Aspects of Janus ABJM Models with Mass Deformation

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In collaboration with
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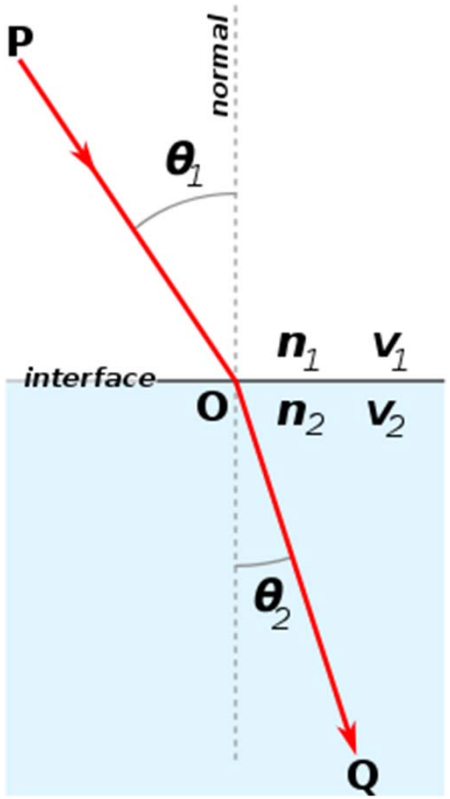
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and work in progress

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Outline

- Motivation and introduction
- Mass-deformed ABJM theory and Janus deformations
- Gravity dual of the Janus ABJM with mass deformation
- Summary

Motivation/Introduction



- Physics on clean systems is easy, but defects and interfaces are everywhere.
- Many interesting physics come from defects or interfaces, for instance, Josephson Junction, Edge currents, varying dielectric constant in condensed matter physics, D-branes,…
- In QFT??

Snell's law: $\frac{\mathrm{Sin}\theta_1}{\mathrm{Sin}\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$

- In usual quantum field theory, we consider a homogeneous background (Minkowski vacuum), for example, 4-dimensional Yang-Mills theory:

$$S = \int d^4x \left(-\frac{1}{4{g_{\mathrm{YM}}}^2} \eta^{\mu\nu}\eta^{\rho\sigma} F_{\mu\rho}F_{\nu\sigma} \right)$$

- QFT in a curved space − time

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{4{g_{\mathrm{YM}}}^2(x)} g^{\mu\nu}g^{\rho\sigma} F_{\mu\rho}F_{\nu\sigma} \right)$$

$$\left(\text{gravitation } \eta_{\mu\nu} \longrightarrow g_{\mu\nu}(x)\right)$$

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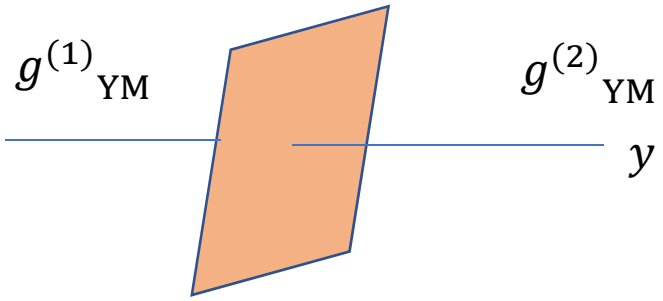
- Spacetime − dependent coupling constant

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{4{g_{\mathrm{YM}}}^2(x)} g^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} \right)$$

$${g_{\mathrm{YM}}}^2 \longrightarrow \mathbf{{g_{YM}}^2(x)} \text{ (Janus field theory)}$$

Janus $\mathcal{N} = 4$ SU(N) super Yang – Mills theory

$$S_{\text{Janus}} = \int_{-\infty}^{0} \mathrm{dy} \int d^3x \mathcal{L}_{\text{SYM}}\left(g^{(1)}{}_{\text{YM}}\right) + \int_{y=0} \int d^3x \mathcal{L}_{\text{interface}} + \int_{0}^{\infty} \mathrm{dy} \int d^3x \mathcal{L}_{\text{SYM}}\left(g^{(2)}{}_{\text{YM}}\right)$$



