Amplitudes for EFT

Alex Pomarol, IFAE & UAB (Barcelona)

Outline

- **EFT (EFfective Theories) from amplitudes, instead of Lagrangians**

- **Renormalization of EFT using on-shell methods:**

Loops from tree-level amplitudes

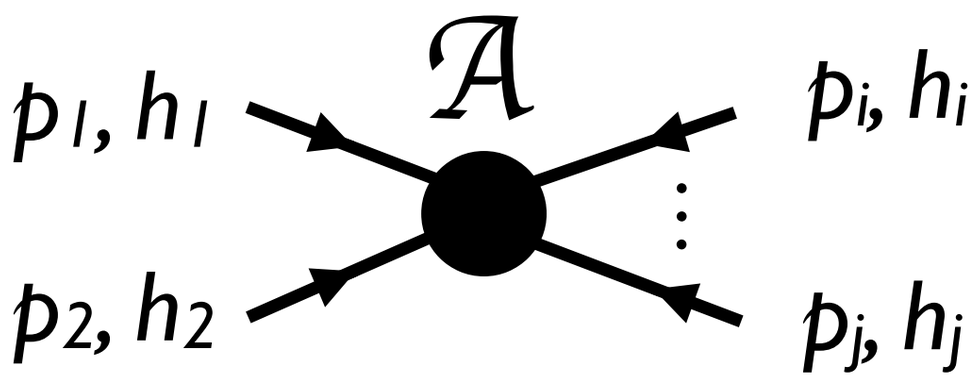
★ **Simple**, elegant, and **efficient**

★ **Selection rules** can explain many **zeros**

★ Clean **relations** between different anomalous dimensions

★ Easy **recycling**: Every calculation can be re-used!

I. EFT (EFfective Theories) from on-shell amplitudes



h_i = helicity of the amplitude

convention: all particles incoming

Ordinary EFT approach

$$\mathcal{L}_{\text{eff}} = \frac{\Lambda^4}{g_*^2}\mathcal{L}\left(\frac{D_\mu}{\Lambda}\,,\ \frac{g_H H}{\Lambda}\,,\ \frac{g_{f_{L,R}} f_{L,R}}{\Lambda^{3/2}}\,,\ \frac{g F_{\mu\nu}}{\Lambda^2}\right) \simeq \mathcal{L}_4 + \mathcal{L}_6 + \cdots$$

SM

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SM

(assuming lepton & baryon number)

$$\mathcal{O}_H = \tfrac{1}{2}(\partial^\mu |H|^2)^2$$
$$\mathcal{O}_T = \tfrac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$$
$$\mathcal{O}_6 = \lambda |H|^6$$

$$\mathcal{O}_W = \tfrac{ig}{2}\left(H^\dagger \sigma^a \overleftrightarrow{D^\mu} H\right) D^\nu W^a_{\mu\nu}$$
$$\mathcal{O}_B = \tfrac{ig'}{2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right) \partial^\nu B_{\mu\nu}$$

$$\mathcal{O}_{2W} = -\tfrac{1}{2}(D^\mu W^a_{\mu\nu})^2$$
$$\mathcal{O}_{2B} = -\tfrac{1}{2}(\partial^\mu B_{\mu\nu})^2$$
$$\mathcal{O}_{2G} = -\tfrac{1}{2}(D^\mu G^A_{\mu\nu})^2$$

$$\mathcal{O}_{BB} = g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu}$$
$$\mathcal{O}_{GG} = g_s^2 |H|^2 G^A_{\mu\nu} G^{A\mu\nu}$$

$$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}$$
$$\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$$

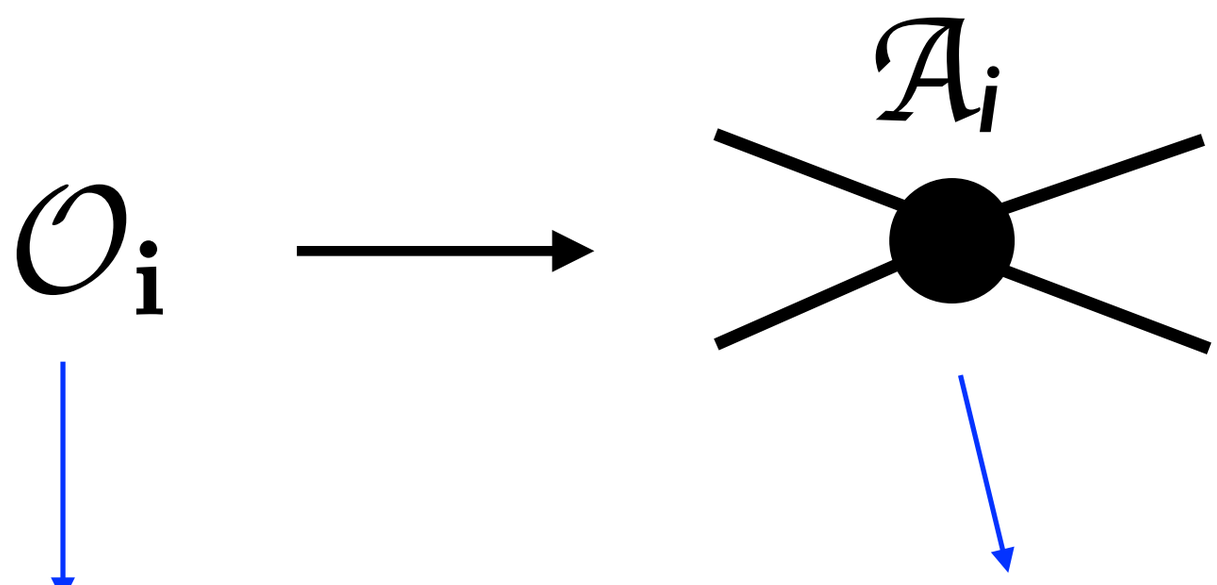
$$\mathcal{O}_{3W} = \tfrac{1}{3!} g \epsilon_{abc} W^{a\,\nu}_{\mu} W^{b}_{\nu\rho} W^{c\,\rho\mu}$$
$$\mathcal{O}_{3G} = \tfrac{1}{3!} g_s f_{ABC} G^{A\,\nu}_{\mu} G^{B}_{\nu\rho} G^{C\,\rho\mu}$$

$\mathcal{O}_{y_u} = y_u \lvert H\rvert^2 \bar{Q}_L \widetilde{H} u_R$	$\mathcal{O}_{y_d} = y_d \lvert H\rvert^2 \bar{Q}_L H d_R$	$\mathcal{O}_{y_e} = y_e \lvert H\rvert^2 \bar{L}_L H e_R$
$\mathcal{O}_R^u = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}_R \gamma^\mu u_R)$	$\mathcal{O}_R^d = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{d}_R \gamma^\mu d_R)$	$\mathcal{O}_R^e = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}_R \gamma^\mu e_R)$
$\mathcal{O}_L^q = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{Q}_L \gamma^\mu Q_L)$		$\mathcal{O}_L^l = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{L}_L \gamma^\mu L_L)$
$\mathcal{O}_L^{(3)\,q} = (iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H)(\bar{Q}_L \gamma^\mu \sigma^a Q_L)$		$\mathcal{O}_L^{(3)\,l} = (iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H)(\bar{L}_L \gamma^\mu \sigma^a L_L)$
$\mathcal{O}_{LR}^u = (\bar{Q}_L \gamma^\mu Q_L)(\bar{u}_R \gamma^\mu u_R)$	$\mathcal{O}_{LR}^d = (\bar{Q}_L \gamma^\mu Q_L)(\bar{d}_R \gamma^\mu d_R)$	$\mathcal{O}_{LR}^e = (\bar{L}_L \gamma^\mu L_L)(\bar{e}_R \gamma^\mu e_R)$
$\mathcal{O}_{LR}^{(8)\,u} = (\bar{Q}_L \gamma^\mu T^A Q_L)(\bar{u}_R \gamma^\mu T^A u_R)$	$\mathcal{O}_{LR}^{(8)\,d} = (\bar{Q}_L \gamma^\mu T^A Q_L)(\bar{d}_R \gamma^\mu T^A d_R)$	
$\mathcal{O}_{RR}^u = (\bar{u}_R \gamma^\mu u_R)(\bar{u}_R \gamma^\mu u_R)$	$\mathcal{O}_{RR}^d = (\bar{d}_R \gamma^\mu d_R)(\bar{d}_R \gamma^\mu d_R)$	$\mathcal{O}_{RR}^e = (\bar{e}_R \gamma^\mu e_R)(\bar{e}_R \gamma^\mu e_R)$
$\mathcal{O}_{LL}^q = (\bar{Q}_L \gamma^\mu Q_L)(\bar{Q}_L \gamma^\mu Q_L)$		$\mathcal{O}_{LL}^l = (\bar{L}_L \gamma^\mu L_L)(\bar{L}_L \gamma^\mu L_L)$
$\mathcal{O}_{LL}^{(8)\,q} = (\bar{Q}_L \gamma^\mu T^A Q_L)(\bar{Q}_L \gamma^\mu T^A Q_L)$		
$\mathcal{O}_{LL}^{ql} = (\bar{Q}_L \gamma^\mu Q_L)(\bar{L}_L \gamma^\mu L_L)$		
$\mathcal{O}_{LL}^{(3)\,ql} = (\bar{Q}_L \gamma^\mu \sigma^a Q_L)(\bar{L}_L \gamma^\mu \sigma^a L_L)$		
$\mathcal{O}_{LR}^{qe} = (\bar{Q}_L \gamma^\mu Q_L)(\bar{e}_R \gamma^\mu e_R)$		
$\mathcal{O}_{LR}^{lu} = (\bar{L}_L \gamma^\mu L_L)(\bar{u}_R \gamma^\mu u_R)$	$\mathcal{O}_{LR}^{ld} = (\bar{L}_L \gamma^\mu L_L)(\bar{d}_R \gamma^\mu d_R)$	
$\mathcal{O}_{RR}^{ud} = (\bar{u}_R \gamma^\mu u_R)(\bar{d}_R \gamma^\mu d_R)$		
$\mathcal{O}_{RR}^{(8)\,ud} = (\bar{u}_R \gamma^\mu T^A u_R)(\bar{d}_R \gamma^\mu T^A d_R)$		
$\mathcal{O}_{RR}^{ue} = (\bar{u}_R \gamma^\mu u_R)(\bar{e}_R \gamma^\mu e_R)$	$\mathcal{O}_{RR}^{de} = (\bar{d}_R \gamma^\mu d_R)(\bar{e}_R \gamma^\mu e_R)$	
$\mathcal{O}_R^{ud} = y_u^\dagger y_d (i\widetilde{H}^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}_R \gamma^\mu d_R)$		
$\mathcal{O}_{y_u y_d} = y_u y_d (\bar{Q}_L^r u_R)\epsilon_{rs}(\bar{Q}_L^s d_R)$		
$\mathcal{O}_{y_u y_d}^{(8)} = y_u y_d (\bar{Q}_L^r T^A u_R)\epsilon_{rs}(\bar{Q}_L^s T^A d_R)$		
$\mathcal{O}_{y_u y_e} = y_u y_e (\bar{Q}_L^r u_R)\epsilon_{rs}(\bar{L}_L^s e_R)$		
$\mathcal{O}'_{y_u y_e} = y_u y_e (\bar{Q}_L^{r\,\alpha} e_R)\epsilon_{rs}(\bar{L}_L^s u_R^\alpha)$		
$\mathcal{O}_{y_e y_d} = y_e y_d^\dagger (\bar{L}_L e_R)(\bar{d}_R Q_L)$		
$\mathcal{O}_{DB}^u = y_u \bar{Q}_L \sigma^{\mu\nu} u_R \widetilde{H} g' B_{\mu\nu}$	$\mathcal{O}_{DB}^d = y_d \bar{Q}_L \sigma^{\mu\nu} d_R H g' B_{\mu\nu}$	$\mathcal{O}_{DB}^e = y_e \bar{L}_L \sigma^{\mu\nu} e_R H g' B_{\mu\nu}$
$\mathcal{O}_{DW}^u = y_u \bar{Q}_L \sigma^{\mu\nu} u_R \sigma^a \widetilde{H} g W^a_{\mu\nu}$	$\mathcal{O}_{DW}^d = y_d \bar{Q}_L \sigma^{\mu\nu} d_R \sigma^a H g W^a_{\mu\nu}$	$\mathcal{O}_{DW}^e = y_e \bar{L}_L \sigma^{\mu\nu} e_R \sigma^a H g W^a_{\mu\nu}$
$\mathcal{O}_{DG}^u = y_u \bar{Q}_L \sigma^{\mu\nu} T^A u_R \widetilde{H} g_s G^A_{\mu\nu}$	$\mathcal{O}_{DG}^d = y_d \bar{Q}_L \sigma^{\mu\nu} T^A d_R H g_s G^A_{\mu\nu}$	

One must eliminate redundancies

Many missed in original papers (Buchmuller, Wyler, …)!

From operators to on-shell amplitudes



fields containing
Ghosts, Golstones,…
($p^2 \neq 0$)

only physical states ($p^2=0$)

↳ **definite helicity**
($h = \mp$)

An important gain in simplicity

the power of being on-shell !

Spinor-helicity formalism

Lorentz ⟶ SL(2,C)

p^μ ⟶ $p_{\alpha\dot\alpha} = |p\rangle_\alpha [p|_{\dot\alpha}$

(1/2,1/2) =(1/2,0)⊗(0,1/2)

"angle" spinor | **"squared" spinor**

Momenta & helicities as a function of spinors:

$$u_\mp(p) = P_\mp \begin{pmatrix} |p\rangle_\alpha \\ |p]^{\dot\alpha} \end{pmatrix}, \qquad \bar v_\mp(p) = \big(\langle p|^\alpha \; [p|_{\dot\alpha}\big) P_\mp, \qquad P_\mp = (1 \pm \gamma_5)/2$$

$$\epsilon^+_\mu = \frac{\langle q|\sigma_\mu|p]}{\sqrt{2}\langle qp\rangle}, \qquad \epsilon^-_\mu = -\frac{\langle p|\sigma_\mu|q]}{\sqrt{2}[qp]}$$

$$2\, p_i \cdot p_j = \langle ij\rangle [ji]$$

keep complex momenta!

