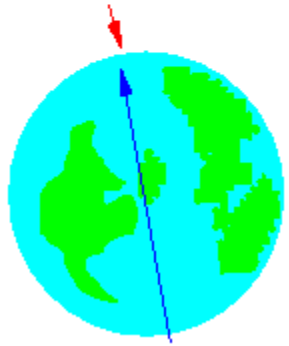
Three Neutrino Oscillations

Lesson Learned from CKM: 3 mixing angles and a phase

Call them $\theta_{12}, \theta_{23}, \theta_{13}, \delta$ if $s_{ij} = \sin\theta_{ij}, c_{ij} = \cos\theta_{ij}$, then

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

U=  Reactor and/or Accelerator ν_e 

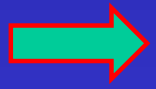
Three-Neutrino Oscillations

- Electron neutrino survival probability

$$P_{\nu_e \to \nu_e} = \left| \sum_{k=1}^{3} |U_{ek}|^2 e^{-i m_k^2 L/2E} \right|^2$$

$$|U_{e3}|^2 \ll |U_{e1}|^2, |U_{e2}|^2 \quad \Longrightarrow \quad |U_{e1}|^2 \simeq \cos\vartheta_{12}\,, \; |U_{e2}|^2 \simeq \sin\vartheta_{12}$$

$$P_{\nu_e \to \nu_e} = 1 - \sin^2 2\vartheta_{12} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$



It is decoupled from atmospheric ν osc.

Three-Neutrino Oscillations

- In the case that $|\Delta m^2_{21}| \ll |\Delta m^2_{31}| \cong |\Delta m^2_{32}|$

$$P_{\nu_\alpha \to \nu_\beta} = \left| U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2} + U^*_{\alpha 3} U_{\beta 3} \exp\left(-i \frac{\Delta m^2_{31} L}{2E} \right) \right|^2$$

$$U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2} = \delta_{\alpha\beta} - U^*_{\alpha 3} U_{\beta 3}$$

$$= \delta_{\alpha\beta} - 4|U_{\alpha 3}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 3}|^2 \right) \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

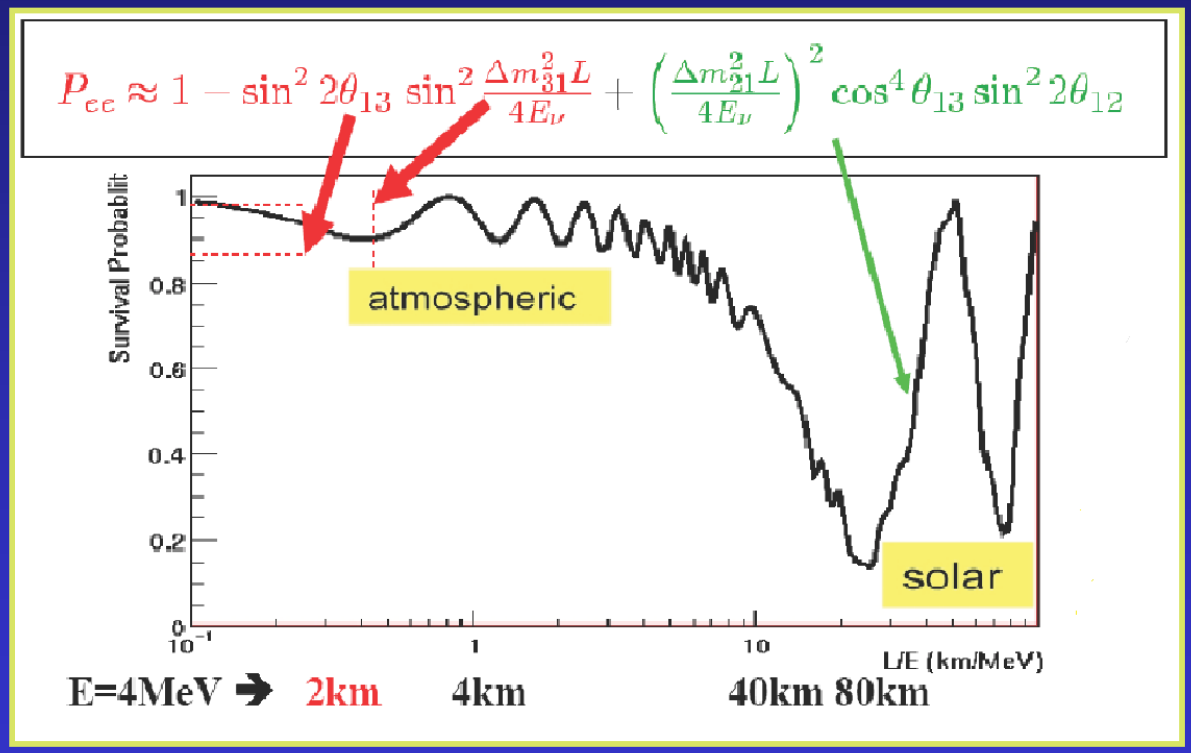
$$\alpha \neq \beta \implies P_{\nu_\alpha \to \nu_\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2 \left(\frac{\Delta m^2_{31} L}{4E} \right)$$

$$\alpha = \beta \implies P_{\nu_\alpha \to \nu_\alpha} = 1 - 4|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2 \right) \sin^2 \left(\frac{\Delta m^2_{31} L}{4E} \right)$$

Atmospheric & reactor exp.

Three-Neutrino Oscillations

- Example for oscillation from reactor neutrinos with near and far detectors



How can we detect Neutrinos?

Neutrino detectors

Super Kamiokande

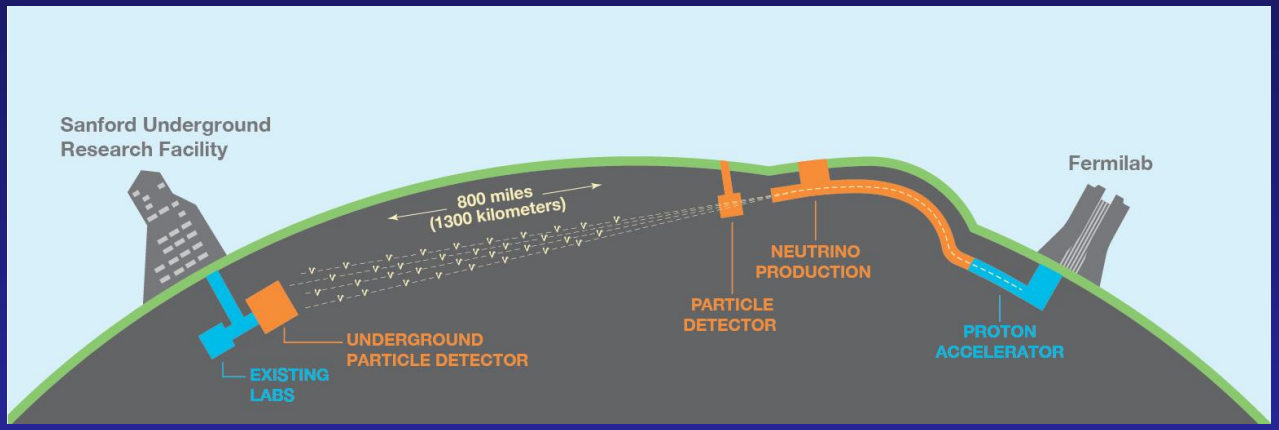




300 events/yr

detect about 1 solar neutrino
per day per 100 tons of water

How can we detect Neutrinos?



Deep Underground Neutrino Exp.

RENO

T2KK and T2KO

- Longer baseline than T2K to enhance sensitivity to the MH
- Wide band beam for sensitivity to $\cos\delta_{CP}$ (phase) as well as $\sin\delta_{CP}$ (amplitude).

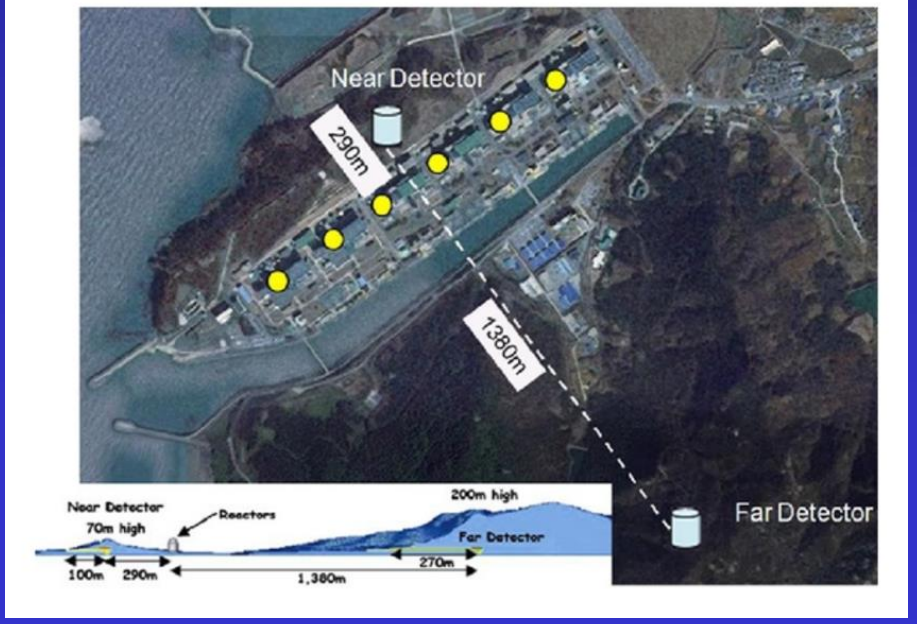


T2KK: 100 kton in Korea and **SK** [hep-ph/0410229, 0504026, 0901.1517, 1001.5165]

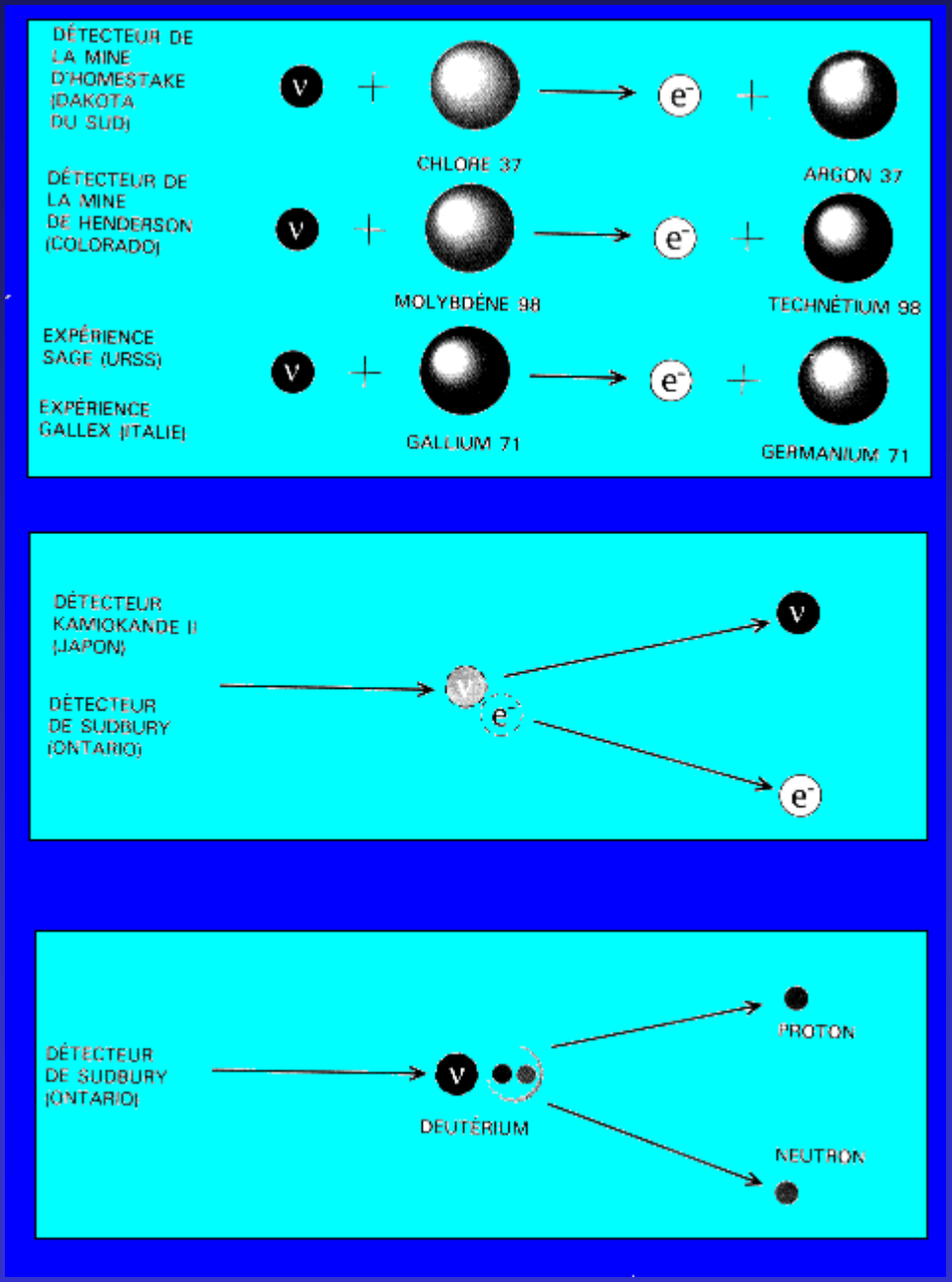
T2KO: 100 kton in Oki island and **SK** [0804.2111, 1209.2763]

* **SK** (22.5kton) is also used as a second detector

16

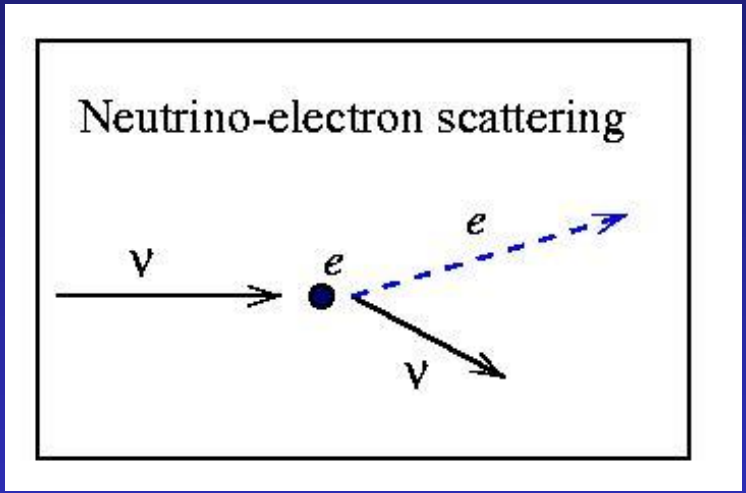


How can we detect Neutrinos?