$(2+1)-$ dimensional interface

- $\mathcal{N} = 0, 1, 2, 4$ **interface supersymmetries**
  [D'Hoker-Estes-Gutperle 2006][Kim-Koh-Lee 2008]

- Supersymmetry is helpful in quantitative comparison in AdS/CFT correspondence

- Dual geometry: asymptotically AdS space deformed by a spacetime dependent massless scalar (dilaton) $\phi_0 \longrightarrow \phi(x)$

  [Bak – Gutperle – Hirano 2003]

- $\mathcal{N} = 6, U_k(N) \times U_{-k}(N)$ **ABJM theory** $\Leftrightarrow$ 11dim. SUGRA on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

  or 10dim. IIA SUGRA on $\mathrm{AdS}_4 \times \mathrm{CP}_3$

  - ABJM theory is a Chern-Simons matter theory with coupling constant $\frac{N}{k}$
  - Original ABJM theory cannot have different coupling constants along spacetime ➔ Janus deformation is not possible
  - Supersymmetry preserving mass deformation is possible!
  - Mass is a self-interacting coupling constant in QFT and a continuous number. ➔ Janus deformation is possible $m_0 \longrightarrow m(x)$
  - **Janus deformation of the massive ABJM theory and its gravity dual**

Mass-deformed ABJM

- N=6 Aharony-Bergman-Jafferis-Maldacena(ABJM) theory : low energy effective action of N coincident M2-branes on the C^4/Z_k orbifold fixed point

***N* =6 ABJM** → **M-theory on** $\mathrm{AdS}_4\times\mathrm{S}^7/\mathrm{Z}_k$

dual gravity(large N)

$$S=\int d^3x\,\left(\mathcal{L}_0+\mathcal{L}_{\mathrm{CS}}-V_{\mathrm{ferm}}-V_{\mathrm{bos}}\right)$$

Mass-deformed ABJM

- N=6 Aharony-Bergman-Jafferis-Maldacena(ABJM) theory : low energy effective action of N coincident M2-branes on the C^4/Z_k orbifold fixed point

N =6 ABJM

4 complex scalars Y^A → become massive

4 complex fermions ψ_A → become massive

2 Chern − Simons terms $A_\mu, \hat{A}_\mu$

SUSY preserving mass deformation

N =6 mABJM

[Hosomichi et al 08]
[Gomis et al 08]

→ Effective action of M2-branes in the presence of the transverse self-dual 4-form flux

Janus ABJM with mass deformation

Introduce
***a space − dependent mass parameter*: $m(x)$**



What is the maximal supersymmetry
and possible form of $m(x)$?

- supersymmetric transformation rule of the original ABJM:

$$
\begin{aligned}
&\delta_1 Y^A = i\omega^{\mathrm{AB}}\psi_B, \qquad \delta_1 Y_A^\dagger = i\psi^{\dagger B}\omega_{AB},\\
&\delta_1\psi_A = \gamma^\mu\omega_{AB}D_\mu Y^B, \qquad \delta_1\psi^{\dagger A} = -D_\mu Y_B^\dagger\omega^{AB}\gamma^\mu,\\
&\delta_2\psi_A = \frac{2\pi}{k}\omega_{AB}\left(Y^B Y_C^\dagger Y^C - Y^C Y_C^\dagger Y^B\right) + \frac{4\pi}{k}\omega_{BC}Y^B Y_A^\dagger Y^C,\\
&\delta_2\psi^{\dagger A} = \frac{2\pi}{k}\omega^{AB}\left(Y_C^\dagger Y^C Y_B^\dagger - Y_B^\dagger Y^C Y_C^\dagger\right) - \frac{4\pi}{k}\omega^{BC}Y_B^\dagger Y^A Y_C^\dagger,\\
&\delta_A A_\mu = -\frac{2\pi}{k}\left(Y^A\psi^{\dagger B}\gamma_\mu\omega_{AB} + \omega^{AB}Y_A^\dagger\gamma_\mu\psi_B\right),\\
&\delta_A\hat{A}_\mu = -\frac{2\pi}{k}\left(\psi^{\dagger B}\gamma_\mu Y^A\omega_{AB} + \omega^{AB}Y_A^\dagger\gamma_\mu\psi_B\right).
\end{aligned}
$$