B.1 Conventions

We start with the conventions taken in this article. We choose the metric $\eta_{\mu\nu} = \text{diag}(+,-,-,-)$, and the 2-component spinors with $h = \mp 1/2$ to be denoted respectively by $|p\rangle_\alpha$ and $|p]^{\dot\alpha}$. The momentum is given by $p_{\alpha\dot\alpha} = |p\rangle_\alpha [p|_{\dot\alpha}$, and the contractions are

$$\langle pq\rangle \equiv \langle p|^\alpha |q\rangle_\alpha \quad \text{and} \quad [pq] \equiv [p|_{\dot\alpha}|q]^{\dot\alpha}\,, \tag{73}$$

where we follow the conventions of Ref. [28] for raising and lowering indices. We also define $\langle i|\sigma_\mu|j] \equiv \langle i|^\alpha (\sigma_\mu)_{\alpha\dot\alpha}|j]^{\dot\alpha}$, that fulfill the property $\langle i|\sigma_\mu|j] = [j|\sigma_\mu|i\rangle$. We also have

$$p_i^\mu = \frac{1}{2}\langle i|\sigma^\mu|i]\,, \qquad 2\,p_i\cdot p_j = \langle ij\rangle[ji]\,, \tag{74}$$

the Fierz relation

$$\langle i|\sigma^\mu|j]\langle k|\sigma^\mu|l] = -2\langle ik\rangle[jl]\,, \tag{75}$$

and the Schouten identity

$$\langle ij\rangle\langle kl\rangle = \langle ik\rangle\langle jl\rangle - \langle il\rangle\langle jk\rangle\,. \tag{76}$$

We are considering amplitudes with all states incomming. Outgoing states can be related to incomming states with opposite momentum and helicity, replacing particle $\leftrightarrow$ antiparticle. When we encounter spinors with negative momenta, it is convenient to write them back with positive momenta. Following the appendix of Ref. [29], we define

$$|-p\rangle_\alpha = i|p\rangle_\alpha\,, \quad |-p]^{\dot\alpha} = i|p]^{\dot\alpha}\,, \tag{77}$$

that consistently leads to $|-p\rangle[-p| = -p$.

The convention Eq. (77) fixes the factorization of an amplitude $\mathcal{A}$ into a product of lower-point amplitudes. For the case of a 4-point amplitude with a pole in the s-channel (see Fig. 13), this is given by

$$\lim_{s_{12}\to 0} s_{12}\mathcal{A}(1,2,3,4) = i^{F[\ell]}\; i\mathcal{A}_L(1,2,\ell)\; i\mathcal{A}_R(-\ell,3,4)\,, \tag{78}$$

The SM as an EFT

= ***EF****fective* ***T****heory*

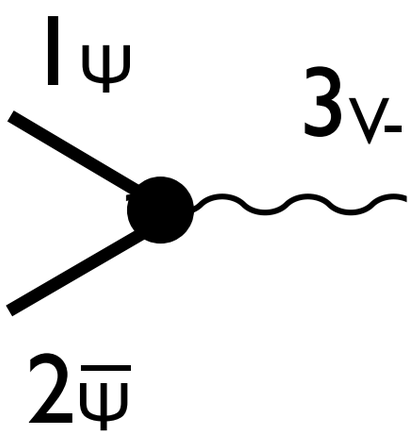
Expansion: $\langle ij\rangle/\Lambda$, $[ij]/\Lambda$

The SM as an EFT

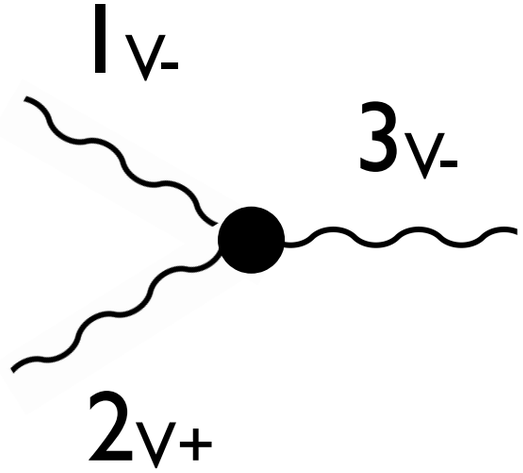
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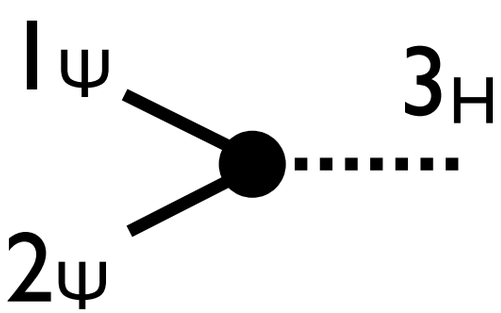
At O(1/Λ⁰): **SM "Building-blocks"**



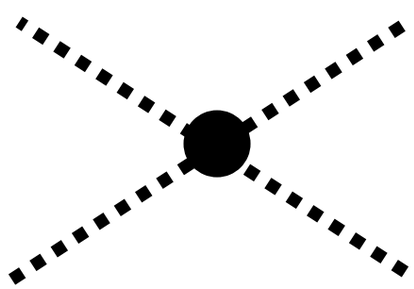
$$= \frac{\langle 13\rangle^2}{\langle 12\rangle}\, g\, T^a_{ij}$$



$$= \frac{\langle 13\rangle^3}{\langle 12\rangle\langle 23\rangle}\, g f_{abc}$$



$$= \langle 12\rangle\, Y_\psi$$



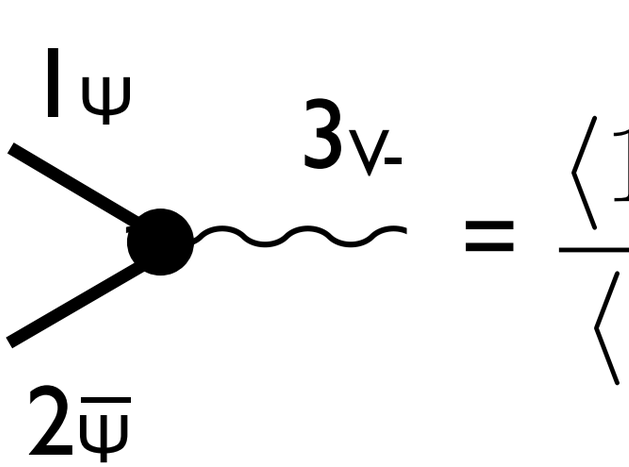
$$= \lambda$$

The SM as an EFT

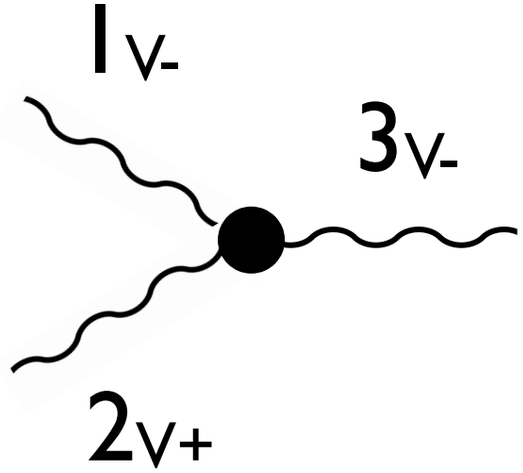
= ***EF****fective* ***T****heory*

Expansion: $\langle ij\rangle/\Lambda$, $[ij]/\Lambda$

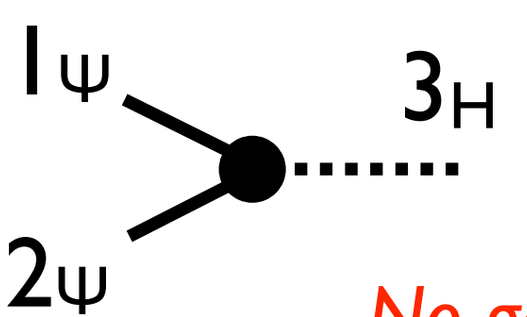
At O(1/Λ⁰): **SM "Building-blocks"**



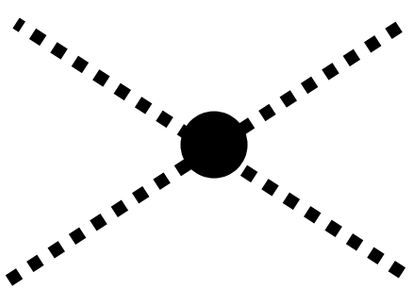
$$= \frac{\langle 13\rangle^2}{\langle 12\rangle}\, g\, T^a_{ij}$$



$$= \frac{\langle 13\rangle^3}{\langle 12\rangle\langle 23\rangle}\, g f_{abc}$$



$$= \langle 12\rangle\, Y_\psi$$

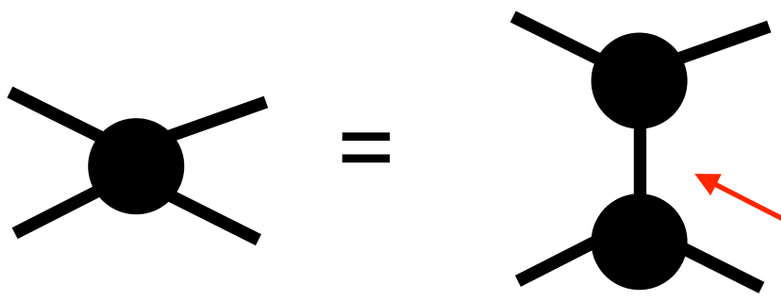


$$= \lambda$$

No gauge invariance, only inv. under global symmetries

From these 3pt+4pt
we can build higher-point SM Amplitudes

☞ at tree-level, singularities only from single poles



= + cross channels + contact int.

single poles: ~1/t

☞ on the pole, amplitudes must factorize in smaller amplitudes

☞ see example

At O(1/Λ): $$\mathcal{A}(1_H, 2_H, 3_\ell, 4_\ell) = \frac{C}{\Lambda}\langle 34\rangle$$

New "Building-block"

At O(1/Λ²):

***n* = number of external states**
***h* = helicity of the amplitude**

$$\mathcal{A}_{F^3}(1_{V_-}, 2_{V_-}, 3_{V_-}) = \frac{C_{F^3}}{\Lambda^2}\langle 12\rangle\langle 23\rangle\langle 31\rangle$$

} n=3, h=-3

$$\mathcal{A}_{F^2\phi^2}(1_{V_-}, 2_{V_-}, 3_\phi, 4_\phi) = \frac{C_{F^2\phi^2}}{\Lambda^2}\langle 12\rangle^2\,,$$

$$\mathcal{A}_{F\psi^2\phi}(1_{V_-}, 2_\psi, 3_\psi, 4_\phi) = \frac{C_{F\psi^2\phi}}{\Lambda^2}\langle 12\rangle\langle 13\rangle\,,$$

$$\mathcal{A}_{\psi^4}(1_\psi, 2_\psi, 3_\psi, 4_\psi) = \left(C_{\psi^4}\langle 12\rangle\langle 34\rangle + C'_{\psi^4}\langle 13\rangle\langle 24\rangle\right)\frac{1}{\Lambda^2}$$

} n=4, h=-2

$$\mathcal{A}_{\Box\phi^4}(1_\phi, 2_\phi, 3_\phi, 4_\phi) = \left(C_{\Box\phi^4}\langle 12\rangle[12] + C'_{\Box\phi^4}\langle 13\rangle[13]\right)\frac{1}{\Lambda^2}$$

$$\mathcal{A}_{\psi\bar{\psi}\phi^2}(1_\psi, 2_{\bar{\psi}}, 3_\phi, 4_\phi) = \frac{C_{\psi\bar{\psi}\phi^2}}{\Lambda^2}\langle 13\rangle[23]\,,$$

$$\mathcal{A}_{\psi^2\bar{\psi}^2}(1_\psi, 2_\psi, 3_{\bar{\psi}}, 4_{\bar{\psi}}) = \frac{C_{\psi^2\bar{\psi}^2}}{\Lambda^2}\langle 12\rangle[34]\,.$$

} n=4, h=0

At O(1/Λ²):

***n* = number of external states**
***h* = helicity of the amplitude**

$$\mathcal{A}_{F^3}(1_{V_-},2_{V_-},3_{V_-}) = \frac{C_{F^3}}{\Lambda^2}\langle 12\rangle\langle 23\rangle\langle 31\rangle$$

} n=3, h=-3

$$\mathcal{A}_{F^2\phi^2}(1_{V_-},2_{V_-},3_{\phi},4_{\phi}) = \frac{C_{F^2\phi^2}}{\Lambda^2}\langle 12\rangle^2,$$

$$F^{\mu\nu}F_{\mu\nu}\phi^2$$

$$\mathcal{A}_{F\psi^2\phi}(1_{V_-},2_{\psi},3_{\psi},4_{\phi}) = \frac{C_{F\psi^2\phi}}{\Lambda^2}\langle 12\rangle\langle 13\rangle\,,$$

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$$\mathcal{A}_{\Box\phi^4}(1_{\phi},2_{\phi},3_{\phi},4_{\phi}) = \left(C_{\Box\phi^4}\langle 12\rangle[12] + C'_{\Box\phi^4}\langle 13\rangle[13]\right)\frac{1}{\Lambda^2}$$

$$\mathcal{A}_{\psi\bar{\psi}\phi^2}(1_{\psi},2_{\bar{\psi}},3_{\phi},4_{\phi}) = \frac{C_{\psi\bar{\psi}\phi^2}}{\Lambda^2}\langle 13\rangle[23]\,,$$

$$\mathcal{A}_{\psi^2\bar{\psi}^2}(1_{\psi},2_{\psi},3_{\bar{\psi}},4_{\bar{\psi}}) = \frac{C_{\psi^2\bar{\psi}^2}}{\Lambda^2}\langle 12\rangle[34]\,.$$

} n=4, h=0

At O(1/Λ²):

n **= number of external states**
h **= helicity of the amplitude**

$$\mathcal{A}_{F^3}(1_{V_-}, 2_{V_-}, 3_{V_-}) = \frac{C_{F^3}}{\Lambda^2}\langle 12\rangle\langle 23\rangle\langle 31\rangle$$

