AdS black holes and deconfinement

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Black holes in AdS/CFT

- Black holes in AdS = large N thermodynamic saddle points.
  - BH's in global AdS_5 = plasma of N^2 deconfined gluons of gauge theories on $S^3 \times R$.

- CFTs can confine at low T on $S^3 \times R$.
  - Confines at $T \ll r^{-1}$ (r: radius of S^3): due to Gauss law (even at weak coupling)
  - Large N CFT w/ gravity dual: Expect deconfinement transition at Tr of order 1.

- Hawking-Page transition (1983) of AdS Schwarzshild BH's at $T = \frac{3}{2\,\pi\ell}$ (ℓ: AdS radius)
  - Low T : thermal gas of gravitons in AdS. Doesn't see $1/\mathrm{G_N} \sim N^2$. So $F_{grav} \sim O(N^0)$
  - High T : AdS black holes see N^2, since $F_{BH} \sim 1/G_N \sim O(N^2)$.

- We shall study BPS AdS black holes from SCFT, w/ various chemical potentials.
  - At general "temperature", many saddles compete. High T is simple ~ "Cardy regime".

AdS BH's from CFT in various dimensions

- At high T, we have "large" AdS BH's: size of BH $\gg$ AdS radius
- AdS_3: large BH's counted by 2d Cardy formula [Strominger, Vafa] ……

$$Z(\tau) \sim \mathrm{Tr}\left[e^{2\pi i \tau L_0}\right] \sim \exp\left[\frac{\pi i c}{12\tau}\right] \quad \text{at } \tau \to i0^+ \qquad e^{S(P,c)} = \oint d\tau Z(\tau) e^{-2\pi i \tau P} \sim \exp\left[2\pi\sqrt{\frac{cP}{6}}\right] \quad \text{at } P \gg c$$

- Conformal symmetries in $D > 2$ are much weaker. However,
  - In $SCFT_{D\geq 3}$, we shall find analogues of the 2d Cardy formulae.
  - In BPS sectors, chemical potentials conjugate to charges play the role of T^{-1}.

- We studied large BPS BH's in all $AdS_{4,5,6,7}$, at least for simplest models.
  - $AdS_4 \times S^7$: 3d maximal SCFT on N M2's → $N^{3/2}$ d.o.f. → partial confinement even at high T
  - $AdS_5 \times S^5$: 4d maximal SYM → N^2 d.o.f.
  - $AdS_6 \times_w S^4/Z_2$: rank N 5d SCFTs on N D4's → $N^{5/2}$
  - $AdS_7 \times S^4$: 6d (2,0) SCFT on N M5's → N^3

  → more exotic/drastic deconfinements

- Today: BH's & deconfinement mechanisms of AdS_5/CFT_4 & AdS_4/CFT_3

BPS black holes in $AdS_5 \times S^5$

- Complicated solutions: Solutions of 5d SUGRA ← consistent truncation on $AdS_5 \times S^5$

  [Gutowski,Reall] (2004) [Chong,Cvetic,Lu,Pope] (2005) [Kunduri,Lucietti,Reall] (2006) .

- Preserves 2 real SUSY (1/16-BPS).
- Carries $SO(6)$ charges $Q_{I=1,2,3}$ on S^5 & $SO(4)$ spins $J_{i=1,2}$ on AdS_5 .
- BPS mass (= energy) given by: $M\ell = Q_1 + Q_2 + Q_3 + J_1 + J_2$
- Known solutions have a charge relation: Solutions w/ 4 independent parameters

$$Q_1Q_2Q_3 + \frac{N^2}{2}J_1J_2 = \left(\frac{N^2}{2} + Q_1 + Q_2 + Q_3\right)\left(Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \frac{N^2}{2}(J_1 + J_2)\right)$$

- There are many possibilities of generalization in full 10d type IIB SUGRA
- E.g. 10d uplifts of 5d solutions are homogeneous along $U(1)^3$ circles of S^5.

$$ds_{10}^2 = \Delta^{\frac{1}{2}} ds_5^2 + \frac{\ell^2}{\Delta^{\frac{1}{2}}} \sum_{I=1}^{3} X^I \left(d\mu_I^2 + \mu_I^2 \left(d\phi^I + \ell^{-1} A^I\right)^2\right) \qquad \Delta = \sum_I X^I \mu_I^2 \qquad \mu_1^2 + \mu_2^2 + \mu_3^2 = 1$$

- Hairy BH's: charge condensates outside the horizon~ inhomogeneous along $U(1)^3$
- Both non-BPS & BPS (numerical) [Markeviciute,Santos] [Bhattacharyya,Minwalla,Papadodimas]
- The general set of BPS black holes in $AdS_5 \times S^5$ is probably unknown.

