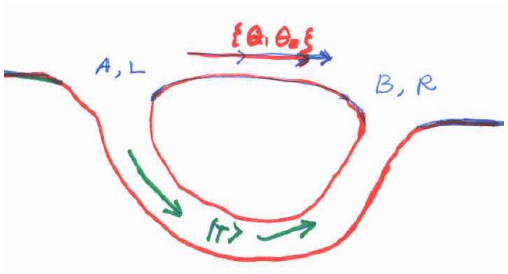
- At this stage the total state becomes

$$|T\rangle|\Psi\rangle_{LR} = \frac{1}{2}\sum_{\theta_1,\theta_2,i,k} |\theta_1\theta_2\rangle_M (U^{-1}_{\{\theta_1\theta_2\}})_{ik} c_k |i\rangle_R$$

  where $U_{\{\theta_1\theta_2\}} = \sigma_1^{\theta_1}\sigma_3^{\theta_2}$.

- Now Alice on the left side makes a measurement in the M basis ending up with a particular state $|\theta_1\theta_2\rangle_M$ and sends its result $\{\theta_1, \theta_2\}$ to Bob on the right side via a classical channel. At this stage, the state becomes

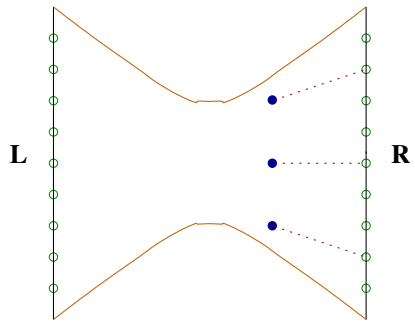
$$\sum_{i,k} |\theta_1\theta_2\rangle_M (U^{-1}_{\{\theta_1\theta_2\}})_{ik} c_k |i\rangle_R$$



- Once Bob gets the message, he performs a unitary transform of his state by the action $V_\theta = U_{\{\theta_1\theta_2\}}$ and then the resulting state becomes

$$|\theta_1\theta_2\rangle_M \sum_k c_k |k\rangle_R$$

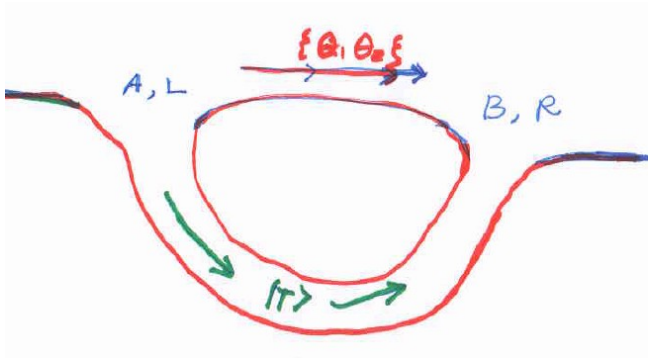
- Of course one can consider more general setup where one has an L-R entanglement involving many qubits and teleports more than one qubit.
- In particular when one uses our Einstein-Rosen bridge as the L-R entanglement based on the so called ER=EPR relation [Maldacena, Susskind 2013], there will be in general a thermailzation of $|T\rangle$ state after its inclusion to the left side.



- Measurement can be made by picking up an arbitrary qubit of L system. Alice again sends the result of measurement to Bob. Bob then recovers the $|T\rangle$ by the action of an appropriate unitary transformation.

Measurement M_θ and Recovery V_θ

- The measurement in general makes the L-R entanglement reduced and the L-T system entangled instead.
- The second essential feature is the L-R coupling by the measurement M_θ on the left side and the recovery action V_θ on the right side.



- This coupling basically makes the wormhole traversable as we shall see in the following.

Gravity description [Jackiw-Teitelboim]

- The AdS2 dilaton gravity is described by

$$I = I_{top} - \frac{1}{16\pi G}\int_M d^2x\sqrt{g}\,\phi\left(R+\frac{2}{\ell^2}\right) + I_M(g,\chi)$$

where

$$\begin{aligned} I_{top} &= -\frac{\phi_0}{16\pi G}\int_M d^2x\sqrt{g}R \\ I_M &= \frac{1}{2}\int_M d^2x\sqrt{g}\left(\nabla\chi\cdot\nabla\chi + m^2\chi^2\right) \end{aligned}$$