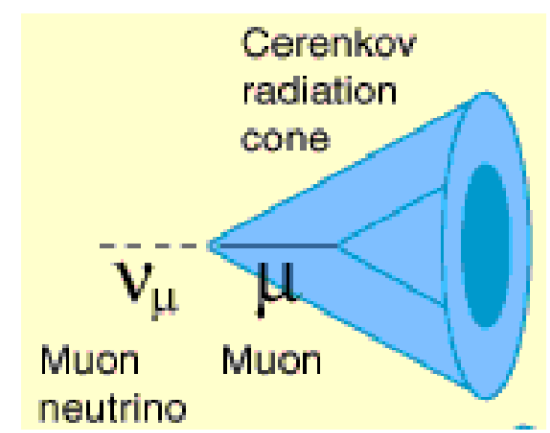


Water Cerenkov method:

$$\nu_x + e^- \longrightarrow \nu_x + e^-$$



Cerenkov radition : emitted when a charged particle passes through a medium at a speed greater than phase velocity of light



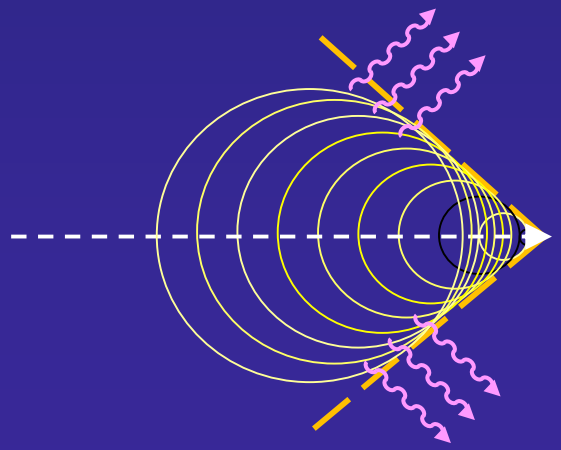


An F-18 Breaking the Sound Barrier

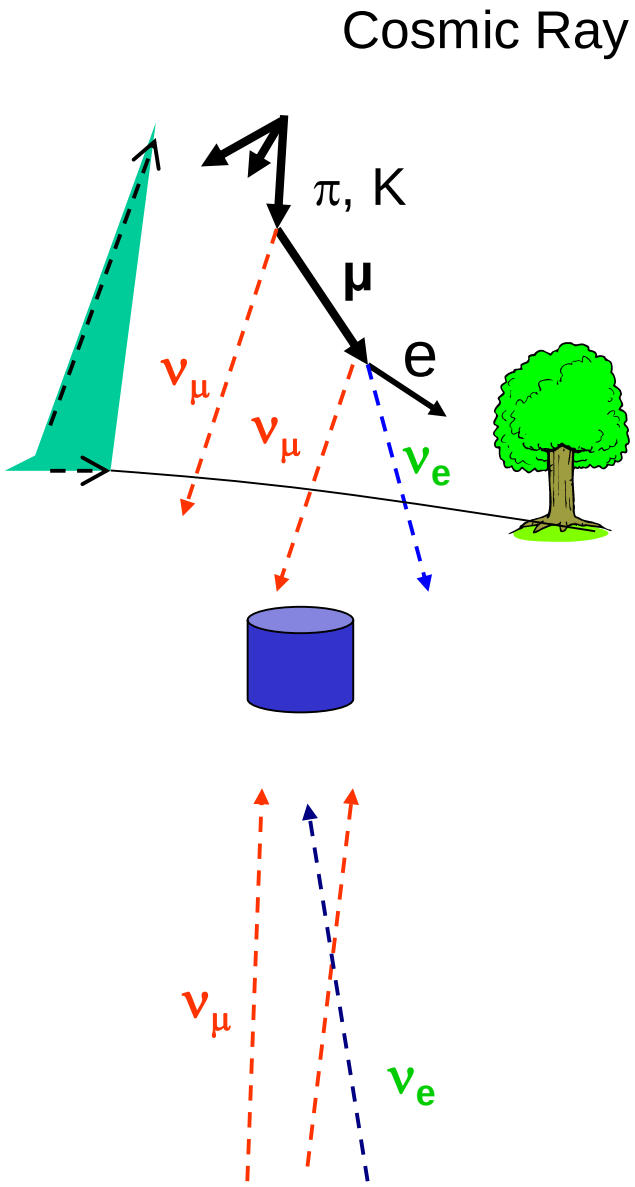
(similar to sonic boom)

Cherenkov Radiation: Shockwave of Light

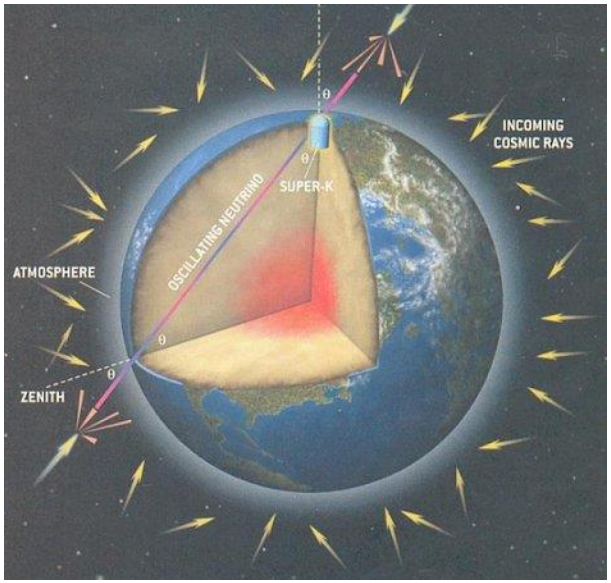
- In a material that has index of refraction n, light moves with speed $V_L = c/n$ (c is the speed of light in vacuum = $3x10^8$ m/s).
  - For water, n = 1.33
  - Light moves ~30% slower in water than in vacuum.
- It's very common for a charged particle, like an electron, to go faster than this.
  - If its energy is high enough (a few MeV) then it will have a speed V very close to c.



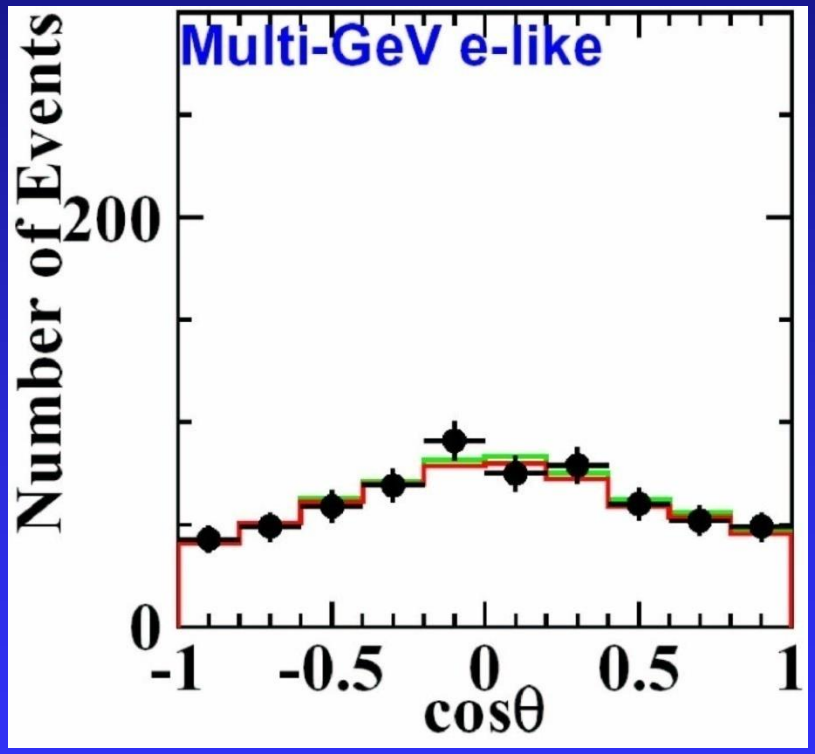
Atmospheric Neutrinos

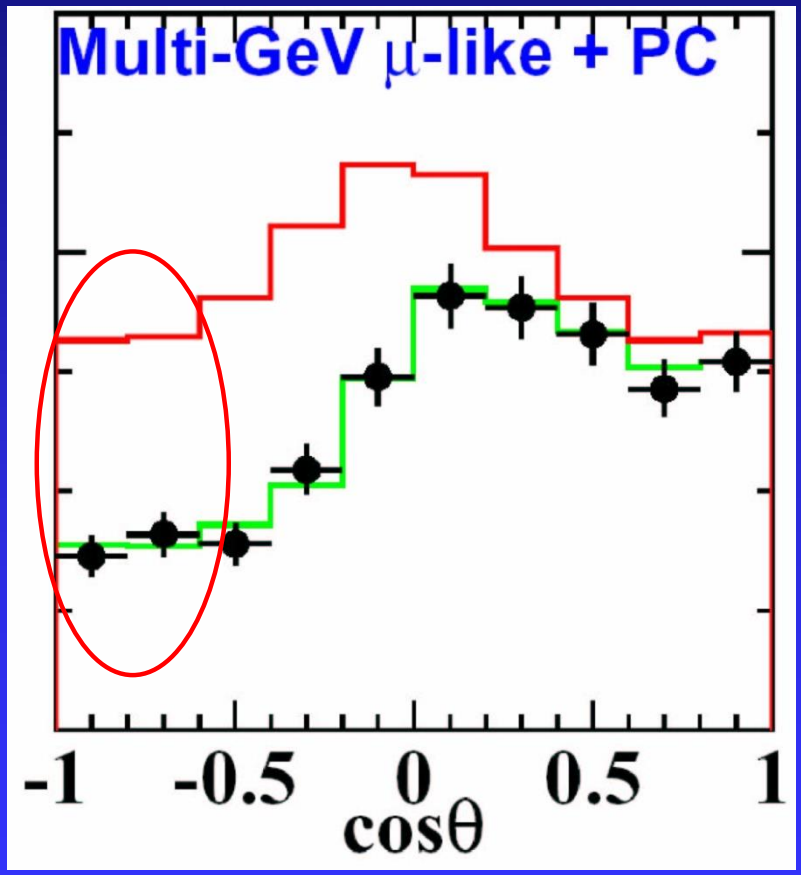


Neutrinos from the other side of the Earth.



A half of ν_μ lost !!



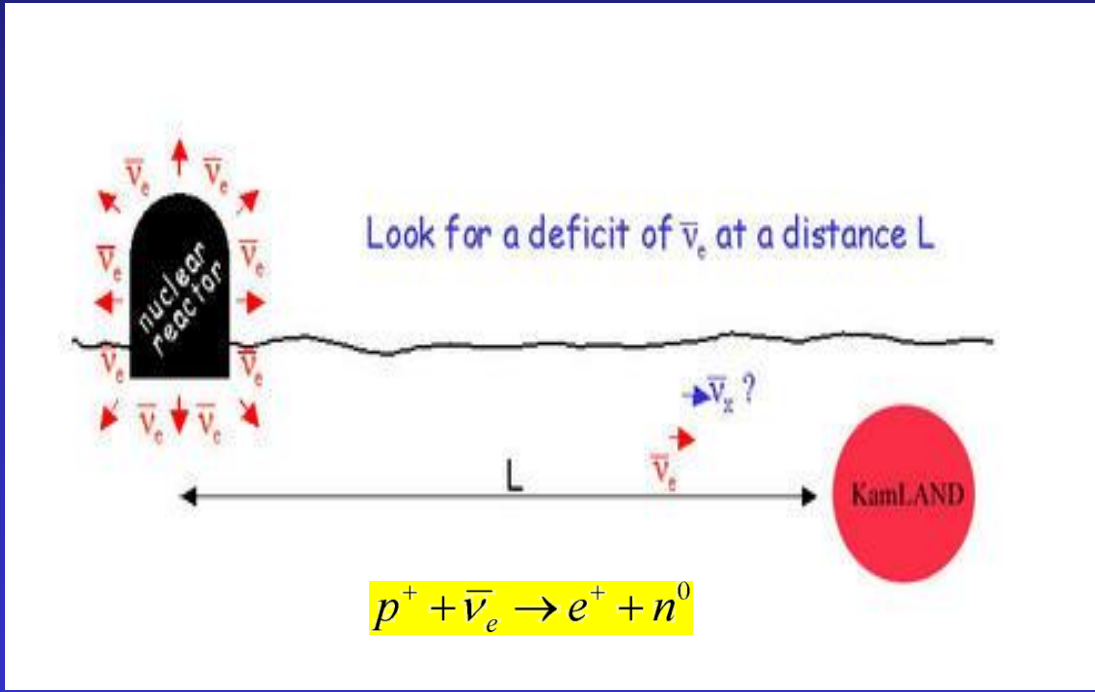


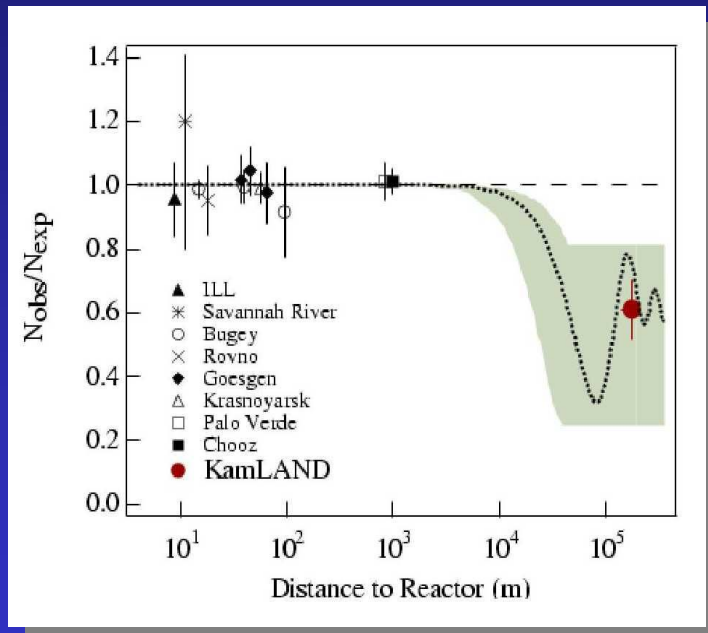
$$\phi(\text{관측}) = P(survival) \cdot \phi(\text{생성})$$

