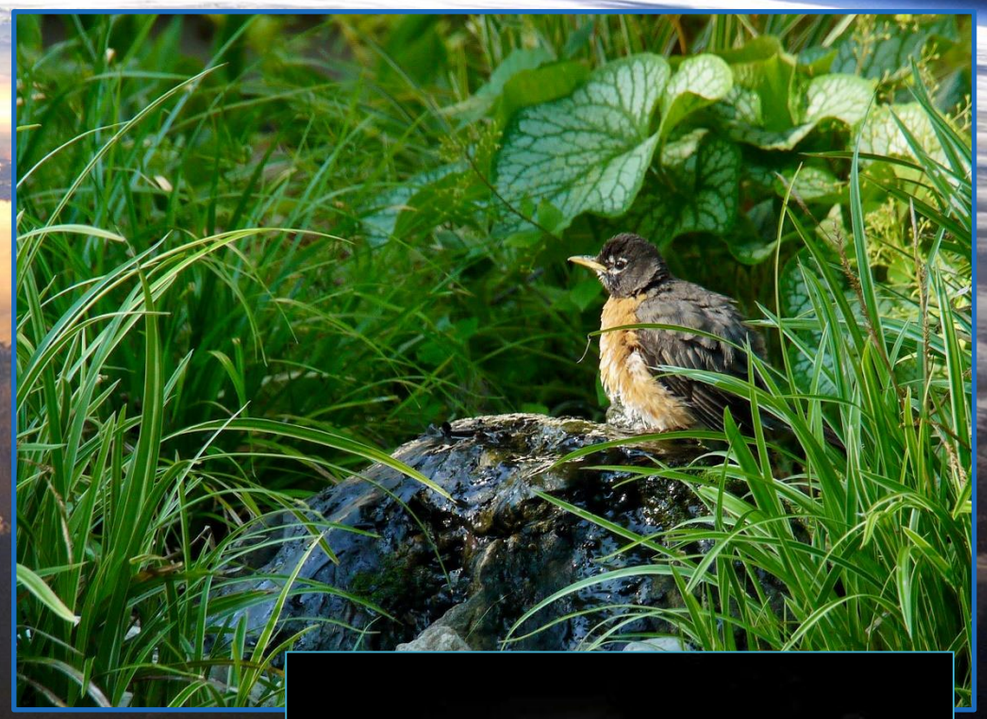
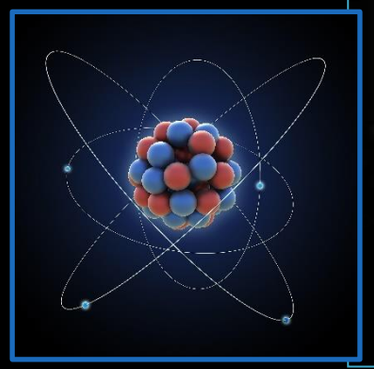
EFFECTIVE FIELD THEORY AND COSMOLOGY

THE VIEW FROM BELOW







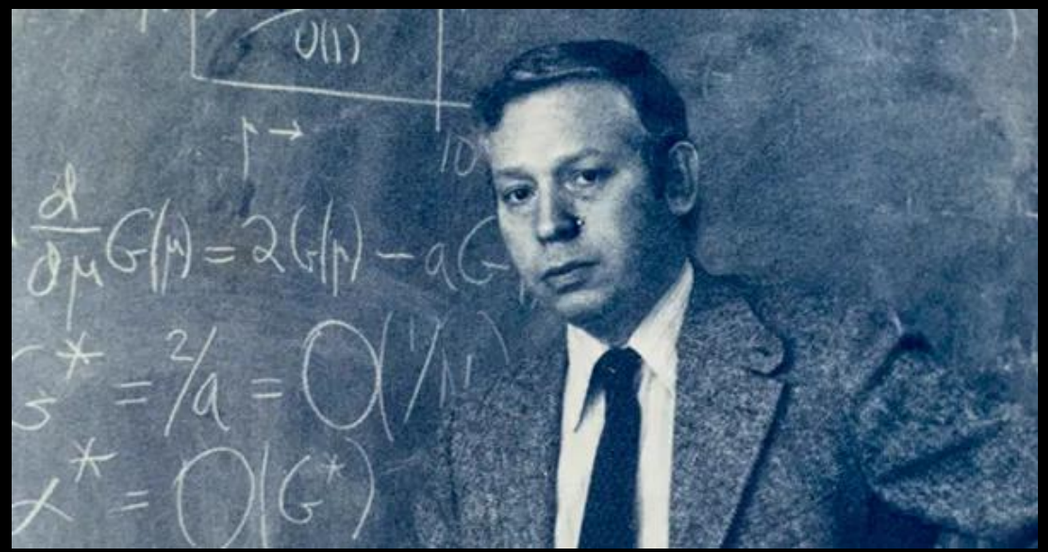


AI IN HIGH ENERGY PHYSICS
QUANTUM UNIVERSE CENTER SCHOOL
KIAS, JULY 2022

CP BURGESS

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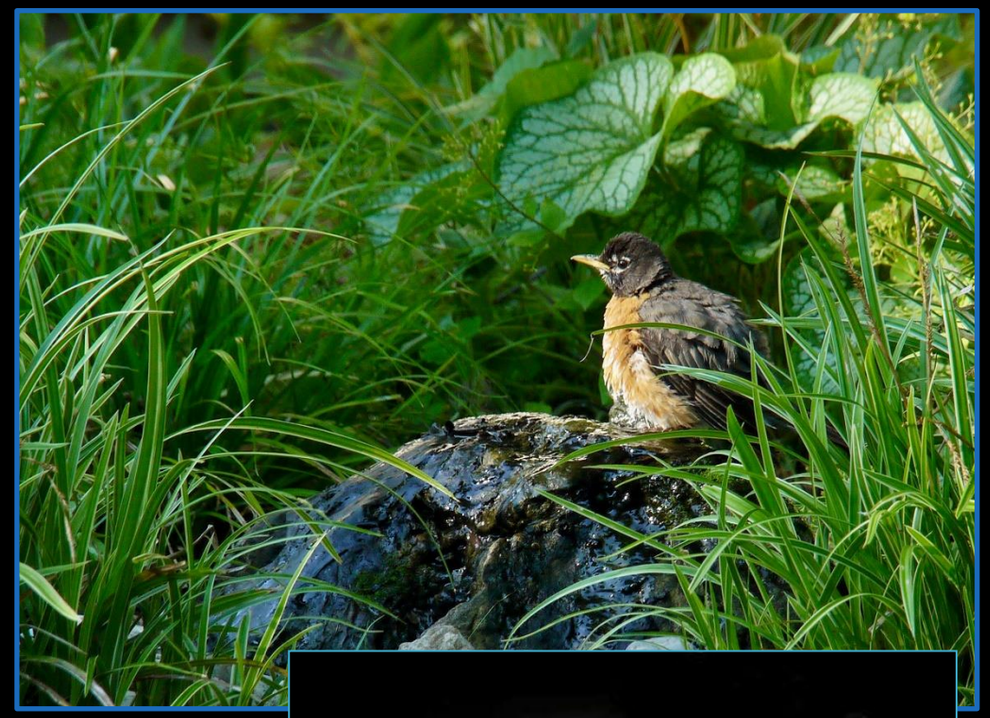
ON THE SHOULDERS OF GIANTS…

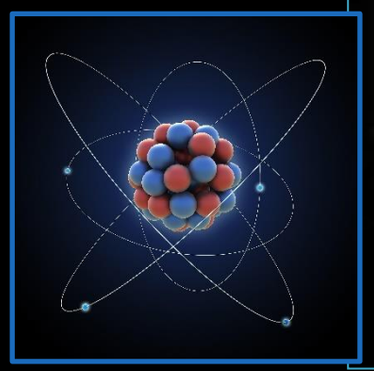


Steven Weinberg 1933-2021

Nobel Prize 1979



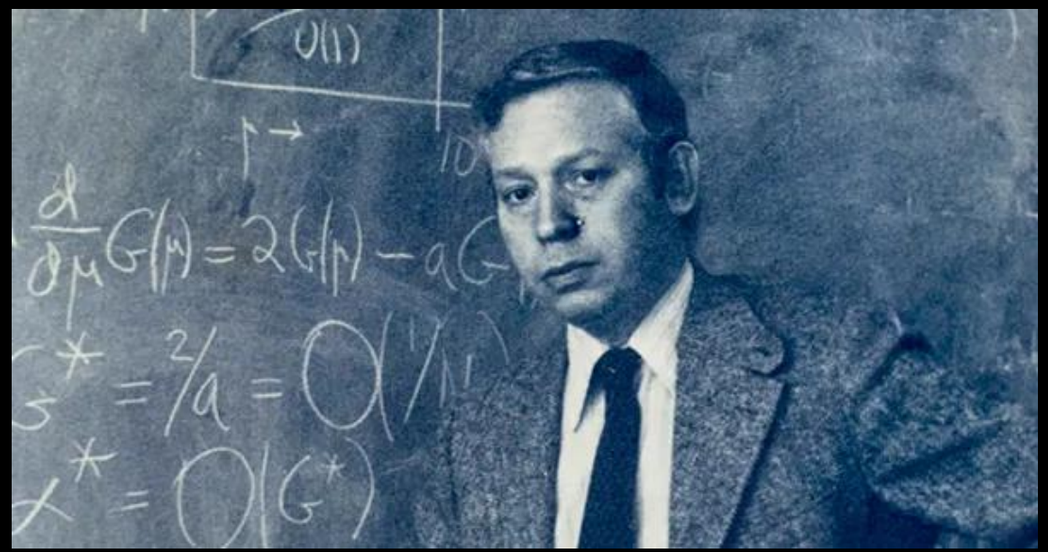






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ON THE SHOULDERS OF GIANTS…



Physica **96A** (1979) 327–340 © *North-Holland Publishing Co.*

PHENOMENOLOGICAL LAGRANGIANS*

STEVEN WEINBERG

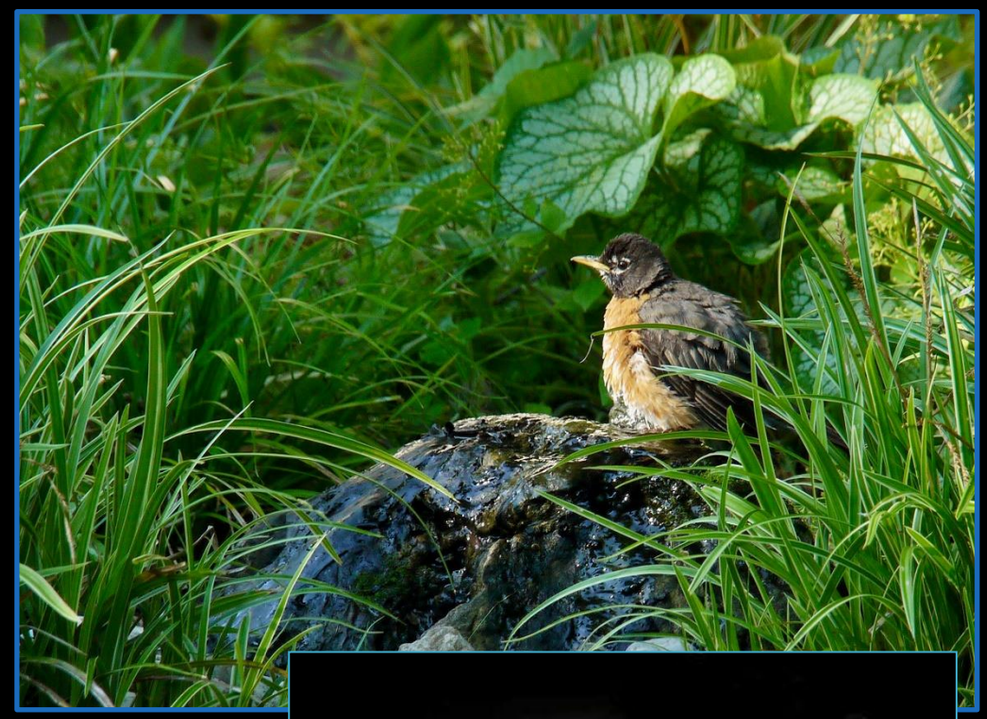
Lyman Laboratory of Physics, Harvard University

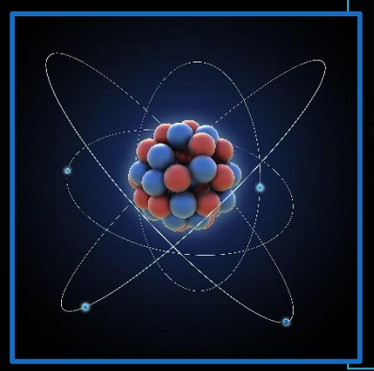
and

Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138, USA

1. Introduction: A reminiscence

Julian Schwinger's ideas have strongly influenced my understanding of phenomenological Lagrangians since 1966, when I made a visit to Harvard. At that time, I was trying to construct a phenomenological Lagrangian which would allow one to obtain the predictions of current algebra for soft pion matrix elements with less work, and with more insight into possible corrections. It was necessary to arrange that the pion couplings in the



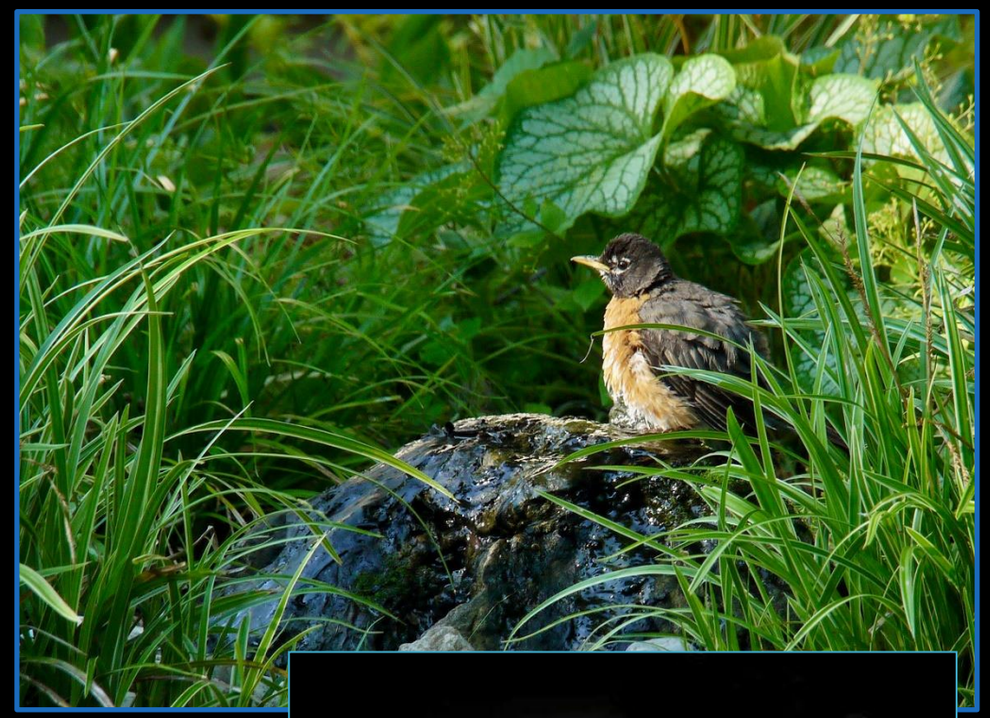


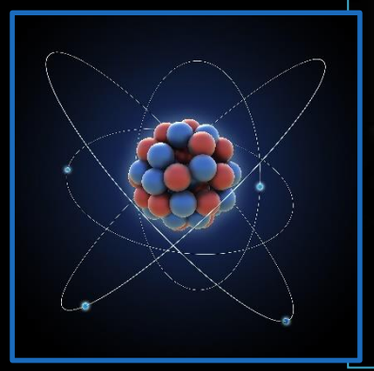


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CONTENTS

- EFTs: GENERAL FRAMEWORK
- COSMOLOGY BACKGROUND (AND FLUCTUATIONS)
- EFTs IN COSMOLOGY



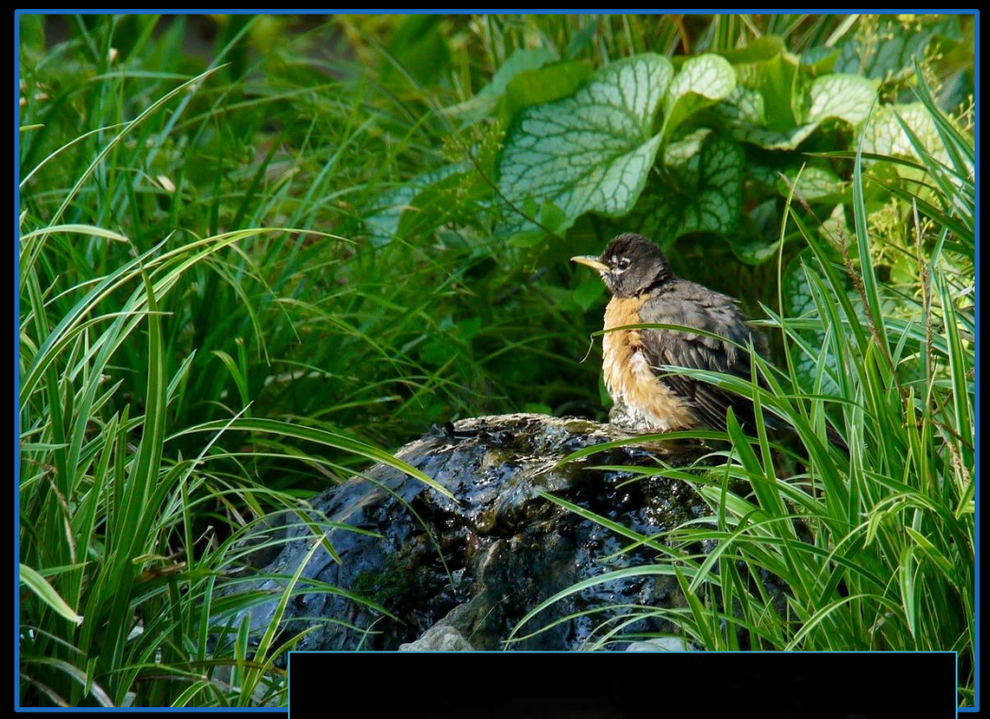


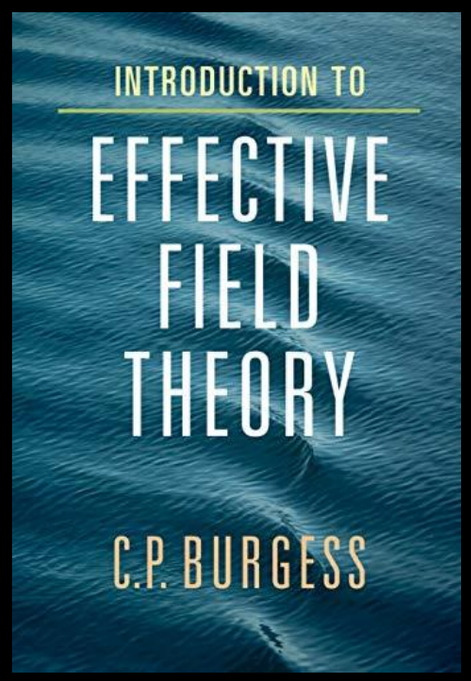


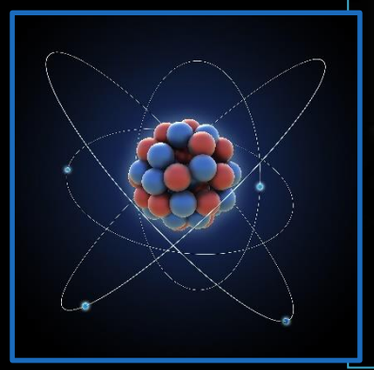
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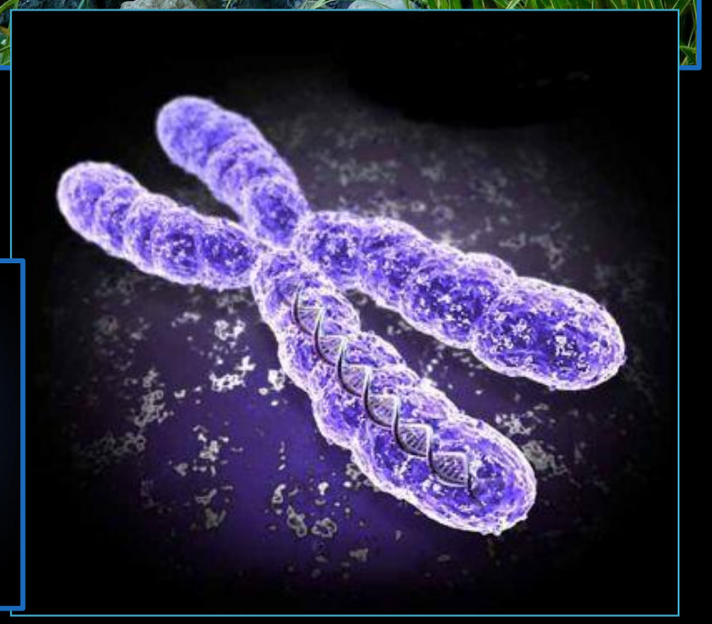
GENERAL FRAMEWORK

- Decoupling
- Exploiting hierarchies
- Why renormalization is a Good Thing
- Time-dependent fields
- Examples





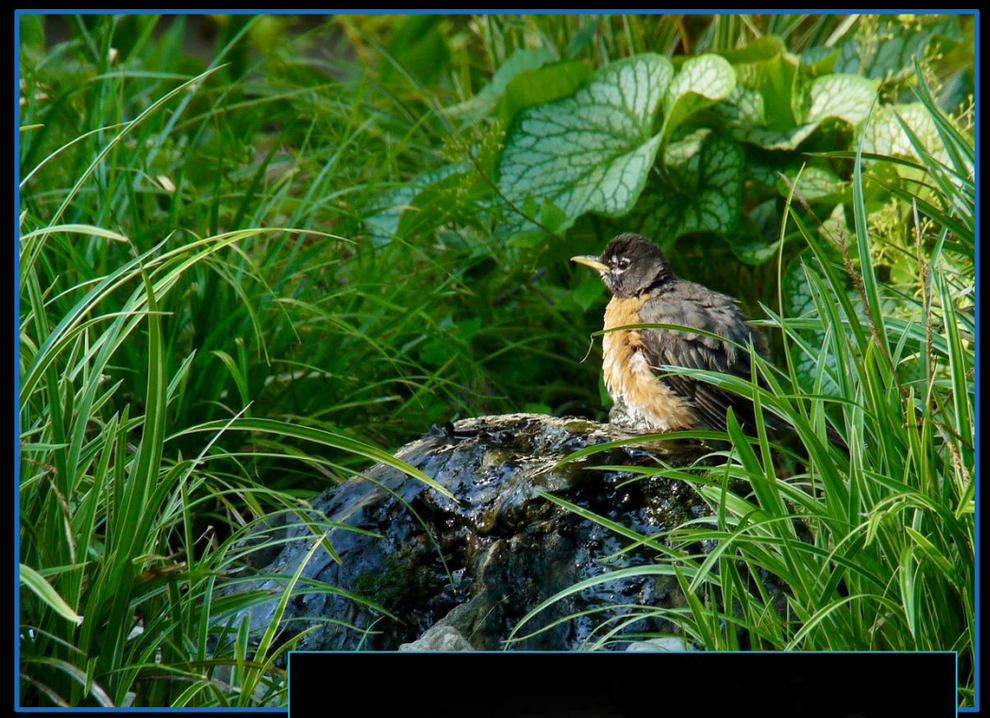


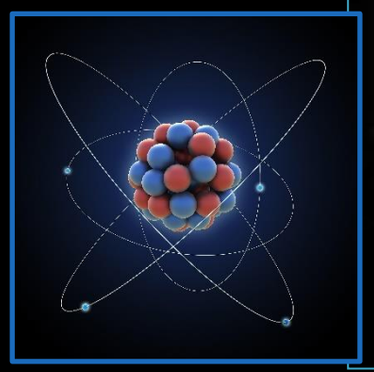


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COSMOLOGY

- Background story
- Structure formation
- Primordial fluctuations



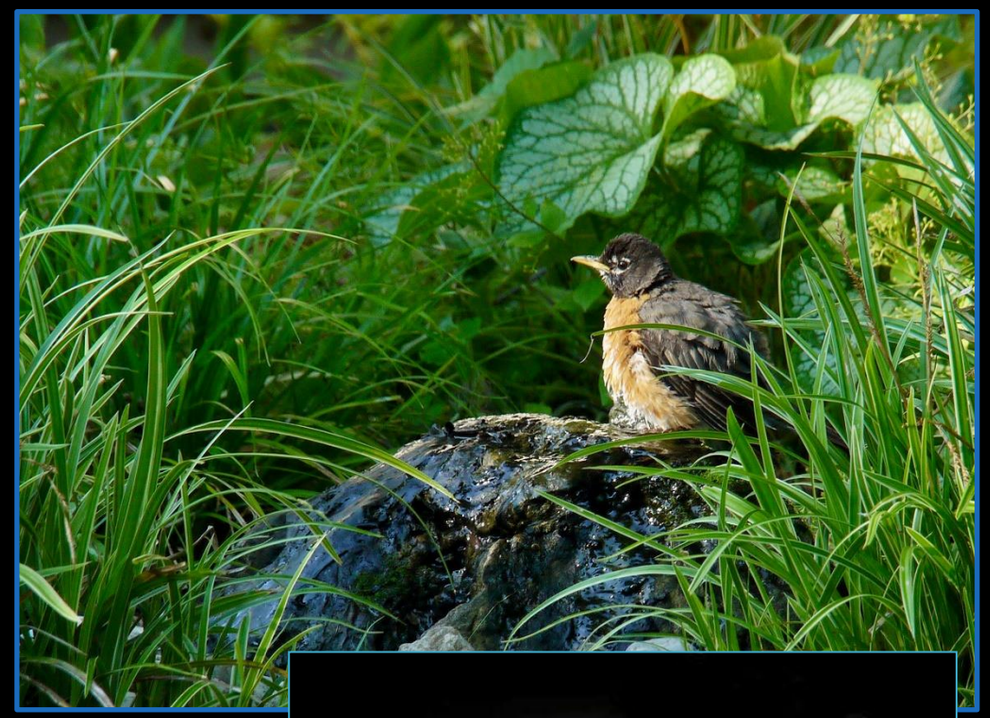


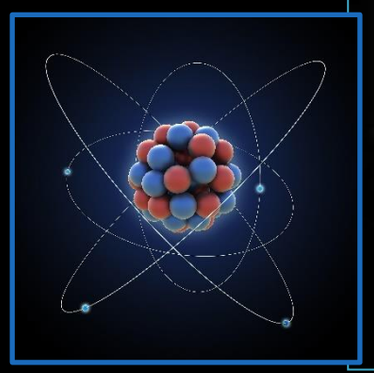


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EFTS AND COSMOLOGY

- GREFT
- Power counting
- Conceptual issues



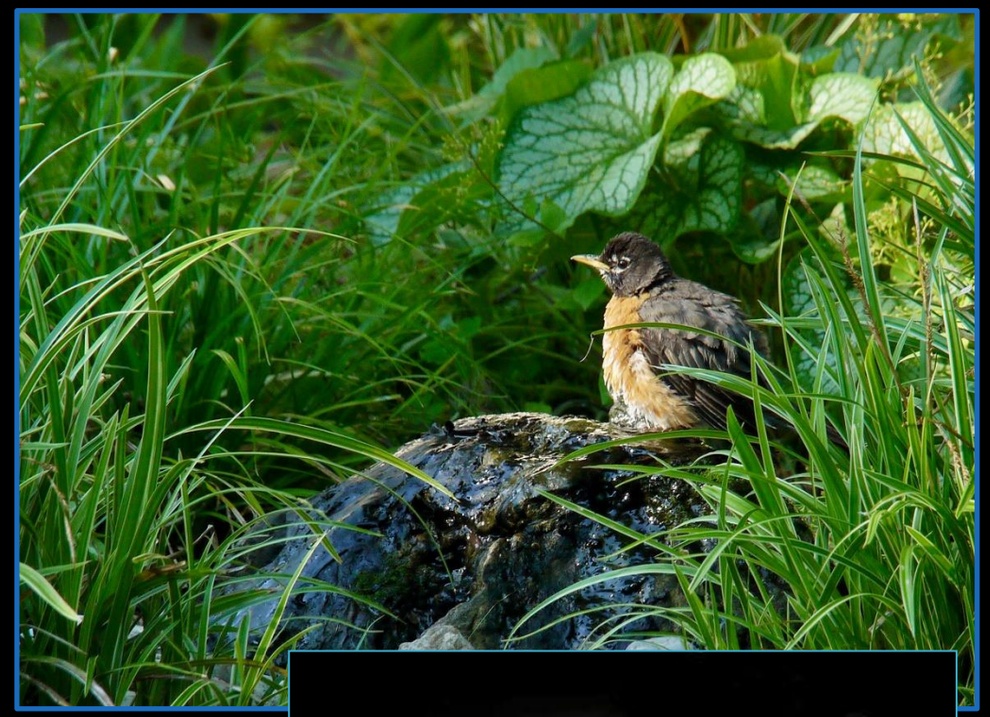


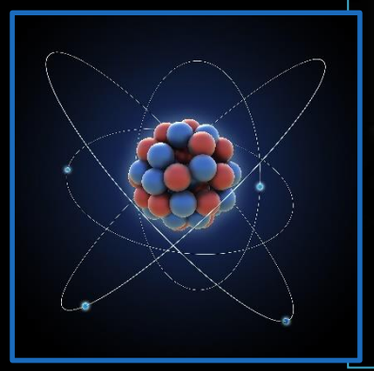


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EFT GENERAL FRAMEWORK

DECOUPLING

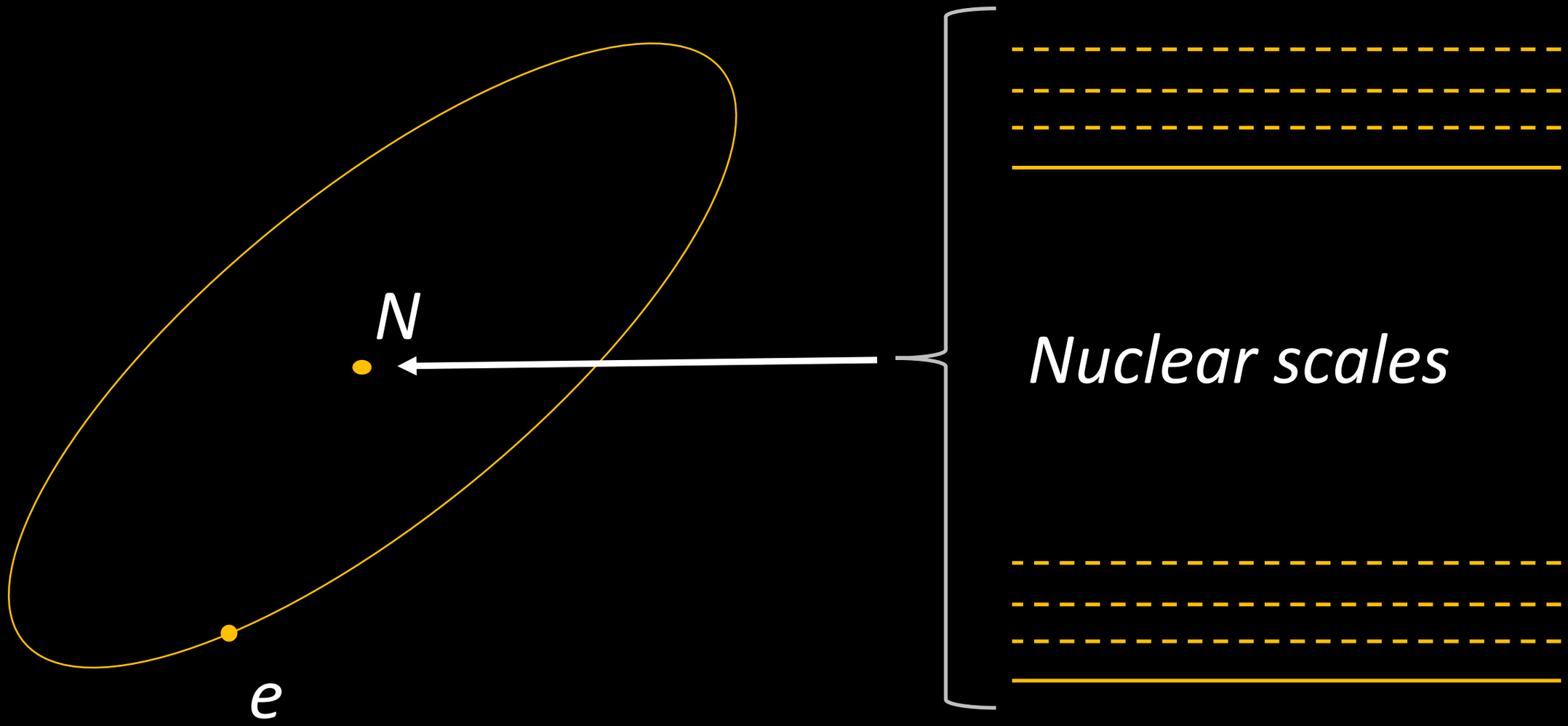




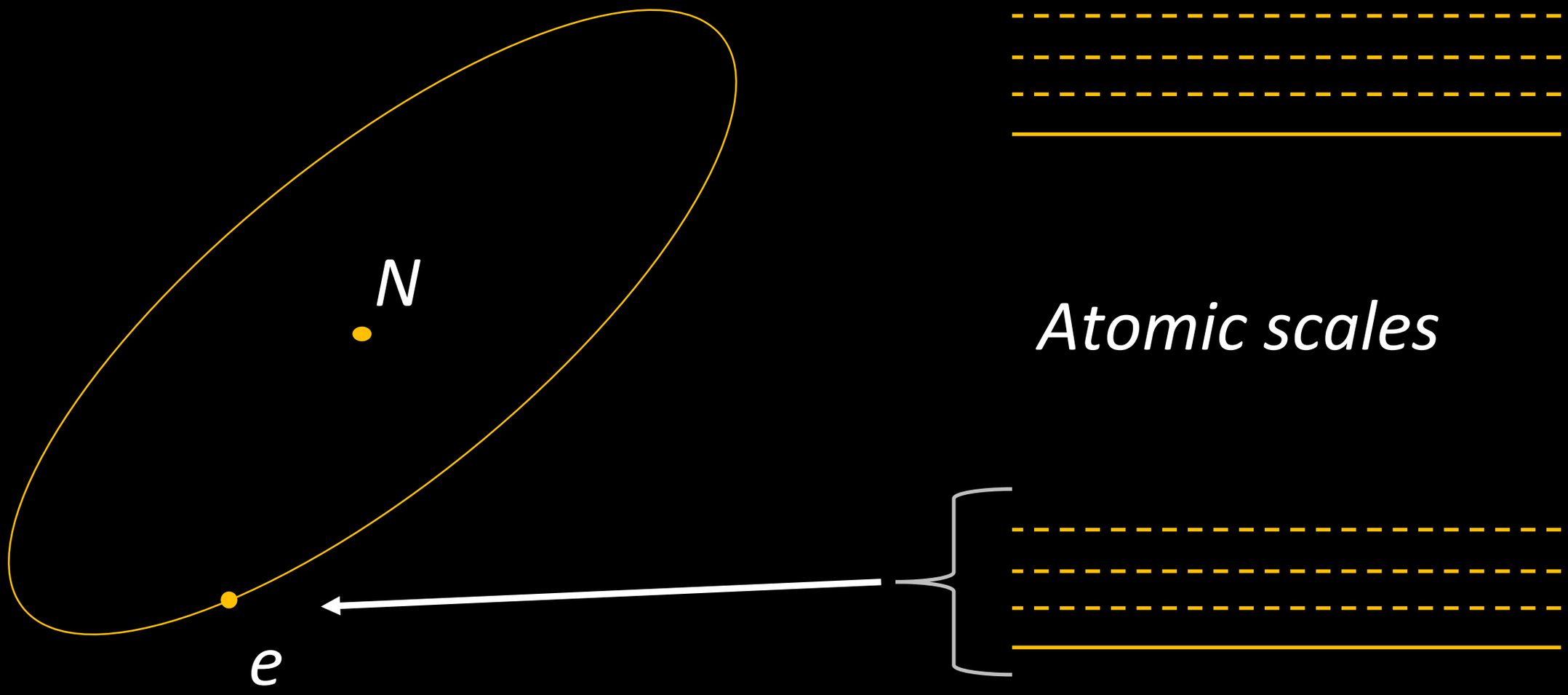


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FACT: *Nature comes with many hierarchies of scale, and details of small distances are not needed to understand long distances*



