The SMEFT/geoSMEFT- the modern EFT analysis of collider data

Prepared for: QUC summer school

Michael Trott, NBI

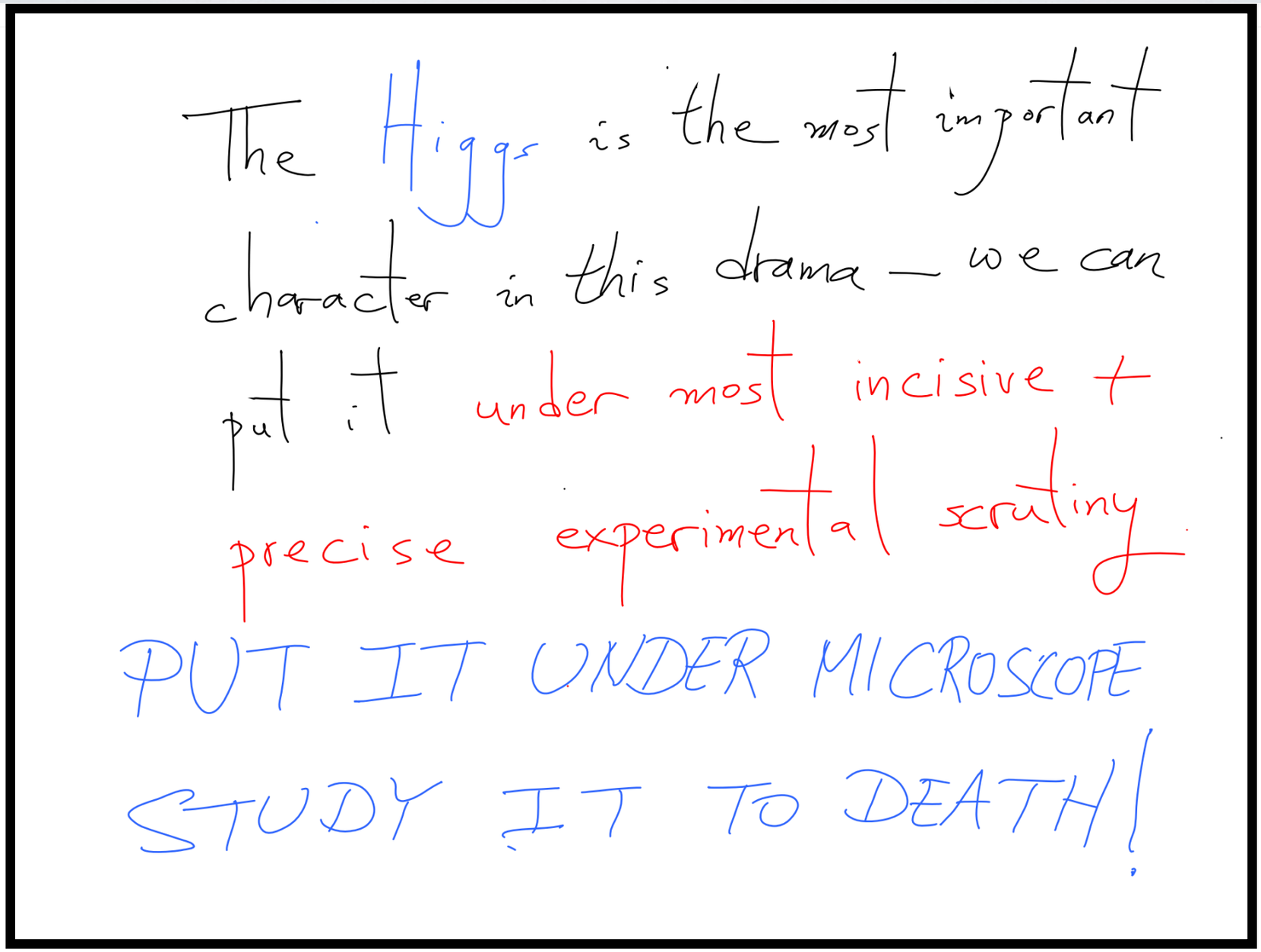
Lecture four

Outline for Lecture four

Fourth lecture

- Motivation for going beyond leading order in the SMEFT
- The RGE of the SMEFT at $\mathcal{L}^{(6)}$
- Problems at one loop using $\Gamma(h \to \gamma\gamma)$ as an example
- Ward identities in the SMEFT related to field space geometry.
- $\mathcal{L}^{(8)}$ Corrections correlations between the op expansion and the loop expansion

Lecture four

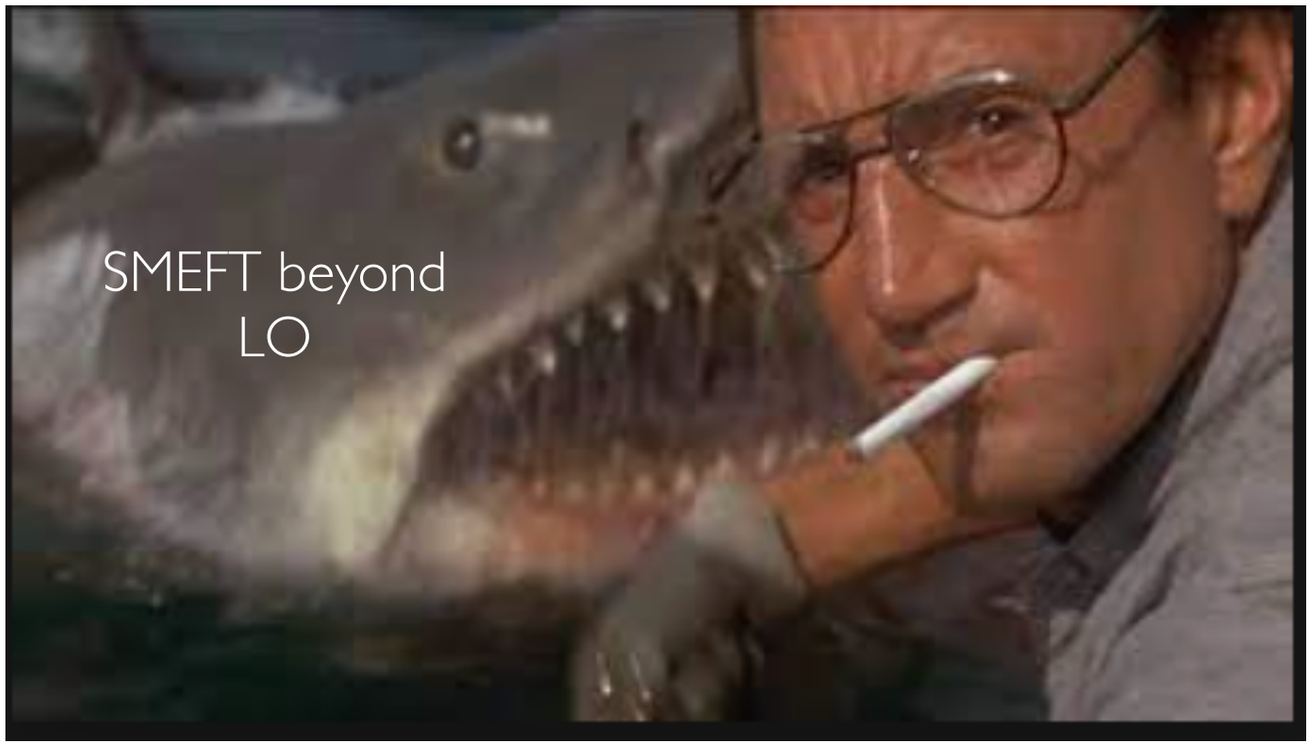


Nima Arkani-Hamed Higgs@10

- The SMEFT is exactly a QFT microscope to study the Higgs consistent with other Global measurements.

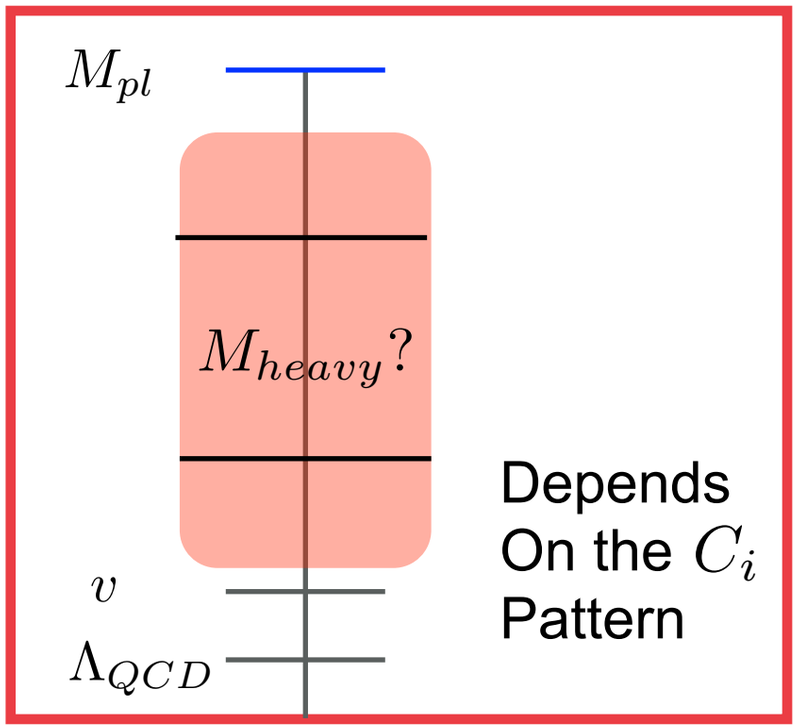
Lecture four

- How good a microscope do we need?



- We are going to have to calculate beyond $\mathcal{L}^{(6)}$ In the SMEFT. Why?

We need Quantum corrections

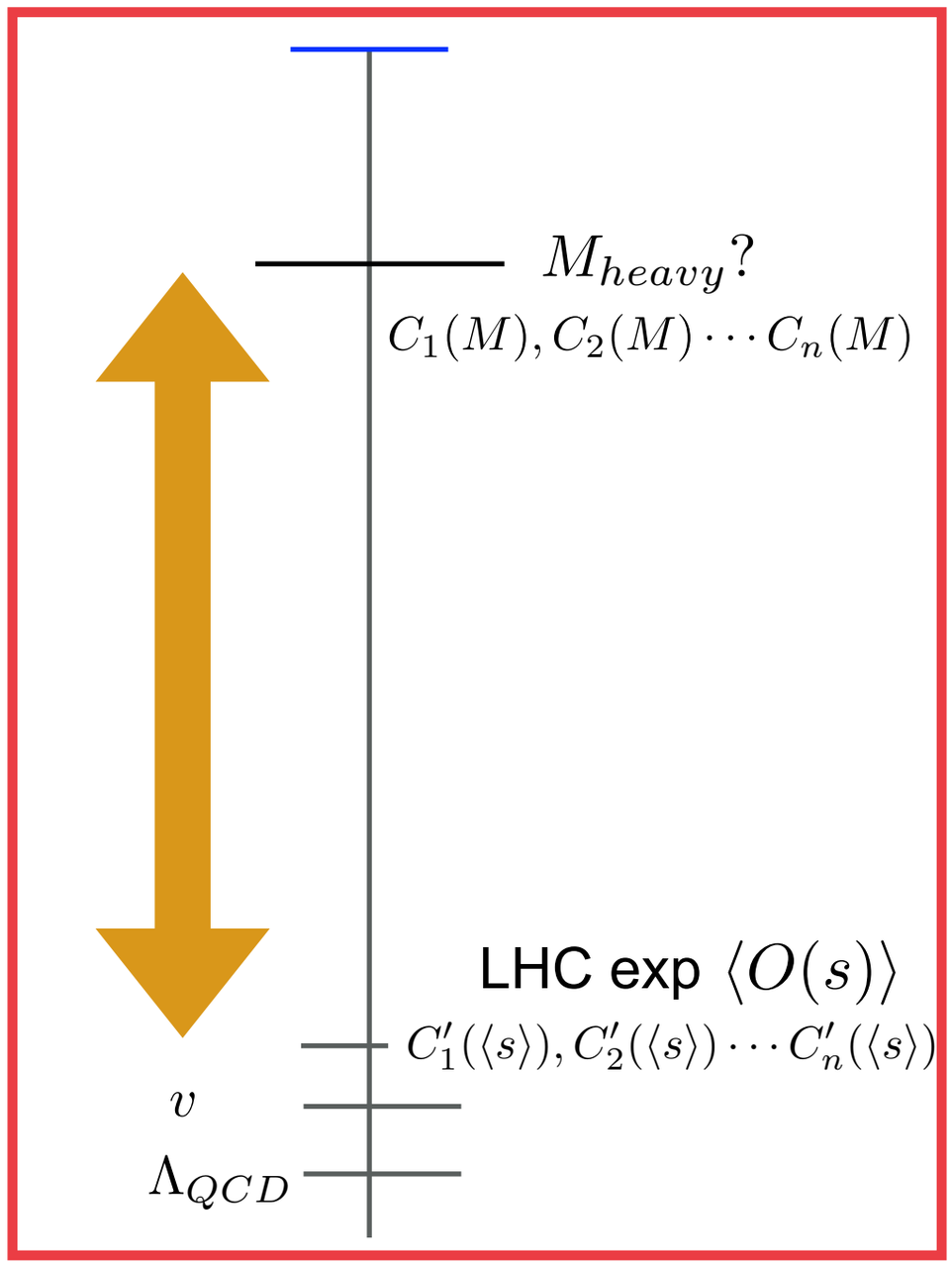


- General structure of corrections:

$$\mathcal{A} = \mathcal{A}_{SM} + \tilde{C}_i^{(6)}\, a_i + \tilde{C}_j^{(6)}\, \tilde{C}_k^{(6)}\, b_{jk} + \tilde{C}_l^{(8)}\, c_l,$$
$$+ \frac{1}{16\pi^2}\left[d_m\, \tilde{C}_m^{(6)} + e_n\, \tilde{C}_n^{(6)}\; log\left(\frac{p_i \cdot p_j}{M^2}\right)\right] + \cdots$$

Needed!

$$\tilde{C}_i^{(6)} = \frac{\bar{v}_T^2}{\Lambda^2} C_i^{(6)}, \quad \tilde{C}_j^{(8)} = \frac{\bar{v}_T^4}{\Lambda^4} C_j^{(8)}$$



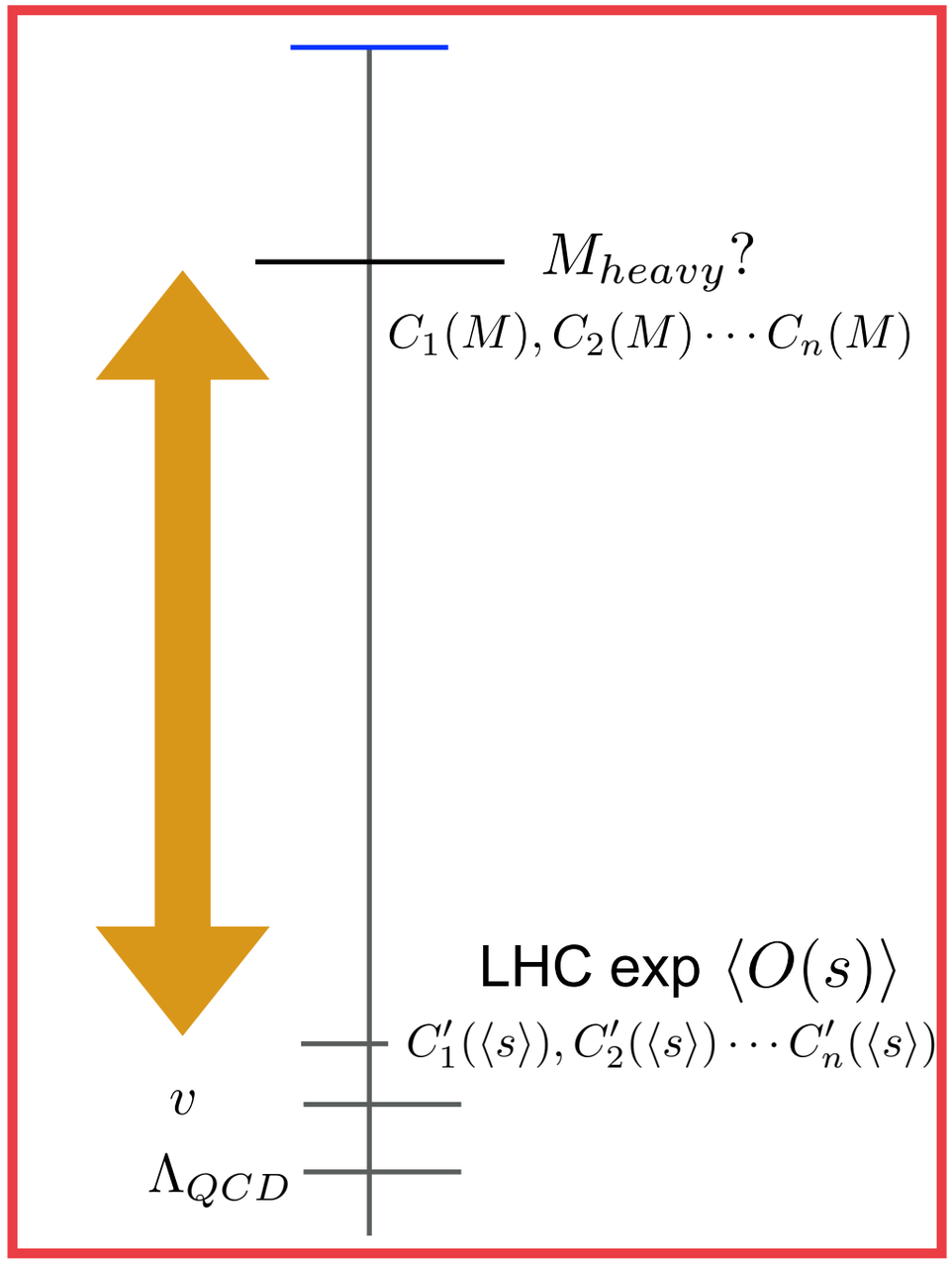
We need a good (NLO) microscope

- General structure of corrections:

$$\mathcal{A} = \mathcal{A}_{SM} + \tilde{C}_i^{(6)}\, a_i + \tilde{C}_j^{(6)}\, \tilde{C}_k^{(6)}\, b_{jk} + \tilde{C}_l^{(8)}\, c_l,$$
$$+ \frac{1}{16\pi^2}\left[d_m\, \tilde{C}_m^{(6)} + e_n\, \tilde{C}_n^{(6)}\; log\left(\frac{p_i \cdot p_j}{M^2}\right)\right] + \cdots$$

Is the log enough? No. For two reasons

- The pattern of the $\tilde{C}_j^{(6)} > \tilde{C}_i^{(6)}$ can have mild hierarchies making loops more important
- The finite terms d_m of the loops are not Much smaller than the log terms Unless $p_i \cdot p_j << M^2$
- If that was true, $M \gtrsim 3\,\mathrm{TeV}$ effects too small to experimentally measure.