KamLAND results

Reactor long baseline experiment
150 - 210 km **(E_{pr} > 2.6 MeV)**

[K. Eguchi et al., Phys. Rev. Lett., 90, 021802 (2003)]





N_{obs}/N_{exp} = 0.611 + 0.094

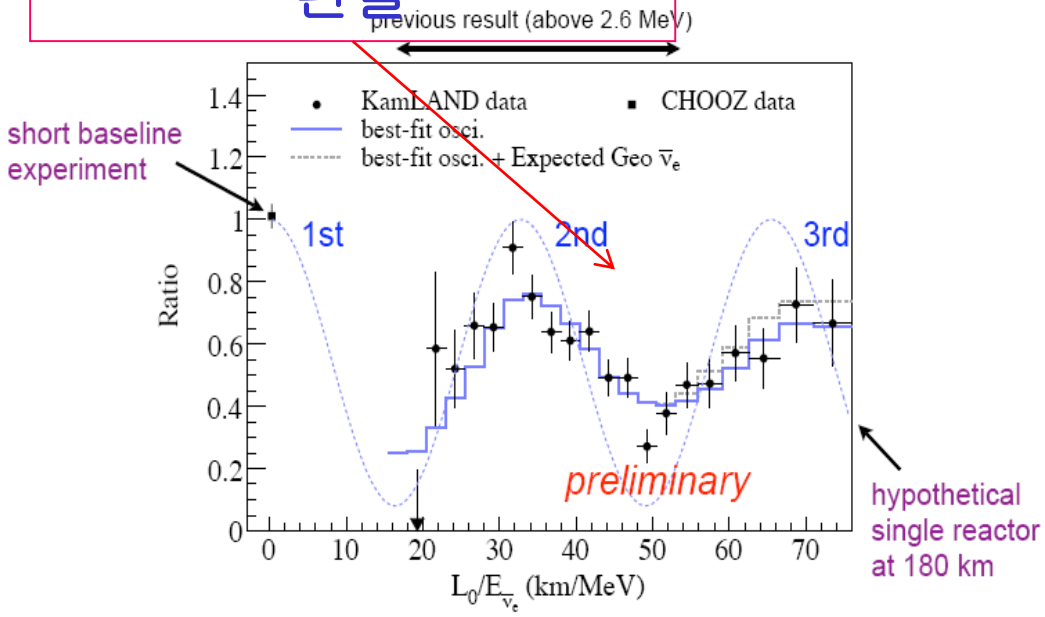
- in consistent with the results of solar neutrino oscillation

KAMLAND & SK (2008)

L/E distribution of events

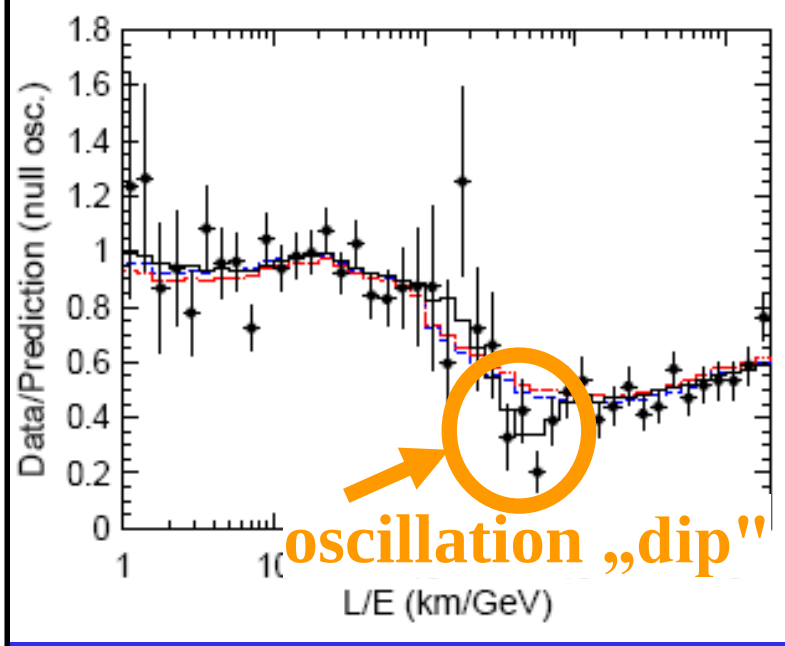
한 주기 oscillation 관찰

Itaru Shimizu



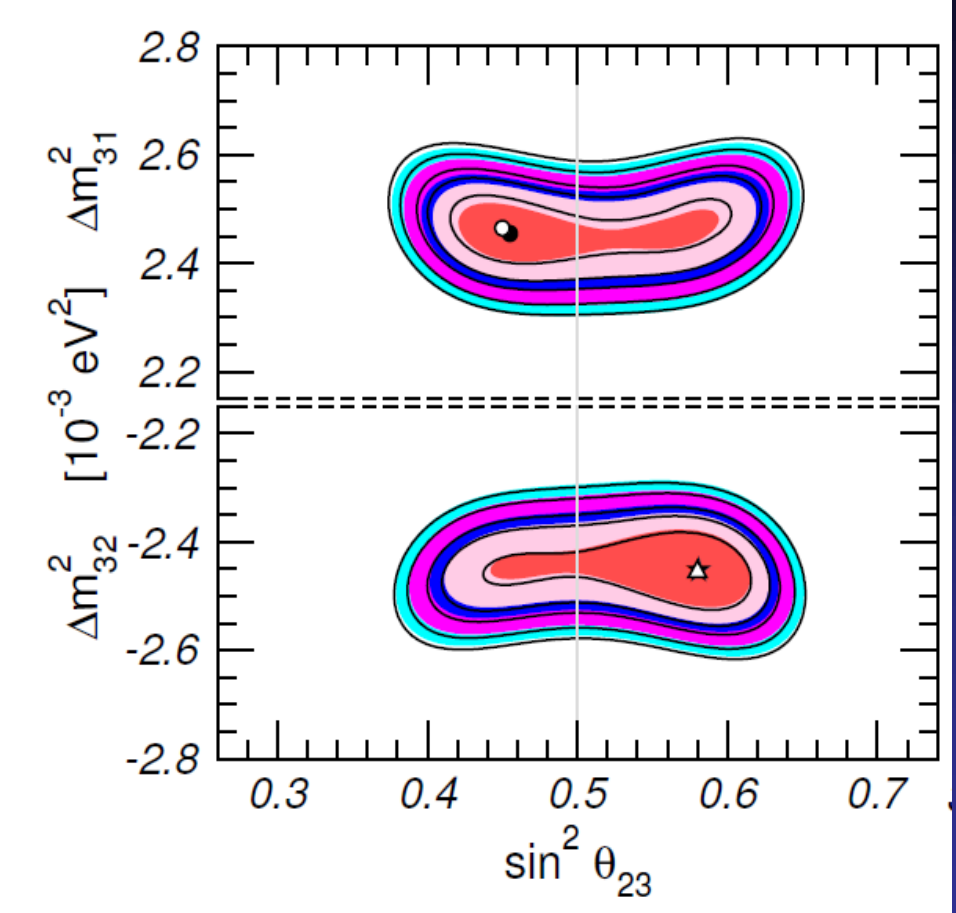
KamLAND covers the 2nd and 3rd maximum

→ characteristic of neutrino oscillation



진동이외의 다른 가설은 기각됨

Results

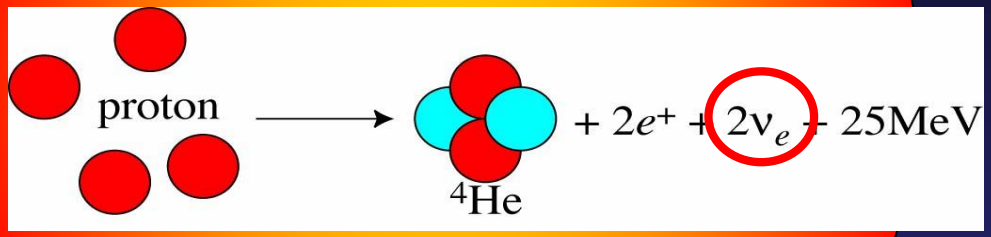


Evidence for the existence of neutrino masses and mixing

Solar Neutrinos

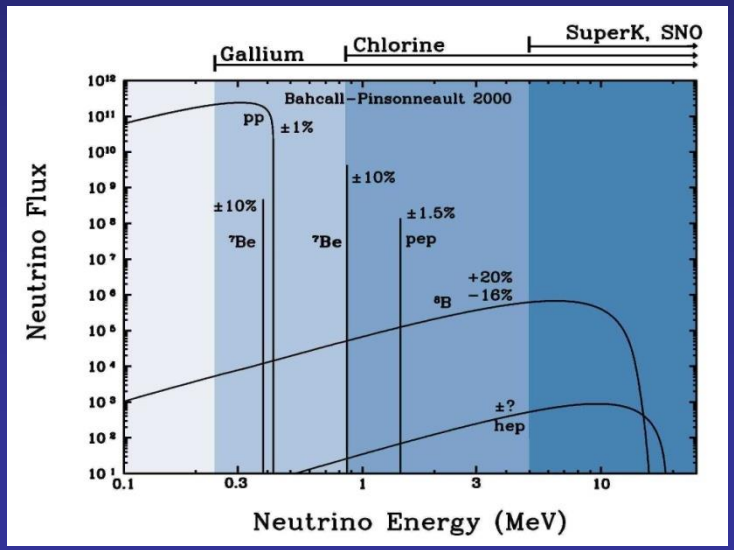
핵융합

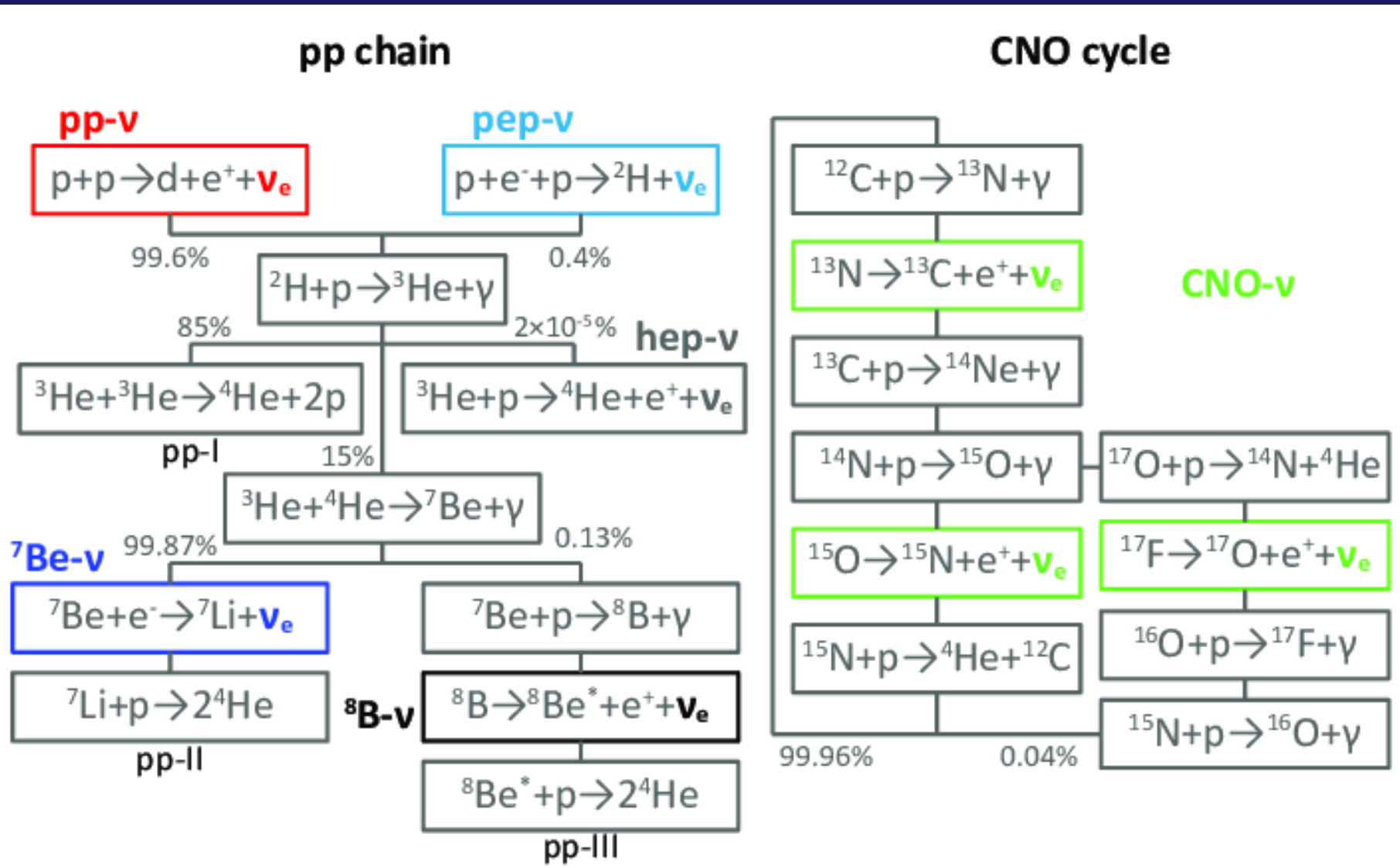
Sun burns!!!



