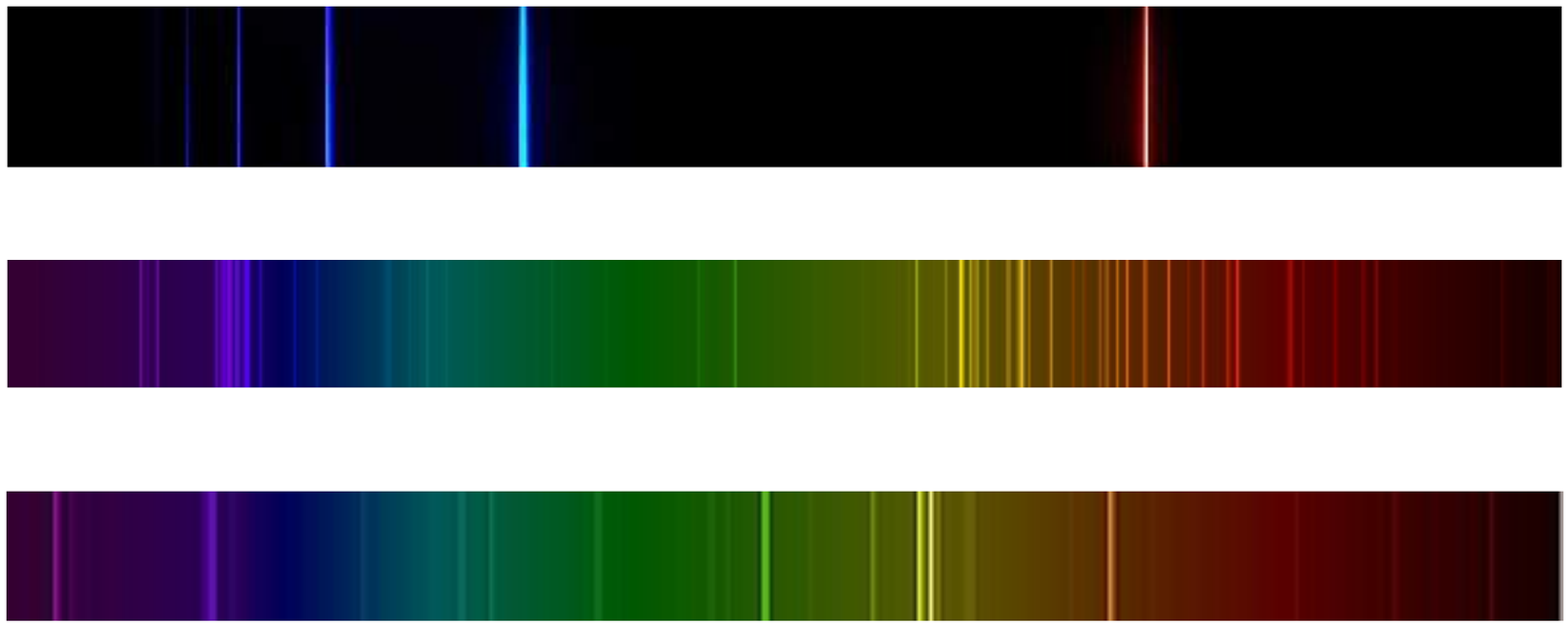
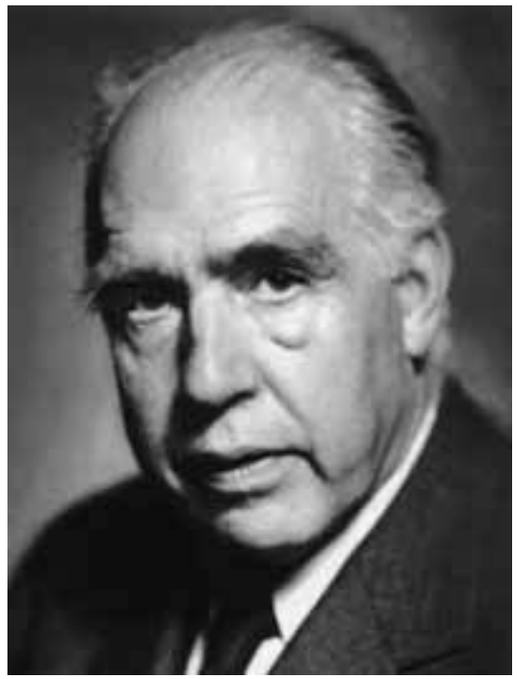
Quantum Gravity and the Swampland



Gary Shiu
University of Wisconsin-Madison

Conjecturing is part of the scientific process in fundamental physics, e.g.





Drawing lessons from the measured spectral lines of atomic transitions, and the proposal that energy is quantized (Planck), Bohr **conjectured**:

$$L = n\hbar, \quad n \in \mathbb{Z}$$

This led to the Bohr atomic model. This conjecture thus gives rise to ***other predictions*** which have been subsequently tested against experiments.

His conjecture was eventually understood as a consequence of **quantum mechanics**, but nonetheless was a stepping stone for its advent.

The **Swampland Conjectures** were made in a similar spirit.

While we don't currently have strong experimental hints on what lies beyond the Standard Model(s), we have lots of **theoretical data.**

These theoretical data suggest some **universal features** of UV complete theories. They are the basis of these conjectures.

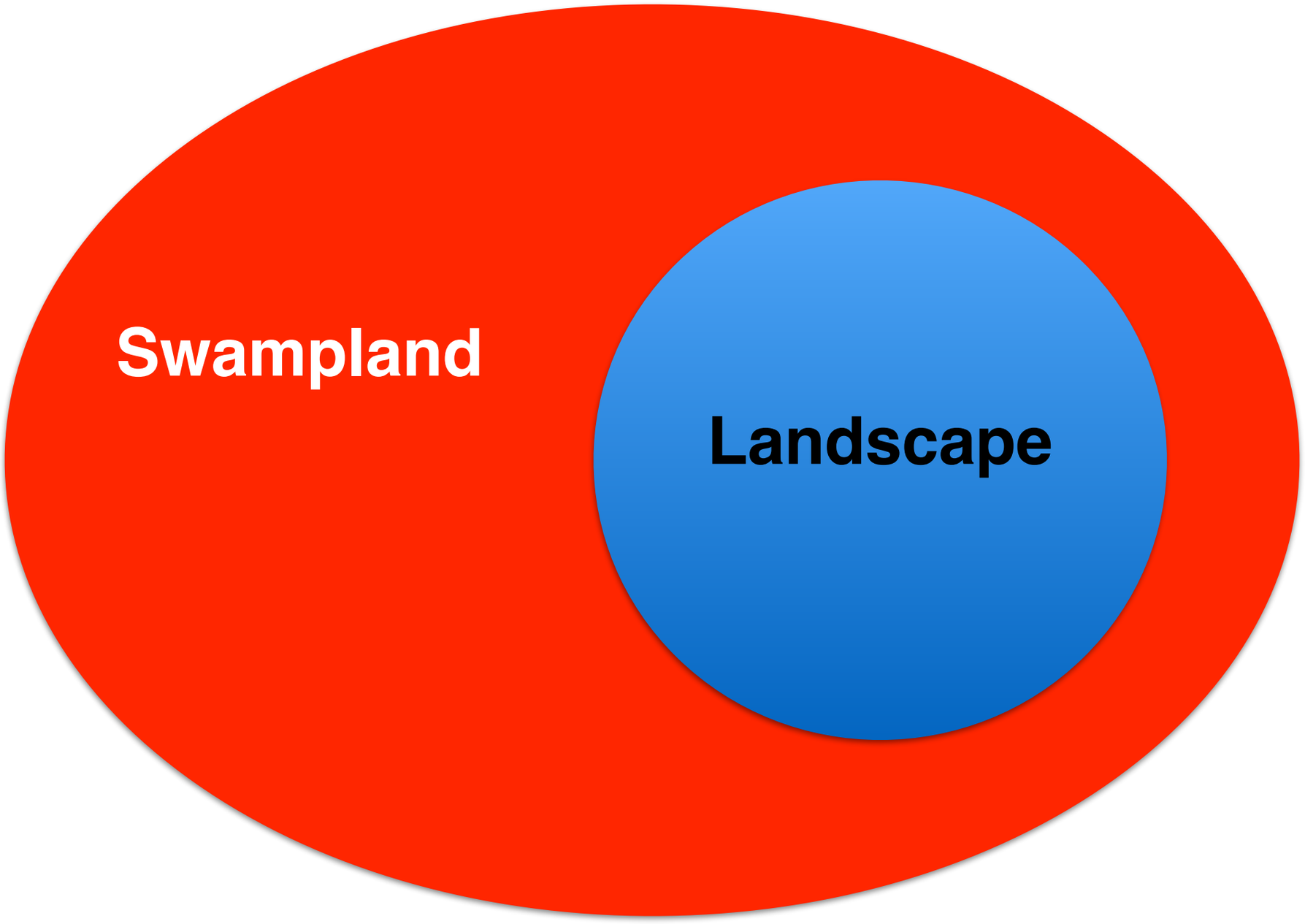
These conjectures led to ***interesting predictions*** in **cosmology** (e.g., inflationary B-mode, dark energy) and **particle physics** (e.g., Higgs physics, neutrino masses and type, axions, and other BSM physics).

Testing these conjectures and seeking the **fundamental reason** behind them is an active research area. The analogue of quantum mechanics is still out there, but that makes this research program particularly exciting.