- Supersymmetric structure of ABJM:

$$
\begin{aligned}
&\delta_1\mathcal{L}_0 + \delta\mathcal{L}_{\mathrm{CS}} = 0,\\
&\delta_A\mathcal{L}_0 + \delta_2\mathcal{L}_0 = \delta_1 V_{\mathrm{ferm}},\\
&\delta_2 V_{\mathrm{ferm}} + \delta_1 V_{\mathrm{bos}} = 0,
\end{aligned}
$$

- additional transformation for fermion fields:

$$\delta_J \psi_A = \mu_A^B(x)\omega_{BC}Y^C,$$
$$\delta_J \psi^{\dagger A} = \mu_B^A(x)\omega^{BC}Y_C^\dagger,$$

- Supersymmetric transformation

$$\delta_{\rm tot}\left(\mathcal{L}_0 + \mathcal{L}_{\rm CS} - V_{\rm ferm} - \hat{V}_{\rm ferm} - V_{\rm bos} - \hat{V}_{\rm flux} - \hat{V}_{\rm mass}\right)$$
$$= -i\left(\partial_\mu \mu_B^A\right)\omega^{BC}Y_C^\dagger\gamma^\mu\psi_A + i\left(\partial_\mu\mu_A^B\right)\psi^{\dagger A}\gamma^\mu\omega_{BC}Y^C$$

$$\hat{V}_{\rm ferm} = i\mu_A^B\psi^{\dagger A}\psi_B$$
$$\hat{V}_{\rm flux} = -\frac{4\pi}{k}\mu_A^B Y_B^\dagger\left(Y^A Y_C^\dagger Y^C - Y^C Y_C^\dagger Y^A\right)$$
$$\hat{V}_{\rm mass} = M_A^B Y_B^\dagger Y^A$$

- Conditions: $\mu_A^A = 0, \qquad \mu_A^B\mu_B^C\omega_{CD} - M_D^B\omega_{AB} = 0$

$\mathcal{N} = 6$ mass-deformed ABJM with constant mass m:

$$\mu_A^B = \mathrm{diag}(m, m, -m, -m),$$
$$M_A^B = \mathrm{diag}(m^2, m^2, m^2, m^2)$$

$\mathcal{N}=1$ Janus ABJM [C.Kim, K.Kim, Y.Kim, OK, work in progress]

$$\mu_A^B = \mathrm{diag}(m_1, m_2, m_3, m_4) \quad \text{with } m_1+m_2+m_3+m_4=0$$

$$M_A^B = \mathrm{diag}(m_2^2, m_1^2, m_4^2, m_3^2),$$

$$\omega_{12} \neq 0, \quad \omega_{13} = \omega_{14} = 0 \quad \rightarrow \quad \mathcal{N}=2 \text{ supersymmetry.}$$

$$\delta_{\mathrm{tot}} \left(\mathcal{L}_0 + \mathcal{L}_{\mathrm{CS}} - V_{\mathrm{ferm}} - \hat{V}_{\mathrm{ferm}} - V_{\mathrm{bos}} - \hat{V}_{\mathrm{flux}} - \hat{V}_{\mathrm{mass}} \right)$$

$$= -i \left(\partial_\mu \mu_B^A\right) \omega^{BC} Y_C^\dagger \gamma^\mu \psi_A + i \left(\partial_\mu \mu_A^B\right) \psi^{\dagger A} \gamma^\mu \omega_{BC} Y^C$$

$$m_i = m_i(x)$$

$$\gamma^1 \omega_{12} = -\omega_{12}, \quad \gamma^1 \omega_{34} = -\omega_{34} \qquad \mathcal{N}=1 \text{ Janus ABJM}$$

$$\hat{V}_J = m_1' Y_2^\dagger Y^2 + + m_2' Y_1^\dagger Y^1 + m_3' Y_4^\dagger Y^4 + m_4' Y_3^\dagger Y^3$$

