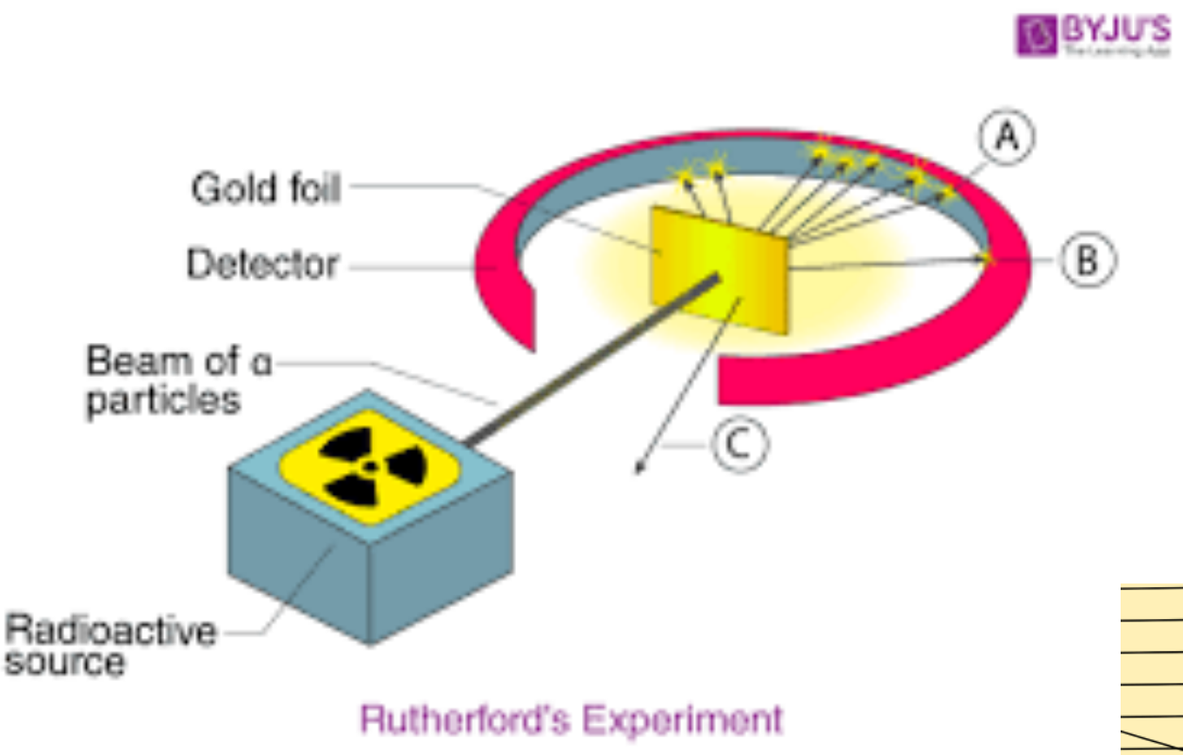
KIAS Particle Physics Summer Camp 2022
August 22 - 25, 2022

중성미자 이론

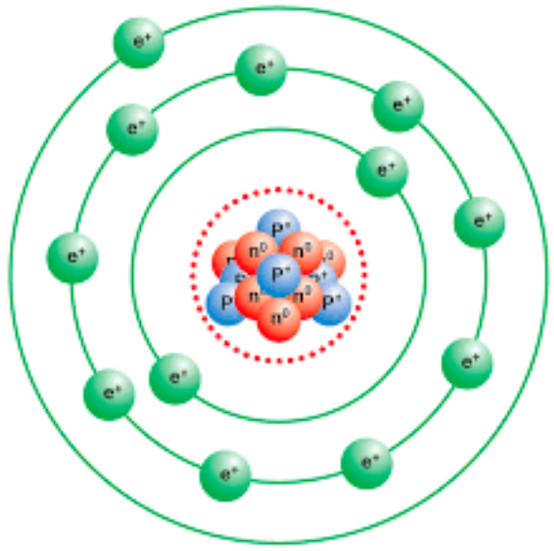
송정현 (건국대학교)