FACT: *Nature comes with many hierarchies of scale, and details of small distances are not needed to understand long distances*



FACT: *Quantum field theory shares this property that small distance physics drops out of long distance physics*



$$A(\theta, m, M) = f\,(\theta, m/M)$$

$$\simeq f(\theta, 0)\Big[1 + O(m/M)\Big]$$



(modulo logarithms)

Behooves us to exploit this simplicity as early as possible in a calculation: EFTs

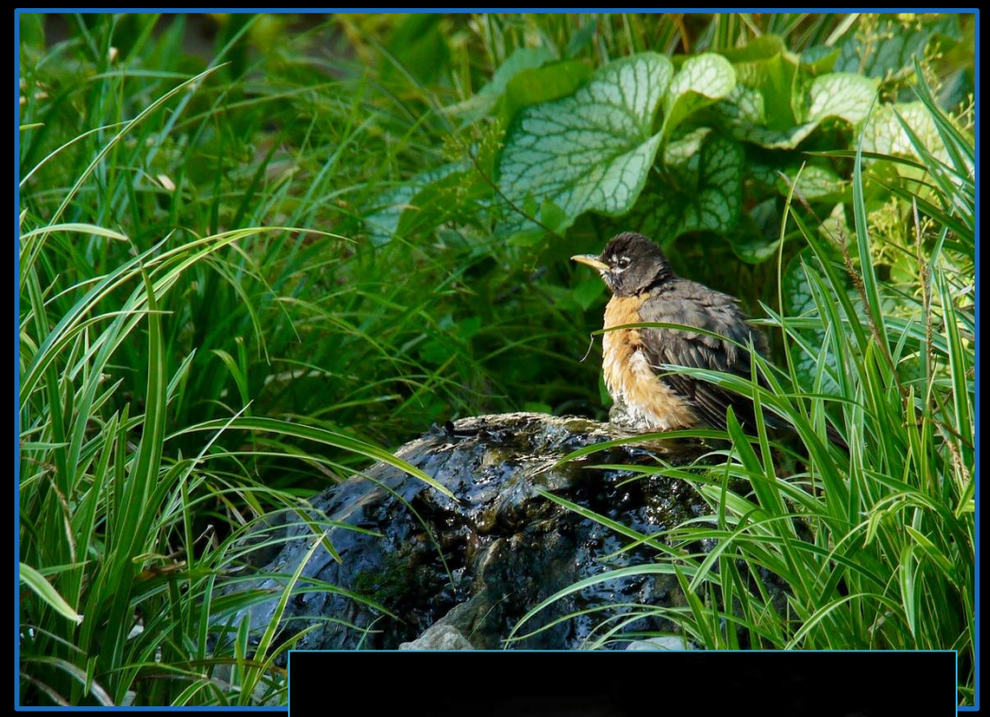
$$A(\theta, m, M) \simeq f(\theta, 0)\Big[1 + O(m/M)\Big]$$

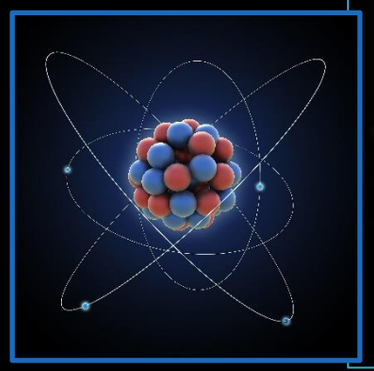
Applicable everywhere because in quantum physics we cannot help probing very short distances

$$E_k \simeq \langle k|H_I|k\rangle + \sum_n \frac{|\langle n|H_I|k\rangle|^2}{E_k - E_n} + \cdots$$

EFT GENERAL FRAMEWORK

EXPLOITING HIERARCHIES







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Simple example: two spinless fields

$$S := -\int \mathrm{d}^4x \left[\partial_\mu \phi^* \partial^\mu \phi + V(\phi^*\phi)\right]$$

$$V(\phi^*\phi) = \tfrac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$

Perturbative treatment: $\phi = v + \frac{1}{\sqrt{2}}\Big(R + iI\Big)$

$$S_0 := -\tfrac{1}{2}\int \mathrm{d}^4x \left[\partial_\mu R\, \partial^\mu R + \partial_\mu I\, \partial^\mu I + \lambda\, v^2\, R^2\right]$$

$$S_{\mathrm{int}} := -\int \mathrm{d}^4x \left[\tfrac{\lambda v}{2\sqrt{2}}\, R(R^2+I^2) + \tfrac{\lambda}{16}(R^2+I^2)^2\right]$$

Simple example: two spinless fields

Particle masses

$$m_R^2 = m^2 = \lambda v^2 \qquad m_I^2 = 0$$

Hierarchy $E \ll m$

$\phi)\Big]$

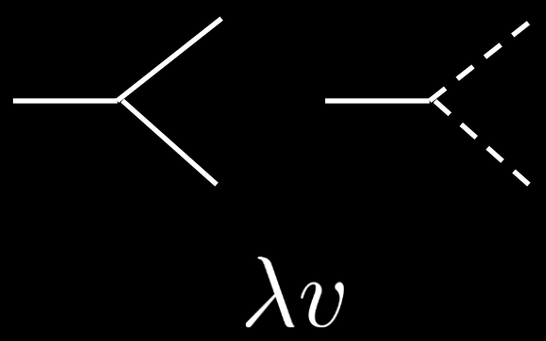
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Simple example: two spinless fields

Interactions



$$= \frac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$

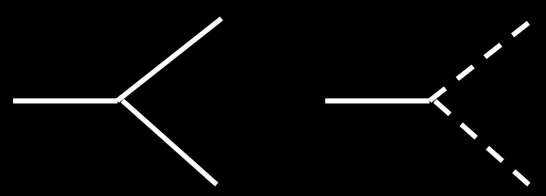
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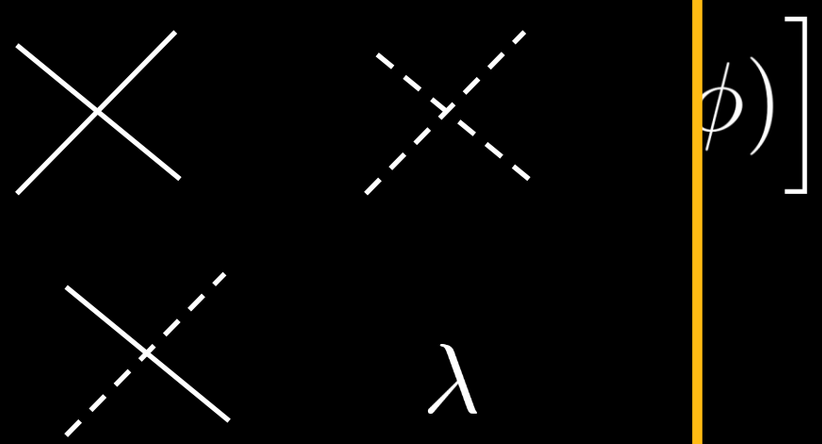
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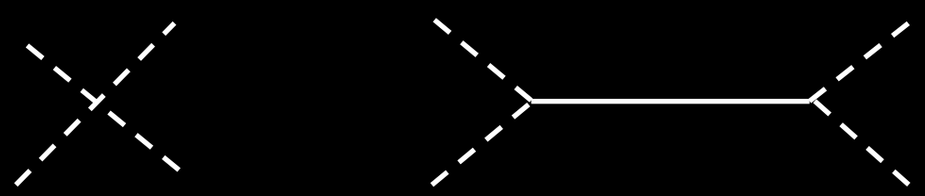


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Low-energy scattering: $I(q) + I(p) \to I(q') + I(p')$



plus ‘crossed’ graphs

has (on shell) amplitude

$$\mathcal{A} = -\frac{3i\lambda}{2} + \frac{i(\lambda v)^2}{2}\left[\frac{1}{m^2+2p\cdot q} + \frac{1}{m^2-2q\cdot q'} + \frac{1}{m^2-2p\cdot q'}\right]$$

which at low energies becomes

$$\mathcal{A} \simeq 2i\lambda\left[\frac{(p\cdot q)^2+(p\cdot q')^2+(q\cdot q')^2}{m^4}\right] + O\left(m^{-6}\right)$$

Low-energy scattering: $I(q) + I(p) \to I(q') + I(p')$

Weaker than expected at energies $E \ll m_R$

Turns out to persist to higher orders in λ

has (on shell) amplitude

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Weaker than expected at energies $E \ll m_R$

Turns out to persist to higher orders in λ

has (on shell) amplitude

$\mathcal{A} =$ [illegible]

which

Also true for low-energy R+I scattering:

$$\mathcal{A}(R + I \to R + I) \simeq 2i\lambda \left(\frac{q \cdot q'}{m^2}\right) + O\left(m^{-4}\right)$$

$$\mathcal{A} \simeq 2i\lambda \left[\frac{(p \cdot q)^2 + (p \cdot q')^2 + (q \cdot q')^2}{m^4}\right] + O\left(m^{-6}\right)$$

Underlying symmetry $\phi \to e^{i\omega}\,\phi$

$$S := -\int \mathrm{d}^4x \left[\partial_\mu\phi^*\partial^\mu\phi + V(\phi^*\phi)\right]$$

Better low-E variables: $\phi = \left(v + \frac{\chi}{\sqrt{2}}\right) e^{i\xi/\sqrt{2}\,v}$

$$\chi \to \chi \qquad \boxed{\xi \to \xi + \sqrt{2}\,\omega}$$

$$S_0 := -\tfrac{1}{2}\int \mathrm{d}^4x \left[\partial_\mu\chi\,\partial^\mu\chi + \boxed{\partial_\mu\xi\,\partial^\mu\xi} + \lambda\,v^2\chi^2\right]$$

$$S_{\text{int}} = -\int \mathrm{d}^4x \left[\left(\frac{\chi}{\sqrt{2}\,v} + \frac{\chi^2}{4\,v^2}\right)\boxed{\partial_\mu\xi\partial^\mu\xi} + \frac{\lambda v}{2\sqrt{2}}\,\chi^3 + \frac{\lambda}{16}\,\chi^4\right]$$

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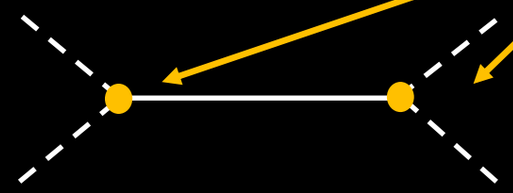
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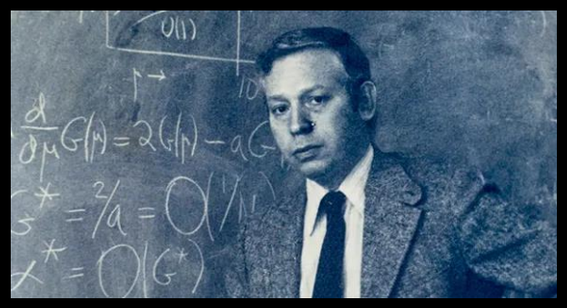
$$S_{\mathrm{int}} = -\int \mathrm{d}^4x \left[\left(\frac{\chi}{\sqrt{2}\,v} + \frac{\chi^2}{4\,v^2}\right)\partial_\mu\xi\partial^\mu\xi + \frac{\lambda v}{2\sqrt{2}}\,\chi^3 + \frac{\lambda}{16}\,\chi^4\right]$$





plus ‘crossed’ graphs

Moral: make symmetries manifest $\phi \to e^{i\omega}\,\phi$



You can use any variables you like, but if you use the wrong ones you will be sorry.
(one of Weinberg's 3 Laws of Theoretical Physics)

Symmetries cannot always be realized linearly when restricted to low energy variables

$$\begin{pmatrix} R \\ I \end{pmatrix} \to \begin{pmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} R \\ I \end{pmatrix}$$

vs

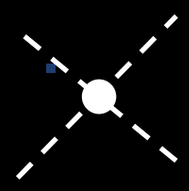
$$\chi \to \chi \qquad \xi \to \xi + \sqrt{2}\,\omega$$

Another Moral: the leading low-energy I-I scattering amplitude

$$\mathcal{A} \simeq 2i\lambda \left[\frac{(p\cdot q)^2+(p\cdot q')^2+(q\cdot q')^2}{m^4} \right] + O\left(m^{-6}\right)$$

is precisely as would have arisen from 'effective' interaction

$$S_{\rm eff} = \frac{\lambda}{4m^4} \int \mathrm{d}^4x \, (\partial_\mu I \, \partial^\mu I)^2$$

through the Feynman graph  plus 'crossed' graphs

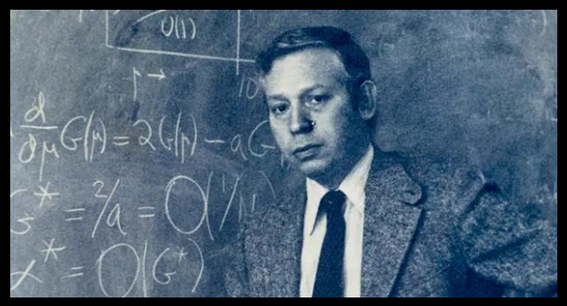
More remarkably: the order (E/m)^4 contributions to ANY low energy observable are captured by this same interaction, possibly with λ-corrected coefficient

Why does this work?

A low-energy lagrangian involving only the light field must exist because energy conservation ensures projecting onto low-energy is consistent with time-evolution

$$P_\Lambda e^{-iHt} P_\Lambda = e^{-iH_{\rm eff} t}$$

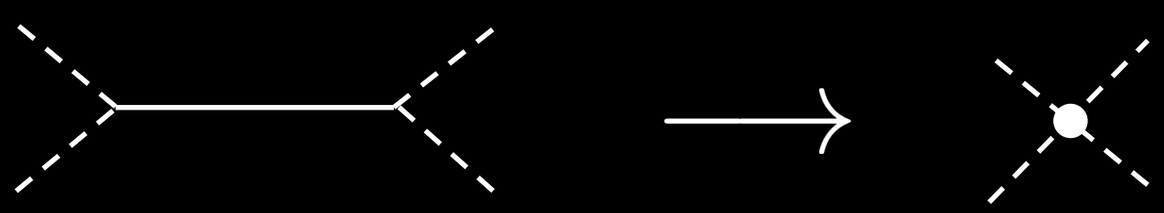
It can always be written in terms of the low energy field because this is a complete set of QFT operators at low energy



another of Weinberg's insights: QFT in itself contains no content beyond encoding things like special relativity, unitarity, cluster decomposition, etc in QM

But why is H_{eff} so simple? (eg why local? why no $1/m^2$ terms?)

Locality is a consequence of the uncertainty principle



Because energy conservation forbids actually producing them, heavy states can only influence light states as virtual particles.

Uncertainty relations allow temporary production of a state with energy *m* only for time intervals $\Delta t < 1/m$

$$G(x,y) = -i \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2+m^2}$$
$$\simeq -\frac{i}{m^2} \sum_{k=0}^{\infty} \left(\frac{\partial^2}{m^2}\right)^k \delta^4(x-y)$$

More complicated interactions dimensionally cost more powers of 1/m, so should be less important at low energies

$$\mathcal{L}_{\rm eff} \simeq -\frac{1}{2}(\partial_\mu \xi\, \partial^\mu \xi) + \frac{\lambda}{m^4}(\partial_\mu \xi\, \partial^\mu \xi)^2 + \frac{a_8}{m^8}(\partial_\mu \xi\, \partial^\mu \xi)^3 + \cdots$$

so working to fixed order in E/m only involves a fixed number of interactions

Explains special role played by renormalizable interactions (unsuppressed by 1/m) in describing Nature

$$\mathcal{L}_{\rm ren} = g_3 m\, \xi^3 + g_4 \xi^4$$

(renormalizable intns forbidden for toy model by symmetry)

Why not start at *1/m²*?

$$\mathcal{L} = \frac{a_1}{m^2}(\partial_\mu \xi \partial^2 \partial^\mu \xi) + \frac{a_2}{m^2}(\partial_\mu \partial_\nu \xi \, \partial^\mu \partial^\nu \xi)$$

$1/m^2$ terms (and other $1/m^4$ terms) are all *redundant*

$$\mathcal{L} = \frac{(a_1 - a_2)}{m^2}(\partial_\mu \xi \partial^2 \partial^\mu \xi) + \frac{a_2}{m^2}\partial_\nu(\partial_\mu \xi \, \partial^\mu \partial^\nu \xi)$$

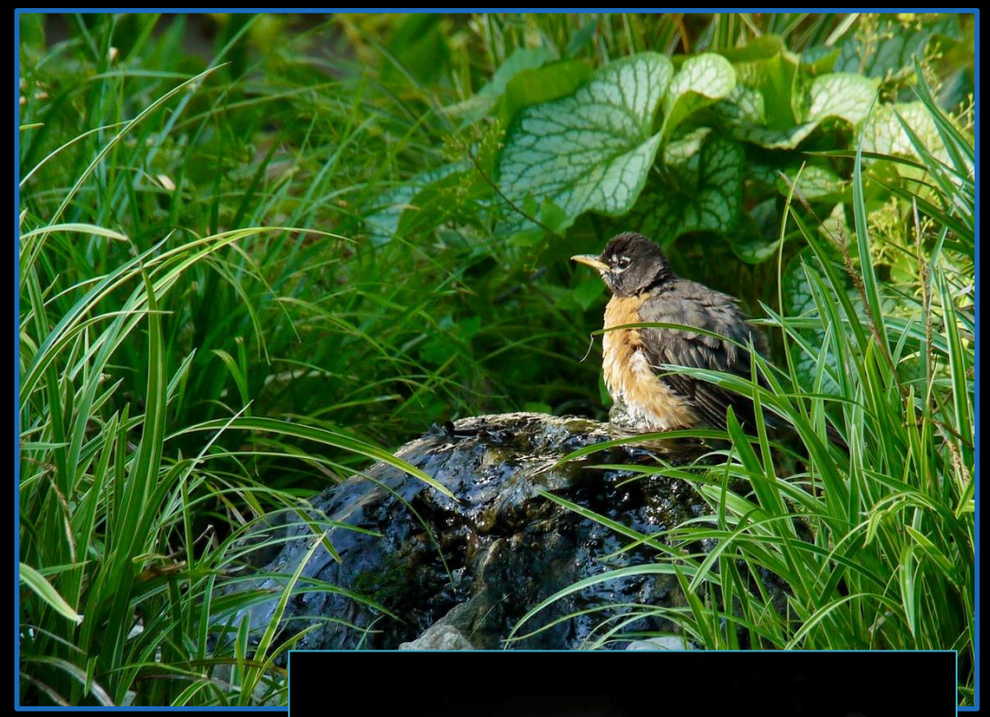
ie a total derivative or can be removed with a field redefinition

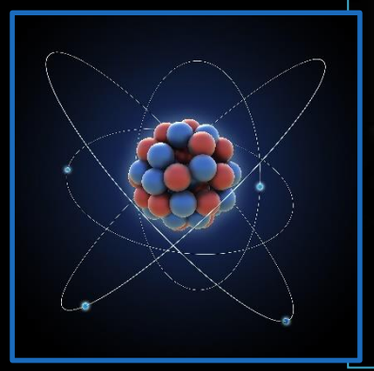
$$\xi \to \xi + \frac{a_2 - a_1}{m^2}\,\partial^2 \xi \quad \text{for which}$$

$$-\frac{1}{2}\,\partial_\mu \xi \partial^\mu \xi \to -\frac{1}{2}\,\partial_\mu \xi \partial^\mu \xi + \frac{a_2 - a_1}{m^2}\,(\partial_\mu \xi \partial^2 \partial^\mu \xi) + \cdots$$

EFT GENERAL FRAMEWORK

WHY RENORMALIZATION IS A GOOD THING







IMAGES: CB, GENEGEEK.CA, QUORA

WILSON ACTION

Suppose heavy and light degrees of freedom exist but only the light ones are ever measured:

$$\langle O(\ell)\rangle = \int \mathcal{D}\ell\,\mathcal{D}h\; e^{iS[\ell,h]}\; O(\ell)$$

Most simple if m/M expansion is done as early as possible

Define low-energy by $E < \Lambda$ and the Wilson action by

$$e^{iS_\Lambda[\ell]} = \int_\Lambda \mathcal{D}h\; e^{iS[\ell,h]}$$

so

$$\langle O(\ell)\rangle = \int^\Lambda \mathcal{D}\ell\; e^{iS_\Lambda[\ell]}\; O(\ell)$$

Important Properties follow from the definition:

1. Same low-energy expansion for S_Λ applies to all low-energy observables

$$\langle O(\ell)\rangle = \int^{\Lambda} \mathcal{D}\ell\; e^{iS_\Lambda[\ell]}\; O(\ell)$$

2a. The precise form for S_Λ depends in detail on precisely how the high- and low-energy sectors get split up

$$e^{iS_\Lambda[\ell]} = \int_\Lambda \mathcal{D}h\; e^{iS[\ell,h]}$$

2b. The details in S_Λ precisely cancel their counterparts in the measure once physical quantities are computed

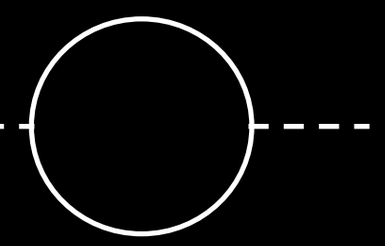
$$\langle O(\ell)\rangle = \int \mathcal{D}\ell\,\mathcal{D}h\; e^{iS[\ell,h]}\; O(\ell)$$

Why renormalization is a good thing:

S_Λ appears in the path integral in the same way as does the traditional classical action

$$\langle O(\ell) \rangle = \int^{\Lambda} \mathcal{D}\ell \; e^{iS_\Lambda[\ell]} \; O(\ell)$$

The cancellation of Λ-dependence of integral with the dependence in S_Λ sounds exactly like traditional way of describing renormalization



$$\frac{1}{\alpha_{\rm phys}} = \frac{1}{\alpha_0(\Lambda)} + b \, \ln\left(\frac{\Lambda}{m}\right)$$

Surely the classical action is really just a Wilson action for a still higher UV completion?

EFTs & dimensional regularization: Can use freedom of definition to use dim-reg (rather than cutoffs like Λ) in the effective theory (with couplings fixed by 'matching')

Cutoff split

UV: ℓ, h

Λ

EFT: ℓ

Dim-reg split

UV: ℓ, h

M

EFT: ℓ

m

EFTs & dimensional regularization: Can use freedom of definition to use dim-reg (rather than cutoffs like Λ) in the effective theory (with couplings fixed by 'matching')

Cutoff Definition:

High-energy sector:
all modes of *h* fields
& E > Λ modes of ℓ

Low-energy sector:
E < Λ modes of ℓ

Dim-Reg Definition:

High-energy sector:
all modes of *h*

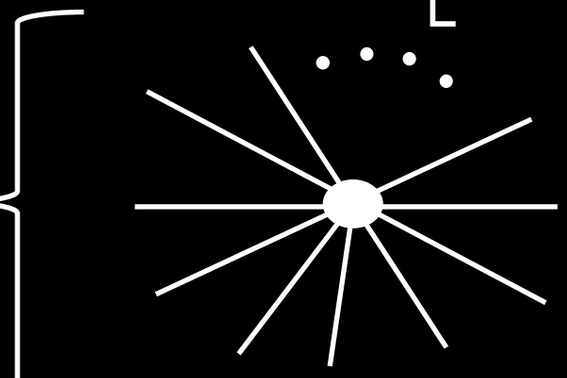
Low-energy sector:
all modes of ℓ

These differ only in how they treat high-energy modes (E > Λ modes of ℓ) and so diff. can be absorbed into eff. couplings.

Dimensional regularization allows more precise identification of which interactions contribute at each order in energy/m.

$$\mathcal{L}_{\rm int} = \mu^4 \sum_n O_n \left[(\partial/M)^{d_n}, (\phi/v)^{f_n} \right]$$

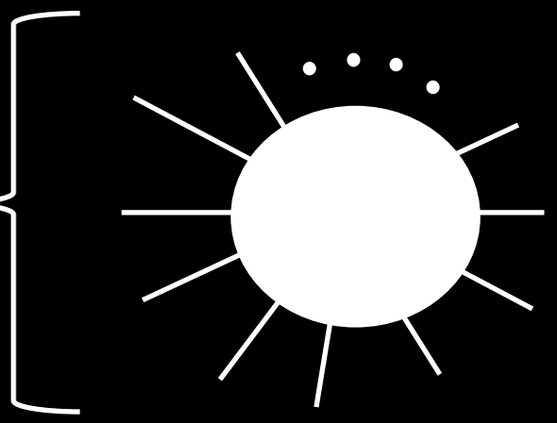
f_n fields



d_n derivatives

Use these to build a Feynman graph with E external lines, L loops and V_n vertices coming from interaction O_n

E external lines



L loops, V_n vertices

Use relations amongst *E, I, L, V* coming from fact they connect together to make a graph

$$E + 2I = \sum_n f_n V_n \quad \text{(conservation of ends)}$$

$$L = 1 + I - \sum_n V_n \quad \text{(definition of \# of loops)}$$

Track factors of μ, *M* and *v* from vertices and use dimensional analysis to determine power of external energy scale *Q*

$$\mathcal{L}_{\rm int} = \mu^4 \sum_n O_n \left[(\partial/M)^{d_n}, (\phi/v)^{f_n} \right]$$

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$$\mathcal{A}_E \sim \mu^4 \left(\frac{1}{v}\right)^E \left(\frac{MQ}{4\pi\mu^2}\right)^{2L} \prod_n \left(\frac{Q}{M}\right)^{2+(d_n-2)V_n}$$

1. Only positive powers of external energy scale Q implies systematic low-energy expansion beyond leading order.

Use relations amongst *E, I, L, V* coming from fact they connect together to make a graph

$$E + 2I = \sum_n f_n V_n \quad \text{(conservation of ends)}$$

$$L = 1 + I - \sum_n V_n \quad \text{(definition of \# of loops)}$$

Track factors of μ, *M* and *v* from vertices and use dimensional analysis to determine power of external energy scale *Q*

$$\mathcal{A}_E \sim \mu^4 \left(\frac{1}{v}\right)^E \left(\frac{MQ}{4\pi\mu^2}\right)^{2L} \prod_n \left(\frac{Q}{M}\right)^{2+(d_n-2)V_n}$$

2. Notice 1/v only appears with external lines: low-energy expansion (Q/M) logically independent of the field expansion

For example in the toy model we had $v = M = \mu = m$.