$\mathcal{N}=2$ Janus ABJM [C.Kim, K.Kim, Y.Kim, OK, work in progress]

$$\mu_A^B = \mathrm{diag}(m_1, -m_1, m_2, -m_2),$$
$$M_A^B = \mathrm{diag}(m_2^2, m_2^2, m_1^2, m_1^2).$$

$\omega_{12} = 0, \quad \omega_{13} \neq 0 \ \& \ \omega_{14} \neq 0$ $\quad\quad \mathcal{N} = 4$ supersymmetry.

$$\gamma^1 \omega_{13} = \omega_{13}, \quad \gamma^1 \omega_{14} = -\omega_{14}$$

$$\hat{V}_J = -m_1' \delta(Y_3^\dagger Y^3 - Y_4^\dagger Y^4) - m_2' \delta(Y_1^\dagger Y^1 - Y_2^\dagger Y^2)$$

$\mathcal{N} = 3$ Janus ABJM [K.Kim, OK 2018]

$$\mu_A^B = \mathrm{diag}(m, m, -m, -m),$$
$$M_A^B = \mathrm{diag}(m^2, m^2, m^2, m^2)$$

No restriction on $\omega_{AB} \Longrightarrow \mathcal{N} = 6$ supersymmetry

- Reduction of supersymmetry $\mathcal{N} = 6 \longrightarrow \mathcal{N} = 3$

$$\begin{aligned} \gamma^1 \omega_{ab} = -\omega_{ab} \quad &\Longleftrightarrow \quad \omega^{ab}\gamma^1 = \omega^{ab}, \\ \gamma^1 \omega_{ai} = \omega_{ai} \quad &\Longleftrightarrow \quad \omega^{ai}\gamma^1 = -\omega^{ai}, \end{aligned} \qquad a = 1, 2 \text{ and } i = 3, 4,$$

- Deformation of the Lagrangian:

$$-\frac{4\pi m}{k} M_B^{\ D} \left(Y_C^\dagger Y^C Y_D^\dagger Y^B - Y^C Y_C^\dagger Y^B Y_D^\dagger \right) \qquad : \textbf{flux term}$$

$$+ \left(m^2 \delta_A^B + \boxed{m' M_A^{\ B}} \right) Y^A Y_B^\dagger \qquad : \textbf{mass term}$$

- Shape of the mass function: **m = m(x)**

⇒ **arbitrary** function but it depends on only **one spatial coordinate**

- M-theory origin of the Janus mass deformation:
  space-dependent 4-form field strength (but we don't discuss here)

Vacuum solution (N=3 Janus)

[C. Kim, K. Kim,Y. Kim, OK]

The energy expression is given by

$$E = \int d^2x \left(|D_0 Y^A|^2 + |D_i Y^A|^2 + V_{\rm bos} \right)$$

$$E = \int d^2x \left(|Y^{A'}|^2 + V_{\rm bos} \right)$$

$$= \int d^2x \left[\frac{2}{3} \left| \delta_A^{[B} Y^{C]\prime} + s^{BC} G_A^{BC} \right|^2 + \left(\frac{1}{2} \beta_a^{ac} Y_c^\dagger + \frac{1}{2} \beta_i^{ik} Y_k^\dagger - \frac{1}{3} \beta_i^{ic} Y_c^\dagger + m M_A{}^B Y_B^\dagger Y^A \right)' \right]$$

$$= \frac{2}{3} \int d^2x \left| \delta_A^{[B} Y^{C]\prime} + s^{BC} G_A^{BC} \right|^2$$