We need a good (NLO) microscope

- General structure of corrections:

$$\mathcal{A} = \mathcal{A}_{SM} + \tilde{C}_i^{(6)}\, a_i + \tilde{C}_j^{(6)}\, \tilde{C}_k^{(6)}\, b_{jk} + \tilde{C}_l^{(8)}\, c_l,$$
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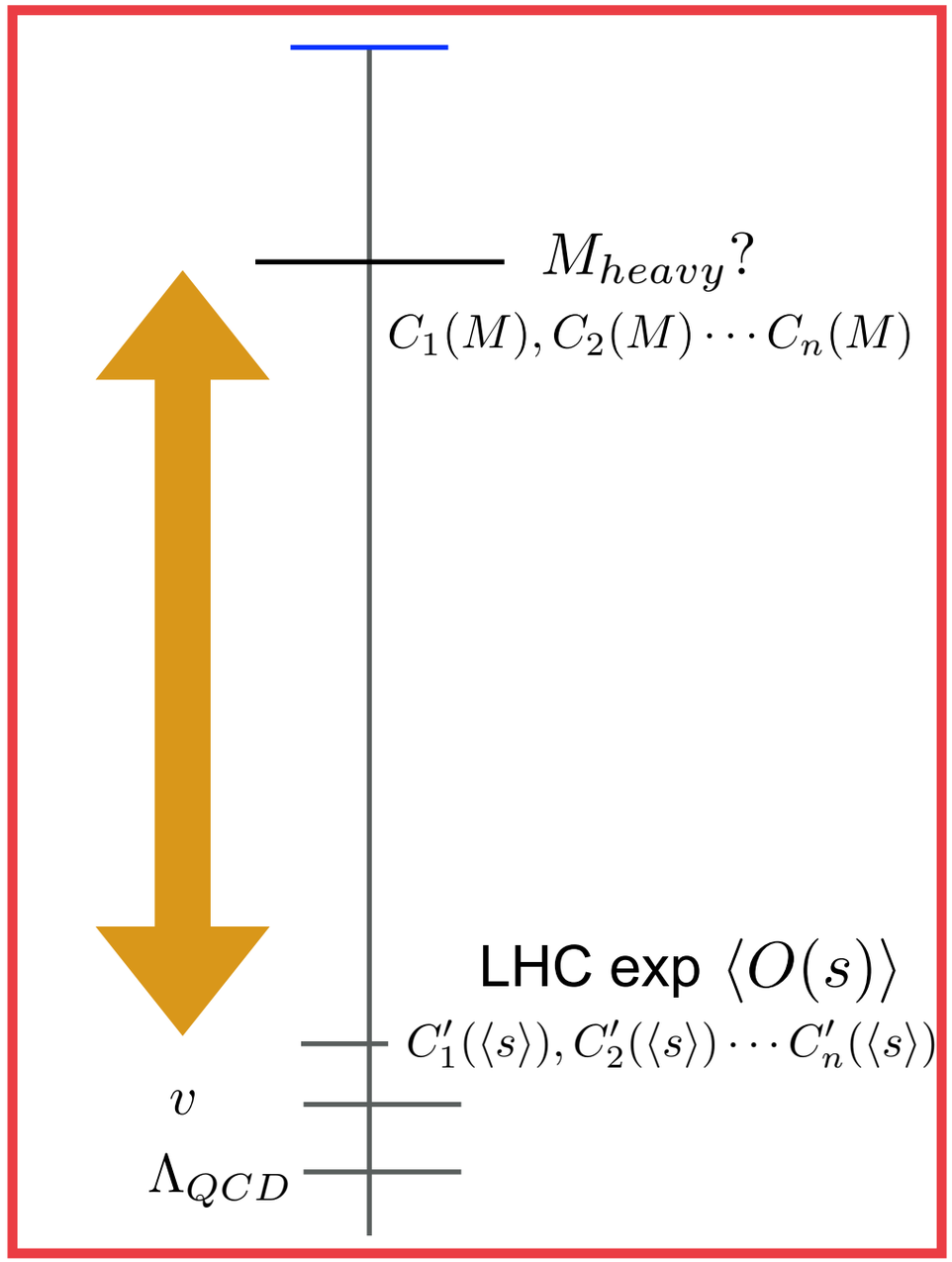
Is $\mathcal{L}^{(6)}$ with loops enough? Not if the SMEFT effects are observable.

- It can be (usually is) true that

$$\frac{v^2}{\Lambda^2}\frac{v^2}{\Lambda^2} > \frac{g_{sm}^2\, v^2}{16\pi^2\Lambda^2}$$

We need the c_l in general.

Therefore we need the b_{ij}



We need a good (NLO) microscope

- General structure of corrections:

$$\mathcal{A} = \mathcal{A}_{SM} + \tilde{C}_i^{(6)} a_i + \tilde{C}_j^{(6)} \tilde{C}_k^{(6)} b_{jk} + \tilde{C}_l^{(8)} c_l,$$
$$+ \frac{1}{16\pi^2}\left[d_m \tilde{C}_m^{(6)} + e_n \tilde{C}_n^{(6)} \; log\left(\frac{p_i \cdot p_j}{M^2}\right)\right] + \cdots$$

- What do we have?

SMEFTsim 1709.06492, 2012.11343

UFO, interfaces with Madgraph gives ***full set*** of

$$a_i$$

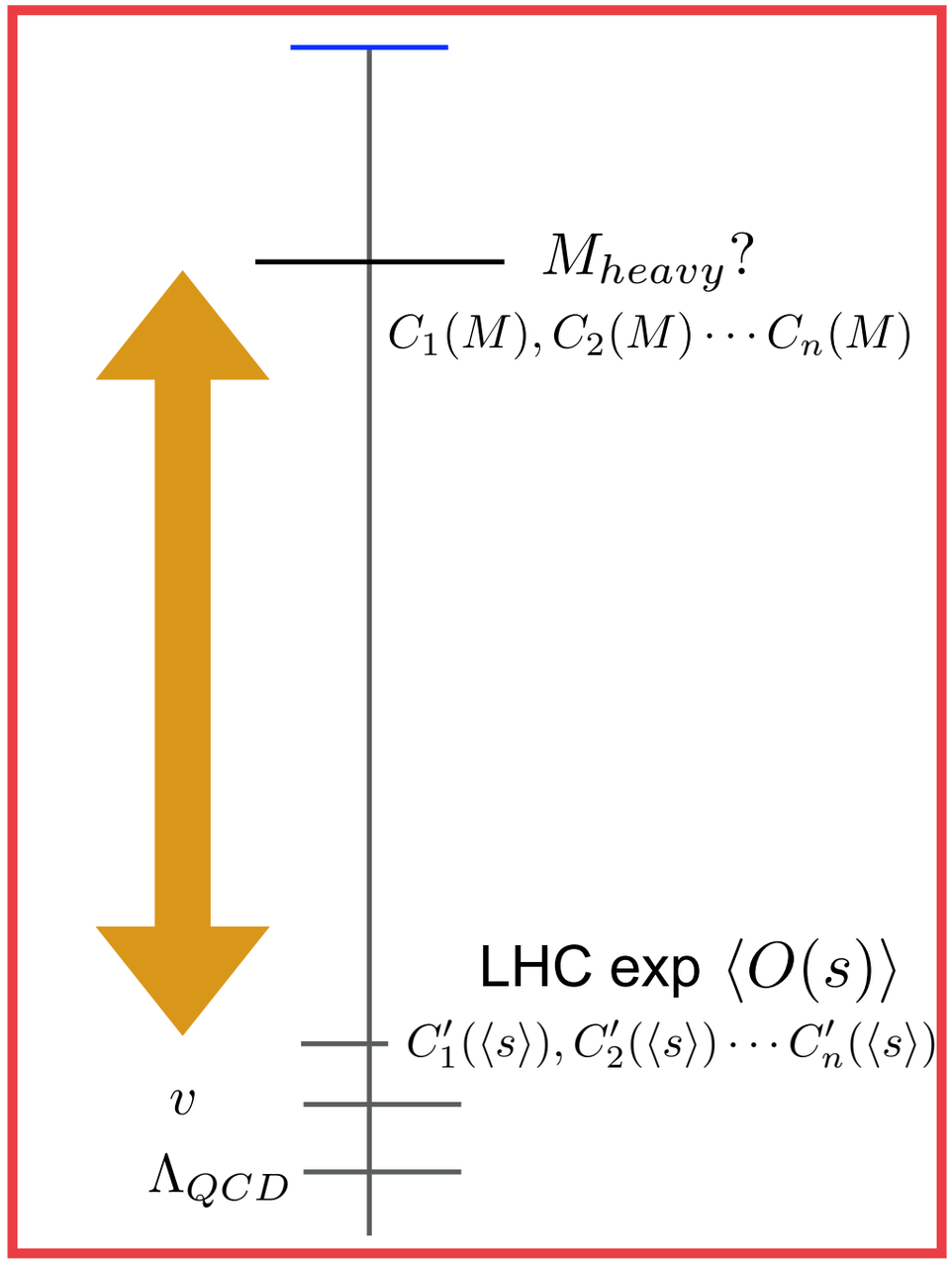
For two input parameter schemes

$\{\hat{\alpha}_{ew}, \hat{M}_Z, \hat{G}_F, \hat{M}_h\}$ $\{\hat{M}_W, \hat{M}_Z, \hat{G}_F, \hat{M}_h\}$

For flavour sym assumptions:

$U(3)^5$ MFV , Gen flavour (2499) :1709.06492

`top`, `topU3l`: $U(2)^3$ symmetry in the quark sector :2012.11343



We need a good (NLO) microscope

- General structure of corrections:

$$\mathcal{A} = \mathcal{A}_{SM} + \tilde{C}_i^{(6)}\, a_i + \tilde{C}_j^{(6)}\, \tilde{C}_k^{(6)}\, b_{jk} + \tilde{C}_l^{(8)}\, c_l,$$
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- What do we have?

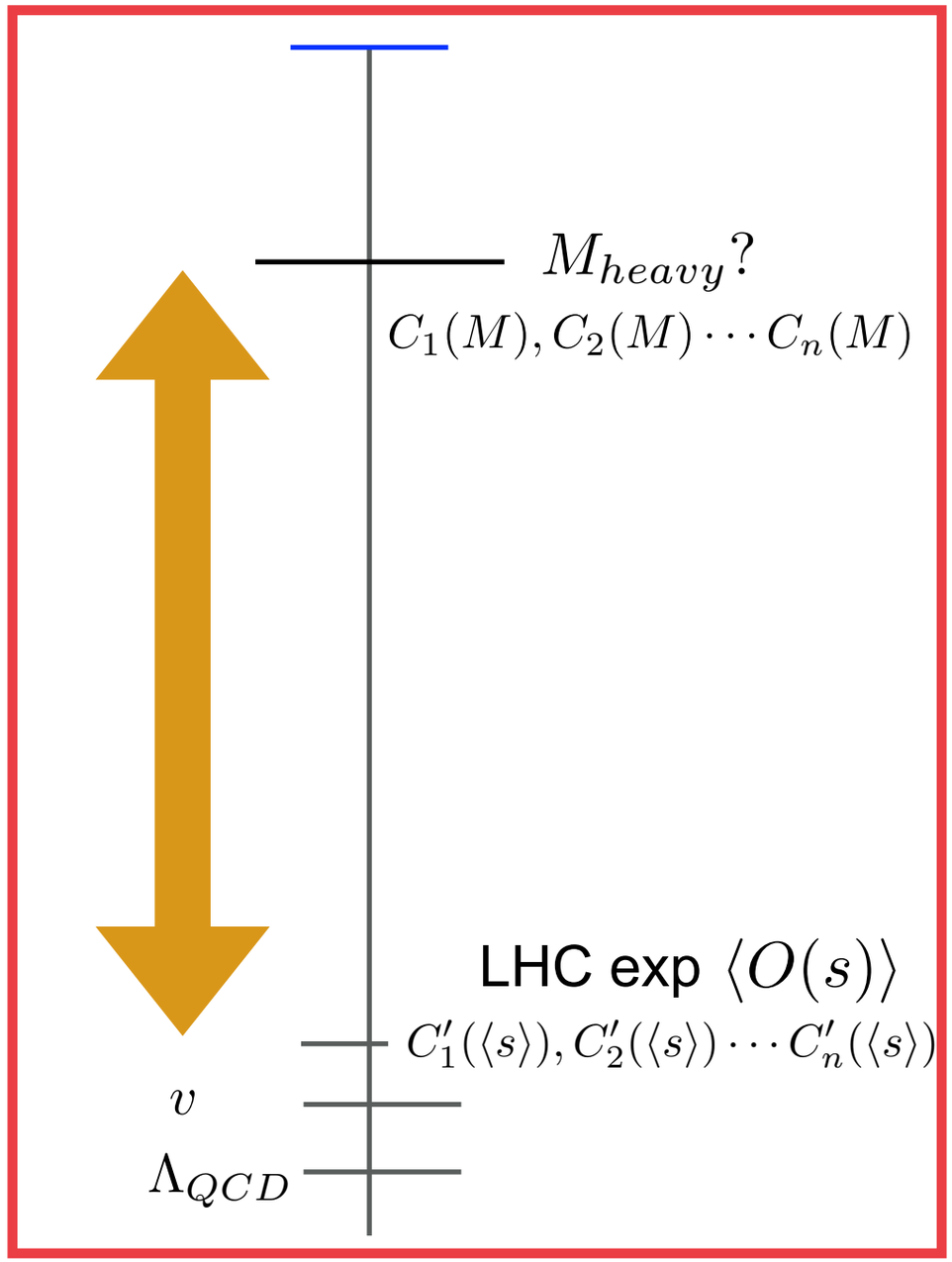
SMEFTsim 1709.06492, 2012.11343

UFO, interfaces with Madgraph gives ***full set*** of

$$a_i$$

For same set of input parameters and flavour assumptions a ***<u>subset</u>*** of the

$$b_{jk}$$



We need a good (NLO) microscope

- General structure of corrections:

$$\mathcal{A} = \mathcal{A}_{SM} + \tilde{C}_i^{(6)}\, a_i + \tilde{C}_j^{(6)}\, \tilde{C}_k^{(6)}\, b_{jk} + \tilde{C}_l^{(8)}\, c_l,$$
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- $\mathcal{L}^{(8)}$ results increasingly available:

