Standard Model and inflation with Weyl symmetry

Hyun Min Lee

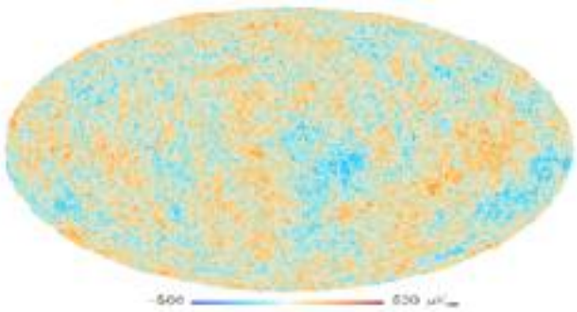
Chung-Ang University, Korea



Based on D. Ghilencea, HML, arXiv: 1809.09174

The 3rd IBS-KIAS Joint Workshop
High-1 resort, Feb 15, 2019

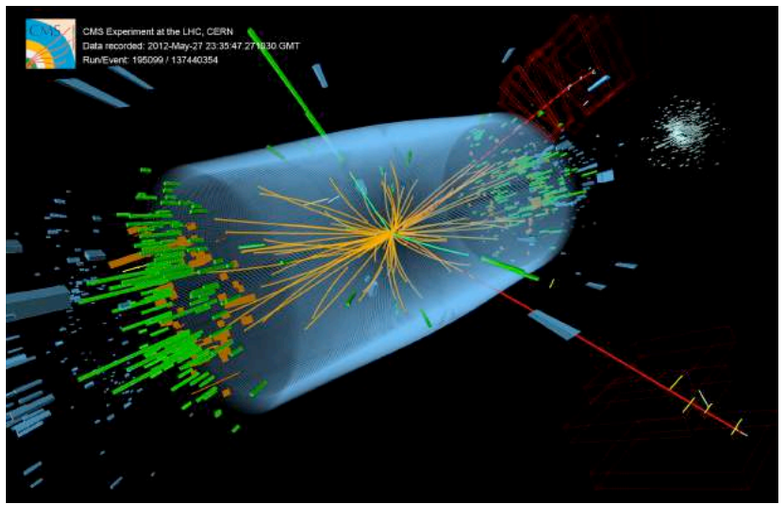
Particle physics in cosmology



Cosmic Microwave Background

"Slow-roll inflation"

$m_I \ll H$



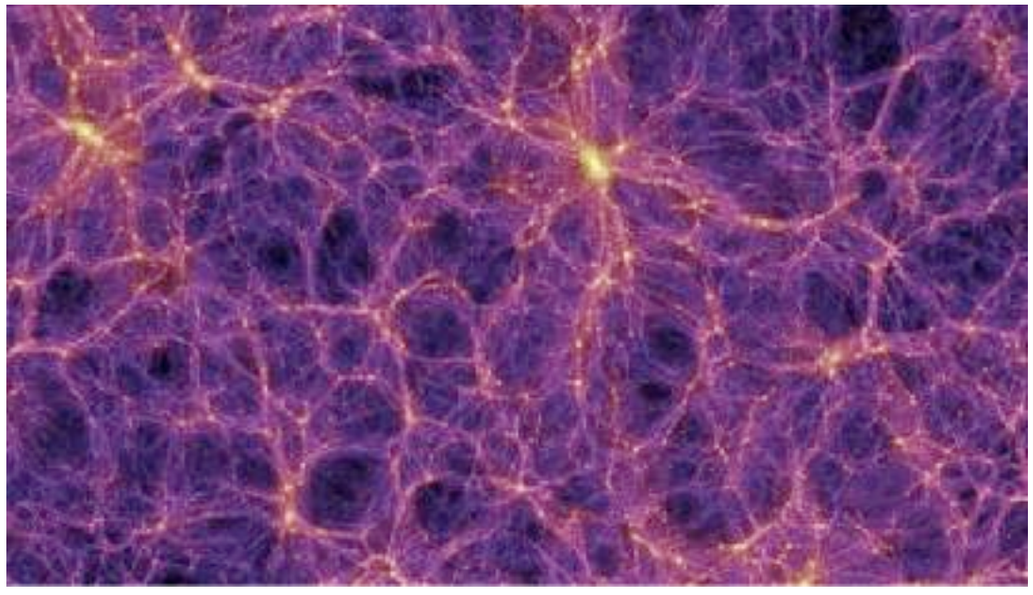
Mass generation

"Higgs boson" $m_H \ll M_P$



?

What are the origins of inflation, symmetry breaking, and dark matter?



Large-scale structure

"Dark matter" $\tau_X > 10^{16}\,\mathrm{sec}$

July 5, 2012 | World News | RayKrebs

Higgs Boson May Be Key to Understanding Dark Matter & New Physics?



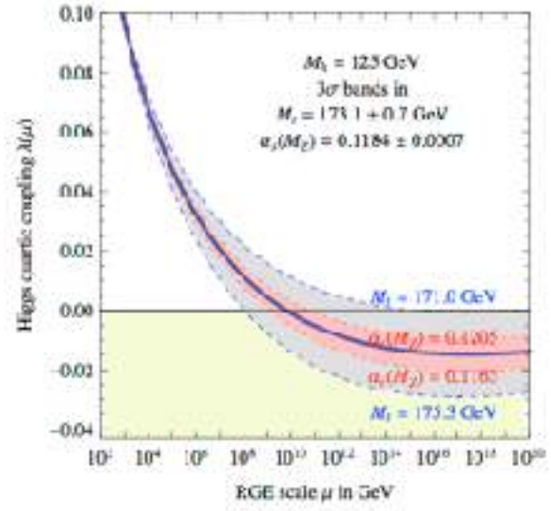
Discovered Higgs boson has a key to open the door to new symmetries and interactions beyond the SM.

Knocking on Higgs's door

- ✓ Hierarchy problem, vacuum instability

$$V = m_H^2 |H|^2 + \lambda |H|^4$$

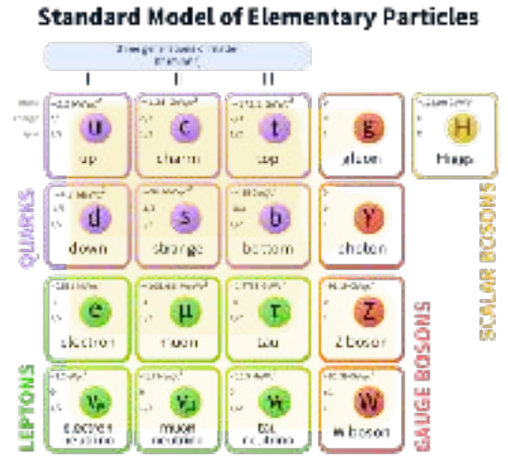
$$m_H^2 = m_{H,0}^2 - \frac{\kappa^2}{16\pi^2} M_{\rm heavy}^2 \sim (100\,{\rm GeV})^2, \;\; \lambda_H > 0$$



- ✓ Electroweak symmetry breaking $m_H^2 < 0$?