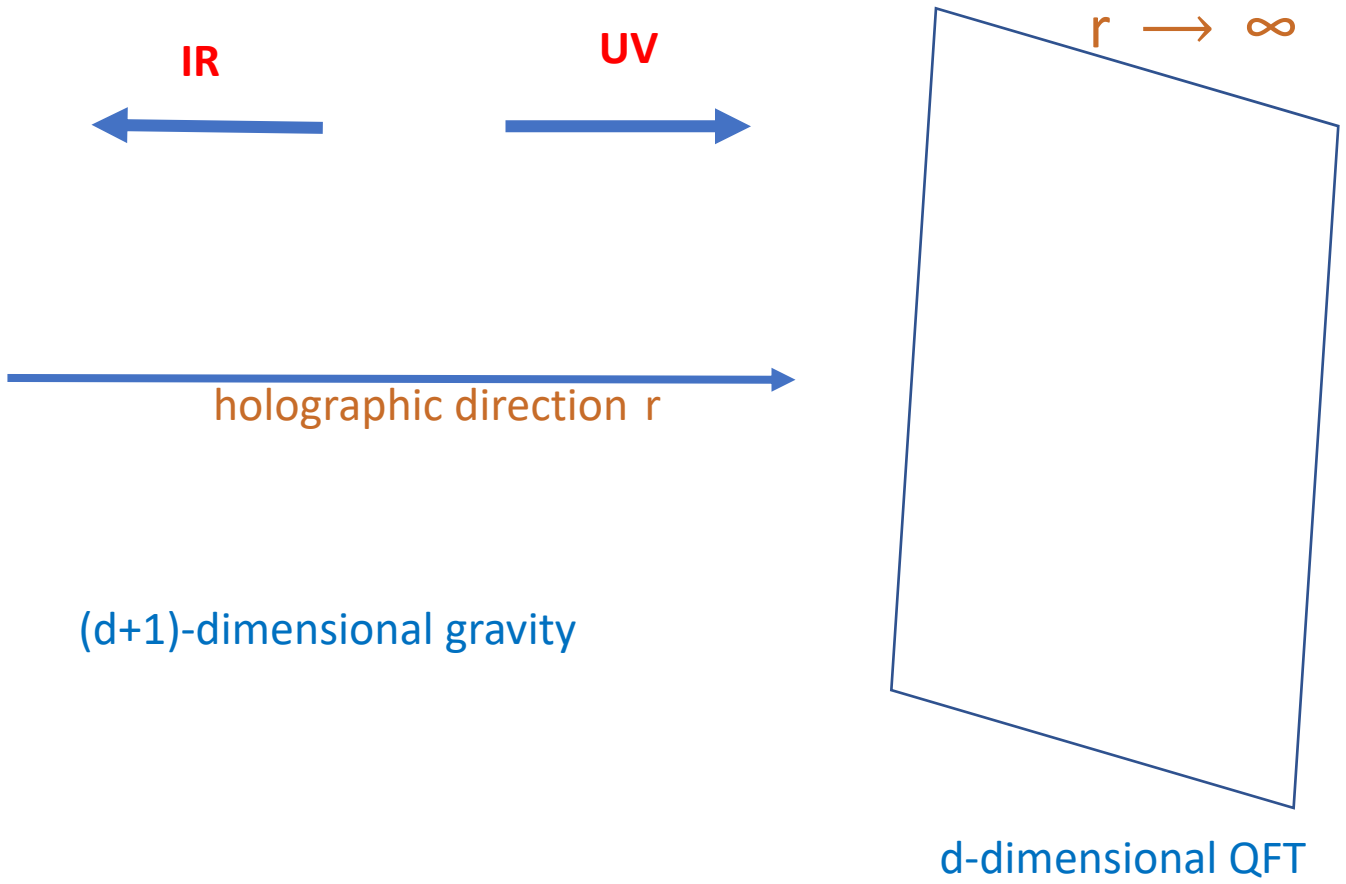
$$+ \int dy \left[\frac{1}{2} \beta_a^{ac} Y_c^\dagger + \frac{1}{2} \beta_i^{ik} Y_k^\dagger - \frac{1}{3} \beta_i^{ic} Y_c^\dagger + m M_A{}^B Y_B^\dagger Y^A \right]_{x=-\infty}^{x=+\infty}. \qquad (3.5$$