$\mathcal{L}^{(8)}$ basis. Murphy 2005.00059

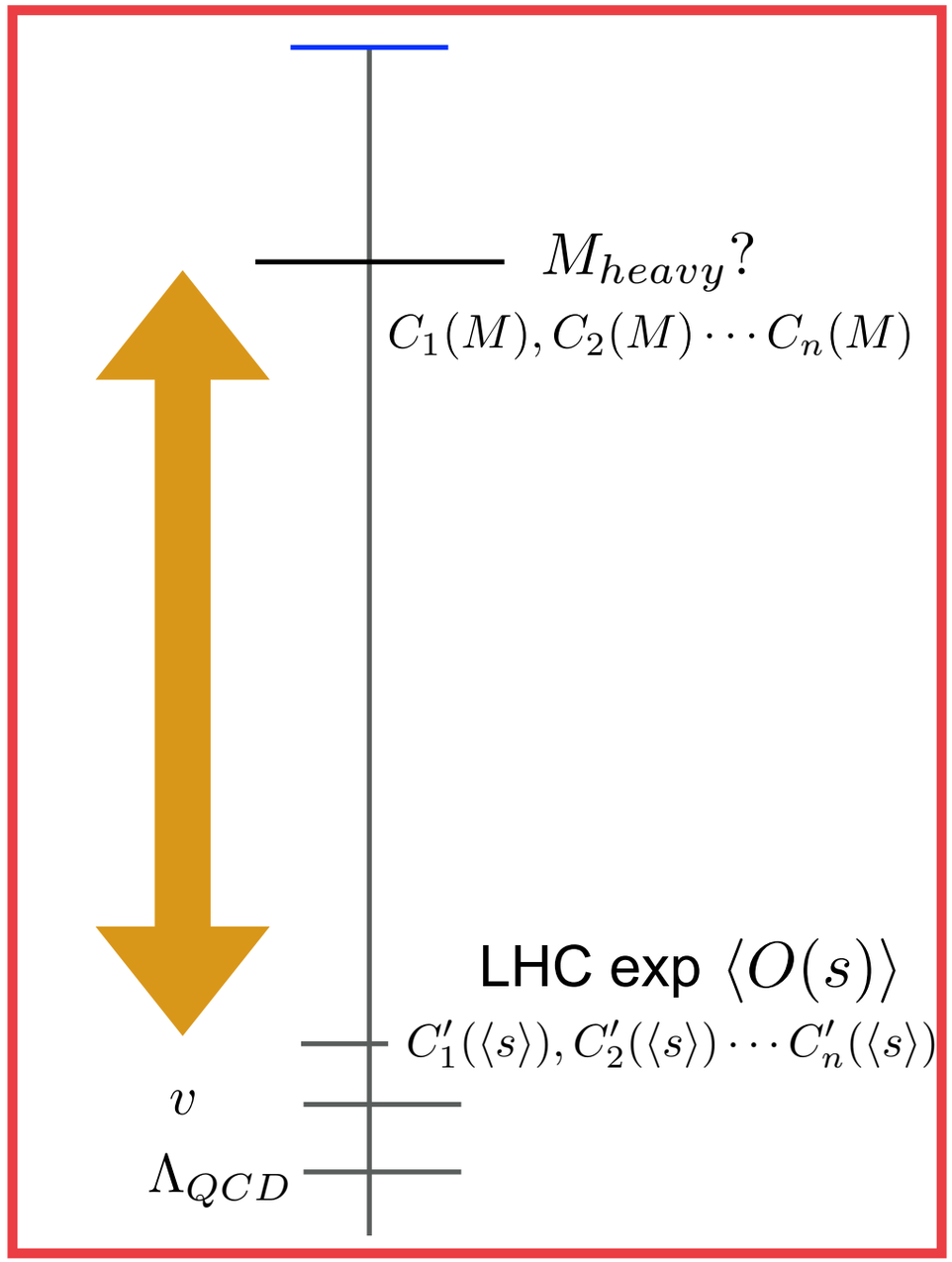
GeoSMEFT enables many dim 8 results

1803.08001, 1909.08470,
2001.01453, 2007.00565

Some specific processes known

Drell Yan/di-lepton: 2003.11615, 2106.05337

mono-lepton: 2203.11976



We need a good (NLO) microscope

- General structure of corrections:

$$\mathcal{A} = \mathcal{A}_{SM} + \tilde{C}_i^{(6)}\, a_i + \tilde{C}_j^{(6)}\, \tilde{C}_k^{(6)}\, b_{jk} + \tilde{C}_l^{(8)}\, c_l,$$
$$+ \frac{1}{16\pi^2}\left[d_m\, \tilde{C}_m^{(6)} + e_n\, \tilde{C}_n^{(6)}\; log\left(\frac{p_i \cdot p_j}{M^2}\right)\right] + \cdots$$

- $\mathcal{L}^{(6)}$ one loop results developed:

$\mu \to e\gamma$ Pruna, Signer 1408.3565

$h \to \gamma\gamma$ 1505.03706, 1505.02646,1507.3568
1807.11504, 2107.07470

$h \to ZZ, Z\gamma$ 1801.01136, 1903.12046

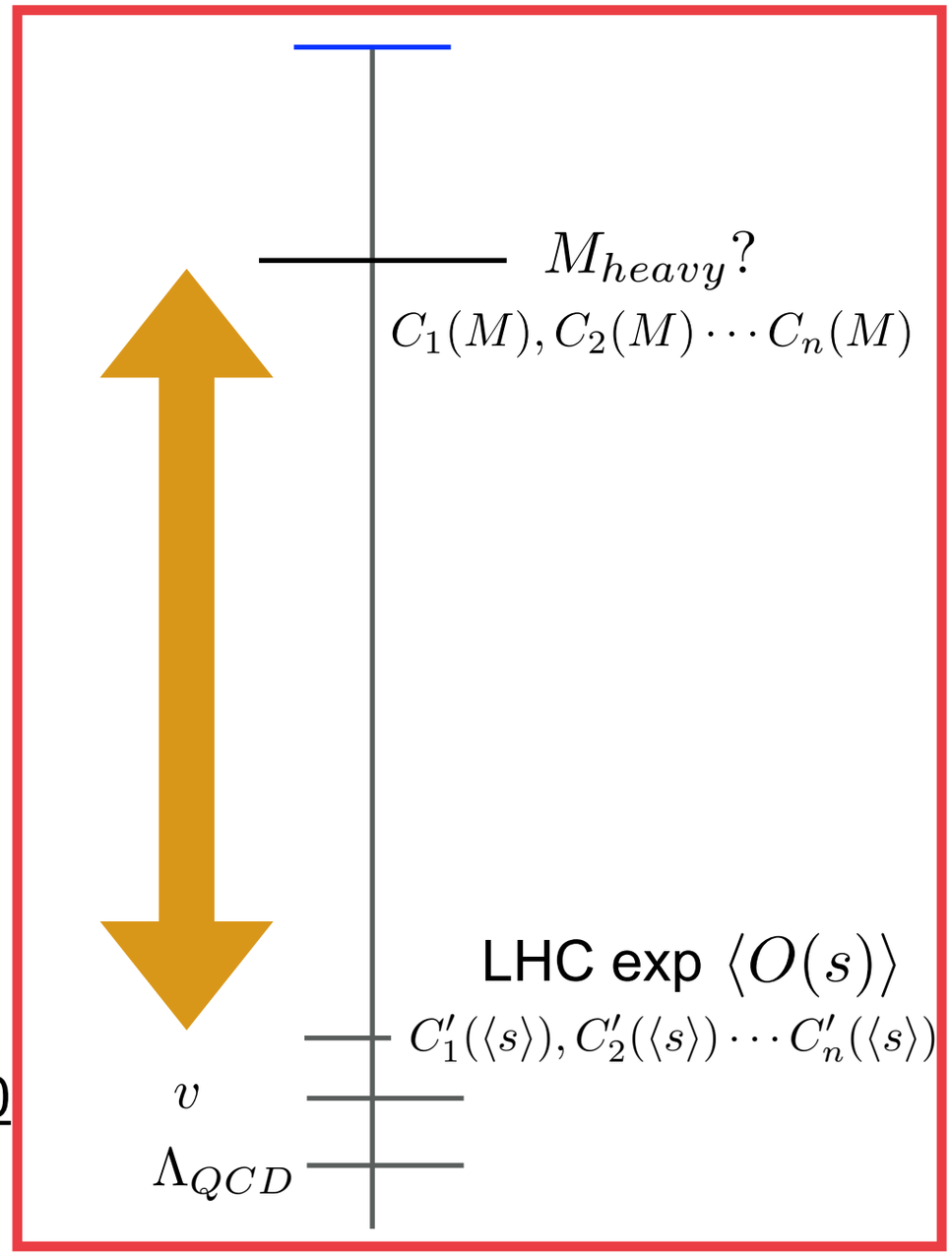
$h \to \bar{f}\, f$ 1512.02508, 1904.06358, 2007.15238

$g\, g \to h\, (j)$ 1708.00460,1806.08832, 2107.07470

Γ_Z 1611.09879, 1808.05948, 1909.02000

$t \to bW$ 1907.00997

A rather limited QCD NLOcode 2008.11743



2107.07470 Denkens Stoffer one loop matching to LEFT

Note cross correlation at sub-leading order

- The operator and loop expansion are not independent.

$$\mathcal{A} = \mathcal{A}_{SM} + \tilde{C}_i^{(6)}\, a_i + \cdots$$

- If you choose to rescale (or not) the Wilson coefficient at L6 by SM parameters, it changes the one loop result and the dimension 8 result in a

$$\mathcal{A} = \mathcal{A}_{SM} + \tilde{C}_i^{(6)}\, g_1^2 + \cdots$$

$$\mathcal{A} = \mathcal{A}_{SM} + \tilde{C}_i^{(6)}\, \hat{g}_1^2 \left[1 + \frac{d_0 + d_j \tilde{C}_j^{(6)}}{16\pi^2} + \frac{e_0 + e_k \tilde{C}_k^{(6)} log(\frac{p_i \cdot p_j}{M^2})}{16\pi^2} + \cdots\right]$$

- So sub-leading terms in the series ***are correlated*** with L.O. conventions, and one another, in a consistent calc:

$$\mathcal{A} = \mathcal{A}_{SM} + \tilde{C}_i^{(6)}\, a_i + \tilde{C}_j^{(6)}\, \tilde{C}_k^{(6)}\, b_{jk} + \tilde{C}_l^{(8)}\, c_l,$$
$$+ \frac{1}{16\pi^2}\left[d_m\, \tilde{C}_m^{(6)} + e_n\, \tilde{C}_n^{(6)}\, \log\left(\frac{\mu^2}{\Lambda^2}\right)\right] + \cdots$$

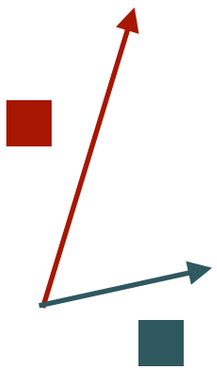
RGE basics

Modification of Higgs properties

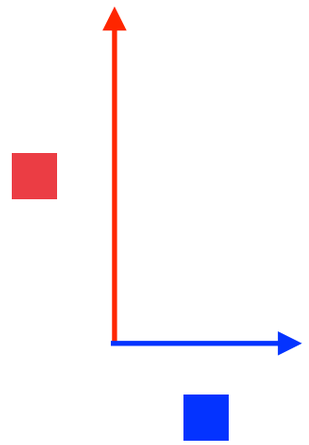
The “interesting scales”

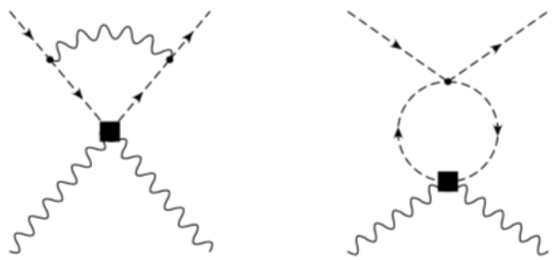
$\sim 125\,GeV$ — Running — $\sim 2000\,GeV$

$$c_i(m_h) = \left(\delta_{ij} - \gamma_{ij}\log\left(\frac{\Lambda}{m_h}\right)\right) c_j(\Lambda)$$



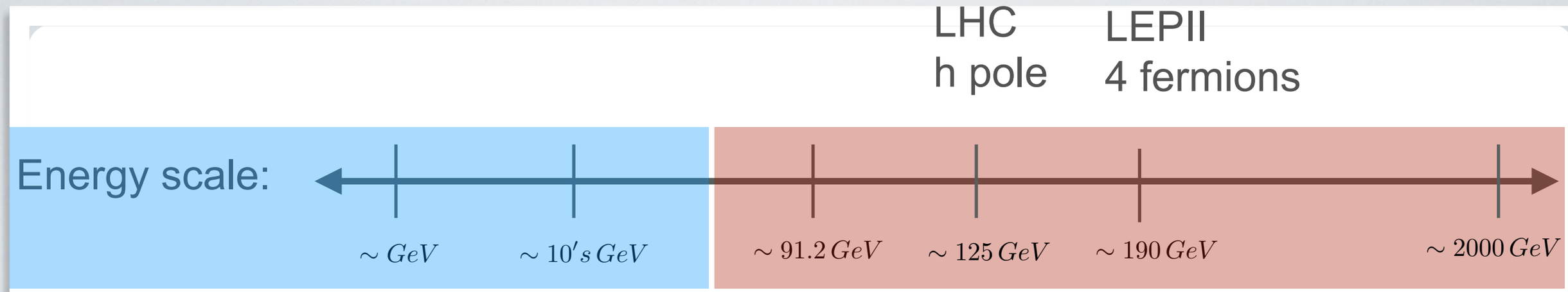
$$u\frac{d}{d\mu}\begin{pmatrix}\blacksquare\\ \blacksquare\end{pmatrix} = \frac{g^2}{16\pi^2}\begin{pmatrix}5 & -5\\ 0 & 0.1\end{pmatrix}\begin{pmatrix}\blacksquare\\ \blacksquare\end{pmatrix}$$





This is a quantum “operator mixing” effect

RGE results



Complete LEFT RGE:

1711.05270 Jenkins, Manohar, Stoffer

HEFT/SMEFT matching LEFT:

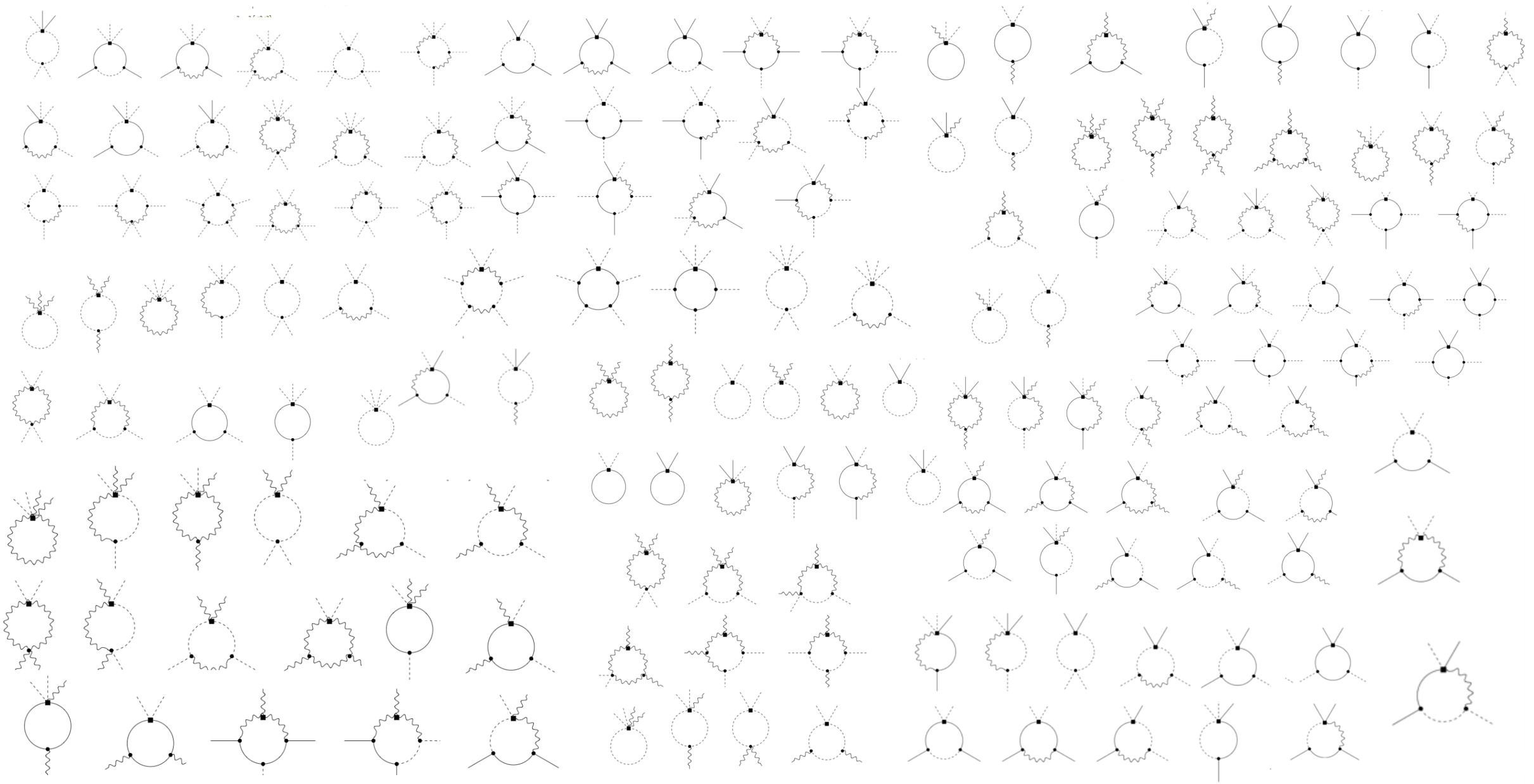
1709.04486 Jenkins, Manohar, Stoffer

Complete SMEFT RGE:

arXiv:1301.2588 Grojean, Jenkins, Manohar, Trott
arXiv:1308.2627, 1309.0819, 1310.4838 Jenkins, Manohar, Trott
arXiv: 1312.2014 Alonso, Jenkins, Manohar, Trott