- The equations of motion read

$$R+\frac{2}{\ell^2}=0, \qquad \nabla^2\chi - m^2\chi = 0$$

$$\nabla_a\nabla_b\phi - g_{ab}\nabla^2\phi + g_{ab}\phi = -8\pi G T_{ab}$$

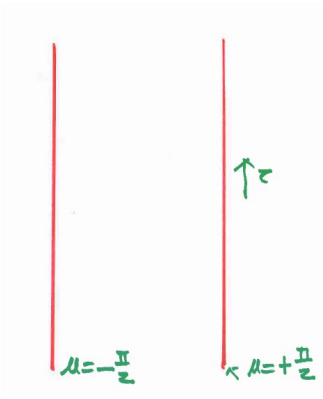
with

$$T_{ab} = \nabla_a\chi\nabla_b\chi - \frac{1}{2}g_{ab}\left(\nabla\chi\cdot\nabla\chi + m^2\chi^2\right)$$

- The geometry is solved by the AdS_2 space

$$ds^2 = \frac{\ell^2}{\cos^2\mu}\left(-d\tau^2 + d\mu^2\right)$$

where the coordinate μ is ranged from $-\pi/2$ to $+\pi/2$.



- The most general vacuum solution for the dilaton field is given by

$$\phi = \bar{\phi}\, L\, \frac{(b+b^{-1})\cos(\tau-\tau_B) - (b-b^{-1})\sin\mu}{2\cos\mu}$$

- Make the coordinate transformation

$$\frac{r}{L} = \frac{(b+b^{-1})\cos(\tau-\tau_B) - (b-b^{-1})\sin\mu}{2\cos\mu}$$

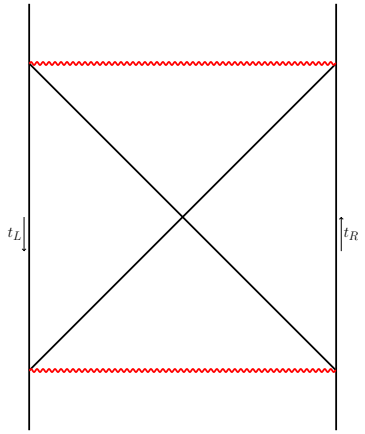
$$\tanh\frac{tL}{\ell^2} = \frac{2\sin(\tau-\tau_B)}{(b+b^{-1})\sin\mu - (b-b^{-1})\cos(\tau-\tau_B)}$$

AdS2 Black holes

- One is led to the corresponding AdS black hole metric and the dilaton field:

$$\begin{aligned} ds^2 &= -\frac{r^2-L^2}{\ell^2}dt^2 + \frac{\ell^2}{r^2-L^2}dr^2 \\ \phi &= \bar{\phi}\, r \end{aligned}$$

We shall set $b = 1$ and $\tau_B = 0$.



- The singularity is determined by $\Phi^2 = \phi_0 + \phi = 0$ where Φ^2 plays the role of radius squared in the transverse space when viewed from the point of view of the dimensional reduction from the higher dimensions as in

$$ds_4^2 = ds_{AdS_2}^2 + \Phi^2(d\theta^2 + sin^2\theta d\varphi^2)$$

AdS2 Black holes

- The Gibbons-Hawking temperature of this black hole can be identified as

$$T = \frac{1}{2\pi}\frac{L}{\ell^2}$$

- The energy and entropy are then

$$\begin{aligned} E &= \frac{1}{2}\mathcal{C}T^2 \\ S &= S_0 + \mathcal{C}T \end{aligned}$$

  which is expected with the AdS2 conformal symmetries.

- This entropy can be written as a Beckenstein formula

$$S = \frac{\phi_0 + \bar{\phi}\,L}{4G}$$

- This two-sided AdS black hole is dual to the so-called thermofield double of $\mathrm{CFT1}_L \times \mathrm{CFT1}_R$

$$|\Psi(0)\rangle = \frac{1}{\sqrt{Z}}\sum_{n,n'}\langle n|\mathcal{U}|n'\rangle\,|n'\rangle\otimes|n\rangle = \frac{1}{\sqrt{Z}}\sum_{n} e^{-\frac{\beta}{2}E_n}\,|n\rangle\otimes|n\rangle$$

Deformations

- Let us turn on a scalar field χ with mass m. This matter field is dual to a scalar primary operator $O_\Delta(t)$, where its dimension is related to the mass squared by

$$\Delta = \frac{1}{2}\left(1+\sqrt{1+4m^2}\right)$$

for nontachyonic case of $m^2 \geq 0$.

- When $0 > m^2 > -1/4$, both possibilities of operator dimensions are allowed

$$\Delta = \Delta_\pm = \frac{1}{2}\left(1\pm\sqrt{1+4m^2}\right)$$

- For the double trace deformation of the traversable wormhole, Δ_- will be the relevant one.