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$$\mathcal{A}_{F^2\phi^2}(1_{V_-}, 2_{V_-}, 3_{\phi}, 4_{\phi}) = \frac{C_{F^2\phi^2}}{\Lambda^2}\langle 12\rangle^2\,,$$

$$\mathcal{A}_{F\psi^2\phi}(1_{V_-}, 2_{\psi}, 3_{\psi}, 4_{\phi}) = \frac{C_{F\psi^2\phi}}{\Lambda^2}\langle 12\rangle\langle 13\rangle\,,$$

$$F^{\mu\nu}\psi\sigma_{\mu\nu}\psi\, H$$

$$\mathcal{A}_{\psi^4}(1_{\psi}, 2_{\psi}, 3_{\psi}, 4_{\psi}) = \left(C_{\psi^4}\langle 12\rangle\langle 34\rangle + C'_{\psi^4}\langle 13\rangle\langle 24\rangle\right)\frac{1}{\Lambda^2}$$

$$\mathcal{A}_{\Box\phi^4}(1_{\phi}, 2_{\phi}, 3_{\phi}, 4_{\phi}) = \left(C_{\Box\phi^4}\langle 12\rangle[12] + C'_{\Box\phi^4}\langle 13\rangle[13]\right)\frac{1}{\Lambda^2}$$

$$\mathcal{A}_{\psi\bar{\psi}\phi^2}(1_{\psi}, 2_{\bar{\psi}}, 3_{\phi}, 4_{\phi}) = \frac{C_{\psi\bar{\psi}\phi^2}}{\Lambda^2}\langle 13\rangle[23]\,,$$

$$\mathcal{A}_{\psi^2\bar{\psi}^2}(1_{\psi}, 2_{\psi}, 3_{\bar{\psi}}, 4_{\bar{\psi}}) = \frac{C_{\psi^2\bar{\psi}^2}}{\Lambda^2}\langle 12\rangle[34]\,.$$

} n=4, h=0

At O(1/Λ²):

$$\mathcal{A}_{\psi^2\phi^3}(1_\psi, 2_\psi, 3_\phi, 4_\phi, 5_\phi) = \frac{C_{\psi^2\phi^3}}{\Lambda^2}\langle 12\rangle$$

n=5
h=-1

$$\mathcal{A}_{\phi^6}(1_\phi, 2_\phi, 3_\phi, 4_\phi, 5_\phi, 6_\phi) = \frac{C_{\phi^6}}{\Lambda^2}$$

n=6
h=0

II. One-loop renormalization from amplitude methods

Ordinary EFT approach

(assuming lepton & baryon number)

$$\mathcal{L}_{\text{eff}} = \frac{\Lambda^4}{g_*^2}\mathcal{L}\left(\frac{D_\mu}{\Lambda}\,,\ \frac{g_H H}{\Lambda}\,,\ \frac{g_{f_{L,R}} f_{L,R}}{\Lambda^{3/2}}\,,\ \frac{g F_{\mu\nu}}{\Lambda^2}\right) \simeq \mathcal{L}_4 + \mathcal{L}_6 + \cdots$$

SM

Λ —— $\mathcal{O}_i, \mathcal{O}_j, \ldots$

mw —— $\mathcal{O}_i$

me ——

One-loop operator mixing important:

(tells us how BSM enter in observables)

$$\gamma_{c_i} = \frac{dc_i}{d\log\mu} = \gamma_{c_i}(c_j)$$

c_i = Wilson coefficient

anomalous dimensions ☞ tell me the "running" of the Wilson coeff.

Ordinary EFT approach

(assuming lepton & baryon number)

$$\mathcal{L}_{\text{eff}} = \frac{\Lambda^4}{g_*^2}\mathcal{L}\left(\frac{D_\mu}{\Lambda}\,,\ \frac{g_H H}{\Lambda}\,,\ \frac{g_{f_{L,R}} f_{L,R}}{\Lambda^{3/2}}\,,\ \frac{g F_{\mu\nu}}{\Lambda^2}\right) \simeq \mathcal{L}_4 + \mathcal{L}_6 + \cdots$$

SM

Λ —— $\mathcal{O}_i, \mathcal{O}_j, \ldots$

One-loop operator mixing important:

(tells us how BSM ent…

m_W —— $\mathcal{O}_i$

m_e ——

Many non-trivial zeros in γ_i found from explicit calculations !

$$\frac{dc_i}{d\log\mu} = \gamma_{c_i}(c_j)$$

c_i = Wilson coefficient

One-loop anomalous dimension of dim-6 operators

$$\gamma_{c_i}(c_j) = \begin{array}{c} \\ \mathcal{O}_{3F_+} \\ \mathcal{O}_{FF_+} \\ \mathcal{O}_D \\ \mathcal{O}_{yy} \\ \mathcal{O}_y \\ \mathcal{O}_R^{ud} \\ \mathcal{O}_6 \\ \mathcal{O}_+ \\ \mathcal{O}_- \\ \mathcal{O}_{4f} \\ \mathcal{O}_{Hf} \end{array} \begin{array}{c} \begin{array}{ccccccccccc} \mathcal{O}_{3F_+} & \mathcal{O}_{FF_+} & \mathcal{O}_D & \mathcal{O}_{yy} & \mathcal{O}_y & \mathcal{O}_R^{ud} & \mathcal{O}_6 & \mathcal{O}_+ & \mathcal{O}_- & \mathcal{O}_{4f} & \mathcal{O}_{Hf} \end{array} \\ \left(\begin{array}{ccccccccccc} & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \end{array} \right. \end{array}$$

holomorphic

vanishing entries

$$\frac{\partial \gamma_{c_i}}{\partial c_j^*} = 0$$

vanishing entries

arXiv:1412.7151

Very practical example:

Renormalization of electron EDM

Recent strong bound by ACME experiment:

$$|d_e| < 1.1 \cdot 10^{-29}\,\mathrm{e}\cdot\mathrm{cm}$$

Can provide important constraints even if BSM enters at the 2-loop level!

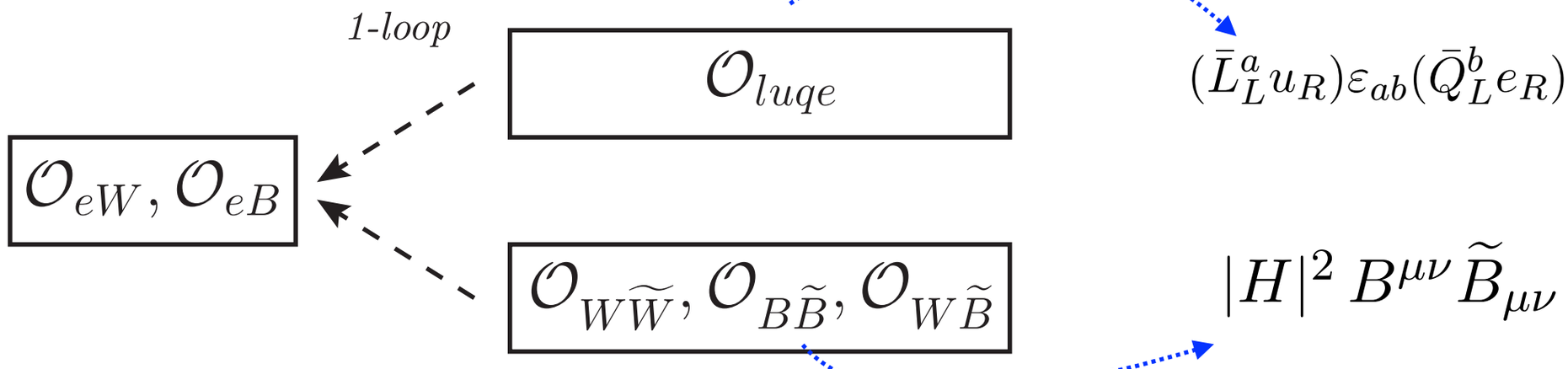
$$\frac{d_e}{e} \sim \frac{1}{(16\pi^2)^2}\frac{m_e}{\Lambda^2} \qquad \rightarrow \Lambda > 3\,\mathrm{TeV}$$

Best weapon of BSM mass destruction!

or even on dimension-8 operators!

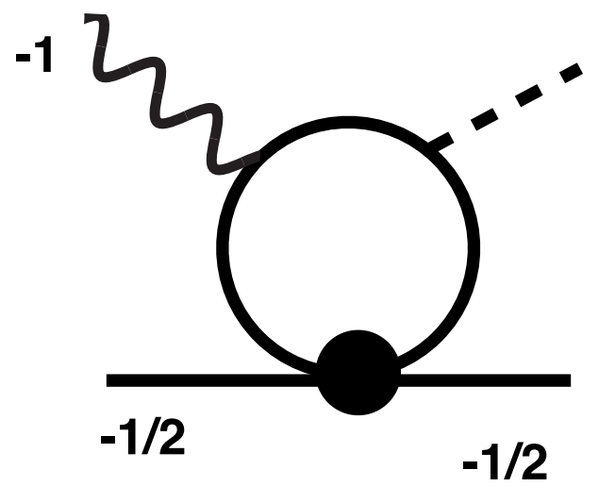
One-loop mixing:

Panico, AP, Riembau arXiv:1810.09413



out of 59 operators

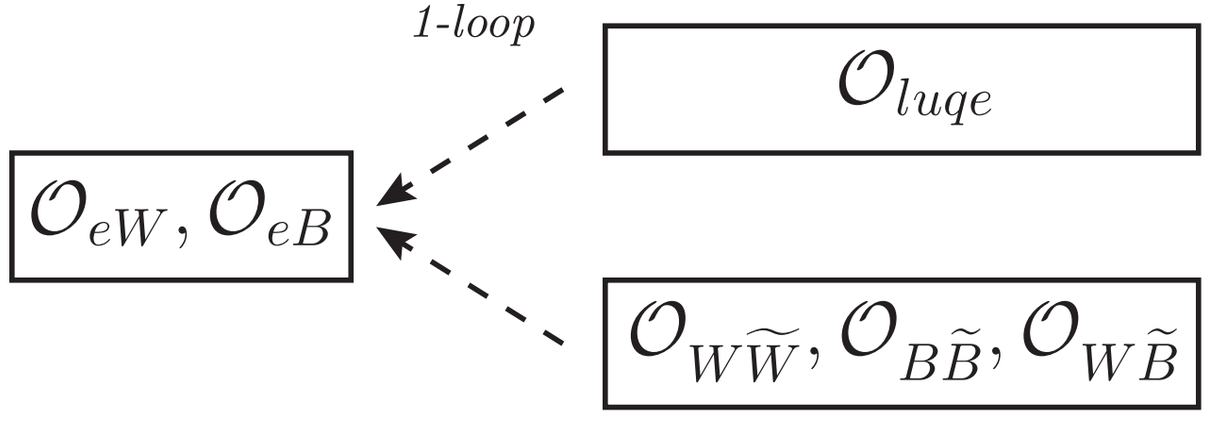
For example, no 4-fermion $(\psi\gamma^\mu\bar{\psi})^2$ contributions



Mysterious from Feynman approach!

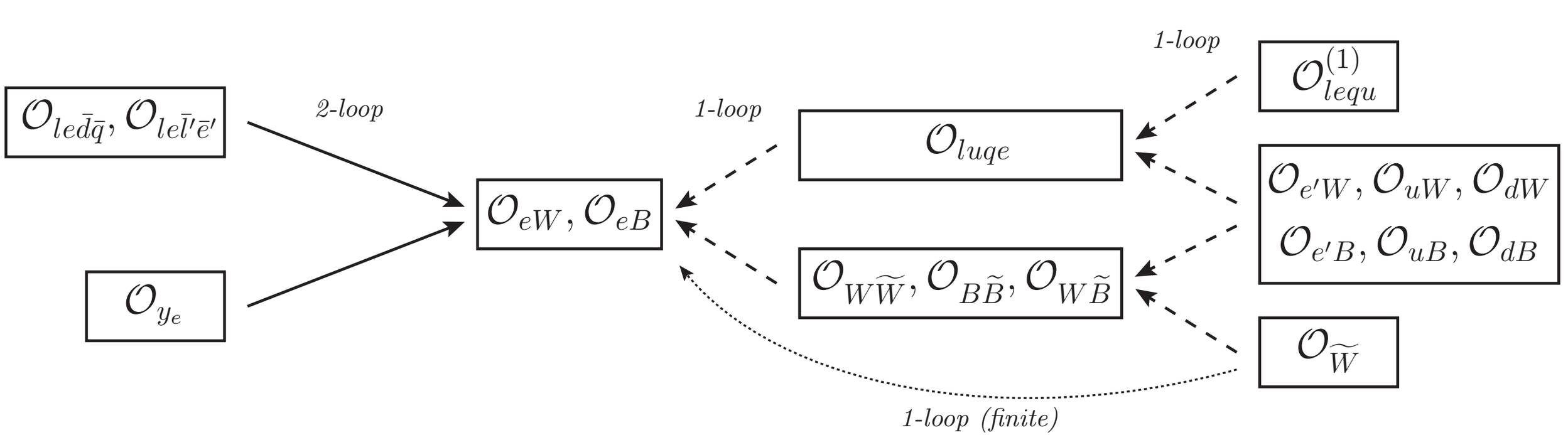
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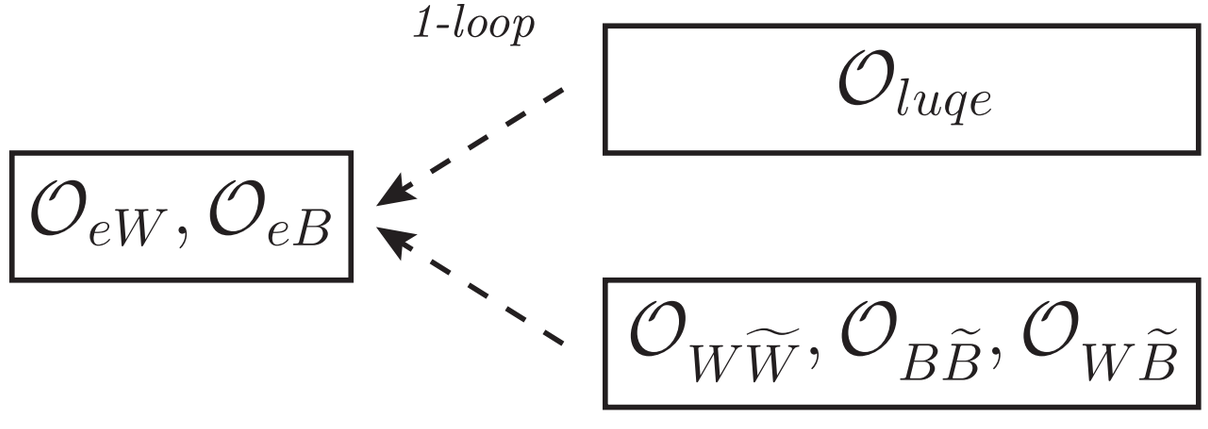
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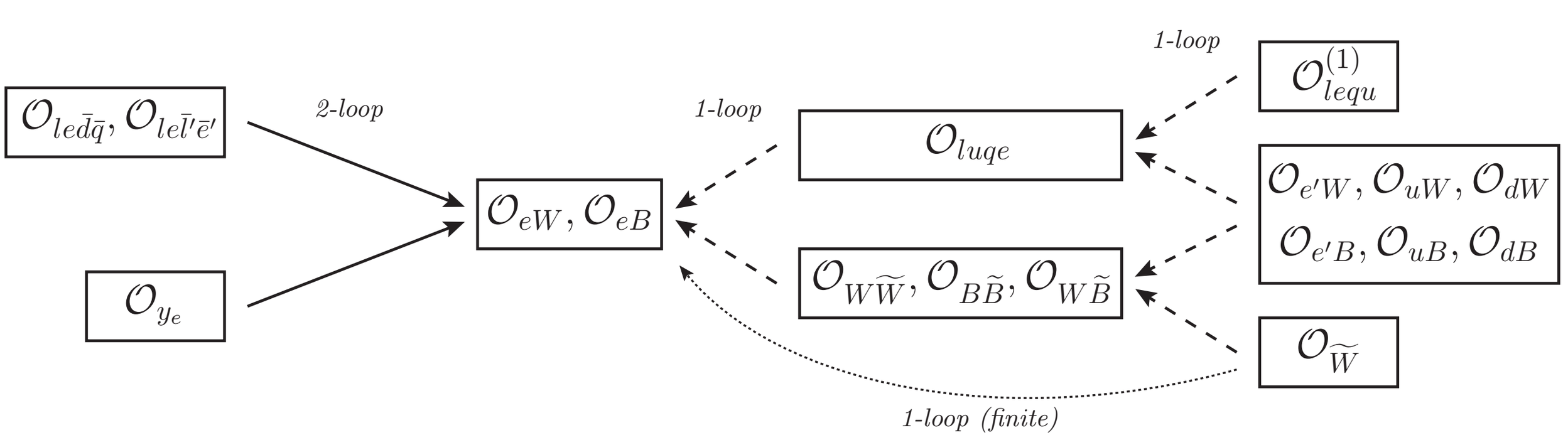
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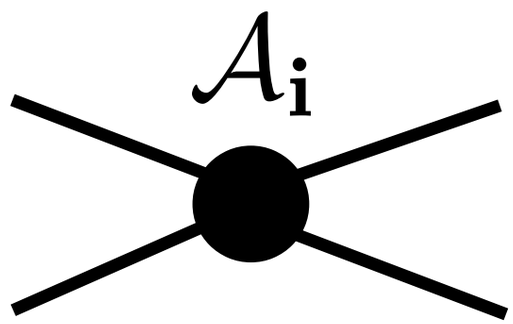
Just from **explicit calculations**!