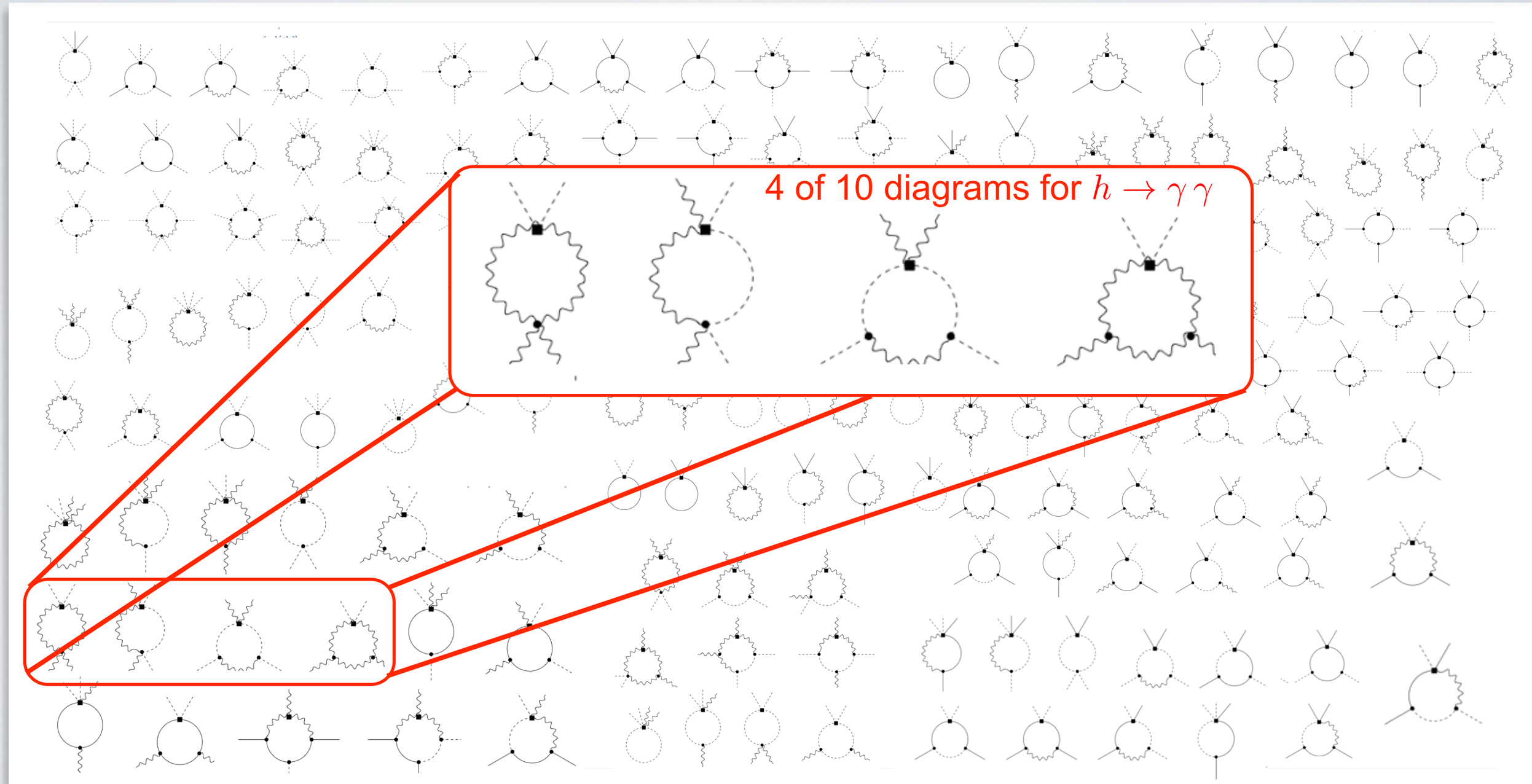
Not a trivial exercise

- 2013

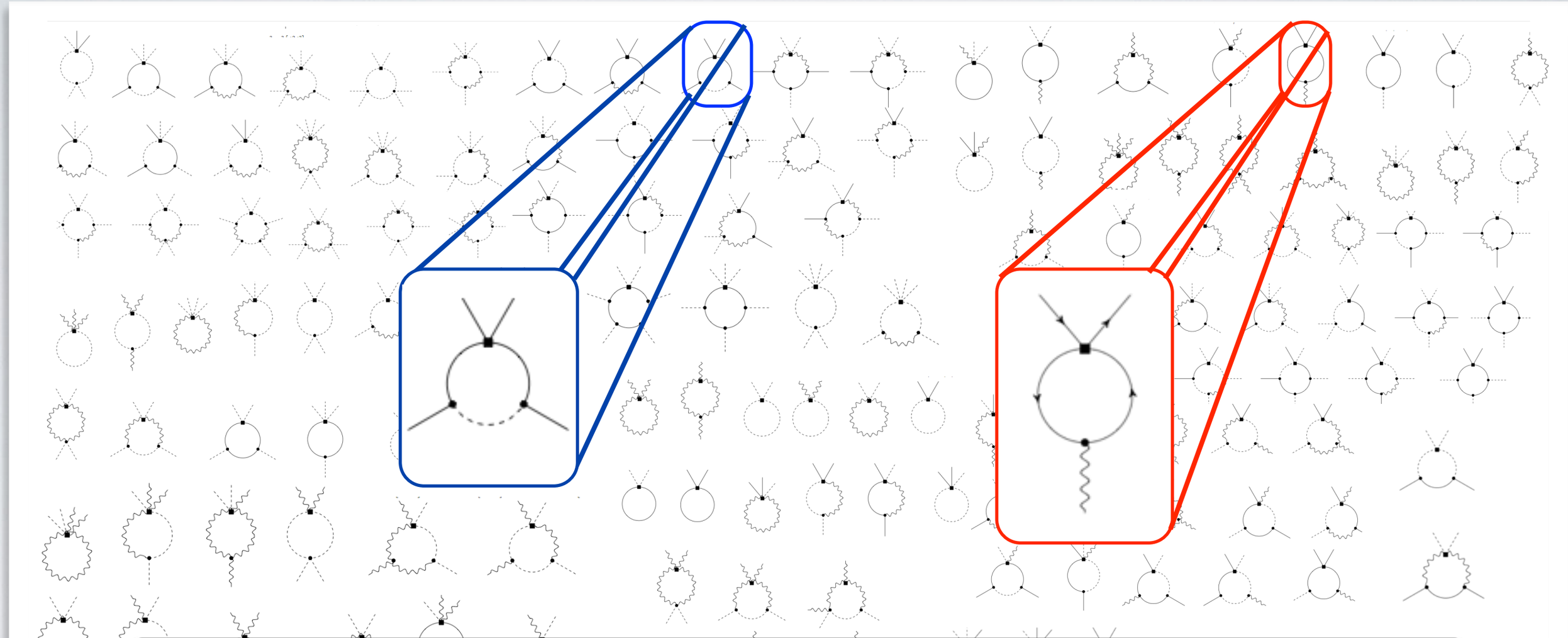


- Each dot can be 59 types operator

Not a trivial exercise



Not a trivial exercise



- Two diagrams that were challenging. 25 4 fermion ops (neglecting flavour)

  Hard part is the group theory, not the divergence.

  Had to keep all indicies for true EFT generality and testing MFV etc..

It is the SMEFT not Higgs EFT.

- It does not really make sense to think of just RGE improving a sector like "the Higgs sector". We need the whole RGE evolution.

Consider the SM equations of motion:

Higgs: $$D^2 H_k - \lambda v^2 H_k + 2\lambda (H^\dagger H) H_k + \overline{q}^j Y_u^\dagger u \epsilon_{jk} + \overline{d} Y_d q_k + \overline{e} Y_e l_k = 0$$

Gauge field:
$$i \not{D} q_j = Y_u^\dagger u \widetilde{H}_j + Y_d^\dagger d H_j \,, \qquad i \not{D} d = Y_d q_j H^{\dagger j} \,, \qquad i \not{D} u = Y_u q_j \widetilde{H}^{\dagger j}$$
$$i \not{D} l_j = Y_e^\dagger e H_j \,, \qquad i \not{D} e = Y_e l_j H^{\dagger j} \,,$$

Fermion: $$[D^\alpha, G_{\alpha\beta}]^A = g_3 j_\beta^A, \qquad [D^\alpha, W_{\alpha\beta}]^I = g_2 j_\beta^I, \qquad D^\alpha B_{\alpha\beta} = g_1 j_\beta,$$

$$j_\beta^A = \sum_{\psi = u,d,q} \overline{\psi} T^A \gamma_\beta \psi \,,$$
$$j_\beta^I = \frac{1}{2} \overline{q} \tau^I \gamma_\beta q + \frac{1}{2} \overline{l} \tau^I \gamma_\beta l + \frac{1}{2} H^\dagger i \overleftrightarrow{D}_\beta^I H \,,$$
$$j_\beta = \sum_{\psi = u,d,q,e,l} \overline{\psi} \mathbf{y}_i \gamma_\beta \psi + \frac{1}{2} H^\dagger i \overleftrightarrow{D}_\beta H \,,$$

SMEFT Renormalisation

- Alonso Jenkins,Manohar,Trott 1312.2014
- Jenkins,Manohar,Trott 1310.4838
- Jenkins,Manohar,Trott 1308.2627

EOM effects essential in RGE

- The EOM have been used EXTENSIVELY in reducing the basis to 59 operators.
  Our intuition does not accommodate that, but it is a fact.
  Here is one way this non-intuitive physics shows up.

You renormalize and obtain a divergence, for example

(People met this in flavour physics Gilman-Wise Phys.Rev. D20 (1979) 2392)

$$E_{H\Box} = [H^\dagger H][H^\dagger (D^2 H) + (D^2 H^\dagger) H]$$

This operator form is not retained in the basis, so remove it:

$$\mathcal{L}_{\rm SM} + \frac{c}{\Lambda^2} E_{H\Box} \to \mathcal{L}_{\rm SM} + \frac{c}{\Lambda^2} \widetilde{E}_{H\Box} + \mathcal{O}\left(\frac{1}{\Lambda^4}\right) \quad \text{via field redefinition} \quad H \to H + \frac{c}{\Lambda^2}(H^\dagger H) H$$

$$\widetilde{E}_{H\Box} = 2\lambda v^2 (H^\dagger H)^2 - 4\lambda Q_H - \left([Y_u^\dagger]_{rs} \underset{rs}{Q_{uH}} + [Y_d^\dagger]_{rs} \underset{rs}{Q_{dH}} + [Y_e^\dagger]_{rs} \underset{rs}{Q_{eH}} + \text{h.c.} \right)$$

- An operator O_1 can mix with an operator O_2 when

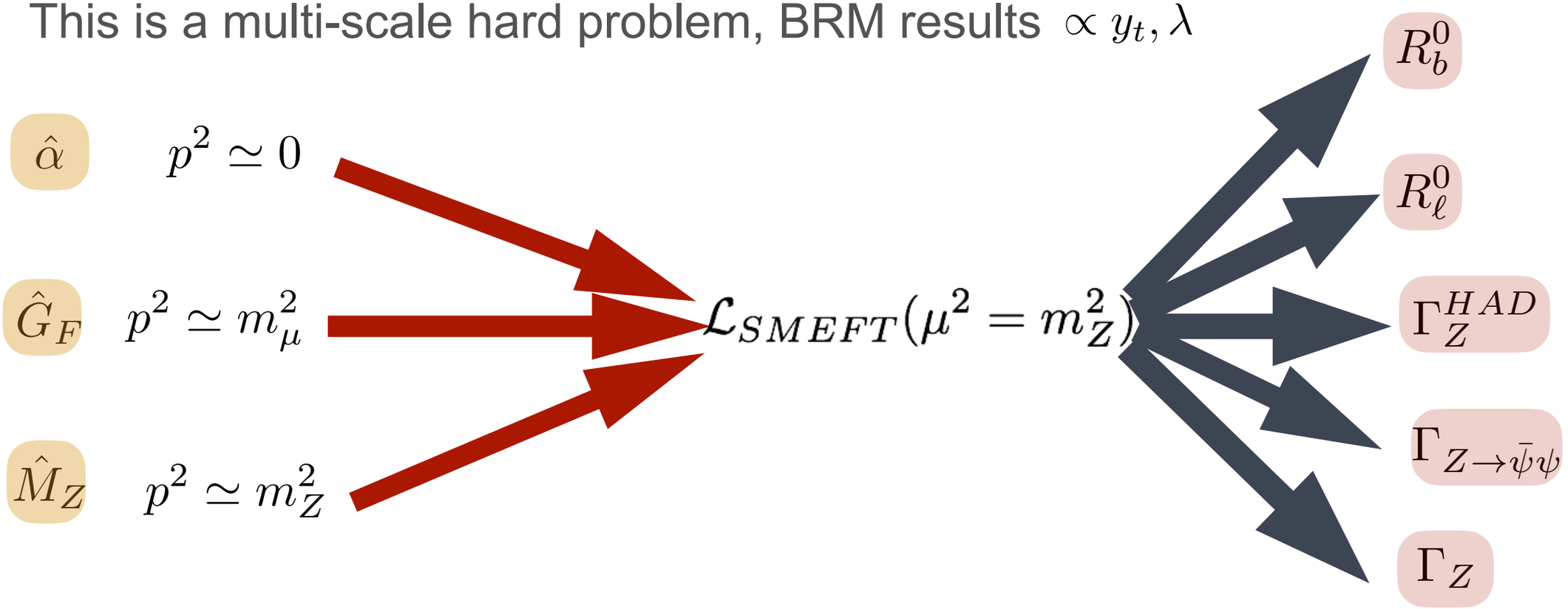
NO 1PI diagram exists that corresponds to the mixing.

An example set of loop calcs

SMEFT decay widths of the Z at one loop

arXiv:1611.09879 One Loop Z C. Hartmann, W. Shepherd, MT

- This is a multi-scale hard problem, BRM results $\propto y_t, \lambda$



- LSZ defn: $\langle Z|S|\bar{\psi}_i\,\psi_i\rangle = (1+\frac{\Delta R_Z}{2})(1+\Delta R_{\psi_i})\,i\,\mathcal{A}_{Z\bar{\psi}_i\psi_i}.$
- Need to loop improve the extraction of parameters AND the decay process of interest.

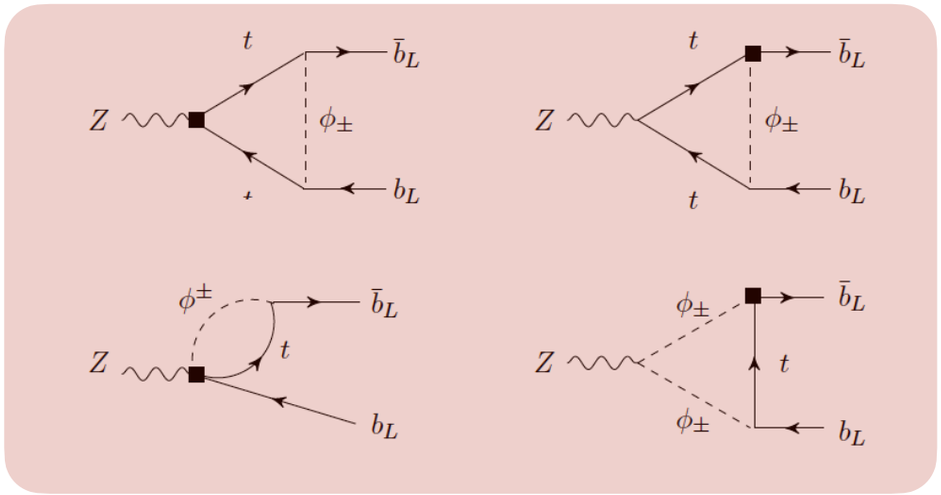
input shifts decay process (wavefunction&process)

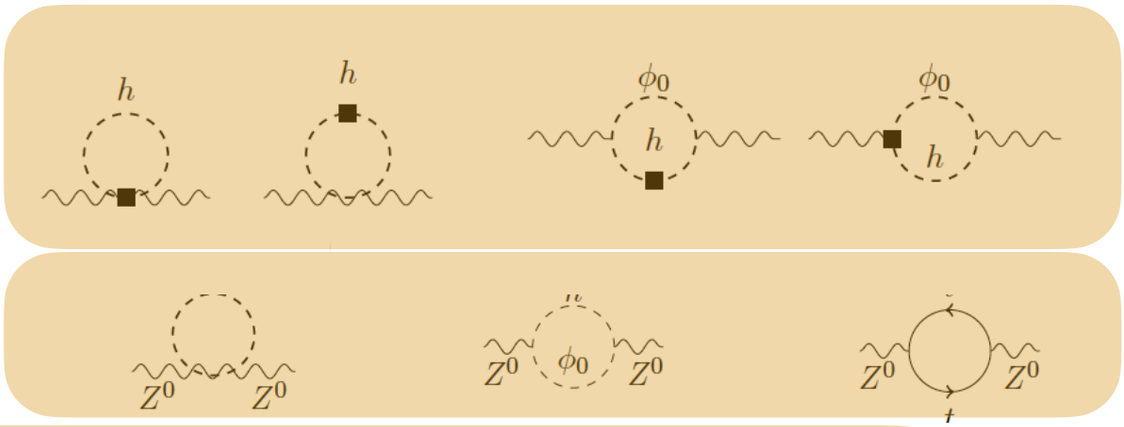
see also : Passarino et al arXiv:1607.01236 , arXiv:1505.03706