Production of electron neutrinro

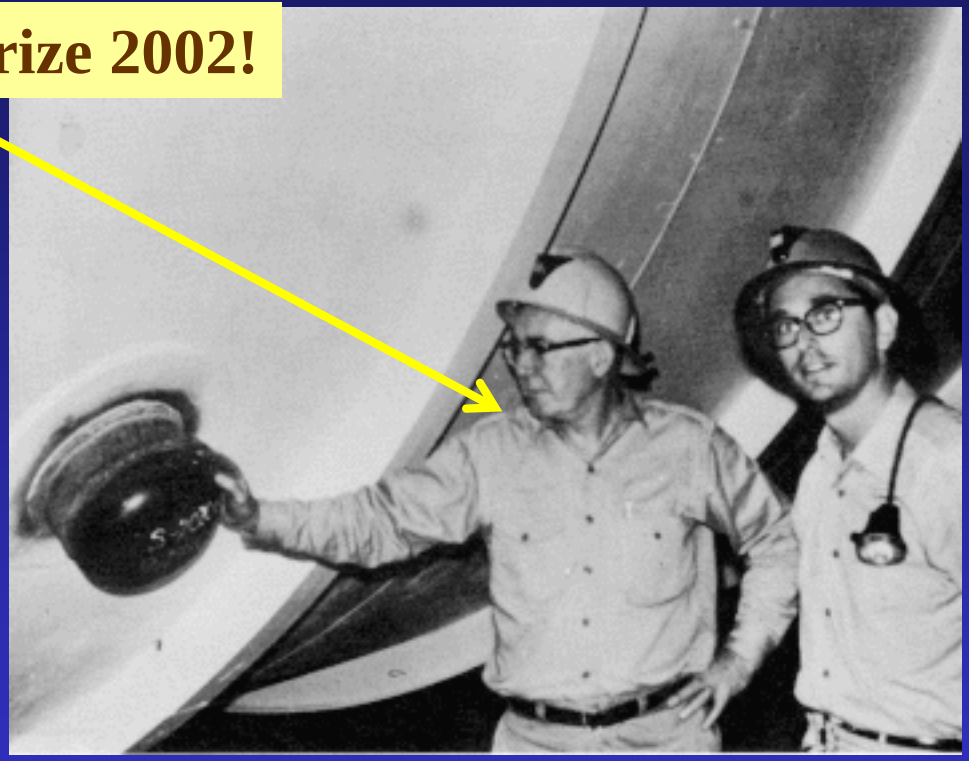
Satndard Solar Model (Bahcall)




pp chain
pp-ν
p+p→d+e⁺+νe
pep-ν
p+e⁻+p→²H+νe
99.6%
0.4%
²H+p→³He+γ
85%
2×10⁻⁵%
hep-ν
³He+³He→⁴He+2p
³He+p→⁴He+e⁺+νe
pp-I
15%
³He+⁴He→⁷Be+γ
⁷Be-ν
99.87%
0.13%
⁷Be+e⁻→⁷Li+νe
⁷Be+p→⁸B+γ
⁷Li+p→2⁴He
⁸B-ν
⁸B→⁸Be*+e⁺+νe
pp-II
⁸Be*+p→2⁴He
pp-III
CNO cycle
¹²C+p→¹³N+γ
¹³N→¹³C+e⁺+νe
CNO-ν
¹³C+p→¹⁴Ne+γ
¹⁴N+p→¹⁵O+γ
¹⁷O+p→¹⁴N+⁴He
¹⁵O→¹⁵N+e⁺+νe
¹⁷F→¹⁷O+e⁺+νe
¹⁵N+p→⁴He+¹²C
¹⁶O+p→¹⁷F+γ
¹⁵N+p→¹⁶O+γ
99.96%
0.04%

Solar Neutrino Experiments

Won Nobel Prize 2002!

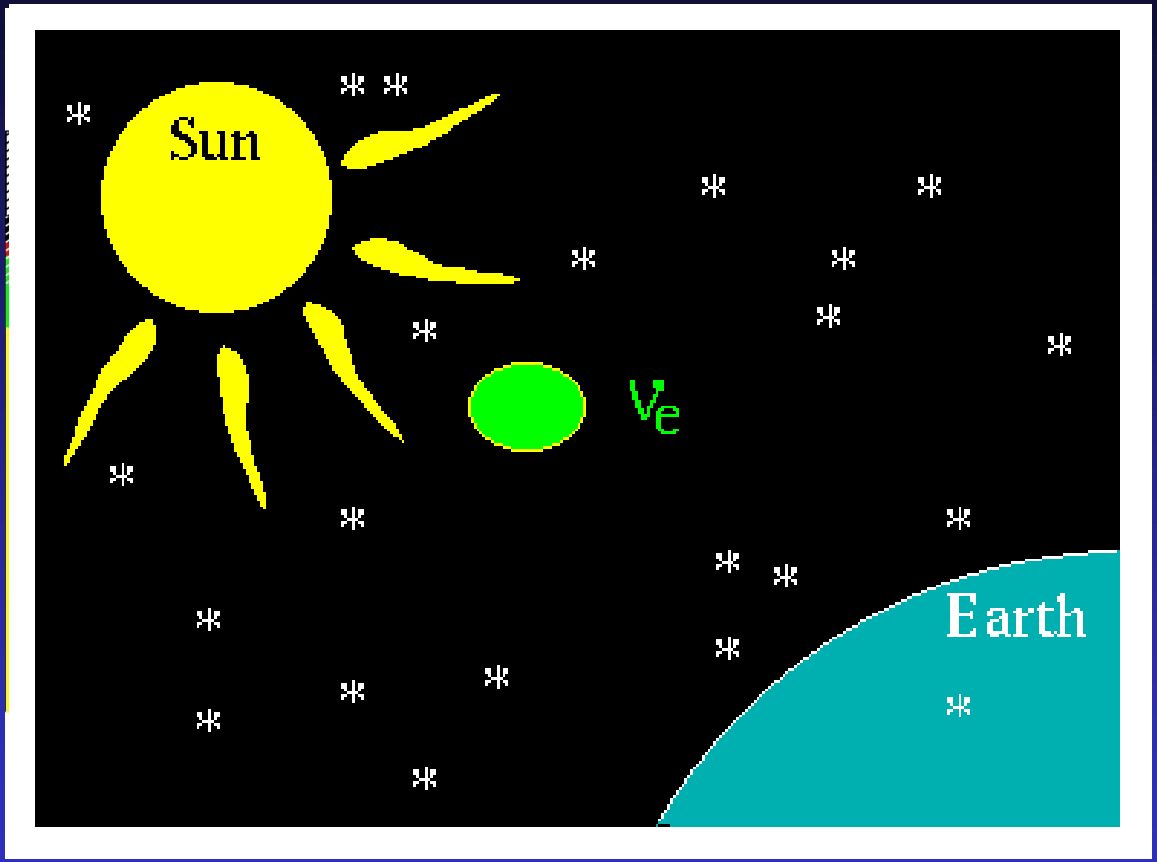


First experiment by Davis *et al.* in 1960's

Radiochemical Method (Chlorine): $\nu_e + {}^{37}Cl \longrightarrow {}^{37}Ar + e^-$.

Found ~ 1/3 of expected rate !

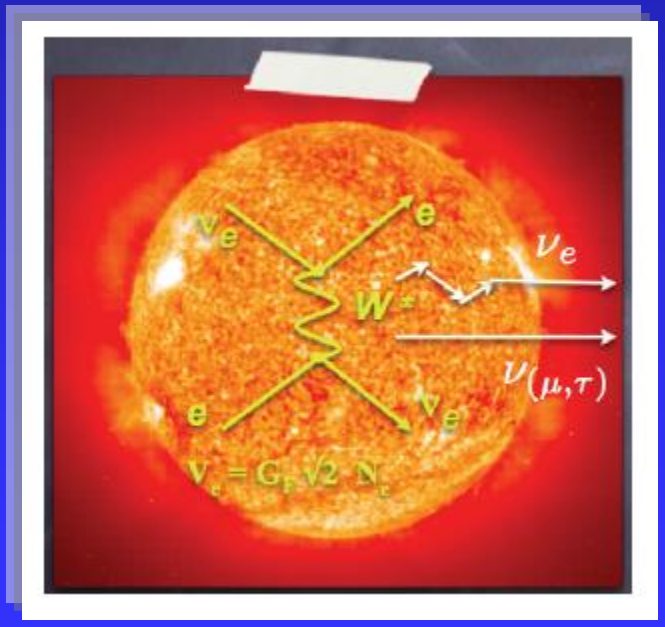
Solar Neutrino Problem

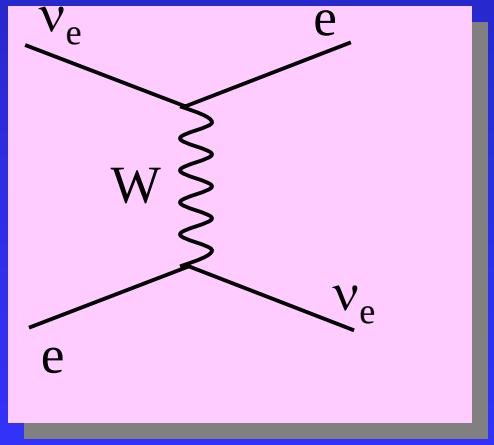


- ν_e flux: observed fluxes are half of SSM pred.

Matter Effect (Mikheyev, Smirnov, Wolfestein)

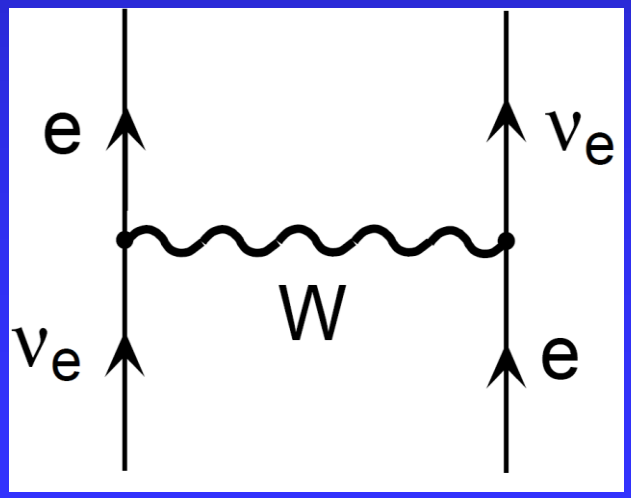
- When neutrinos travel through a medium, they interact with the background of electron, proton and neutron and acquire effective mass.

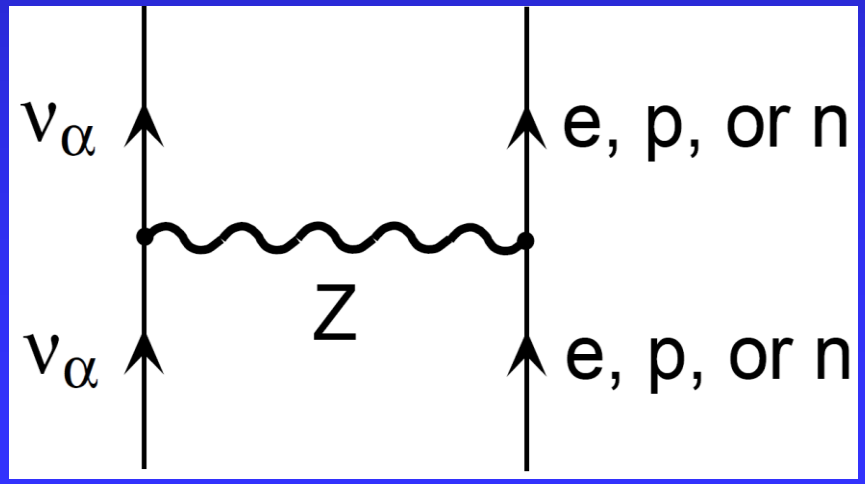




Matter Effect (Mikheyev, Smirnov, Wolfestein)

- This modifies mixing between flavor states and mass (propagation) states, and eigenvalues of Hamiltonian, leading to different osc. probablity
- ν_e has C.C. and N.C. while ν_μ, ν_τ have only N.C.





Matter Effect (Mikheyev, Smirnov, Wolfestein)

- The Hamiltonian in matter can be obtained by adding the potential terms

$$\mathcal{H}_{\rm mat} = \begin{bmatrix} V_e & 0 \\ 0 & V_\mu \end{bmatrix} = \begin{bmatrix} V & 0 \\ 0 & 0 \end{bmatrix} + \lambda'' \, \mathbf{I}$$

irrelevant for flavor evolution

- Crucial role is played by difference of V

$$V \equiv V_{\nu_e} - V_{\nu_\mu} = V_{\nu_e} - V_{\nu_\tau} = +\sqrt{2}\, G_F \, N_e$$

Matter Effect (Mikheyev, Smirnov, Wolfestein)

- In vacuum, time evolution of neutrino states

$$
\begin{aligned}
i\frac{d}{dt}\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} &= U \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} U^\dagger \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} \\
&= \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix} \begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix}
\end{aligned}
$$

- In matter,

$$
i\frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}
$$

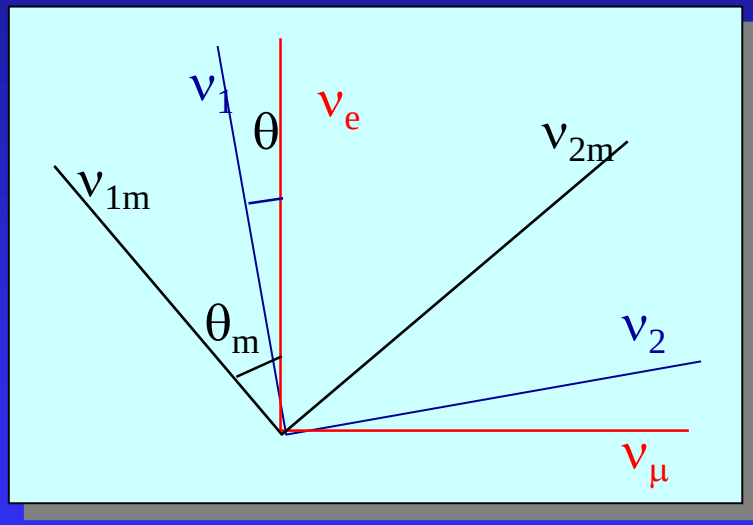
- For antineutrinos, new term has opposite sign

Matter Effect (Mikheyev, Smirnov, Wolfestein)

- If N_e is constant (earth), diagonalizaing Hamiltonian

$$\tan 2\theta_M = \frac{\tan 2\theta}{1 - \frac{A_{CC}}{\Delta m^2 \cos 2\theta}}, \qquad A_{CC} = 2\sqrt{2} G_F N_e E$$