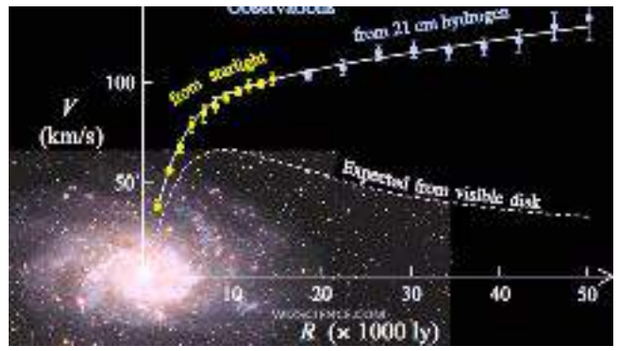
- ✓ Flavor puzzles, baryogenesis

$$H_1 Q_i U_j^c + H_2 Q_i D_j^c + \cdots$$ More Higgs?



- ✓ Dark matter

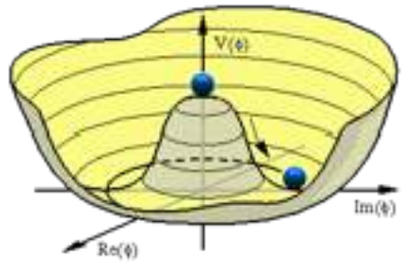
$$\lambda_{HS} |H|^2 S^2$$ WIMP, FIMP, SIMP, …



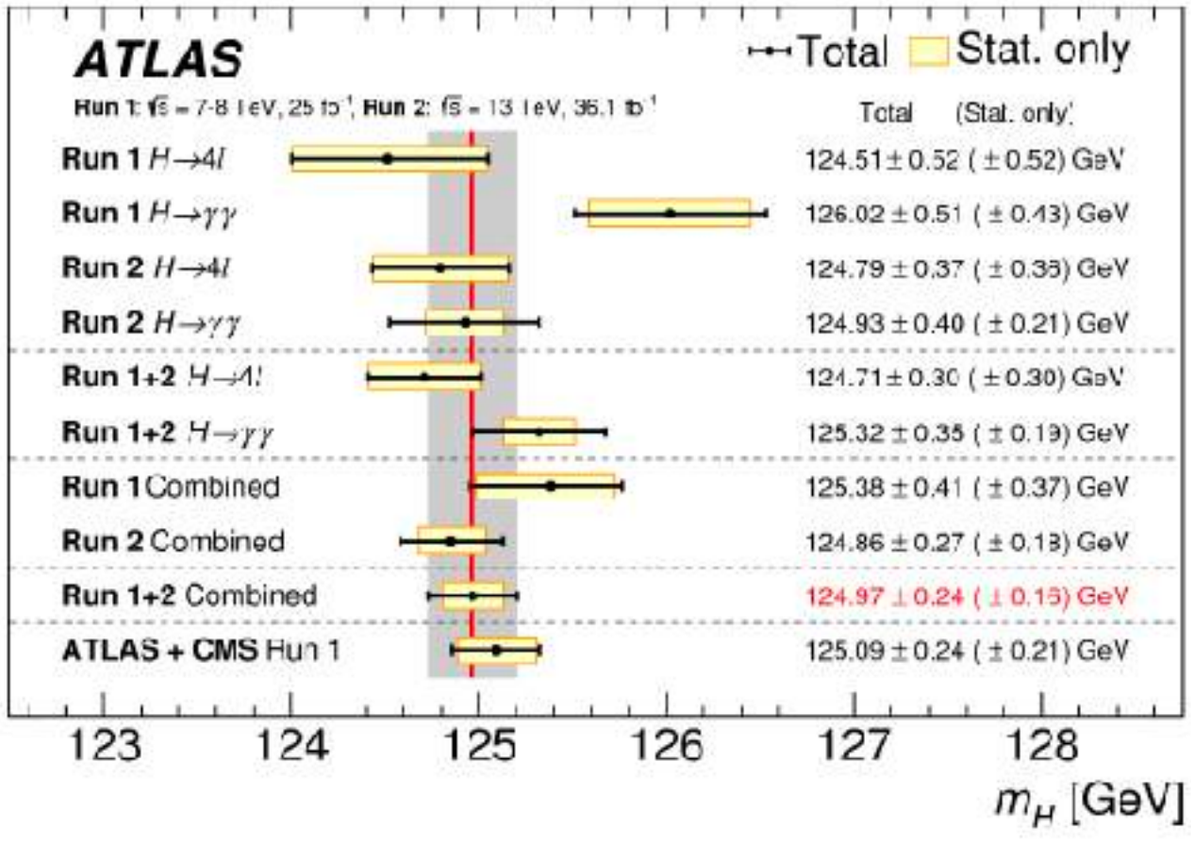
- ✓ Inflation

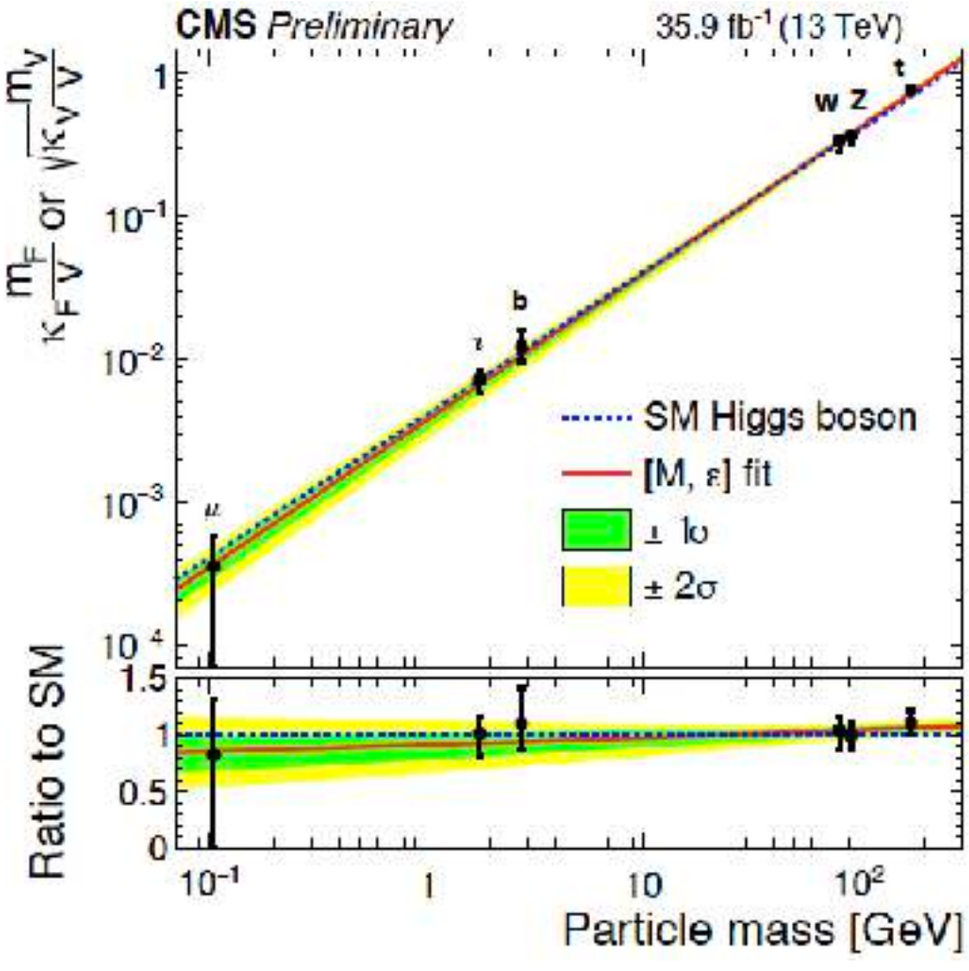
$$\xi |H|^2 \mathcal{R}$$ Higgs inflation?