Amplitude with E external ξ particles depends on external energy scale Q by an amount

$$\mathcal{A}_E \sim m^4 \left(\frac{1}{m}\right)^E \left(\frac{Q}{4\pi m}\right)^{2L} \prod_n \left(\frac{Q}{m}\right)^{2+(d_n-2)V_n}$$

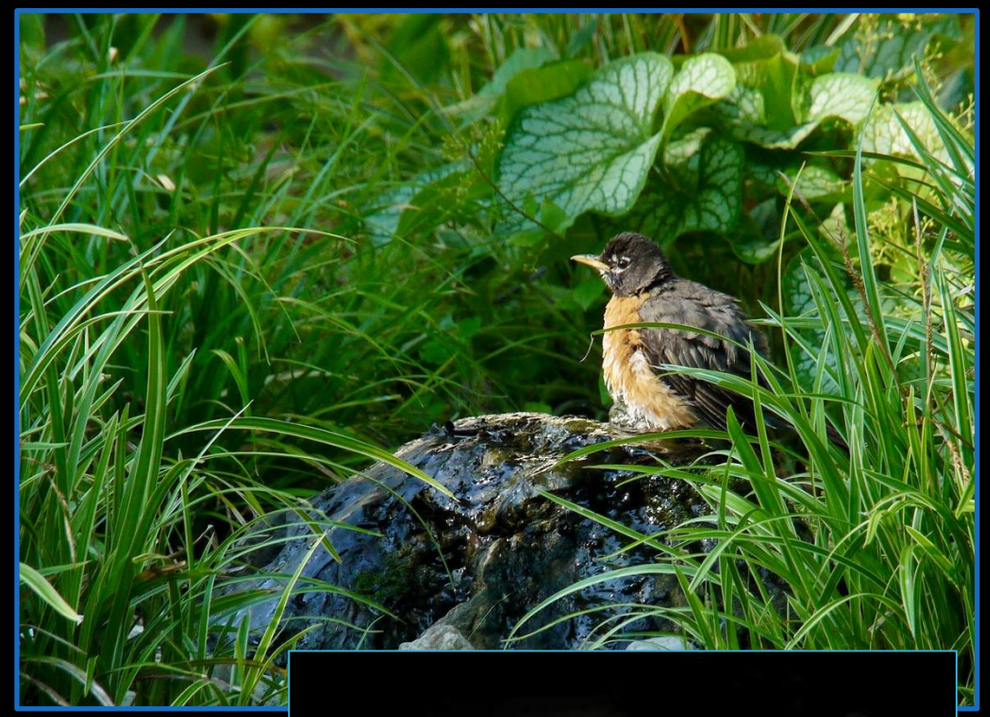
where all interactions satisfy $d_n \geq f_n \geq 4$

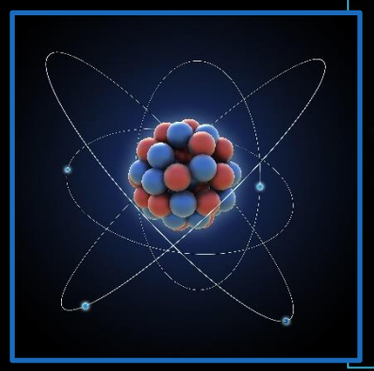
When $E = 4$ largest contribution has: $L = 0$ and $V_n = 1$ only if $d_n = f_n = 4$

for which $$\mathcal{A}_E \sim \left(\frac{Q}{m}\right)^4$$

EFT GENERAL FRAMEWORK

TIME-DEPENDENT FIELDS







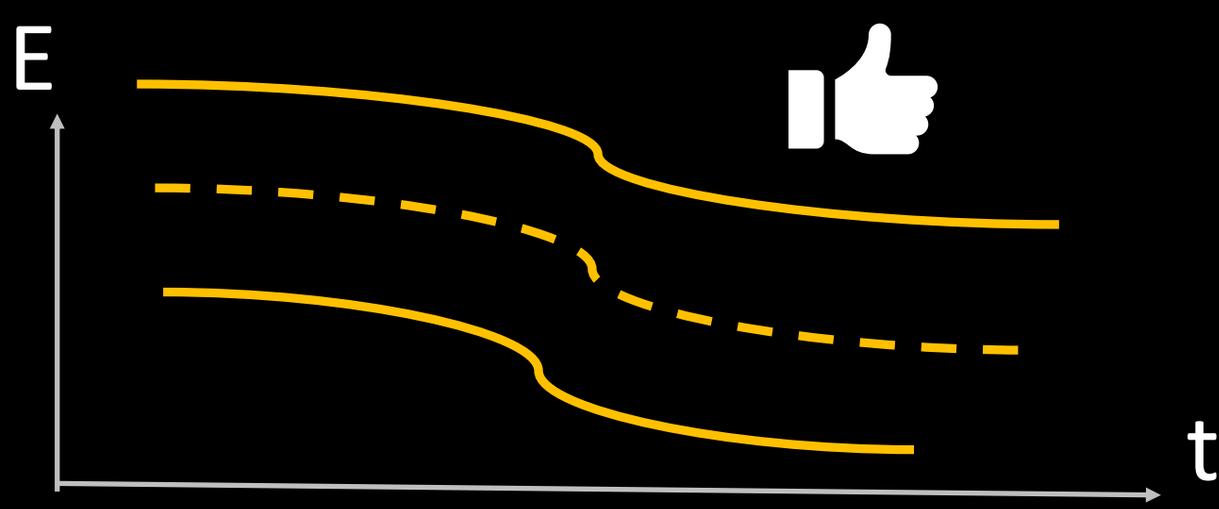
IMAGES: CB, GENEGEEK.CA, QUORA

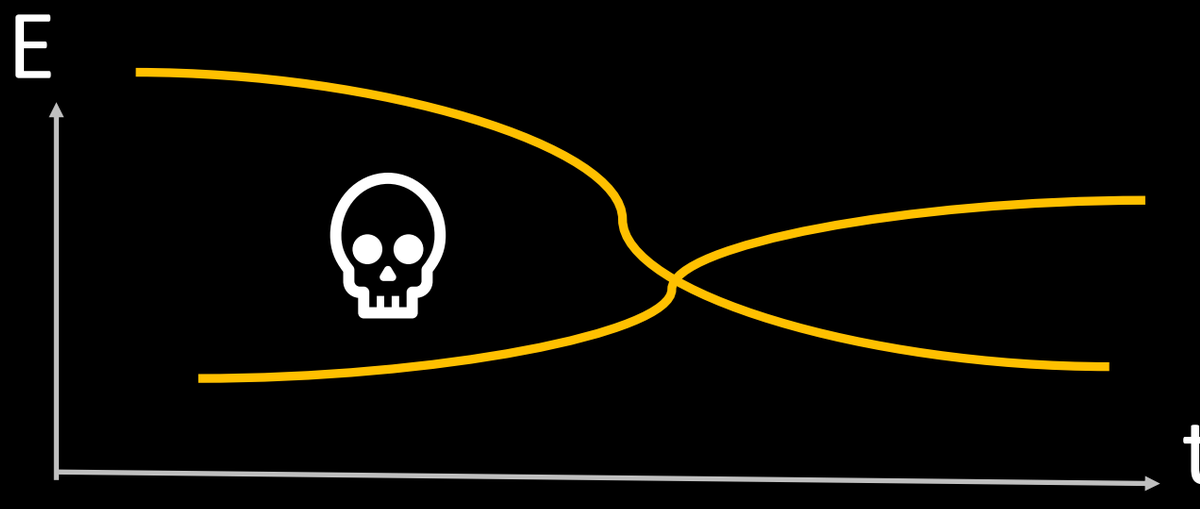
EVOLVING BACKGROUND FIELDS

Time-dependent background fields: Can EFTs be used with time-dependent backgrounds?

Naively not, because energy conservation is central while energy for fluctuations need not be conserved for time-dependent backgrounds.

They can if evolution is adiabatic, so $A^{-1}\, dA/dt$ << UV scales.
Then energy levels vary parametrically with time, $E_n = E_n(t)$.
Must also demand high and low energy levels do not cross.





Toy model of two spinless fields

$$S := -\int \mathrm{d}^4x \left[\partial_\mu \phi^* \partial^\mu \phi + V(\phi^*\phi)\right]$$

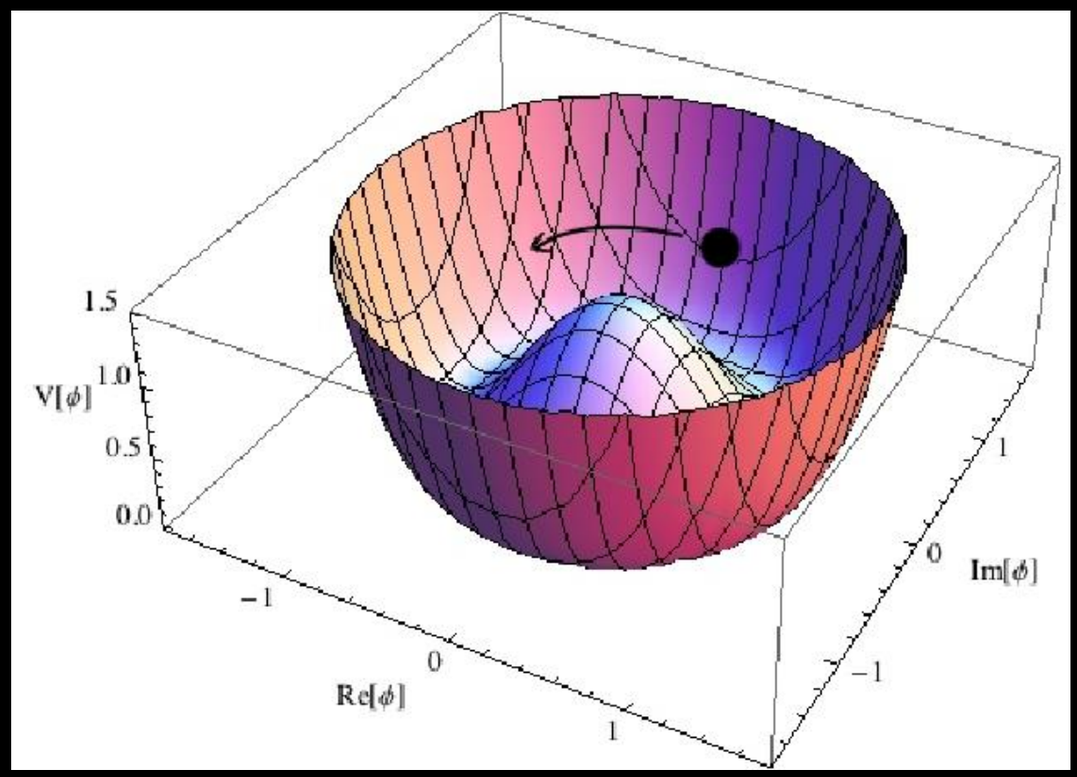
$$V(\phi^*\phi) = \tfrac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$

Low energy EFT: $S_{\mathrm{eff}} = \frac{\lambda}{4m^4}\int \mathrm{d}^4x \left(\partial_\mu \xi\, \partial^\mu \xi\right)^2$

Time dependent classical solution:

$$\phi_{\mathrm{cl}} = \rho_0\, e^{i\omega t}$$

EOM: $\rho_0 = \sqrt{v^2 + \frac{2\omega^2}{\lambda}}$

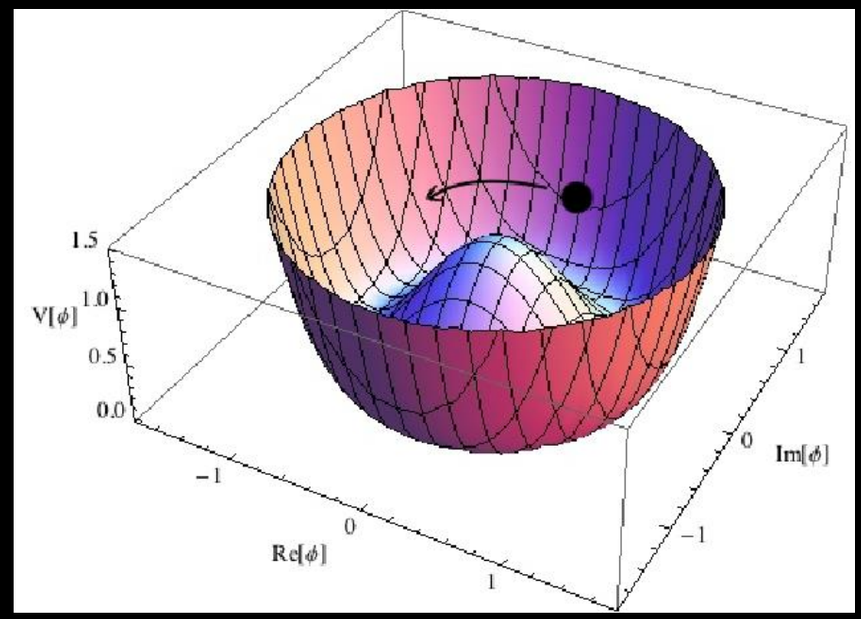


Classical energy partly kinetic and partly due to field climbing the potential

$$\varepsilon = \dot{\phi}^* \dot{\phi} + V = \omega^2 \left(v^2 + \frac{3\omega^2}{\lambda} \right)$$

The EFT has no V, so how does it account for the potential energy?

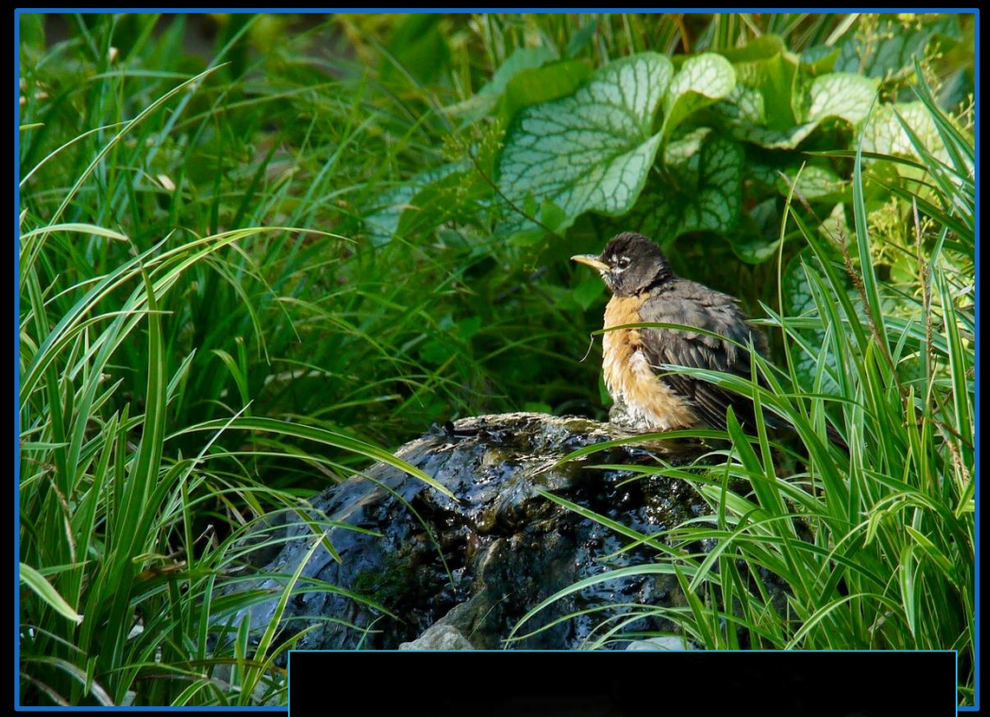
$$\varepsilon = \frac{1}{2} \dot{\xi}^2 + \frac{3\lambda}{4m^4} \dot{\xi}^4$$
$$= \omega^2 \left(v^2 + \frac{3\omega^2}{\lambda} \right)$$

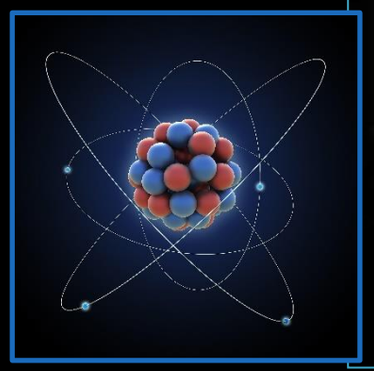


This is why EFT methods can be applied eg in cosmological applications (which proves important for consistency in GR)

EFT GENERAL FRAMEWORK

EXAMPLES





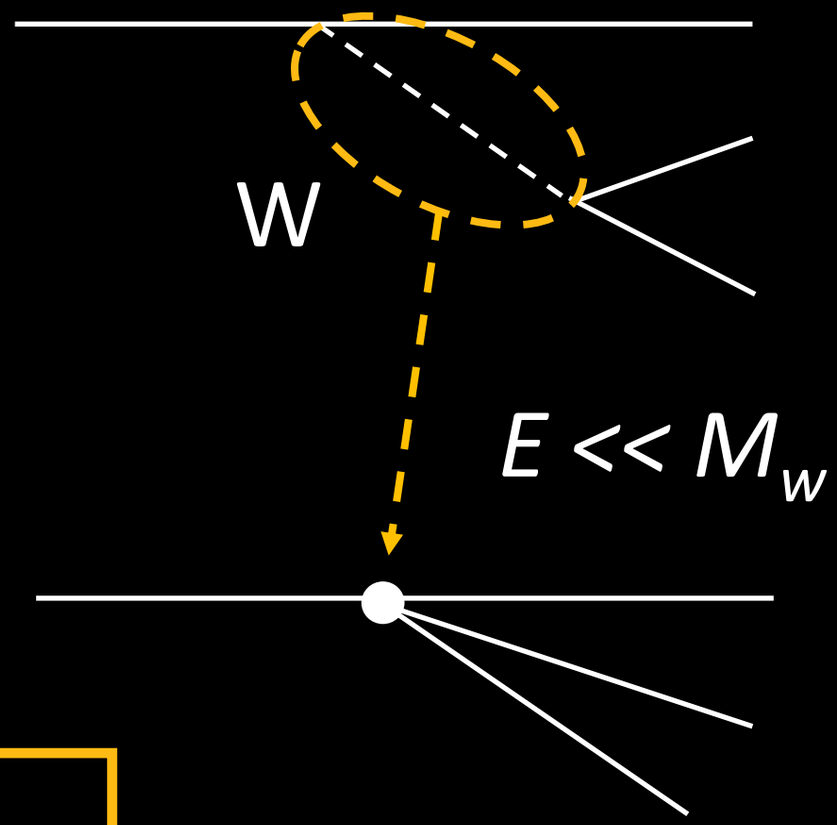


IMAGES: CB, GENEGEEK.CA, QUORA

The Standard Model provides many examples of hierarchies of scale (some of which helped EFT methods to be discovered).

Weak decays proceed through W exchange

$$\mathcal{L}_w = \frac{g}{2\sqrt{2}} W_\mu J^\mu + \text{c.c.}$$



Fermi theory describes weak interactions at low energies

$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}} J^*_\mu J^\mu$$

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_w^2}}$$

Massless spin-1 particle

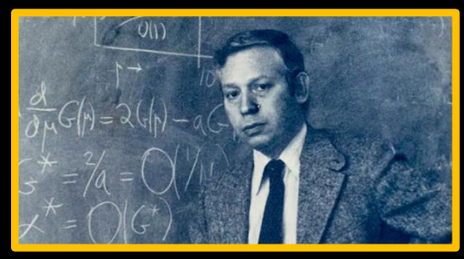
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Free action

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

A_μ not a 4-vector!

$$A_\mu \to A_\mu + \partial_\mu \omega$$



Nonderivative couplings to other matter

$$S_{\rm mat} = e \int \mathrm{d}^4 x \, A_\mu J^\mu(\psi) \quad \Rightarrow \quad \partial_\mu J^\mu(\psi) = 0$$

Gauge invariance and conserved current both required by QM + Lorentz invariance!

At low energies renormalizable interactions should dominate. What are possibilities for electron – photon?

$$\mathcal{L}_{\text{ren}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \overline{\psi}(\gamma^\mu\partial_\mu + m)\psi - ieA_\mu\overline{\psi}\gamma^\mu\psi$$

All other interactions involve couplings with dimensions of inverse length (so are not renormalizable).

QED emerges at low energies as the most general form allowed for static massless spin-one coupling to matter

Massless spin-2 particle

$$R_{\mu\nu\lambda\rho} = \partial_\mu \partial_\lambda h_{\nu\rho} -$$

$h_{\mu\nu}$ not a sym-tensor!

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \omega_\nu + \partial_\nu \omega_\mu$$

Free action

$$\mathcal{L}_0/\sqrt{-g} = -\tfrac{1}{2}\, g^{\mu\nu} g^{\lambda\rho} R_{\mu\lambda\nu\rho} \qquad g_{\mu\nu} = \eta_{\mu\nu} + 2h_{\mu\nu}/M_p$$

Nonderivative couplings to other matter

$$S_{\rm mat} = \frac{1}{M_p} \int \mathrm{d}^4x\, h_{\mu\nu} T^{\mu\nu}(\psi) \;\Rightarrow\; \partial_\mu T^{\mu\nu}(\psi) = 0$$

Gauge invariance and conserved current both required by QM + Lorentz invariance!

At low energies renormalizable interactions should dominate. What are possibilities for electron – graviton?

$$\mathcal{L}_{\rm ren} = \mathcal{L}_{\rm graviton} - \overline{\psi}(\gamma^\mu \partial_\mu + m)\psi$$

All interactions involve couplings with dimensions of inverse mass (so are not renormalizable).

Inability to formulate renormalizable interactions for graviton are consistent with gravity being weakest known force

COSMOLOGY

BACKGROUND STORY

NASA HUBBLE DEEP FIELD

SPATIALLY FLAT FLRW UNIVERSE

Cosmology starts with two observations and an inference

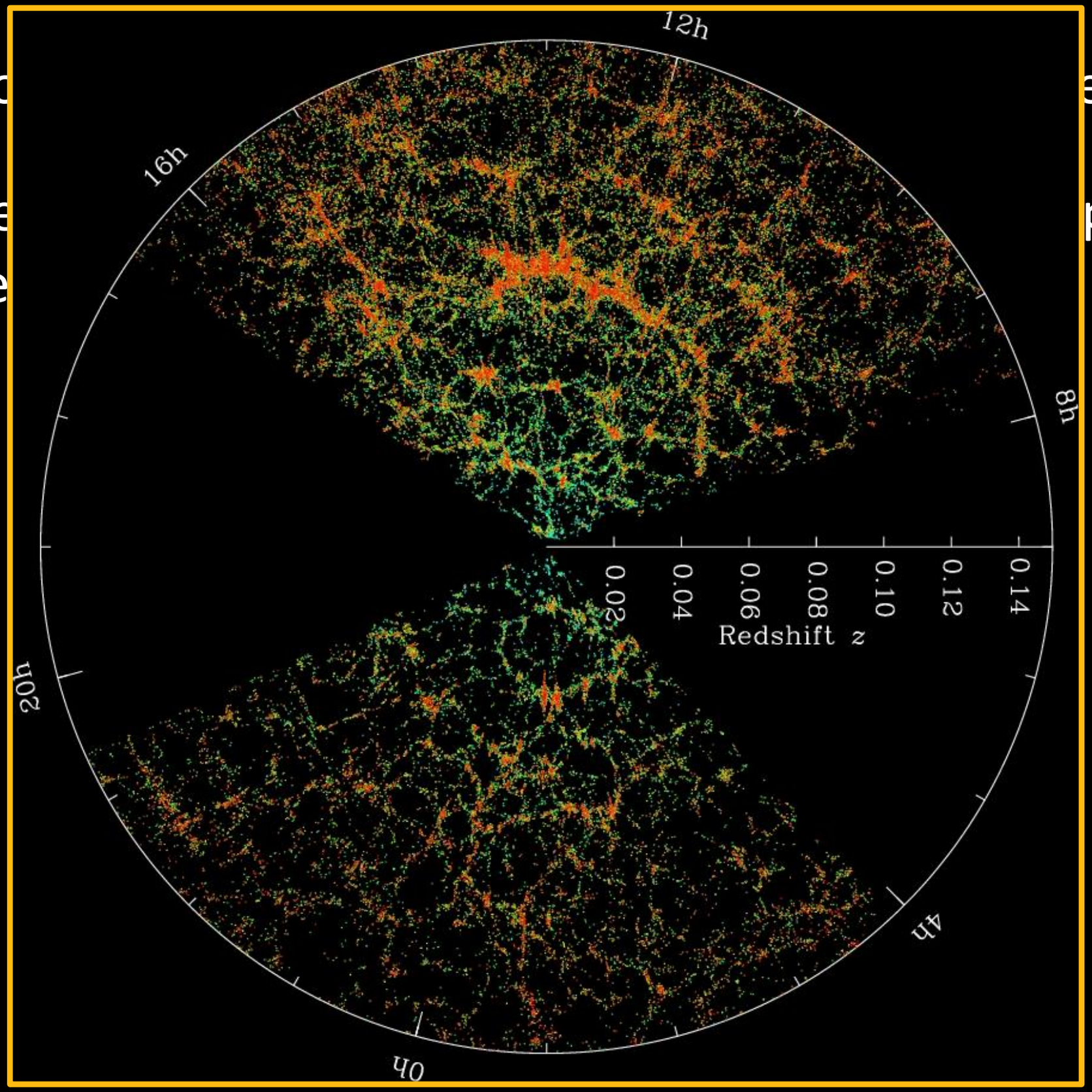
1. Matter, on average, seems to be distributed isotropically and homogeneously around us

SLOAN DIGITAL SKY SURVEY

SPATIALLY FLAT FLRW UNIVERSE

Cosmolo ence

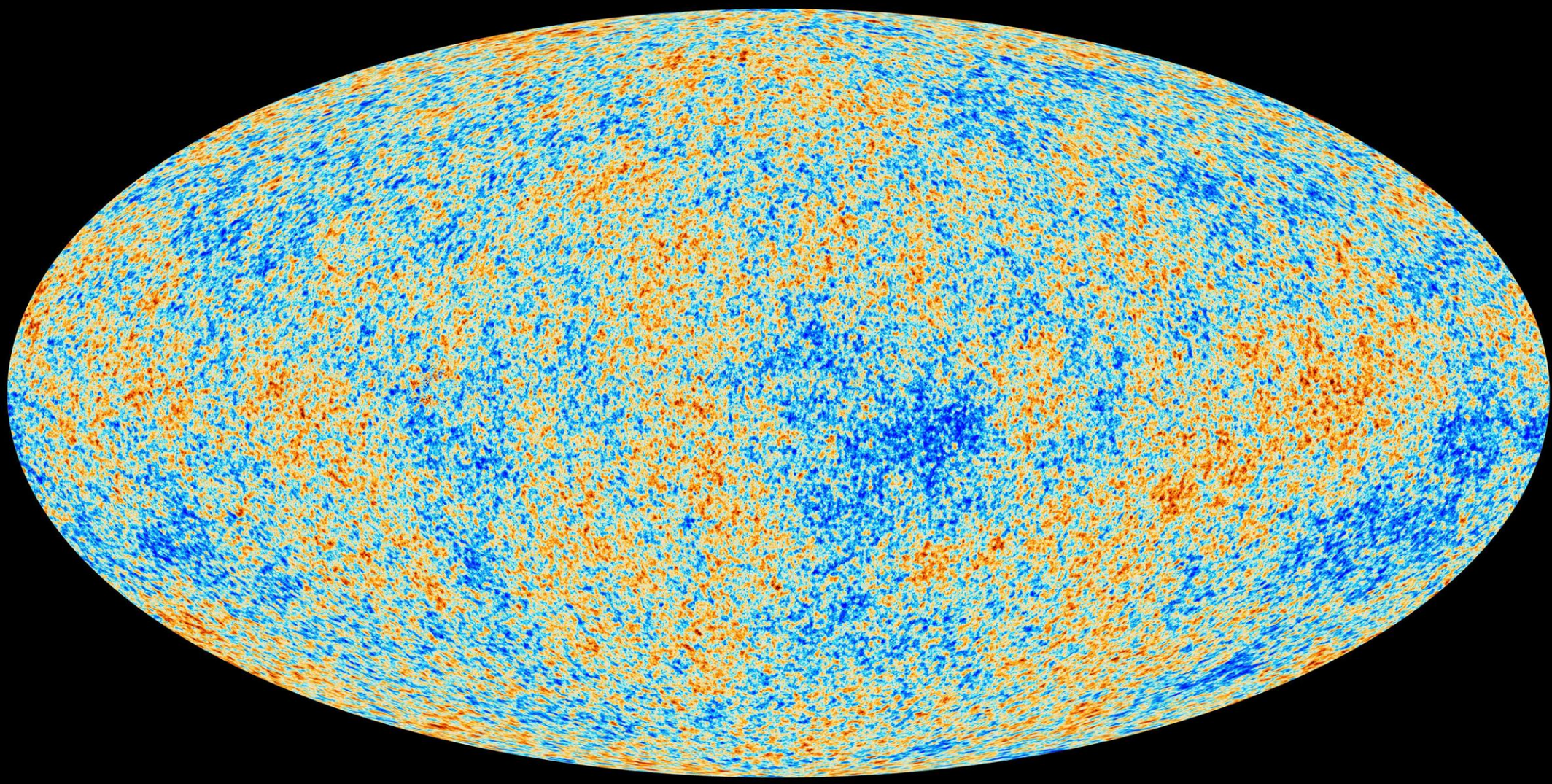
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SLOAN DIGITAL SKY SURVEY

Cosmology starts with two observations and an inference

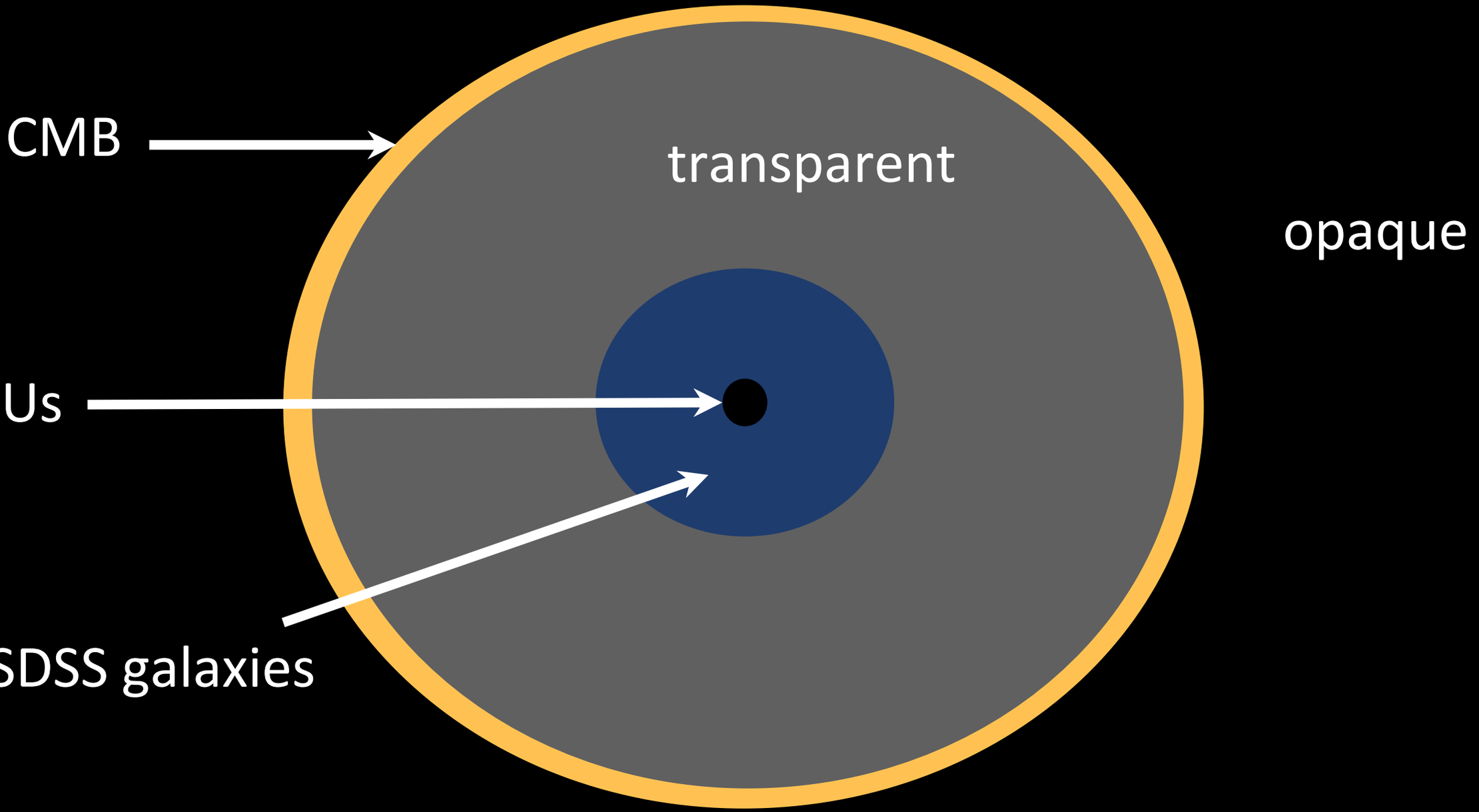
1. Matter, on average, seems to be distributed isotropically and homogeneously around us ***(eg cosmic microwave background temperature fluctuations are 1 part in 100,000)***



PLANCK CMB MAP

Cosmology starts with two observations and an inference

1. Matter, on average, seems to be distributed isotropically and homogeneously around us (eg cosmic microwave background)