$$\beta_C^{AB} = \frac{4\pi}{k} Y^{[A} Y_C^\dagger Y^{B]} \qquad G_A^{BC} = \beta_A^{BC} + \delta_A^{[B} \beta_D^{C]D} + m M_A{}^{[B} Y^{C]}$$

Gauge/gravity duality

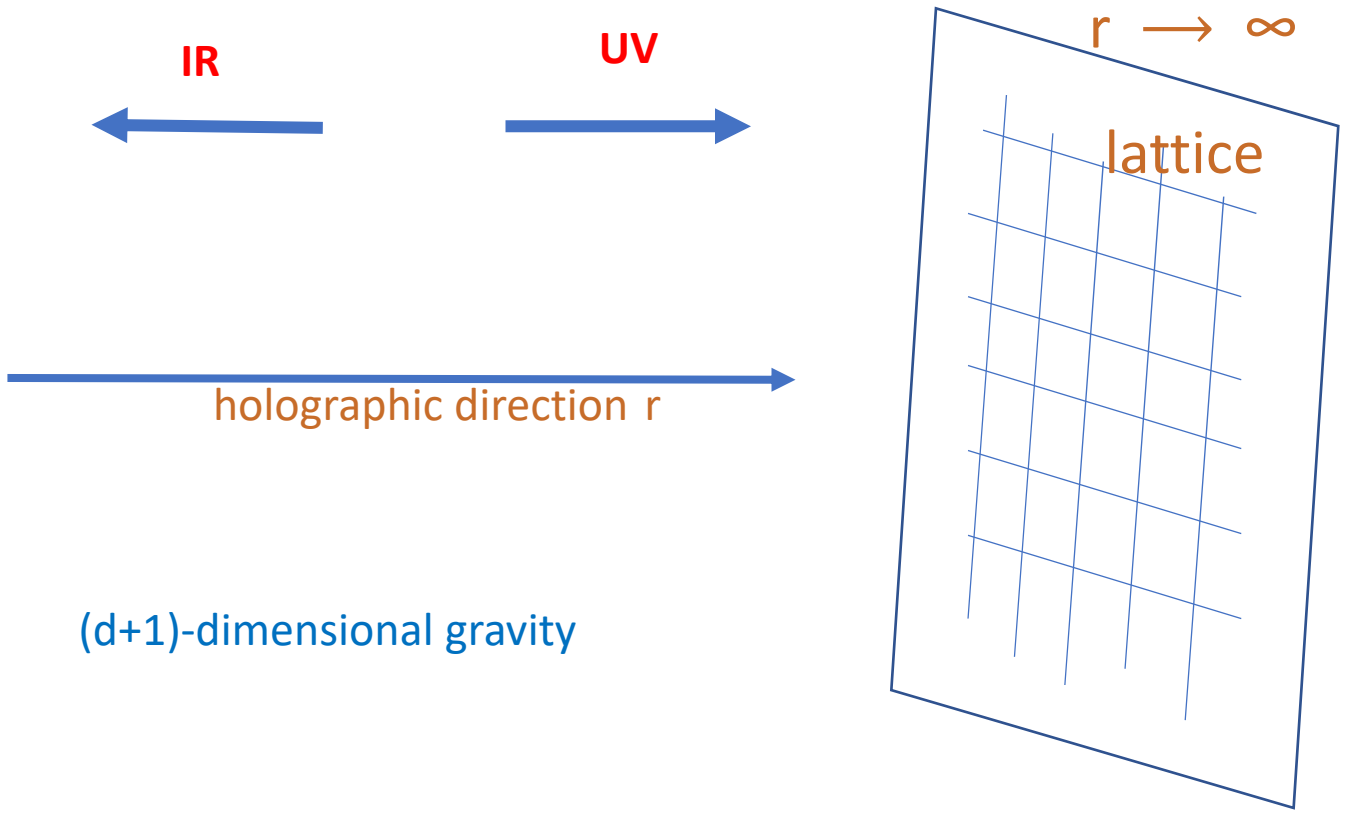
one way of describing strongly coupled QFT