Higgs boson at LHC



- 125GeV Higgs boson is (close to) a fundamental scalar particle, consistent with Higgs couplings predicted in SM.





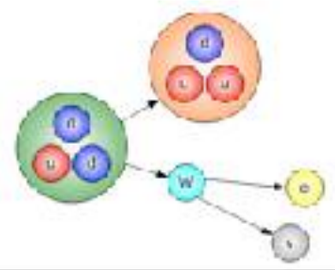
- Higgs precision is important at HL-LHC and future colliders.
- Is Higgs boson the last piece of the puzzles?

Scale invariance and Higgs mass

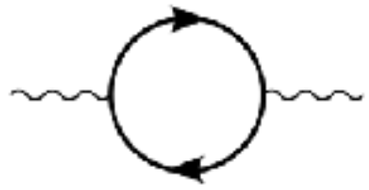
- Scale invariance is broken by explicit mass scales.

  - Planck mass M_P, Higgs mass m_H, etc





  - Regularization of divergent loop corrections
    (cutoff Λ or renormalization scale μ)



- Classical scale invariance

  No heavy particles coupled to Higgs.

  ⇒ No UV sensitivity of Higgs mass corrections in DR.

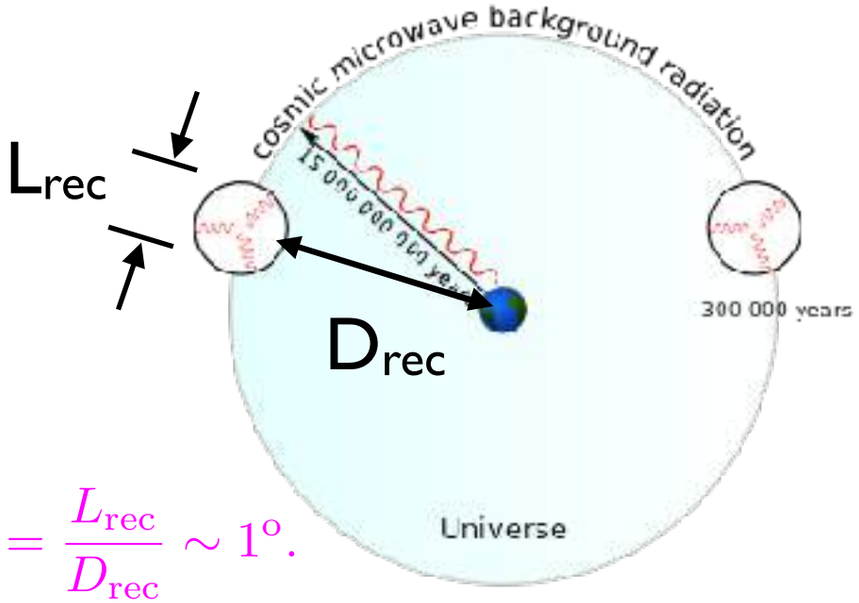
- Electroweak symmetry breaking cf. Coleman-Weinberg

  Scale-invariant potential: $V = \frac{1}{4}\lambda(h^2 - \epsilon\phi^2)^2, \qquad \epsilon \ll 1$

  Technically natural!

Cosmic inflation

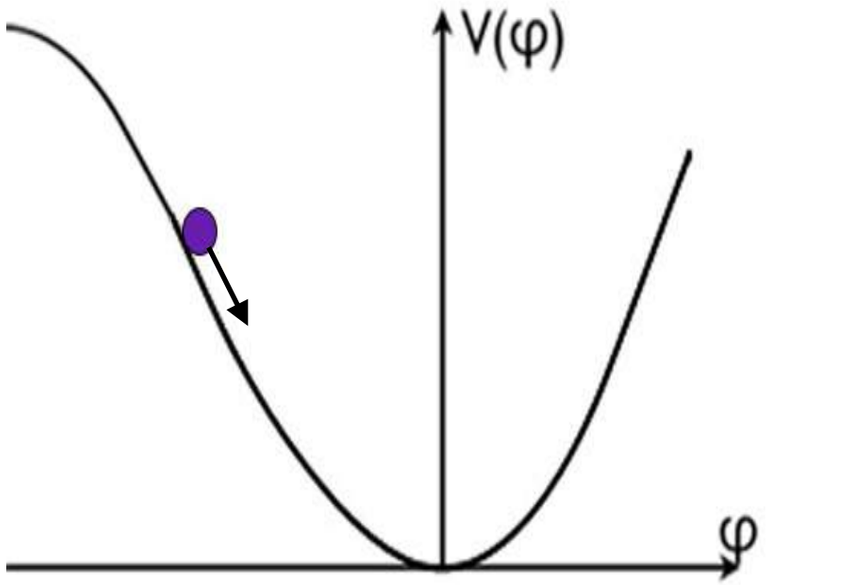
- Exponential expansion with a scalar field "inflaton" explains the initial conditions for Big Bang Cosmology.



$$\theta = \frac{L_{\rm rec}}{D_{\rm rec}} \sim 1^{\rm o}.$$

Causally connected at recombination

How uniform CMB 1 in 10^4 ?