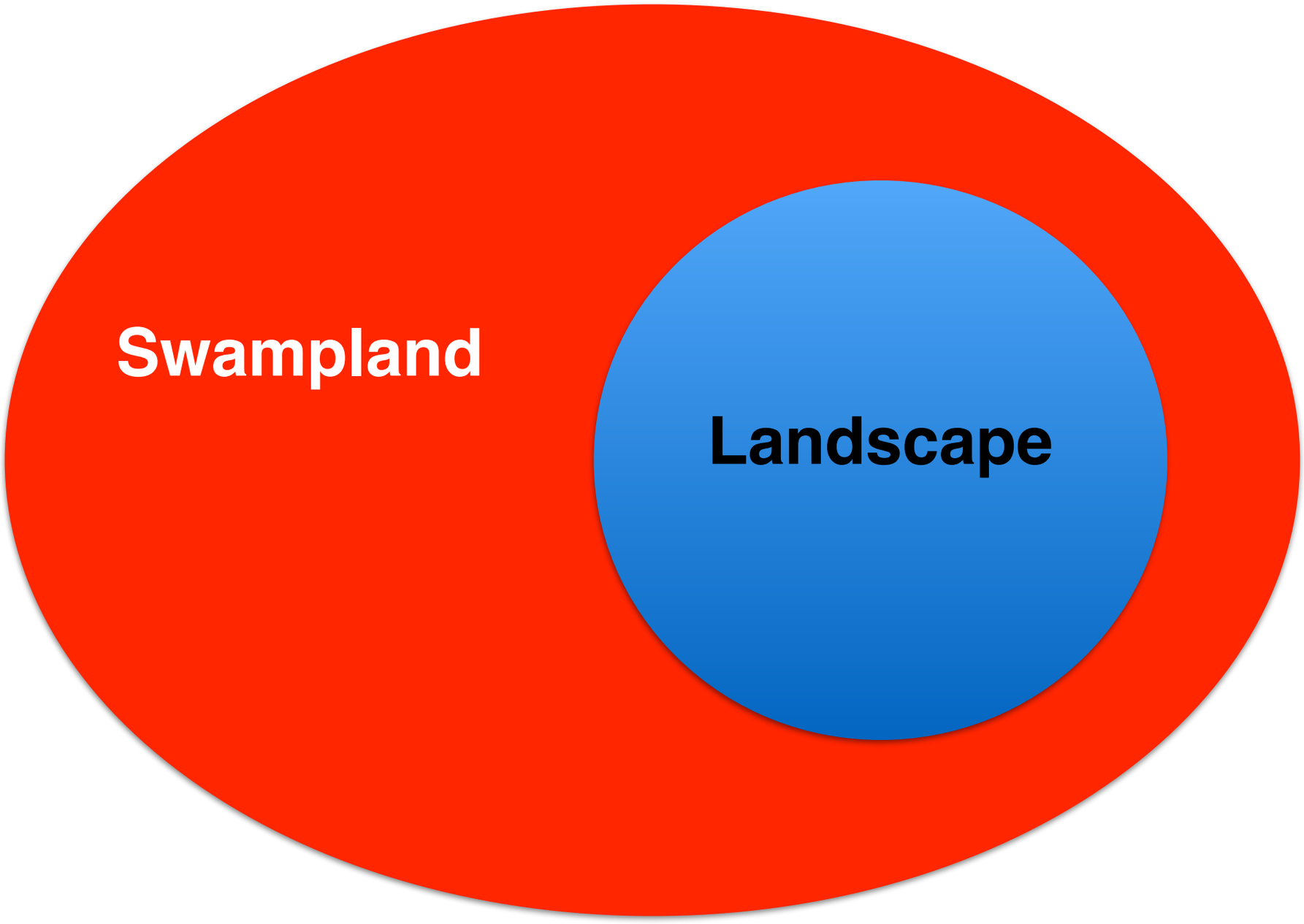
Landscape vs Swampland



Landscape vs Swampland



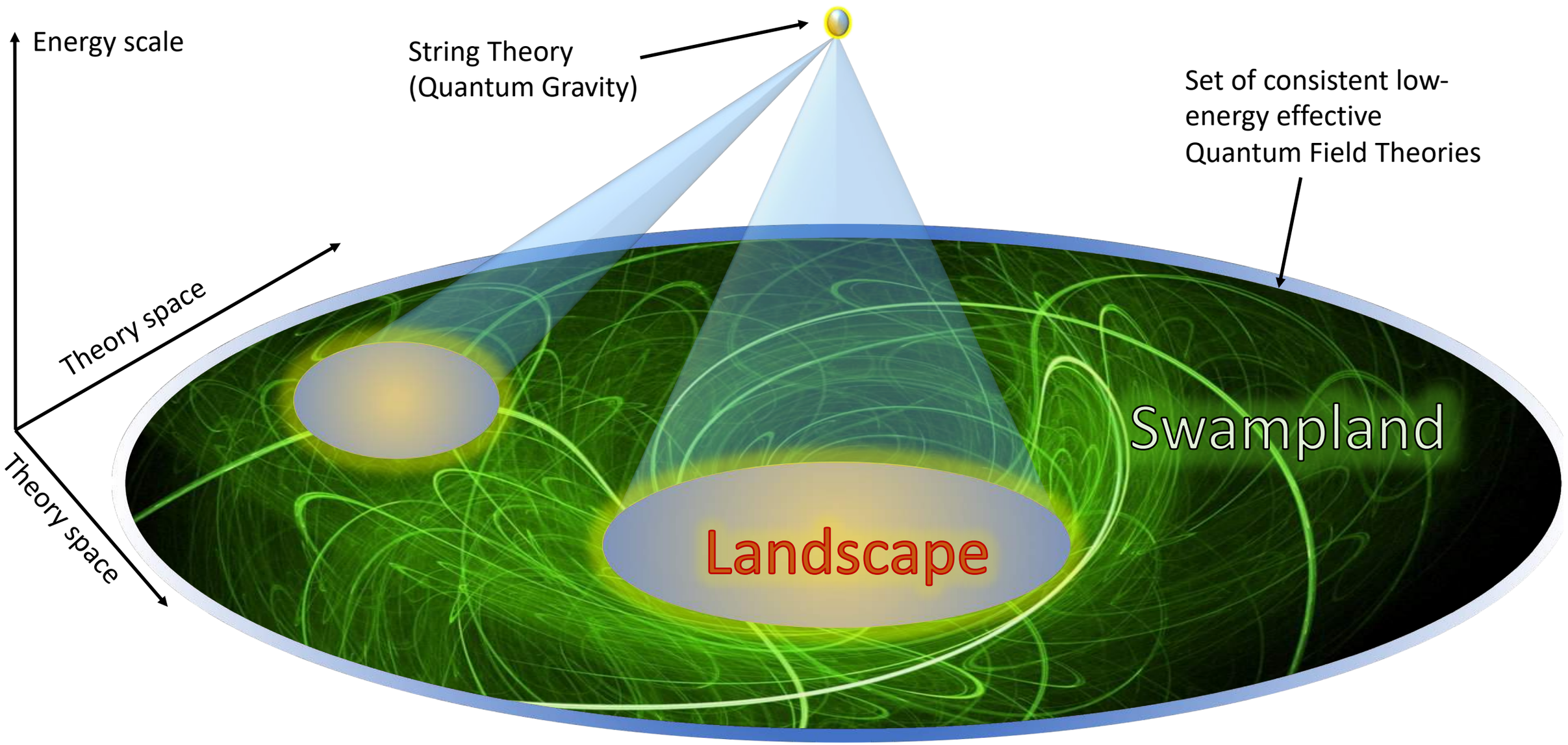
Landscape vs Swampland



We refer to the space of quantum field theories which are incompatible with quantum gravity as the ***swampland***. **[Vafa, ’05]; [Ooguri, Vafa, ’06]**

Landscape vs Swampland

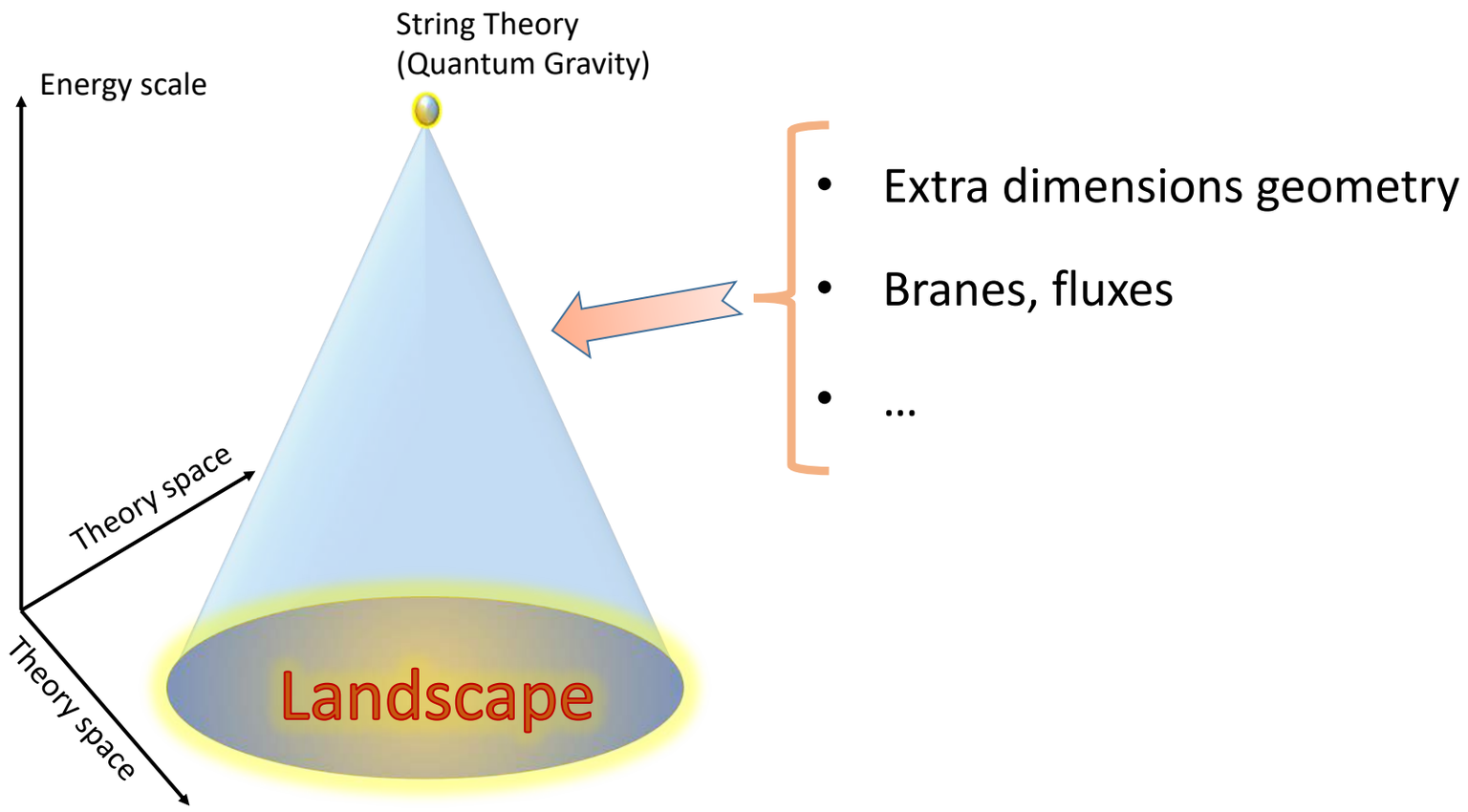
The Swampland program is about extracting possible universal predictions from string theory, and the rules governing them.