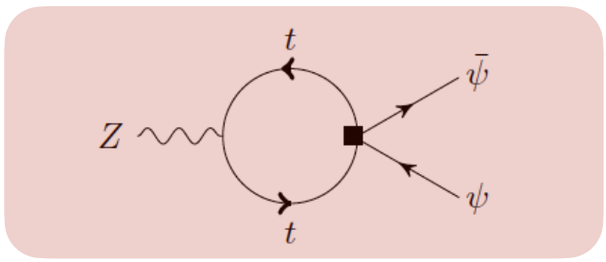
Loops present

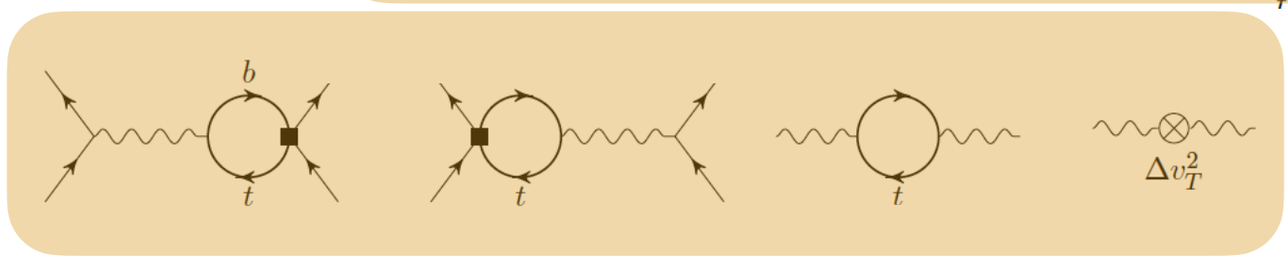
- ~ 30 massive loops in addition to the RGE dim reg results of inputs and outputs need loops

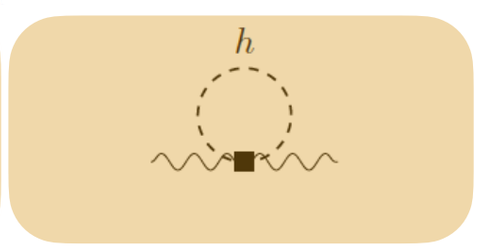
arXiv:1308.2627,1309.0819,1310.4838 Jenkins, Manohar, Trott
arXiv: 1312.2014 Alonso, Jenkins, Manohar, Trott

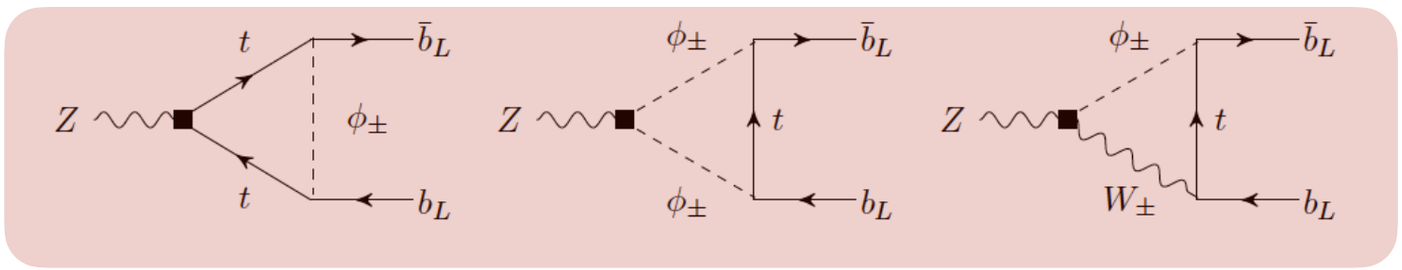


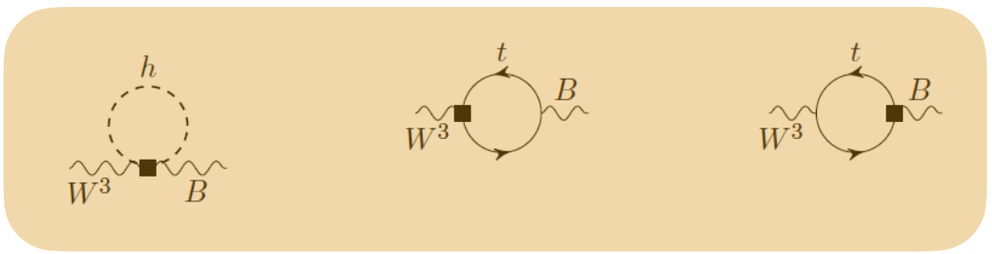










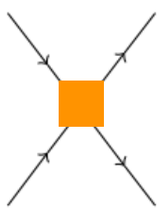


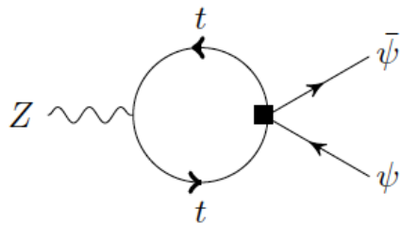
Main conclusions

- ***MORE PARAMETERS*** (At least) the following operators contribute at one loop to EWPD, that are not present at tree level.

$$\{C_{qq}^{(1)}, C_{qq}^{(3)}, C_{qu}^{(1)}, C_{uu}, C_{qd}^{(1)}, C_{ud}^{(1)}, C_{\ell q}^{(1)}, C_{\ell q}^{(3)}, C_{\ell u}, C_{qe}, C_{HB} + C_{HW}, C_{uB}, C_{uW}, C_{uH}\}.$$

- Distinctions between operators made at LO not relevant





- Need to combine data sets carefully due to hierarchies in experimental precision and different scales of measurements

SMEFT Subtleties

- $\Gamma(h \to \gamma\,\gamma)$ Defining the basis of operators as

$$\mathcal{O}_i = (\mathcal{O}_{HB}, \mathcal{O}_{HW}, \mathcal{O}_{HWB}, \mathcal{O}_W, \mathcal{O}_{eB}, \mathcal{O}^*_{eB}, \mathcal{O}_{uB}, \mathcal{O}^*_{uB}, \mathcal{O}_{dB}, \mathcal{O}^*_{dB}, \mathcal{O}_{eW}, \mathcal{O}^*_{eW}, \mathcal{O}_{uW}, \mathcal{O}^*_{uW}, \mathcal{O}_{dW}, \mathcal{O}^*_{dW})$$

$$\begin{aligned}\mathcal{L}_6^{(0)} &= Z_{SM}\, Z_{i,j}\, C_i\, \mathcal{O}_j^{(r)}, \\ &= Z_{SM}\, \mathcal{N}_{HB}\, \mathcal{O}_{HB}^{(r)} + Z_{SM}\, \mathcal{N}_{HW}\, \mathcal{O}_{HW}^{(r)} + Z_{SM}\, \mathcal{N}_{HWB}\, \mathcal{O}_{HWB}^{(r)}.\end{aligned}$$

- 3x3 sub-matrix of ops that contribute at tree level and first at one loop

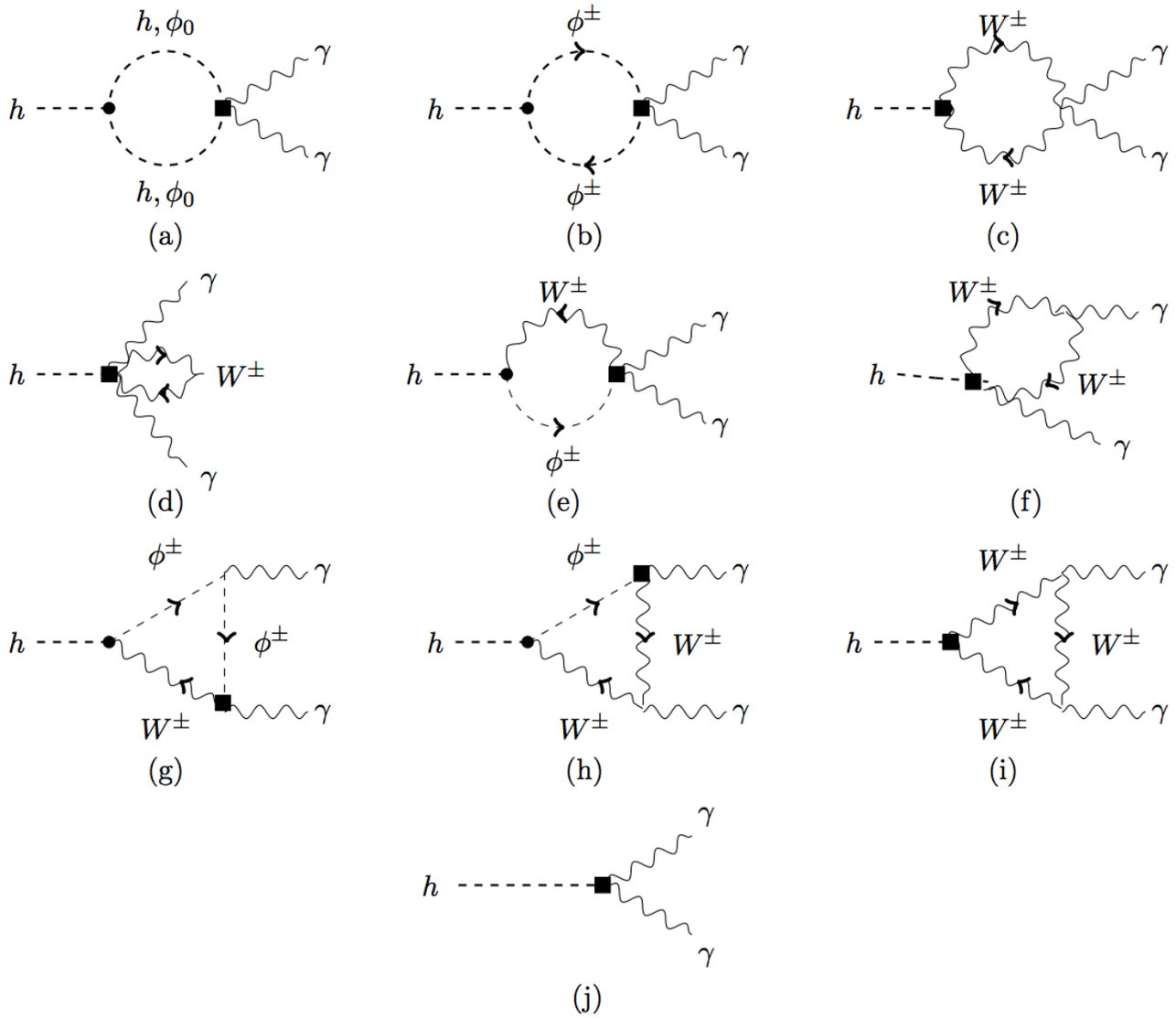
$$Z_{i,j} = \frac{1}{16\,\pi^2} \cdot \begin{pmatrix} \frac{g_1^2}{4} - \frac{9g_2^2}{4} + 6\lambda + Y & 0 & g_1^2 \\ 0 & -\frac{3g_1^2}{4} - \frac{5g_2^2}{4} + 6\lambda + Y & g_2^2 \\ \frac{3g_2^2}{2} & \frac{g_1^2}{2} & -\frac{g_1^2}{4} + \frac{9g_2^2}{4} + 2\lambda + Y \end{pmatrix}$$

$$\begin{pmatrix} 0 & -\frac{15}{2}\,g_2^4 & \frac{3}{2}\,g_2^4 \\ -(y_l + y_e)\,Y_e & 0 & -\frac{1}{2}\,Y_e \\ -(y_l + y_e)\,Y_e^\dagger & 0 & -\frac{1}{2}\,Y_e^\dagger \\ -N_c\,(y_q + y_u)\,Y_u & 0 & \frac{1}{2}\,N_c\,Y_u \\ -N_c\,(y_q + y_u)\,Y_u^\dagger & 0 & \frac{1}{2}\,N_c\,Y_u^\dagger \\ -N_c\,(y_q + y_d)\,Y_d & 0 & -\frac{1}{2}\,N_c\,Y_d \\ -N_c\,(y_q + y_d)\,Y_d^\dagger & 0 & -\frac{1}{2}\,N_c\,Y_d^\dagger \\ 0 & -\frac{1}{2}\,Y_e & -(y_l + y_e)\,Y_e \\ 0 & -\frac{1}{2}\,Y_e^\dagger & -(y_l + y_e)\,Y_e^\dagger \\ 0 & -\frac{1}{2}\,N_c\,Y_u & N_c\,(y_q + y_u)\,Y_u \\ 0 & -\frac{1}{2}\,N_c\,Y_u^\dagger & N_c\,(y_q + y_u)\,Y_u^\dagger \\ 0 & -\frac{1}{2}\,N_c\,Y_d & -N_c\,(y_q + y_d)\,Y_d \\ 0 & -\frac{1}{2}\,N_c\,Y_d^\dagger & -N_c\,(y_q + y_d)\,Y_d^\dagger \end{pmatrix}$$

- Counter-term subtraction is proportional to v

The required loops.

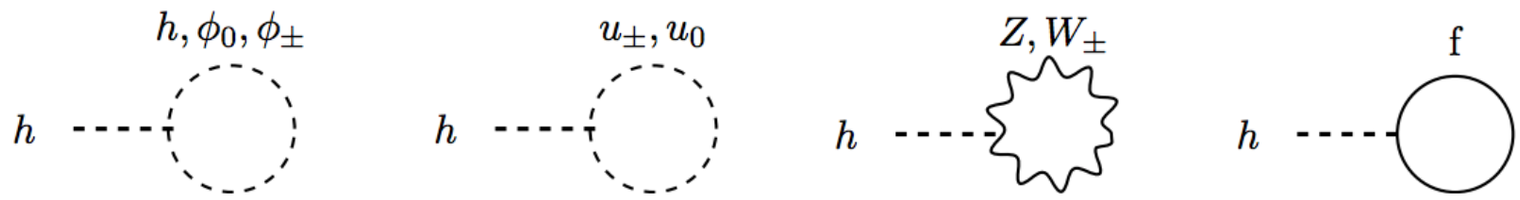
- Calculate in BF method, in R_ξ gauge, for operators that contribute at tree level



- Gauge dependence cancels ☑ remaining divergences cancel exactly ☑

NLO EFT - fix finite terms

- Define vev of the theory as the one point function vanishing - fixes δv



$$
\begin{aligned}
T = \; & m_h^2\, h\, v\, \frac{1}{16\pi^2}\left[-16\pi^2\,\frac{\delta v}{v} + 3\,\lambda\left(1+\log\left[\frac{\mu^2}{m_h^2}\right]\right) + \frac{m_W^2}{v^2}\xi\left(1+\log\left[\frac{\mu^2}{\xi\, m_W^2}\right]\right)\right., \\
& + \frac{1}{2}\frac{m_Z^2}{v^2}\xi\left(1+\log\left[\frac{\mu^2}{\xi\, m_Z^2}\right]\right) - \frac{1}{2}\sum_i y_i^4\, N_c \frac{1}{\lambda}\left(1+\log\left[\frac{\mu^2}{m_i^2}\right]\right), \\
& \left. + \frac{g_2^2}{2}\frac{m_W^2}{m_h^2}\left(1+3\log\left[\frac{\mu^2}{m_W^2}\right]\right) + \frac{1}{4}(g_1^2+g_2^2)\frac{m_Z^2}{m_h^2}\left(1+3\log\left[\frac{\mu^2}{m_Z^2}\right]\right)\right].
\end{aligned}
$$

- The finite terms that are fixed by renormalization conditions (at one loop) in the theory enter as