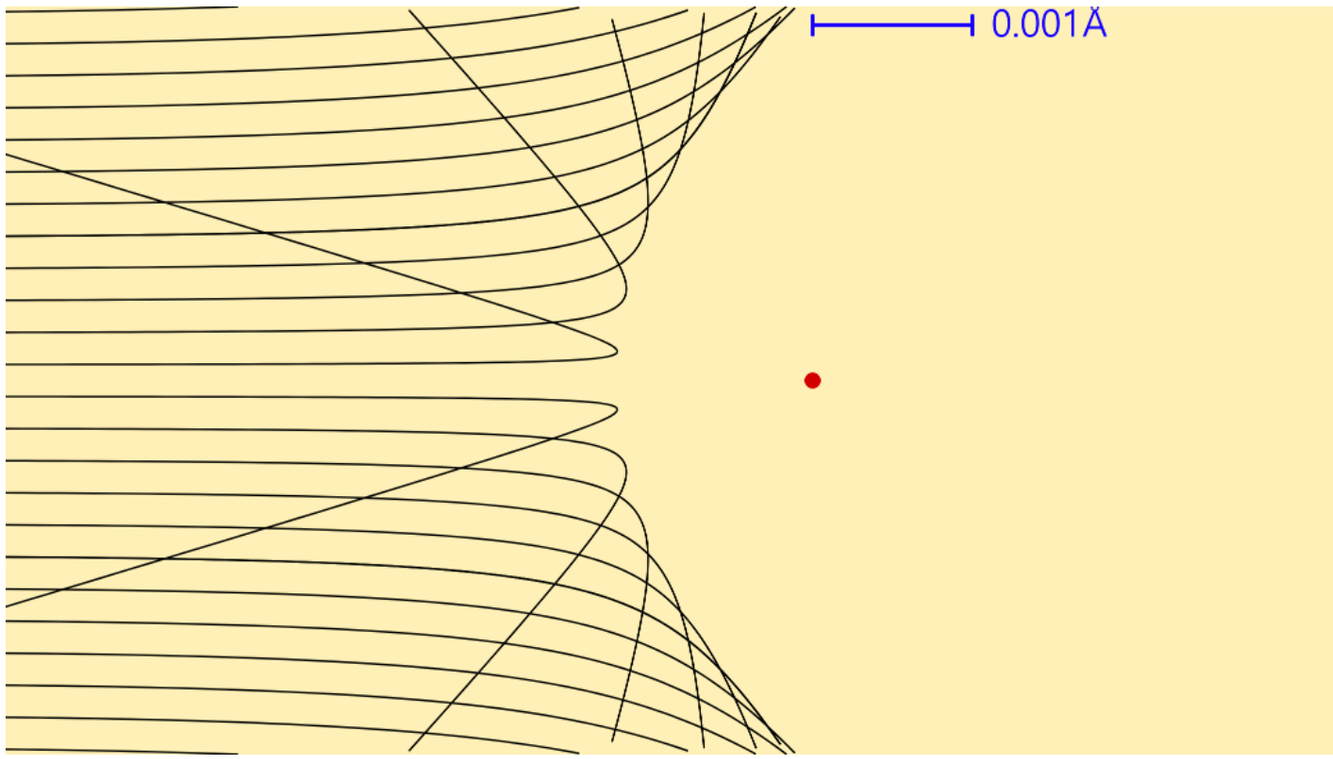
Neutron

Rutherford Atomic model



Rutherford's Experiment



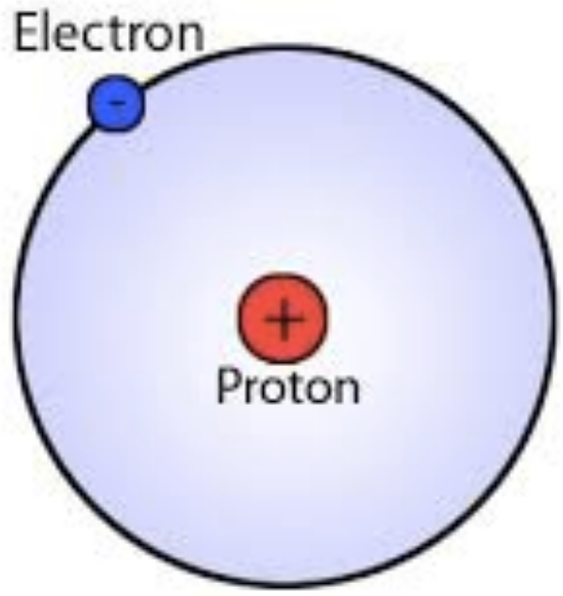


원자