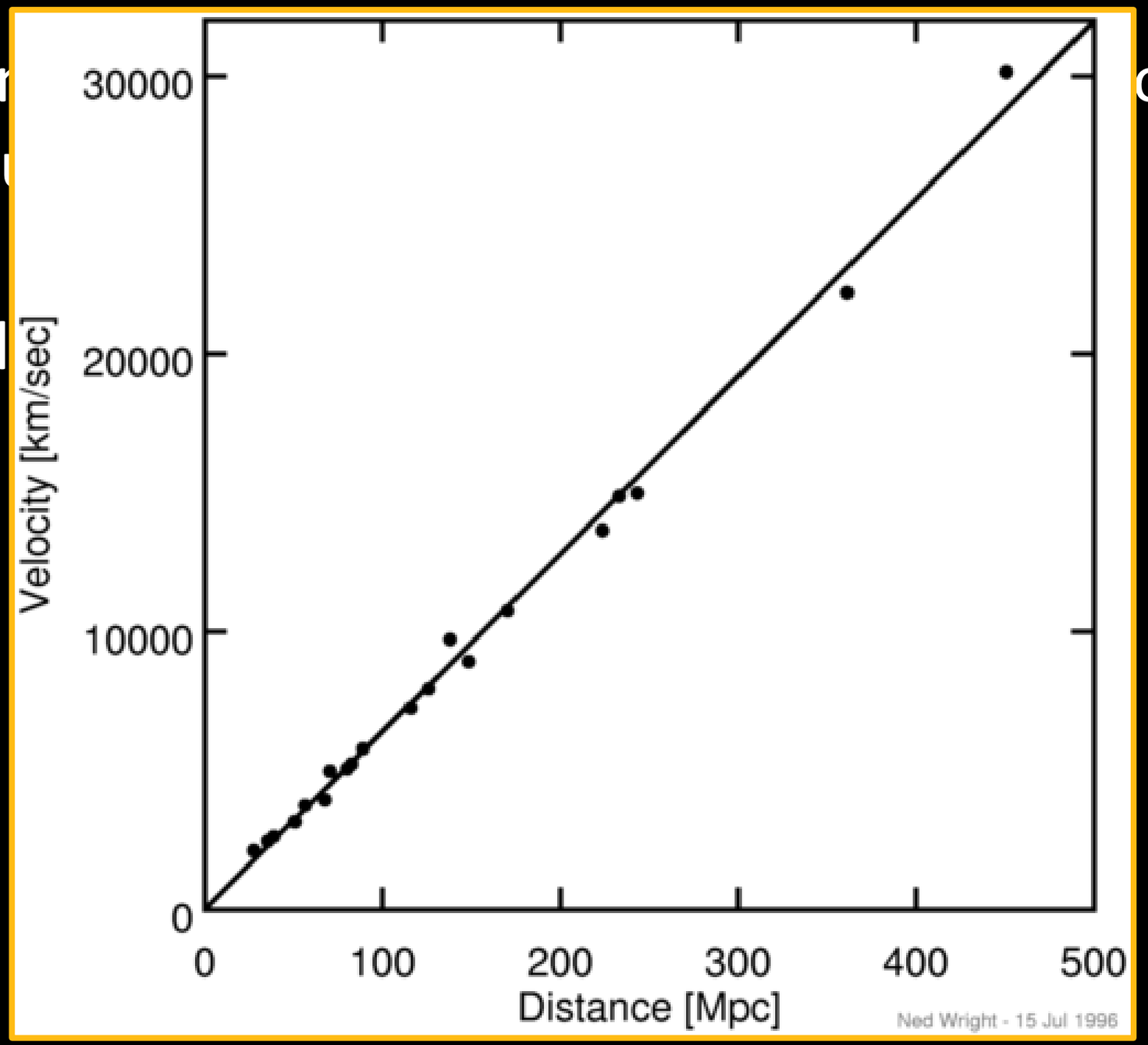


SPATIALLY FLAT FLRW UNIVERSE

Cosmology starts with two observations and an inference

1. Matter, or opically and homogeneou

2. Sufficientl from us



NED WRIGHT'S COSMOLOGY PAGE

Cosmology starts with two observations and an inference

1. Matter, on average, seems to be distributed isotropically and homogeneously around us

2. Sufficiently distant galaxies redshift as if receding from us

$$v_i \simeq H_0\, d_i \quad \text{(Hubble Law)}$$

Suggests an explosion. *(Where is the crater?)*

Cosmology starts with two observations and an inference

1. Matter, on average, seems to be distributed isotropically and homogeneously around us

2. Sufficiently distant galaxies redshift as if receding from us

$$v_i \simeq H_0\, d_i \quad \text{(Hubble Law)}$$

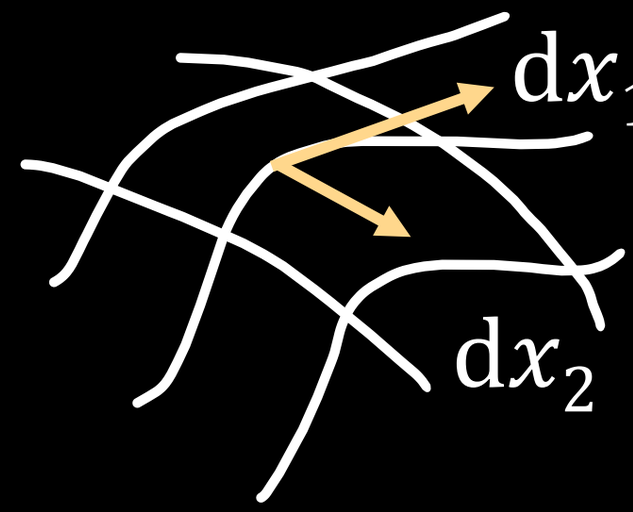
There is no crater: ***can regard space itself as expanding!***
Predicted by general relativity, which relates overall geometry of spacetime to its overall energy content

Homogeneous & isotropic expanding universe:

$$\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}\cdot\mathrm{d}\vec{x}$$

All lengths stretch with time

$$d_{\mathrm{phys}}(t) = a(t)\,\Delta x$$



In particular true for wavelengths of light whose redshift

$$z := \frac{\lambda_{\mathrm{obs}} - \lambda_{\mathrm{em}}(t)}{\lambda_{\mathrm{em}}(t)} = \frac{a_0}{a(t)} - 1$$

is a useful proxy for expansion and for time.

Homogeneous & isotropic expanding universe:

$$\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}\cdot\mathrm{d}\vec{x}$$

Notice important time-dependent scales

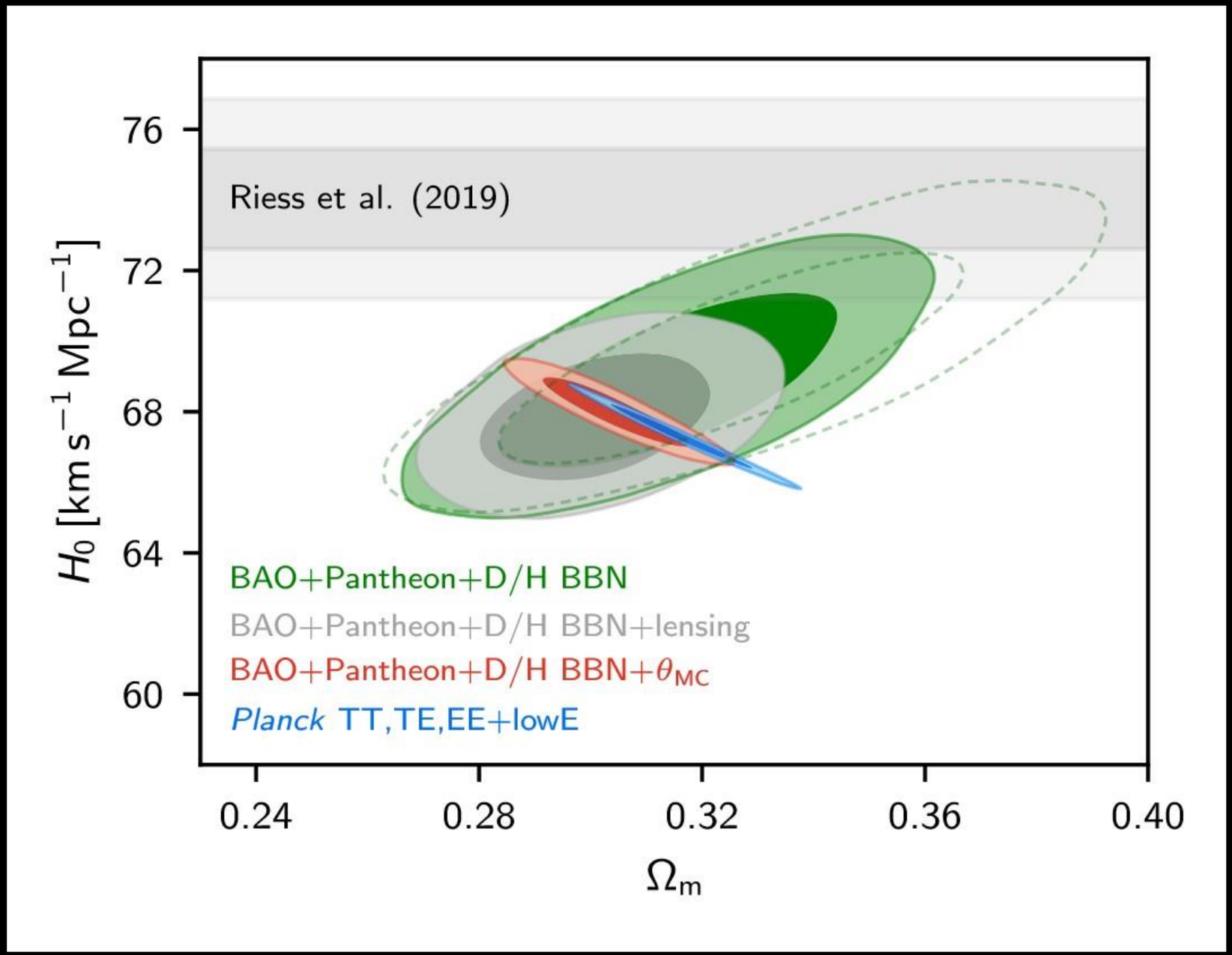
$$H(t) = \dot{a}/a \qquad H_0 = H(t_0)$$

$$d_{\mathrm{phys}} = a(t)\,\Delta x$$

grow with different speeds during expansion

$$a(t) \propto t^\alpha \implies d_H(t) := H^{-1} = t/\alpha$$

There are hints of a problem in latest measurements of H_0

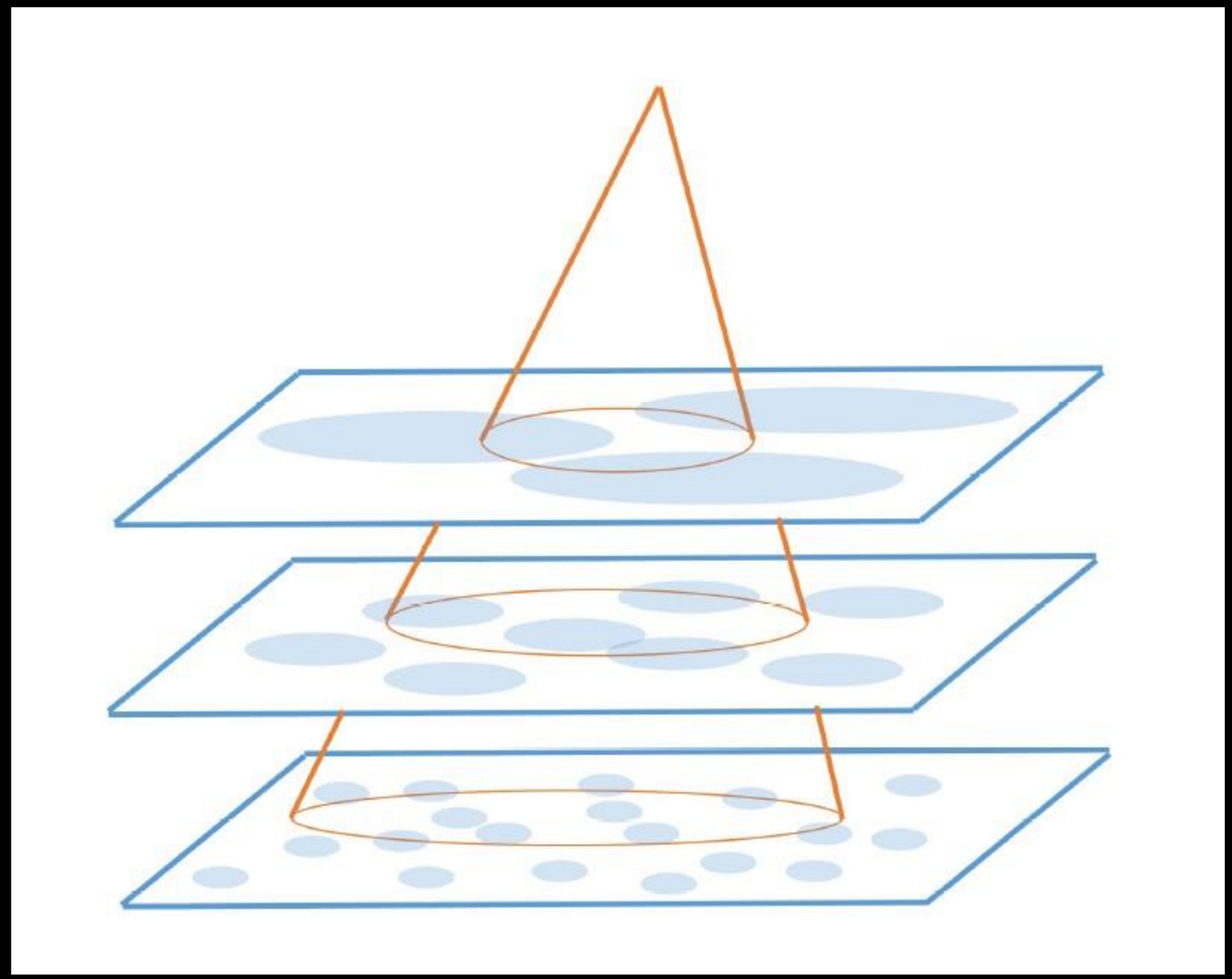


Planck Collaboration

Our view on the universe is along past light cone

This view intersects different sized
eg Hubble patches at different times

$$d_H(t) := H^{-1}$$



SPATIALLY FLAT FLRW UNIVERSE

Our view on the universe is along past light cone

Correlations across these Hubble patches
require explanation

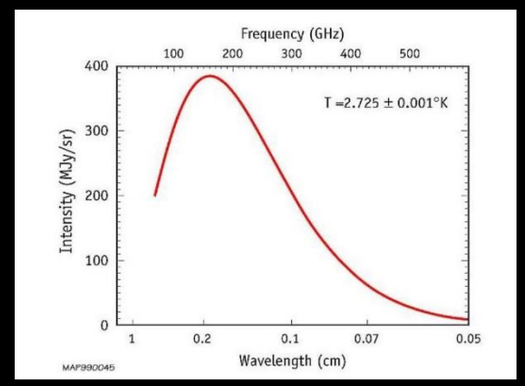


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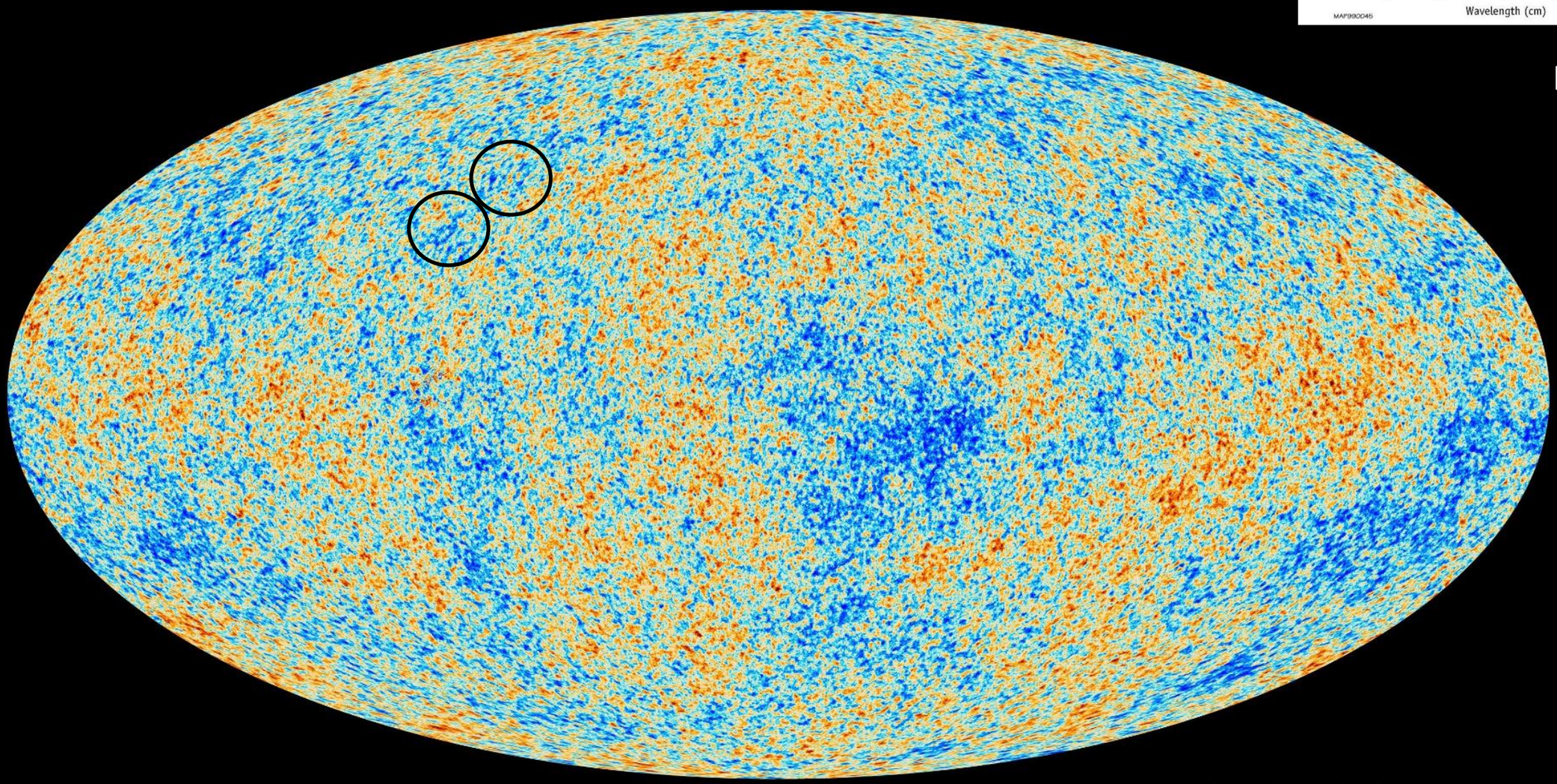
SPATIALLY FLAT FLRW UNIVERSE

Our view on the universe is along past light cone

Correlations across these Hubble patches require explanation



FIRAS



Planck Collaboration

eg why the same T?

Evolving Hubble size has implications for how other fields evolve (and so also on how to explain correlations)

$$-\Delta\phi = \ddot{\phi} + 3H\,\dot{\phi} - \nabla^2\phi/a^2$$

eg in momentum space

$$\phi(x,t) = \phi_k(t)\, e^{i\vec{k}\cdot\vec{x}}$$

$$-\Delta\phi = \ddot{\phi} + 3H\,\dot{\phi} + \boxed{k^2\phi/a^2}$$

Modes redshift

Damped oscillations when $k/a > H$ (Hubble friction)

Evolving Hubble size has implications for how other fields evolve (and so also on how to explain correlations)

$$-\Delta\phi = \ddot{\phi} + 3H\,\dot{\phi} - \nabla^2\phi/a^2$$

eg in momentum space

$$\phi(x,t) = \phi_k(t)\, e^{i\vec{k}\cdot\vec{x}}$$

$$-\Delta\phi = \ddot{\phi} + \boxed{3H\,\dot{\phi}} + k^2\phi/a^2$$

Modes redshift

Modes over-damp ‘freeze’ when $k/a \ll H$ (Hubble friction)

EINSTEIN FIELD EQUATIONS

General relativity relates expansion rate to energy content

$$R_{\mu\nu} - \tfrac{1}{2}\, R\, g_{\mu\nu} + T_{\mu\nu}/M_p^2 = 0$$

Homogeneity & isotropy imply

$$T_{\mu\nu} = (p+\rho)\, U_\mu U_\nu + p\, g_{\mu\nu}$$

where pressure p and energy density ρ depend only on time and 4-velocity of matter fluid U^μ satisfies

$$g_{\mu\nu}\, U^\mu U^\nu = -1$$

For spatially flat FLRW universes this implies *Friedmann eq*

$$H^2 = (\dot{a}/a)^2 = \rho/(3M_p^2)$$

and local energy conservation

$$\dot{\rho} + 3H(\rho + p) = 0$$

Compare this 2nd equation to

$$\mathrm{d}\Big(\rho\, a^3\Big) + p\,\mathrm{d}(a^3) = \mathrm{d}U + p\,\mathrm{d}V = 0$$

For spatially flat FLRW universes this implies *Friedman eq*

$$H^2 = (\dot{a}/a)^2 = \rho/(3M_p^2)$$

and local energy conservation

$$\dot{\rho} + 3H(\rho + p) = 0$$

Together these imply cosmic acceleration is

$$\ddot{a}/a = -(\rho + 3p)/(6M_p^2)$$

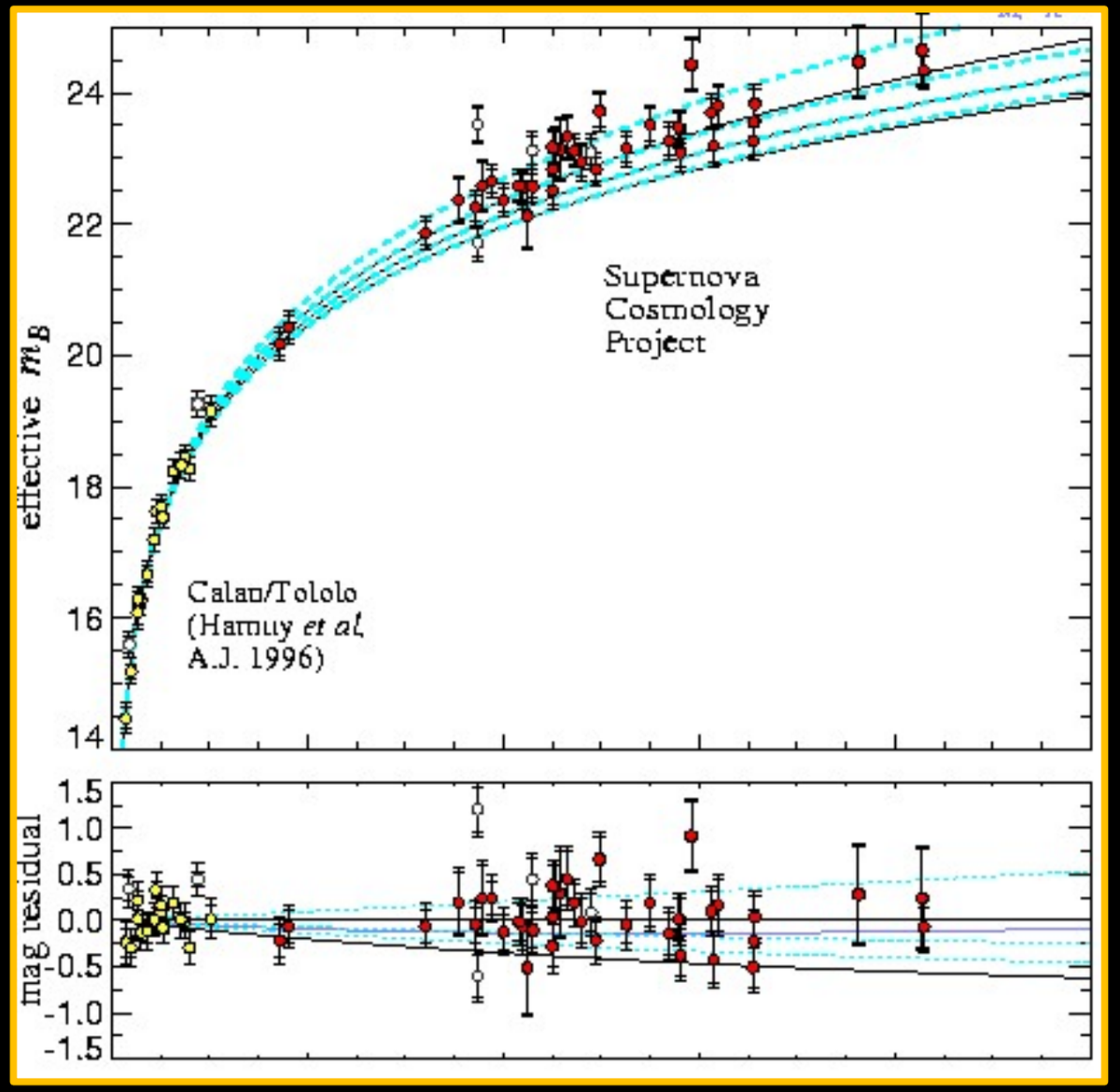
which is negative whenever

$$p \geq -\rho/3$$

EINSTEIN FIELD EQUATIONS

THIS IS IMPORTANT BECAUSE THE UNIVERSAL EXPANSION NOW APPEARS TO BE ACCELERATING!





What can provide negative pressure?

The vacuum (which we know is Lorentz invariant):

$$T^{\mu\nu}_{\rm vac} = -\rho_{\rm vac}\, g^{\mu\nu}$$

and conservation implies ρ_{vac} must be constant

Comparing to general fluid implies p = - ρ

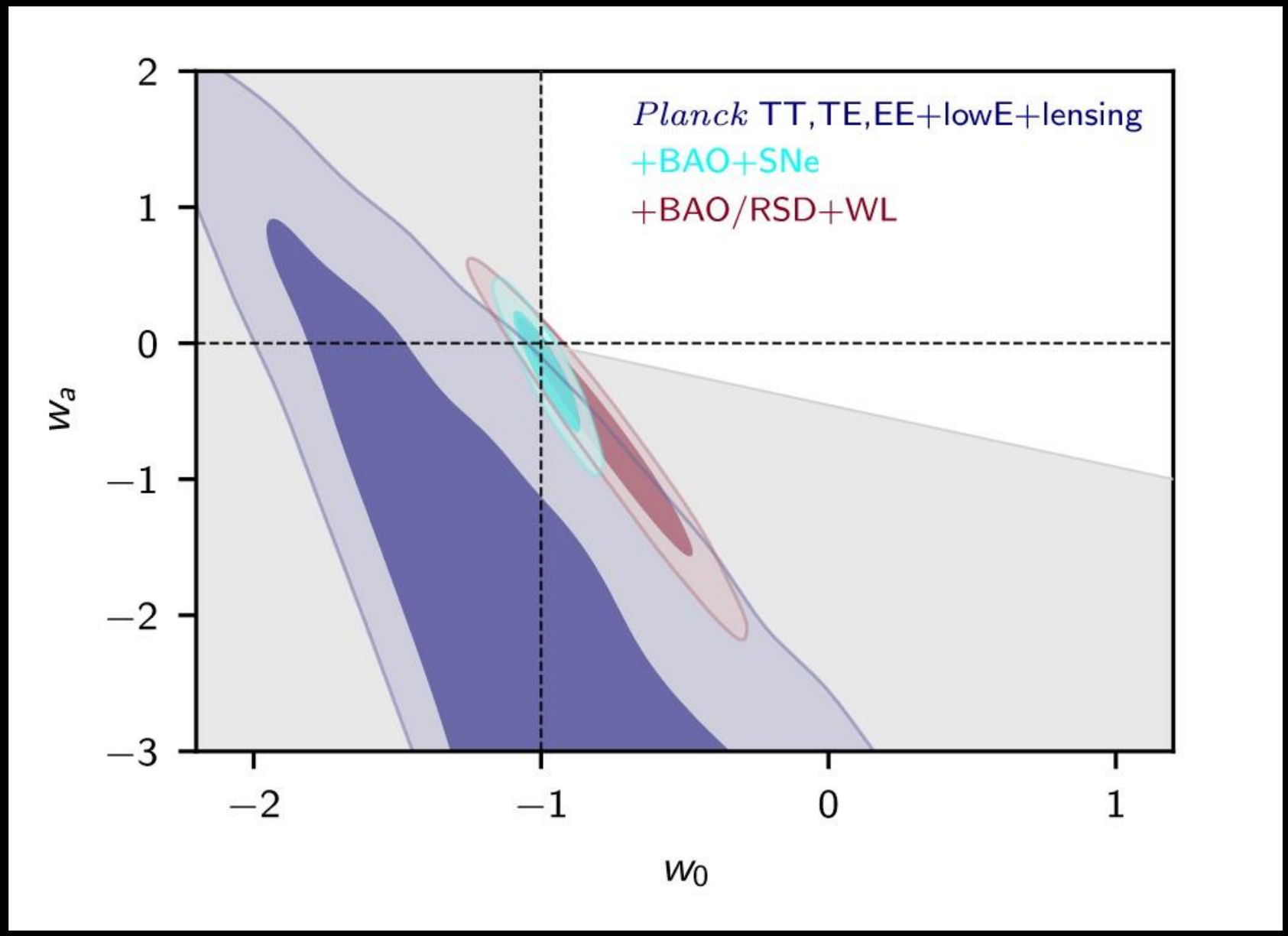
$$T_{\mu\nu} = (p+\rho)\, U_\mu U_\nu + p\, g_{\mu\nu}$$

Eq of state is as required for energy to be conserved despite having constant density in an expanding universe

$$\mathrm{d}\Big(\rho\, a^3\Big) + p\, \mathrm{d}(a^3) = \mathrm{d}U + p\, \mathrm{d}V = 0$$

Observations are consistent with the vacuum e.o.s. $w := p/\rho = -1$

$$w(a) := w_0 + w_a(a - a_0)$$



Planck Collaboration

HOT BIG BANG

Expansion of universe suggests smaller, hotter past.

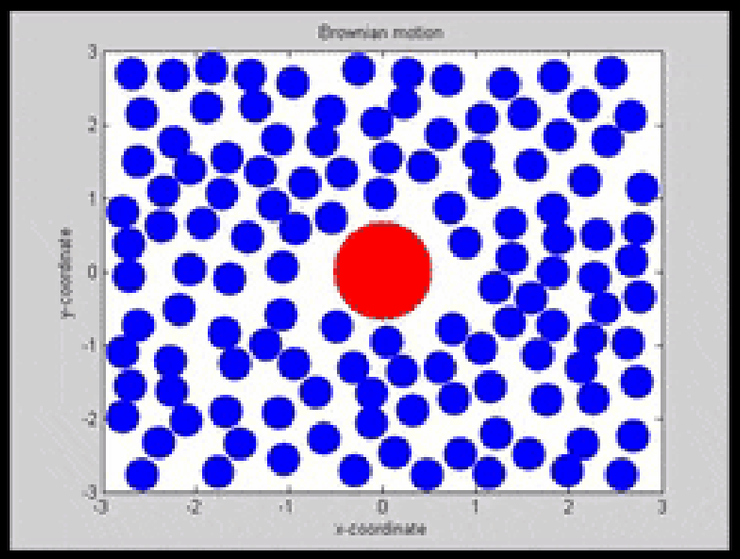
The Hot Big Bang Model: Assume this was a hot gas of elementary particles in remote past; evolve forward to see if it looks like what we see *(it does)*

Main predictions: cosmic relics. Bound states (atoms, nuclei, etc) do not survive at high temperatures but start to survive once temperatures are low enough.