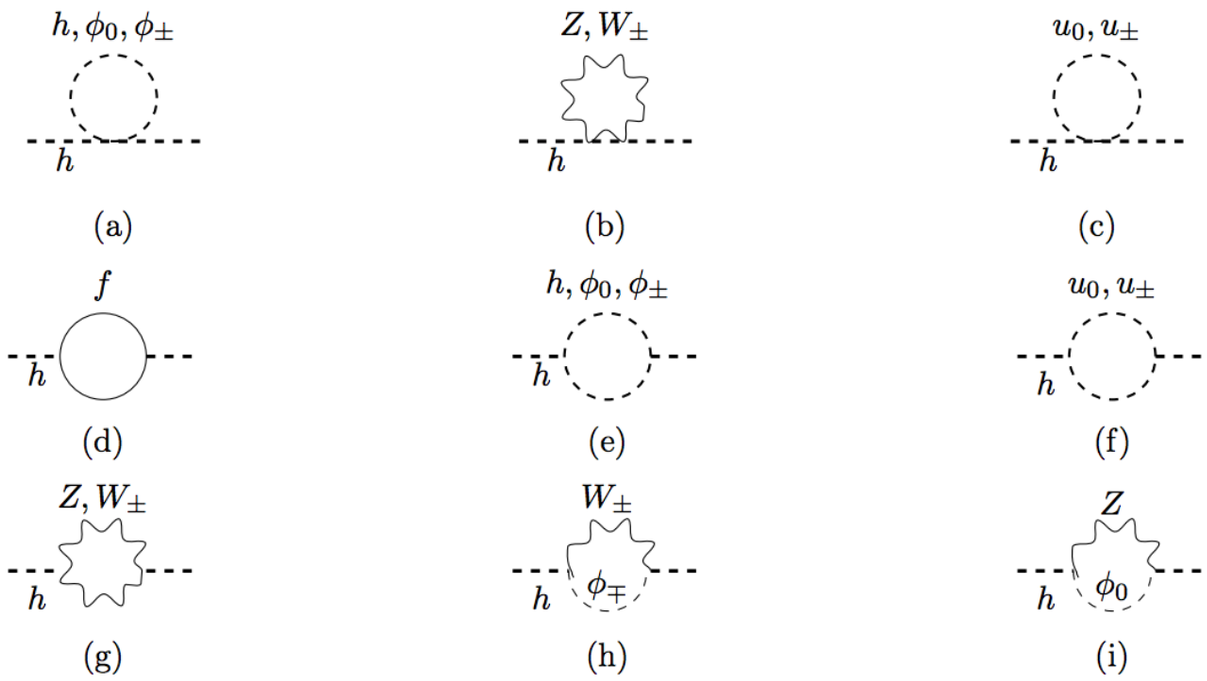
$$
\langle h(p_h)|S|\gamma(p_a,\alpha),\gamma(p_b,\beta)\rangle_{BSM} = (1+\frac{\delta R_h}{2})\,(1+\delta R_A)\,(1+\delta R_e)^2\, i \sum_{x=a..o} \mathcal{A}_x.
$$

NLO EFT - fix finite terms

- Remaining finite terms fixed by defining in renormalization conditions on the couplings and two point function residues and poles

$$\delta R_h = -\frac{\partial \Pi_{hh}(p^2)}{\partial p^2}|_{p^2=m_h^2} \qquad \delta R_e = -\frac{1}{2}\delta R_A,$$

So also calc:



Renormalization conditions

- The finite terms that are fixed by renormalization conditions (at one loop) in the theory enter as

$$\langle h(p_h)|S|\gamma(p_a,\alpha),\gamma(p_b,\beta)\rangle_{BSM} = (1+\frac{\delta R_h}{2})\,(1+\delta R_A)\,(1+\delta R_e)^2\, i \sum_{x=a..o} \mathcal{A}_x.$$

Cancels!

- Remaining finite terms fixed by defining in renormalization conditions on the couplings and two point function residues and poles

$$\delta R_h = -\frac{\partial \Pi_{hh}(p^2)}{\partial p^2}|_{p^2=m_h^2} \qquad \delta R_e = -\frac{1}{2}\delta R_A,$$

This relation follows from a Ward identity using BFM.

NLO SMEFT

1505.02646 Hartmann, Trott

- The final tree result is of the form

$$\frac{i\,\mathcal{A}^{NP}_{total}}{i\,v\,e^2\,A^{h\gamma\gamma}_{\alpha\beta}} = C_{\gamma\gamma}\left(1+\frac{\delta R_h}{2}+\frac{\delta\,v}{v}\right),$$
$$+\left(\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\frac{g_1^2}{4}+\frac{3\,g_2^2}{4}+6\,\lambda\right)+\frac{C_{HWB}}{16\,\pi^2}\left(-3\,g_2^2+4\,\lambda\right)\right)\log\left(\frac{m_h^2}{\Lambda^2}\right),$$
$$+\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\left(\frac{g_1^2}{4}+\frac{g_2^2}{4}+\lambda\right)\mathcal{I}[m_Z^2]+\left(\frac{g_2^2}{2}+2\lambda\right)\mathcal{I}[m_W^2]+(\sqrt{3}\,\pi-6)\,\lambda\right),$$
$$+\frac{C_{HWB}}{16\,\pi^2}\left(2e^2\left(1+6\frac{m_W^2}{m_h^2}\right)-2\,g_2^2\left(1+\log\left(\frac{m_W^2}{m_h^2}\right)\right)+(4\,\lambda-g_2^2)\,\mathcal{I}[m_W^2],\right.$$
$$\left.+4\left(3e^2-g_2^2-6e^2\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right),$$
$$-\frac{g_2^2\,C_{HW}}{4\,\pi^2}\left(3\,\frac{m_W^2}{m_h^2}+\left(4-\frac{m_h^2}{m_W^2}-6\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right).$$

Where $C^{NP}_{\gamma\gamma} = C_{HB}+C_{HW}-C_{HWB}$

$$\mathcal{I}[m^2]\equiv\int_0^1 dx\,\log\left(\frac{m^2-m_h^2\,x\,(1-x)}{m_h^2}\right),\quad \mathcal{I}_y[m^2]\equiv\int_0^{1-x}dy\int_0^1 dx\,\frac{m^2}{m^2-m_h^2\,x\,(1-x-y)},$$

NLO SMEFT

1505.02646 Hartmann, Trott

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$$+\left(\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\frac{g_1^2}{4}+\frac{3\,g_2^2}{4}+6\,\lambda\right)+\frac{C_{HWB}}{16\,\pi^2}\left(-3\,g_2^2+4\,\lambda\right)\right)\log\left(\frac{m_h^2}{\Lambda^2}\right),$$
$$+\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\left(\frac{g_1^2}{4}+\frac{g_2^2}{4}+\lambda\right)\mathcal{I}[m_Z^2]+\left(\frac{g_2^2}{2}+2\lambda\right)\mathcal{I}[m_W^2]+(\sqrt{3}\,\pi-6)\,\lambda\right),$$
$$+\frac{C_{HWB}}{16\,\pi^2}\left(2e^2\left(1+6\frac{m_W^2}{m_h^2}\right)-2\,g_2^2\left(1+\log\left(\frac{m_W^2}{m_h^2}\right)\right)+(4\,\lambda-g_2^2)\,\mathcal{I}[m_W^2],\right.$$
$$\left.+4\left(3e^2-g_2^2-6e^2\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right),$$
$$-\frac{g_2^2\,C_{HW}}{4\,\pi^2}\left(3\,\frac{m_W^2}{m_h^2}+\left(4-\frac{m_h^2}{m_W^2}-6\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right).$$

Fixed by renorm conditions

Where $C^{NP}_{\gamma\gamma} = C_{HB}+C_{HW}-C_{HWB}$

$$\mathcal{I}[m^2]\equiv\int_0^1 dx\,\log\left(\frac{m^2-m_h^2\,x\,(1-x)}{m_h^2}\right),\quad \mathcal{I}_y[m^2]\equiv\int_0^{1-x}dy\int_0^1 dx\,\frac{m^2}{m^2-m_h^2\,x\,(1-x-y)},$$

NLO SMEFT

1505.02646 Hartmann, Trott

- The final tree result is of the form

$$\frac{i\,\mathcal{A}^{NP}_{total}}{i\,v\,e^2\,A^{h\gamma\gamma}_{\alpha\beta}} = C_{\gamma\gamma}\left(1+\frac{\delta R_h}{2}+\frac{\delta\,v}{v}\right),$$
$$+\left(\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\frac{g_1^2}{4}+\frac{3\,g_2^2}{4}+6\,\lambda\right)+\frac{C_{HWB}}{16\,\pi^2}\left(-3\,g_2^2+4\,\lambda\right)\right)\log\left(\frac{m_h^2}{\Lambda^2}\right),$$
$$+\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\left(\frac{g_1^2}{4}+\frac{g_2^2}{4}+\lambda\right)\mathcal{I}[m_Z^2]+\left(\frac{g_2^2}{2}+2\lambda\right)\mathcal{I}[m_W^2]+(\sqrt{3}\,\pi-6)\,\lambda\right),$$
$$+\frac{C_{HWB}}{16\,\pi^2}\left(2e^2\left(1+6\frac{m_W^2}{m_h^2}\right)-2\,g_2^2\left(1+\log\left(\frac{m_W^2}{m_h^2}\right)\right)+(4\,\lambda-g_2^2)\,\mathcal{I}[m_W^2],\right.$$
$$\left.+4\left(3e^2-g_2^2-6e^2\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right),$$
$$-\frac{g_2^2\,C_{HW}}{4\,\pi^2}\left(3\,\frac{m_W^2}{m_h^2}+\left(4-\frac{m_h^2}{m_W^2}-6\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right).$$

"(not so) Large" log terms consistent with RGE

Where $C^{NP}_{\gamma\gamma} = C_{HB}+C_{HW}-C_{HWB}$

$$\mathcal{I}[m^2]\equiv\int_0^1 dx\,\log\left(\frac{m^2-m_h^2\,x\,(1-x)}{m_h^2}\right),\quad \mathcal{I}_y[m^2]\equiv\int_0^{1-x}dy\int_0^1 dx\,\frac{m^2}{m^2-m_h^2\,x\,(1-x-y)},$$

NLO SMEFT

1505.02646 Hartmann, Trott

- The final tree result is of the form

$$\frac{i\,\mathcal{A}^{NP}_{total}}{i\,v\,e^2\,A^{h\gamma\gamma}_{\alpha\beta}} = C_{\gamma\gamma}\left(1+\frac{\delta R_h}{2}+\frac{\delta\,v}{v}\right),$$
$$+\left(\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\frac{g_1^2}{4}+\frac{3\,g_2^2}{4}+6\,\lambda\right)+\frac{C_{HWB}}{16\,\pi^2}\left(-3\,g_2^2+4\,\lambda\right)\right)\log\left(\frac{m_h^2}{\Lambda^2}\right),$$
$$+\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\left(\frac{g_1^2}{4}+\frac{g_2^2}{4}+\lambda\right)\mathcal{I}[m_Z^2]+\left(\frac{g_2^2}{2}+2\lambda\right)\mathcal{I}[m_W^2]+(\sqrt{3}\,\pi-6)\,\lambda\right),$$
$$+\frac{C_{HWB}}{16\,\pi^2}\left(2e^2\left(1+6\frac{m_W^2}{m_h^2}\right)-2\,g_2^2\left(1+\log\left(\frac{m_W^2}{m_h^2}\right)\right)+(4\,\lambda-g_2^2)\,\mathcal{I}[m_W^2],\right.$$
$$\left.+4\left(3e^2-g_2^2-6e^2\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right),$$
$$-\frac{g_2^2\,C_{HW}}{4\,\pi^2}\left(3\,\frac{m_W^2}{m_h^2}+\left(4-\frac{m_h^2}{m_W^2}-6\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right).$$

Finite terms with associated logs terms

Where $C^{NP}_{\gamma\gamma} = C_{HB}+C_{HW}-C_{HWB}$

$$\mathcal{I}[m^2]\equiv\int_0^1 dx\,\log\left(\frac{m^2-m_h^2\,x\,(1-x)}{m_h^2}\right),\quad \mathcal{I}_y[m^2]\equiv\int_0^{1-x}dy\int_0^1 dx\,\frac{m^2}{m^2-m_h^2\,x\,(1-x-y)},$$

NLO SMEFT

1505.02646 Hartmann, Trott

- The final tree result is of the form

$$\frac{i\,\mathcal{A}^{NP}_{total}}{i\,v\,e^2\,A^{h\gamma\gamma}_{\alpha\beta}} = C_{\gamma\gamma}\left(1+\frac{\delta R_h}{2}+\frac{\delta\,v}{v}\right),$$
$$+\left(\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\frac{g_1^2}{4}+\frac{3\,g_2^2}{4}+6\,\lambda\right)+\frac{C_{HWB}}{16\,\pi^2}\left(-3\,g_2^2+4\,\lambda\right)\right)\log\left(\frac{m_h^2}{\Lambda^2}\right),$$
$$+\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\left(\frac{g_1^2}{4}+\frac{g_2^2}{4}+\lambda\right)\mathcal{I}[m_Z^2]+\left(\frac{g_2^2}{2}+2\lambda\right)\mathcal{I}[m_W^2]+(\sqrt{3}\,\pi-6)\,\lambda\right),$$
$$+\frac{C_{HWB}}{16\,\pi^2}\left(2e^2\left(1+6\frac{m_W^2}{m_h^2}\right)-2\,g_2^2\left(1+\log\left(\frac{m_W^2}{m_h^2}\right)\right)+(4\,\lambda-g_2^2)\,\mathcal{I}[m_W^2],\right.$$
$$\left.+4\left(3e^2-g_2^2-6e^2\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right),$$
$$-\frac{g_2^2\,C_{HW}}{4\,\pi^2}\left(3\,\frac{m_W^2}{m_h^2}+\left(4-\frac{m_h^2}{m_W^2}-6\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right).$$

"Pure" finite terms not in $C_{\gamma\,\gamma}$ and no associated log

Where $C^{NP}_{\gamma\gamma} = C_{HB}+C_{HW}-C_{HWB}$

$$\mathcal{I}[m^2]\equiv\int_0^1 dx\,\log\left(\frac{m^2-m_h^2\,x\,(1-x)}{m_h^2}\right),\quad \mathcal{I}_y[m^2]\equiv\int_0^{1-x}dy\int_0^1 dx\,\frac{m^2}{m^2-m_h^2\,x\,(1-x-y)},$$

NLO EFT - Physics lessons

- Operators can contribute a “pure finite term” at NLO and not have a corresponding RGE log. This fact consistent with results in 1505.03706 Ghezzi et al.

- Finite terms are not small in general compared to the log terms

$$R_{CHWB/CHW} \simeq \frac{C_{HWB}}{C_{HW}}\left(0.5 + 0.7\,\log\frac{m_h^2}{\Lambda^2}\right) \qquad R_{CHWB} \simeq 1 + 0.7\,\log^{-1}\frac{m_h^2}{\Lambda^2}$$

- Log mu dependence of RGE consistent with full one loop result, but important modification due to mass scales running (vev not 0)

- <u>The RGE is not a good proxy for the full one loop structure of the SMEFT.</u>

(0’s in the rge do not mean 0’s guaranteed at one loop for finite terms)

NLO SMEFT

1505.02646 Hartmann, Trott

- An unusual feature of the calculation

$$
\begin{aligned}
\mathcal{L}_{eff}(v^2) &= 2\,e^4\,v^2\,C_{HWB}\,\left(A_\mu A^\mu W^+_\nu W^\nu_- - A_\mu A_\nu W^\mu_+ W^\nu_-\right) - i\,e\,g_2^2\,v^2\,C_{HWB}\,A_{\mu\,\nu} W^+_\mu\,W^\nu_-, \\
&+ i\,\frac{e^3\,v^2\,C_{HWB}}{\xi}\,\left(A_\mu W^\mu_+ \partial_\nu W^\nu_- - A_\mu W^\mu_- \partial_\nu W^\nu_+ + 2\,e\,W^+_\mu\,W^\nu_-\,A_\nu\,A^\mu\right), \\
&+ i\,e^3\,v^2\,C_{HWB}\,\left(A_\mu W^+_\nu W^{\mu\nu}_- - A_\mu W^-_\nu W^{\mu\nu}_+ + W^+_\mu\,W^-_\nu\,A^{\mu\nu}\right), \\
&+ e^3\,v^2\,C_{HWB}\,\left(i\,\phi_-\,A_\mu\,\partial^\mu\phi_+ - i\,\phi_+\,A_\mu\,\partial^\mu\phi_- - 2\,e\,\phi_+\,\phi_-\,A_\mu\,A^\mu\right), \\
&- \frac{1}{2}\,e^3\,g_2\,v^2\,C_{HWB}\,A_\mu\,\left(W^\mu_-\phi_+\phi_0 + W^\mu_+\phi_-\phi_0\right) - 2\,e^4\,v^2\,C_{HWB}\,A_\mu\,A^\mu\,\left(\bar{u}^+\,u^+ + \bar{u}^-\,u^-\right), \\
&+ ie^3v^2C_{HWB}\,\left(\left(\bar{u}^+\,A^\mu\,\partial_\mu u^+ - \bar{u}^-\,A^\mu\,\partial_\mu u^-\right) + \partial_\mu A^\mu\,\left(\bar{u}^+ u^+ - \bar{u}^- u^-\right)\right).
\end{aligned}
$$