String Theory and the Swampland

The primary challenges include:

- No complete definition of String Theory — how can we be sure?



- String theory constructions live on a spectrum, where to draw the line?

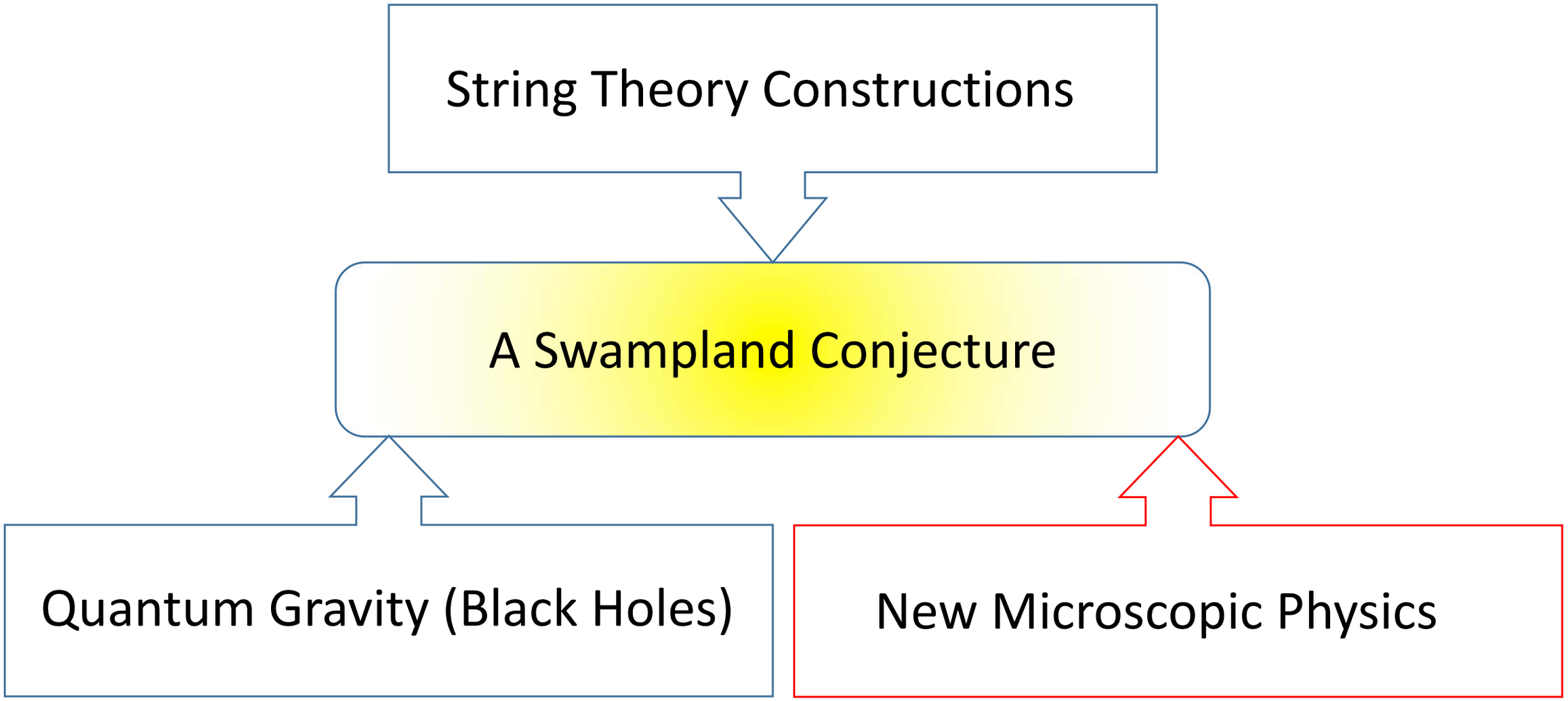


“String Constructed”
+ quantum corrections under control

“String Inspired”

String Theory and the Swampland

To make progress, we combine inputs from different fronts:



Web of Conjectures

No global symmetry

[Banks, Dixon,'88];[Harlow, Ooguri,'18]

Lecture 1

$g \neq 0$

Weak Gravity Conjecture (WGC)

[Arkani-Hamed, Motl, Nicolis, Vafa. '06]

Lectures 1 and 2

$\geqq \rightarrow >$

Non-SUSY AdS

[Ooguri, Vafa,'16]

Lecture 3

non-SUSY

Distance Conjecture

[Ooguri, Vafa,'06]

Lecture 4

$g \rightarrow 0$

Tower WGC

[Andriolo, Junghans, Noumi, GS, '18]

sLWGC

[Montero, GS, Soler '16];[Heidenreich et al, '16]

[Ooguri, Palti, GS, Vafa,'18]

Lecture 4

dS Conjecture

[Obied, Ooguri, Spodyneiko, Vafa,'18]

Quantum Gravity and Global Symmetries

Quantum Gravity and Global Symmetries

- **Global symmetries** are expected to be violated by gravity:



  - **No hair theorem:** Hawking radiation is insensitive to Q.
    - ➡ Infinite number of states (remnants) with $m \lesssim M_p$
    - ➡ Violation of entropy bounds. At finite temperature (e.g. in Rindler space), the density of states blows up. **Susskind '95**

- **Swampland conjecture**: theories with exact global symmetries are not UV-completable.

- In (perturbative) string theory, all symmetries are gauged **[Banks, Dixon '88].**

- There is a recent proof using holography **[Harlow, Ooguri, '18].**

- Many phenomenological ramifications, e.g., mini-charged DM comes with a new massless gauge boson **[GS, Soler, Ye, '13].**

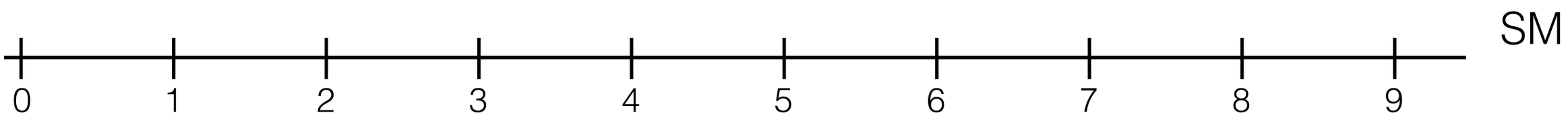
Quantum Gravity and Global Symmetries

- **Pheno application:** mini-charged DM

What if DM was not electrically neutral?

$$\mathcal{L} = -\frac{1}{4e} F_{\mu\nu} F^{\mu\nu} + A_\mu \left(J^\mu_{\rm SM} + \epsilon \, J^\mu_{\rm DM} \right)$$

- Experimental constraints: $\epsilon \lesssim 10^{-6}$
- The SM charges are quantized (in units of e). DM charges are proportional to $\epsilon \, e$
- Imagine ϵ is irrational: charges are **not quantized.**



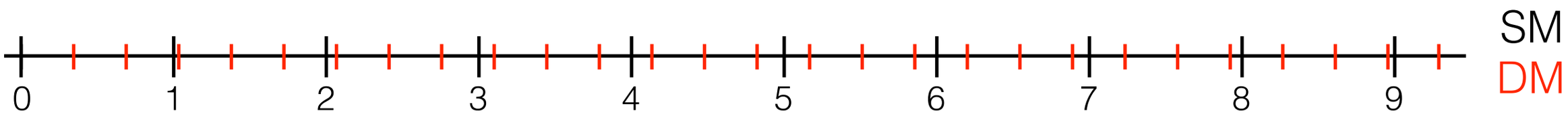
Quantum Gravity and Global Symmetries

- **Pheno application:** mini-charged DM

What if DM was not electrically neutral?

$$\mathcal{L} = -\frac{1}{4e} F_{\mu\nu} F^{\mu\nu} + A_\mu \left(J^\mu_{\rm SM} + \epsilon \, J^\mu_{\rm DM} \right)$$

- Experimental constraints: $\epsilon \lesssim 10^{-6}$
- The SM charges are quantized (in units of e). DM charges are proportional to $\epsilon \, e$
- Imagine ϵ is irrational: charges are **not quantized.**



Quantum Gravity and Global Symmetries

- **Pheno application:** mini-charged DM

What if DM was not electrically neutral?

$$\mathcal{L} = -\frac{1}{4e} F_{\mu\nu} F^{\mu\nu} + A_\mu \left(J^\mu_{\rm SM} + \epsilon\, J^\mu_{\rm DM} \right)$$

- Problem: the theory contains an **exact global symm.**
- Gauge symmetry:

$$A_\mu \to A_\mu + e\, \partial_\mu \alpha$$
$$\Psi_{\rm SM} \to e^{i\alpha}\, \Psi_{\rm SM}$$
$$\Psi_{\rm DM} \to e^{i\epsilon\alpha}\, \Psi_{\rm DM}$$

- Global symmetry:

$$A_\mu \to A_\mu$$
$$\Psi_{\rm SM} \to \Psi_{\rm SM}$$
$$\Psi_{\rm DM} \to e^{i\beta}\, \Psi_{\rm DM}$$

Every gauge invariant operator, is invariant under global β-transformations

Quantum Gravity and Global Symmetries

- **Pheno application:** mini-charged DM

What if DM was not electrically neutral?

$$\mathcal{L} = -\frac{1}{4e} F_{\mu\nu} F^{\mu\nu} + A_\mu \left(J^\mu_{\rm SM} + \epsilon\, J^\mu_{\rm DM} \right)$$

- Conclusion: simplest minicharged DM models $\in$ **Swampland**
- Possible ways out:
  - Small **rational** charges ("FCHAMP"): $\epsilon \in \mathbb{Q}$ Naturalness???
  - Gauge extra global symmetry $\Longrightarrow$ **New massless gauge boson!**

$$\mathcal{L} = -\frac{1}{4e} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4\tilde{e}} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + A_\mu \left(J^\mu_{\rm SM} + \epsilon\, J^\mu_{\rm DM} \right) + \tilde{A}_\mu\, J^\mu_{\rm DM}$$

Holdom '86; GS, Soler, Ye '13

The Weak Gravity Conjecture



The Weak Gravity Conjecture

- We have argued that global symmetries are in conflict with Quantum Gravity

- Global symmetry = gauge symmetry at g=0

  - It is not unreasonable to expect problems for gauge theories in the weak coupling limit: $g \to 0$

- When do things go wrong? How? …

The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa '06

- The conjecture:

"Gravity is the Weakest Force"

- For every long range gauge field there exists a particle of charge q and mass m, s.t.

$$\frac{q}{m} M_P \geq \text{“1”}$$

- Seems to hold for all known string theory models.