There is evidence this occurred both for atoms and nuclei

Atoms form once $T = 10^3$ K

Nuclei form once $T = 10^{10}$ K



gfycat.com

Universe contains several kinds of hot cosmic fluids, distinguished by equations of state.

$$\rho = \sum\nolimits_i \rho_i \qquad \text{and} \qquad p = \sum\nolimits_i p_i$$

1. Dark energy (vacuum): $p_{\rm vac} = -\rho_{\rm vac}$

2. Relativistic matter (radiation): $p_r = \frac{1}{3}\,\rho_r = \sigma\,T^4$

3. Nonrelativistic matter: $p_m \simeq n\,T \ll \rho_m \simeq n\,m$

Current abundances of each component can be inferred by success of Big Bang cosmology

$$\Omega_i = \rho_i(t_0)/\rho_{\rm tot}(t_0)$$

$$\rho_{\rm tot}(t_0) \simeq 10^{-29}\,{\rm g/cm}^3$$

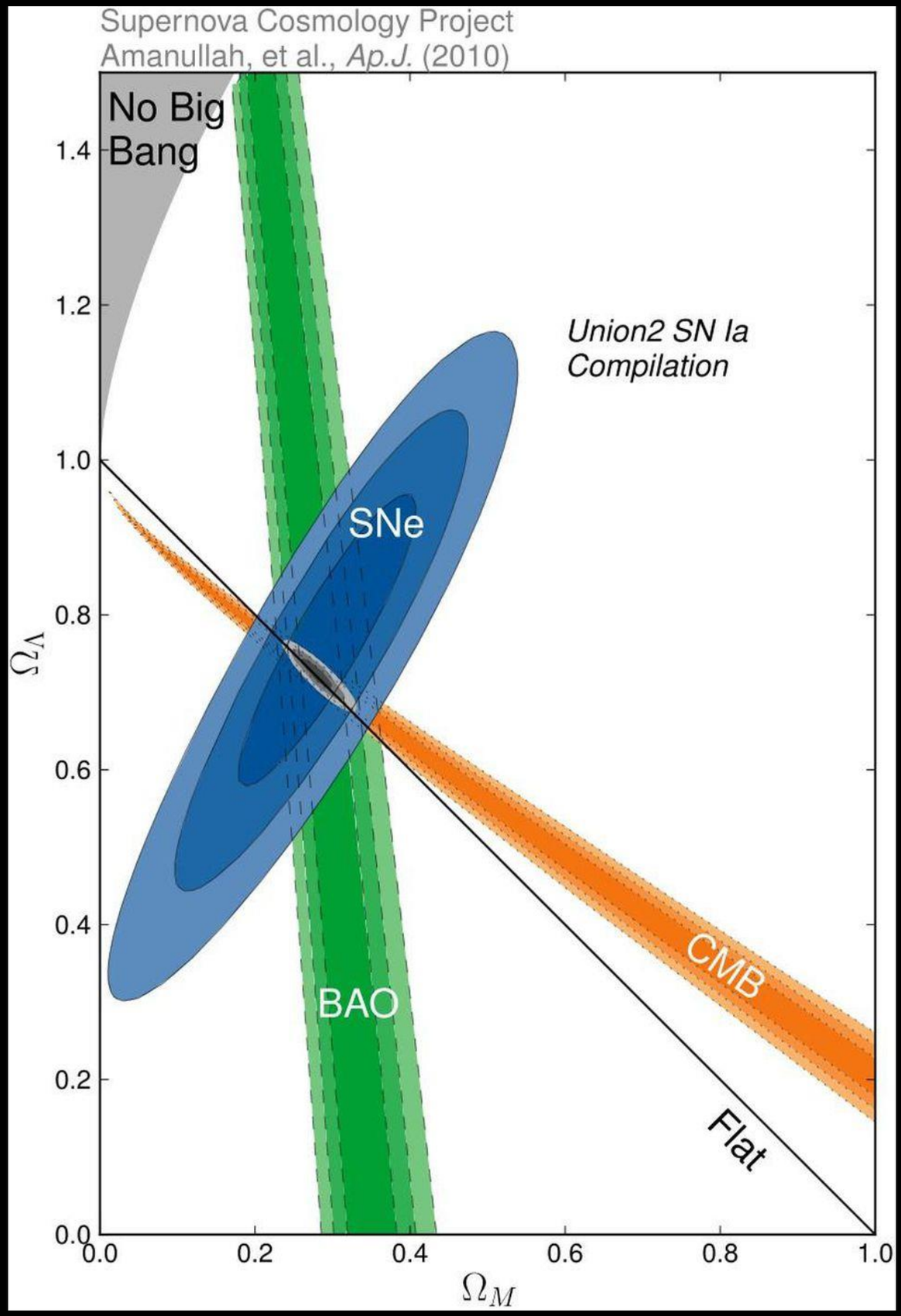
$$\Omega_{\rm vac} \simeq 0.7$$

$$\Omega_m \simeq 0.3$$

$$\Omega_b \simeq 0.05$$

DM vs baryons

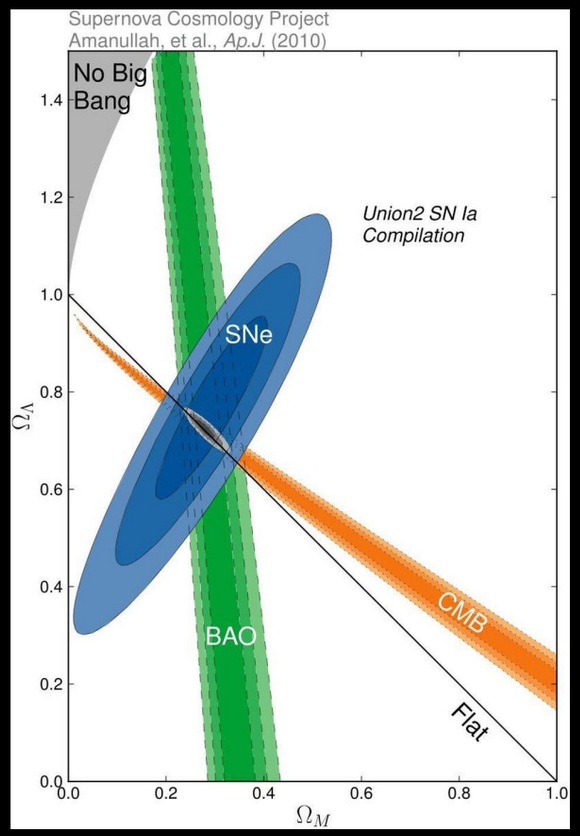
$$\Omega_r \simeq 8 \times 10^{-5}$$

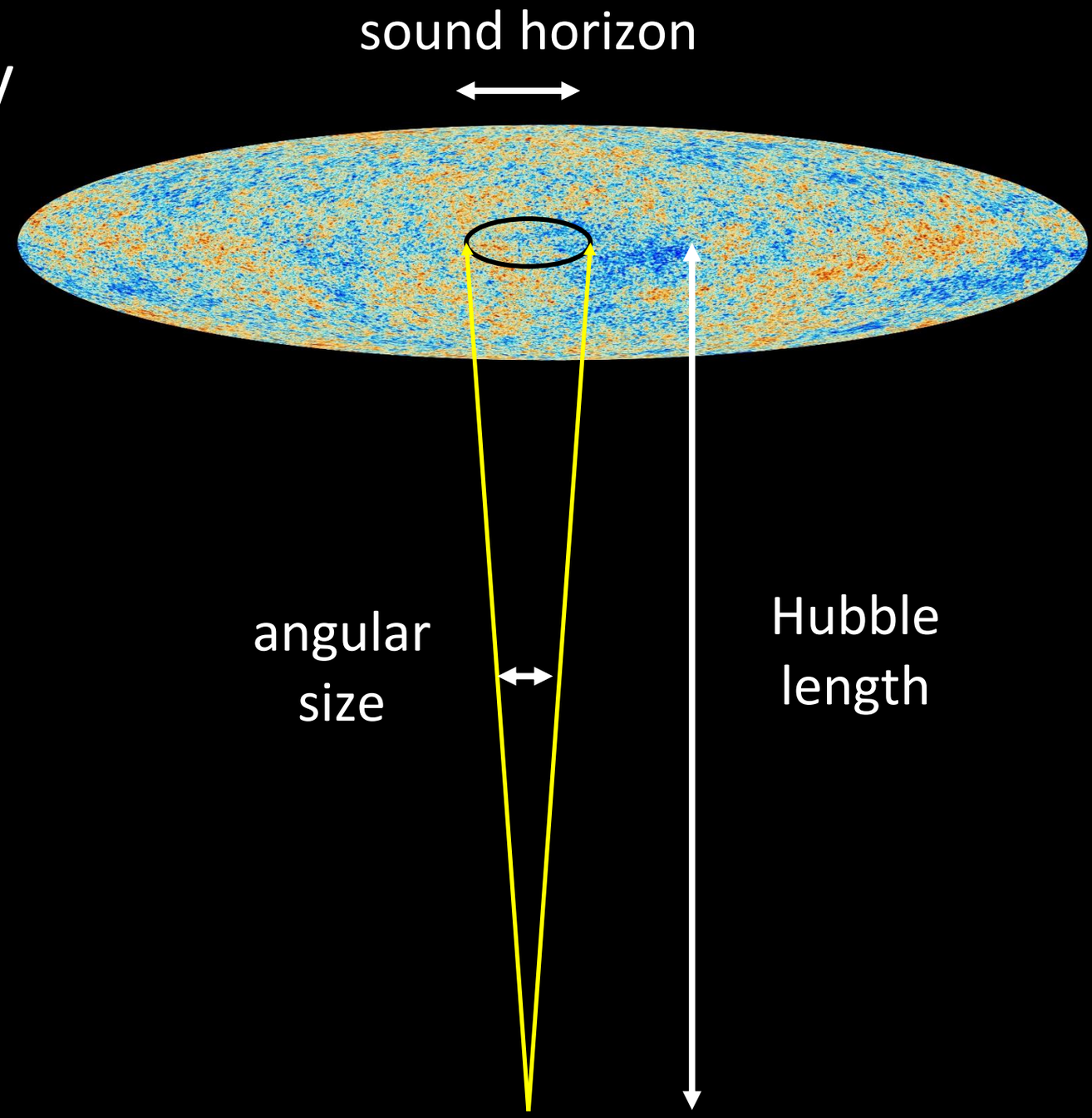


Current abundances of each component can be inferred by success of cosmology

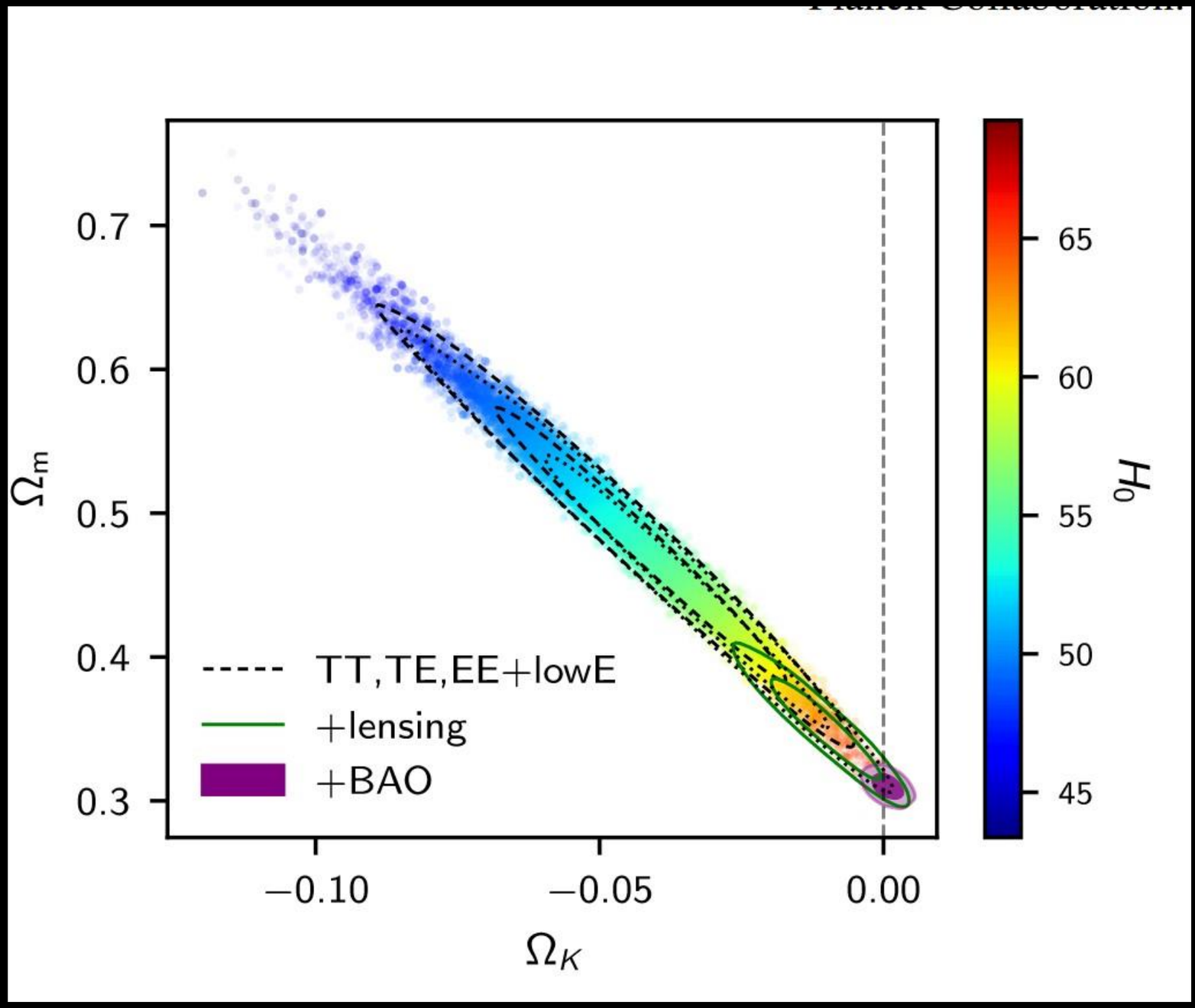
$$\rho_{\rm tot}(t_0) \simeq 10^{-29}\,{\rm g/cm}^3$$

$$\Omega_{\rm vac} \simeq 0.7 \qquad \Omega_m \simeq 0.3$$





Metric used assumes curvature of spatial slices vanishes ($\Omega_k = 0$)



Planck Collaboration

Precise values are obtained by fitting handful of parameters of LCDM Hot Big Bang cosmology to ***redundant*** observations

Parameter	[1] *Planck* TT+lowP	[2] *Planck* TE+lowP	[3] *Planck* EE+lowP	[4] *Planck* TT,TE,EE+lowP	$([1]-[4])/\sigma_{[1]}$
$\Omega_b h^2$	0.02222 ± 0.00023	0.02228 ± 0.00025	0.0240 ± 0.0013	0.02225 ± 0.00016	−0.1
$\Omega_c h^2$	0.1197 ± 0.0022	0.1187 ± 0.0021	$0.1150^{+0.0048}_{-0.0055}$	0.1198 ± 0.0015	0.0
$100\theta_{MC}$	1.04085 ± 0.00047	1.04094 ± 0.00051	1.03988 ± 0.00094	1.04077 ± 0.00032	0.2
τ	0.078 ± 0.019	0.053 ± 0.019	$0.059^{+0.022}_{-0.019}$	0.079 ± 0.017	−0.1
$\ln(10^{10} A_s)$	3.089 ± 0.036	3.031 ± 0.041	$3.066^{+0.046}_{-0.041}$	3.094 ± 0.034	−0.1
n_s	0.9655 ± 0.0062	0.965 ± 0.012	0.973 ± 0.016	0.9645 ± 0.0049	0.2
H_0	67.31 ± 0.96	67.73 ± 0.92	70.2 ± 3.0	67.27 ± 0.66	0.0
Ω_m	0.315 ± 0.013	0.300 ± 0.012	$0.286^{+0.027}_{-0.038}$	0.3156 ± 0.0091	0.0
σ_8	0.829 ± 0.014	0.802 ± 0.018	0.796 ± 0.024	0.831 ± 0.013	0.0
$10^9 A_s e^{-2\tau}$	1.880 ± 0.014	1.865 ± 0.019	1.907 ± 0.027	1.882 ± 0.012	−0.1

Planck Collaboration

In GR eq of state determines cosmic evolution:

$$H^2 = (\dot{a}/a)^2 = \rho/(3M_p^2)$$

$$\boxed{\dot{\rho} + 3H(\rho + p) = 0}$$

$$p = w\,\rho \quad \text{implies} \quad \rho = \rho_0(a_0/a)^{3(1+w)}$$

$$w = -1 \quad \text{implies} \quad \rho = \rho_0$$

$$w = 0 \quad \text{implies} \quad \rho = \rho_0(a_0/a)^3$$

$$w = \tfrac{1}{3} \quad \text{implies} \quad \rho = \rho_0(a_0/a)^4$$

In GR eq of state determines cosmic evolution:

$$H^2 = (\dot{a}/a)^2 = \rho/(3M_p^2)$$

$$\dot{\rho} + 3H(\rho + p) = 0$$

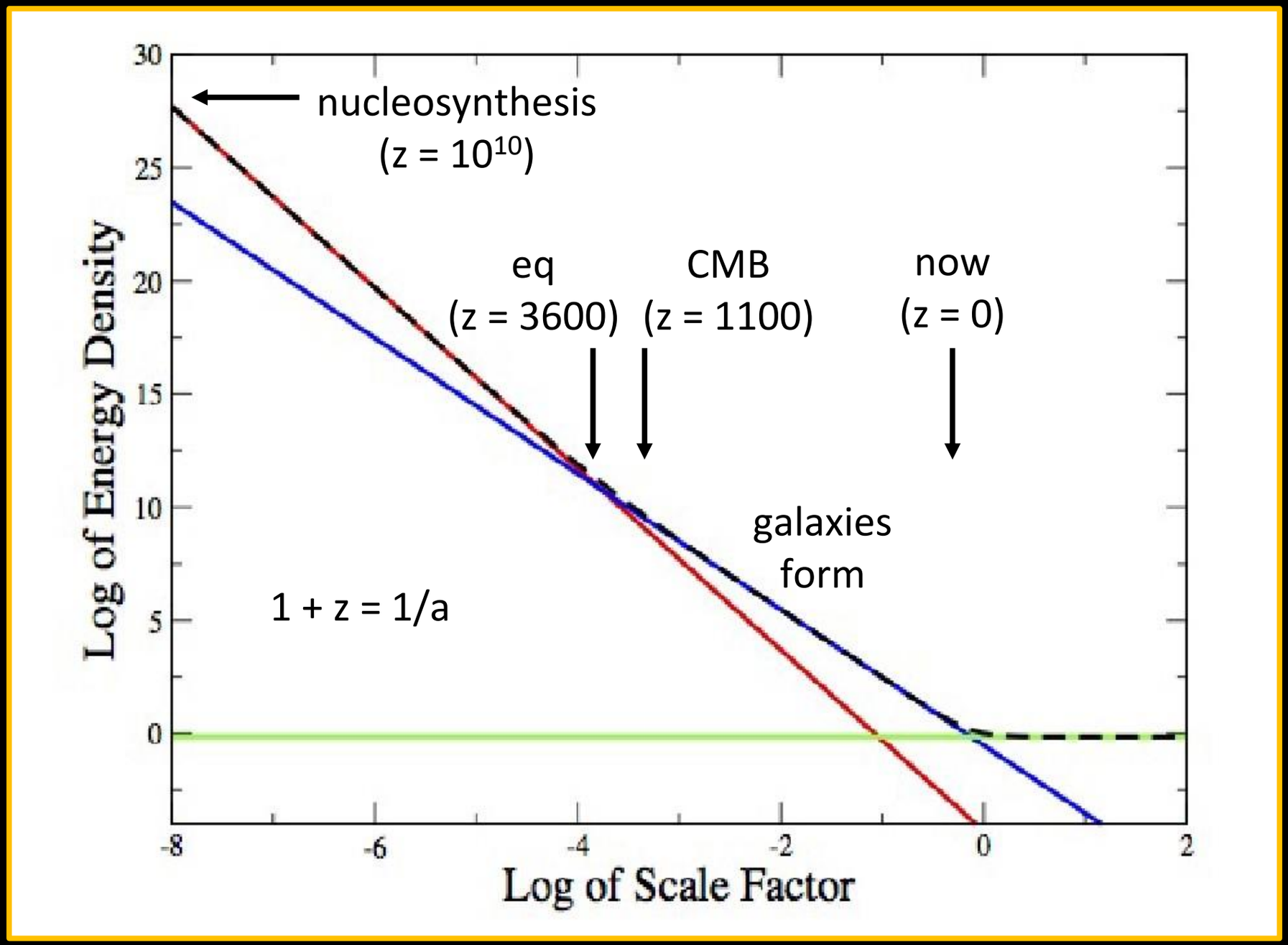
$$\rho = \rho(a) \quad \text{implies} \quad a = a_0(t/t_0)^{2/[3(1+w)]}$$

$$w = -1 \quad \text{implies} \quad a = a_0\, e^{H(t-t_0)}$$

$$w = 0 \quad \text{implies} \quad a = a_0(t/t_0)^{2/3}$$

$$w = \tfrac{1}{3} \quad \text{implies} \quad a = a_0(t/t_0)^{1/2}$$

Key events: nuclei form; radiation-matter equality; atoms form; galaxies form; present epoch



COSMOLOGY

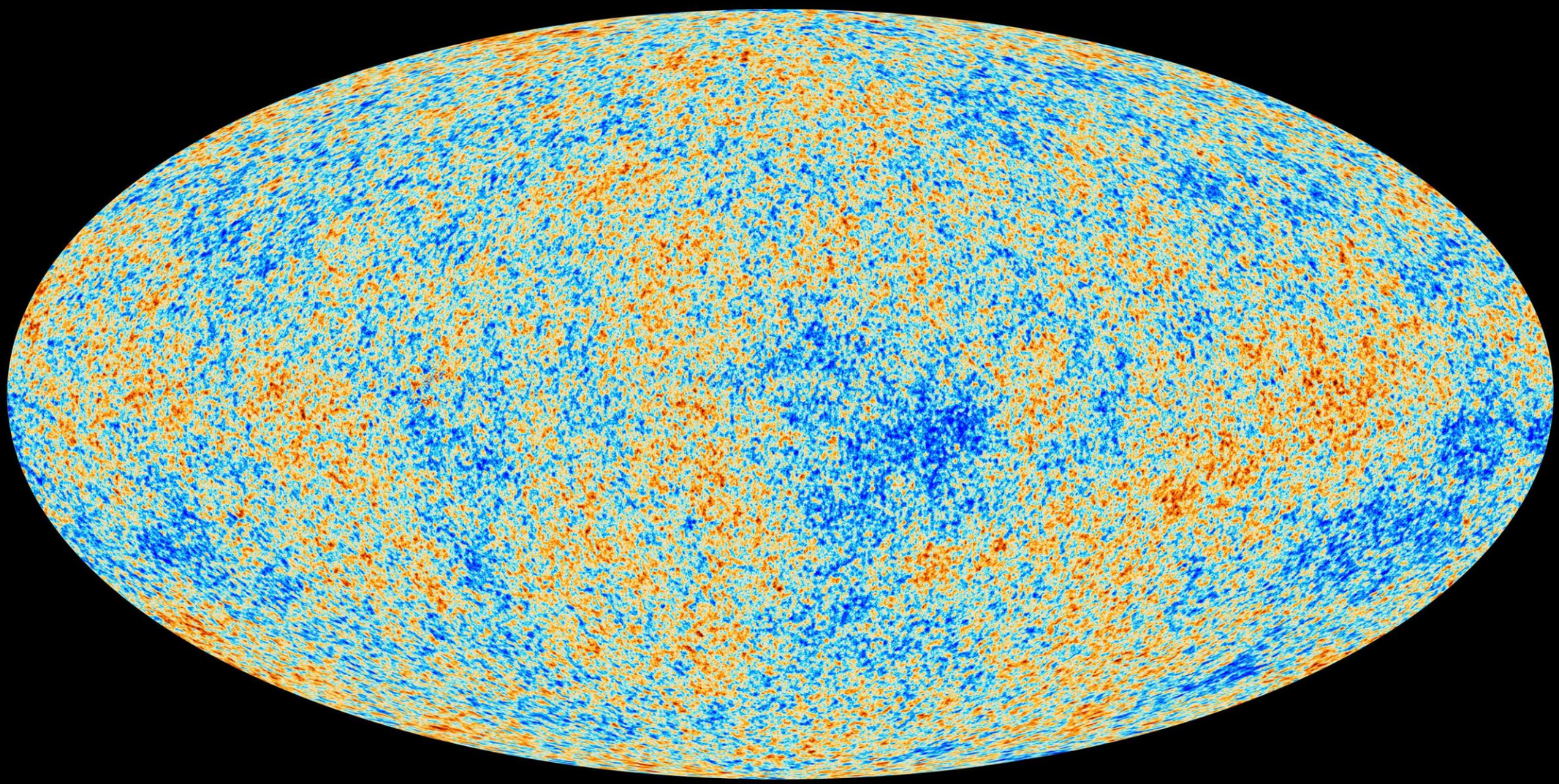
STRUCTURE FORMATION

NASA HUBBLE DEEP FIELD

Where did all the galaxies come from?

Gravitational instability: matter is attracted to small initial inhomogeneities, which are amplified as matter accretes

Universe was initially much more homogeneous: (CMB temperature the same in all directions to 1 part in 100,000)



PLANCK CMB MAP

Galaxies eventually arise and their distribution is measured

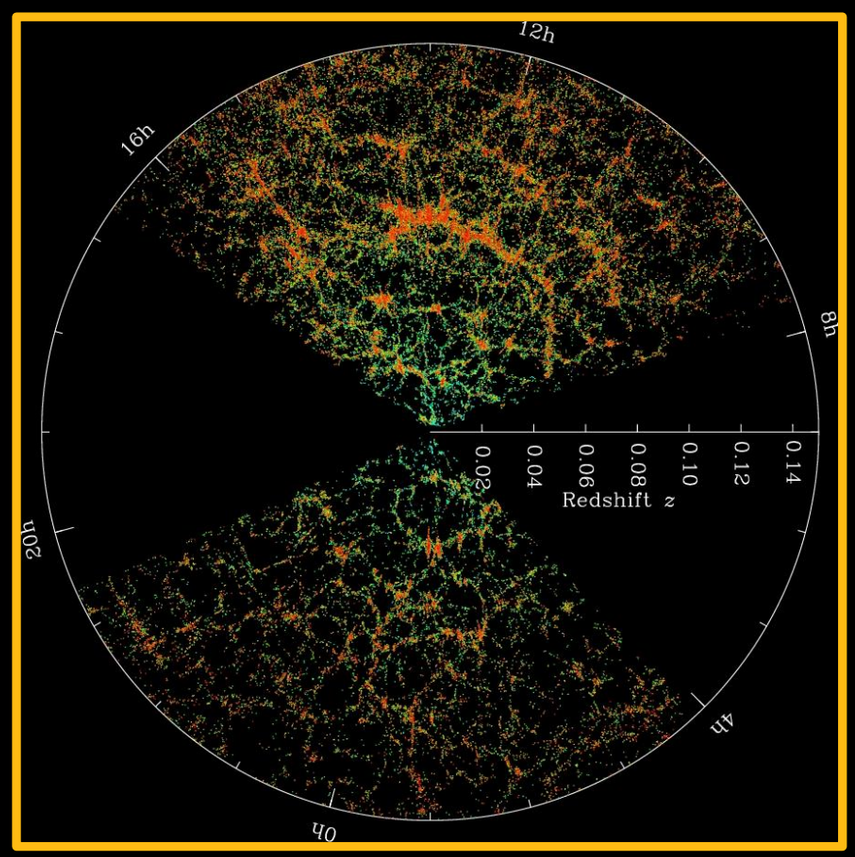
Distribution is characterized by its statistical properties

$$\delta(\mathbf{r}) = \delta\rho(\mathbf{r})/\langle\rho\rangle = \int \mathrm{d}^3k\, \delta_k\, e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\langle\delta(\mathbf{r})\,\delta(\mathbf{r}')\rangle = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\, P_\rho(k)\, e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}$$

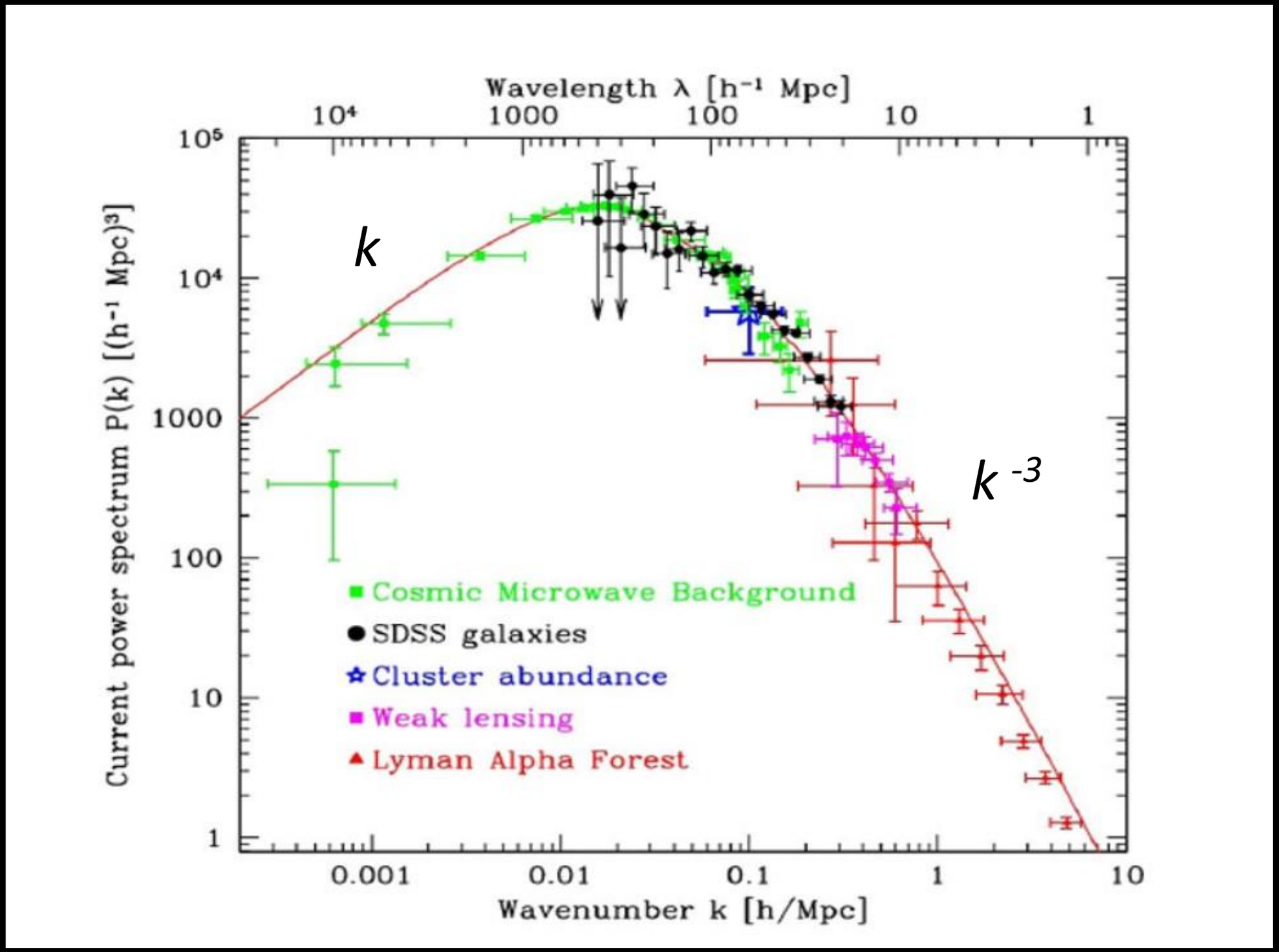
Power spectrum:

$$P_\rho(k) = \langle|\delta_k|^2\rangle$$



SLOAN DIGITAL SKY SURVEY

Power spectrum is measured for wide range of k



SLOAN DIGITAL SKY SURVEY

Approximate phenomenological parameterization

$$P_\rho(k) \simeq Ak^{n_s}/(1+\alpha k+\beta k^2)^2$$

$$n_s \simeq 0.97 \qquad A \simeq 3\times 10^{-10}$$

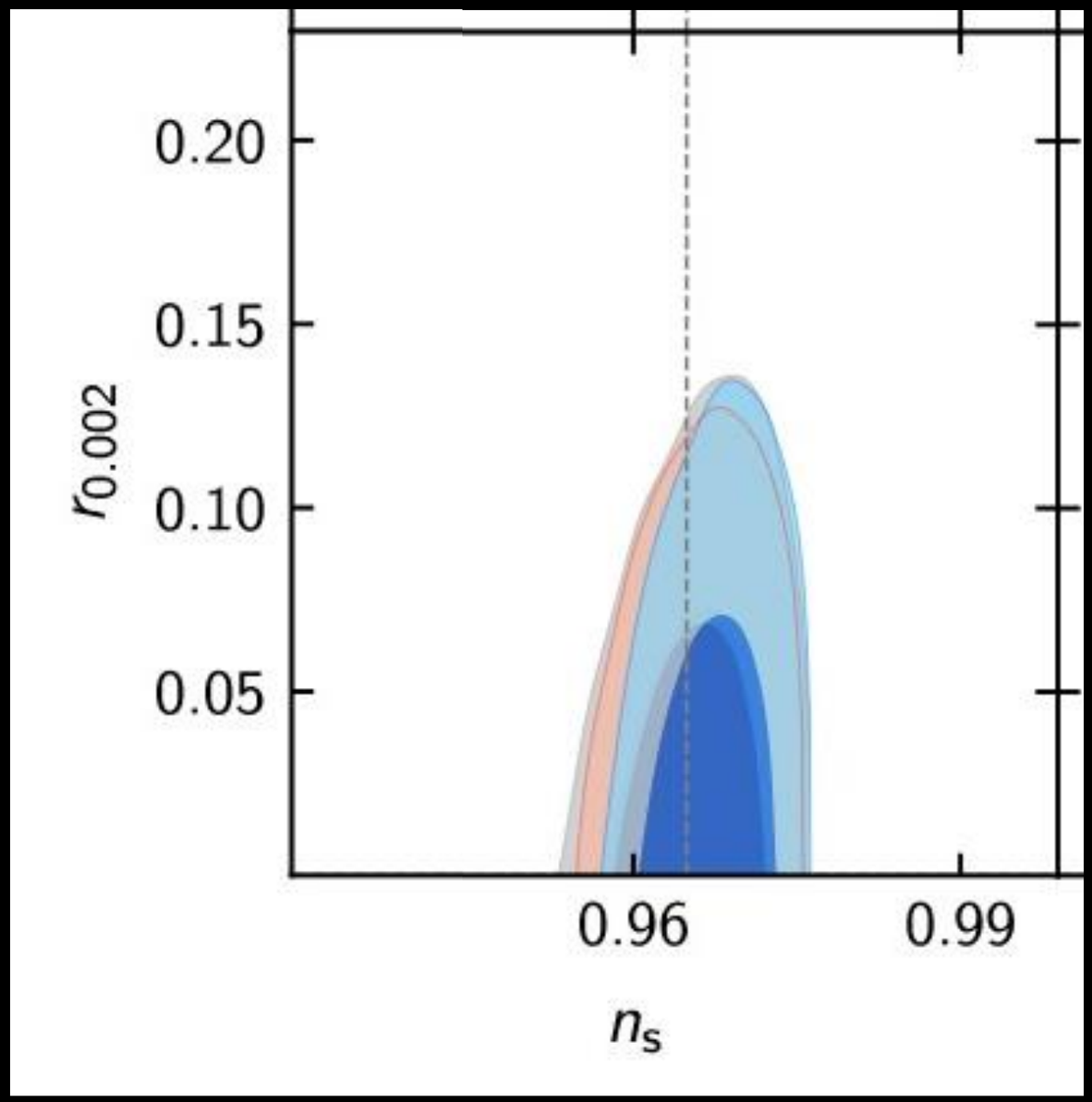
$$\alpha \simeq 16\,\mathrm{Mpc} \qquad \beta \simeq 19\,\mathrm{Mpc}^2$$