But why? Amplitude method needed!

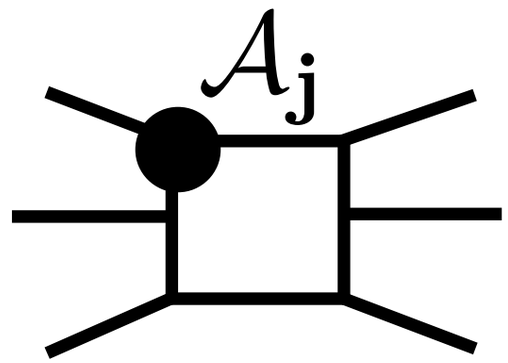
(or susy: see arXiv:1412.7151)

One-loop mixing (anomalous dimensions)

$$\mathcal{A}_i^{1-\text{loop}} =$$

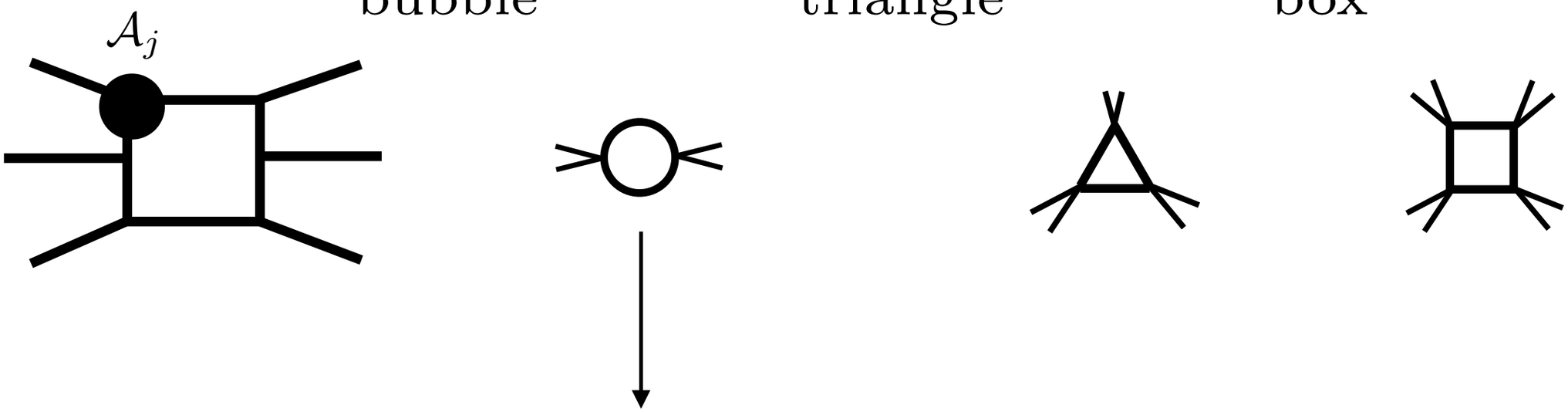


+



After one-loop reduction to Passarino-Veltman integrals

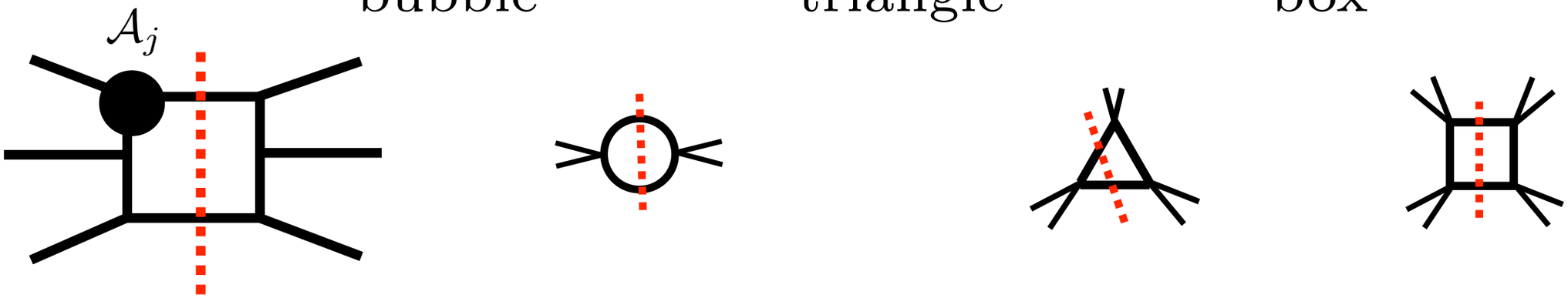
$$\mathcal{A}_\mathbf{i} = \sum_{\text{bubble}} \mathbf{c_2 I_2} + \sum_{\text{triangle}} \mathbf{c_3 I_3} + \sum_{\text{box}} \mathbf{c_4 I_4} + \mathbf{rational}$$



UV divergent ☞ c_2 = anomalous dimensions

After one-loop reduction to Passarino-Veltman integrals

$$\mathcal{A}_\mathbf{i} = \sum_{\text{bubble}} \mathbf{c_2 I_2} + \sum_{\text{triangle}} \mathbf{c_3 I_3} + \sum_{\text{box}} \mathbf{c_4 I_4} + \mathbf{rational}$$



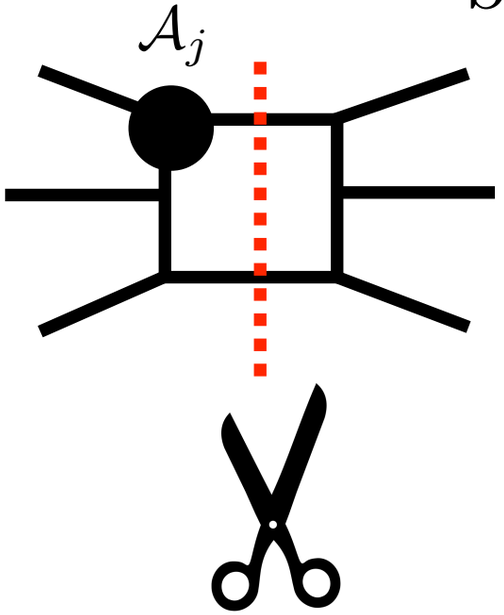


double cut

(internal particles on-shell)

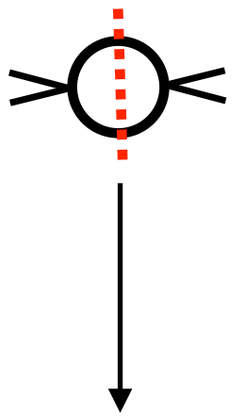
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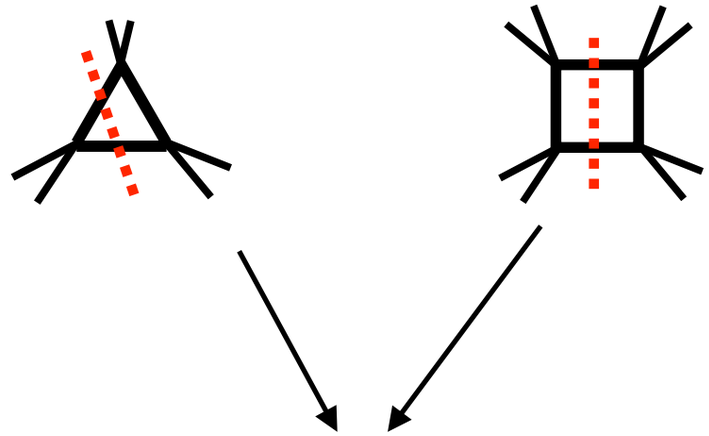


double cut

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c_2



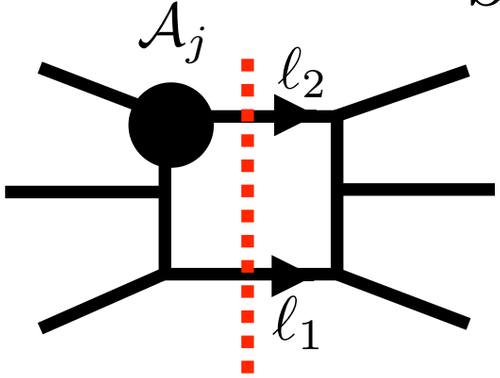
Zero contribution

(after extracting IR-div)

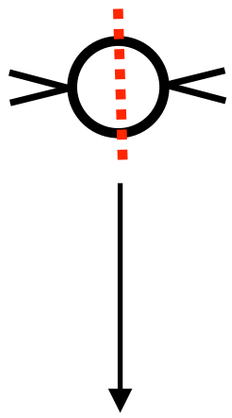
P. Baratella, C. Fernandez, AP 2005.07129

After one-loop reduction to Passarino-Veltman integrals

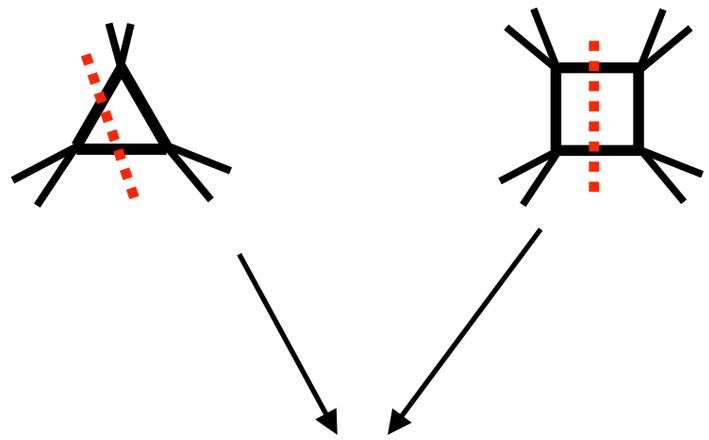
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$\widehat{\mathcal{A}}_j$ $\mathcal{A}_{\rm SM}$



c_2



Zero contribution
(after extracting IR-div)

P. Baratella, C. Fernandez, AP 2005.07129

$$\gamma_{ij}\,\mathcal{A}_i(1,2,...,n) = -\frac{1}{4\pi^3}\frac{C_i}{C_j}\int d\text{LIPS}\sum_{\substack{\text{ext. legs}\\ \text{distrib}}}\sum_{\ell_1,\ell_2}\sigma_{\ell_1,\ell_2}\widehat{\mathcal{A}}_j(...,-\ell_1,-\ell_2)\times\mathcal{A}_{\rm SM}(\ell_2,\ell_1,\dots)$$

phase-space integration & sum over internal states

Example:

Renormalization of the dipole

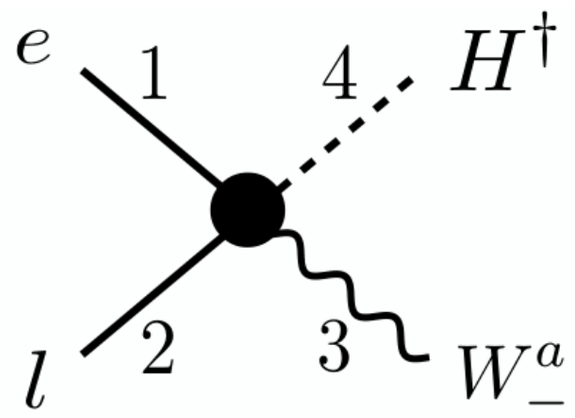
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Renormalization of the dipole