The 4d index

- We wish to study these BH's (& predict new ones) from a QFT index on $S^3 \times R$.
- BPS states at strong coupling: "index" on $S^3 \times R$ is coupling independent.

  [Romelsberger] [Kinney,Maldacena,Minwalla,Raju] ≡ [KMMR] (2005)

$$Z(\Delta_I, \omega_i) = \mathrm{Tr}\left[(-1)^F e^{-\sum_{I=1}^3 \Delta_I Q_I - \sum_{i=1}^2 \omega_i J_i}\right] \quad \longleftrightarrow \quad \mathrm{Tr}\left[e^{-\sum_{I=1}^3 \Delta_I Q_I - \sum_{i=1}^2 \omega_i J_i}\right]$$

$$\Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 0 \ (\mathrm{mod}\ 4\pi i) \qquad \Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 2\pi i \ (\mathrm{mod}\ 4\pi i)$$

Use $(-1)^F = e^{-2\pi i Q_1}$ and shift $\Delta_1 \to \Delta_1 - 2\pi i$

- U(N) group:

$$Z = \frac{1}{N!} \int \prod_{a=1}^{N} \frac{d\alpha_a}{2\pi} \prod_{a<b} \left(2\sin\frac{\alpha_{ab}}{2}\right)^2 \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} \left(1 - \frac{\prod_{I=1}^3 2\sinh\frac{n\Delta_I}{2}}{2\sinh\frac{n\omega_1}{2} \cdot 2\sinh\frac{n\omega_2}{2}}\right) \sum_{a,b=1}^{N} e^{in\alpha_{ab}}\right]$$

Or the version with $\Delta_1 \to \Delta_1 - 2\pi i$ shift

- Large N saddle points? Many saddle points can compete.

- Simpler set-up: high "temperature" limit. (Recall $E\ell = Q_1 + Q_2 + Q_3 + J_1 + J_2$.)

- Study sectors w/ large angular momenta $J_i \gg N^2$ (for large BH's)
- Can use either/both $1/N \ll 1$, $N^2/J_i \ll 1$ for semi-classical approximation of the integral.

High “temperature” Cardy limit

- “high T limit” : $J_i \gg N^2 (\gg 1)$, $|\omega_i| \ll 1$. Similar to 2d Cardy limit, $P \gg c (\gg 1)$, $\tau \to i0^+$.

- Should also take $Re(\Delta_I) \to 0^+$, but generically keep finite $\mathrm{Im}(\Delta_I) \sim O(1)$:

$$\mathrm{Tr}\left[e^{-\sum_{I=1}^{3}\Delta_I Q_I - \sum_{i=1}^{2}\omega_i J_i}\right] \qquad \Delta_1+\Delta_2+\Delta_3-\omega_1-\omega_2 = 2\pi i \ (\mathrm{mod}\ 4\pi i)$$

- The matrix integral becomes:

$$Z \sim \frac{1}{N!}\oint \prod_{a=1}^{N}\frac{d\alpha_a}{2\pi}\exp\left[-\frac{1}{\omega_1\omega_2}\sum_{a\neq b}\sum_{s_1,s_2,s_3=\pm 1} s_1 s_2 s_3 \mathrm{Li}_3\left(-e^{\frac{s_I\Delta_I}{2}}e^{i\alpha_{ab}}\right)\right] \qquad \mathrm{Li}_3(x) \equiv \sum_{n=1}^{\infty}\frac{x^n}{n^3}$$

- Lots of evidence that “maximally deconfining” saddle $\alpha_1 = \alpha_2 = \cdots = \alpha_N$ is dominant

  (Assume $Im(\omega_1\omega_2) < 0$ & confirm self-consistently later). [Choi, Joonho Kim, SK, Nahmgoong]≡[CKKN] (2018) [Honda] [Ardehali] [J. Kim, SK, Song] [Cabo Bizet, Cassani, Martelli, Murthy] (2019)

- This is natural, since deconfined quarks/gluons are effectively massless at high T.