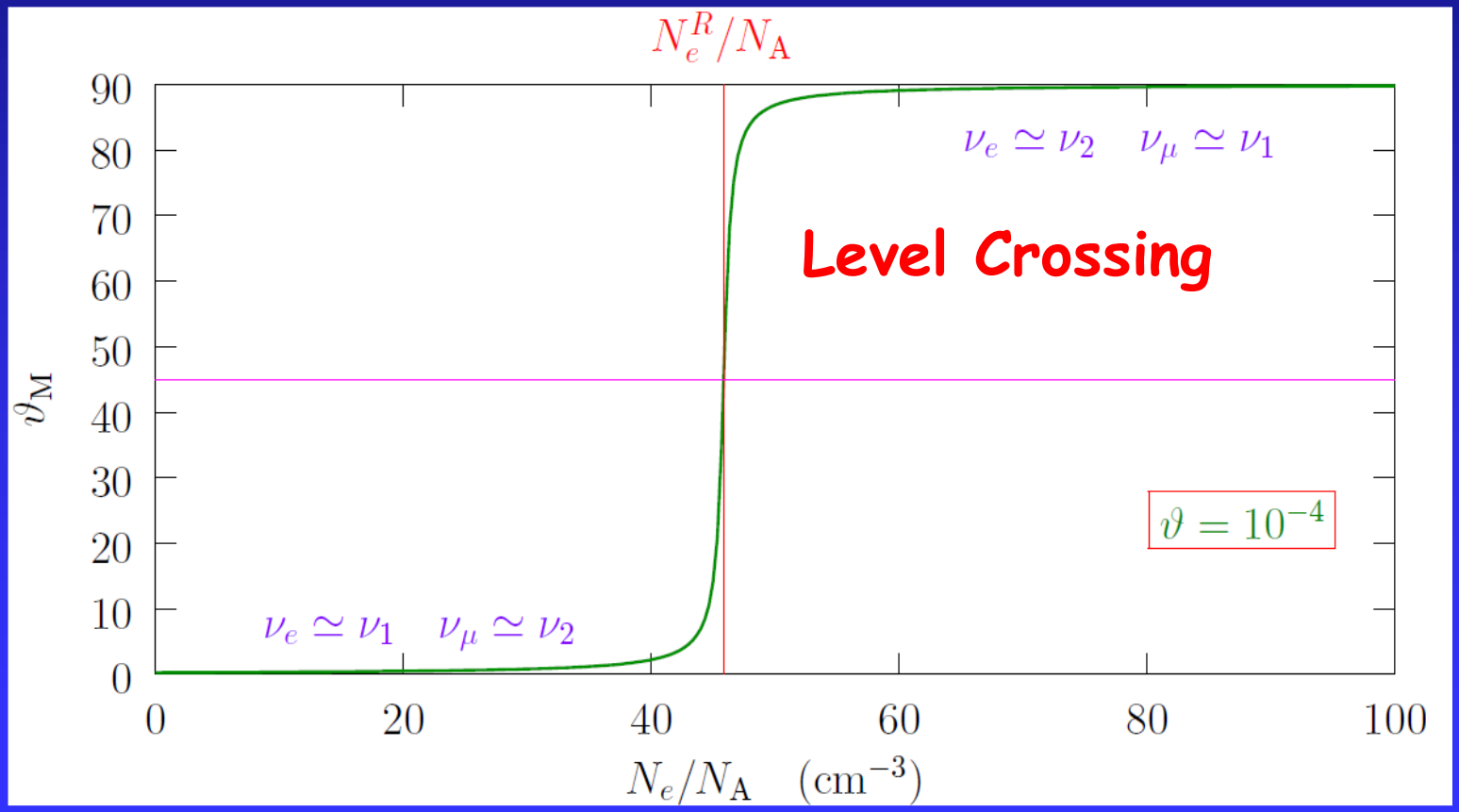
$$\nu_e = \cos\theta_M \nu_{1m} + \sin\theta_M \nu_{2m}$$
$$\nu_\mu = -\sin\theta_M \nu_{1m} + \cos\theta_M \nu_{2m}$$



$$\Delta m^2_{\mathrm{M}} = \sqrt{(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}})^2 + (\Delta m^2 \sin 2\vartheta)^2}$$

Matter Effect

$$\nu_e = \cos\theta_M \, \nu_{1m} + \sin\theta_M \nu_{2m}$$
$$\nu_\mu = -\sin\theta_M \, \nu_{1m} + \cos\theta_M \, \nu_{2m}$$



Matter Effect (Mikheyev, Smirnov, Wolfestein)

- If $A_{CC} = \Delta m^2 \cos 2\theta$ resonance occurs, mixing is maximal $\theta_M = \frac{\pi}{4}$.

Oscillations in matter

- In medium with constant density

  - The flavors of the eigenstates do not change
  - There is no $\nu_{1m} \leftrightarrow \nu_{2m}$ transitions, ν_{1m}, ν_{2m} are the eigenstates of propagation

$$P(\nu_e \to \nu_\mu; t) = \sin^2(2\theta_m) \sin^2 \frac{(E_A - E_B)L}{2}$$

Matter Effect (Mikheyev, Smirnov, Wolfestein)

- In case that matter density varies with time, (Sun, Supernova), it is hard to solve analytically.
- ν_{1m}, ν_{2m} are not propagation eigenstates and transition between them occurs.

- Adiabatic limit : evolution is sufficiently slow
  → each component evolves independently.

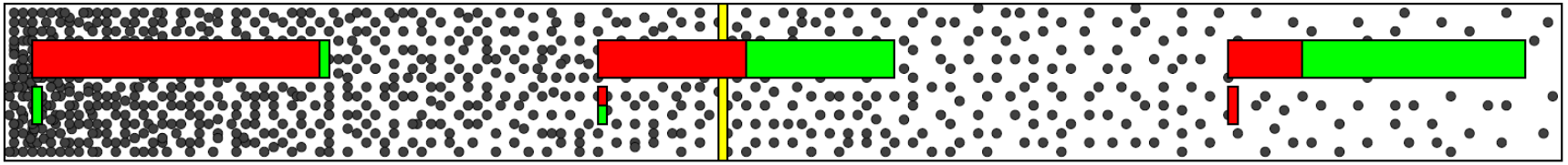
  $\nu_{1m} \leftrightarrow \nu_{2m}$ transitions are neglected

- 3 possibilities of adiabatic conversion

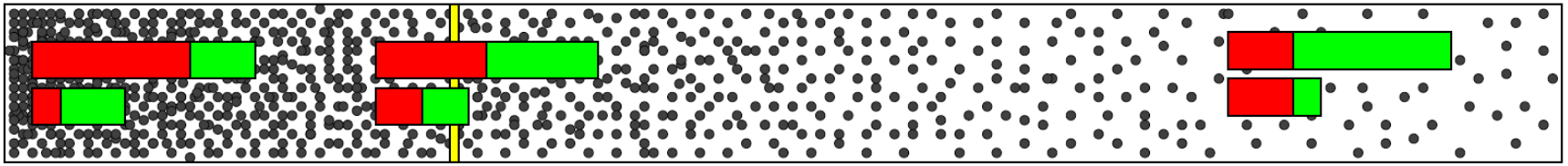
Matter Effect

(Mikheyev, Smirnov, Wolfestein)

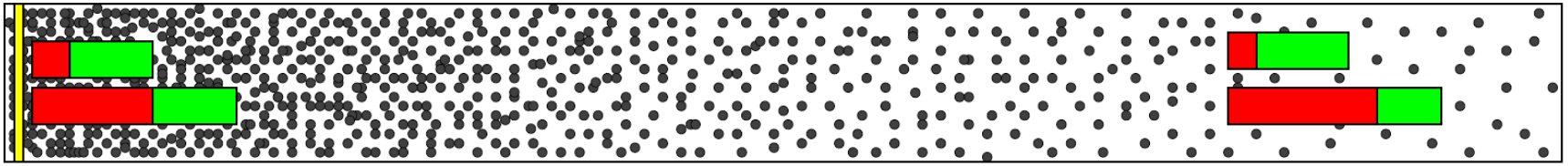
$n_e^o >> n_e^R$ non−oscillatory conversion



$n_e^o > n_e^R$ conversion + oscillations



$n_e^o < n_e^R$ oscillations + small matter effect



Matter Effect on solar neutrinos

$\nu_e = \cos\theta_M \nu_{1m} + \sin\theta_M \nu_{2m}$: production

$\nu_e(x) = \cos\theta\, \nu_1(x) + \sin\theta\, \nu_2(x)$:detection

Neglect interference (averaged over E spectrum)

$$\overline{P}_{\nu_e\to\nu_e}(x) = |\langle\psi_e(x)\rangle|^2 = \cos^2\vartheta\, \cos^2\vartheta^0_{\rm M}\, |\mathcal{A}^{\rm R}_{11}|^2 + \cos^2\vartheta\, \sin^2\vartheta^0_{\rm M}\, |\mathcal{A}^{\rm R}_{21}|^2 + \sin^2\vartheta\, \cos^2\vartheta^0_{\rm M}\, |\mathcal{A}^{\rm R}_{12}|^2 + \sin^2\vartheta\, \sin^2\vartheta^0_{\rm M}\, |\mathcal{A}^{\rm R}_{22}|^2$$

$$|\mathcal{A}^{\rm R}_{12}|^2 = |\mathcal{A}^{\rm R}_{21}|^2 = P_{\rm c} \qquad |\mathcal{A}^{\rm R}_{11}|^2 = |\mathcal{A}^{\rm R}_{22}|^2 = 1 - P_{\rm c}$$

$P_{\rm c} \equiv$ crossing probability non-adiabatic

$$\overline{P}_{\nu_e\to\nu_e}(x) = \frac{1}{2} + \left(\frac{1}{2} - P_{\rm c}\right) \cos 2\vartheta^0_{\rm M}\, \cos 2\vartheta$$

(S. Parke, PRL57, '86)

Matter Effect on solar neutrinos

- Production in the center of the sun : matter effects dominate at high E, negligible at low E
- When matter effects are negligible (low E)

$$P(\nu_e \rightarrow \nu_e) = 1 - \frac{1}{2}\sin^2(2\theta)$$

Assuming adiabatic

- When matter effects dominate (high E)

(i.e., $\boldsymbol{\nu_e \approx \nu_{2m}}$ at production,

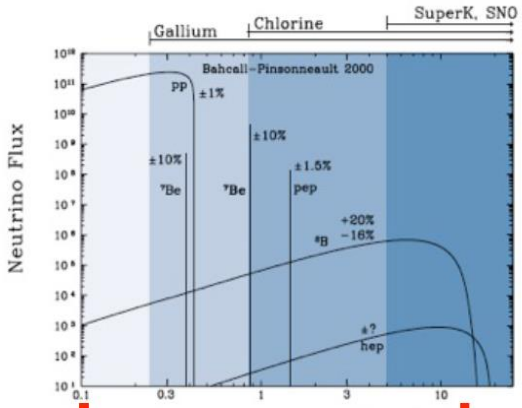
$\boldsymbol{\nu_{2m}}$ =$1/\sqrt{2}(\nu_e+\nu_\mu)$ in resonance region

$\boldsymbol{\nu_{2m} = \cos\theta\ \nu_\mu + \sin\theta\ \nu_e}$: no matter effect

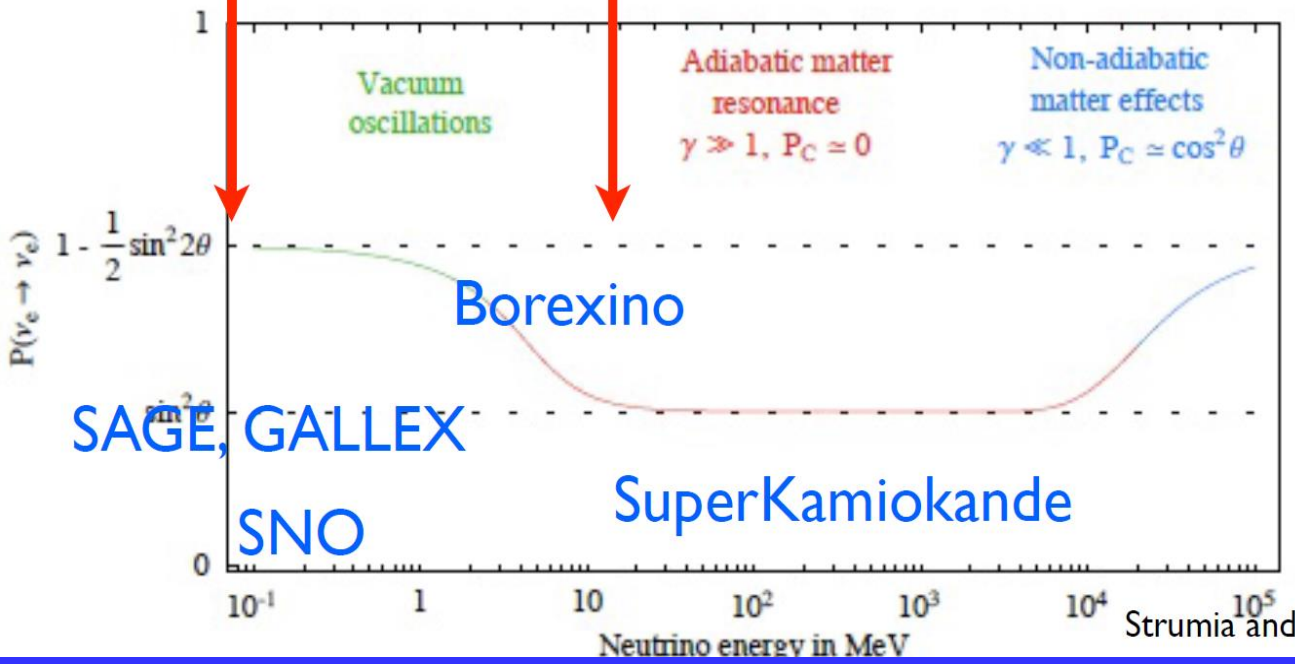
$$P(\nu_e \rightarrow \nu_e) = \sin^2\theta$$

non-oscillatory transition

Matter Effect on solar neutrinos

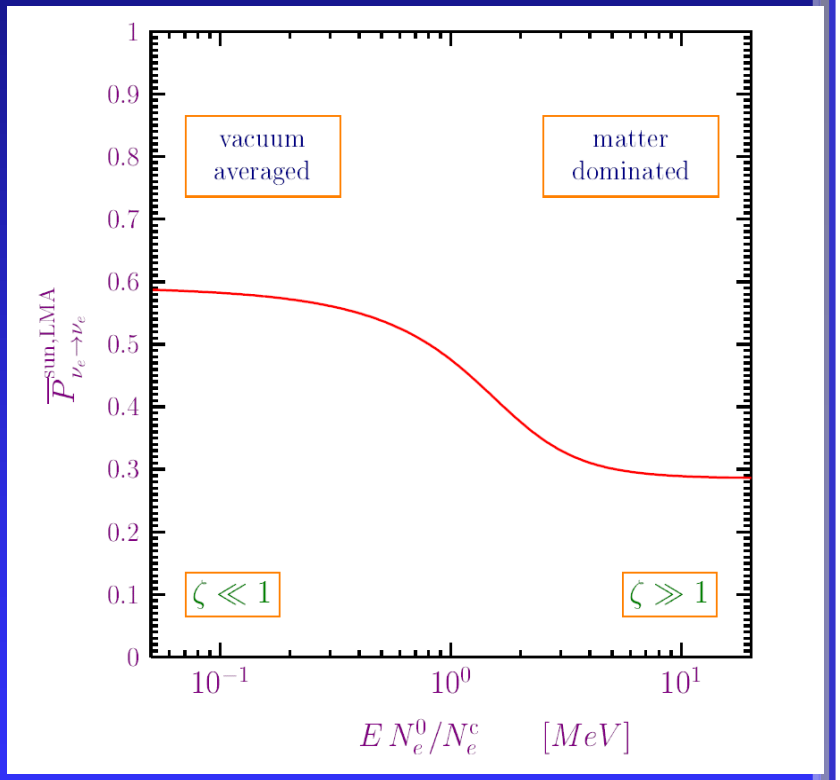


Solar neutrinos have energies which go from vacuum oscillations to adiabatic resonance.