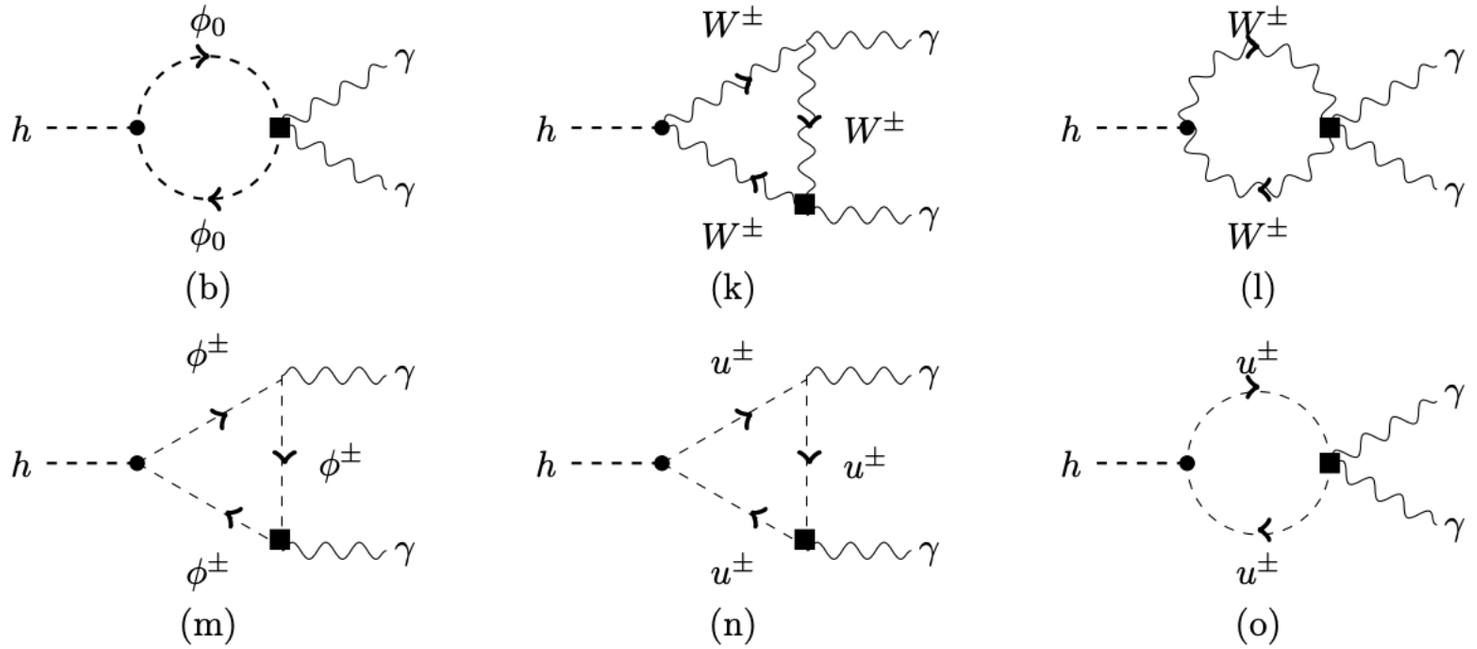


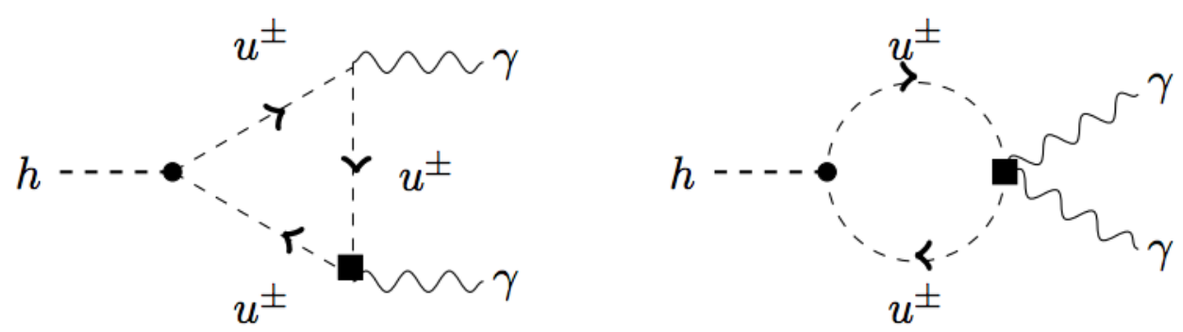
Figure 3. Diagrams contributing to $H \to \gamma\gamma$ decay due to $\mathcal{L}_{eff}(v^2)$. The divergent terms exactly cancel in this class of contributions, as expected. $u^\pm$ are ghost fields.

SMEFT gauge fixing issues.

- The fields are redefined at each order in the power counting, this leads to the appearance of L6 Wilson coefficients in the gauge fixing term.

$$\mathcal{L}_{FP} = -\bar{u}^{\alpha} \frac{\delta G^{\alpha}}{\delta \theta^{\beta}} u^{\beta}.$$

Some operators in $\mathcal{L}_6$ then source ghosts!



- This cancels the unusual divergences in $\Gamma_{SMEFT}(h \to \gamma\gamma)$ exactly.
- The mismatch of the mass eigenstates in the SMEFT with the SM means gauge fixing in the former also results in some interesting local contact operators

$$-\frac{c_w\, s_w}{\xi_B\, \xi_W}(\xi_B - \xi_W)\,(\partial^{\mu} A_{\mu}\, \partial^{\nu}\, Z_{\nu}) - \frac{C_{HWB} v^2 (s_w^2 - c_w^2)(s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B\, \xi_W}\,(\partial^{\mu} A_{\mu}\, \partial^{\nu}\, Z_{\nu})\,.$$

Another geoSMEFT clue

$$-\frac{c_w\, s_w}{\xi_B\, \xi_W}(\xi_B - \xi_W)\,(\partial^\mu A_\mu\, \partial^\nu\, Z_\nu) - \frac{C_{HWB} v^2 (s_w^2 - c_w^2)(s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B\, \xi_W}\,(\partial^\mu A_\mu\, \partial^\nu\, Z_\nu)\,.$$

- *This was extremely confusing.* The BFM usually removes such terms by design. Not just at tree level but at one loop. In other gauge fixing approaches, this term is removed by a gauge fixing condition.

  What is going on here with the vev redefinitions?

SMEFT Ward identities in BFM

Aside on the BFM

Abbott Acta Phys.Polon.B 13 (1982) 33

- Here we are gauge fixing in the BFM: $F \to \hat{F} + F$

  Double the fields, introduce background fields that have gauge sym unbroken when gauge fixing quantum fields

$$\mathcal{L}_{GF} = -\frac{1}{2\,\xi_W}\sum_a \left[\partial_\mu W^{a,\mu} - g_2\,\epsilon^{abc}\hat{W}_{b,\mu}W_c^\mu + i\,g_2\,\frac{\xi}{2}\left(\hat{H}_i^\dagger\sigma_{ij}^a H_j - H_i^\dagger\sigma_{ij}^a\hat{H}_j\right)\right]^2,$$
$$-\frac{1}{2\,\xi_B}\left[\partial_\mu B^\mu + i\,g_1\,\frac{\xi}{2}\left(\hat{H}_i^\dagger H_i - H_i^\dagger\hat{H}_i\right)\right]^2.$$

From the gauge fixing term, choosing $\xi_B = \xi_W$, one directly finds

$$\mathcal{L}_{GF} = -\frac{1}{2\,\xi}\left[(G^A)^2 + (G^Z)^2 + 2\,G^+\,G^-\right],$$

where

$$G^A = \partial_\mu A^\mu + ie\left(\hat{W}_\mu^+ W_\mu^- - W_\mu^+\hat{W}_\mu^-\right) + ie\,\xi\left(\hat{\phi}^-\phi^+ - \hat{\phi}^+\phi^-\right),$$
$$G^Z = \partial_\mu Z^\mu + ie\frac{c_w}{s_w}\left(\hat{W}_\mu^+ W_\mu^- - W_\mu^+\hat{W}_\mu^-\right) + ie\xi\frac{1}{2c_w\,s_w}(c_w^2 - s_w^2)\left(\hat{\phi}^-\phi^+ - \hat{\phi}^+\phi^-\right),$$
$$- e\,\xi\frac{1}{2\,c_w s_w}\left(\hat{\phi}_0\,h - \hat{h}\phi_0 - v\phi_0\right),$$
$$G^\pm = \partial^\mu W_\mu^\pm \pm i\,e\left[\hat{A}^\mu + \frac{c_w}{s_w}\hat{Z}^\mu\right]W_\mu^\pm \mp ie\left(A^\mu + \frac{c_w}{s_w}Z^\mu\right)\hat{W}_\mu^\pm,$$
$$- e\,\xi\,\frac{1}{2s_w}\left((v + \hat{h} \mp i\hat{\phi}_0)\phi^\pm - (h \mp i\phi_0)\hat{\phi}^\pm\right).$$

SMEFT gauge fixing solution!

- Problem: contact operators change mass and weak eigenstate relationship.
  This can be understood as a “curved field space” due to the contact operators.

- Solution:define a gauge fixing term on the “curved field space”

$$\mathcal{L}_{\rm GF} = -\frac{\hat{g}_{AB}}{2\,\xi}\mathcal{G}^A\,\mathcal{G}^B,$$

$$\mathcal{G}^X \equiv \partial_\mu \mathcal{W}^{X,\mu} - \tilde{\epsilon}^X_{\ CD}\hat{\mathcal{W}}^C_\mu \mathcal{W}^{D,\mu} + \frac{\xi}{2}\hat{g}^{XC}\phi^I\,\hat{h}_{IK}\,\tilde{\gamma}^K_{C,J}\hat{\phi}^J.$$

$$\int \mathcal{D}F \det\left[\frac{\Delta\mathcal{G}^A}{\Delta\alpha^B}\right] e^{i\left(S[F+\hat{F}]+\mathcal{L}_{\rm GF}+\hat{g}_{CD}\,J^C_\mu\,\mathcal{W}^{D,\mu}+\hat{h}_{IJ}\,J^I_\phi\,\phi^J\right)}.$$

arXiv:1803.08001 Helset, Paraskevas, Trott

SMEFT gauge fixing solution!

$$\mathcal{L}_{\text{scalar,kin}} = (D_\mu H)^\dagger (D^\mu H) + C_{H\Box} (H^\dagger H) \Box (H^\dagger H) + C_{HD} (H^\dagger D_\mu H)^* (H^\dagger D^\mu H),$$

$$\equiv \frac{1}{2} h_{IJ}(\phi) (D_\mu \phi)^I (D^\mu \phi)^J .$$

$$\mathcal{L}_{\text{GF}} = -\frac{\hat{g}_{AB}}{2\xi} \mathcal{G}^A \mathcal{G}^B,$$

$$\mathcal{G}^X \equiv \partial_\mu \mathcal{W}^{X,\mu} - \tilde{\epsilon}^X_{CD} \hat{\mathcal{W}}^C_\mu \mathcal{W}^{D,\mu} + \frac{\xi}{2} \hat{g}^{XC} \phi^I \hat{h}_{IK} \tilde{\gamma}^K_{C,J} \hat{\phi}^J .$$

$$\int \mathcal{D}F \det \left[\frac{\Delta \mathcal{G}^A}{\Delta \alpha^B} \right] e^{i\left(S[F+\hat{F}] + \mathcal{L}_{\text{GF}} + \hat{g}_{CD} J^C_\mu \mathcal{W}^{D,\mu} + \hat{h}_{IJ} J^I_\phi \phi^J \right)}.$$

SMEFT gauge fixing solution!

$$\mathcal{L}_{\text{scalar,kin}} = (D_\mu H)^\dagger (D^\mu H) + C_{H\Box} (H^\dagger H) \Box (H^\dagger H) + C_{HD} (H^\dagger D_\mu H)^* (H^\dagger D^\mu H), \equiv \frac{1}{2} h_{IJ}(\phi) (D_\mu \phi)^I (D^\mu \phi)^J .$$

$$\mathcal{L}_{\text{WB}} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{C_{HB}}{\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu} + \frac{C_{HW}}{\Lambda^2} H^\dagger H W^a_{\mu\nu} W^{a,\mu\nu} + \frac{C_{HWB}}{\Lambda^2} H^\dagger \sigma^a H W^a_{\mu\nu} B^{\mu\nu}, \equiv -\frac{1}{4} g_{AB}(H) \mathcal{W}^A_{\mu\nu} \mathcal{W}^{B,\mu\nu},$$

$$\mathcal{L}_{\text{GF}} = -\frac{\hat{g}_{AB}}{2\xi} \mathcal{G}^A \mathcal{G}^B,$$

$$\mathcal{G}^X \equiv \partial_\mu \mathcal{W}^{X,\mu} - \tilde{\epsilon}^X_{CD} \hat{\mathcal{W}}^C_\mu \mathcal{W}^{D,\mu} + \frac{\xi}{2} \hat{g}^{XC} \phi^I \hat{h}_{IK} \tilde{\gamma}^K_{C,J} \hat{\phi}^J .$$

$$\int \mathcal{D}F \det\left[\frac{\Delta \mathcal{G}^A}{\Delta \alpha^B}\right] e^{i\left(S[F+\hat{F}] + \mathcal{L}_{\text{GF}} + \hat{g}_{CD} J^C_\mu \mathcal{W}^{D,\mu} + \hat{h}_{IJ} J^I_\phi \phi^J\right)}.$$

$$\gamma^I_{1,J} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \gamma^I_{2,J} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \gamma^I_{3,J} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \gamma^I_{4,J} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Another geoSMEFT clue

$$-\frac{c_w\, s_w}{\xi_B\, \xi_W}(\xi_B - \xi_W)\,(\partial^\mu A_\mu\, \partial^\nu\, Z_\nu) - \frac{C_{HWB} v^2 (s_w^2 - c_w^2)(s_w^2 \xi_B + c_w^2 \xi_W)}{\xi_B\, \xi_W}\,(\partial^\mu A_\mu\, \partial^\nu\, Z_\nu)\,.$$

The geometric BFM gauge fixing cancels this mixing at all orders in the vev expansion! Exactly as it should be. ***You need to gauge fix geometrically to have the usual BFM benefits.***

GeoSMEFT based loop corrections?

- Will this simplify the NLO SMEFT radiative correction program?(Yes)

  Immediate BFM Ward Identities have already been derived:

$$\frac{\delta\Gamma[\hat{F},0]}{\delta\hat{\alpha}^B}=0.$$

Background field gauge transformation

1909.08470 Corbett, Helset, Trott

$$0=\left(\partial^\mu\delta^A_B-\tilde{\epsilon}^A_{BC}\,\hat{\mathcal{W}}^{C,\mu}\right)\frac{\delta\Gamma}{\delta\hat{\mathcal{W}}^\mu_A}-\frac{\tilde{\gamma}^I_{B,J}}{2}\hat{\phi}^J\frac{\delta\Gamma}{\delta\hat{\phi}^I}$$
$$+\sum_j\left(\bar{f}_j\bar{\Lambda}^j_{B,i}\frac{\delta\Gamma}{\delta\bar{f}_i}-\frac{\delta\Gamma}{\delta f_i}\Lambda^i_{B,j}f_j\right).$$

Photon identities:

$$0=\partial^\mu\frac{\delta^2\Gamma}{\delta\hat{\mathcal{A}}^{4\mu}\delta\hat{\mathcal{A}}^{Y\nu}},\qquad 0=\partial^\mu\frac{\delta^2\Gamma}{\delta\hat{\mathcal{A}}^{4\mu}\delta\hat{\Phi}^I}.$$

$$\Sigma^{\hat{\mathcal{A}},\hat{\mathcal{A}}}_{L,\mathrm{SMEFT}}(k^2)=0,\qquad \Sigma^{\hat{\mathcal{A}},\hat{\mathcal{A}}}_{T,\mathrm{SMEFT}}(0)=0.$$

One loop behaviour works!

Z identities:

Geometric mass

$$0=\partial^\mu\frac{\delta^2\Gamma}{\delta\hat{\mathcal{A}}^{3\mu}\delta\hat{\mathcal{A}}^{Y\nu}}-\bar{M}_Z\frac{\delta^2\Gamma}{\delta\hat{\Phi}^3\delta\hat{\mathcal{A}}^{Y\nu}},$$
$$0=\partial^\mu\frac{\delta^2\Gamma}{\delta\hat{\mathcal{A}}^{3\mu}\delta\hat{\Phi}^I}-\bar{M}_Z\frac{\delta^2\Gamma}{\delta\hat{\Phi}^3\delta\hat{\Phi}^I}$$
$$+\frac{\bar{g}_Z}{2}\frac{\delta\Gamma}{\delta\hat{\Phi}^4}\left(\sqrt{h}_{[4,4]}\sqrt{h}^{[3,3]}-\sqrt{h}_{[4,3]}\sqrt{h}^{[4,3]}\right)\delta^3_I$$
$$-\frac{\bar{g}_Z}{2}\frac{\delta\Gamma}{\delta\hat{\Phi}^4}\left(\sqrt{h}_{[4,4]}\sqrt{h}^{[3,4]}-\sqrt{h}_{[4,3]}\sqrt{h}^{[4,4]}\right)\delta^4_I,$$

GeoSMEFT based loop corrections?