- True meaning: Eigenvalues asymptotically get close at high T.

$$|\alpha_a - \alpha_b| \sim |\omega_1\omega_2|^{\frac{1}{2}} \ll 1$$

- In fact true for “all” $N = 1$ SCFTs (checked for numerous examples) [J. Kim, SK, Song]

Cardy limit *(continued)*

- Plug in the maximally deconfining saddle to the matrix integrand:

$$Z \sim \frac{1}{N!}\oint \prod_{a=1}^{N}\frac{d\alpha_a}{2\pi}\exp\left[-\frac{1}{\omega_1\omega_2}\sum_{a\neq b}\sum_{s_1,s_2,s_3=\pm 1} s_1s_2s_3\mathrm{Li}_3\left(-e^{\frac{s_I\Delta_I}{2}}e^{i\alpha_{ab}}\right)\right]$$

$$\longrightarrow \log Z \sim -\frac{N^2}{\omega_1\omega_2}\sum_{s_1s_2s_3=+1}\left[\mathrm{Li}_3\left(-e^{\frac{s_I\Delta_I}{2}}\right)-\mathrm{Li}_3\left(-e^{-\frac{s_I\Delta_I}{2}}\right)\right]$$

- Here, $Re(\Delta_I)$, $Re(\omega_i)$ naturally originate from chemical potentials conjugate to charges.
- $Im(\Delta_I)$, $Im(\omega_i)$: Forced to be turned on by $(-1)^F$. Should optimally choose them to have maximal entropy when one Legendre transforms to microcanonical ensemble.
- The dominant region is $-\pi < \frac{1}{2}Im(s_I\Delta_I) < \pi$: Final result [CKKN]

Use: $\mathrm{Li}_3(-e^x)-\mathrm{Li}_3(-e^{-x}) = -\frac{x^3}{6}-\frac{\pi^2 x}{6}$, $-\pi < \mathrm{Im}(x) < \pi$

$$\log Z \sim -\frac{N^2}{\omega_1\omega_2}\sum_{s_1s_2s_3=+1}\left[\mathrm{Li}_3\left(-e^{\frac{s_I\Delta_I}{2}}\right)-\mathrm{Li}_3\left(-e^{-\frac{s_I\Delta_I}{2}}\right)\right] \longrightarrow \log Z \sim \frac{N^2\Delta_1\Delta_2\Delta_3}{2\omega_1\omega_2}$$

- Few subleading corrections in ω_i's computable [Cabo-Bizet, et.al.] [Nahmgoong] …

$$\log Z \sim \frac{N^2\Delta_1\Delta_2\Delta_3}{2\omega_1\omega_2} \qquad \text{with} \quad \Delta_1+\Delta_2+\Delta_3-\omega_1-\omega_2 = 2\pi i$$

- Exact large N saddle pt. with this free energy at finite ω_i [Cabo-Bizet, Murthy].

Legendre transformation & interpretation

- Further take large N & compute entropy: Legendre transform at macroscopic charge

$$S(\Delta_I, \omega_i; Q_I, J_i) = \frac{N^2}{2}\frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2} + \sum_{I=1}^{3} Q_I\Delta_I + \sum_{i=1}^{2} J_i\omega_i \qquad \Delta_1+\Delta_2+\Delta_3-\omega_1-\omega_2 = 2\pi i$$

- Already discussed in the context of BH solutions [Hosseini, Hristov, Zaffaroni] (2017)

[Interpretation of the calculus] … purely within QFT [CKKN] (2018) [Choi, SK] (2019), ……

- $S_*(Q_I, J_i)$ is in general complex.
- Index may have overall sign. $e^{i\,Im(S_*)}$ imitates $\pm$ sign alternations as charges change.
- More precisely, ∃ c.c. saddle point: $\sim e^{Re(S)}\cos[Im(s)+\cdots]$
- $Re(S_*)$: One should find solutions which maximizes this.

- E.g., at $Q_1 = Q_2 = Q_3, J_1 = J_2$, one can set $\omega_1 = \omega_2 \equiv \omega$, $\Delta_1 = \Delta_2 = \Delta_3 \equiv \Delta = \frac{2\pi i + 2\,\omega}{3}$

$$S(Q, J; \omega) = \frac{N^2\left(\frac{2\pi i}{3} + \frac{2\omega}{3}\right)^3}{2\omega^2} + 2\omega(Q+J) + 2\pi i Q$$

- Extremize → cubic eqn. in ω: choose the solution with largest $Re[S_*(Q+J)]$.