Primordial in origin

SLOAN DIGITAL SKY SURVEY

Accuracy of the determination of n_s

$$n_s = 0.9649 \pm 0.0042 \quad \text{(Planck)}$$



Planck Collaboration

Approximate phenomenological parameterization

$$P_\rho(k) \simeq Ak^{n_s}/(1+\alpha k+\beta k^2)^2$$

$$n_s \simeq 0.97 \qquad A \simeq 3\times 10^{-10}$$

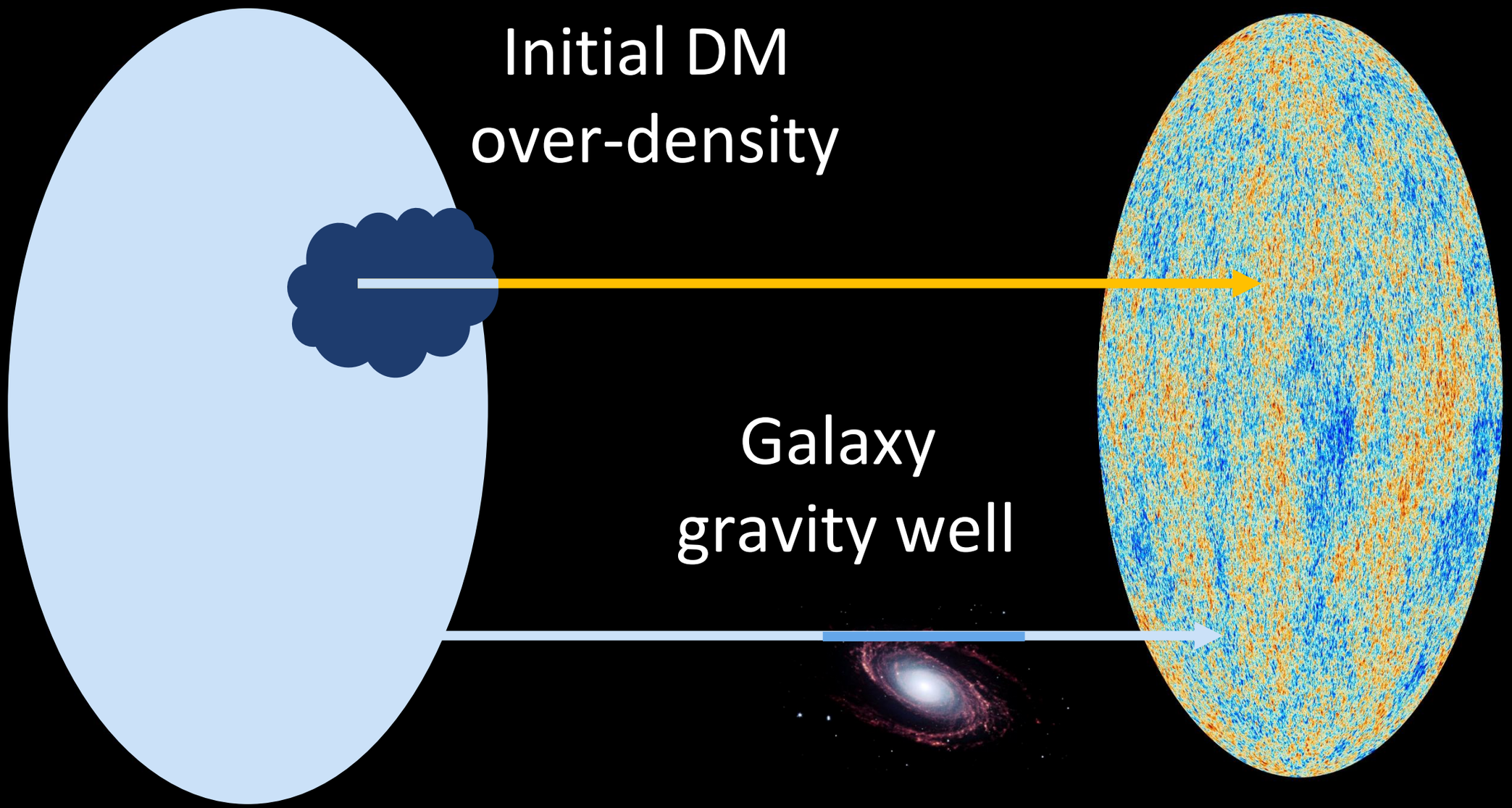
$$\alpha \simeq 16\,\mathrm{Mpc} \qquad \beta \simeq 19\,\mathrm{Mpc}^2$$

Primordial in origin

Due to evolution in the later universe

$P_\rho(k)$ is probed in the CMB, since temperature fluctuations related to redshift of light climbing out of gravitational potential wells (*Sachs-Wolfe effect*).

Relates δT to Newtonian potential $\Phi_k = \delta_k/k^2$



13directioner.deviantart.com

Can perturbatively evolve initially small density fluctuations

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + T_{\mu\nu}/M_p^2 = 0$$

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + \delta g_{\mu\nu}$$

$$T_{\mu\nu} = \overline{T}_{\mu\nu} + \delta T_{\mu\nu}$$

FLRW solution

Can perturbatively evolve initially small density fluctuations

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + T_{\mu\nu}/M_p^2 = 0$$

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + \delta g_{\mu\nu}$$

$$T_{\mu\nu} = \overline{T}_{\mu\nu} + \delta T_{\mu\nu}$$

Considerable freedom to perform gauge transformations

$$\delta g_{\mu\nu} \to \delta g_{\mu\nu} + \overline{\nabla}_\mu \omega_\nu + \overline{\nabla}_\nu \omega_\mu$$

Classify fluctuations using rotational properties

$$\delta_S g_{\mu\nu} = a^2 \begin{pmatrix} 2\phi & \partial_j \mathcal{B} \\ \partial_i \mathcal{B} & 2\psi\,\delta_{ij} + \partial_i \partial_j \mathcal{E} \end{pmatrix}$$

$$\delta_V g_{\mu\nu} = a^2 \begin{pmatrix} 0 & \mathcal{V}_j \\ \mathcal{V}_i & \partial_i \mathcal{W}_j + \partial_j \mathcal{W}_i \end{pmatrix}$$

$$\delta_T g_{\mu\nu} = a^2 \begin{pmatrix} 0 & 0 \\ 0 & h_{ij} \end{pmatrix}$$

where

$$\partial^i \mathcal{V}_i = \partial^i \mathcal{W}_i = \partial^i h_{ij} = \delta^{ij} h_{ij} = 0$$

The following are gauge invariant at linear order

$$\Phi = \phi - \frac{1}{a}\left[a(\mathcal{B} - \mathcal{E}')\right]'$$

$$\Psi = \psi + \frac{a'}{a}(\mathcal{B} - \mathcal{E}')$$

$$V_i = \mathcal{V}_i - \mathcal{W}_i \qquad \text{and} \qquad h_{ij}$$

Can use gauge transformations to set $\mathcal{B} = \mathcal{E} = \mathcal{W}_i = 0$
in which case $\Phi = \phi\,, \quad \Psi = \psi\,, \quad V_i = \mathcal{V}_i$

V_i , Ψ , Φ are constrained variables; h_{ij} describes grav. waves

The following are gauge invariant at linear order

$$\Phi = \phi - \frac{1}{a}\left[a(\mathcal{B} - \mathcal{E}')\right]'$$

$$\Psi = \psi + \frac{a'}{a}(\mathcal{B} - \mathcal{E}')$$

$$V_i = \mathcal{V}_i - \mathcal{W}_i \qquad \text{and} \qquad h_{ij}$$

Can use gauge transformations to set $\mathcal{B} = \mathcal{E} = \mathcal{W}_i = 0$
in which case $\Phi = \phi\,, \quad \Psi = \psi\,, \quad V_i = \mathcal{V}_i$

V_i and $\Psi - \Phi$ decay away quickly if $\delta T^0{}_i = \partial_i f$

$$\delta T^i{}_j = \delta p\, \delta^i_j$$

Tensor fluctuation describes gravitational waves and satisfies

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + (k^2 h_{ij}/a^2) = 0$$

Scalar fluctuation contains Newtonian potential Φ and satisfies a constraint (Poisson's equation in the nonrelativistic limit)

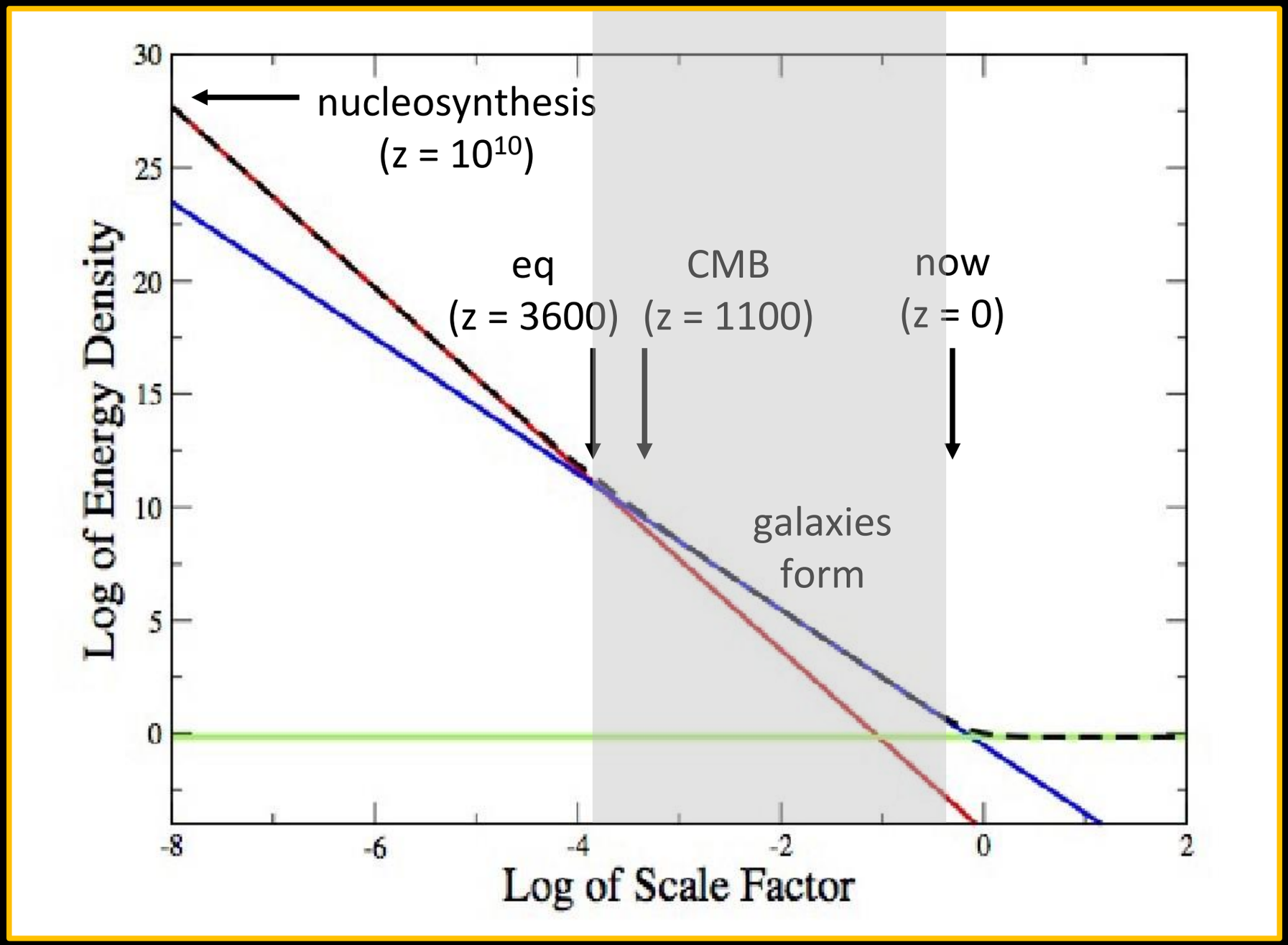
$$\nabla^2\phi - 4\pi G\rho = 0 \qquad \text{Poisson eq}$$

$$\partial_t\rho + \nabla\cdot(\rho\mathbf{v}) = 0 \qquad \text{energy cons}$$

$$\rho\left[\partial_t\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] + \nabla p + \rho\nabla\phi = 0 \qquad \text{mom. cons}$$

$$\partial_t s + \nabla\cdot(s\mathbf{v}) = 0 \qquad \text{entropy cons}$$

Nonrelativistic limit turns out to be the relevant one because gravity mainly amplifies fluctuations when matter dominates



Uniformly expanding cold ($s_0 = 0$) fluid

$$\mathbf{v}_0 = H(t)\,\mathbf{r}$$

satisfies the nonrelativistic field equations provided

$$\rho_0 \propto a^{-3}$$ energy cons

$$\nabla\phi_0 = (\rho_0/6M_p^2)\,\mathbf{r}$$ Poisson eq

and

$$H(t) = 2/(3t)$$ mom. cons

as would be expected if $H(t) = \dot{a}/a$

for matter-dominated expansion $a = a_0(t/t_0)^{2/3}$

Can now linearize field equations around expanding background

$$\mathbf{v} = \mathbf{v}_0 + \delta\mathbf{v} \qquad \rho = \rho_0 + \delta\rho \qquad s = \delta s$$
$$p = p_0 + \delta p \qquad \phi = \phi_0 + \delta\phi$$

Leading to the following fluctuation equations: $D_t\, \delta\, s = 0$ and

$$\nabla^2\delta\phi - \delta\rho/(6M_p^2) = 0 \qquad \text{Poisson eq}$$

$$D_t\,\delta\rho + 3H\,\delta\rho + \rho_0\nabla\cdot\delta\mathbf{v} = 0 \qquad \text{energy cons}$$

$$\rho_0(D_t\,\delta\mathbf{v} + H\,\delta\mathbf{v}) + \nabla\delta p + \rho_0\nabla\delta\phi = 0 \qquad \text{mom. cons}$$

where $\qquad D_t = \partial_t + \mathbf{v}_0\cdot\nabla$

Eliminating v between energy and momentum conservation eqs:

$$D_t^2\delta + 2H\, D_t\delta - c_s^2\,\nabla^2\delta - 4\pi G\rho_0\,\delta = \xi\,\delta s/\rho_0$$

where

$$\delta = \delta\rho/\rho_0\,, \quad c_s^2 = (\partial p/\partial\rho)_{s0}\,, \quad \xi = (\partial p/\partial s)_{\rho 0}$$

In momentum space this becomes

$$\ddot{\delta}_k + 2H\,\dot{\delta}_k + \left[(c_s^2\,k^2/a^2) - 4\pi G\rho_0\right]\delta_k = \xi\,\delta s_k/\rho_0$$

Solutions to this equation describe density fluctuation evolution

$$\ddot{\delta}_k + 2H\,\dot{\delta}_k + \left[(c_s^2\,k^2/a^2) - 4\pi G\rho_0\right]\delta_k = \xi\,\delta s_k/\rho_0$$

eg for short wavelengths relative to Jeans length

$$(k/a)^2 \gg p_J^2 := 4\pi G\rho_0/c_s^2 = 3H^2/(2c_s^2)$$

solutions describe damped oscillations (sound waves)

$$\delta_k(t) = a^{-1/2}\exp\left[\pm ikc_s\int^t \mathrm{d}t'/a(t')\right]$$

Jeans instability: density fluctuation equation

$$\ddot{\delta}_k + 2H\,\dot{\delta}_k + \left[(c_s^2\,k^2/a^2) - 4\pi G\rho_0\right]\delta_k = \xi\,\delta s_k/\rho_0$$

has unstable runaway solutions if

$$(k/a)^2 < p_J^2 := 4\pi G\rho_0/c_s^2 = 3H^2/(2c_s^2)$$

NR derivation cannot be trusted if p_J is order H (ie if c_s not small)

Approximate solutions when k << p_J

$$\delta \propto t^{2/3} \propto a \qquad \text{and} \qquad \delta \propto t^{-1} \propto a^{-3/2}$$

Jeans instability: density fluctuation equation

$$\ddot{\delta}_k + 2H\,\dot{\delta}_k + \left[(c_s^2\,k^2/a^2) - 4\pi G\rho_0\right]\delta_k = \xi\,\delta s_k/\rho_0$$

has unstable runaway solutions if

$$(k/a)^2 < p_J^2 := 4\pi G\rho_0/c_s^2 = 3H^2/(2c_s^2)$$

This equation also applies for nonrelativistic matter during radiation domination if *H(a)* is proportional to a^{-2} rather than $a^{-3/2}$. Approximate solutions when $k \ll p_J$ then grow less dramatically

$$\delta \propto \ln a \quad \text{and} \quad \delta \propto a^0$$

Jeans instability summary:

Instability only occurs if all three of the following hold:

1. Mode is sub-Hubble: $(k/a) \ll H$

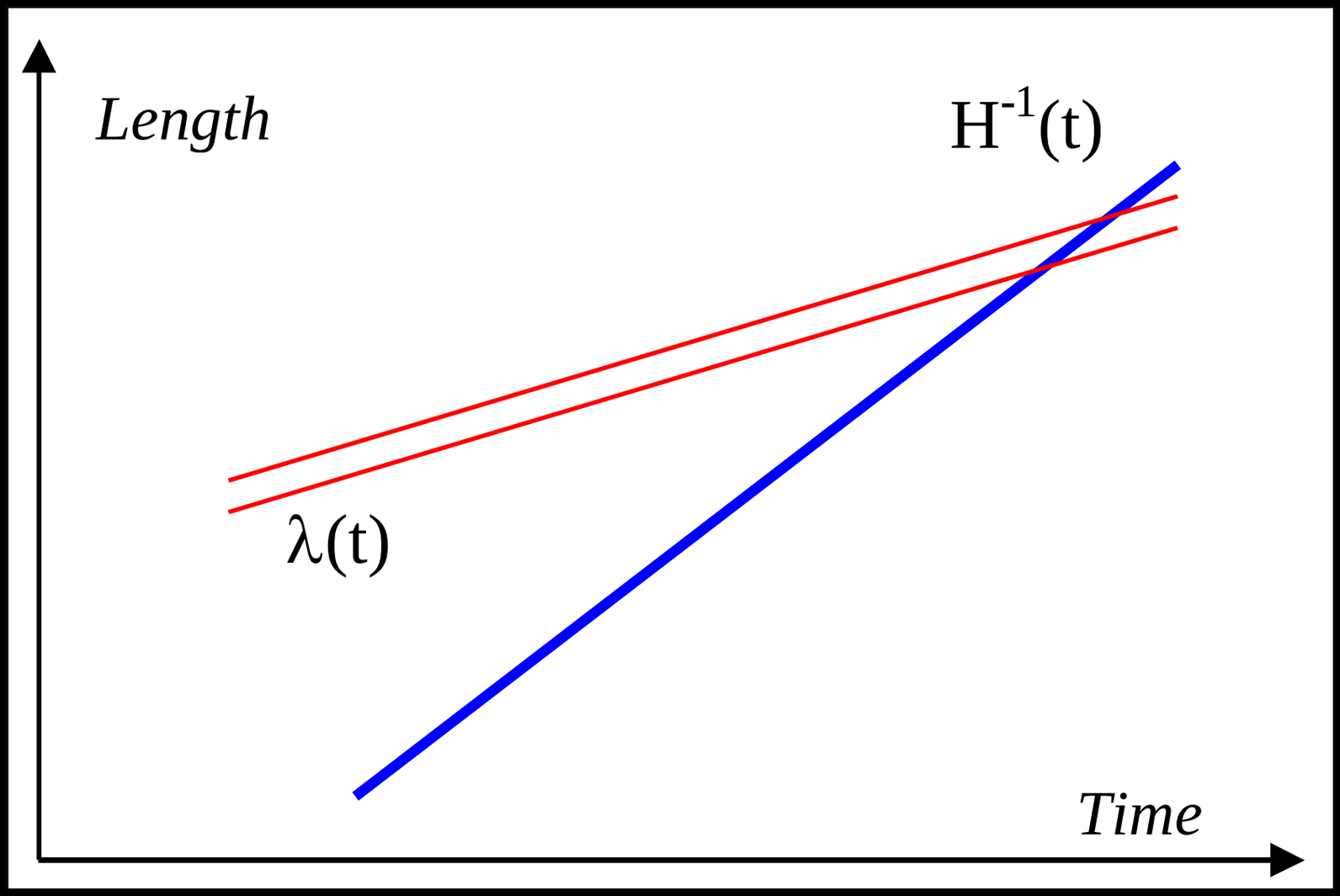
2. Mode is in nonrelativistic fluid

3. Universe is matter dominated

Unstable modes grow proportional to scale factor: a

These properties suffice to explain shape of power spectrum

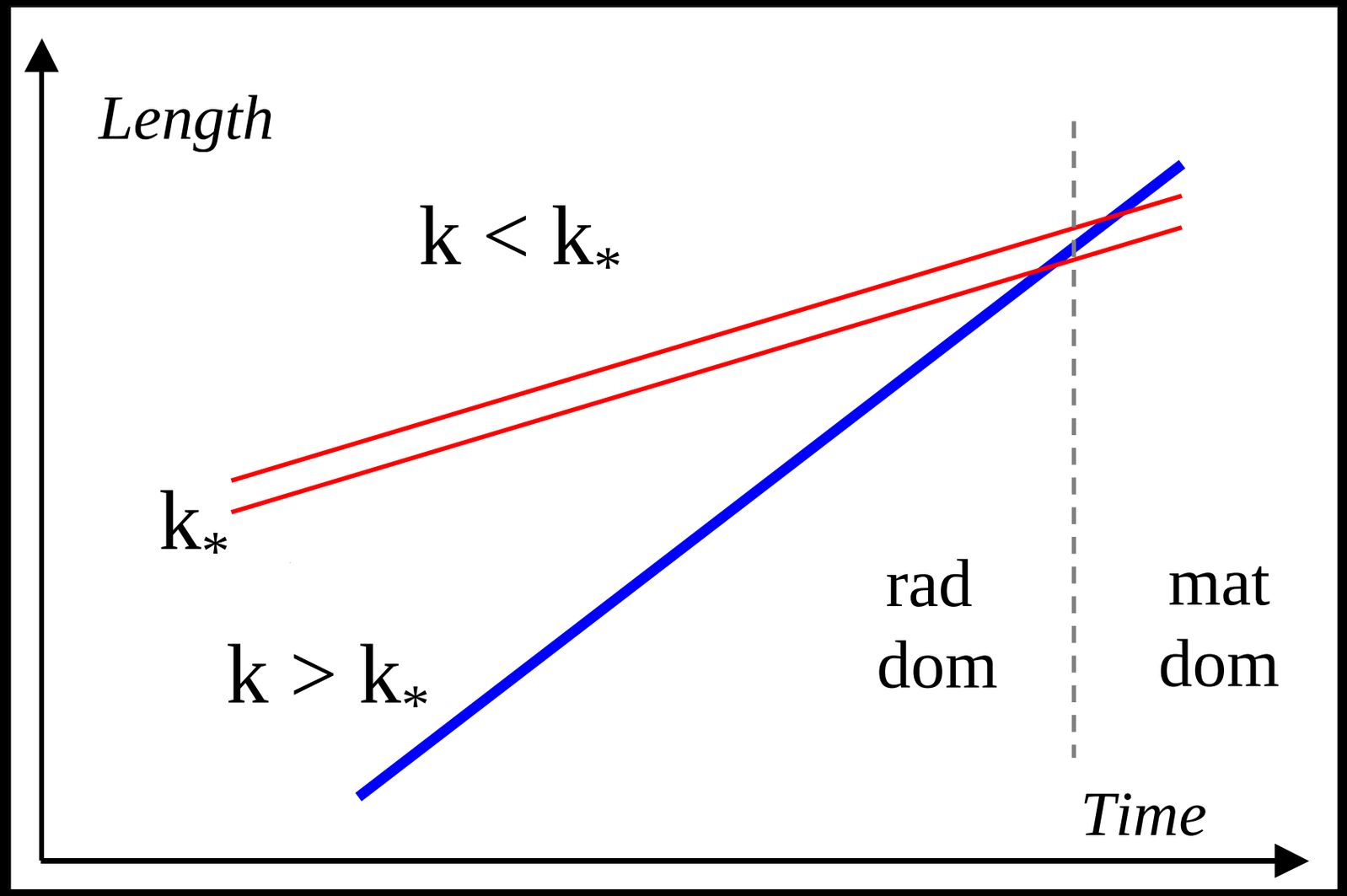
During radiation and matter domination *a* grows more slowly than does Hubble length, so modes transit from super-Hubble to sub-Hubble. Super-Hubble modes are overdamped and frozen so cannot become unstable until after this happens.



Mode becomes sub-Hubble when $k/a = H$. Suppose k_* is mode that becomes sub-Hubble at radiation/matter equality.

Modes with $k < k_*$ become sub-Hubble during matter domination and so immediately start to grow proportional to a.

Modes with $k > k_*$ become sub-Hubble during rad dom and so are stunted because they must wait for mat dom before they can grow.



How stunted are the large-k modes?

Instability is linear in *a*. Compare modes with $k > k_* > k_0$

$$\frac{\delta_k(a)}{\delta_{k_0}(a)} \propto \frac{\delta_k(a_k)(a/a_{\rm eq})}{\delta_{k_0}(a_{k_0})(a/a_{k_0})} \propto \frac{\delta_k(a_k)(a/a_k)}{\delta_{k_0}(a_{k_0})(a/a_{k_0})} \left(\frac{a_k}{a_{\rm eq}}\right)$$

but $a_k/a_{\rm eq} = (k_{\rm eq}/k)^2$

because $k/a_k = H_k$ implies $k = a_k H_k \propto a_k^{-1/2}$

since $H \propto t^{-1} \propto a^{-3/2}$ during matter domination

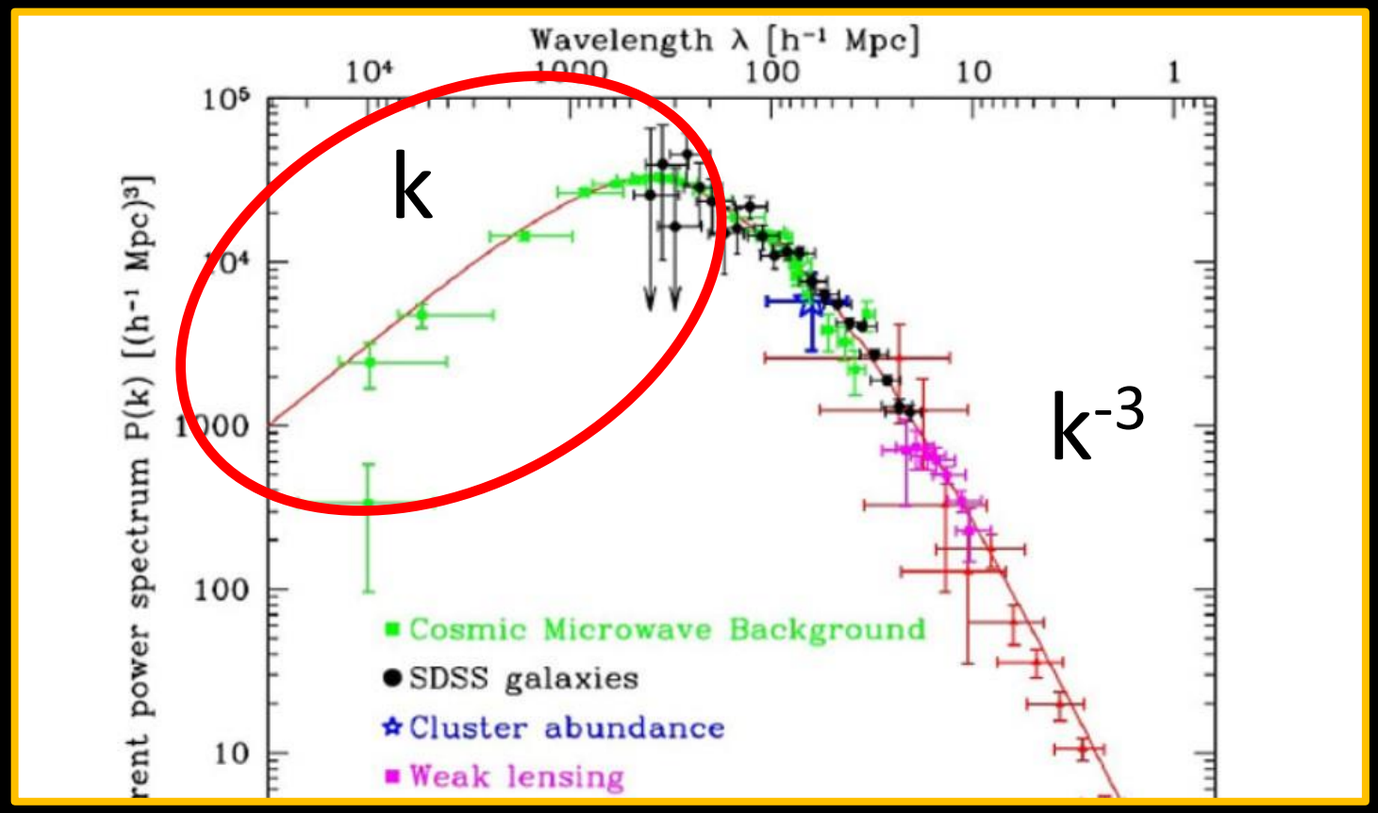
Therefore $$\frac{\delta_k(a)}{\delta_{k_0}(a)} \propto \frac{\delta_k(a_k)(a/a_k)}{\delta_{k_0}(a_{k_0})(a/a_{k_0})} \left(\frac{k_{\rm eq}}{k}\right)^2$$

But power spectrum is quadratic in δ and so $k > k_* > k_0$ is suppressed relative to k_0 by $(k_*/k)^4$

$$P_\rho(k) = \langle |\delta_k|^2 \rangle$$

What determines the primordial form?

$$P_\rho^{(0)}(k) \simeq A k^{n_s}$$



SLOAN DIGITAL SKY SURVEY

Primordial spectrum nearly scale invariant

$$P_\rho^{(0)}(k) \simeq A k^{n_s} \quad \text{with} \quad n_s \simeq 0.97$$

Scale invariance measured by k-independence of Δ rather than P

$$\langle \delta(\mathbf{r})\delta(\mathbf{0})\rangle = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,\langle|\delta_k|^2\rangle\, e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$= \frac{1}{2\pi^2}\int_0^\infty \frac{\mathrm{d}k}{k}\, k^3\, P_\rho\,(k)\left(\frac{\sin kr}{kr}\right)$$

suggests $\quad \Delta_\rho(k) := k^3\, P_\Phi\,(k)/(2\pi^2)$

Primordial spectrum nearly scale invariant

$$P_\rho^{(0)}(k) \simeq A k^{n_s} \quad \text{with} \quad n_s \simeq 0.97$$

Also: interest is usually in correlations of the Newtonian potential rather than the density (eg for CMB)

$$k^2 \Phi_k \propto \delta_k \quad \text{so} \quad \Delta_\Phi(k) \propto \Delta_\rho(k)/k^4 \propto P_\rho(k)/k$$

has primordial form $\Delta_\Phi^{(0)}(k) \simeq \tilde{A}\, k^{n_s - 1}$

COSMOLOGY

PRIMORDIAL FLUCTUATIONS

NASA HUBBLE DEEP FIELD

COSMOLOGICAL INITIAL CONDITIONS

Hot Big Bang Cosmology *assumes specific initial conditions*

Nearly scale-invariant spectrum of primordial fluctuations.