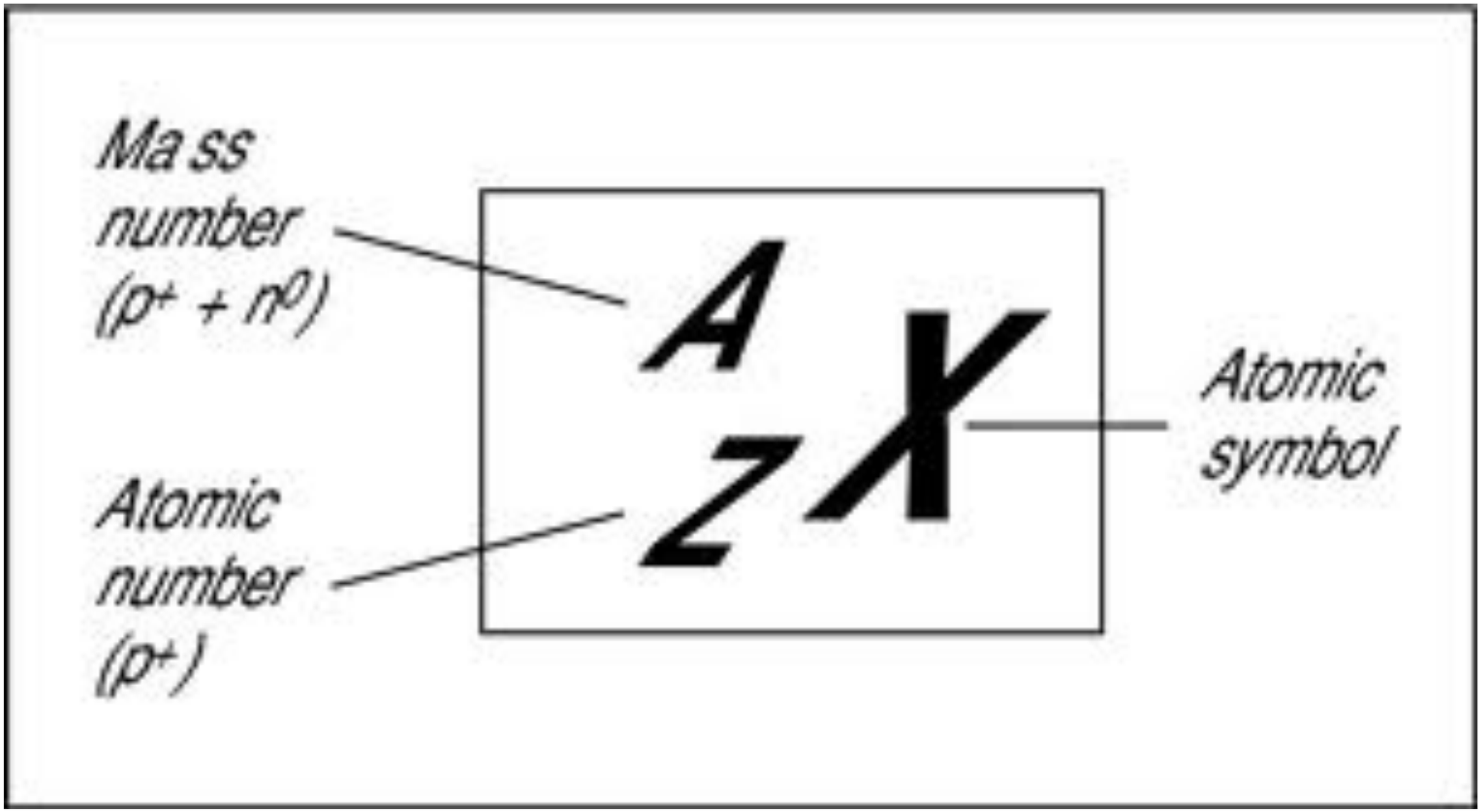
- 양전하를 띤 무거운 핵
  + 빈 공간
  + 음전하를 띤 가벼운 전자

If the atom is the size of Giants Stadium,
the nucleus is a marble at the center.