Evidence of the interpretation

- E.g. consider U(5) index. ($N = 5$ is a compromise of reasonably large $N^2 = 25$ & fast calculus.)
- Consider an unrefined index $Z(x) = \sum_j \Omega_j x^j$ (where $j \equiv 6(Q + J)$).

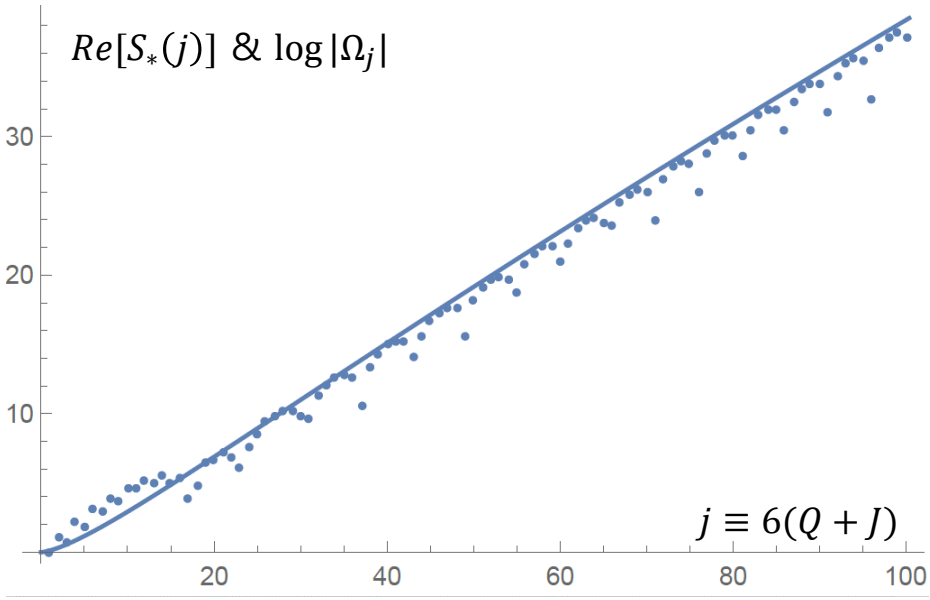
$$e^{-\omega} = x^3,\ e^{-\Delta} = x^2$$
$$\Delta_1 = \Delta_2 = \Delta_3 \equiv \Delta,\ \omega_1 = \omega_2 \equiv \omega$$

$$\begin{aligned} Z \ = \ & 1 + 3x^2 - 2x^3 + 9x^4 - 6x^5 + 21x^6 - 18x^7 + 48x^8 - 42x^9 + 99x^{10} - 96x^{11} + 172x^{12} - 156x^{13} + 252x^{14} - 160x^{15} \\ & + 195x^{16} + 48x^{17} - 127x^{18} + 612x^{19} - 783x^{20} + 1258x^{21} - 948x^{22} + 450x^{23} + 1921x^{24} - 5430x^{25} + 11793x^{26} \\ & - 18812x^{27} + 26379x^{28} - 27750x^{29} + 17809x^{30} + 15648x^{31} - 78324x^{32} + 175030x^{33} - 285576x^{34} + 366024x^{35} \\ & - 323807x^{36} + 38856x^{37} + 624894x^{38} - 1718016x^{39} + 3094992x^{40} - 4226862x^{41} + 4098270x^{42} - 1210728x^{43} \\ & - 5968935x^{44} + 18061488x^{45} - 33152565x^{46} + 44941584x^{47} - 41448422x^{48} + 6241896x^{49} + 75761478x^{50} \\ & - 205993284x^{51} + 354209109x^{52} - 440168670x^{53} + 328572109x^{54} + 142704804x^{55} - 1079522706x^{56} \\ & + 2385844062x^{57} - 3584202447x^{58} + 3694263972x^{59} - 1331772481x^{60} - 4771857420x^{61} + 14697077445x^{62} \\ & - 25833114276x^{63} + 31549909440x^{64} - 21264664440x^{65} - 16439430686x^{66} + 86286819246x^{67} - 174750537792x^{68} \\ & + 238416590234x^{69} - 201108631665x^{70} + \mathcal{O}(x^{71}) \end{aligned}$$

- Ω_j grows at large j, but signs alternate.
- Can't absorb all −'s to independent fugacity.
- $S_j \equiv \log|\Omega_j| \sim N^2$ at $j \sim N^2$.

line: $\mathrm{Re}[S_*(j)]$ from our free energy, inserting $N^2 \to 25$
dots: $S_j = \log|\Omega_j|$ from the index

Note: $Im[S_*(j)]$ is nontrivial in this calculus.