- Expanding out the Ward ID you get expressions like this:

$$0 = \Sigma_L^{\hat{Z}\hat{Z}}(k^2) - i\bar{M}_Z \Sigma^{\hat{Z}\hat{\chi}}(k^2),$$

Geometric mass

Consider the operator C_{HWB}

$$-i\bar{M}_Z\Sigma^{\hat{Z}\hat{\chi}}(k^2) = -i\,\frac{\sqrt{g_1^2+g_2^2}\,\bar{v}_T}{2}\,\left[\Sigma^{\hat{Z}\hat{\chi}}(k^2)\right]^{div}_{\tilde{C}_{HWB}} - i\frac{g_1\,g_2\,\tilde{C}_{HWB}\,\bar{v}_T}{2\,\sqrt{g_1^2+g_2^2}}\,\left[\Sigma^{\hat{Z}\hat{\chi}}(k^2)\right]^{div}_{SM},$$

$$= -\tilde{C}_{HWB}\,\bar{v}_T^2\,(\xi+3)\left[\frac{g_1\,g_2\,(3\,g_1^2+5\,g_2^2)}{256\,\pi^2\,\epsilon} + \frac{g_1\,g_2\,(g_1^2+3g_2^2)}{256\pi^2\epsilon}\right],$$

$$= -\tilde{C}_{HWB}\,\bar{v}_T^2\,(\xi+3)\frac{g_1^2(g_1^2+2g_2^2)}{64\,\pi^2\,\epsilon}.$$

2010.08451 Corbett, Trott

2010.15852 Corbett

Which exactly cancels: $\left[\Sigma_L^{\hat{Z}\hat{Z}}(k^2)\right]^{div}_{\tilde{C}_{HWB}} = \tilde{C}_{HWB}\,\bar{v}_T^2\,(\xi+3)\frac{g_1^2(g_1^2+2g_2^2)}{64\,\pi^2\,\epsilon}.$

All one and two point ward ID working out at one loop. Powerful new NLO code tool is being developed using this as a theory cross check by Tyler Corbett.

Current state of the art

Best practice example in geoSMEFT

$$\frac{\Gamma^{\hat{m}_W}_{SMEFT}}{\Gamma^{\hat{m}_W}_{\rm SM}} \simeq 1 - 788 f_1^{\hat{m}_W}, \tag{5.6}$$
$$+ 394^2\,(f_1^{\hat{m}_W})^2 - 351\,(\tilde{C}^{(6)}_{HW} - \tilde{C}^{(6)}_{HB})\,f_3^{\hat{m}_W} + 2228\,\delta G_F^{(6)}\,f_1^{\hat{m}_W},$$
$$+ 979\,\tilde{C}^{(6)}_{HD}(\tilde{C}^{(6)}_{HB} + 0.80\,\tilde{C}^{(6)}_{HW} - 1.02\,\tilde{C}^{(6)}_{HWB}) - 788\left[\left(\tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{4}\right) f_1^{\hat{m}_W} + f_2^{\hat{m}_W}\right],$$
$$+ 2283\,\tilde{C}^{(6)}_{HWB}(\tilde{C}^{(6)}_{HB} + 0.66\,\tilde{C}^{(6)}_{HW} - 0.88\,\tilde{C}^{(6)}_{HWB}) - 1224\,(f_1^{\hat{m}_W})^2,$$
$$- 117\,\tilde{C}^{(6)}_{HB} - 23\,\tilde{C}^{(6)}_{HW} + \left[51 + 2\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\tilde{C}^{(6)}_{HWB} + \left[-0.55 + 3.6\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\tilde{C}^{(6)}_{W},$$
$$+ \left[27 - 28\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{uB\\33}} + \left[14 - 15\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{uW\\33}} + 0.56\,\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{uH\\33}},$$
$$- 0.31\,\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{dH\\33}} + 2\,\tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{2} + 2.0\,\tilde{C}^{(6)}_{HD} - 7.5\,\tilde{C}^{(6)}_{HWB} - 3\sqrt{2}\,\delta G_F^{(6)}.$$

$$\frac{\Gamma^{\hat{\alpha}_{ew}}_{SMEFT}}{\Gamma^{\hat{\alpha}_{ew}}_{\rm SM}} \simeq 1 - 758 f_1^{\hat{\alpha}_{ew}},$$
$$+ 379^2\,(f_1^{\hat{\alpha}_{ew}})^2 - 350\,(\tilde{C}^{(6)}_{HW} - \tilde{C}^{(6)}_{HB})^2 - 1159\,(f_1^{\hat{\alpha}_{ew}})^2$$
$$- 61\,\tilde{C}^{(6)}_{HWB}\left(\tilde{C}^{(6)}_{HB} + 7.2\tilde{C}^{(6)}_{HW} - 9.2\tilde{C}^{(6)}_{HWB}\right) - 13.5\,\tilde{C}^{(6)}_{HD}\left(\tilde{C}^{(6)}_{HB} + 16\tilde{C}^{(6)}_{HW} - 15\tilde{C}^{(6)}_{HWB}\right)$$
$$+ 1383\,\delta G_F^{(6)}\left(\tilde{C}^{(6)}_{HB} - 0.13\tilde{C}^{(6)}_{HW} - 0.15\tilde{C}^{(6)}_{HWB}\right) - 758\left[\left(\tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{4}\right) f_1^{\hat{\alpha}_{ew}} + f_2^{\hat{\alpha}_{ew}}\right],$$
$$- 218\,\tilde{C}^{(6)}_{HB} + 22\,\tilde{C}^{(6)}_{HW} + \left[-17 + 2.0\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\tilde{C}^{(6)}_{HWB} + \left[-0.60 + 3.6\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\tilde{C}^{(6)}_{W},$$
$$+ \left[26 - 27\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{uB\\33}} + \left[14 - 15\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{uW\\33}} + 0.56\,\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{uH\\33}},$$
$$- 0.31\,\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{dH\\33}} + 2\,\tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{2} - \sqrt{2}\,\delta G_F^{(6)}.$$

- State of the art $\Gamma(h \to \gamma\gamma)$ has $\mathcal{L}^{(6)}$ at tree and loop Level and also $\mathcal{L}^{(8)}$

$\{\hat{M}_W, \hat{M}_Z, \hat{G}_F, \hat{M}_h\}$ Scheme

$\{\hat{\alpha}_{ew}, \hat{M}_Z, \hat{G}_F, \hat{M}_h\}$ Scheme

- Here

$$\delta G_F^{(6)} = \frac{1}{\sqrt{2}}\left(\tilde{C}^{(3)}_{\substack{Hl\\ee}} + \tilde{C}^{(3)}_{\substack{Hl\\\mu\mu}} - \frac{1}{2}(\tilde{C}'_{\substack{ll\\\mu ee\mu}} + \tilde{C}'_{\substack{ll\\e\mu\mu e}})\right),$$
$$f_1^{\hat{m}_W} = \left[\tilde{C}^{(6)}_{HB} + 0.29\,\tilde{C}^{(6)}_{HW} - 0.54\,\tilde{C}^{(6)}_{HWB}\right],$$
$$f_2^{\hat{m}_W} = \left[\tilde{C}^{(8)}_{HB} + 0.29\,(\tilde{C}^{(8)}_{HW} + \tilde{C}^{(8)}_{HW,2}) - 0.54\,\tilde{C}^{(8)}_{HWB}\right],$$
$$f_3^{\hat{m}_W} = \left[\tilde{C}^{(6)}_{HW} - \tilde{C}^{(6)}_{HB} - 0.66\,\tilde{C}^{(6)}_{HWB}\right].$$

Best practice example in geoSMEFT

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$$+ 394^2\,(f_1^{\hat{m}_W})^2 - 351\,(\tilde{C}^{(6)}_{HW} - \tilde{C}^{(6)}_{HB})\,f_3^{\hat{m}_W} + 2228\,\delta G_F^{(6)}\,f_1^{\hat{m}_W},$$

$$+ 979\,\tilde{C}^{(6)}_{HD}(\tilde{C}^{(6)}_{HB} + 0.80\,\tilde{C}^{(6)}_{HW} - 1.02\,\tilde{C}^{(6)}_{HWB}) - 788\left[\left(\tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{4}\right) f_1^{\hat{m}_W} + f_2^{\hat{m}_W}\right],$$

$$+ 2283\,\tilde{C}^{(6)}_{HWB}(\tilde{C}^{(6)}_{HB} + 0.66\,\tilde{C}^{(6)}_{HW} - 0.88\,\tilde{C}^{(6)}_{HWB}) - 1224\,(f_1^{\hat{m}_W})^2,$$

$$- 117\,\tilde{C}^{(6)}_{HB} - 23\,\tilde{C}^{(6)}_{HW} + \left[51 + 2\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\tilde{C}^{(6)}_{HWB} + \left[-0.55 + 3.6\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\tilde{C}^{(6)}_{W},$$

$$+ \left[27 - 28\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{uB\\33}} + \left[14 - 15\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{uW\\33}} + 0.56\,\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{uH,\\33}}$$

$$- 0.31\,\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{dH\\33}} + 2\,\tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{2} + 2.0\,\tilde{C}^{(6)}_{HD} - 7.5\,\tilde{C}^{(6)}_{HWB} - 3\sqrt{2}\,\delta G_F^{(6)}.$$

- State of the art $\Gamma(h \to \gamma\gamma)$ has $\mathcal{L}^{(6)}$ at tree and loop Level and also $\mathcal{L}^{(8)}$

- The various contributions

$$|\mathcal{A}^{a,ij}_{SM} + \mathcal{A}^{a,ij}_{SMEFT}|^2 = |\mathcal{A}^{a,ij}_{SM} + \frac{\mathcal{A}^{a,ij}_{SMEFT,6}}{\Lambda^2} + \frac{\mathcal{A}^{a,ij}_{SMEFT,8}}{\Lambda^4} + \cdots|^2$$

$$= |\mathcal{A}^{a,ij}_{SM}|^2 + \mathcal{A}^{a,ij}_{SM}\,\frac{\mathcal{A}^{a,ij}_{SMEFT,6}}{\Lambda^2} + |\frac{\mathcal{A}^{a,ij}_{SMEFT,6}}{\Lambda^2}|^2 + \mathcal{A}^{a,ij}_{SM}\,\frac{\mathcal{A}^{a,ij}_{SMEFT,8}}{\Lambda^4} + \mathrm{h.c} + \cdots$$

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$$+ 979\,\tilde{C}^{(6)}_{HD}(\tilde{C}^{(6)}_{HB} + 0.80\,\tilde{C}^{(6)}_{HW} - 1.02\,\tilde{C}^{(6)}_{HWB}) - 788\left[\left(\tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{4}\right) f_1^{\hat{m}_W} + f_2^{\hat{m}_W}\right],$$

$$+ 2283\,\tilde{C}^{(6)}_{HWB}(\tilde{C}^{(6)}_{HB} + 0.66\,\tilde{C}^{(6)}_{HW} - 0.88\,\tilde{C}^{(6)}_{HWB}) - 1224\,(f_1^{\hat{m}_W})^2,$$

$$- 117\,\tilde{C}^{(6)}_{HB} - 23\,\tilde{C}^{(6)}_{HW} + \left[51 + 2\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\tilde{C}^{(6)}_{HWB} + \left[-0.55 + 3.6\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\tilde{C}^{(6)}_{W},$$

$$+ \left[27 - 28\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{uB\\33}} + \left[14 - 15\log\left(\frac{\hat{m}_h^2}{\Lambda^2}\right)\right]\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{uW\\33}} + 0.56\,\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{uH,\\33}}$$

$$- 0.31\,\mathrm{Re}\,\tilde{C}^{(6)}_{\substack{dH\\33}} + 2\,\tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{2} + 2.0\,\tilde{C}^{(6)}_{HD} - 7.5\,\tilde{C}^{(6)}_{HWB} - 3\sqrt{2}\,\delta G_F^{(6)}.$$

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- The various contributions

$$|\mathcal{A}^{a,ij}_{SM} + \mathcal{A}^{a,ij}_{SMEFT}|^2 = |\mathcal{A}^{a,ij}_{SM} + \frac{\mathcal{A}^{a,ij}_{SMEFT,6}}{\Lambda^2} + \frac{\mathcal{A}^{a,ij}_{SMEFT,8}}{\Lambda^4} + \cdots|^2$$

$$= |\mathcal{A}^{a,ij}_{SM}|^2 + \mathcal{A}^{a,ij}_{SM}\,\frac{\mathcal{A}^{a,ij}_{SMEFT,6}}{\Lambda^2} + |\frac{\mathcal{A}^{a,ij}_{SMEFT,6}}{\Lambda^2}|^2 + \mathcal{A}^{a,ij}_{SM}\,\frac{\mathcal{A}^{a,ij}_{SMEFT,8}}{\Lambda^4} + \mathrm{h.c} + \cdots$$

- "Quadratic" Terms obtained in SMEFTsim only!

Consistency checks at one loop/dim8

Benefits of the Background Field method one loop approach in SMEFT.

- Many cross checks afforded (Ward identities and more).
- Clean understanding of ward identities.
- One loop redefinition of input parameters INDIVIDUALLY gauge independent.
- Cross checks of

$$\Delta Z_e = -\tfrac{1}{2}\Delta Z_{\hat{\mathcal{A}}},$$
$$\Delta R_e = -\tfrac{1}{2}\Delta R_{\hat{\mathcal{A}}}.$$

Our calc in 2107.07470
Stoffer/Denkens in 1908.05295

$$\Delta R_{\hat{\mathcal{A}}} = \frac{\bar{g}_1^2\bar{g}_2^2}{(\bar{g}_1^2+\bar{g}_2^2)}\left[-\frac{7}{16\pi^2}\log\left(\frac{\mu^2}{\bar{m}_W^2}\right)+\sum_\psi \frac{N_c^\psi Q_\psi^2}{12\pi^2}\log\left(\frac{\mu^2}{\bar{m}_\psi^2}\right)-\frac{1}{24\pi^2}\right].$$

Consistency checks at one loop/dim8

Cancelation of large mt dependent logs in relations between observables: Expected and anticipated in Hartmann/Trott. 1505.02646

- Expected cancelation confirmed in 2107.07470 and 1908.05295

$$\bar{v}_T = \hat{v}_T \left[1 + \frac{2y_t^2}{16\pi^2} N_C \frac{m_f^2}{\bar{m}_h^2} \left[1 + \log\left(\frac{\mu^2}{m_f^2} \right) \right] + \cdots \right].$$

$$\frac{\Delta v}{\bar{v}_T} \propto -\frac{2y_t^2}{16\pi^2} N_C \frac{m_f^2}{\bar{m}_h^2} \left[1 + \log\left(\frac{\mu^2}{m_f^2} \right) \right].$$

- Cancelation in single Higgs, single dev observables with tadpole term and GF extraction.

Consistency checks at one loop/dim8

Gauge independence of a common partial matrix element in single Higgs processes in BFM:

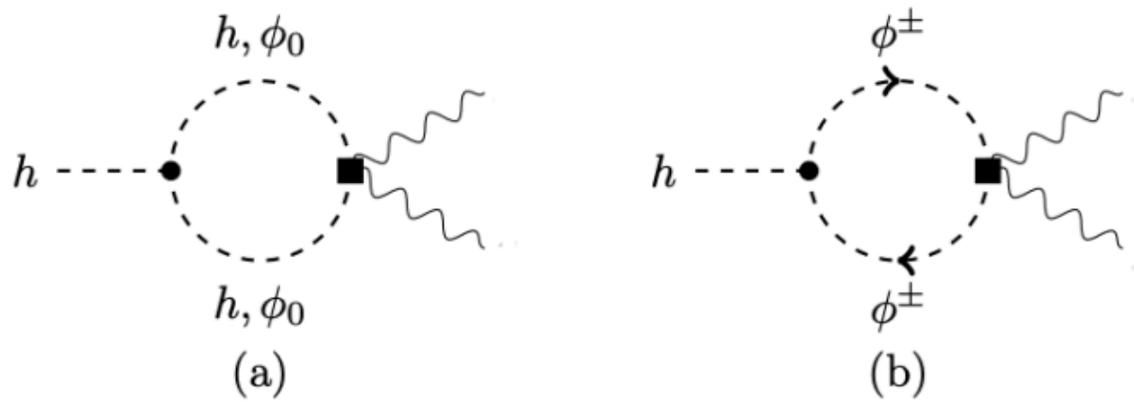


Figure 2. One loop contributions to $\langle \phi_4 | F\, F \rangle \langle \frac{\delta M_{AB}}{\delta \phi_4} \rangle$.

$$\frac{\langle \phi_4 F(p_1) F(p_2) \rangle^1}{\langle \phi_4 F^{\mu\nu} F_{\mu\nu} \rangle^0 \langle \frac{\delta M_{AB}(\phi)}{\delta \phi_4} \rangle^0} \propto M_1$$

Gauge independent combination in result:

$$M_1 \equiv \left(\frac{\Delta R_h}{2} + \frac{\Delta v}{v} + \frac{(\sqrt{3}\pi - 6)\lambda}{16\pi^2} + \frac{1}{16\pi^2} \left(\frac{\bar{g}_1^2}{4} + \frac{3\bar{g}_2^2}{4} + 6\lambda \right) \log \left[\frac{\bar{m}_h^2}{\mu^2} \right] \right),$$
$$+ \frac{1}{16\pi^2} \left(\frac{\bar{g}_1^2}{4} \mathcal{I}[\bar{m}_Z] + (\frac{\bar{g}_2^2}{4} + \lambda)(\mathcal{I}[\bar{m}_Z] + 2\mathcal{I}[\bar{m}_W]) \right).$$

There is so much SMEFT/geoSMEFT to do!