Especially for strongly coupled condensed matter system



- Holographic lattice models ➔ explicitly break translation invariance using a spatially periodic deformation

G.T. Horowitz, J.E. Santos and D. Tong, *Optical conductivity with holographic lattices*, *JHEP* **07** (2012) 168 [arXiv:1204.0519] [INSPIRE].



- Many works for AdS/CMT correspondence in bottom-up approaches

➔ useful tool in figuring out the qualitative properties of strongly coupled condensed matter system

➔Difficult in quantitative calculations or predictions

(no explicit QFT in bottom-up approaches)

- Top-down approaches have explicit QFTs and gravity theories. But examples are very rare and not realistic.

Gravity dual for Janus field theory??

➔ top-down and more realistic!!

gravity dual of the Janus ABJM with mass deformation?

Recent work:

arXiv: 1808.02448, JHEP1810: 066

Susy Q and spatially modulated deformations of ABJM theory

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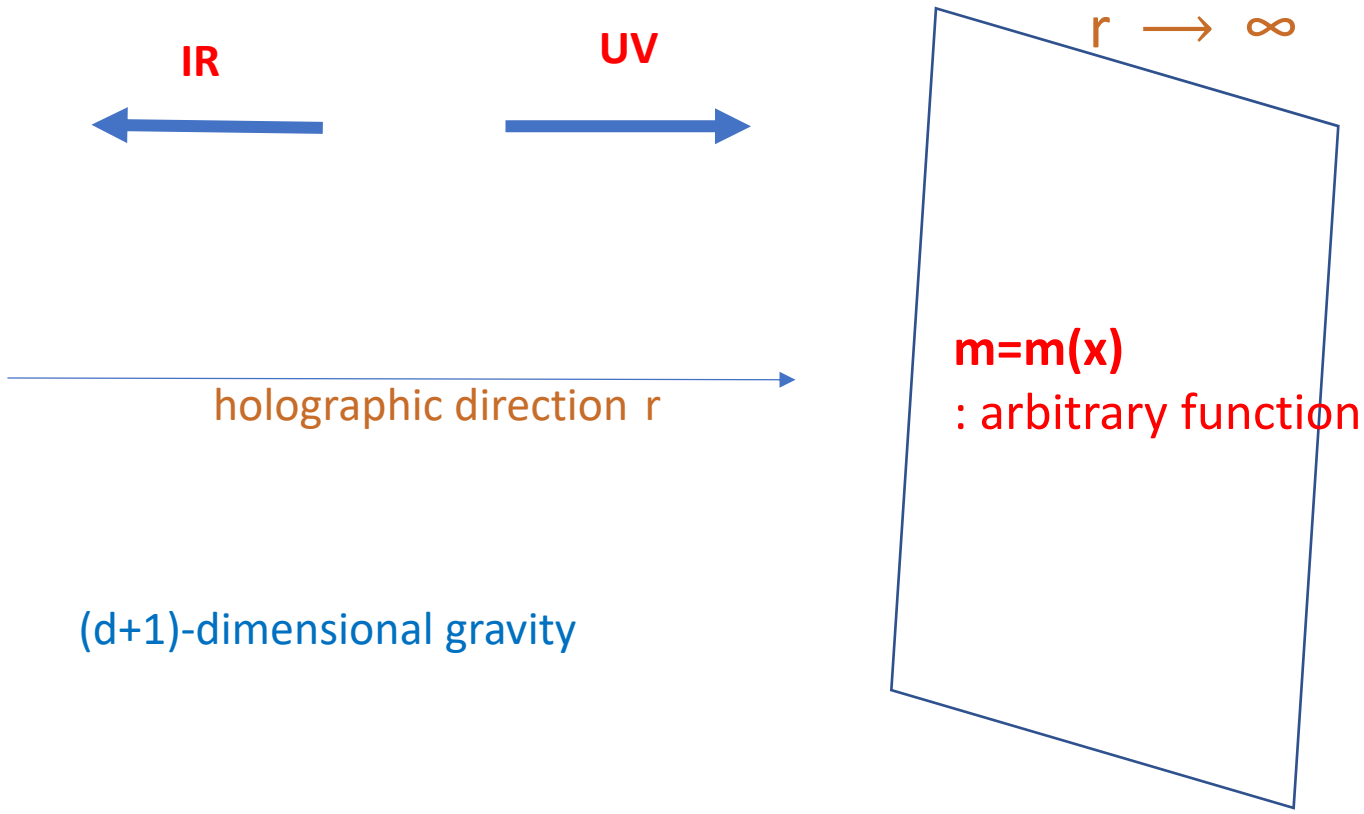
arXiv: 1812.11159