The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa '06

- The conjecture:

"Gravity is the Weakest Force"

- For every long range gauge field there exists a particle of charge q and mass m, s.t.

$$\frac{q}{m} M_P \geq \text{“1”} \equiv \frac{Q_{Ext}}{M_{Ext}} M_P$$

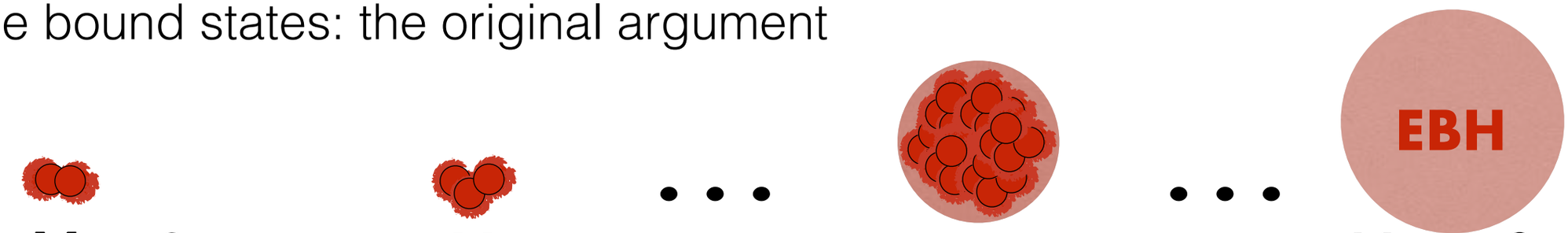
- Seems to hold for all known string theory models.

The Weak Gravity Conjecture

- Take U(1) gauge theory and a scalar with $m > q\,M_p$



- Stable bound states: the original argument



$2m > M_2 > 2q$ $\quad 3m > M_3 > 3q$ $\quad \cdots \quad Nm > M_N > Nq$ $\quad \cdots \quad M_\infty \to Q_\infty$

- All these BH states are **exactly stable**. In particular, large bound states (charged black holes) do not Hawking radiate once they reach the extremal limit M=Q, equiv. T=0.

> "...there should not exist a large number of exactly stable objects (extremal black holes) whose stability is not protected by any symmetries."

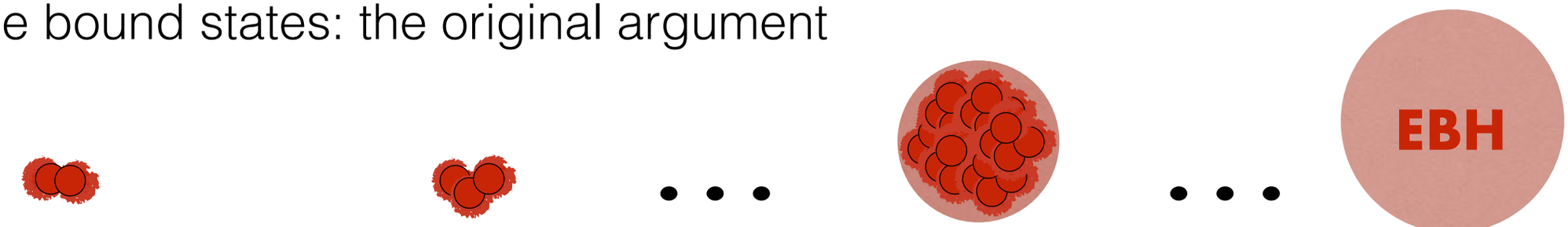
Arkani-Hamed et al. '06

The Weak Gravity Conjecture

- Take U(1) gauge theory and a scalar with $m > q\,M_p$



- Stable bound states: the original argument



$2m > M_2 > 2q$ $\quad 3m > M_3 > 3q$ $\quad \cdots \quad Nm > M_N > Nq$ $\quad \cdots \quad M_\infty \to Q_\infty$

- All these BH states are **exactly stable**. In particular, large bound states (charged black holes) do not Hawking radiate once they reach the extremal limit M=Q, equiv. T=0.

- In order to avoid a large number of exactly stable states one must demand the existence of some particle with

$$\frac{q}{m} \geq \frac{Q_{ext}}{M_{ext}} = \frac{1}{M_p}$$

Evidences for the WGC

Evidences for the Weak Gravity Conjecture

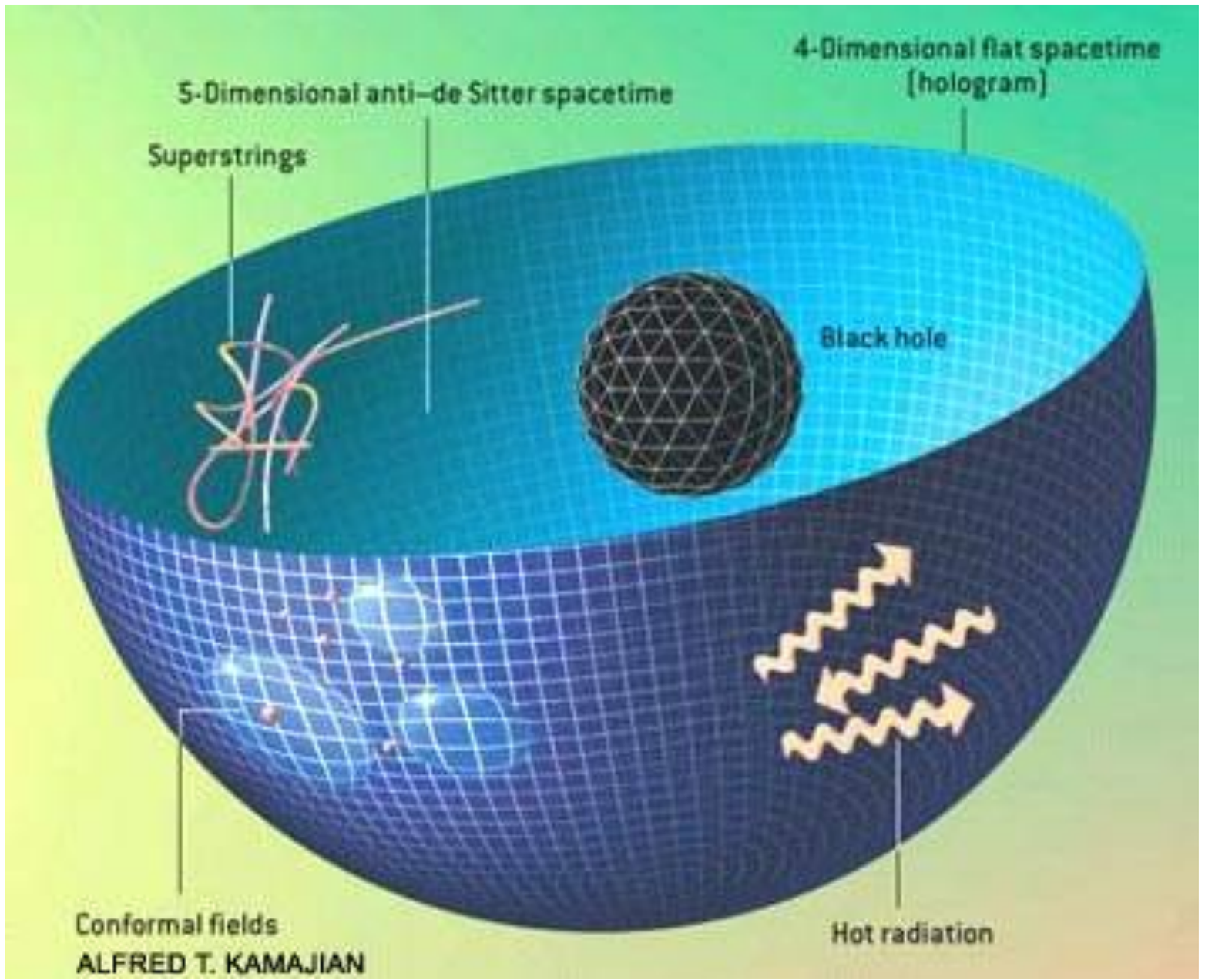
Several lines of argument have been taken (so far):

- Holography [Nakayama, Nomura, ’15];[Harlow, ‘15];[Benjamin, Dyer, Fitzpatrick, Kachru, ‘16];[Montero, GS, Soler, ’16];[Bae, Lee, Song, ’18];[Montero, ’18]
- Cosmic Censorship [Horowitz, Santos, Way, ’16];[Cottrell, GS, Soler, ’16];[Crisford, Horowitz, Santos, ’17]
- Entropy considerations [Cottrell, GS, Soler, ’16] [Fisher, Mogni, ’17]; [Cheung, Liu, Remmen, ’18];[Hamada, Noumi, GS, ‘18]
- IR Consistencies (unitarity & causality) [Cheung, Remmen, ’14] [Andriolo, Junghans, Noumi, GS,’18];[Hamada, Noumi, GS, ’18]

Evidences for *stronger* versions of the WGC:

- Consistencies with T-duality [Brown, Cottrell, GS, Soler, ‘15] and dimensional reduction [Heidenreich, Reece, Rudelius ’15].
- Modular invariance + charge quantization suggest a **sub-lattice WGC** [Montero, GS, Soler, ‘16] (see also [Heidenreich, Reece, Rudelius ’16])

WGC and Holography



WGC and Holography

- The WGC in AdS_{d+1} gives a bound on the conformal dimension of operators in the dual CFT_d.
- In particular, the **enhanced symmetry algebra** for **d=2 CFT** imposes strong constraints on the spectrum of operators.
- **Modular invariance** and **charge quantization** imply invariance under spectral flow:

$$L_0 \to L_0 + Q + \frac{k}{2}, \quad Q \to Q + k, \quad \tilde{L}_0 \to \tilde{L}_0$$

- Acting the spectral flow on the vacuum $L_0 = \tilde{L}_0 = Q = 0$ imply such a bound for a sub-lattice of charges (**sublattice WGC**).

$$\Delta \leq \frac{c}{12} + \frac{Q^2}{2k}$$ **[Montero, GS, Soler, '16]**

which has been verified using **modular bootstrap** **[Bae, Lee, Song, '18]**

Completeness Conjecture

Completeness Conjecture

- All charged states allowed by Dirac quantization are populated in theories that couple consistently with gravity **[Polchinski, ’03].**
- A heuristic argument can be made with charged black holes:

$$S = \frac{A}{4G}$$

- The BH entropy has a statistical interpretation of BH microstates. As the BH has arbitrary charge Q, all allowed charged states should exist in the spectrum **[Banks, Seiberg, ’10]; [Brennan, Carta, Vafa, ’17].**
- A less heuristic argument can be made by requiring factorization of correlators in AdS/CFT **[Harlow, ’15].**
- Make no reference to the masses of these states: may not be accessible in the low energy effective field theory.