$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$$

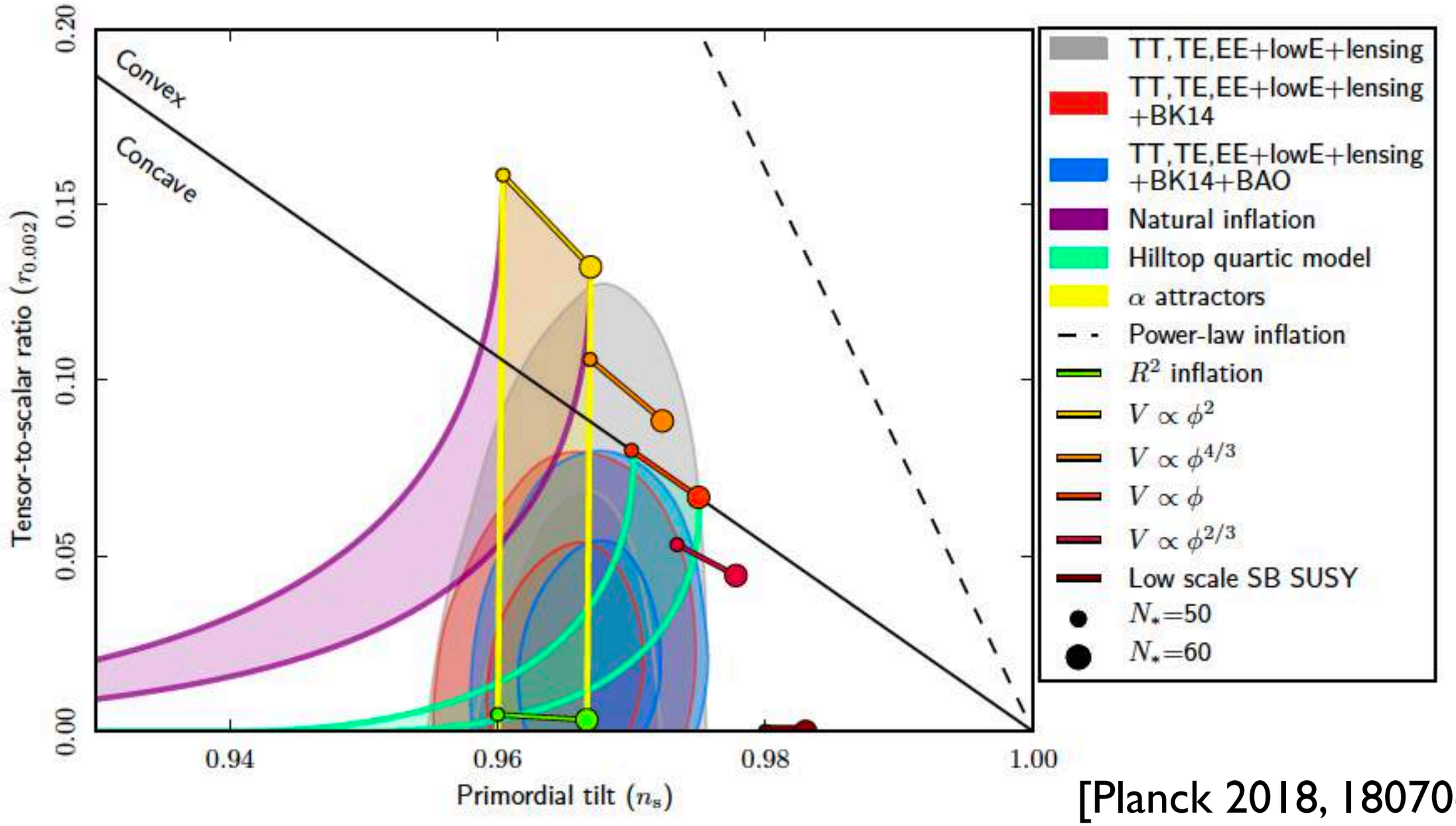
Number of efoldings:

$$N = \int_{t_i}^{t_f} H dt = 60$$

- Slow-roll inflation is confirmed by CMB anisotropies.

$$n_s = 1 - 6\epsilon + 2\eta, \qquad r = 16\epsilon, \qquad \epsilon \sim \frac{(V')^2}{V^2} \ll 1, \quad \eta \sim \frac{V''}{V} \ll 1.$$

Planck data



[Planck 2018, 180706211]

$n_s = 0.9659 \pm 0.0041$ $\quad r_{0.002} < 0.10$ (95 % CL, *Planck* TT+lowE+lensing)

$n_s = 0.9653 \pm 0.0041$ $\quad r_{0.002} < 0.064$ (95 % CL, *Planck* TT,TE,EE +lowE+lensing+BK14) .

Scale invariance and inflation

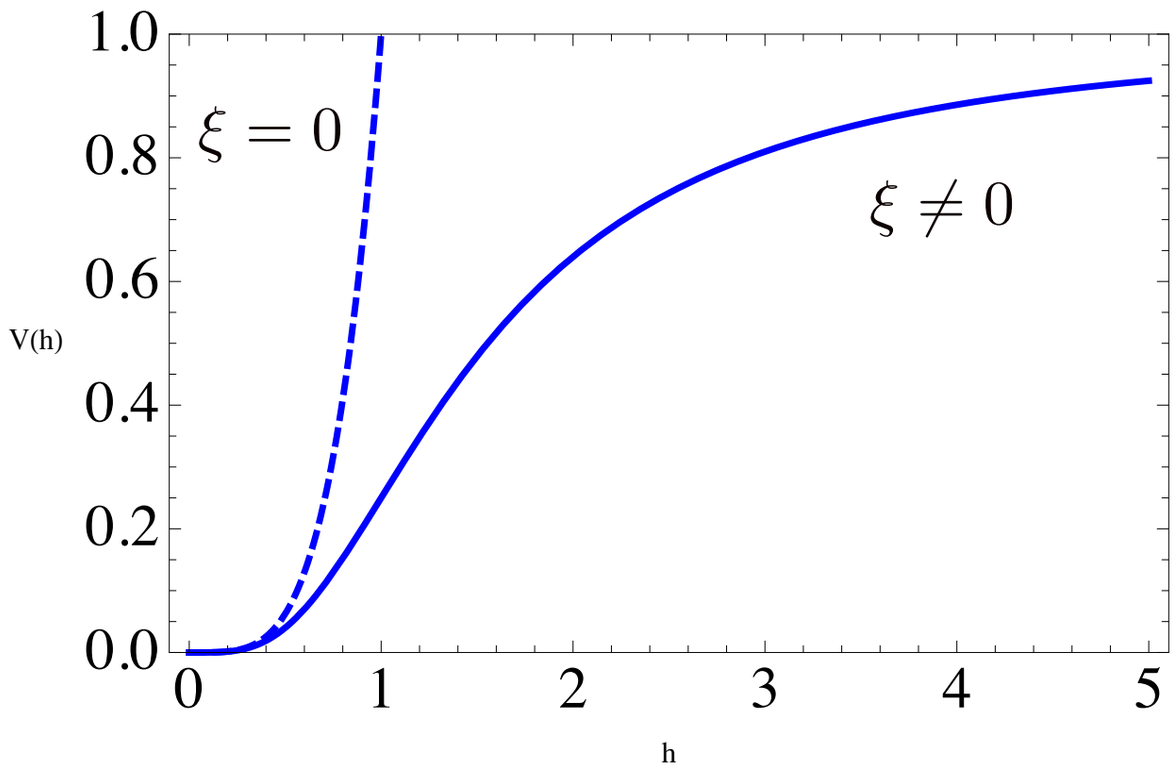
- Scale symmetry is restored in Higgs inflation.