Spatially modulated and supersymmetric deformations of ABJM theory

Igal Arav, Jerome P. Gauntlett
Matthew Roberts and Christopher Rosen

They found *a* gravity dual geometry in 11 dim. SUGRA for *a* special mass function:

$m(x) \sim \sin(\alpha x)$: reflect lattice structure in AdS/CFT applications



Various holographic models
to study condensed matter system in top-down approaches

Discussion

- Is there some supersymmetry for $m = (x, y)$ case?
  (more realistic model in condenced matter physics)

- Physical meanings for various forms of mass functions
  m~δ(x), sin(αx), tanh (bx), etc...

- Are there well-defined supersymmetric vacua and soliton solutions and other lower SUSY Janus models?
  [C. Kim, K. Kim,Y. Kim, OK]

- Dual gravity geometry for arbitrary function of m=m(x)?
  Some deformation of the LLM geometry?
  [K. Kim, OK, H. Shin]

Supersymmetric boomerang solutions in D = 11
Uplifting 4-dimensional solution to 11-dimensional solution

$$ds_{11}^2 = (Z\tilde{Z})^{1/3} ds_4^2 + 4(Z\tilde{Z})^{1/3} \left[d\xi^2 + \frac{\cos^2\xi}{Z} d\Omega_3^2 + \frac{\sin^2\xi}{\tilde{Z}} d\tilde{\Omega}_3^2 \right]$$

$d\Omega_3$ and $d\tilde{\Omega}_3^2$ are metrics of S^3 with unit radius

$$Z = \sin^2\xi + \cos^2\xi(\cosh\lambda + \cos kx \sinh\lambda)$$

$$\tilde{Z} = \cos^2\xi + \sin^2\xi(\cosh\lambda - \cos kx \sinh\lambda)$$

$$F_{(4)} = F_{(4)}^{\mathrm{FR}} + F_{(4)}^{I_1} + F_{(4)}^{I_2},$$

$$F_{(4)}^{\mathrm{FR}} = -(2 + \cosh\lambda + \cos 2\xi \sinh\lambda \cos kx)\, \mathrm{vol}_4,$$

$$F_{(4)}^{I_1} = \sin 2\xi \left(-\cos kx \star d\lambda + \frac{1}{2} \sin kx \sinh 2\lambda \star d(kx) \right) \wedge d\xi,$$

$$F_{(4)}^{I_2} = d\left(\sinh\lambda \sin kx \frac{\cos^4\xi}{Z} \right) \wedge \mathrm{vol}_{\Omega_3} - d\left(\sinh\lambda \sin kx \frac{\sin^4\xi}{\tilde{Z}} \right) \wedge \mathrm{vol}_{\tilde{\Omega}_3}$$