Some WGC Generalizations

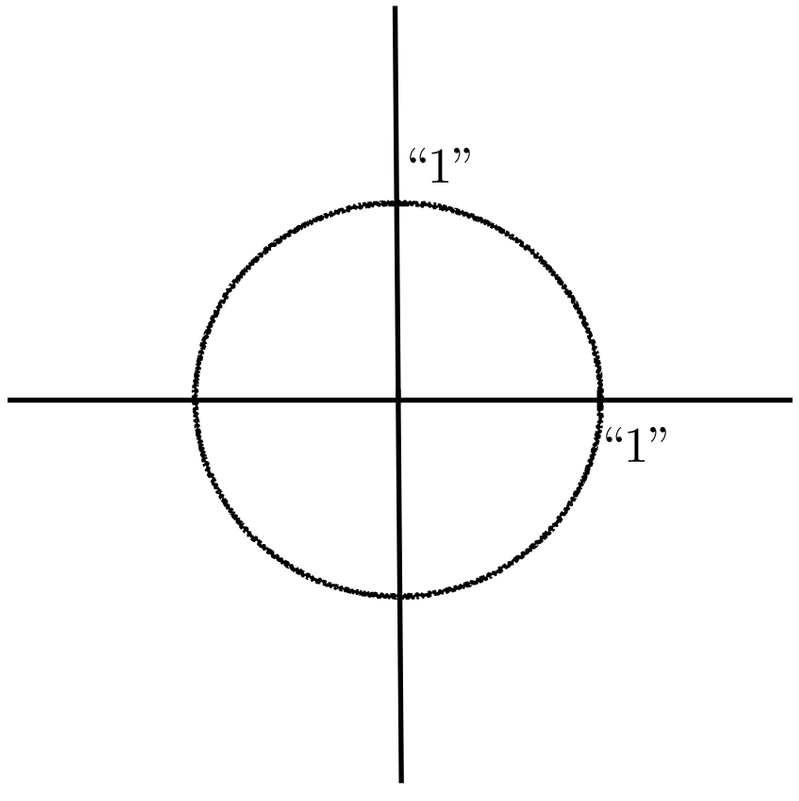
WGC for Multiple U(1)'s

[Cheung, Remmen '14];[Rudelius '15];[Brown, Cottrell, GS, Soler '15]

- Consider two U(1) bosons: there must be at least 2 particles i=1,2, so that BH's can decay (we will see how this generalizes to axions in Lecture 3).

$$\vec{z}_i \equiv \frac{M_P}{M_i}\begin{pmatrix} Q_i^1 & Q_i^2 \end{pmatrix}$$

$|\vec{z}_{EBH}| \equiv$ "1"



WGC

WGC for Multiple U(1)'s

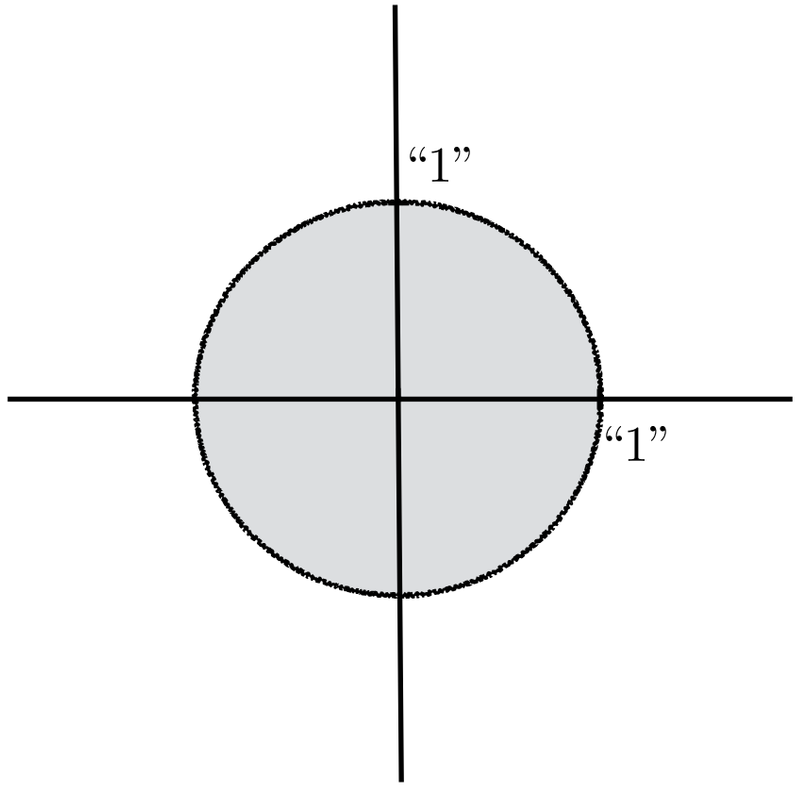
[Cheung, Remmen '14];[Rudelius '15];[Brown, Cottrell, GS, Soler '15]

- Consider two U(1) bosons: there must be at least 2 particles i=1,2, so that BH's can decay (we will see how this generalizes to axions in Lecture 3).

$$\vec{z}_i \equiv \frac{M_P}{M_i}\begin{pmatrix} Q_i^1 & Q_i^2 \end{pmatrix}$$

$$|\vec{z}_{EBH}| \equiv \text{“1”}$$

$$|\vec{z}_{BH}| \leq \text{“1”}$$



WGC

WGC for Multiple U(1)'s

[Cheung, Remmen '14];[Rudelius '15];[Brown, Cottrell, GS, Soler '15]

- Consider two U(1) bosons: there must be at least 2 particles i=1,2, so that BH's can decay (we will see how this generalizes to axions in Lecture 3).

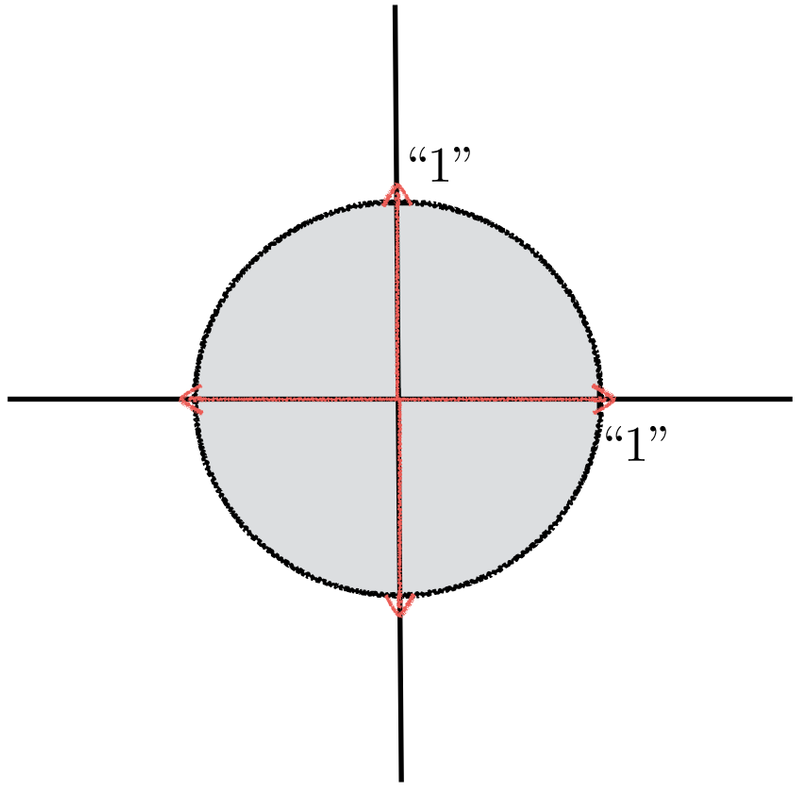
$$\vec{z}_i \equiv \frac{M_P}{M_i}\begin{pmatrix} Q_i^1 & Q_i^2 \end{pmatrix}$$

$$|\vec{z}_{EBH}| \equiv \text{“1”}$$

$$|\vec{z}_{BH}| \leq \text{“1”}$$

$$\vec{z}_{p1} \geq (\text{“1”} \;\; 0)$$

$$\vec{z}_{p2} \geq (0 \;\; \text{“1”})$$



WGC

WGC for Multiple U(1)'s

[Cheung, Remmen '14];[Rudelius '15];[Brown, Cottrell, GS, Soler '15]

- Consider two U(1) bosons: there must be at least 2 particles i=1,2, so that BH's can decay (we will see how this generalizes to axions in Lecture 3).