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$$\mathcal{A}(1_e, 2_{l_j}, 3_{W^a_-}, 4_{H^\dagger_i})$$

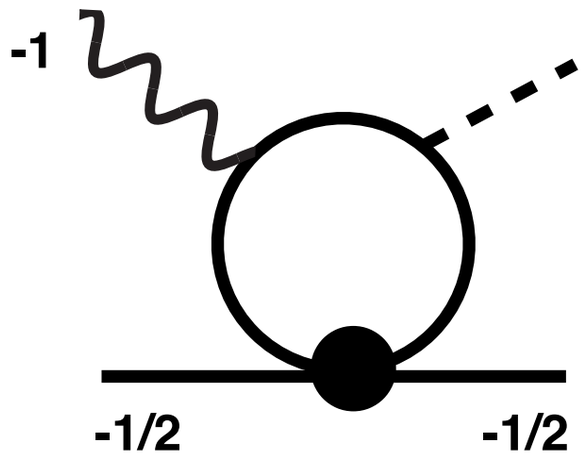


“Emergent” selection rules

1505.01844 (also by susy techniques:1412.7151)

No 4-fermion $(\psi\bar{\gamma}^{\mu}\psi)^2$ corrections to dipoles

$$\mathcal{A}(1_e, 2_{l_j}, 3_{W_-^a}, 4_{H_i^\dagger})$$



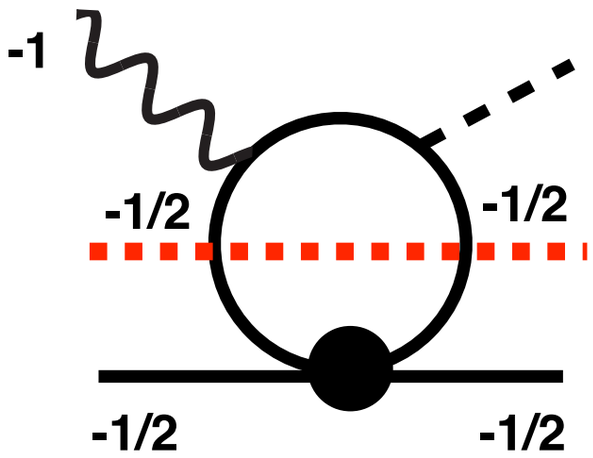
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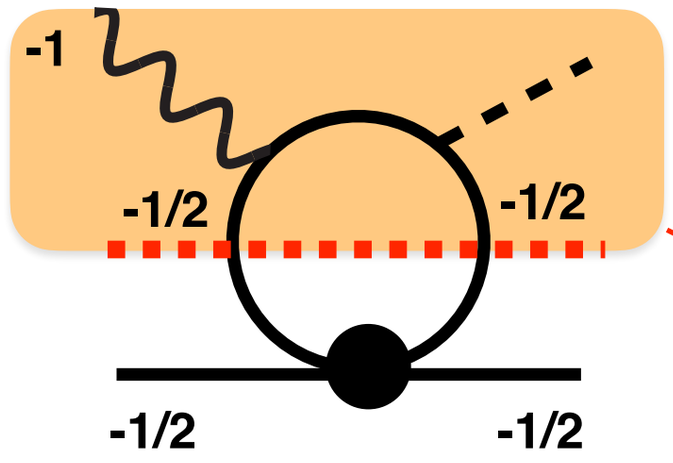
$$\gamma\mathcal{A}_{WHle} = -\frac{1}{4\pi^3}\int d\mathrm{LIPS}\,\mathcal{A}_{luqe}(1_e, 2_l, 3'_{\bar{e}}, 4'_{\bar{l}}) \times \mathcal{A}_{\mathrm{SM}}(4'_e, 3'_l, 3_{W_-^a}, 4_{H^\dagger})$$

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$h_{total} = -2$

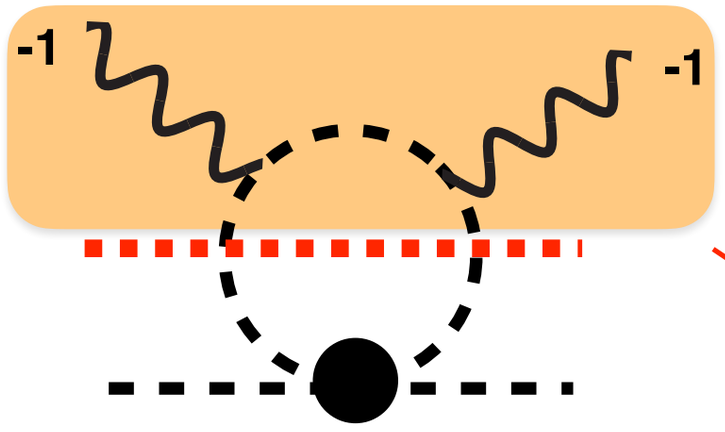
Absent in the SM

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No p^2H^4 corrections to Hγγ

e.g.

$$(H^\dagger D_\mu H)^2$$



$$F_{\alpha\beta}F^{\alpha\beta}h^2$$

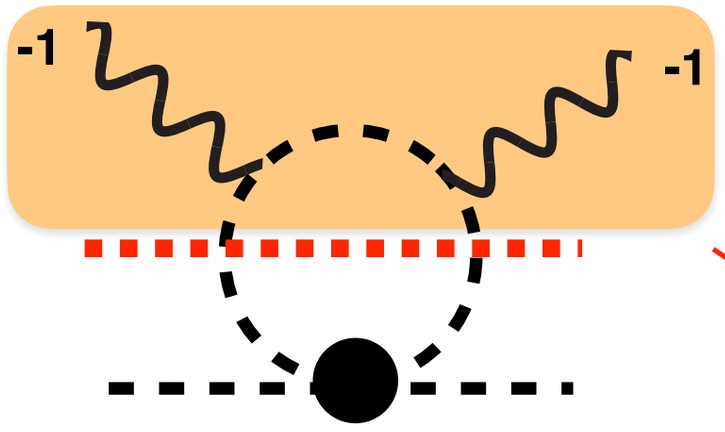
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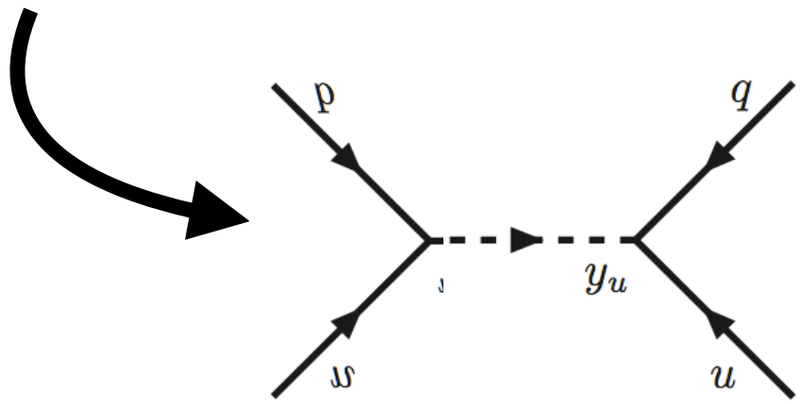


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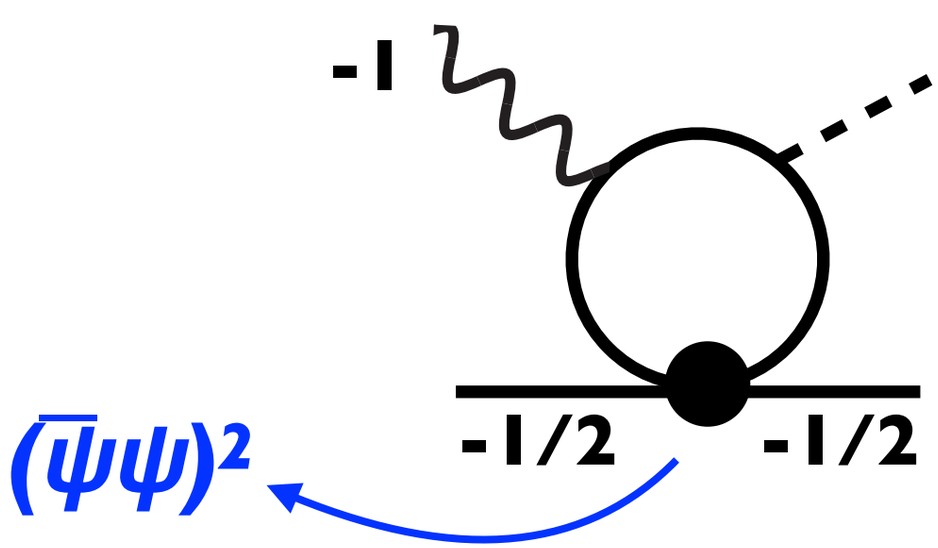
Only exception in the SM of |h|=2 amplitude:

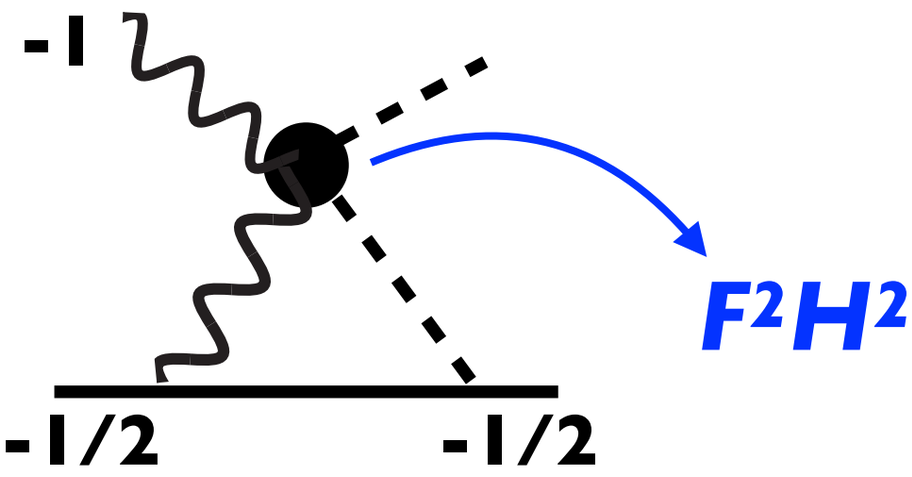


But the **on-shell methods** also tell us about the non-zero result

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about the non-zero result

From **Feynman** approach:

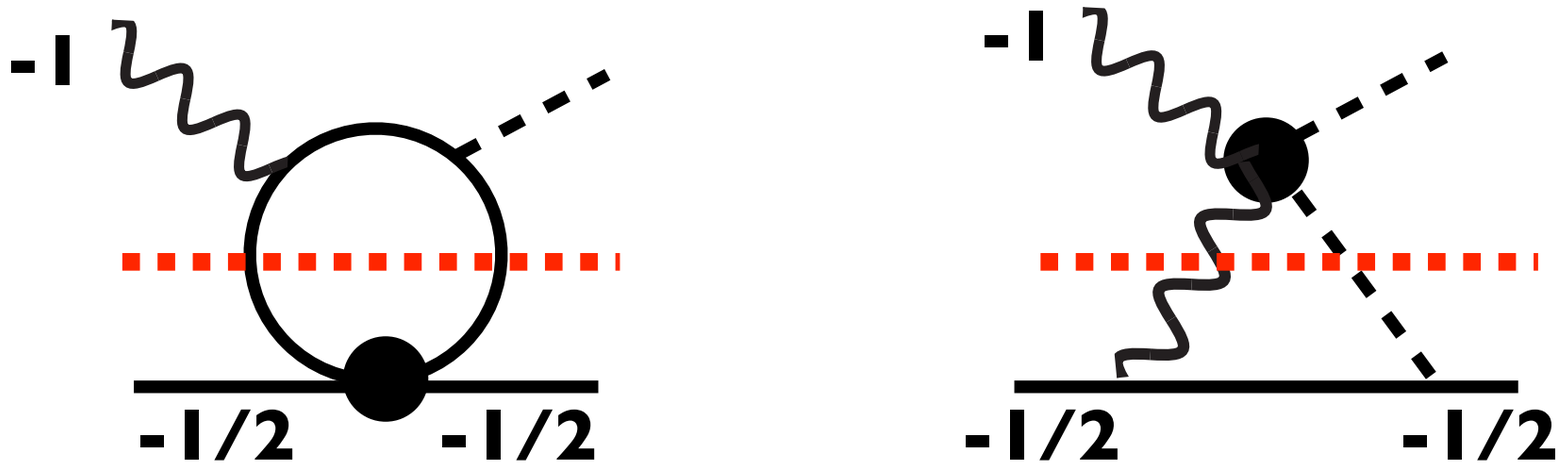




very **different** contributions

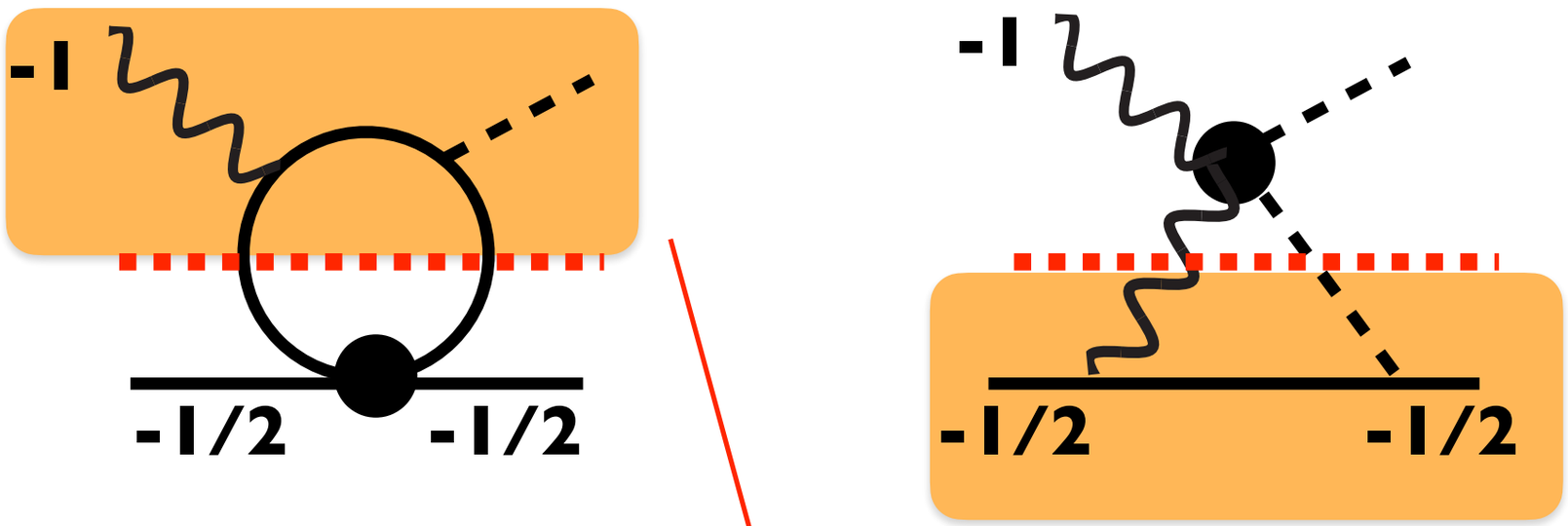
But the **on-shell methods** also tell us about the non-zero result