Black hole microstates

- General Legendre transformation is cumbersome but analytically doable.
- We want to see if $Re(S_*)$ accounts for the Bekenstein-Hawking entropy of known BH solutions. (Analytic solution found from 5d SUGRA)

- Here, recall that the known analytic BH solutions satisfy a charge relation

$$Q_1Q_2Q_3 + \frac{N^2}{2}J_1J_2 = \left(\frac{N^2}{2} + Q_1 + Q_2 + Q_3\right)\left(Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \frac{N^2}{2}(J_1 + J_2)\right)$$

- I don't know what it means in QFT (yet), though I have a natural conjecture.
- Impose it by hand on $Re[S_*]$.
- One can now obtain various (equivalent) expressions for the entropy in terms of 5 charges.
- In particular, one obtains [CKKN]

$$Re[S_*(Q_I, J_i)] = 2\pi\sqrt{Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \frac{N^2}{2}(J_1 + J_2)}$$
$$= 2\pi\sqrt{\frac{Q_1Q_2Q_3 + \frac{N^2}{2}J_1J_2}{\frac{N^2}{2} + Q_1 + Q_2 + Q_3}}$$

known expression for S_{BH} [SK, Kimyeong Lee] (2006)

Compatibility of two expressions: charge relation of known BH's

3d QFTs & deconfinement

- We now study BH's and deconfinement in the Cardy limit of AdS_4/CFT_3.
- We focus on maximal SCFT on N M2-branes. There are many QFT descriptions

- ABJM: $U(N)_1 \times U(N)_{-1}$ Chern-Simons matter CFT. [Aharony,Bergman,Jafferis,Maldacena]

- 3d maximal SYM: $U(N)$ Yang-Mills with $N = 8$ SUSY

  Stack of D2-branes probing flat space R^7 . Flows to maximal SCFT in IR. Very hard to use.

- "Mirror dual" of maximal SYM: N D2-branes probing 1 D6-brane in flat space

  $N = 4$ SUSY in UV. $U(N)$ gauge theory w/ 1 fundamental & 1 adjoint hypers.

  Flows to same maximal SCFT.

- They all have $N \times N$ matrix degrees of freedom.
- But "maximally deconfining" saddle w/ $\log Z \sim N^2$ should be forbidden: We want $N^{3/2}$.
- Need to understand this "partial confinement" mechanism (even at high T)

Yang-Mills & the index

- We use $N = 4$ Yang-Mills description. (ABJM is also OK, although a bit uglier.)

$$Z(\Delta_I, \beta) = \mathrm{Tr}\left[(-1)^F e^{-\sum_{I=1}^4 \Delta_I Q_I - 2\beta J}\right] \qquad \sum_I \Delta_I - 2\beta = 0 \ (\mathrm{mod}\ 4\pi i)$$

$$\Delta_1 \equiv -\hat{\xi} + \frac{T}{2} + \frac{\beta}{2}\,, \quad \Delta_2 \equiv \hat{\xi} + \frac{T}{2} + \frac{\beta}{2}\,, \quad \Delta_3 \equiv f - \frac{T}{2} + \frac{\beta}{2}\,, \quad \Delta_4 \equiv -f - \frac{T}{2} + \frac{\beta}{2}$$

(β: $N = 2$ R-charge & J, T: extra Cartan of $N = 4$ R-charge, $\hat{\xi}$: topological charge, f: flavor of adj. hyper)