$$\mathcal{L} = \sqrt{-g}\left(-\frac{1}{2}M_P^2\mathcal{R} - \xi|H|^2\mathcal{R} + |D_\mu H|^2 - V\right)$$

$$\Rightarrow \quad \mathcal{L} = \sqrt{-g_E}\left(-\frac{1}{2}M_P^2\mathcal{R}(g_E) + \frac{3\xi^2}{\Omega^2}(\partial_\mu|H|^2)^2 + \frac{1}{\Omega}|D_\mu H|^2 - \frac{V}{\Omega^2}\right)$$

$$g_{\mu\nu} = \Omega^{-1}g_{E,\mu\nu}, \quad \Omega = 1 + 2\xi|H|^2/M_P^2$$

[Fezrukov, Shaposhnikov, 2007]



$|H|^2 \gg M_P^2/(2\xi)$:

Planck mass, Higgs mass negligible

$$\frac{V}{\Omega^2} = \frac{\lambda_H|H|^4}{(1+2\xi|H|^2)^2} \sim \frac{\lambda_H}{4\xi^2}\left(1 - e^{-\frac{2}{\sqrt{6}}\chi}\right)$$

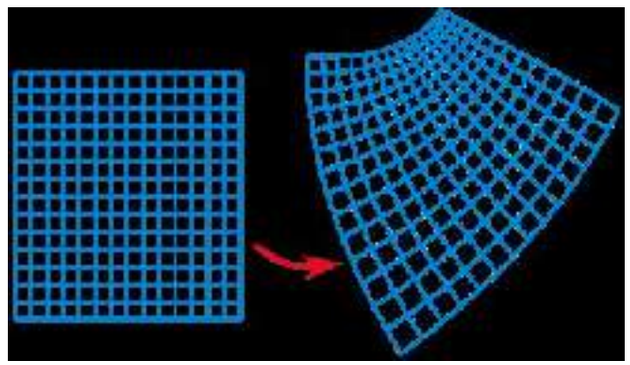
"approximately scale invariant"

Weyl symmetry

- Weyl transformations: "local" scale transform.

$$g_{\mu\nu} \to e^{2\alpha(x)}\, g_{\mu\nu}, \qquad \psi \to e^{-\frac{3}{2}\alpha(x)}\, \psi.$$

$$\phi \to e^{-\alpha(x)}\, \phi,$$



- Weyl current

$$\mathcal{L} = \sqrt{-g}\left(-\frac{1}{12}\xi\phi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)$$: global scale invariant

$$\Rightarrow \quad \delta\mathcal{L} = \sqrt{-g}\,\frac{1}{2}\partial^\mu\alpha(x)\,K_\mu, \qquad K_\mu = \partial_\mu[(\xi+1)\phi^2].$$

But, Weyl symmetry broken unless $\xi = -1$.

Note: Weyl current vanishes and preserved for $\phi = \text{const.}$

[C. Hill, G. Ross, 2016, 2018]

cf. approximate Weyl symmetry for Higgs inflation

Weyl gauge field

- Couple a Weyl gauge field to the Weyl current.

$$\Delta\mathcal{L} = \sqrt{-g}\,\frac{q}{2}\omega_\mu K^\mu, \qquad \omega_\mu \to \omega_\mu - \frac{2}{q}\,\partial_\mu\alpha(x).$$

$$\Rightarrow \qquad \delta(\mathcal{L}+\Delta\mathcal{L}) = 0! \quad \text{for any} \quad \xi.$$

- Fully Weyl-invariant Lagrangian:

$$\mathcal{L} = \sqrt{-g}\left(-\frac{1}{12}\xi\phi^2 R - \frac{1}{2}\xi\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}(1+\xi)g^{\mu\nu}\tilde{D}_\mu\phi\tilde{D}_\nu\phi \; -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right),$$

with $\boxed{\tilde{D}_\mu\phi = \left(\partial_\mu - \frac{q}{2}\,\omega_\mu\right)\phi,}$ $\quad F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu.$

i) "Right-sign" kinetic terms $\quad(\xi > 0)$

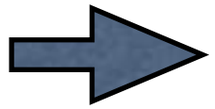
ii) Even real scalar field couples to Weyl gauge field.

SM with Weyl symmetry

- Higgs and extra scalar fields

$$L = \sqrt{g}\Big\{ -\frac{\xi_j}{2}\Big[\frac{1}{6}\,\phi_j^2\,R + g^{\mu\nu}\,\partial_\mu\phi_j\,\partial_\nu\phi_j\Big] + (1+\xi_j)\,\frac{1}{2}g^{\mu\nu}\,\tilde{D}_\mu\phi_j\,\tilde{D}_\nu\phi_j - V(\phi_j)\Big\}$$

$$V(\phi_j) = \phi_k^4\,V(\phi_j/\phi_k), \qquad k = \text{fixed}.$$



i) Weyl gauge couplings exist!

ii) No mass parameters!

- Fermion kinetic terms are Weyl invariant, with no coupling to Weyl gauge field.

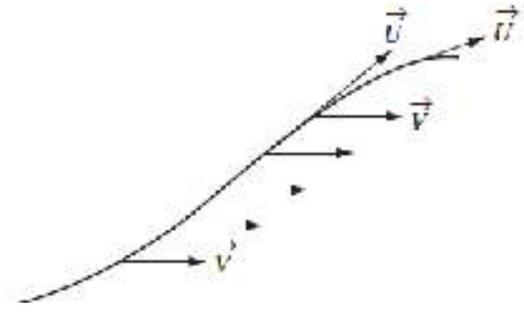
$$L_f = \sqrt{g}\,\bar{\psi}\,i\gamma^a\,e^\mu{}_a D_\mu\psi, \qquad D_\mu\psi = \Big(\partial_\mu + \frac{1}{2}\omega_\mu^{ab}\sigma_{ab}\Big)\,\psi$$

- SM gauge field kinetic terms are Weyl invariant.