From **on-shell** approach: $\gamma \mathcal{A}_{\mathbf{i}} \sim \sum\!\!\!\!\!\!\int \mathcal{A}_{\mathbf{j}} \mathcal{A}_{\mathbf{SM}}$



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From **on-shell** approach: $\gamma\mathcal{A}_{\rm i} \sim \sum\!\!\!\!\!\!\int \mathcal{A}_{\rm j}\mathcal{A}_{\rm SM}$

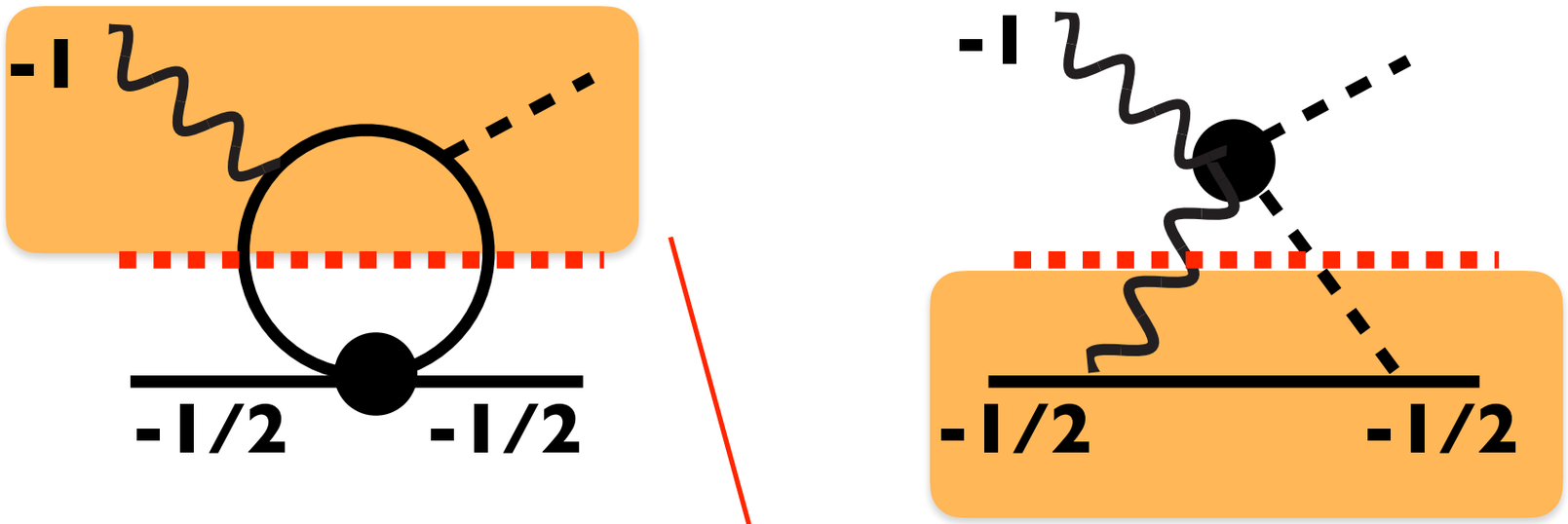


$$\mathcal{A}_{\rm SM}(1_{\bar{\psi}}, 2_{\bar{\psi}}, 3_{V_-}, 4_{H^\dagger})$$

from the same SM amplitude!

But the **on-shell methods** also tell us about the non-zero result

From **on-shell** approach: $\gamma \mathcal{A}_{\mathrm{i}} \sim \sum\!\!\!\!\!\!\int \mathcal{A}_{\mathrm{j}} \mathcal{A}_{\mathrm{SM}}$





No calculation wasted in the on-shell method

$$\mathcal{A}_{\mathrm{SM}}(1_{\bar{\psi}}, 2_{\bar{\psi}}, 3_{V_-}, 4_{H^\dagger})$$

from the same SM amplitude!

At O(E²/Λ²):

$$\mathcal{A}_{F^3}(1_{V_-}, 2_{V_-}, 3_{V_-}) = \frac{C_{F^3}}{\Lambda^2}\langle 12\rangle\langle 23\rangle\langle 31\rangle$$

$$\mathcal{A}_{\rm SM}(1_{\bar\psi}, 2_{\bar\psi}, 3_{V_-}, 4_{H^\dagger})$$

$$\begin{aligned}
\mathcal{A}_{F^2\phi^2}(1_{V_-}, 2_{V_-}, 3_\phi, 4_\phi) &= \frac{C_{F^2\phi^2}}{\Lambda^2}\langle 12\rangle^2\,, \\
\mathcal{A}_{F\psi^2\phi}(1_{V_-}, 2_\psi, 3_\psi, 4_\phi) &= \frac{C_{F\psi^2\phi}}{\Lambda^2}\langle 12\rangle\langle 13\rangle\,, \\
\mathcal{A}_{\psi^4}(1_\psi, 2_\psi, 3_\psi, 4_\psi) &= \left(C_{\psi^4}\langle 12\rangle\langle 34\rangle + C'_{\psi^4}\langle 13\rangle\langle 24\rangle\right)\frac{1}{\Lambda^2}
\end{aligned}$$

n=4
h=-2

$$\begin{aligned}
\mathcal{A}_{\Box\phi^4}(1_\phi, 2_\phi, 3_\phi, 4_\phi) &= \left(C_{\Box\phi^4}\langle 12\rangle[12] + C'_{\Box\phi^4}\langle 13\rangle[13]\right)\frac{1}{\Lambda^2} \\
\mathcal{A}_{\psi\bar\psi\phi^2}(1_\psi, 2_{\bar\psi}, 3_\phi, 4_\phi) &= \frac{C_{\psi\bar\psi\phi^2}}{\Lambda^2}\langle 13\rangle[23]\,, \\
\mathcal{A}_{\psi^2\bar\psi^2}(1_\psi, 2_\psi, 3_{\bar\psi}, 4_{\bar\psi}) &= \frac{C_{\psi^2\bar\psi^2}}{\Lambda^2}\langle 12\rangle[34]\,.
\end{aligned}$$

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\end{aligned}$$

$$\mathcal{A}_{SM}(1_{\bar{\psi}},2_{\psi},3_{H},4_{H^\dagger})$$

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\end{aligned}$$

n=4
h=0

But there is more to say by

angular-momentum decomposition (partial-waves)

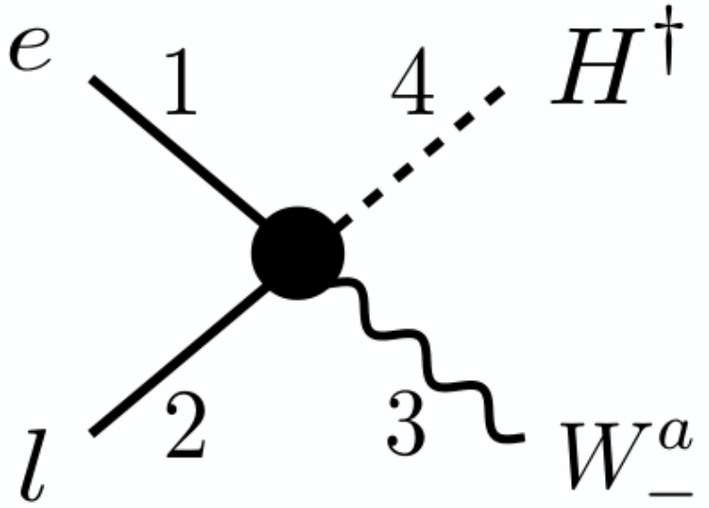
$$\mathcal{A}(1_{h_1}, 2_{h_2}, 3_{h_3}, 4_{h_4}) = e^{i\phi(h_{12}-h_{43})} \left(\frac{\sqrt{s}}{\Lambda}\right)^{w} \sum_{J} n_J d^J_{h_{12}h_{43}}(\theta) a^J$$

But there is more to say by **angular-momentum decomposition (partial-waves)**

$$\mathcal{A}(1_{h_1}, 2_{h_2}, 3_{h_3}, 4_{h_4}) = e^{i\phi(h_{12}-h_{43})} \left(\frac{\sqrt{s}}{\Lambda}\right)^{w} \sum_{J} n_J d^J_{h_{12}h_{43}}(\theta) a^J$$

Example of dipoles:

J=1

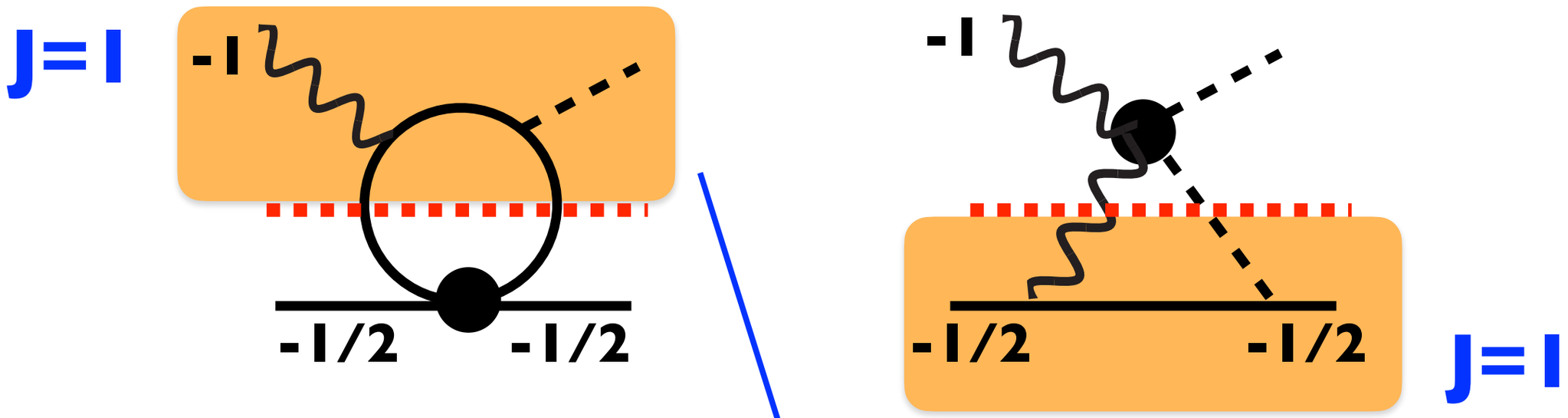


J=1

$$\mathcal{A}(1_e, 2_l, 3_{W_-}, 4_{H^\dagger}) = 3e^{-i\phi} d^{J=1}_{01}(\theta)\, a^{J=1}$$

only one partial-wave!

But there is more to say by **angular-momentum decomposition (partial-waves)**



Not needed the full SM amplitude, only: $a_{\rm SM}^{J=1}$

B. vonHarling, P. Baratella, C. Fernandez, AP 2010.13809

☞ **angular-momentum selection rules:**

see also arXiv:2001.04481

Amplitudes with J≠1 cannot contribute to dipoles

Anomalous Dimensions as a product of partial-waves

B. vonHarling, P. Baratella, C. Fernandez, AP 2010.13809

$$\gamma_i \sim a^J_{\rm SM} a^J_{\rm BSM}$$

$1/\Lambda^2$ amplitude

$$\mathcal{A}_{WHle}(1_e, 2_{l_j}, 3_{W_-^a}, 4_{H_i^\dagger}) = \frac{C_{WHle}}{\Lambda^2}\langle 31\rangle\langle 32\rangle (T^a)_{ij}\,,$$

$$\mathcal{A}_{eluq,0}(1_e, 2_{l_i}, 3_u, 4_{q_j}) = \frac{C_{eluq,0}}{\Lambda^2}\langle 12\rangle\langle 34\rangle\,\epsilon_{ij}\,,$$

$$\mathcal{A}_{eluq,1}(1_e, 2_{l_i}, 3_u, 4_{q_j}) = \frac{C_{eluq,1}}{\Lambda^2}\frac{1}{2}\left(\langle 23\rangle\langle 41\rangle + \langle 13\rangle\langle 42\rangle\right)\epsilon_{ij}\,,$$

$$\mathcal{A}_{W^2H^2}(1_{W_-^a}, 2_{H_i}, 3_{W_-^a}, 4_{H_i^\dagger}) = \frac{C_{W^2H^2}}{\Lambda^2}\langle 13\rangle^2\,.$$

n=4
h=-2

Anomalous dimension mixings:

$$\begin{pmatrix} \gamma_{WHle}\ C^{-1}_{WHle}a^1_{WHle} \\ \gamma_{eluq,1}\ C^{-1}_{eluq,1}a^1_{eluq,1} \\ \gamma_{W^2H^2}\ C^{-1}_{W^2H^2}a^1_{W^2H^2} \end{pmatrix} = -\frac{\widetilde{a}^{J=1}_{\rm SM}}{8\pi^2}\begin{pmatrix} \times & -N_c y_u & y_e \\ -y_u & \times & 0 \\ y_e & 0 & \times \end{pmatrix}\begin{pmatrix} a^1_{WHle} \\ a^1_{eluq,1} \\ a^1_{W^2H^2} \end{pmatrix}$$