- matrix integral & charge sum for the monopole operators ($q \equiv e^{-\beta}$, $t \equiv e^{T}$, $z \equiv e^{f}$)

$$\begin{aligned} Z \;=\; & \sum_{\{m\}=-\infty}^{\infty} \frac{1}{\mathrm{Weyl}(\{m\})} \oint \frac{d\alpha_a}{2\pi} e^{-\hat{\xi}\sum_{a=1}^N m_a} \prod_{a=1}^N (qt^{-1})^{\frac{|m_a|}{2}} \frac{(e^{-i\alpha_a} q^{|m_a|} t^{-\frac{1}{2}} q^{\frac{3}{2}}; q^2)(e^{i\alpha_a} q^{|m_a|} t^{-\frac{1}{2}} q^{\frac{3}{2}}; q^2)}{(e^{i\alpha_a} q^{|m_a|} t^{\frac{1}{2}} q^{\frac{1}{2}}; q^2)(e^{-i\alpha_a} q^{|m_a|} t^{\frac{1}{2}} q^{\frac{1}{2}}; q^2)} \\ & \times \prod_{a\neq b} q^{-\frac{|m_{ab}|}{2}} (1 - e^{i\alpha_{ab}} q^{|m_{ab}|}) \prod_{a,b=1}^N t^{\frac{|m_{ab}|}{2}} \frac{(e^{i\alpha_{ab}} t q^{1+|m_{ab}|}; q^2)}{(e^{i\alpha_{ab}} t^{-1} q^{1+|m_{ab}|}; q^2)} \\ & \times \prod_{a,b=1}^N (qt^{-1})^{\frac{|m_{ab}|}{2}} \frac{(e^{i\alpha_{ab}} q^{|m_{ab}|} z^{-1} t^{-\frac{1}{2}} q^{\frac{3}{2}}; q^2)(e^{i\alpha_{ab}} q^{|m_{ab}|} z t^{-\frac{1}{2}} q^{\frac{3}{2}}; q^2)}{(e^{i\alpha_{ab}} q^{|m_{ab}|} z t^{\frac{1}{2}} q^{\frac{1}{2}}; q^2)(e^{i\alpha_{ab}} q^{|m_{ab}|} z^{-1} t^{\frac{1}{2}} q^{\frac{1}{2}}; q^2)} \end{aligned} \qquad (x; q^2) \equiv \prod_{n=0}^{\infty} (1 - x q^{2n})$$

- $(\cdots)^{|m|}$ are ground state "energy" factors of BPS states in monopole background
- Identity [Dimofte,Gaiotto,Gukov] : Allows holomorphic/anti-holomorphic combinations of m, α.

$$(e^{-i\rho(\alpha)} q^{1-R} y^{-1})^{\frac{|\rho(m)|}{2}} \frac{(e^{-i\rho(\alpha)} q^{2-R+|\rho(m)|} y^{-1}; q^2)}{(e^{i\rho(\alpha)} q^{R+|\rho(m)|} y; q^2)} = (e^{-i\rho(\alpha)} q^{1-R} y^{-1})^{-\frac{\rho(m)}{2}} \frac{(e^{-i\rho(\alpha)} q^{2-R-\rho(m)} y^{-1}; q^2)}{(e^{i\rho(\alpha)} q^{R-\rho(m)} y; q^2)}$$

Cardy limit: naïve saddle point

- Take $\beta \to 0$, $\Delta_I \sim$ pure imaginary & finite. Approximate the integrand using

$$(xq^a;q^2) \xrightarrow{\beta\to 0} \exp\left[-\frac{\mathrm{Li}_2(x)}{2\beta}\right]$$

- Similar to 4d, one naively finds "maximally deconfining" saddle with $\alpha_1 = \cdots = \alpha_N = 0$.
- However, sum over fluxes m_a changes the fate of this naïve saddle point.

- Flux sum around the "saddle point" of α_a, m_a^* yields [Choi, Hwang, SK] (2019)≡[CHK]

$$\prod_{a=1}^{N} 2\pi\delta\left(-\frac{i}{2}\log\frac{(1-e^{\beta m_a^*+i\alpha_a}t^{-\frac{1}{2}})(1-e^{\beta m_a^*-i\alpha_a}t^{-\frac{1}{2}})}{(1-e^{\beta m_a^*+i\alpha_a}t^{\frac{1}{2}})(1-e^{\beta m_a^*-i\alpha_a}t^{\frac{1}{2}})}+i\hat{\xi}-\frac{iT}{2}\right)$$

- Peak of delta function yields 1 solution: demands all α_a's to be exactly the same.
- So at this "saddle", Haar measure yields exact zero, suppressing its contribution.

$$\prod_{a\neq b}\left(1-e^{i\alpha_{ab}}\right)=0$$

- "Exclusion" principle for the matrix eigenvalues in 3d.
- First discovered in $U(N)_k$ vector-Chern-Simons theory: no conformal fixed point at $N > k$

[Aharony,Giombi,Gur-Ari,Maldacena,Yacobi] (2012) [Jain,Minwalla,Sharma,Takimi,Wadia] (2013)

Cardy limit: continuum approximation

- In our M2 QFT, there are more nontrivial saddles.