Relation to Weyl geometry

- Metric compatibility in Weyl geometry

$$D_\mu g_{\nu\lambda} = 0 \quad \Rightarrow \quad \tilde{D}_\mu g_{\nu\lambda} = -q\,\omega_\mu\, g_{\nu\lambda}$$



Weyl connection:

$$\tilde{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + \frac{q}{2}\left[\delta^\rho_\mu\,\omega_\nu + \delta^\rho_\nu\,\omega_\mu - g_{\mu\nu}\omega^\rho\right]$$

"Torsion-free"

- Weyl gravity

$$\tilde{R} = R - 3q\left[\partial_\mu\omega^\mu + \frac{1}{2}\omega^\rho g^{\lambda\beta}\partial_\rho g_{\lambda\beta}\right] - \frac{3}{2}q^2\omega^\mu\omega_\nu$$

$$\tilde{R} \to e^{-2\alpha(x)}\,\tilde{R}, \qquad \text{Weyl-covariant}$$

$$\mathcal{L}_w = -\sqrt{-g}\,\frac{1}{12}\xi\phi^2\tilde{R} \quad \longleftrightarrow \quad \mathcal{L} = \sqrt{-g}\left(-\frac{1}{12}\xi\phi^2 R - \frac{1}{2}\xi\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)$$

up to total derivative

Stueckelberg mechanism

- Jordan frame action for one real scalar.

$$K = (\xi + 1)\phi^2, \quad K_\mu = \partial_\mu K$$

[D. Ghilencea, HML, 2018]

$$\mathcal{L} = \sqrt{-g}\left(-\frac{1}{12}\xi\phi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{q^2}{8}K\omega_\mu\omega^\mu - \frac{q}{4}\omega^\mu K_\mu - \frac{1}{4!}\lambda\phi^4 \right)$$

- Stueckelberg mechanism for Weyl symmetry

Einstein frame: $\hat{g}_{\mu\nu} = \Omega\, g_{\mu\nu}, \qquad \Omega = \frac{\xi}{6}\frac{\phi^2}{\langle\phi\rangle^2}.$ Gauge choice: $\langle\phi\rangle = M_P$

$$\hat{L} = \sqrt{\hat{g}}\left\{ -\frac{1}{2}\langle\phi\rangle^2 \hat{R} + \frac{3}{4}\langle\phi\rangle^2(\partial_\mu \ln\Omega)^2 + \frac{1}{\Omega}\left[\frac{1}{2}(\partial_\mu\phi)^2 + \frac{q^2}{8}K\omega_\mu\omega^\mu - \frac{q}{4}\omega^\mu K_\mu\right] - \frac{V}{\Omega^2}\right\}$$

$$= \sqrt{\hat{g}}\left[-\frac{1}{2}\langle\phi\rangle^2 \hat{R} + \frac{3}{4}q^2\langle\phi\rangle^2\left(1 + \frac{1}{\xi}\right)\omega'_\mu\omega'^\mu - \frac{3\lambda}{2\xi^2}\langle\phi\rangle^4\right]$$

Weyl gauge boson mass

$$\omega'_\mu = \omega_\mu - \frac{2}{q}\partial_\mu \ln\phi$$ ← SSB : dilaton eaten by Weyl gauge field.

A nonzero C.C. is a general consequence of Weyl symmetry.

The case with two scalars

- Jordan frame action for two scalars [D. Ghilencea, HML, 2018]

$$K = (1+\xi_1)\,\phi_1^2 + (1+\xi_2)\,\phi_2^2, \qquad V(\phi_1,\phi_2) = \frac{\lambda_1}{4!}\phi_1^4 + \frac{\lambda_{12}}{4}\phi_1^2\,\phi_2^2 + \frac{\lambda_2}{4!}\,\phi_2^4.$$

- Generalized Stueckelberg mechanism

Einstein frame: $\hat{g}_{\mu\nu} = \Omega\, g_{\mu\nu}, \qquad \Omega = \frac{1}{6\,v^2}\,(\xi_1\phi_1^2 + \xi_2\phi_2^2), \qquad v^2 \equiv \langle \xi_1\phi_1^2 + \xi_2\phi_2^2 \rangle.$

$$\hat{L} = \sqrt{\hat{g}}\Big[-\frac{1}{2}v^2\,\hat{R} + \frac{3}{4}v^2\,(\partial_\mu \ln\Omega)^2 + \frac{1}{\Omega}\Big(\frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 + \frac{q^2}{8}K\,\omega_\mu\omega^\mu - \frac{q}{4}\omega^\mu K_\mu\Big) - \hat{V}\Big]$$

$$= \sqrt{\hat{g}}\Big[\frac{-1}{2}v^2\,\hat{R} + \frac{1}{2}F(\theta)\;v^2\,(\partial_\mu \tan\theta)^2 + \frac{1}{2}m^2(\theta)\hat{g}^{\mu\nu}\omega'_\mu\omega'_\nu - \hat{V}(\theta)\Big]$$

$$\phi_1 = \frac{1}{\sqrt{1+\xi_1}}\,\rho\,\sin\theta, \quad \phi_2 = \frac{1}{\sqrt{1+\xi_2}}\,\rho\,\cos\theta.$$: polar basis

$\omega'_\mu = \omega_\mu - \frac{1}{q}\partial_\mu \ln K$, $K = \rho^2$: modulus eaten by Weyl gauge field.

(Weyl current along modulus)

Weyl gauge boson mass & potential depends only on θ.

Quantum scale invariance

[Wetterich, 1988; Shaposhnikov, Zenhausern, 2008; Armilis, Monin, Shaposhnikov, 2013]

- Scale anomalies in Dimensional Regularization : $d = 4 - \epsilon$

$$S = \int d^d x \left(|D_\mu H|^2 + \bar{\psi} i \gamma^\mu D_\mu \psi - \frac{1}{4g^2}\, \mu^{d-4} F_{\mu\nu} F^{\mu\nu} - V(H) - y \mu^{2-d/2} H \bar{\psi}\psi + \text{h.c.} \right),$$

$$V(H) = \lambda \mu^{4-d} |H|^4.$$

$$\delta_A, \delta_y, \delta_\lambda \propto \frac{1}{\epsilon} \quad \Longrightarrow \quad \frac{1}{(g_R)^2}, y_R, \lambda_R \sim \beta \ln \mu + \cdots$$

Beta functions

- Scale symmetry is respected by dilaton at quantum level.

$$\mu \longrightarrow \phi\, f(|H|^2/\phi^2), \quad \phi : \text{dilaton.}$$ [D. Ghilencea et al, 2016]

But, theory is not renormalizable: $\frac{|H|^6}{\phi^2}, \frac{|H|^8}{\phi^4}, \text{etc}$

- Angular field = Higgs, Technically small Higgs mass!