Color factors & signs from fermions

After treating IR-div, *shocking* **simplicity** for **gravity**:

$\mathbf{a^J}$	$J=0$	$J=2$	$J=4$	
$\mathcal{A}_{\mathrm{GR}_{+-}}$	0	-6	$-\frac{25}{3}$	
$\mathcal{A}_{\mathrm{GR}_{-}}$	$\frac{5}{3}$	$\frac{1}{15}$	0	$\times \frac{r}{16\pi^2}$
$\widehat{\mathcal{A}}_{R^3}$	$\frac{1}{6}$	$-\frac{1}{30}$	0	$\times C_{R^3}$
$\mathcal{A}_{R^4}$	$\frac{7}{5}$	$\frac{4}{35}$	$\frac{1}{315}$	$\times C_{R^4}$

$$\gamma_{R^3}\widehat{\mathcal{A}}_{R^3} = \frac{1}{4\pi^2}\frac{r}{16\pi^2}\frac{s^3}{M_P^6}P_2\left(\frac{t-u}{s}\right) + \text{crossing}$$

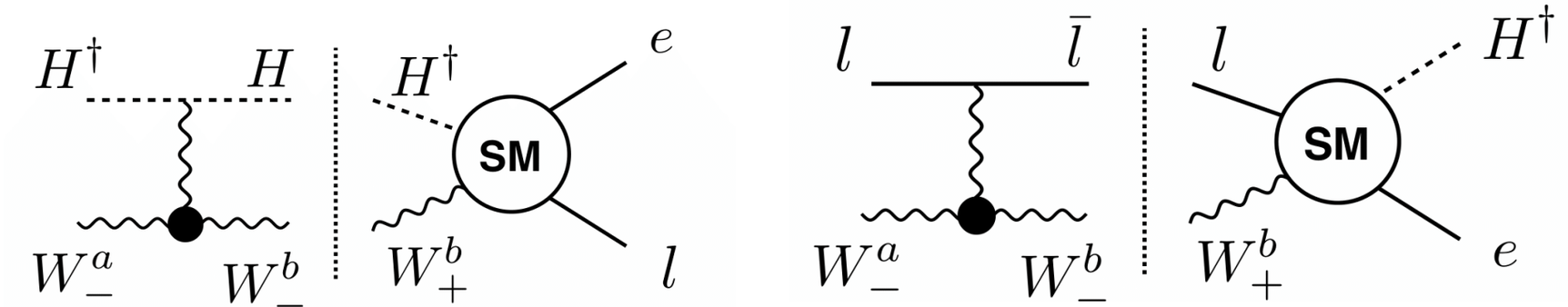
$$\gamma_{R^4}\mathcal{A}_{R^4} = -C_{R^4}\frac{5}{8\pi^2}\left(\frac{s}{M_P^2}\right)^4 a_{R^3}^{J=2}\, a_{\mathrm{GR}_{+-}}^{J=2}|_{\mathrm{reg}}\, P_2\left(\frac{t-u}{s}\right) + \text{crossing}$$

Some contributions are not so simple:

- Renormalization increasing the legs:

$$\mathcal{A}_{W^3} \to \text{dipole}$$

n=3 n=4

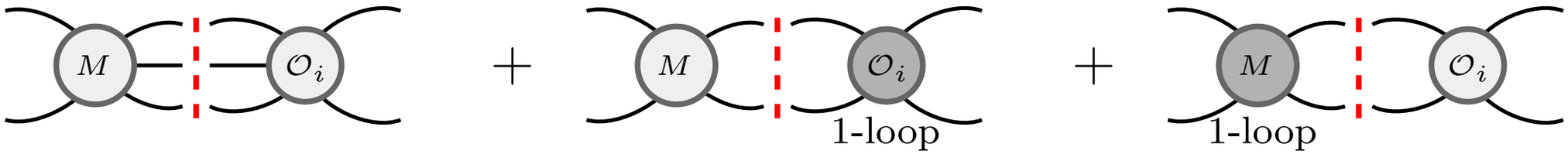


appear **logarithms** (e.g. Log[s/t]) that, although cancel in the total sum, complicate the calculation

Beyond one-loop

2005.06983
2005.12917
2112.12131

Two-loop:

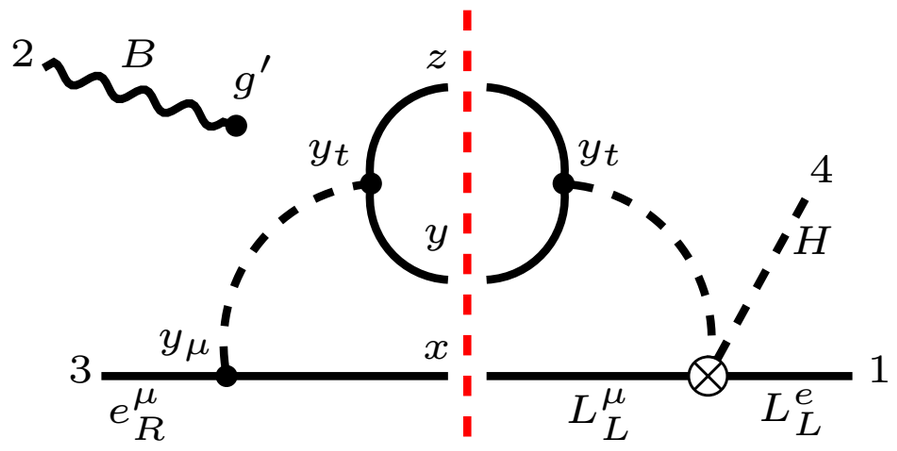


Two-loops for μ→eγ

J. Elias-Miro, C. Fernandez, M. Gümüs, AP 2112.12131

$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_R\gamma^\mu\mu_R)$ affects μ→eγ at the two-loop level:

↳ **Z→μe**



product of tree-level amplitudes

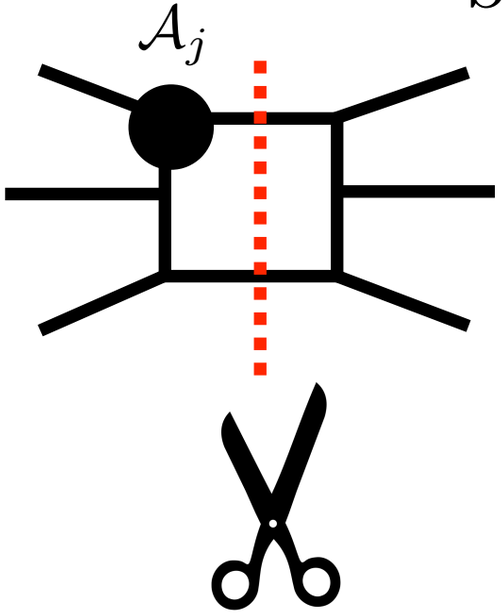
Finite terms?

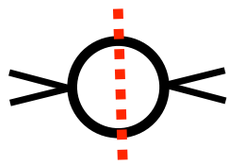
Difficult in general, **but** simplifies a lot
for BSM calculations, where new physics scale **M >> E_{exp}**

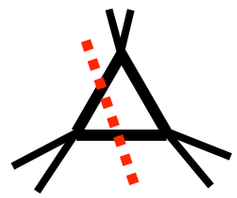
New insights from the **amplitude** method!

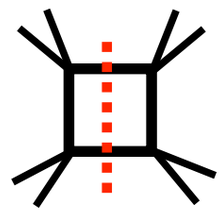
After one-loop reduction to Passarino-Veltman integrals

$$\mathcal{A}_\mathbf{i} = \sum_{\text{bubble}} \mathbf{c_2 I_2} + \sum_{\text{triangle}} \mathbf{c_3 I_3} + \sum_{\text{box}} \mathbf{c_4 I_4} + \mathbf{rational}$$









double cut

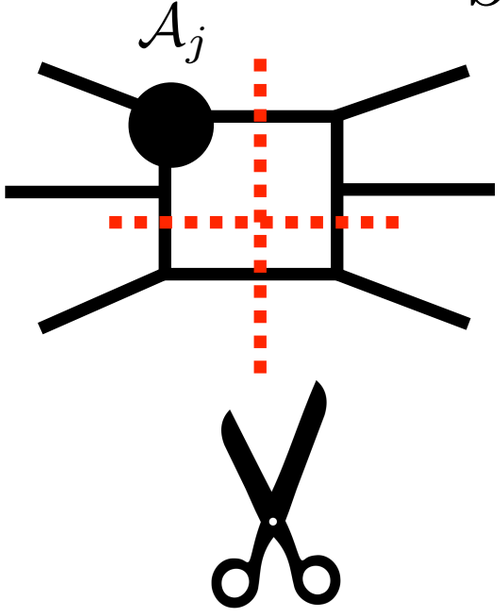
(internal particles on-shell)

It can be shown
to be zero
at order $1/M^2$

L. Delle Rosse, B. von Harling, AP in 2201.10572

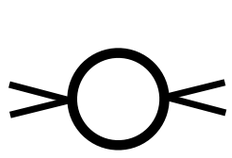
After one-loop reduction to Passarino-Veltman integrals

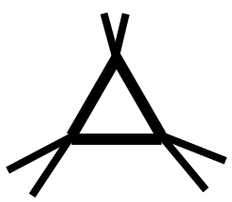
$$\mathcal{A}_\mathbf{i} = \sum_{\text{bubble}} \mathbf{c_2 I_2} + \sum_{\text{triangle}} \mathbf{c_3 I_3} + \sum_{\text{box}} \mathbf{c_4 I_4} + \mathbf{rational}$$



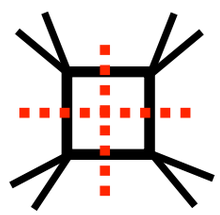
Quadruple cut

(internal particles on-shell)





Not possible
to perform
a 4-cut



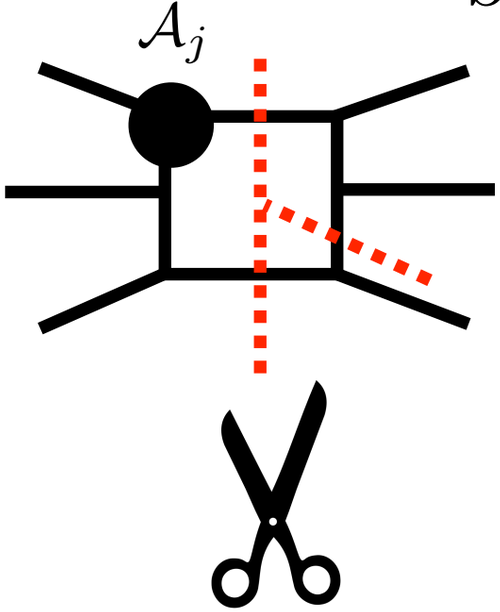


It can be shown
to be zero
at order $1/M^2$

L. Delle Rosse, B. von Harling, AP in 2201.10572

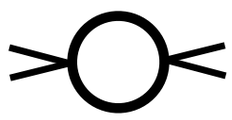
After one-loop reduction to Passarino-Veltman integrals

$$\mathcal{A}_\mathbf{i} = \sum_{\text{bubble}} \mathbf{c_2 I_2} + \sum_{\text{triangle}} \mathbf{c_3 I_3} + \sum_{\text{box}} \mathbf{c_4 I_4} + \mathbf{rational}$$

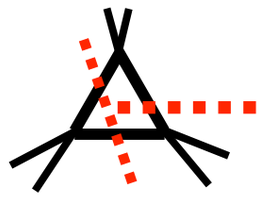


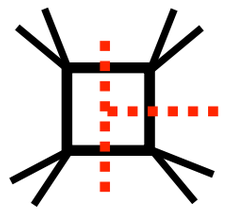
triple cut

(internal particles on-shell)



Not possible
to perform
a 4-cut





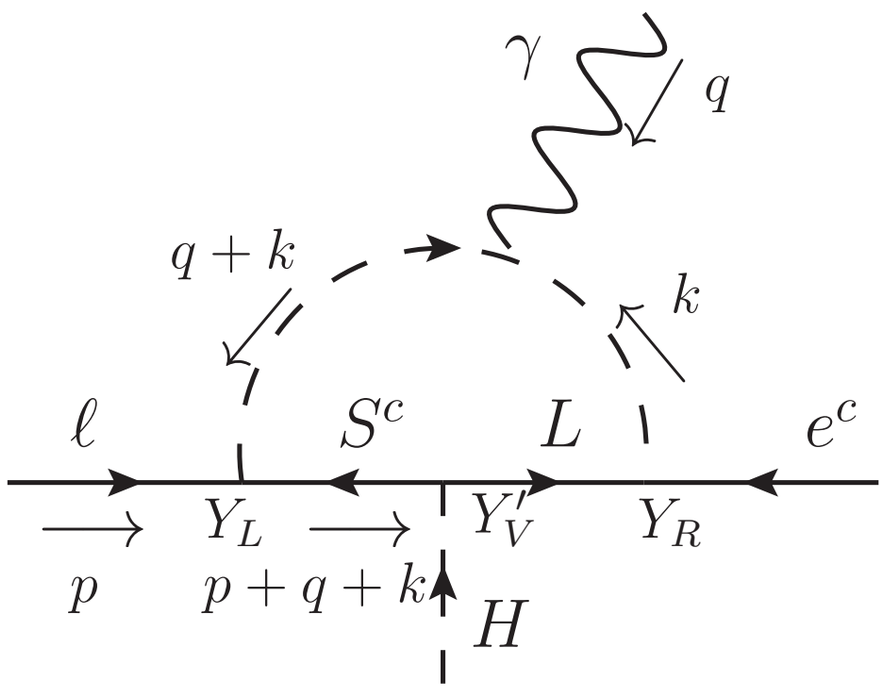


It can be shown
to be zero
at order $1/M^2$

L. Delle Rosse, B. von Harling, AP in 2201.10572

Finite terms to g-2

No contribution $O(1/M^2)$ to **dipoles from** a **heavy singlet + doublet fermion**:



~ $O(1/M^4)$

N. Arkani-Hamed, K. Harigaya 2106.01373

Finite terms to g-2

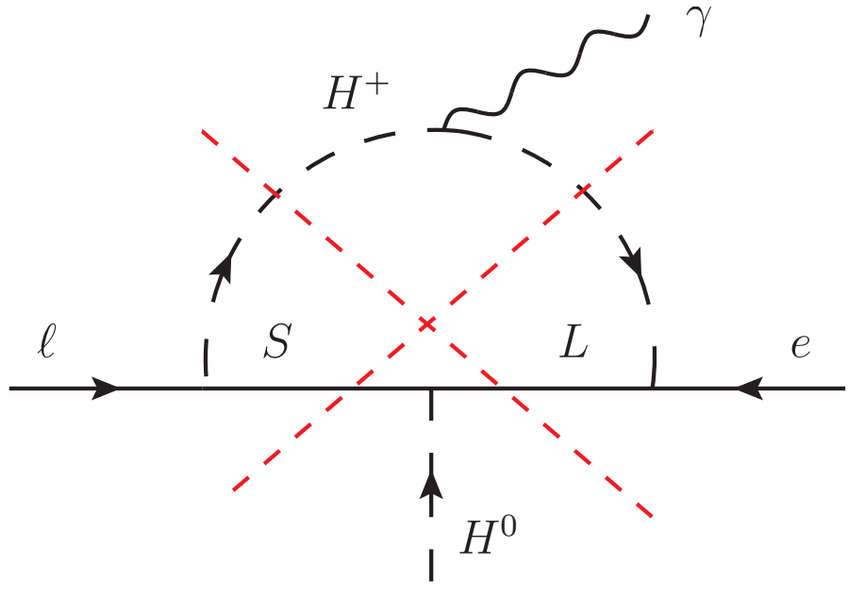
No contribution $O(1/M^2)$ to **dipoles**
from a **heavy singlet + doublet fermion**:

L. Delle Rosse, B. von Harling, AP in 2201.10572

from on-shell methods:

$$p_{H^0} = 0$$

$$M_S^2 \neq M_L^2$$



= 0

Not possible to put
S and L on-shell
if the have
different masses

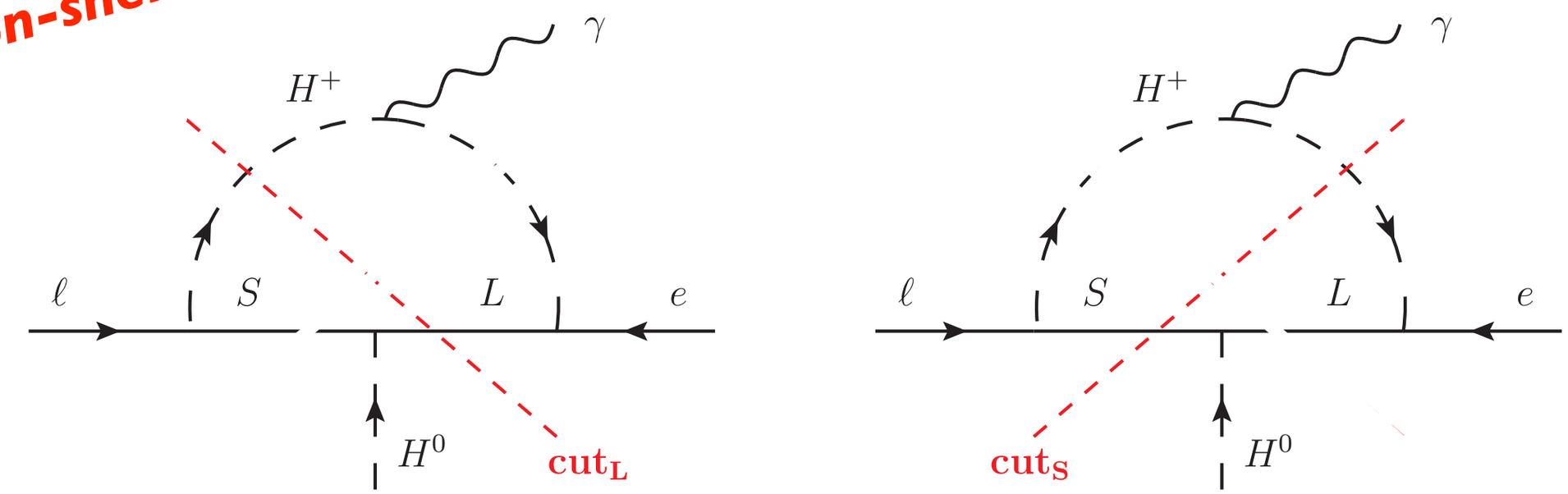
similarly, not possible 3-cuts

Finite terms to g-2

No contribution $O(1/M^2)$ to **dipoles**
from a **heavy singlet + doublet fermion**:

from on-shell methods:

L. Delle Rosse, B. von Harling, AP in 2201.10572



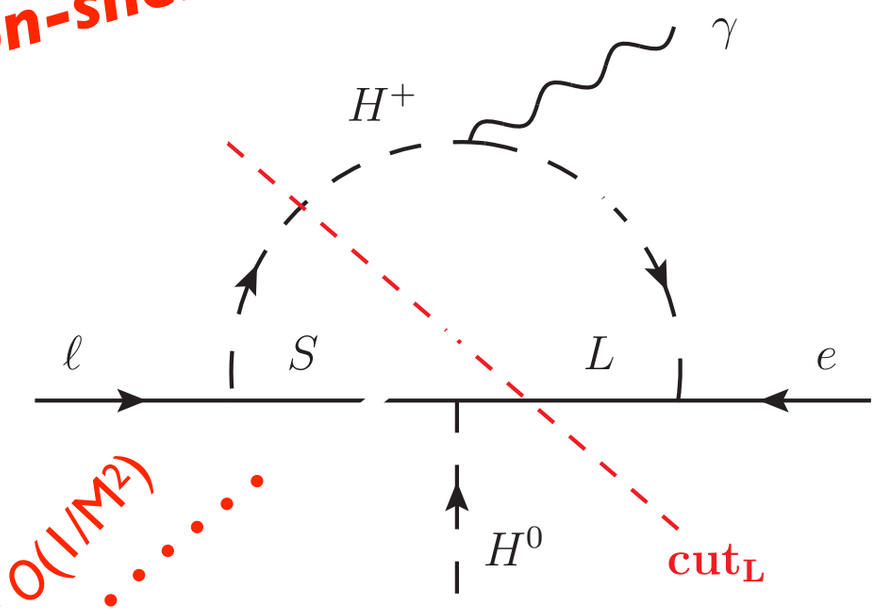
even under S↔L

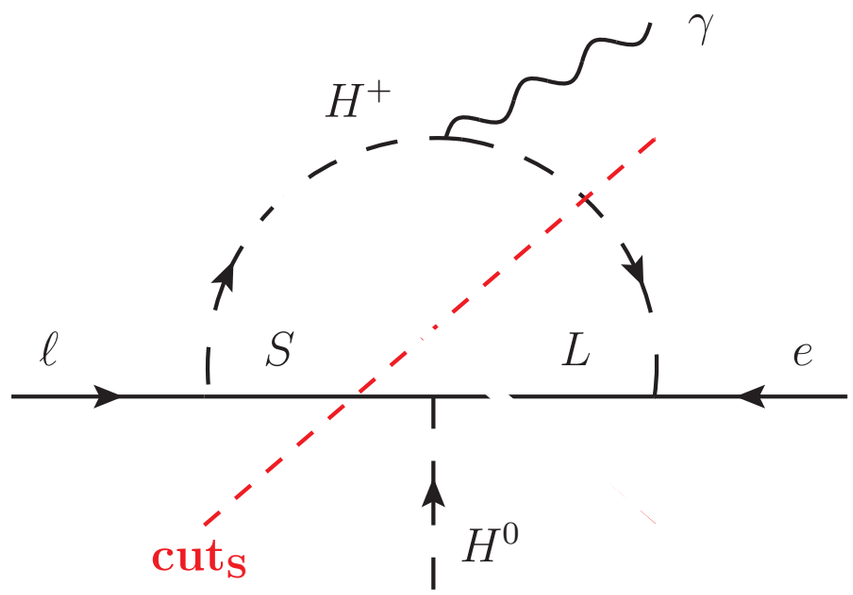
Finite terms to g-2

No contribution $O(1/M^2)$ to **dipoles from** a **heavy singlet + doublet fermion**:

from on-shell methods:

L. Delle Rosse, B. von Harling, AP in 2201.10572





amplitude at $O(1/M^2)$

$$\frac{1}{M_L^2 - M_S^2}$$

even under S↔L

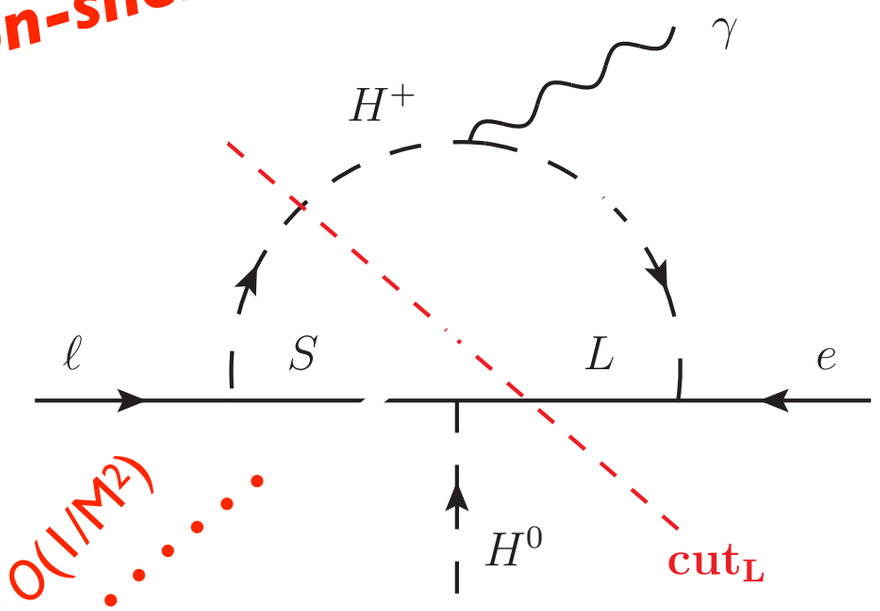
odd under S↔L

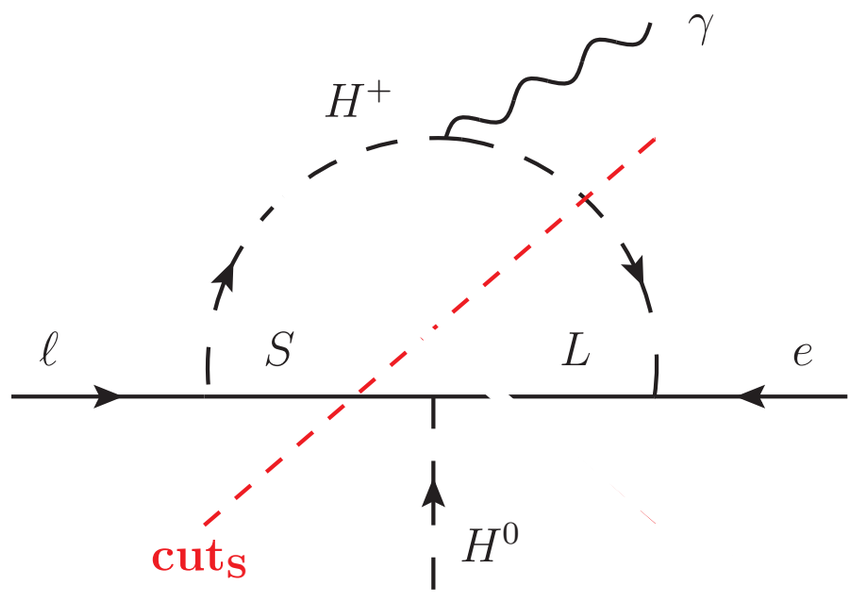
Finite terms to g-2

No contribution $O(1/M^2)$ to **dipoles**
from a **heavy singlet + doublet fermion**:

L. Delle Rosse, B. von Harling, AP in 2201.10572

from on-shell methods:





amplitude at $O(1/M^2)$

$$\frac{1}{M_L^2 - M_S^2}$$

even under S↔L

odd under S↔L

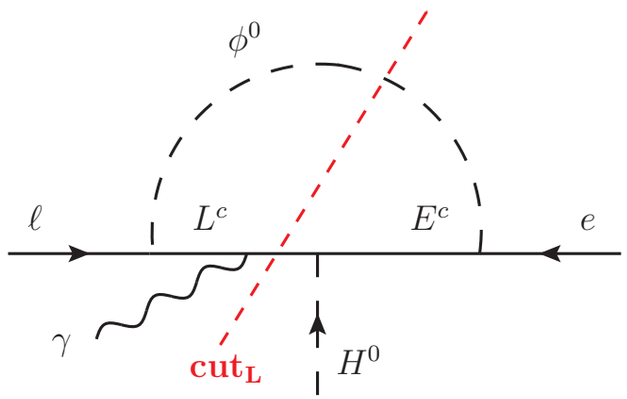
} ☞ **zero**

Finite terms to g-2

L. Delle Rosse, B. von Harling, AP in 2201.10572

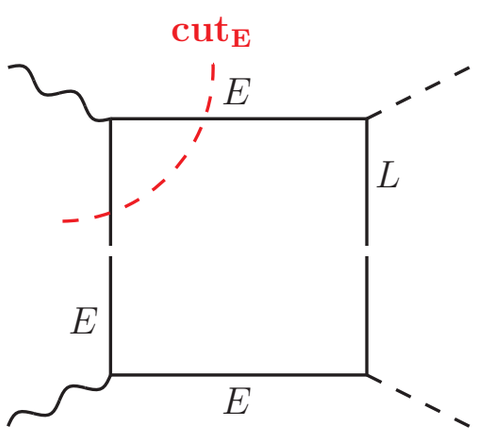
Following the same argument, more zeros can be found:

- **Scalar + heavy doublet + charged fermion**:



+ (L↔E) ☞ **zero**

- **Beyond g-2: Zeros in hγγ**



+ (L↔E) ☞ **zero**

Conclusions

Amplitude methods seems quite **suited for** dealing with EFTs for **BSM**

- Allows to construct **BSM without Lagrangians**
- Calculation of loop effects:

Simpler with easy recycling

☞ many "emergent" **selection rules**

☞ many relations **between anomalous dimensions**

where Feynman approach is quite obscure