- Here, notice that fluxes appear only in the combination of βm_a.

- So in the Cardy limit $\beta \to 0$, sum over $m_a \sim$ integral over βm_a. [Pasquetti, Sacchi] (2019)
- Integral variables: $u_a = \beta m_a + i\alpha_a$ and $\bar{u}_a = \beta m_a - i\alpha_a$. [CHK]

$$Z \sim \int \prod_{a=1}^{N} du_a e^{-\frac{\xi}{2\beta}\sum_{a=1}^{N} u_a} \prod_{a=1}^{N} \frac{(e^{u_a} t^{-\frac{1}{2}} q^{\frac{3}{2}}; q^2)}{(e^{u_a} t^{\frac{1}{2}} q^{\frac{1}{2}}; q^2)} \prod_{a,b=1}^{N} \frac{(e^{u_{ab}} z t^{-\frac{1}{2}} q^{\frac{3}{2}}; q^2)}{(e^{u_{ab}} z t^{\frac{1}{2}} q^{\frac{1}{2}}; q^2)} \frac{\prod_{a\neq b}(e^{u_{ab}}; q^2)}{\prod_{a,b}(e^{u_{ab}} t^{-1} q; q^2)}$$

$$\times \int \prod_{a=1}^{N} d\bar{u}_a e^{-\frac{\xi}{2\beta}\sum_{a=1}^{N} \bar{u}_a} \prod_{a=1}^{N} \frac{(e^{\bar{u}_a} t^{-\frac{1}{2}} q^{\frac{3}{2}}; q^2)}{(e^{\bar{u}_a} t^{\frac{1}{2}} q^{\frac{1}{2}}; q^2)} \prod_{a,b=1}^{N} \frac{(e^{\bar{u}_{ab}} z^{-1} t^{-\frac{1}{2}} q^{\frac{3}{2}}; q^2)}{(e^{\bar{u}_{ab}} z^{-1} t^{\frac{1}{2}} q^{\frac{1}{2}}; q^2)} \frac{\prod_{a,b}(e^{\bar{u}_{ab}} t q; q^2)}{\prod_{a\neq b}(e^{\bar{u}_{ab}} q^2; q^2)}$$

$$(xq^a; q^2) \xrightarrow{\beta\to 0} \exp\left[-\frac{\mathrm{Li}_2(x)}{2\beta}\right]$$

- This is related to the factorization to holomorphic blocks, i.e. two $D_2 \times S^1$ partition functions.
- However, our Cardy factorization only refers to a particular term (particular b.c. on D_2)

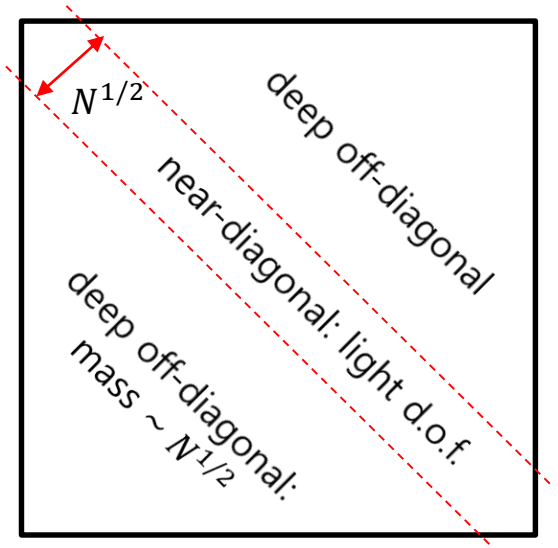
- So the problem reduces to finding Cardy (& large N) saddle points for u_a, ignoring the naïve one with $\log Z \sim N^2$ as explained in the previous slide.