$$\xi_1 \gg 1, \xi_2 = 0 : \quad \hat{V} = \frac{\lambda_2}{4!}\left(h^2 - \frac{\sqrt{\lambda_1}}{\sqrt{\lambda_2}} \frac{v^2}{(\xi_1/6)} \right)^2, \quad 3\lambda_{12} = -\sqrt{\lambda_1 \lambda_2},$$

$$m_h^2 = 2\sqrt{\lambda_1 \lambda_2}\, \frac{v^2}{\xi_1} \ll v^2.$$

Inflation with Weyl symmetry

- Weyl gauge boson mass depends on dilaton only in vacuum.

$$m^2(\theta) = \frac{q^2 K}{4\Omega} = \frac{3q^2}{2}\frac{v^2(1+\xi_1)(1+\xi_2)(1+\tan^2\theta)}{\xi_1(1+\xi_2)\tan\theta^2 + \xi_2(1+\xi_1)} \longrightarrow \boxed{m^2(\langle\theta\rangle) = \frac{3}{2}q^2\langle\rho\rangle^2.}$$

$$\theta = \langle\theta\rangle$$

$$v^2 = \langle\rho\rangle^2\left[\frac{\xi_1}{1+\xi_1}\sin^2\langle\theta\rangle + \frac{\xi_2}{1+\xi_2}\cos^2\langle\theta\rangle\right] = M_P^2.$$

- Angular field = inflaton

eom: $\partial_\mu(\sqrt{-g}F^{\mu\nu}) = -m^2(\theta)\omega'^\nu$

$\omega'_\mu = 0$: Weyl gauge field decoupled during inflation.

⇒ Inflaton effective action:

$$L_{\rm infl} = \sqrt{\hat{g}}\Big[-\frac{1}{2}v^2\hat{R} + \frac{3bv^2}{\xi_2}\frac{\tau^2+\xi_2/\xi_1}{(1+\tau^2)(\tau^2+b)^2}(\partial_\mu\tau)^2 - \hat{V}(\tau)\Big], \qquad \tau \equiv \tan\theta$$

$$\hat{V} = \frac{36v^4(1+\xi_1)^2}{\xi_1^2(\tau^2+b)^2}\left(c_1\tau^4 + c_{12}\tau^2 + c_2\right), \qquad b = \frac{\xi_2(1+\xi_1)}{\xi_1(1+\xi_2)}.$$

Hilltop inflation

- Hilltop inflation limit [D. Ghilencea, HML, 2018]

$$\tau = \tan\theta \gg \max\{\sqrt{b}, 1, \frac{\xi_2}{\xi_1}\},$$

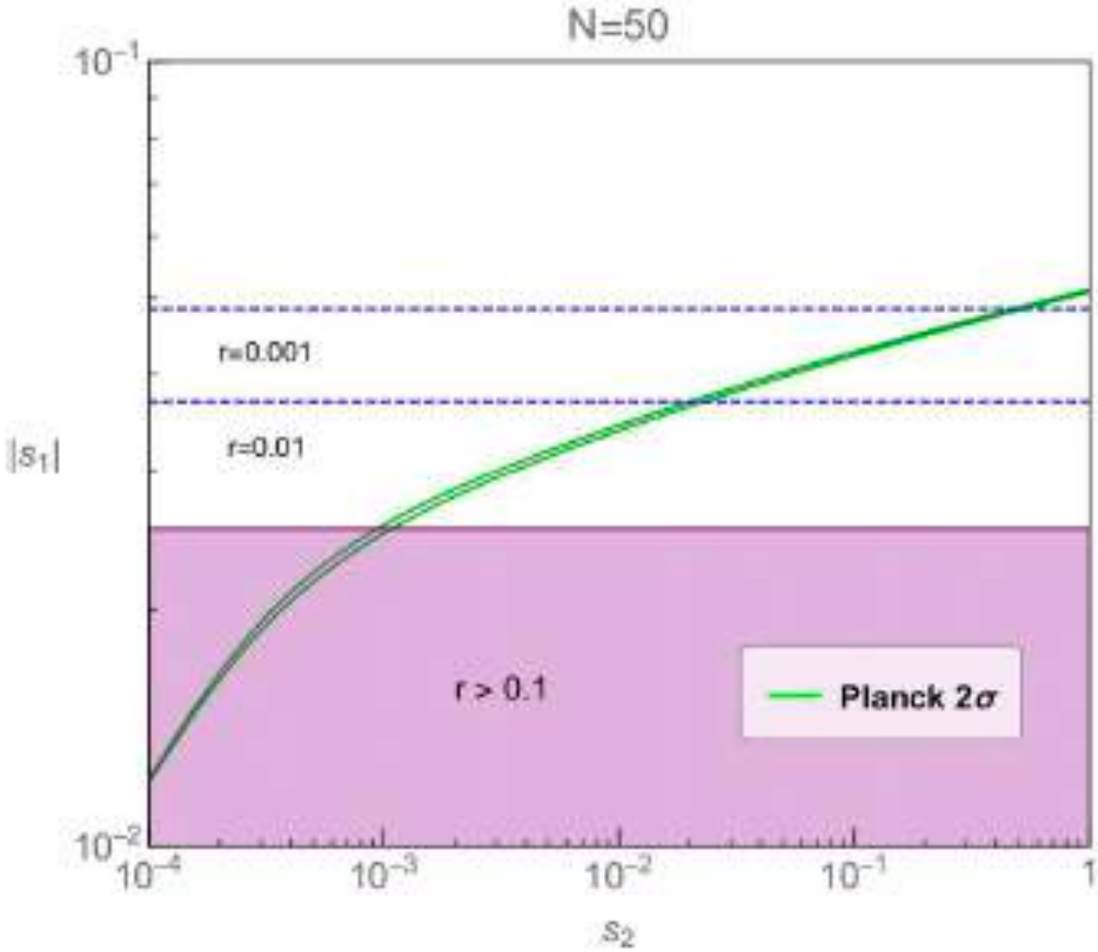
Canonical inflaton: $$\chi = \frac{v}{\tau}\sqrt{\frac{6\,b}{\xi_2}}\left[1+\frac{1}{6}\left(\frac{\xi_2}{\xi_1}-1-2b\right)\frac{1}{\tau^2}\right]$$