Very uniform CMB temperature. (Horizon Problem)

Very flat spatial slices. (Flatness Problem)

Could these have a common dynamical explanation?

YES!! An early accelerated expansion does all three

Easier to see the problems using conformal time: $\mathrm{d}t = a\,\mathrm{d}\tau$

$$\begin{aligned}\mathrm{d}s^2 &= -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}\cdot\mathrm{d}\vec{x} \\ &= a^2(\tau)\left[-\mathrm{d}\tau^2 + \mathrm{d}\vec{x}\cdot\mathrm{d}\vec{x}\right]\end{aligned}$$

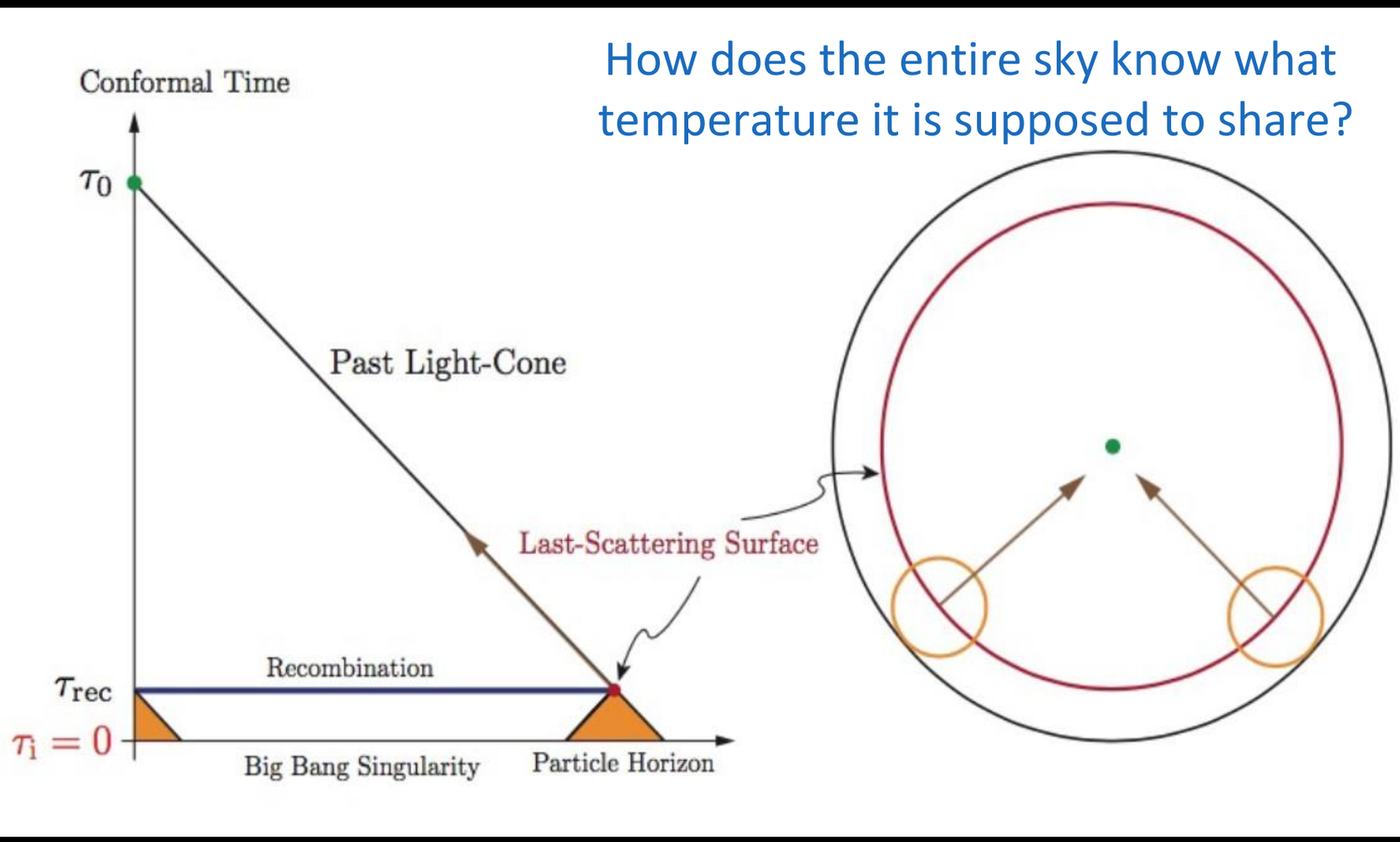
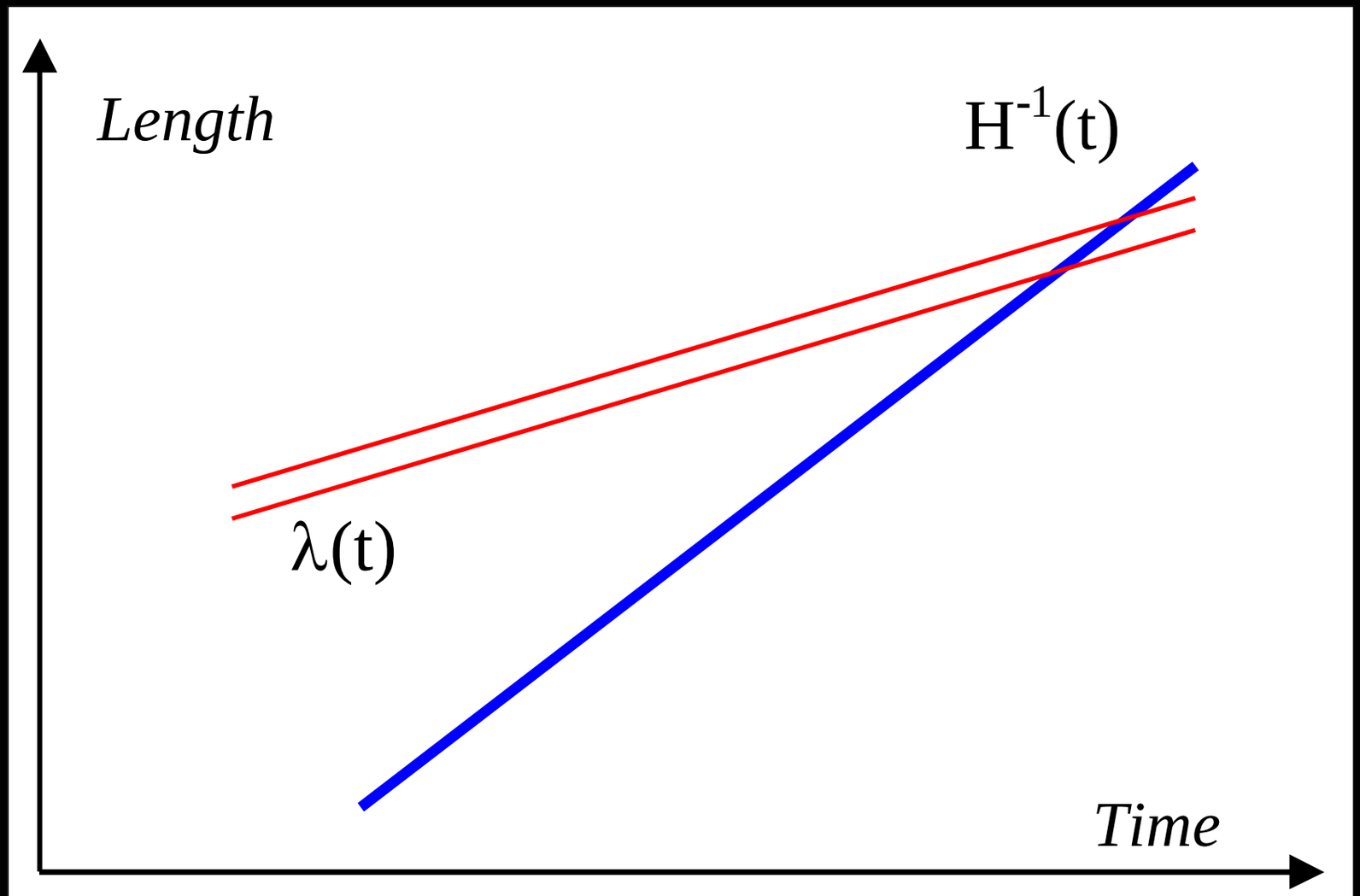


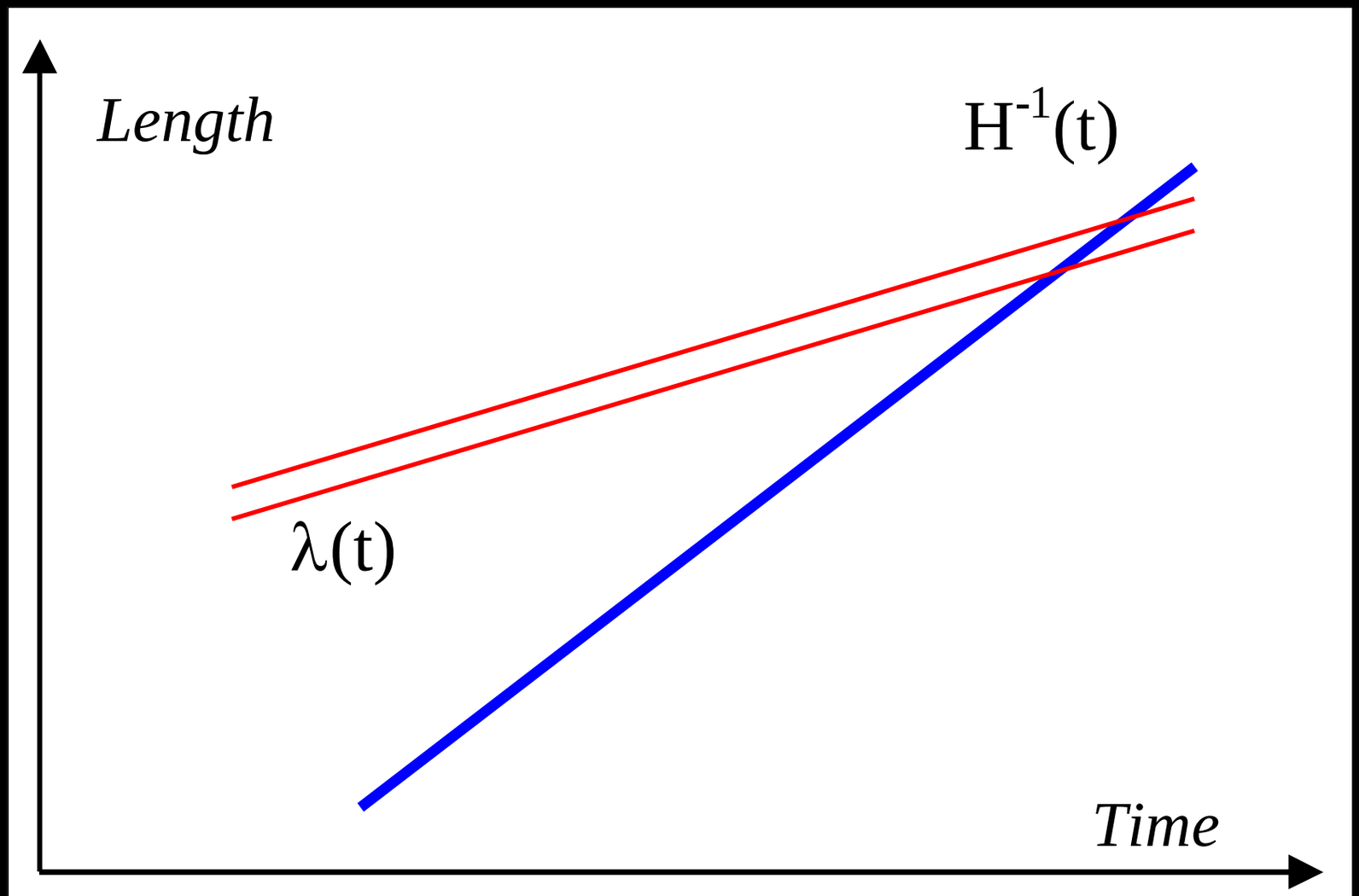
FIGURE: D BAUMANN

It is super-Hubble freezing that makes initial correlations hard to understand



Changing extrapolation to the past can remove the problem:
must change the relative growth of a(t) and $d_H(t)$

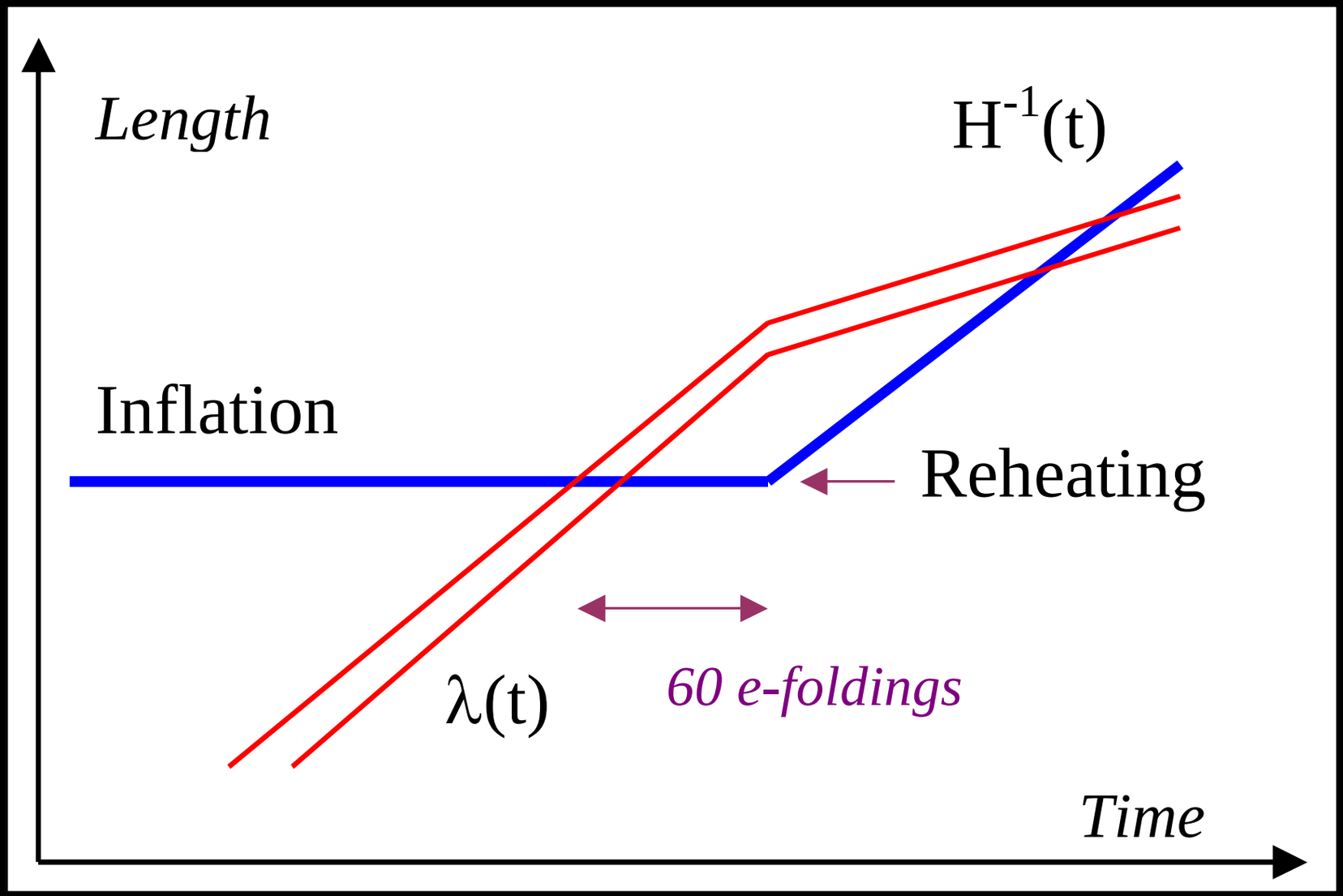
$$a(t)/d_H(t) = a\,H = \dot{a}$$ *should grow rather than shrink*



Inflation: vacuum e.o.s. can provide accelerated expansion: $p = -\rho$

$$H = \text{constant}\,, \quad a(t) = e^{Ht}$$

Require: *H t > 60 e*-foldings to fix Big Bang initial conditions



Guth

How to *temporarily* engineer vacuum equation of state?

Linde
Albrecht
Steinhardt

Introduce new degrees of freedom - usually scalar(s)

$$\mathcal{L} = -\sqrt{-g}\left[\tfrac{1}{2}\,\mathcal{G}_{ab}(\chi)\,\partial^\mu\chi^a\,\partial_\mu\chi^b + V(\chi)\right]$$

For homogeneous roll χ(t)

$$\rho = \tfrac{1}{2}\,\mathcal{G}_{ab}(\chi)\,\dot{\chi}^a\,\dot{\chi}^b + V(\phi)$$

$$p = \tfrac{1}{2}\,\mathcal{G}_{ab}(\chi)\,\dot{\chi}^a\,\dot{\chi}^b - V(\chi)$$

$$p \simeq -\rho < -\tfrac{1}{3}\,\rho \qquad \text{if} \quad \tfrac{1}{2}\,\mathcal{G}_{ab}(\phi)\,\dot{\phi}^a\,\dot{\phi}^b \ll V(\phi)$$

'slow roll'

Specialize to simplest case: single-field models

$$\mathcal{L} = -\sqrt{-g}\left[\tfrac{1}{2}(\partial\chi)^2 + V(\chi)\right]$$

Speed of scalar motion suppressed if V(χ) is very shallow.

How shallow? Slow-roll approximation:

$$0 = \ddot{\chi} + 3H\dot{\chi} + V'(\chi) \simeq 3H\dot{\chi} + V'(\chi)$$

$$3M_p^2 H^2 = \rho \simeq V \simeq \text{constant}$$

which neglects

$$\tfrac{1}{2}\dot{\chi}^2/V \simeq \tfrac{1}{6}M_p^2(V'/V)^2$$

$$\ddot{\chi}/V' \sim \tfrac{1}{3}M_p^2\left[(V''/V) - \tfrac{1}{2}(V'/V)^2\right]$$

Define slow roll parameters

$$\epsilon := \tfrac{1}{2} M_p^2 (V'/V)^2 \quad \text{and} \quad \eta := M_p^2 (V''/V)$$

so $\tfrac{1}{2}\dot{\chi}^2/V \simeq \epsilon/3$ and $\ddot{\chi}/V' \sim (\eta - \epsilon)/3$

are negligible if $\epsilon\,, |\eta| \ll 1$

Alternative definitions often defined directly in terms of variation of *H* and ε over Hubble time:

$$\epsilon_1 := -\dot{H}/H^2 \simeq \tfrac{1}{2} M_p^2 (V'/V)^2 \simeq \epsilon$$

$$\epsilon_2 := \dot{\epsilon}_1/(H\epsilon_1) \simeq 4\epsilon - 2\eta$$

Inflation can be expected to occur in regions of field space for which slow roll parameters are both small. During slow roll can use χ (*a)* and *a(t)* to trade χ for *a* or *t*.

$$\mathrm{d}\chi/\mathrm{d}a = \dot{\chi}/\dot{a} = -V'/(3aH^2) = -M_p^2\,V'/(aV)$$

e.g. the number of *e*-foldings of inflation $a_{\rm end}/a_{\rm he} = e^{N_e}$ can be expressed directly in terms of field values

$$N_e = \int_{a_{\rm he}}^{a_{\rm end}} \frac{\mathrm{d}a}{a} = \int_{\chi_{\rm end}}^{\chi_{\rm he}} \mathrm{d}\chi \left(\frac{V}{M_p^2\,V'}\right) = \frac{1}{M_p}\int_{\chi_{\rm end}}^{\chi_{\rm he}} \frac{\mathrm{d}\chi}{\sqrt{2\epsilon}}$$

Simple examples $\boxed{V(\chi) = \frac{1}{2}\, m^2\, \chi^2}$

$$\epsilon = \eta = 2(M_p/\chi)^2 \ll 1 \quad \text{if} \quad \chi \gg M_p$$

Starting at $\chi_{\rm he}$ leads to the following number of *e*-foldings

$$N_e = \int_{\chi_{\rm end}}^{\chi_{\rm he}} {\rm d}\chi \left(\frac{\chi}{2M_p^2}\right) = \frac{1}{4M_p^2}\left(\chi_{\rm he}^2 - \chi_{\rm end}^2\right)$$

where if inflation ends once ε = ½ then $\chi_{\rm end} = 2M_p$

Sensible EFT understanding will require $V(\chi_{\rm he}) \ll M_p^4$
and so $2(N_e+1)m^2 \ll M_p^2$

$\epsilon \simeq \eta$ starting to be disfavoured by observations

Goldstone boson I:

$$V(\chi) = V_0 + 2\Lambda^4 \sin^2(\chi/2f)$$

$$\epsilon = \frac{M_p^2}{2f^2}\cot^2\left(\frac{\chi}{2f}\right) \quad \text{and} \quad \eta = \epsilon - \frac{M_p^2}{2f^2}$$

Slow roll prefers $f >> M_p$. Agrees with quadratic V if $\chi << f$

Starting at χ_{he} leads to the following number of *e*-foldings

$$N_e = \frac{2f}{M_p^2}\int_{\chi_{\rm end}}^{\chi_{\rm he}} {\rm d}\chi\ \tan\left(\frac{\chi}{2f}\right) = \left(\frac{2f}{M_p}\right)^2 \ln\left|\frac{\sin(\chi_{\rm he}/2f)}{\sin(\chi_{\rm end}/2f)}\right|$$

Sensible EFT understanding requires $\Lambda \ll M_p$

$\epsilon \simeq \eta$ starting to be disfavoured by observations

Goldstone boson II:

$$V(\chi) = V_0 - V_1\, e^{-\chi/f}$$

$$\epsilon = \frac{1}{2}\left(\frac{M_p V_1}{f V_0}\right)^2 e^{-2\chi/f} \quad \text{and} \quad \eta = -\left(\frac{M_p^2 V_1}{f^2 V_0}\right) e^{-\chi/f}$$

Slow roll prefers $\chi >> f$.

Starting at χ_{he} leads to the following number of *e*-foldings

$$N_e = \left(\frac{f V_0}{M_p^2 V_1}\right) \int_{\chi_{\rm end}}^{\chi_{\rm he}} {\rm d}\chi\, e^{\chi/f} = \left(\frac{f^2 V_0}{M_p^2 V_1}\right) \left[e^{\chi_{\rm he}/f} - e^{\chi_{\rm end}/f}\right]$$

Sensible EFT understanding requires $V_0 \ll M_p^4$

Prediction $\epsilon = \frac{1}{2}\left(\frac{f}{M_p}\right)^2 \eta^2 \ll \eta$ observationally successful

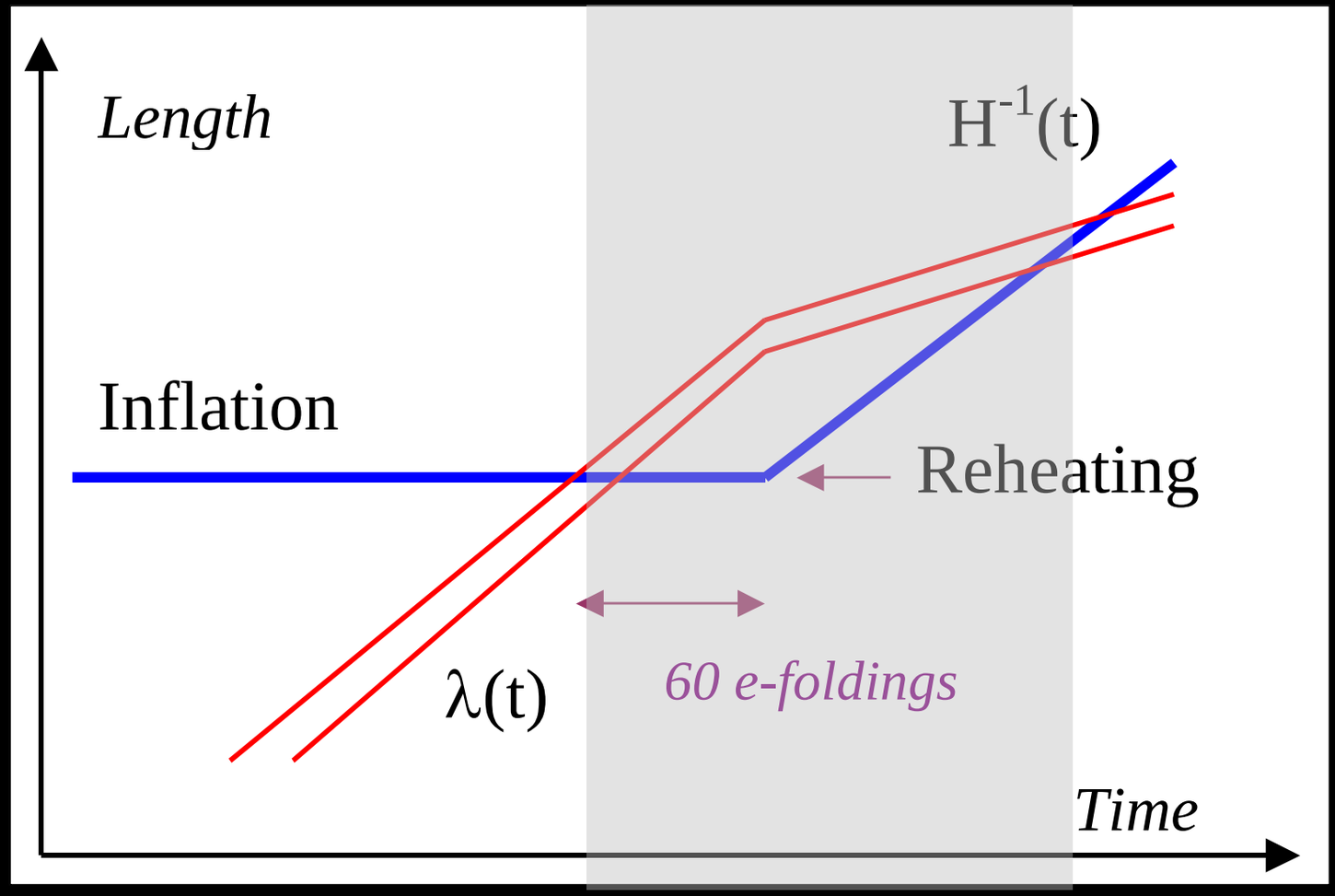
Other models share this form of potential (Higgs, Starobinsky,…)