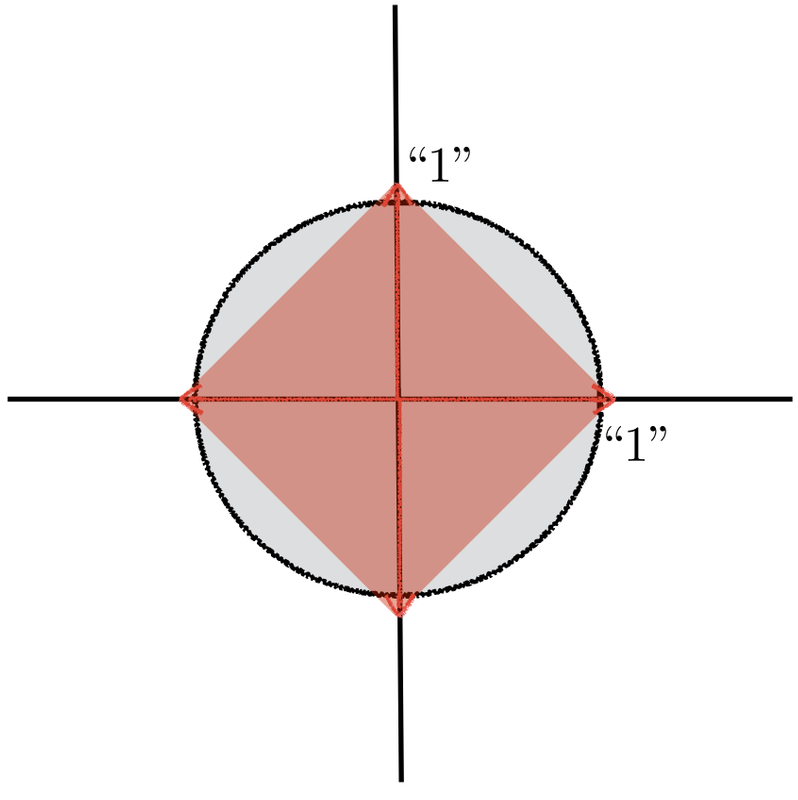
$$\vec{z}_i \equiv \frac{M_P}{M_i}\begin{pmatrix} Q_i^1 & Q_i^2 \end{pmatrix}$$

$$|\vec{z}_{EBH}| \equiv \text{“1”}$$

$$|\vec{z}_{BH}| \leq \text{“1”}$$

$$\vec{z}_{p1} \geq (\text{“1”} \;\; 0)$$

$$\vec{z}_{p2} \geq (0 \;\; \text{“1”})$$



WGC

WGC for Multiple U(1)'s

[Cheung, Remmen '14];[Rudelius '15];[Brown, Cottrell, GS, Soler '15]

- Consider two U(1) bosons: there must be at least 2 particles i=1,2, so that BH's can decay (we will see how this generalizes to axions in Lecture 3).

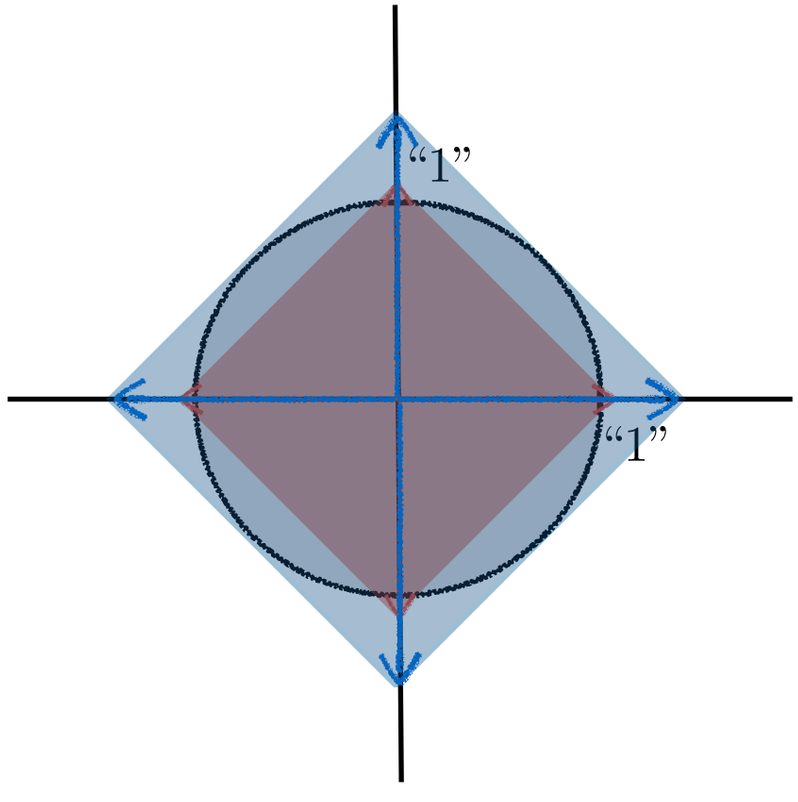
$$\vec{z}_i \equiv \frac{M_P}{M_i}\begin{pmatrix} Q_i^1 & Q_i^2 \end{pmatrix}$$

$$|\vec{z}_{EBH}| \equiv \text{“1”}$$

$$|\vec{z}_{BH}| \leq \text{“1”}$$

$$\vec{z}_{p1} \geq (\text{“1”} \;\; 0)$$

$$\vec{z}_{p2} \geq (0 \;\; \text{“1”})$$



WGC

$$\vec{z}_{p1} \geq (\text{“1”}\sqrt{2} \quad 0)$$

$$\vec{z}_{p2} \geq (0 \quad \text{“1”}\sqrt{2})$$

WGC for Multiple U(1)'s

[Cheung, Remmen '14];[Rudelius '15];[Brown, Cottrell, GS, Soler '15]

- Consider two U(1) bosons: there must be at least 2 particles i=1,2, so that BH's can decay (we will see how this generalizes to axions in Lecture 3).

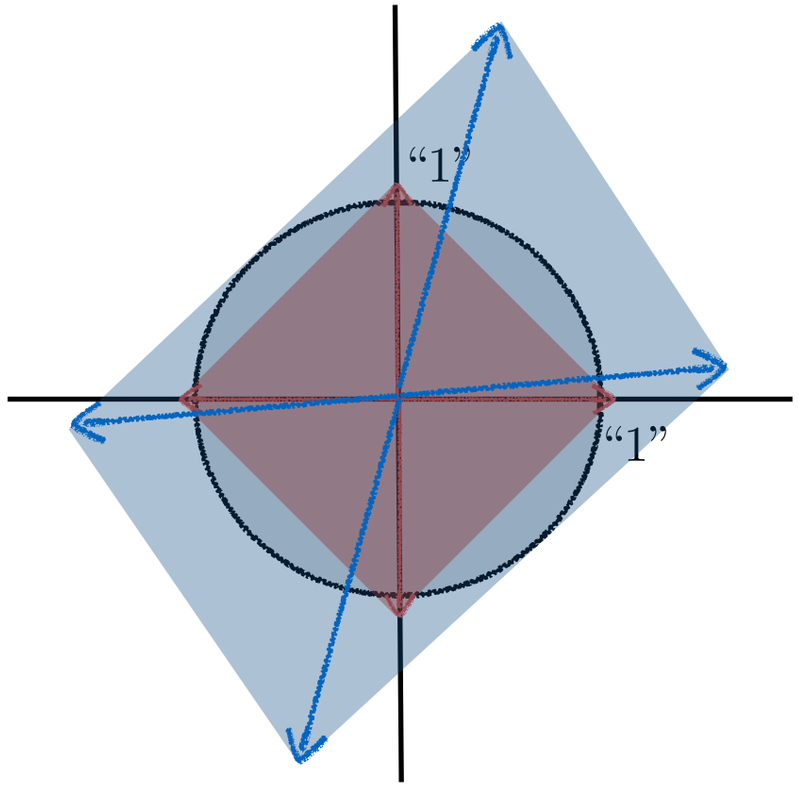
$$\vec{z}_i \equiv \frac{M_P}{M_i}\begin{pmatrix} Q_i^1 & Q_i^2 \end{pmatrix}$$

$$|\vec{z}_{EBH}| \equiv \text{“1”}$$

$$|\vec{z}_{BH}| \leq \text{“1”}$$

$$\vec{z}_{p1} \geq (\text{“1”} \;\; 0)$$

$$\vec{z}_{p2} \geq (0 \;\; \text{“1”})$$



WGC

$$\vec{z}_{p1} \geq (\text{“1”}\sqrt{2} \quad 0)$$

$$\vec{z}_{p2} \geq (0 \quad \text{“1”}\sqrt{2})$$

WGC for Multiple U(1)'s

[Cheung, Remmen '14];[Rudelius '15];[Brown, Cottrell, GS, Soler '15]

- Consider two U(1) bosons: there must be at least 2 particles i=1,2, so that BH's can decay (we will see how this generalizes to axions in Lecture 3).

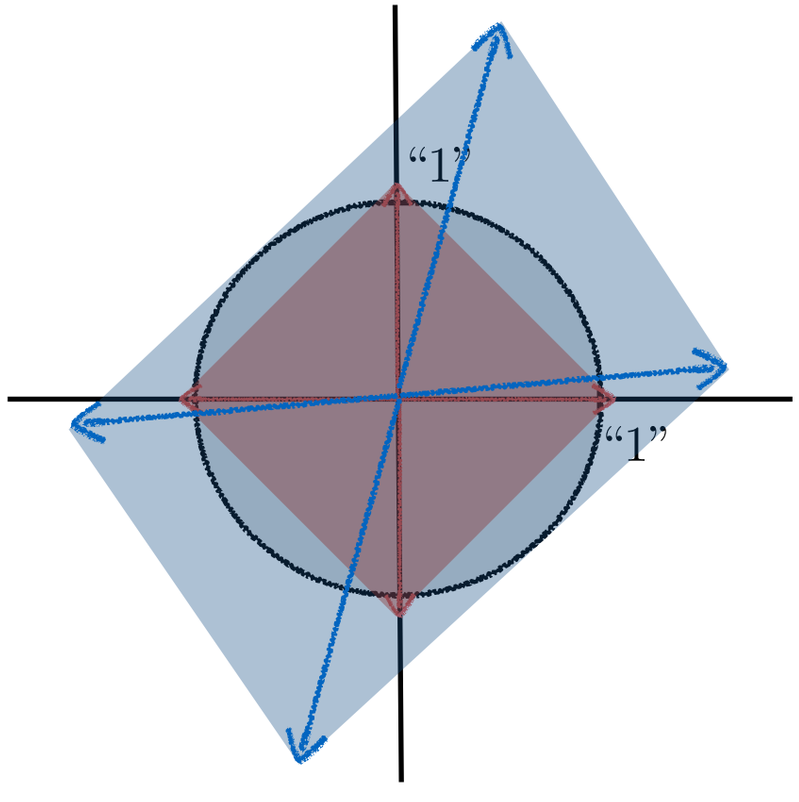
$$\vec{z}_i \equiv \frac{M_P}{M_i}\begin{pmatrix} Q_i^1 & Q_i^2 \end{pmatrix}$$

$$|\vec{z}_{EBH}| \equiv \text{“1”}$$

$$|\vec{z}_{BH}| \leq \text{“1”}$$

$$\vec{z}_{p1} \geq (\text{“1”} \;\; 0)$$

$$\vec{z}_{p2} \geq (0 \;\; \text{“1”})$$



WGC

$$|\vec{z}| = \text{“1”}$$

$$\cap$$

$$\text{Convex Hull}\{\vec{z}_{p1}, \vec{z}_{p2}\}$$

WGC for p-branes

- The WGC is about the existence of **super-extremal** objects, and should have analogues for objects charged under p-forms in d-dim. Consider:

$$S = \frac{1}{2\kappa_d^2}\int d^dx\sqrt{-g}\left(\mathcal{R}_d - \frac{1}{2}(\nabla\phi)^2\right) - \frac{1}{2e_{p;d}^2}\int d^dx\sqrt{-g}e^{-\alpha_{p;d}\phi}F_{p+1}^2\,.$$

  which includes the **dilation** as it appears in string theory & KK reduction.