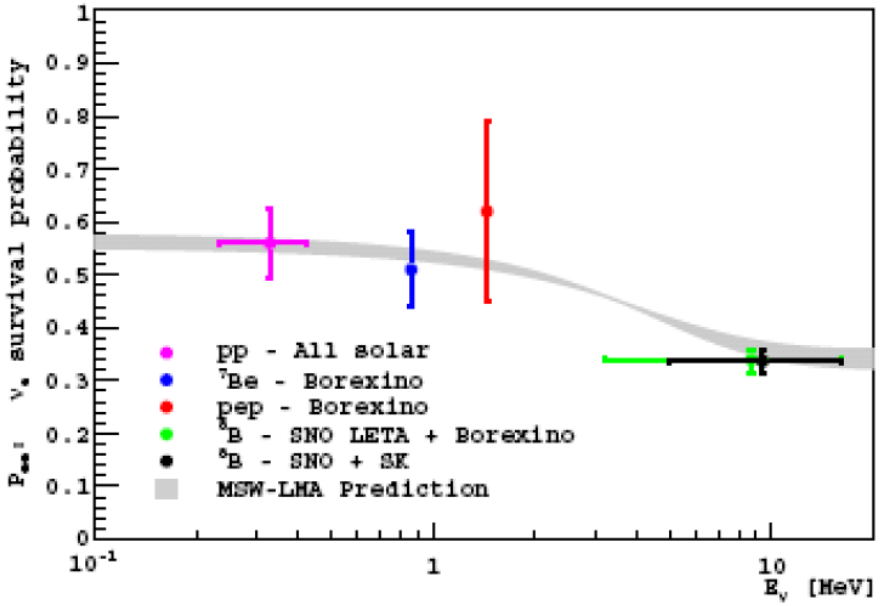
Strumia and Vissani

Proof of MSW matter Effects



$$\zeta = \frac{A^0_{CC}}{\Delta m^2 \cos 2\vartheta}$$

ν_e survival Probability (Pee)



Consistent with MSW-LMA scenario

Borexino(2011)

For Three Neutrino flavors

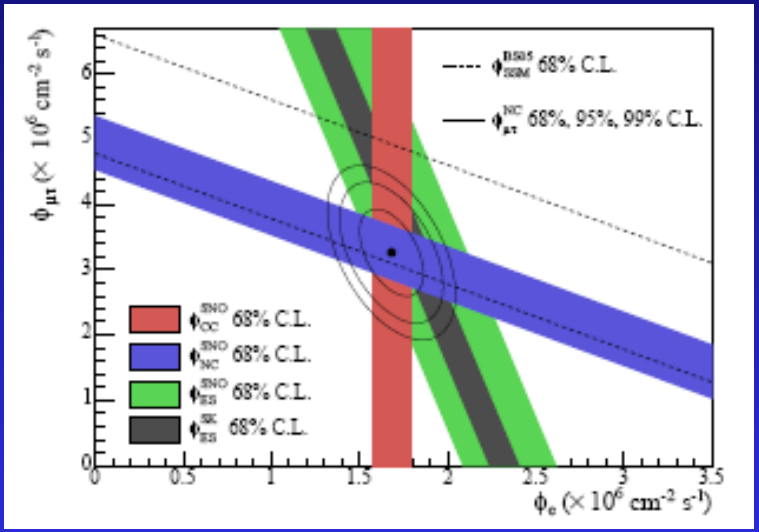
charged current interaction with electron

$$i\frac{\partial}{\partial t}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \left\{ U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m^2_{21}}{2E} & 0 \\ 0 & 0 & \frac{\Delta m^2_{31}}{2E} \end{pmatrix} U^{-1} + \begin{pmatrix} \sqrt{2}G_F N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right\} \begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

electron number density

	Core density ρ	CC effective potential V_C
Earth	~ 10 g/cm^3	~ 10^{-13} eV
Sun	~ 100 g/cm^3	~ 10^{-12} eV
Supernova	~ 10^{14} g/cm^3	~ 1 eV

Solar Neutrino Experiment at SNO



$$\phi_{\mathrm{CC}}^{\mathrm{con}} = 1.72^{+0.05}_{-0.05}(\mathrm{stat})^{+0.11}_{-0.11}(\mathrm{syst})$$
$$\phi_{\mathrm{ES}}^{\mathrm{con}} = 2.34^{+0.23}_{-0.23}(\mathrm{stat})^{+0.15}_{-0.14}(\mathrm{syst})$$
$$\phi_{\mathrm{NC}}^{\mathrm{con}} = 4.81^{+0.19}_{-0.19}(\mathrm{stat})^{+0.28}_{-0.27}(\mathrm{syst}),$$

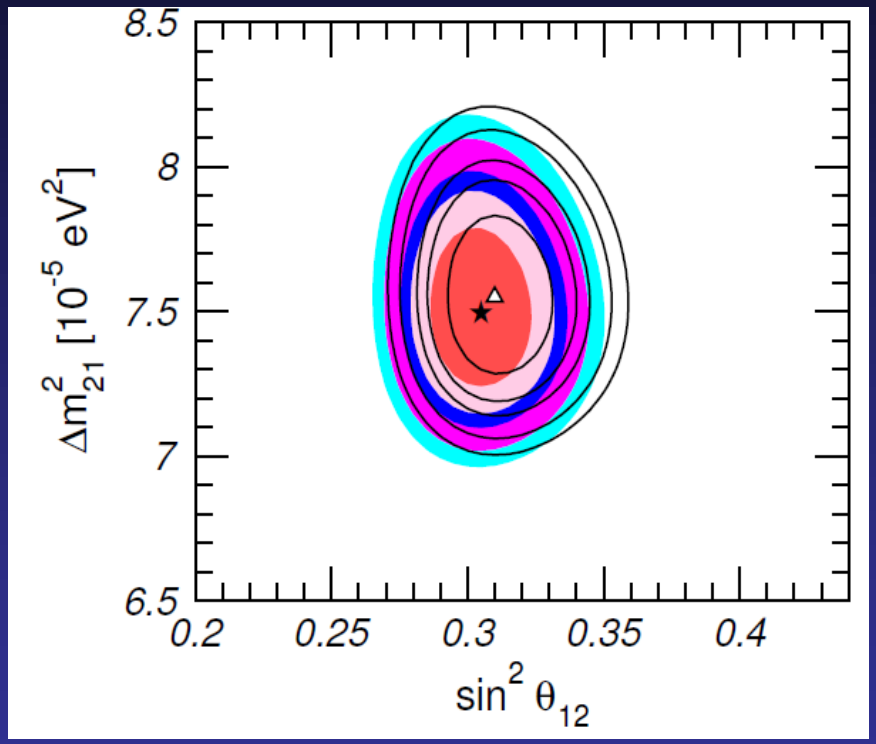
$$\phi(\text{관측}) = P(survival)\cdot\phi(\text{생성})$$

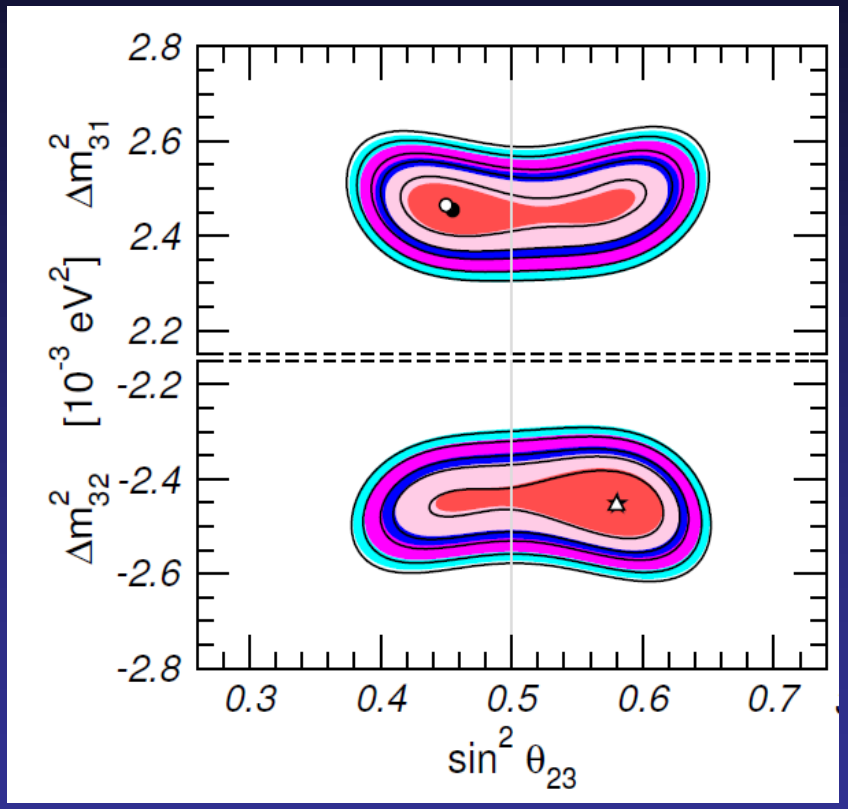
$$\mathrm{SSM(2004)}: \Phi(^8\mathrm{B}) = 5.26(1\pm 0.23)\times 10^6\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$$

Bahcall & Pinsonneault

These support neutrino transition as well as verify SSM

Results





Evidence for the existence of neutrino masses and mixing

What we know from neutrino oscillations

U: Pontecorvo-Maki-Nakagawa-Sakata-matrix (PMNS-matrix)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric — CP phase — reactor/accel. — solar

$$\theta_{12}, \theta_{13}, \theta_{23}, \delta, \Delta m^2_{12}, \Delta m^2_{23}$$

$$U_{PMNS} = \begin{pmatrix} 0.81 - 0.85 & 0.53 - 0.58 & 0.018 - 0.025 \\ 0.32 - 0.49 & 0.52 - 0.69 & 0.60 - 0.76 \\ 0.27 - 0.46 & 0.47 - 0.64 & 0.65 - 0.80 \end{pmatrix}$$

$\theta_{12} = \theta_{sol} \sim 32°$

$\theta_{23} = \theta_{atm} \sim 45°$

$\theta_{13} \sim 9°$

What we know from neutrino oscillations

Table 6.1: Table indicating mixing probabilities in neutrinos

	ν_e	ν_μ	ν_τ
ν_3	$\lvert U_{e3}\rvert^2 \ll 1$	$\lvert U_{\mu 3}\rvert^2 \approx \frac{1}{2}$	$\lvert U_{\tau 3}\rvert^2 \approx \frac{1}{2}$
ν_2	$\lvert U_{e2}\rvert^2 \approx \frac{1}{3}$	$\lvert U_{\mu 2}\rvert^2 \approx \frac{1}{3}$	$\lvert U_{\tau 2}\rvert^2 \approx \frac{1}{3}$
ν_1	$\lvert U_{e1}\rvert^2 \approx \frac{2}{3}$	$\lvert U_{\mu 1}\rvert^2 \approx \frac{1}{6}$	$\lvert U_{\tau 1}\rvert^2 \approx \frac{1}{6}$

Mass scale of neutrinos



- Assume mass hierarchy; $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$
- Atmospheric neutrino; $\Delta m^2_{atm} \approx \Delta m^2_{31} \approx \Delta m^2_{32} \sim 10^{-3} eV^2$
- Solar neutrino; $\Delta m^2_{sol} \approx \Delta m^2_{21} \sim 8 \times 10^{-5} eV^2$
- Sum of 3 Δm^2 should be 0; $\Delta m^2_{12} + \Delta m^2_{23} + \Delta m^2_{31} = 0$

중성미자 진동 실험으로 중성미자 질량들 사이의 차이는 알아내었지만 개별 질량의 크기는 아직 모른다.

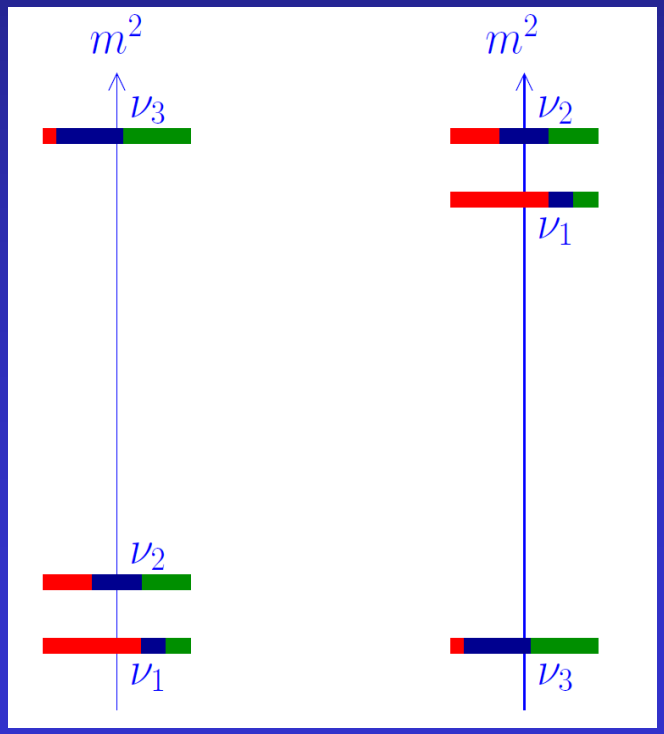
Mass Hierachy of neutrinos

- Oscillation probability in vacuum is invariant under $\Delta m^2 \rightarrow -\Delta m^2$
- Matter effect depends on sign of Δm^2
- Solar neutrino exp. fix $\Delta m^2_{21} > 0$
- But, we do not determine the sign of Δm^2_{31}

Mass Hierachy of neutrinos

- We have 2 options for neutrino mass spectrum

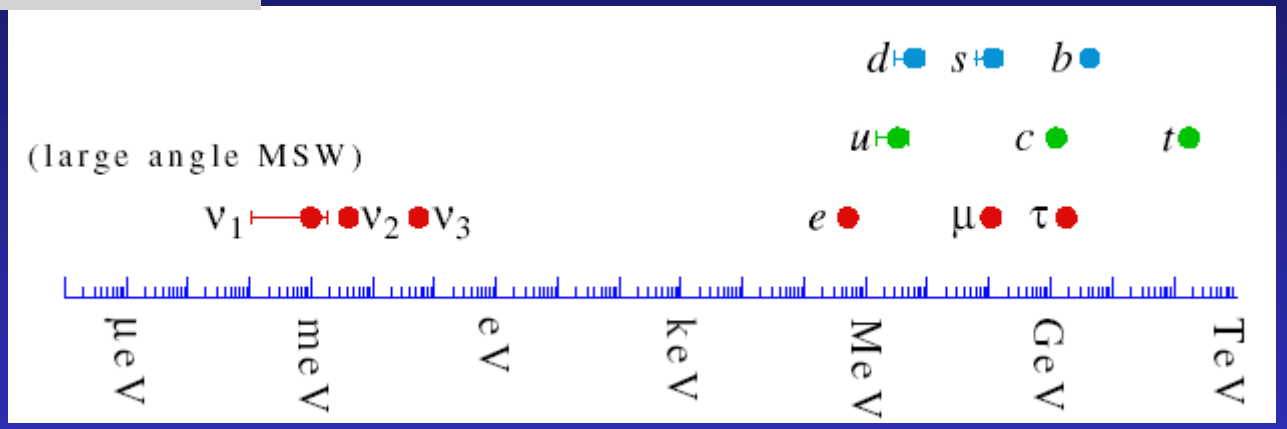
Normal hierarchy



Inverted hierarchy

How small neutrino masses?