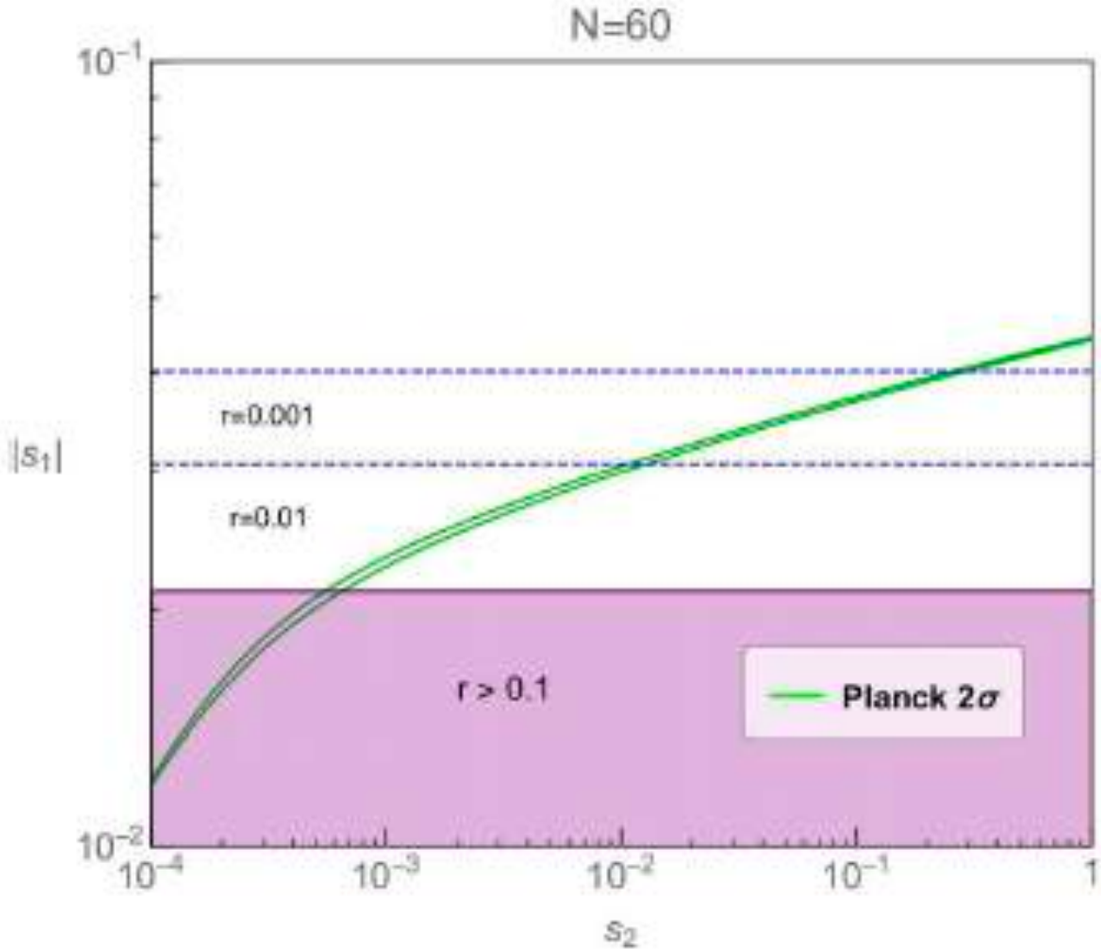
$$\hat{V}(\chi) = V_0\left(1+s_1\frac{\chi^2}{v^2}+s_2\frac{\chi^4}{v^4}\right)+\mathcal{O}\left(\frac{\chi^6}{v^6}\right),$$

$$\begin{aligned}
V_0 &= \frac{3\,v^4}{2}\frac{\lambda_1}{\xi_1^2}, \\
s_1 &= \frac{\xi_1\lambda_{12}}{\lambda_1}-\frac{\xi_2}{3}, \\
s_2 &= \frac{\xi_2^2}{12}-\frac{\xi_1\xi_2}{3}\frac{\lambda_{12}}{\lambda_1}+\frac{\xi_1^2}{36}\frac{\lambda_2}{\lambda_1} \\
&\quad +\frac{\xi_2}{54(1+\xi_1)}\left(1-\frac{3\xi_1}{\xi_2}\frac{\lambda_{12}}{\lambda_1}\right)\left(\xi_2(\xi_2-1)-\xi_1(3\xi_2+1)\right).
\end{aligned}$$

Hilltop inflation

- Successful inflation can be obtained with sizable r.





$$n_s = 1 + 2\eta_* - 6\epsilon_* = 1 + 4\,s_1 + \frac{4}{s_1^2}\,(3\,s_2 - 2\,s_1^2)\;e^{-4|s_1|N}, \qquad r = 16\,\epsilon_* = 16\,e^{-4|s_1|N}.$$

CMB normalization: $\dfrac{\lambda_1}{\xi_1^2} = 3.3 \times 10^{-7}\, e^{-4|s_1|N}.$

$$\xi_2 = 0: \quad s_1 \lesssim 0.1,\; s_2 \lesssim 1 \quad \Rightarrow \quad \frac{\lambda_{12}}{\sqrt{\lambda_1}} \lesssim 10^{-5}, \quad \lambda_2 \lesssim 10^{-6}$$

Higher curvature terms

- Weyl-inv. curvature squared terms in Weyl geometry

[D. Ghilencea, 2018]

$$L = \sqrt{g}\,\frac{\xi_0}{4!}\,\tilde{R}^2 + \frac{\sqrt{g}}{\eta}\,\tilde{C}_{\mu\nu\rho\sigma}\,\tilde{C}^{\mu\nu\rho\sigma} \qquad \xi_0 > 0, \quad q^2 = -\eta/6$$

$$\begin{cases} \tilde{R}^2 \longrightarrow -2\,\phi_0^2\,\tilde{R} - \phi_0^4 & \text{scalar-dual description} \\ \tilde{C}_{\mu\nu\rho\sigma}\,\tilde{C}^{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma}\,C^{\mu\nu\rho\sigma} + \dfrac{3\,q^2}{2}\,F_{\mu\nu}\,F^{\mu\nu} & \text{Weyl kinetic term} \end{cases}$$

$$\tilde{R} = R - 3q\left[\partial_\mu\omega^\mu + \frac{1}{2}\omega^\rho g^{\lambda\beta}\partial_\rho g_{\lambda\beta}\right] - \frac{3}{2}q^2\omega^\mu\omega_\nu \qquad \text{Riemann geometry}$$

$$\Rightarrow \quad \mathcal{L} = \sqrt{-g}\Big(-\frac{1}{12}\xi_0\phi_0^2 R - \frac{1}{2}\xi_0\, g^{\mu\nu}\partial_\mu\phi_0\partial_\nu\phi_0 + \frac{1}{2}(1+\xi_0)g^{\mu\nu}\tilde{D}_\mu\phi_0\tilde{D}_\nu\phi_0 - \frac{1}{4!}\lambda\phi_0^4$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{\eta}\,C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\Big).$$

: generalized Weyl-invariant gravity with a real scalar

Conclusions

- Local scale invariance (Weyl symmetry) needs a Weyl gauge field, which couples to scalar fields.

- We showed that Weyl symmetry is spontaneously broken by Stueckelberg mechanism, with dilaton being absorbed by Weyl gauge field and decoupled.

- Angular mode = Higgs: quantum scale invariance is realized, generating the hierarchy of scales due to dimensionless parameters.

- Angular mode = inflaton: Hilltop type inflation is realized, predicting a sizable tensor-to-scalar ratio.