- Considering **charged black p-brane solution** leads to the WGC:

$$\left[\frac{\alpha^2}{2} + \frac{p(d-p-2)}{d-2}\right]T_p^2 \leq e_{p;d}^2 q^2 M_d^{d-2}.$$

- While this extremality bound is only valid for $1 \leq p \leq d-3$, it is interesting to examine the behavior for all p if we naively extrapolate it.

- For α=0, the extremality bound becomes degenerate for p=0 (**instantons**) and p=d-2 (**D7-branes in 10d**).

- We will discuss in Lecture 3 the interesting case of the WGC for 0-forms (axions) whose charged objects are instantons (where $\alpha \neq 0$ naturally).

WGC and KK Reduction

- Is the WGC robust against dimensional reduction?
- Compactify to **d= D-1 dimensions** on a circle of radius R:

$$ds^2 = e^{\frac{\lambda(x)}{d-2}} d\hat{s}^2(x) + e^{-\lambda(x)} dy^2.$$

- Then the dimensionally reduced action:

$$S = \int d^d x \sqrt{-\hat{g}} \left[\frac{1}{2\kappa_d^2} \left(\hat{R}_d - \frac{1}{2}(\nabla\phi)^2 - \frac{d-1}{4(d-2)}(\nabla\lambda)^2 \right) - \frac{1}{2e_{p,d}^2} e^{-a\phi - b\lambda} F_{p+1}^2 \right]$$

where $(a,b) = (\alpha_{p,D}, \frac{p}{d-2})$ for $p = P$ **reduction ⊥ brane**

$(a,b) = (\alpha_{P,D}, \frac{p+2-d}{d-2})$ for $p = P-1$ **reduction // brane**

- A linear combination of ϕ and λ is decoupled from the p-form gauge field. The orthogonal combination, when canonically normalized (call it ρ) has an effective coupling $e^{-\alpha_{p;d}\rho} F_{p+1}^2$

WGC and KK Reduction

- In both cases:

$$\frac{\alpha_{p;d}^2}{2}+\frac{p(d-p-2)}{d-2}=\frac{\alpha_{p;D}^2}{2}+\frac{P(D-P-2)}{D-2}$$

- Taking into account the R dependences of

$$M_d^{d-2}=(2\pi R)M_D^{D-2} \quad \text{and} \quad \begin{cases} \dfrac{1}{e_{p;d}^2}=\dfrac{2\pi R}{e_{p;D}^2}, & T_p=T_P \quad \text{for } p=P \\ e_{p;d}^2=(2\pi R)e_{P,D}^2, & T_p=(2\pi R)T_P \quad \text{for } p=P-1 \end{cases}$$

- We see that the WGC is **stable under dimensional reduction:**

$$\left[\frac{\alpha^2}{2}+\frac{p(d-p-2)}{d-2}\right]T_p^2\leq e_{p;d}^2q^2M_d^{d-2}.$$

- Moreover, if the dilation or the radion acquires a mass (setting α=0), the WGC bound becomes monotonically weaker in the IR.

WGC and KK Photons

- KK reduction also leads to a KK photon. Consider reduction on a circle:

$$ds^2 = e^{\frac{\lambda(x)}{d-2}} d\hat{s}^2(x) + e^{-\lambda(x)}(dy + RB_1)^2$$

- The dimensionally reduced action:

$$S = \frac{1}{2\kappa_d^2}\int d^dx\sqrt{-\hat{g}}\left[\hat{\mathcal{R}}_d - \frac{d-1}{4(d-2)}(\nabla\lambda)^2 - \frac{R^2}{2}e^{-\frac{d-1}{d-2}\lambda}H_2^2\right], \qquad \text{where} \qquad H_2 = dB_1.$$

- We can read off the couplings of the canonically normalized fields:

$$\frac{1}{e_{\rm KK}^2} = \frac{1}{2}R^2M_d^{d-2} \quad , \quad \alpha_{\rm KK} = \sqrt{\frac{2(d-1)}{d-2}}$$

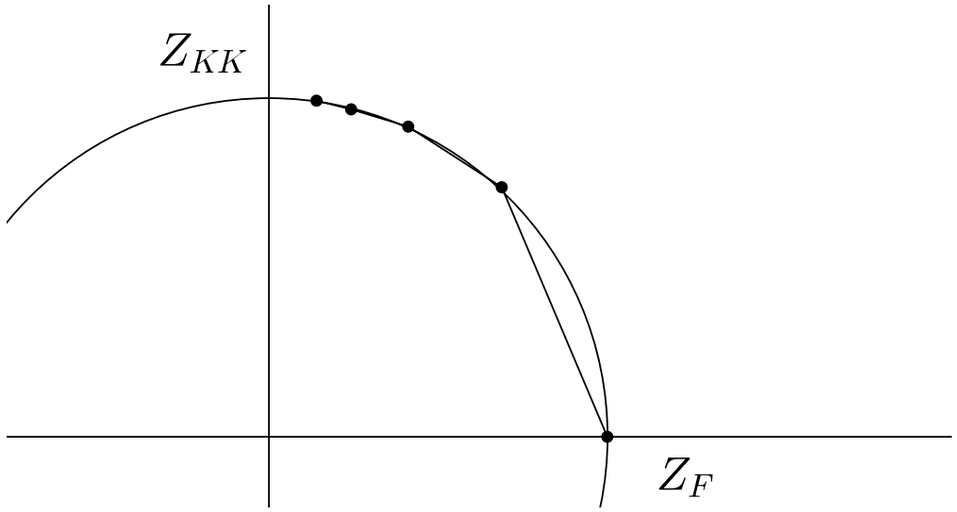
- The WGC for KK photons:

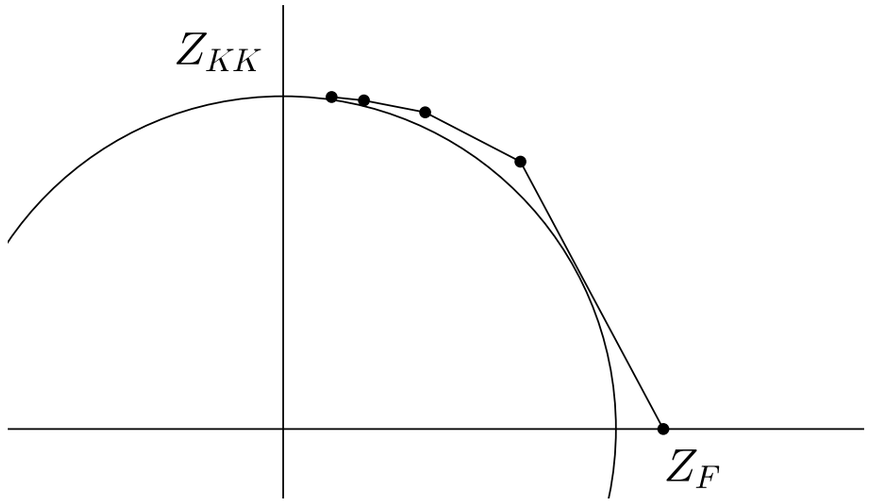
$$\left[\frac{\alpha_{\rm KK}^2}{2} + \frac{d-3}{d-2}\right]m^2 \leq e_{\rm KK}^2q^2M_d^{d-2} \qquad \text{becomes} \qquad m^2 \leq \frac{q^2}{R^2}$$

- thus the KK modes of B **saturate the WGC bound** (no surprise).

KK Photon with Other U(1)'s

- If the BH carries charges under both the KK U(1) and another U(1).





- The KK modes only populate a finite density of points, and so for if R is too small, the convex hull condition is violated.
- This led **[Heidenreich, Reece, Rudelius '15]** to conjecture:

The Lattice Weak Gravity Conjecture (LWGC): For every point $\vec{Q}$ on the charge lattice, there is a particle of charge $\vec{Q}$ with charge-to-mass ratio at least as large as that of a large, semi-classical, non-rotating extremal black hole with charge $\vec{Q}_{\rm BH} \propto \vec{Q}$.

(Sub)Lattice WGC

- This stronger form of the WGC implies the convex hull condition, and is preserved under toroidal compactification on a circle.
- The **Lattice WGC** turns out to be **false** **[Montero, GS, Soler, '16]; [Heidenreich, Reece, Rudelius '16]**. Rather, what seem to survive scrutiny is a weaker form known as **sub lattice WGC.**