- **Cosmological Mass Limit :**

Planck CMB

$$\Omega_\nu h^2 = \sum \frac{m_\nu}{92.5eV} \leq 0.0076$$

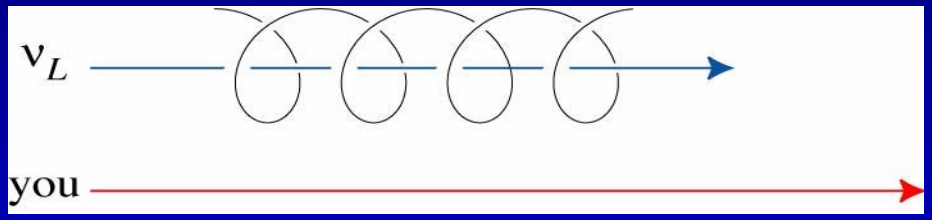
$$\Rightarrow \quad m_\nu \leq 0.7(eV)$$

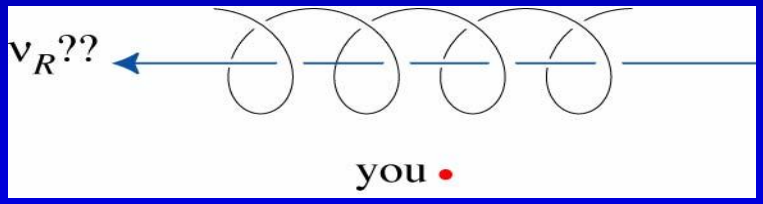
Neutrino Masses



- ***Neutrinos must be massless***

All neutrinos left-handed ⇒ massless





Right-handed Neutrinos ?

No observation⇒ can't have a mass !!

Massive neutrinos imply the standard model is incomplete !

·Fermion Masses

(1) Dirac Mass

For a Dirac field, $\psi = \psi_R + \psi_L$

$$\psi_R = \frac{1}{2}(1-\gamma_5)\psi, \quad \psi_L = \frac{1}{2}(1+\gamma_5)\psi$$

$$\begin{aligned}\bar{\psi}_R &= (\psi_R)^\dagger \gamma^0 = \left[\frac{1}{2}(1-\gamma_5)\psi\right]^\dagger \gamma^0 \\ &= \psi^\dagger \frac{1}{2}(1-\gamma^5)\gamma^0 = \psi^\dagger \gamma^0 \frac{1}{2}(1+\gamma^5) \\ &= \bar{\psi}\frac{1}{2}(1+\gamma^5)\end{aligned}$$

Fermion Masses

(1) Dirac Mass

we can easily check that $\bar{\psi}_R\psi_R = \bar{\psi}_L\psi_L = 0.$

$$\begin{aligned}\bar{\psi}\psi &= \overline{(\psi_R + \psi_L)}(\psi_R + \psi_L) \\ &= \bar{\psi}_R\psi_R + \bar{\psi}_L\psi_L + \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R \\ &= \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R\end{aligned}$$

Thus, we can write down the Dirac mass term

$$\bar{\psi}_R M^\dagger \psi_L + \bar{\psi}_L M \psi_R$$

Fermion Masses

(1) Dirac Mass

- Dirac type mass matrix M is in general N x N complex matrix
- M is diagonalized by biunitary matrix

$$M = U m_D V^\dagger$$

$$M M^\dagger = U m_D^2 U^\dagger$$

$$m_D^2 = \begin{pmatrix} m_1^2 & & & \\ & m_2^2 & & \\ & & m_3^2 & \\ & & & \ddots \end{pmatrix}$$

$$U : a\ unitary\ matrix$$

Fermion Masses

(1) Dirac Mass

$$Y\bar{\psi}_R H^\dagger \psi_L \rightarrow \text{SSB}$$



$$\bar{\psi}_L M \psi_R = \bar{\psi}_L U m_D V^\dagger \psi_R = \overline{(U^\dagger \psi_L)} m_D V^\dagger \psi_R$$

$$\psi_L = U\psi_L^M$$

$$\psi_R = V\psi_R^M$$

This shows that weak eigenstates are different From mass eigenstates.

Fermion Masses

(1) Dirac Mass

- Dirac type mass matrix M is invariant under

$$\psi \rightarrow e^{i\alpha}\psi$$

which implies lepton number conservation.

$$\left(\overline{\nu_{eL}} \quad \overline{\nu_{\mu L}} \quad \overline{\nu_{\tau L}}\right) \begin{pmatrix} m^{\mathrm{D}}_{ee} & m^{\mathrm{D}}_{e\mu} & m^{\mathrm{D}}_{e\tau} \\ m^{\mathrm{D}}_{\mu e} & m^{\mathrm{D}}_{\mu\mu} & m^{\mathrm{D}}_{\mu\tau} \\ m^{\mathrm{D}}_{\tau e} & m^{\mathrm{D}}_{\tau\mu} & m^{\mathrm{D}}_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} + \mathrm{H.c.}$$

L_e, L_μ, L_τ are not conserved

lepton number (L=Le+Lμ+Lτ) conserved.

Fermion Masses

(1) Dirac Mass

- The unitary mixing matrix U occurs in C.C. weak interactions

(in the flavor basis) :

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(\nu_e, \nu_\mu, \nu_\tau)_{\rm L}} \, \gamma^\mu \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\rm L} W_\mu^+ + \text{h.c.}$$

(in the mass basis) :

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(\nu_1, \nu_2, \nu_3)_{\rm L}} \, U^\dagger \gamma^\mu \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\rm L} W_\mu^+ + \text{h.c.}$$

Fermion Masses

(2) Majorana Mass

- Some basics for understanding Majorana mass

$$\left\{ \begin{array}{l} \gamma_\alpha \gamma_\beta + \gamma_\beta \gamma_\alpha = 2\delta_{\alpha\beta} \\ \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 = \gamma_4\gamma_3\gamma_2\gamma_1 \\ C\gamma_\alpha^T C^{-1} = -\gamma_\alpha \end{array} \right\}$$

$$\Longrightarrow \quad C\gamma_5^T C^{-1} = \gamma_5$$

$$C^+ C = \mathbb{1} \ , \quad C^T = -C$$

Fermion Masses

(2) Majorana Mass

- Some basics for understanding Majorana mass

$$\psi^c = C\bar{\psi}^T, \quad (C = i\gamma^2\gamma^0)$$

$$\overline{\psi^c} = \overline{C\bar{\psi}^T} = (C\bar{\psi}^T)^\dagger\gamma^0 = -\psi^T C^{-1}$$

- Exercise 1: Prove that

$$(\psi_R)^c = (\psi^c)_L$$

$$(\psi_L)^c = (\psi^c)_R$$

Fermion Masses

(2) Majorana Mass

- Majorana neutrinos : $\psi = \psi^c$
- Let $\psi_L = \begin{pmatrix} \varphi \\ 0 \end{pmatrix}$

$$(\psi_L)^c = C\gamma^0 (\psi_L^*) = i\gamma^2 \psi_L^* = \begin{pmatrix} 0 \\ -i\sigma^2 \varphi^* \end{pmatrix}$$

- **We need only 2-component field φ to describe a Majorana neutrino.**

$$\psi_M = \psi_L + (\psi_L)^c = \begin{pmatrix} \varphi \\ -i\sigma^2 \varphi^* \end{pmatrix}$$

Fermion Masses

(2) Majorana Mass

- Note that in the same representation, a Dirac fermion can be written as

$$\psi_D = \begin{pmatrix} \varphi \\ -i\sigma^2\chi^* \end{pmatrix}, \quad \varphi \neq \chi,$$

- If $\varphi = \chi$, it is a Majorana fermion.
- For spinors φ, χ

$$(\bar{\chi}\varphi)^\dagger = \varphi^\dagger \gamma^{0\dagger} \chi = \bar{\varphi}\chi$$

- For Majorana spinor

$$\bar{\chi}\varphi = \bar{\varphi}\chi$$

Fermion Masses

(2) Majorana Mass

- Using

$$(\psi_R)^c = (\psi^c)_L$$
$$(\psi_L)^c = (\psi^c)_R$$

- Then, a Majorana fermion can be written

$$\psi_M = \begin{cases} \psi_L + (\psi_L)^c = \psi_L + (\psi^c)_R \\ \psi_R + (\psi_R)^c = \psi_R + (\psi^c)_L \end{cases}$$

Fermion Masses

(2) Majorana Mass

- Majorana mass term :

$$
\begin{aligned}
L^{M}_{\text{mass}} &= -\frac{1}{2}\overline{\psi_M} M \psi_M + h.c. \\
&= -\frac{1}{2}\left(\bar{\psi}_L + \overline{(\psi^c)_R}\right) M \left(\psi_L + (\psi^c)_R\right) + h.c. \\
&= -\frac{1}{2}\left(\bar{\psi}_L M (\psi^c)_R + \overline{(\psi^c)_R} M \psi_L\right) + h.c. \\
&= -\frac{1}{2}\left(\bar{\psi}_L M (\psi_L)^c + \overline{(\psi^c)_L} M \psi_L\right) + h.c.
\end{aligned}
$$

- Prove : $\bar{\psi}_L M (\psi_L)^c = \overline{(\psi^c)_L} M^T \psi_L$ ⇒ $M = M^T$

Fermion Masses

(2) Majorana Mass

- Majorana mass term is not invariant under

$$\psi \rightarrow e^{i\alpha}\psi$$

- So, lepton number is not conserved.

$$\psi_L \rightarrow e^{+i\alpha}\psi_L,$$

$$\psi_R \rightarrow e^{(\ldots)}\psi_R$$

$$\bar{\psi}_L M \left(\psi_L\right)^c \rightarrow e^{-2i\alpha}\bar{\psi}_L M \left(\psi_L\right)^c$$

Fermion Masses

(2) Majorana Mass

- Diagonalization of Majorana mass matrix :
- In general, $X^\dagger M Y = M_D, \rightarrow X^\dagger M M^\dagger X = M_D^2$

Since $M = M^T$, $Y^T M X^* = M_D, \rightarrow Y^T M M^\dagger Y^* = M_D^2$

- Therefore, $X M_D^2 X^\dagger = Y^* M_D^2 Y^T$



$(Y^T X) M_D^2 = M_D^2 (Y^T X)$

Then, $(Y^T X)$ must be diagonal.

define $Y^T X \equiv P^2$ with P being diagonal, and $XP^* \equiv U$.

$U^\dagger M U^* = M_D$

Neutrinos are massless in SM

In SM, only left-handed neutrino states,
only one Higgs SU(2) doublet
SU(2)xU(1) gauge symmetry

- In SM, there is no right-handed neutrino states, so no Dirac-type neutrino masses
- In SM, no way to construct a Majorana mass term with ν_L because it violates gauge symmetry

Neutrinos are massless in SM

- SM particle table

	I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
lepton doublet $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	-1	0 -1
lepton singlet ℓ_R	0	0	-2	-1
Higgs doublet $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$+1$	1 0

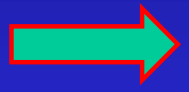
$$m_{LL}\bar{\nu}_L\nu_L^c$$

This term implies $I_3 = 1, Y = -2$

We need a scalar with $I_3 = 1, Y = 2$

Implications of neutrino oscillation

- Neutrinos are massive
- Neutrinos mix



We need new physics beyond SM

A generic neutrino mass Lagrangian :

$$\begin{aligned}\mathcal{L}_\nu &= \overline{\nu_L} M_{\rm D} \nu_R + \frac{1}{2}\overline{\nu_L} M_{\rm L} \left(\nu_L\right)^c + \frac{1}{2}\overline{\left(\nu_R\right)^c} M_{\rm R} \nu_R + \text{h.c.} \\ &= \frac{1}{2}\left[\overline{\nu_L} \quad \overline{\left(\nu_R\right)^c}\right] \begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{bmatrix} \left(\nu_L\right)^c \\ \nu_R \end{bmatrix} + \text{h.c.}\end{aligned}$$

$$m_{LL}\bar{\nu}_L\nu_L^c$$

$$M_{RR}\bar{\nu}_R\nu_R^c$$

Violates L

Violates L_e, L_μ, L_τ

Neutrino=antineutrino

$$m_{LR}\bar{\nu}_L\nu_R$$

Conserves L

Violates L_e, L_μ, L_τ

Neutrino≠antineutrino

- **Possible origins of mass terms**
  - Dirac mass : $Y\overline{L_L}\tilde{H}\nu_R + h.c$ When H gets VEV

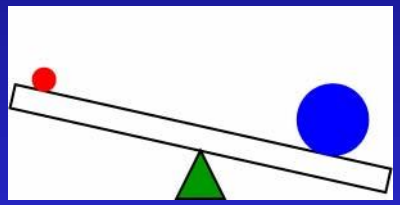
so,characterized by electroweak symmetry breaking

- Light Majorana mass : arisen from dim-5 effective Lagrangian or from SU(2) triplet Higgs
- Heavy Majorana mass : arisen from SU(2) singlet Higgs
- To generate neutrino masses, we need right handed neutrinos, triplet Higgs, etc.