Deformations

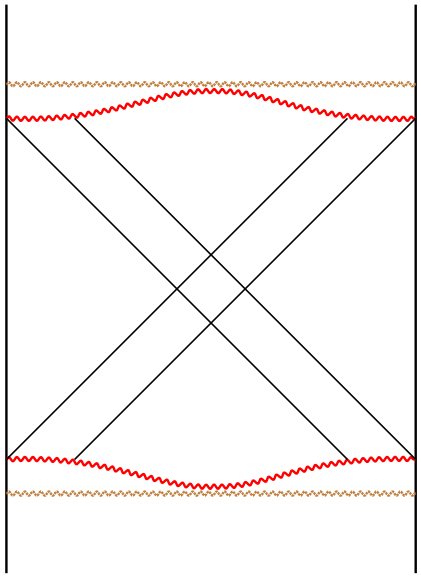
- We solve the scalar field equation for χ with boundary conditions.
- The asymptotic behavior of the scalar field in the right/left wedges in the Penrose diagram is given by

$$\chi(t,r)|_{R/L} = \frac{\alpha_{R/L}}{r^{\Delta}} + \cdots + \frac{\beta_{R/L}}{r^{1-\Delta}} + \cdots$$

  where where α plays a source term and β is then the expectation value in the conventional case.
- Once the matter part is solved, we solve the dilaton with the corresponding energy momentum tensor. Thus the problem is almost linear.

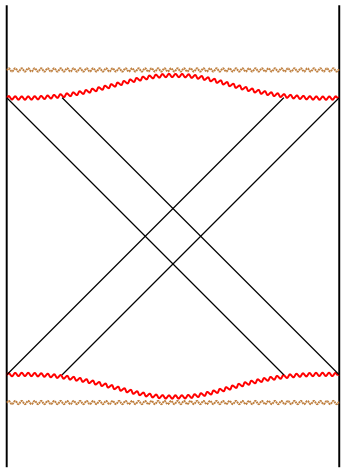
Thermalization of excited states

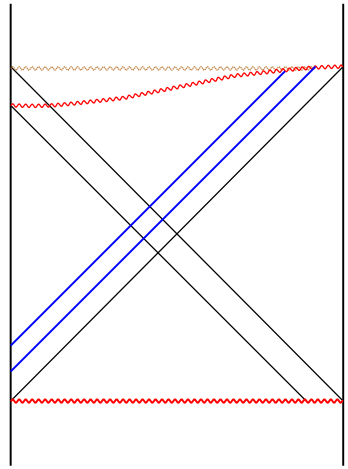
- In this case we turn on the coefficient β without turning on the source term. The Hamiltonian of the system remains undeformed while the thermofield double state will be excited.
- This is describing a thermalization of an initial excitation above the thermal vacuum. The left and the right system remains independent and one cannot send a signal from one side to the other side as an effective elongation of the Penrose diagram horizontally.



Janus deformation [Bak Kim Yi 18]

- We consider $m^2 = 0$ with $\Delta = 1$ and turn on the source term which makes left and right Hamiltonians differ from each other. The Penrose diagram is similar to the above.
- One may consider a time dependent Janus deformation where the left side Hamiltonian is deformed at a certain time. Then this deformation falls into the horizon and singularity and in the right hand side there is no signal of the deformation.





- Later we shall use this to send a teleportee state to the other side.

Double trace deformation and traversable wormholes

- For the double trace deformation, one has the mixed boundary condition

$$\beta_L(t) = h(-t)\alpha_R(-t)\,, \quad \beta_R(t) = h(t)\alpha_L(-t)$$

- According to the AdS/CFT correspondence, this mixed bc corresponds to the double trace deformation in the boundary theory of the form [Witten:2001]

$$\delta H(t) = -h(t)\mathcal{O}_R(t)\mathcal{O}_L(-t)$$

- The change in the two point function $G = \langle \chi(t)\chi(t')\rangle$ can be evaluated by

$$G = i\int_{t_0}^{t} ds\,\langle[\delta H(s),\chi(t)],\chi(t')\rangle + i\int_{t_0}^{t'} ds\,\langle\chi(t)[\delta H(s),\chi(t')]\rangle$$

One-loop stress tensor and ANEC

- The 1-loop stress tensor can be computed through the bulk 2-point function $G(x, x')$ as

$$T_{ab} = \lim_{x' \to x} \left[\partial_a \partial'_b G(x, x') - \frac{1}{2} g_{ab} g^{\rho\sigma} \partial_\rho \partial'_\sigma G(x, x') - \frac{1}{2} g_{ab} m^2 G(x, x') \right]$$

  which is linear in the coupling h.