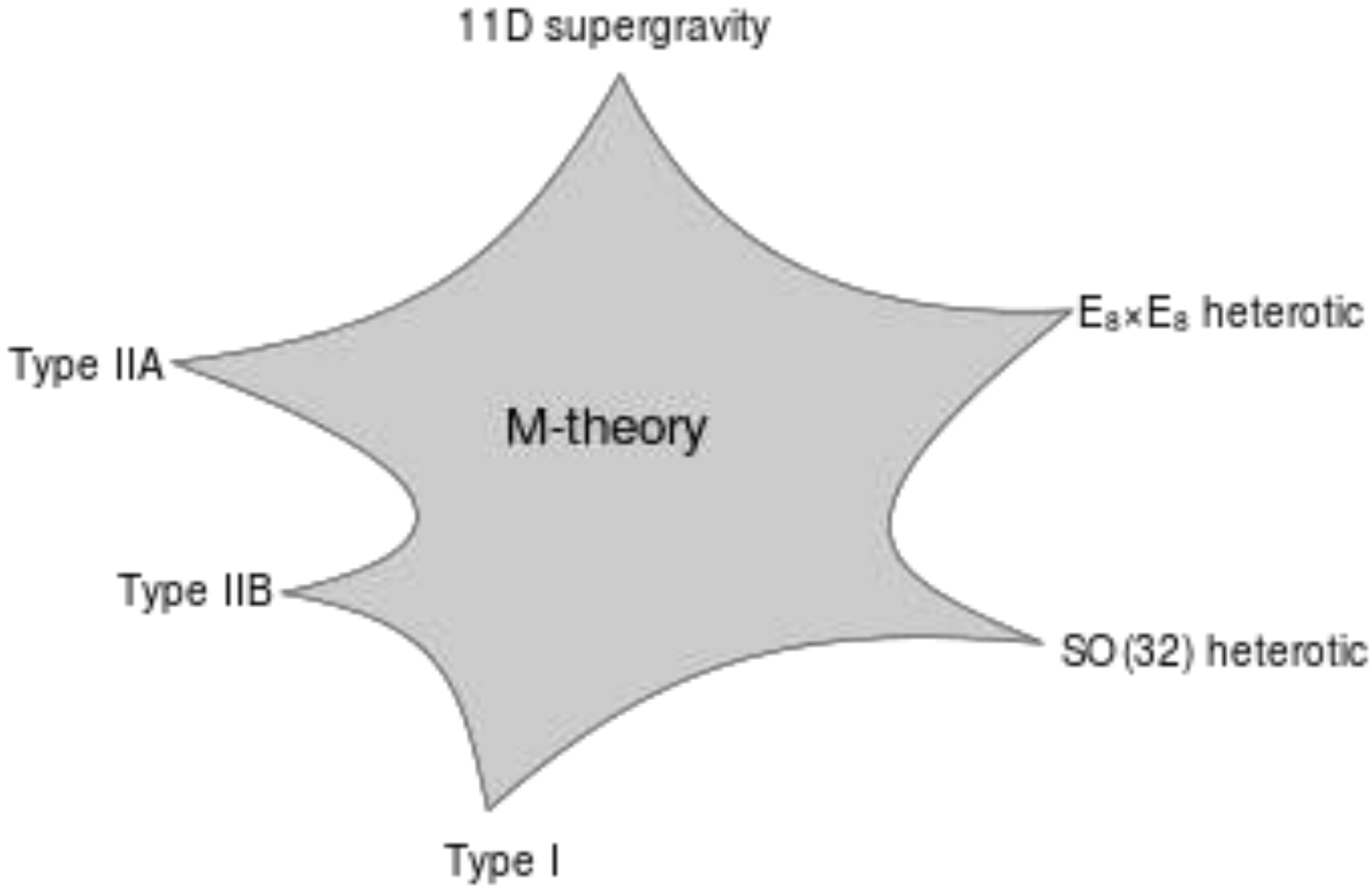
The Sublattice Weak Gravity Conjecture (sLWGC): For a theory with charge lattice Γ, there exists a sublattice $\Gamma_{\rm ext} \subseteq \Gamma$ of finite coarseness such that for each $\vec{q} \in \Gamma_{\rm ext}$, there is a (possibly unstable) superextremal particle of charge $\vec{q}$.

- We have already seen the argument for this sLWGC using holography **[Montero, GS, Soler, '16]** where the index of the sub lattice is level k.
- One can derive the sLWGC for perturbative string theory using again spectral flow on the worldsheet CFT **[Heidenreich, Reece, Rudelius '16]**.
- We will see in Lecture 2 that a similar strong form known as **tower WGC** follows from unitarity and causality **[Andriolo, Junghans, Noumi, GS,'18].**

Swampland Distance Conjecture

Maximum Field Range

- In theories with gravity, there seems to be a maximum field range beyond which a given effective description breaks down, e.g.

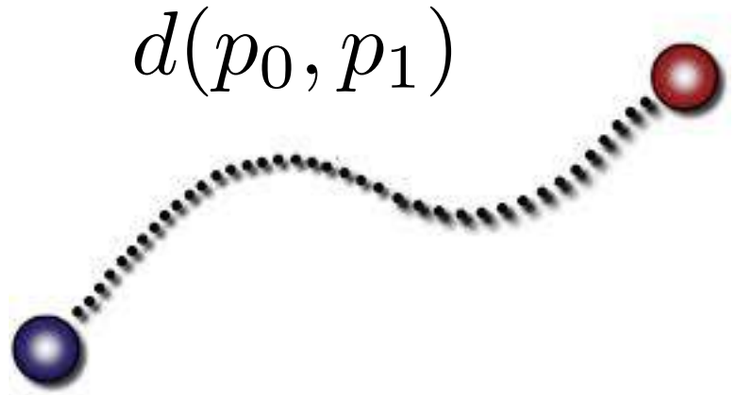


For large Δϕ, new light states emerge, invalidating the EFT.

Swampland Distance Conjecture(s)

- **Swampland Distance Conjecture [Ooguri, Vafa]:**

  In a consistent quantum theory of gravity, for any point p_0 in the field space,∃ point p_1 arbitrarily far away. As the shortest geodesic $d(p_0, p_1)$ diverges, a tower of lights states with m:



$$m \sim m_0 e^{-\lambda d(p_0, p_1)}$$

$$\text{EFT breaks down for } d(p_0, p_1) > \lambda^{-1}$$

- **Refined Swampland Conjecture [Klawer, Palti]:** $\lambda \sim \mathcal{O}(1)$
  more about this later **[Landete, GS] (in Lecture 3)**

Moduli Space Metric

- The EFT of a string compactification contains many scalar fields $\mathbf{\Phi}_i$ whose vevs $\langle \mathbf{\Phi}_i \rangle$ describes the geometry of the compact manifold.
- The algebraic variety parametrized by $\langle \mathbf{\Phi}_i \rangle$ is the **moduli space** $\mathcal{M}$.
- The kinetic term for the scalar fields typically takes the form:

$$\mathcal{L}_{eff} = g_{ij}(\Phi)\partial_\mu \Phi^i \partial_\mu \Phi^j + \dots \,,$$

- We use this metric $\mathbf{g}_{ij}$ to compute distances in the moduli space.
- In known examples, the moduli space in non-compact **[Ooguri, Vafa, '06].**
- Consider the axio-dilaton τ whose moduli space is the fundamental domain of SL(2,Z), the moduli space metric:

$$ds^2 = \frac{d\tau d\bar{\tau}}{(\mathrm{Im}\tau)^2}$$

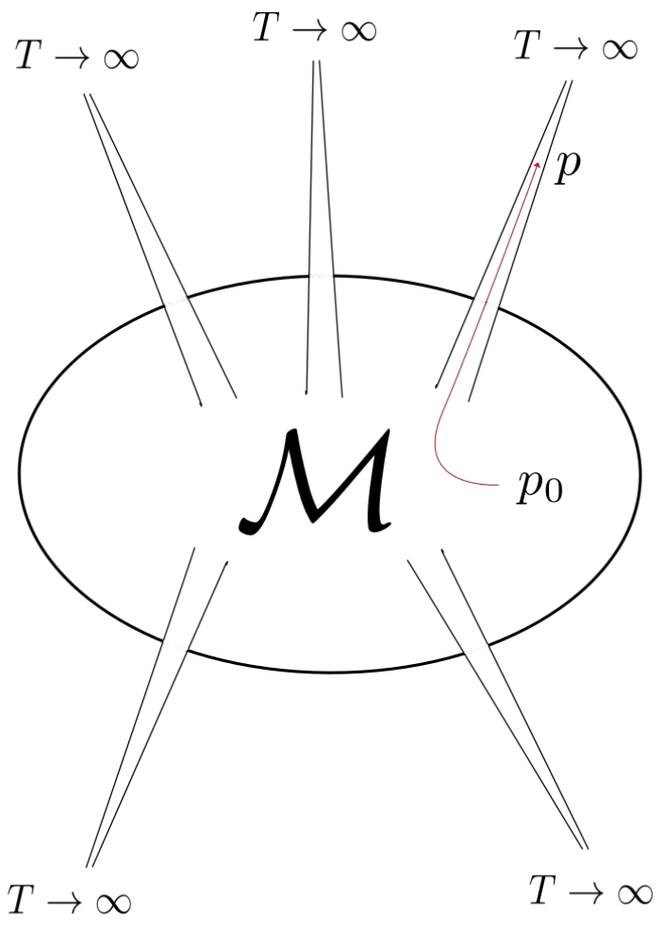
Boundary of Moduli Space

- The geodesic distance indeed diverges logarithmically as $\tau \to i\infty$:

$$T \sim \log(\mathrm{Im}\ \tau / \mathrm{Im}\ \tau_0)$$

- Light stringy states (D-branes) appear at the boundary of moduli space.
- More generally, going to infinities corresponds to going to a singularity where new massless dofs appear.

The new dofs can be
massless particles,
or tensionless strings/branes.



Tower of Light States

- Consider compactification on a circle:

$$\mathcal{L}_{eff} = \int \left(\frac{dR}{R}\right)^2 + ...$$

- The moduli space is 1D. Using the moduli space metric:

$$\int_{R_0}^{\tilde{R}} \frac{dR}{R} = \log(\tilde{R}) - \log(R_0)$$

- The moduli space is **non-compact**. As we go to infinite distances:

$$m \sim \frac{1}{\tilde{R}}$$

- A tower of KK modes becomes light. In terms of the canonically normalized field T, the KK modes are **exponentially light**:

$$m \sim e^{-T}$$

Extended Objects

- *The SDC also implies that a consistent theory of quantum gravity must have* ***extended objects (strings, branes)*** *in the spectrum.*
- Instead of going to infinite radius, consider R→0:

$$\lim_{\tilde{R}\to 0}\int_{\tilde{R}}^{R_0}\frac{dR}{R}$$

- The geodesic distance diverges as well, but all the particle states are KK states which become massive as R→0.
- The only way to get light states as R→0 is to have extended objects winding around the circle which become light in this limit.
- Other than strings wrapping around a circle giving light particles, there are other light states. For example, M2-branes wrapping a R→0 circle become light string states in 10d.

Summary

Summary of Lecture 1

- In this lecture, I have introduced several swampland conjectures:
  - **No global symmetry**
  - **Completeness Conjecture**
  - **Weak Gravity Conjecture**
    - Convex hull condition
    - Sub-lattice/Tower Weak Gravity Conjecture
  - **Swampland Distance Conjecture**

  and presented some **evidences** and **applications** of these conjectures.
- As we will see, these conjectures are related in such intricate way.

Summary of Lecture 1

- Plan for the next few lectures:
- ***Lecture 2:*** Proof of the (mild form of the) WGC from unitarity and causality and the tower WGC.
- ***Lecture 3:*** Applications to inflation and particle physics.
- ***Lecture 4:*** de Sitter vacua in string theory and the Swampland.

Summary of Lecture 1

- Plan for the next few lectures:
- ***Lecture 2:*** Proof of the (mild form of the) WGC from unitarity and causality and the tower WGC.
- ***Lecture 3:*** Applications to inflation and particle physics.
- ***Lecture 4:*** de Sitter vacua in string theory and the Swampland.