Why neutrino mass is so small?

- **Seesaw Mechanism**

$$-\frac{m_D^2}{M}$$



M

tiny neutrino mass

$$m_\nu = m_D M_R^{-1} m_D^T$$

Natural values of mass operators :

- m_D : electroweak scale
- M: L violation scale ↔ embedding into GUT

- The seesaw idea was originally mentioned in a paper's footnote : PLB59 (1975) 256 by Fritzsch, Gell-Mann & Minkowski

- This idea was clearly elaborated by Minkowski in his paper, PLB67 (1977) 421
- But, today, the following papers are cited :

  Minkowski (1977), Yanagida (1979),
  Gell-Mann, Ramond & Slanski (1979)
  Glashow (1979), Mohapatra & Senjanovic(1980)

CP violation in neutrino oscillation

C : charge conjugate

$$\psi^c = C\bar{\psi}^T, \quad (C = i\gamma^2\gamma^0)$$

$$\overline{\psi^c} = \overline{C\bar{\psi}^T} = (C\bar{\psi}^T)^\dagger\gamma^0 = -\psi^T C^{-1}$$

P: parity

$$\psi \to \psi'(x') = S_P\psi(x) = \gamma^0\psi(x)$$

$$\bar{\psi} \to \bar{\psi}' = \bar{\psi}S_P^{-1}$$

$$P : \psi_L \to \gamma^0\psi_L = \gamma^0\tfrac{1}{2}(1-\gamma_5)\psi = \tfrac{1}{2}(1+\gamma_5)\gamma^0\psi = (\psi')_R$$

C $\Longrightarrow$ Particle $\leftrightarrows$ Antiparticle

P $\Longrightarrow$ Left-Handed $\leftrightarrows$ Right-Handed

CP violation in neutrino oscillation

- Oscillation probablity

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\text{CP}} P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

- CP Asymmetry: $A^{\text{CP}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$

$$A^{\text{CP}}_{\alpha\beta}(L,E) = 4 \sum_{k>j} \text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)$$

- Jarskog invariant: $\text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] = \pm J$

$$J = c_{12} s_{12} c_{23} s_{23} c^2_{13} s_{13} \sin\delta_{13}$$

CP violation in neutrino oscillation

- Right-handed antineutrino is CP conjugate of left-handed neutrino.

$$\nu^{\mathrm{CP}} = \gamma^0 \mathcal{C} \overline{\nu}^T = -\mathcal{C} \nu^*$$

- **CP : $U \leftrightarrow U^*$**

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E}\right)$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L,E) = \sum_{k,j} U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E}\right)$$

CP violation in neutrino oscillation

- In the standard parameterization

$$J = c_{12}s_{12}c_{23}s_{23}c_{13}^2 s_{13}\sin\delta_{13}$$

- Jarskog invariant is useful for quantifying CP violation in a parameterization-indep't way

- Nonzero J implies CPV in case that $0 < \theta_i < \frac{\pi}{2}$
- Measurement of CPV is not easy.

CPT symmetry in neutrino oscillation

- Under CPT transformation

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\text{CPT}} P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

- In QFT, CPT is unbroken symmetry

$$P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

- We can test CPT symmetry by measuring both oscillation probability

- It is confirmed that $P_{\nu_e \to \nu_e} = P_{\bar{\nu}_e \to \bar{\nu}_e}$ from solar neutrino exp. & reactor exp.

T violation in neutrino oscillation

- Under time reversal

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\mathsf{T}} P_{\nu_\beta \to \nu_\alpha}$$

- T Asymmetry: $A^{\mathsf{T}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha}$

$$A^{\mathsf{T}}_{\alpha\beta}(L,E) = 4 \sum_{k>j} \mathrm{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)$$

- Due to CPT symmetry $A^{\mathsf{T}}_{\alpha\beta} = A^{\mathsf{CP}}_{\beta\alpha}$

The Big Questions

- 중성미자의 질량과 플레이버 혼합의 근원은 무엇인가 ?
- 물질 기원에 중성미자가 어떤 역할을 하였는가 ?
- 우주 탄생에 중성미자가 어떤 역할을 할 수 있었는가 ?
- 중성미자는 마요라나 타입의 입자인가?
- 중성미자는 물질과 힘의 통합 이론에 관하여 어떤 중대한 힌트를 제공하는가 ?

Conclusions

- 중성미자 진동 현상의 발견은 중성미자가 질량을 가진다는 것을 의미하며 표준 모형을 넘어서는 새로운 물리 이론을 요구한다.
- 중성미자는 우주에 반물질은 거의 없고 주로 물질로만 되어 있도록 하는 데 중요한 역할을 하였을 수도 있다.
- 중성미자에 대해 여전히 알아야 할 것들이 많으며, 입자 물리학과 우주론에서의 풀리지 않은 문제들을 해결하는 데 중요한 역할을 할 수도 있다.

Solar Neutrino Experiments

Radiochemical detectors

- **Homestake :**

$\nu_e + Cl \rightarrow Ar + e$

- **SAGE, GALLEX, GNO**

$\nu_e + Ga \rightarrow Ge + e$





Cerenkov detectors

- **Kamiokande**

$\nu_e + e \rightarrow \nu_e + e$

- **SuperKamiokande**

- **SNO**

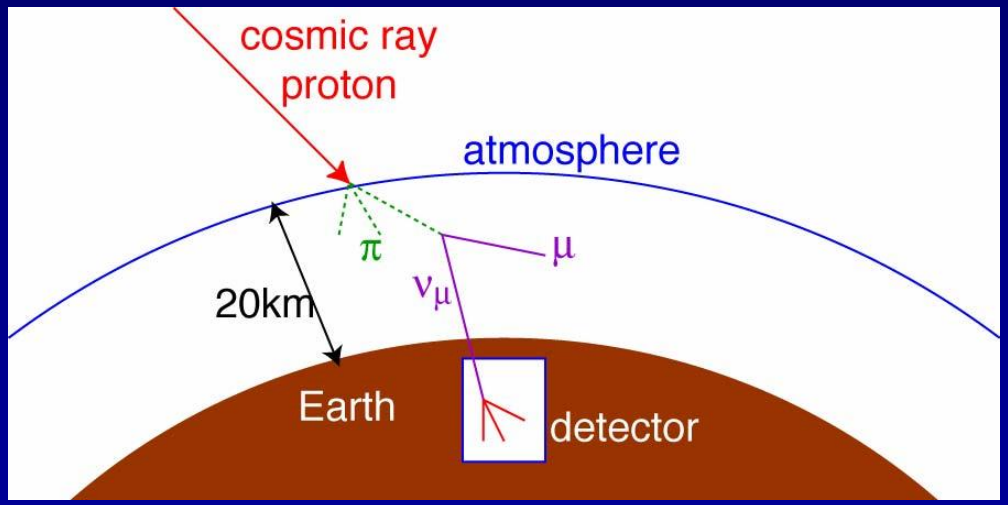
(CC) $\nu_e + d \rightarrow p + p + e$

(ES) $\nu_x + e \rightarrow \nu_x + e$

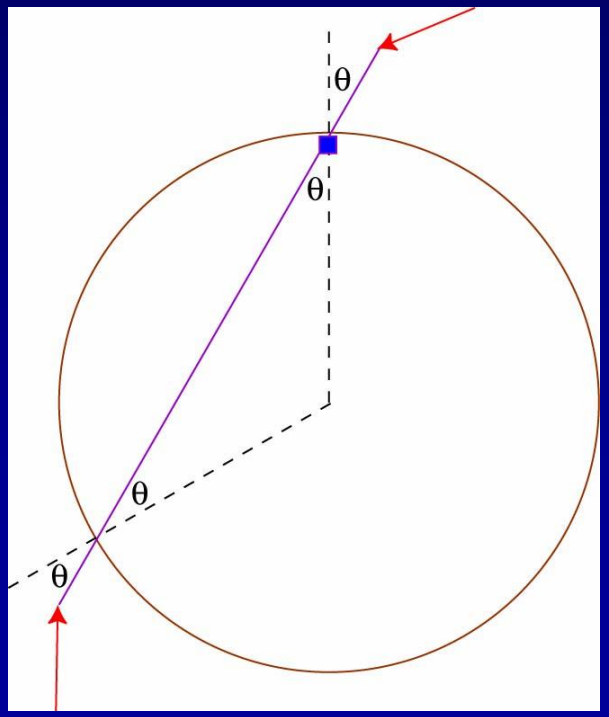
(NC) $\nu_x + d \rightarrow n + p + \nu_x$

Atmospheric Neutrinos

- $p \rightarrow e^+\pi^0$, $K^+\nu$, etc
  - So far not seen
  - Atmospheric neutrino main background



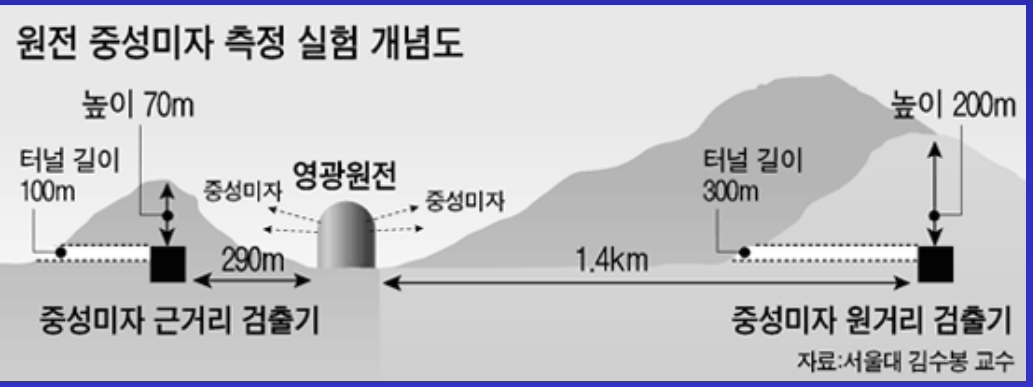
- Cosmic rays isotropic
  - Atmospheric neutrino up-down symmetric



Measurement of θ_{13}

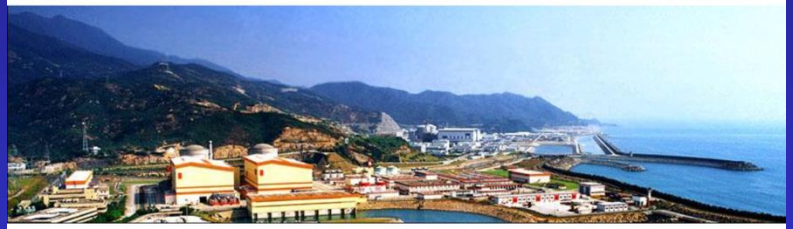
RENO ν θ_{13}





Daya Bay Neutrino Experiment

Jun Cao
Institute of High Energy Physics, Beijing



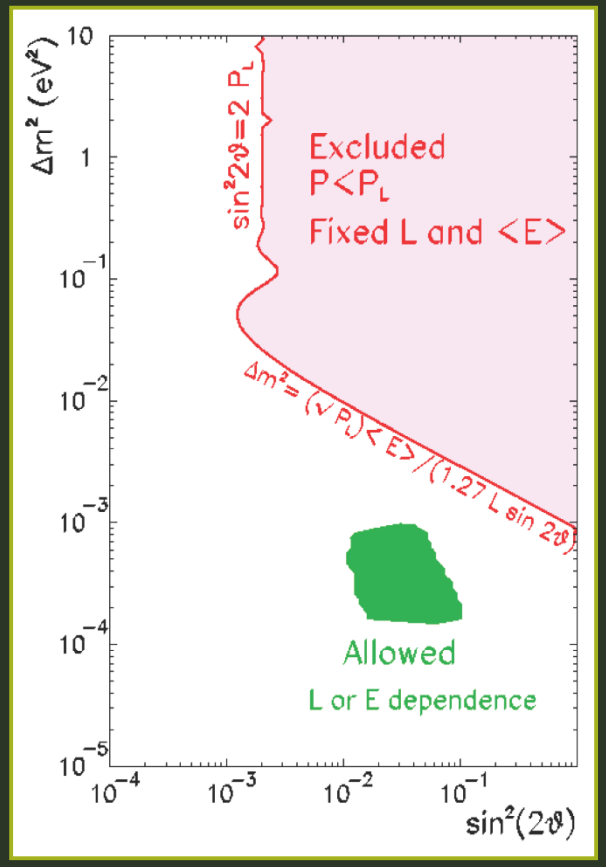
3rd International Conference on Flavor Physics, Oct. 3-8, 2005
National Central University
Chung-li, Taiwan

The measurement of oscillation

- $\langle P_{\alpha\beta} \rangle = \dfrac{\int dE\nu \; d\Phi/dE\nu \; \sigma_{CC}(E\nu) \; P_{\alpha\beta}(E\nu) \; \varepsilon(E\nu)}{\int dE\nu \; d\Phi/dE\nu \; \sigma_{CC}(E\nu) \; \varepsilon(E\nu)}$

  $= \delta_{\alpha\beta} - (2\delta_{\alpha\beta} - 1)\sin^2 2\theta \langle \sin^2 (\Delta m^2 L/4E) \rangle.$

- Taking data at fixed <L> and <E>.
  - For $\Delta m^2 \gg 1/\langle L/E \rangle$, $\langle \sin^2 x \rangle = \frac{1}{2}$. A vertical line at $\sin^2 2\theta = 2P_L$. neutrino telescope.
  - For $\Delta m^2 \ll 1/\langle L/E \rangle$, $\Delta m^2 \sin^2 2\theta = 2\sqrt{P_L}/\langle L/E \rangle$.



Three-Neutrino Oscillations

- Appearance probability including matter

$$
\begin{aligned}
P(\nu_e \to \nu_\mu) \approx\ & \sin^2 2\theta_{13} \sin^2\theta_{23} \frac{\sin^2((1-\hat{A})\Delta)}{(1-\hat{A})^2} \\
& \pm \sin\delta_{CP}\ \alpha \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin(\Delta) \frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\
& + \cos\delta_{CP} \alpha \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \cos(\Delta) \frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\
& + \alpha^2 \sin^2 2\theta_{12} \cos^2\theta_{23} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}
\end{aligned}
$$

$\Delta \equiv \Delta m_{31}^2 L/4E$, $\hat{A} \equiv \pm(2\sqrt{2}\, G_F n_e E)/\Delta m_{31}^2$,

$\alpha = \Delta m_{21}^2 / \Delta m_{31}^2$

Matter Effect (Mikheyev, Smirnov, Wolfestein)

- Level Crossing