Monopole condensation & (de)confinement

- Physical saddle point: u_a spread macroscopically along m_a direction.
- Analysis is very similar to $Z[S^3]$ or the topologically twisted index at large N.
- One finds a distribution in the following range (with $x_{1,2} \sim O(1)$ and $x_1 < 0 < x_2$)

$$N^{\frac{1}{2}} x_1 < \mathrm{Re}(\beta m_a + i\alpha_a) < N^{\frac{1}{2}} x_2$$

- Such monopole condensation breaks $U(N)$.
- Most of the N^2 d.o.f. become massive: acquire mass $\sim N^{1/2}$.
- Except the near-diagonal elements within the width $\sim N^{1/2}$.



- Monopole condensation is the partial confinement mechanism
- Explains $N^{3/2} \sim N^{1/2} \cdot N$ as # of light d.o.f. in the condensate.

- Resulting Cardy & large N free energy completely accounts for the microstates of large BH's in $AdS_4 \times S^7$. [Choi, Hwang, SK]

$$\log Z \sim -i\frac{2\sqrt{2}N^{\frac{3}{2}}\sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}}{3\beta} \qquad \sum_{I=1}^{4}\Delta_I - 2\beta = 2\pi i$$

Comments on other dimensions

- Higher dimensional SCFTs: "What does deconfinement mean?" "Quarks & gluons…?"

- 5d: Index formula on $S^4 \times R$ suggested from 5d SYM intuitions [H.-C. Kim, S.-S. Kim, K. Lee]

$$Z_{S^4\times S^1} = \oint [da] Z_{\mathbb{R}^4\times S^1}(ia, \epsilon_{1,2}, m_a, q) Z_{\mathbb{R}^4\times S^1}(-ia, \epsilon_{1,2}, -m_a, q^{-1})$$

- Uses instanton partition functions on $R^4_{\epsilon_1,\epsilon_2} \times S^1$ in the integrand
- Cardy saddle: Again, Coulomb VEV spread w/ width $\sim N^{1/2}$, but now enhancing d.o.f.
- 5d SCFT on N $D4 + N_f$ $D8 + O8$: [Choi, SK]

$$\log Z \sim -i\frac{\sqrt{2}N^{\frac{5}{2}}}{15\sqrt{8-N_f}}\frac{\Delta^3}{\omega_1\omega_2} \qquad \Delta - \omega_2 - \omega_2 = 2\pi i$$

- 6d: 't Hooft anomalies determine the Cardy index [CKKN] [Nahmgoong]

(see also [Di Pietro, Komargodski] (2014))

- (2,0) theory on N M5-branes

dominant term in the large N limit

$$\log Z \sim \boxed{-\frac{N^3}{24}\frac{\Delta_1^2\Delta_2^2}{\omega_1\omega_2\omega_3}} - \frac{N((\Delta_1+\Delta_2)^2+4\pi^2)((\Delta_1-\Delta_2)^2+4\pi^2)}{192\omega_1\omega_2\omega_3} + \frac{N(\Delta_1^2+\Delta_2^2-4\pi^2)(\omega_1^2+\omega_2^2+\omega_3^2)}{96\omega_1\omega_2\omega_3} + \mathcal{O}(\log\omega)$$

Conclusion & remarks

- Index of $SCFT_D$ sees deconfinement & AdS_{D+1} black holes.
  - High temperature behaviors are simple. Captured by “Cardy formulae”

- In 4d, maximally deconfining saddle at high temperature QFT & large BH’s.
- In non-Cardy regimes, dominant saddle points can be yet unknown **new BH’s**.
  - We studied a 1/8-BPS sector of $N = 4$ SYM (Macdonald index [Gadde,Rastelli,Razamat,Yan]).
  - Known BH’s degenerate to singular solutions, but our QFT free energy scales like N^2.
  - Despite partial evidences, much more studies are required.

- In 3d, $N^{3/2}$ d.o.f. via partial confinement: magnetic monopole condensation.

- In 5d, 6d, various (indirect) methods used to count the dual BH microstates.
  - But not enough intuitions on deconfinement mechanisms.
  - Perhaps, 5d instanton partons hold the key. [Collie, Tong] [Bolognesi, Kimyeong Lee]

- Characterization of (non-)hairiness of BH’s from QFT & charge relations…?