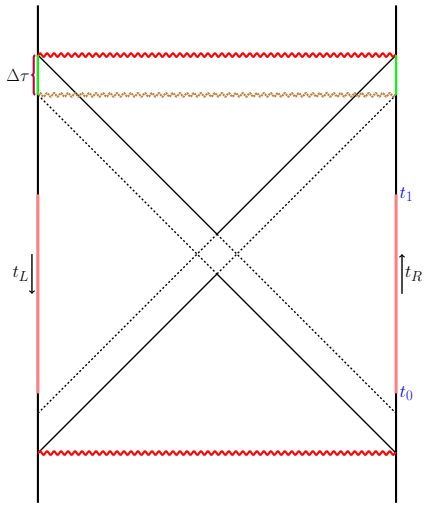
- Thus depending on the signature of h, the averaged null energy condition can be violated

$$\int_{-\infty}^{\infty} T_{UU} dU \; < \; 0$$

  which makes the wormhole traversable. [Jafferis Gao Wall 2016]

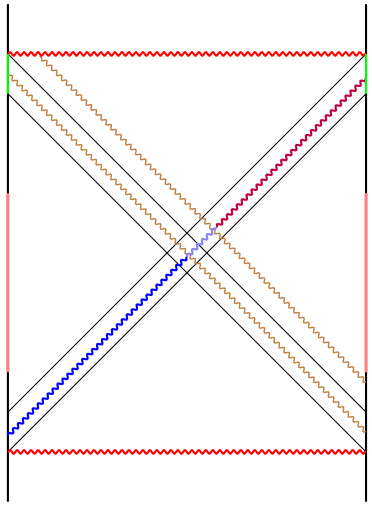
Traversable wormholes

- The change in the dilaton field can be identified explicitly and the location of singularity is delayed by $\Delta\tau$ in τ coordinate leading to the traversable wormhole solution. We shall call this $\Delta\tau$ as a wormhole opening parameter. [Bak Kim Yi 18]



- One can show that the horizon size and the corresponding entropy is reduced if the wormhole becomes traversable. With this traversable wormhole, one can send a signal from the left to the right.

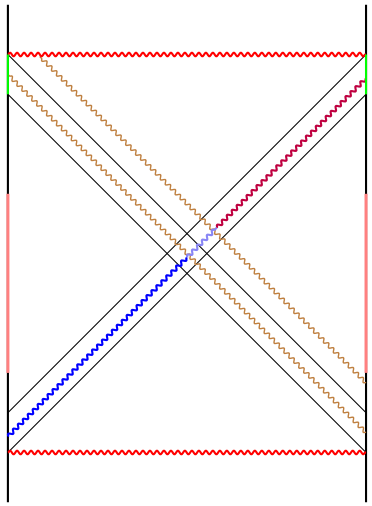
- We add the teleportee state T to the left side, which will appear to the right side after some time which is describing the full quantum teleportation.



- The measurement in general makes the L-R entanglement reduced and the L-T system entangled instead as we discussed previously.
- The essential feature is the L-R coupling by the measurement M_L^θ and the recovery action V_R^θ. This coupling basically makes the wormhole traversable which is modelled by the L-R interaction hO_LO_R.

Observability of bulk!

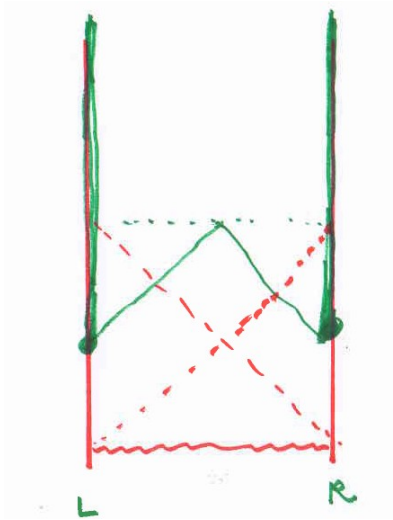
- Let us now turn to the case where Bob also throws a matter into the horizon from the right. If the extra matter is not too big, the wormhole remains traversable and the teleportee can be sent from the left to the right sides.



- It is clear that the teleportee will meet the matter from the right side while transported. It can record and report this encounter to Bob on the right side. Hence one may conclude that the bulk wormhole is experimentally observable. [Susskind 2017]

Eternally traversable wormholes

- The eternal traversable wormhole geometry [Maldacena Qi 18], is quite interesting.



- We simply adjust $h(t)$ in an appropriate manner which makes the wormhole eternally traversable and one may verify explicitly the singularity disappears completely. [Bak Kim Yi]
- Hence there are no trapped region and all the information behind the horizon comes out to L or R. This is an example of black hole evaporation.