PRIMORDIAL FLUCTUATIONS

Inflation and primordial fluctuations? *YES!*

Cartoon: two main epochs to follow

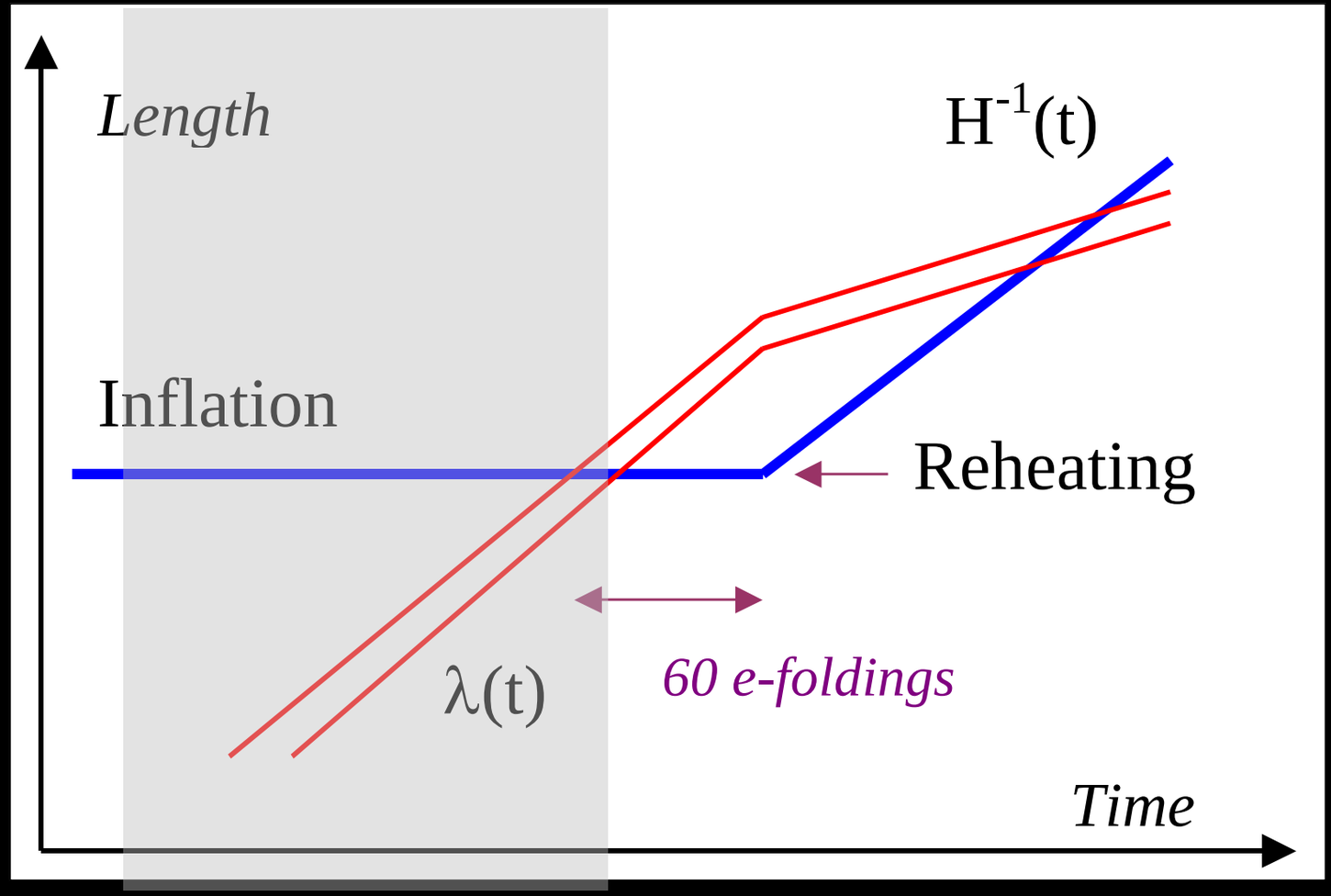
1. super-Hubble epoch



Inflation and primordial fluctuations? *YES!*

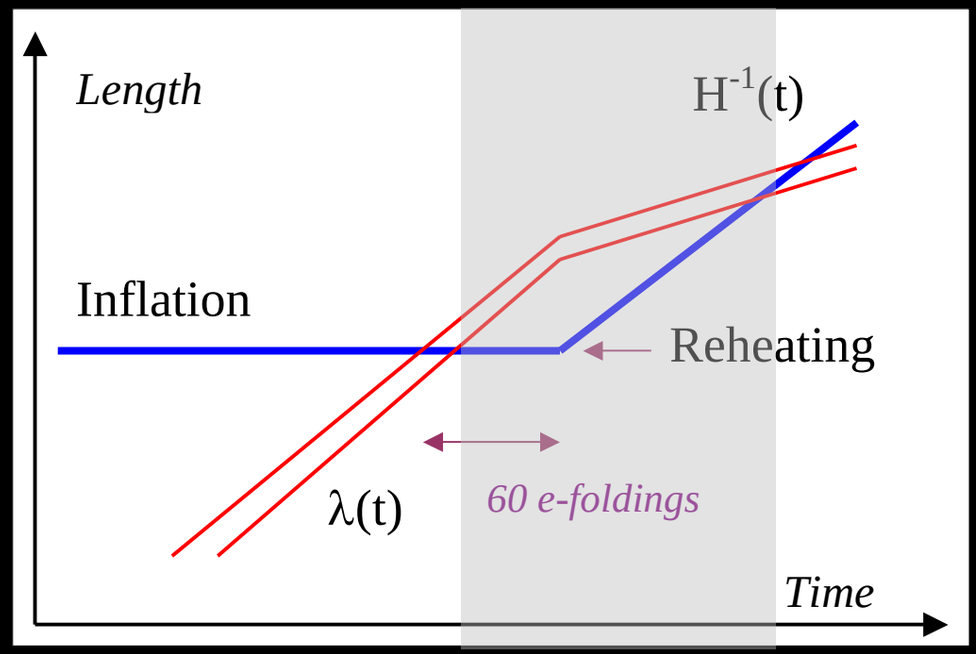
Cartoon: two main epochs to follow

1. super-Hubble epoch
2. pre horizon-exit



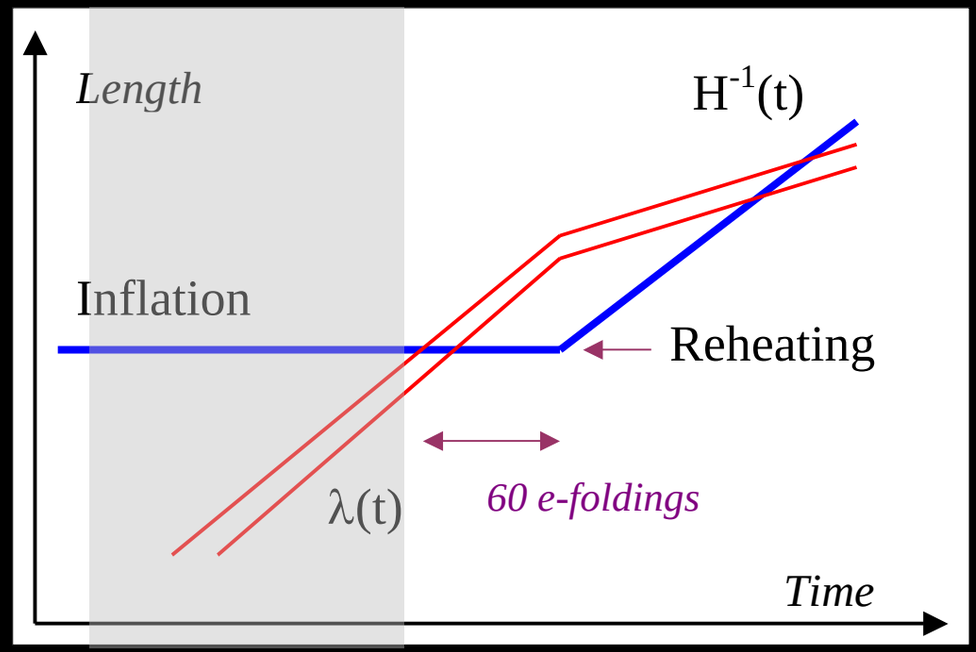
Super-Hubble epoch

For single-scalar models one function ζ of the scalar metric perturbation Φ is time-independent for super-Hubble scales $k/a \ll H$



$$\zeta = \Phi + \frac{2}{3}\left(\frac{\Phi + \dot{\Phi}/H}{1+w}\right)$$

Need only compute Φ at horizon exit to know it also at horizon re-entry



Pre horizon-exit epoch

Metric fluctuation Φ is again constrained in terms of matter density (in this case χ during inflation)

$$\chi = \overline{\chi} + \delta\chi$$

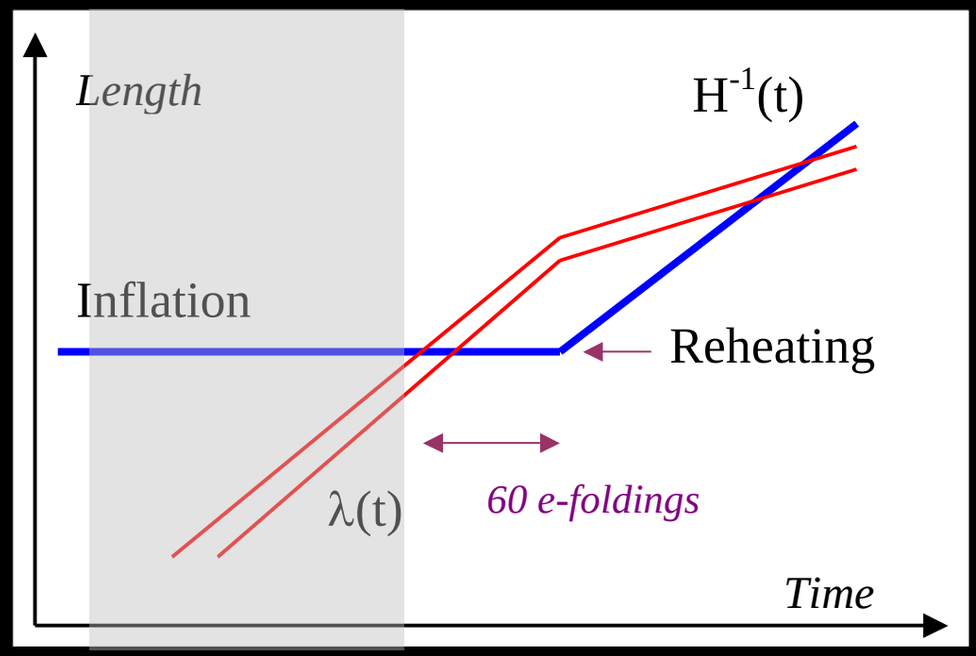
$$\delta\ddot{\chi} + 3H\delta\dot{\chi} + \frac{k^2}{a^2}\delta\chi + V''(\overline{\chi})\delta\chi - 4\dot{\overline{\chi}}\,\dot{\Phi} + 2V'(\overline{\chi})\,\Phi = 0$$

$$\dot{\Phi} + H\,\Phi = \frac{\dot{\overline{\chi}}}{2M_p^2}\,\delta\chi$$

Can compute $\delta\chi$ and Φ at horizon exit in terms of earlier values

Pre horizon-exit epoch

BUT inflation ruthlessly irons away any earlier classical sub-Hubble fluctuations (*k/a >> H*)



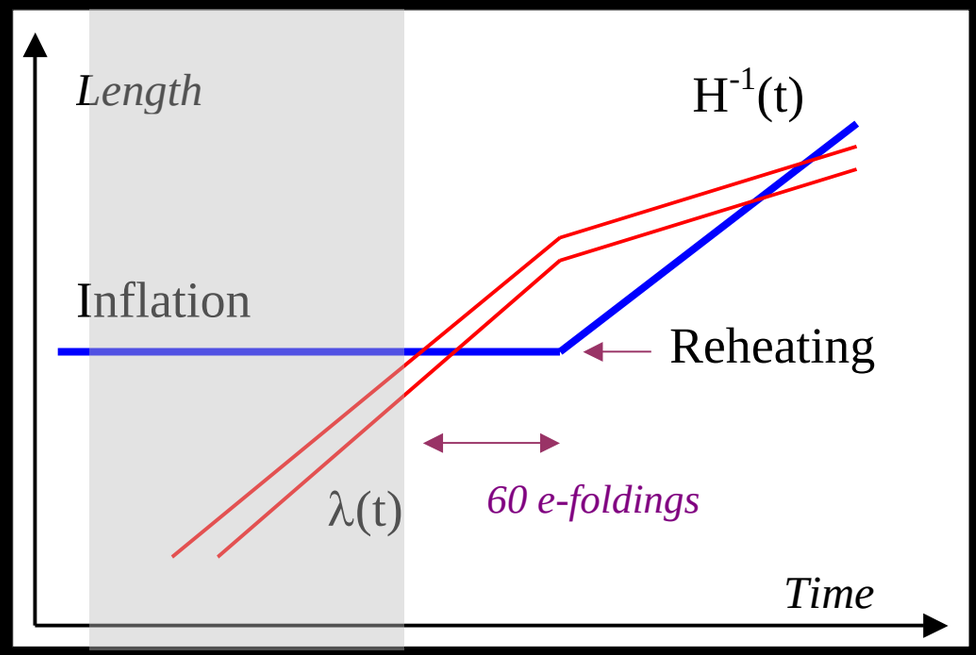
$$\delta\chi_k \propto \boxed{\frac{1}{a\sqrt{k}}} \exp\left[\pm ik \int^t \frac{\mathrm{d}t'}{a(t')}\right]$$

$$|\delta\chi_k|^2 \propto a^{-2} \propto e^{-2Ht} \to 0$$

If inflation runs for very long there are no classical fluctuations left to generate nonzero metric fluctuations Φ

Pre horizon-exit epoch

Quantum fluctuations of h_{ij} and $\delta\chi$ are not damped because these are set by the Hubble scale



$$\langle |\delta\chi_k|^2 \rangle \propto \left(\frac{H}{2\pi}\right)^2$$

$$\langle |h_{ij}(k)|^2 \rangle \propto \left(\frac{H}{2\pi}\right)^2$$

Mukhanov
Starobinsky
Hawking
Guth & Pi
Linde

Tensor fluctuations directly freeze on super-Hubble scales while χ fluctuations drive nonzero scalar metric component Φ

Predicted quantum primordial fluctuations

Both tensor and scalar fluctuations are predicted

$$\Delta_\Phi(k) = \left(\frac{H^2}{8\pi^2 M_p^2\,\epsilon}\right)_{\rm he} \qquad \textit{(density perturbations)}$$

$$\Delta_h(k) = \frac{8}{M_p^2}\left(\frac{H}{2\pi}\right)^2_{\rm he} \qquad \textit{(tensor perturbations)}$$

Quantum effects visible because classical prediction vanishes

Predicted quantum primordial fluctuations

Both tensor and scalar fluctuations are predicted

$$\Delta_\Phi(k) = \left(\frac{H^2}{8\pi^2 M_p^2\,\epsilon}\right)_{\rm he} \qquad \textit{(density perturbations)}$$

$$\Delta_h(k) = \frac{8}{M_p^2}\left(\frac{H}{2\pi}\right)^2_{\rm he} \qquad \textit{(tensor perturbations)}$$

Ratio depends on ε due to necessity of scalar fluctuations to mix with Φ

$$r := \Delta_h(k)/\Delta_\Phi = 16\,\epsilon$$

Fluctuations would be scale invariant if H were constant

Can compute spectral index because χ_{he} depends on *k*

$$n_s - 1 = \left. \frac{\mathrm{d} \ln \Delta^2_\Phi}{\mathrm{d} \ln k} \right|_{\mathrm{he}}$$

Evaluate derivatives using *k/a = H* to infer k-dependence

$$\frac{\mathrm{d}}{\mathrm{d} \ln k} = -M_p^2 \left(\frac{V'}{V} \right) \frac{\mathrm{d}}{\mathrm{d}\chi} = \sqrt{2\epsilon}\, M_p \frac{\mathrm{d}}{\mathrm{d}\chi}$$

To predict scalar and tensor slow-roll spectral indices

$$n_s - 1 = -6\epsilon + 2\eta \qquad n_T = -2\epsilon = -r/8$$

Single-field models are predictive

Only three parameters predict four features of primordial fluctuations

$$\Delta_\Phi(k) = \left(\frac{H^2}{8\pi^2 M_p^2\,\epsilon}\right)_{\rm he}$$

$$n_s - 1 = -6\epsilon + 2\eta$$

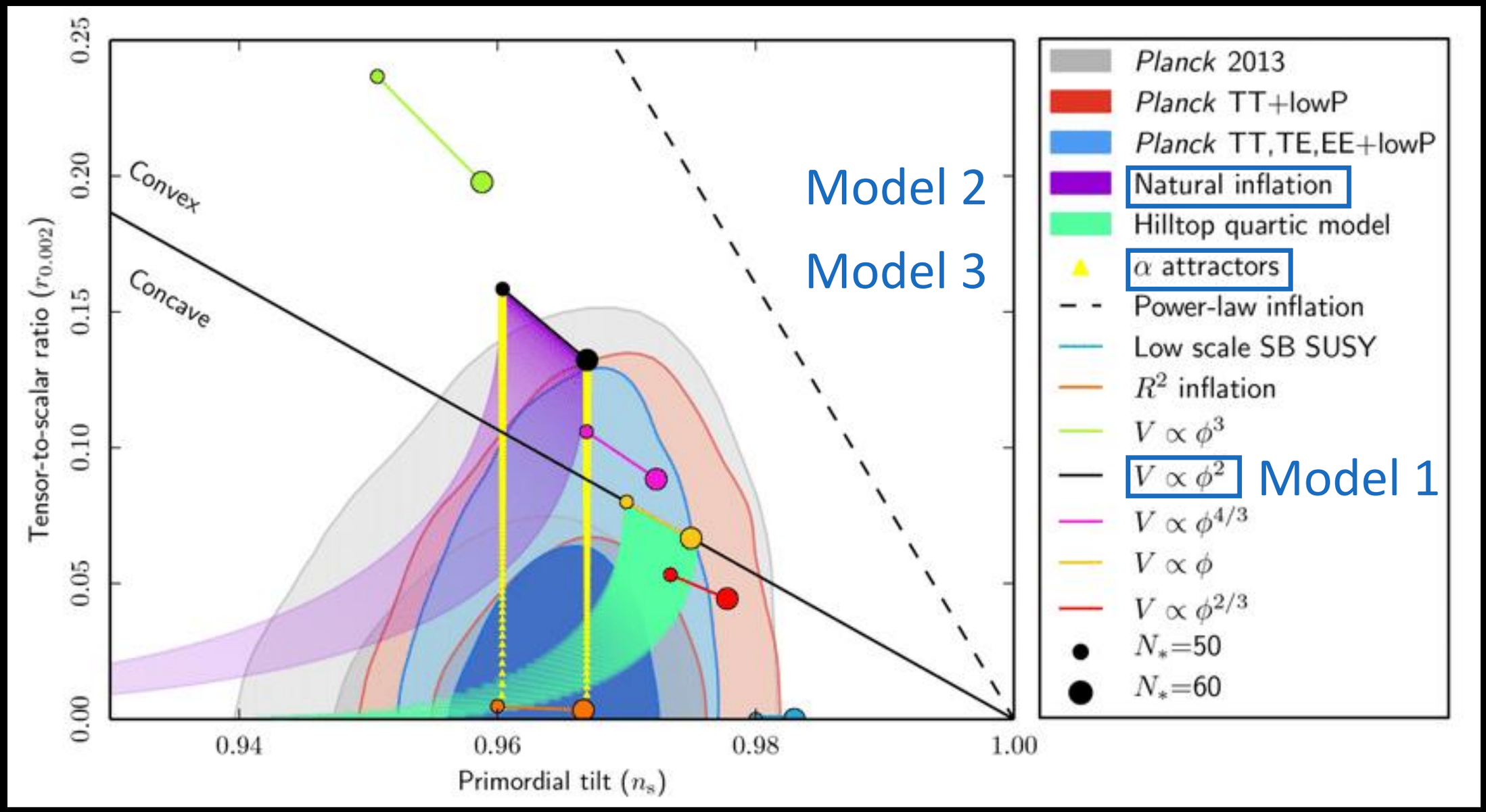
$$\Delta_\Phi(k) = A_s\, k^{n_s - 1}$$

$$\Delta_h(k) = A_T\, k^{n_T}$$

$$\Delta_h(k) = \frac{8}{M_p^2}\left(\frac{H}{2\pi}\right)^2_{\rm he}$$

$$n_T = -2\epsilon = -r/8$$

Scalar amplitude and spectral index are measured



Planck-BICEP 2022 bound: r < 0.038 @ 95% CL

EFTS AND COSMOLOGY

GREFT

IMAGE: CHASE DOLAN VIA PICTURECORRECT.COM

Deser
Weinberg

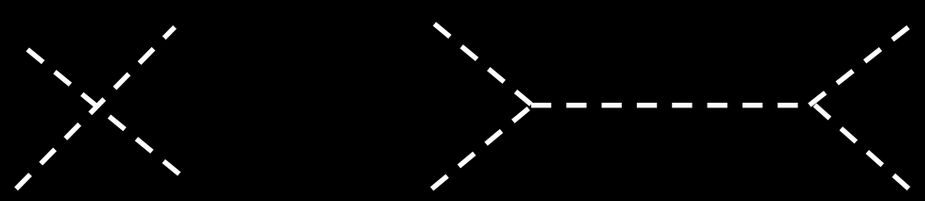
Symmetries dictate nonlinear couplings at two-derivative level to be those of General Relativity

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2h_{\mu\nu}}{M_p}$$

with lagrangian density

$$\mathcal{L} = -\tfrac{1}{2}\, M_p^2 \sqrt{-g}\, \mathcal{R}$$

$$= (\partial h)^2 + \tfrac{1}{M_p}\, h(\partial h)^2 + \tfrac{1}{M_p^2} h^2 (\partial h)^2 + \cdots$$

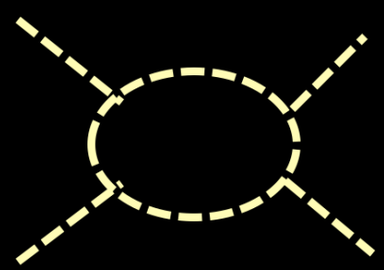


plus ‘crossed’ graphs

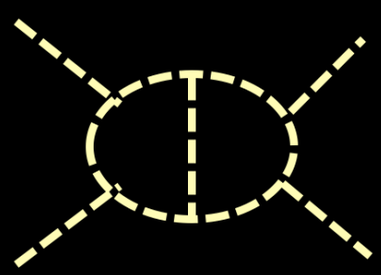
$$\mathcal{A} = \frac{8\pi i G s^3}{tu} \sim \frac{Q^2}{M_p^2}$$

DeWitt

Loop integrals diverge, and higher loops diverge more and more (because the coupling has dimensions of negative powers of mass)



$$A_{1-\text{loop}} \sim \frac{Q^2}{M_p^4} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \, \frac{p^6}{(p^2+Q^2)^4} + \cdots$$

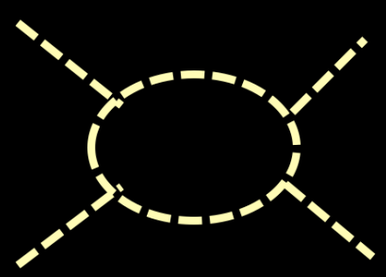


$$A_{2-\text{loop}} \sim \frac{Q^2}{M_p^4} \int \left(\frac{\mathrm{d}^4 p}{(2\pi)^4} \right)^2 \frac{p^{10}}{(p^2+Q^2)^7} + \cdots$$

Corrections modify Newton's constant

$$\delta G \simeq G^2 \Lambda^2 + G^3 \Lambda^4 + \cdots$$

Loop integrals also introduce divergences with a new dependence on external momenta



$$A_{1-\text{loop}} \sim \frac{Q^2}{M_p^4}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\ \frac{p^6}{(p^2+Q^2)^4} + \frac{Q^4}{M_p^4}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\ \frac{p^4}{(p^2+Q^2)^4} + \cdots$$

Some corrections *cannot* be absorbed into Newton's constant (this is what it means to say that GR is not renormalizable)

WHY DOES CLASSICAL GR WORK?

- If quantum contributions to gravity cannot be quantified, why is it meaningful to compare GR with experiment?

Quantum field theory is a precision science:

e.g. QED:

$$a_\mu = 1159652188.4(4.3)\ \ 10^{-12}\ \text{(exp)}$$

$$a_\mu = 1159652140(27.1)\ 10^{-12}\quad \text{(th)}$$

QED's renormalizability is an important part of its calculability, and so also underpins the theory error

WHY DOES CLASSICAL GR WORK?

- If quantum contributions to gravity cannot be quantified, why is it meaningful to compare GR with experiment?

General Relativity is also a precision science:

e.g. solar system tests, binary pulsar, …

$$dP/dt = -2.408(10)\ 10^{-12} \quad \text{(exp)}$$

$$dP/dt = -2.40243(5)\ 10^{-12} \quad \text{(th)}$$

Quantum corrections are controlled once it is recognized GR is only the leading part of the low-energy GR EFT

$$\frac{L}{\sqrt{-g}} = \Lambda + \frac{M_p^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{c_3}{m^2} R^3 + \cdots$$

Each curvature involves two derivatives of the metric

$$\Gamma \sim g^{-1} \partial g \qquad R \sim \partial \Gamma + \Gamma \Gamma$$

Scale m need not be as big as M_p

Using

$$\frac{L}{\sqrt{-g}} = \Lambda + \frac{M_p^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{c_3}{m^2} R^3 + \cdots$$

to power-count amplitude for scattering *E* gravitons at *L* loops using V_n vertices involving f_n fields and d_n derivatives gives

$$A_E(Q) \propto \left(\frac{Q^2}{M_p^{E-2}}\right)\left(\frac{Q}{4\pi M_p}\right)^{2L} \prod_{i;k>2}\left(\frac{Q}{M_p}\right)^{2V_{ik}} \left(\frac{Q}{m}\right)^{(k-4)V_{ik}}$$

Leading contribution: L = 0 and V_n = 0 unless d_n=2 (ie classical GR)

Next-to-leading: L=1 and V_n = 0 unless d_n = 2 (ie one-loop GR)
or L=0 and V=1 for d_n = 4 (ie tree with one R^2 term)

Using

$$\frac{L}{\sqrt{-g}} = \Lambda + \frac{M_p^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{c_3}{m^2} R^3 + \cdots$$

to power-count amplitude for scattering *E* gravitons at *L* loops using V_n vertices involving f_n fields and d_n derivatives gives

$$\left(\frac{Q}{M_p}\right)^{2L} \prod_{i;k>2} \left(\frac{Q}{M_p}\right)^{2V_{ik}} \left(\frac{Q}{m}\right)^{(k-4)V_{ik}}$$

Divergences in these are absorbed into these

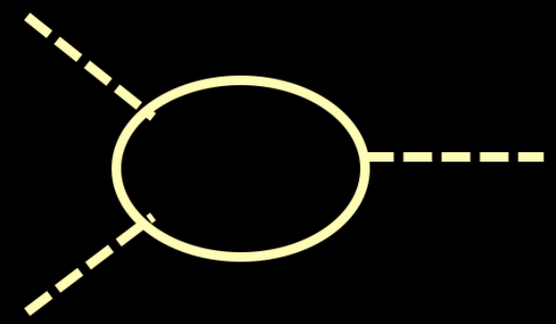
Leading contribution: L = 0 and $V_n = 0$ unless $d_n = 2$ (ie classical GR)

Next-to-leading: L=1 and $V_n = 0$ unless $d_n = 2$ (ie one-loop GR)
or L=0 and V=1 for $d_n = 4$ (ie tree with one R^2 term)

Non-GR terms likely arise as high-energy states are integrated out (such as in string theory).

$$\frac{L}{\sqrt{-g}} = \Lambda + \frac{M_p^2}{2}R + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + \frac{c_3}{m^2}R^3 + \cdots$$

As would be obtained at low energies from loops of particles with m >> Q



Notice smallest m dominates in denominators while largest m dominates in numerators when summing over virtual particles

SUMMARY

EFFECTIVE FIELD THEORY

GREFT provides a unified conceptual framework for all interactions

GREFT provides the only known way to compute quantum effects in GR.

Only known way to quantify quantum ‘theory error’

Against Gravitational Exceptionalism:

Many issues with use of EFTs in cosmology and gravity are special cases of issues that also arise elsewhere.



WEINBERG

IMAGE: PUBLIC DOMAIN

EFTS AND COSMOLOGY

POWER COUNTING

IMAGE: CHASE DOLAN VIA PICTURECORRECT.COM

Repeating the power-counting exercise including also scalar fields (such as the inflaton) shows scalar potential raises new issues

$$-\frac{\mathcal{L}_{\rm eff}}{\sqrt{-g}} = v^4 V(\theta) + \frac{M_p^2}{2}\, g^{\mu\nu}\left[W(\theta)\, R_{\mu\nu} + G_{ij}(\theta)\, \partial_\mu\theta^i \partial_\nu\theta^j\right]$$
$$+A(\theta)(\partial\theta)^4 + B(\theta)\, R^2 + C(\theta)\, R\,(\partial\theta)^2$$
$$+\frac{E(\theta)}{M^2}\,(\partial\theta)^6 + \frac{F(\theta)}{M^2}\, R^3 + \cdots$$

where $g_{\mu\nu} = \overline{g}_{\mu\nu} + h_{\mu\nu}/M_p\,, \quad \theta = \overline{\theta} + \chi/M_p$

and so $\mathcal{L}_{\rm eff} = \overline{\mathcal{L}}_{\rm eff} + M^2 M_p^2 \sum_n \frac{c_n}{M^{d_n}}\, O_n\left(\frac{\chi}{M_p}, \frac{h_{\mu\nu}}{M_p}\right)$

with $V(\phi) = v^4\left[\lambda_0 + \lambda_2\left(\frac{\chi}{M_p}\right)^2 + \lambda_4\left(\frac{\chi}{M_p}\right)^4 + \cdots\right]$

Power-count as before: E external lines; L loops; V_n interactions

Only one low-energy scale (near horizon exit): $Q \sim k/a \sim H$

Compute 'unamputated' correlation functions (so must include external lines)

$$\mathcal{B}_E(H) \simeq \frac{M_p^2}{H^2}\left(\frac{H^2}{M_p}\right)^E \left(\frac{H}{4\pi\, M_p}\right)^{2L} \prod_{d_n=0}\left[\lambda_n\left(\frac{v^4}{H^2 M_p^2}\right)\right]^{V_n}$$

$$\times \left[\prod_{d_n=2} c_n^{V_n}\right] \prod_{d_n\geq 4}\left[g_n\left(\frac{H}{M_p}\right)^2 \left(\frac{H}{M}\right)^{d_n-4}\right]^{V_n}$$

Loops suppressed by $H^2/(4\pi\, M_p)^2$ (*this justifies the semiclassical approx*)

Power-count as before: E external lines; L loops; V_n interactions

Only one low-energy scale (near horizon exit): $Q \sim k/a \sim H$

Compute 'unamputated' correlation functions (so must include external lines)

$$\mathcal{B}_E(H) \simeq \frac{M_p^2}{H^2}\left(\frac{H^2}{M_p}\right)^E \left(\frac{H}{4\pi\, M_p}\right)^{2L} \prod_{d_n=0}\left[\lambda_n\left(\frac{v^4}{H^2 M_p^2}\right)\right]^{V_n}$$

$$\times \left[\prod_{d_n=2} c_n^{V_n}\right] \prod_{d_n\geq 4}\left[g_n\left(\frac{H}{M_p}\right)^2 \left(\frac{H}{M}\right)^{d_n-4}\right]^{V_n}$$

4-derivative and higher terms suppressed as before

Power-count as before: E external lines; L loops; V_n interactions

Only one low-energy scale (near horizon exit): $Q \sim k/a \sim H$

Compute 'unamputated' correlation functions (so must include external lines)

$$\mathcal{B}_E(H) \simeq \frac{M_p^2}{H^2}\left(\frac{H^2}{M_p}\right)^E \left(\frac{H}{4\pi\, M_p}\right)^{2L} \prod_{d_n=0}\left[\lambda_n\left(\frac{v^4}{H^2 M_p^2}\right)\right]^{V_n}$$

$$\times \boxed{\left[\prod_{d_n=2} c_n^{V_n}\right]} \prod_{d_n\geq 4}\left[g_n\left(\frac{H}{M_p}\right)^2 \left(\frac{H}{M}\right)^{d_n-4}\right]^{V_n}$$

2-derivative terms unsuppressed by low-energy expansion *(so no need to linearize GR)*

POWER COUNTING IN COSMOLOGY

Power-count as before: E external lines; L loops; V_n interactions

Only one low-energy scale (near horizon exit): $Q \sim k/a \sim H$

Compute 'unamputated' correlation functions (so must include external lines)

$$\mathcal{B}_E(H) \simeq \frac{M_p^2}{H^2}\left(\frac{H^2}{M_p}\right)^E \left(\frac{H}{4\pi\, M_p}\right)^{2L} \boxed{\prod_{d_n=0}\left[\lambda_n\left(\frac{v^4}{H^2 M_p^2}\right)\right]^{V_n}}$$

$$\times \left[\prod_{d_n=2} c_n^{V_n}\right] \prod_{d_n\geq 4}\left[g_n\left(\frac{H}{M_p}\right)^2 \left(\frac{H}{M}\right)^{d_n-4}\right]^{V_n}$$

0-derivative terms dangerously depend on *1/H*

Not a problem if potential dominates curvature: $H^2 \sim v^4/M_p^2$

Power-count as before: E external lines; L loops; V_n interactions

Only one low-energy scale (near horizon exit): $Q \sim k/a \sim H$

Compute 'unamputated' correlation functions (so must include external lines)

$$\mathcal{B}_E(H) \simeq \frac{M_p^2}{H^2}\left(\frac{H^2}{M_p}\right)^E \left(\frac{H}{4\pi\, M_p}\right)^{2L} \prod_{d_n=0}\left[\lambda_n\left(\frac{v^4}{H^2 M_p^2}\right)\right]^{V_n}$$

$$\times \left[\prod_{d_n=2} c_n^{V_n}\right] \prod_{d_n\geq 4}\left[g_n\left(\frac{H}{M_p}\right)^2 \left(\frac{H}{M}\right)^{d_n-4}\right]^{V_n}$$

eg applied to power spectrum: *E=2, L=0* $\quad \langle h\, h\rangle \sim \langle \chi\, \chi\rangle \sim H^2$

EFTS AND COSMOLOGY

CONCEPTUAL ISSUES

IMAGE: CHASE DOLAN VIA PICTURECORRECT.COM

CONCEPTUAL ISSUES FOR EFTS IN COSMOLOGY

Are time-dependent backgrounds a problem?	NO!
Is use of large fields relative to M_p a problem?	Not necessarily
Is there a trans-Planckian problem when modes descend through a UV cutoff?	Not if adiabatic
What role is there for 'naturalness' arguments?	Technical naturalness is a really big clue
Ostrogradsky ghosts: need we seek reasons to forbid higher-derivative interactions in EFTs?	NO
How to work with EFTs numerically (well-posedness)?	Unknown as yet
Open systems and late-time secular growth?	In development
General formalism capturing dS breaking Goldstone?	YES! EFT inflation