가장 가벼운 원자: 수소



- 기본 원자
- 전하, 질량수
- 수소 원자의 핵: 양성자
- 수소의 질량은 거의 핵의 질량 (전자의 질량의 대략 2000배)


Mass number (p+ + n0)
A
Z
X
Atomic symbol
Atomic number (p+)

$$\text{Mass number} = \frac{\text{Mass of an atom}}{\text{Mass of H}}$$

U.S. Department of Commerce

Physics Laboratory
physics.nist.gov

Standard Reference Data
www.nist.gov/srd

13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
					2 $^1S_0$ **He** Helium 4.002602 $1s^2$ 24.5874
5 $^2P^o_{1/2}$ **B** Boron 10.811 $1s^2 2s^2 2p$ 8.2980	6 $^3P_0$ **C** Carbon 12.0107 $1s^2 2s^2 2p^2$ 11.2603	7 $^4S^o_{3/2}$ **N** Nitrogen 14.0067 $1s^2 2s^2 2p^3$ 14.5341	8 $^3P_2$ **O** Oxygen 15.9994 $1s^2 2s^2 2p^4$ 13.6181	9 $^2P^o_{3/2}$ **F** Fluorine 18.9984032 $1s^2 2s^2 2p^5$ 17.4228	10 $^1S_0$ **Ne** Neon 20.1797 $1s^2 2s^2 2p^6$ 21.5645
13 $^2P^o_{1/2}$ **Al** Aluminum 26.9815386 $[Ne]3s^2 3p$ 5.9858	14 $^3P_0$ **Si** Silicon 28.0855 $[Ne]3s^2 3p^2$ 8.1517	15 $^4S^o_{3/2}$ **P** Phosphorus 30.973762 $[Ne]3s^2 3p^3$ 10.4867	16 $^3P_2$ **S** Sulfur 32.065 $[Ne]3s^2 3p^4$ 10.3600	17 $^2P^o_{3/2}$ **Cl** Chlorine 35.453 $[Ne]3s^2 3p^5$ 12.9676	18 $^1S_0$ **Ar** Argon 39.948 $[Ne]3s^2 3p^6$ 15.7596

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???

- 원자핵의 질량수 ≠ 원자핵의 전하량

- 전자가 원자핵 안에 있어서 중성화?

전자가 원자핵 안에 있을 때 원자핵의 전체 스핀?

- 스핀의 합

$$
\begin{aligned}
\chi_{s=1,m=1} &= \chi_+^{(1)}\chi_+^{(2)} \\
\chi_{s=1,m=0} &= \frac{1}{\sqrt{2}}\left(\chi_+^{(1)}\chi_-^{(2)} + \chi_-^{(1)}\chi_+^{(2)}\right) \\
\chi_{s=1,m=-1} &= \chi_-^{(1)}\chi_-^{(2)} \\
\chi_{s=0,m=0} &= \frac{1}{\sqrt{2}}\left(\chi_+^{(1)}\chi_-^{(2)} - \chi_-^{(1)}\chi_+^{(2)}\right)
\end{aligned}
$$

전자가 원자핵 안에 있을 때 원자핵의 전체 스핀?

- 스핀의 합

Two electrons, each with $s = \frac{1}{2}\hbar$

$$\underbrace{\left|\tfrac{1}{2}-\tfrac{1}{2}\right|}_{0} \leq \Delta \leq \underbrace{\tfrac{1}{2}+\tfrac{1}{2}}_{1}$$

In integer steps, $\Delta = 0, 1$

Product wave functions

$$\chi_1^1 = \uparrow_1 \uparrow_2$$

$$\chi_1^0 = \frac{1}{\sqrt{2}}\left(\uparrow_1 \downarrow_2 + \uparrow_2 \downarrow_1\right)$$

$$\chi_1^{-1} = \downarrow_1 \downarrow_2$$

} $S = 1$ Symmetric

$$\chi_0^0 = \frac{1}{\sqrt{2}}\left(\uparrow_1 \downarrow_2 - \uparrow_2 \downarrow_1\right)$$

} $S = 0$ antisymmetric

Addition of angular momentum with spin

$$\hat{\mathbf{J}} = \mathbf{L} + \hat{\mathbf{S}}$$

$$j = \left| l - \tfrac{1}{2} \right|, l + \tfrac{1}{2}$$

$$m_j = -j, \ldots, +j$$

Eigenvalues of $\hat{J}^2$ are $j(j+1)\hbar^2$

total J ? Scattering Experiment

스핀-1/2 입자 3개

$$j = j_1 \times j_2 = 1 \times \frac{1}{2}$$

$$|j_1 - j_2| \le j \le j_1 + j_2$$

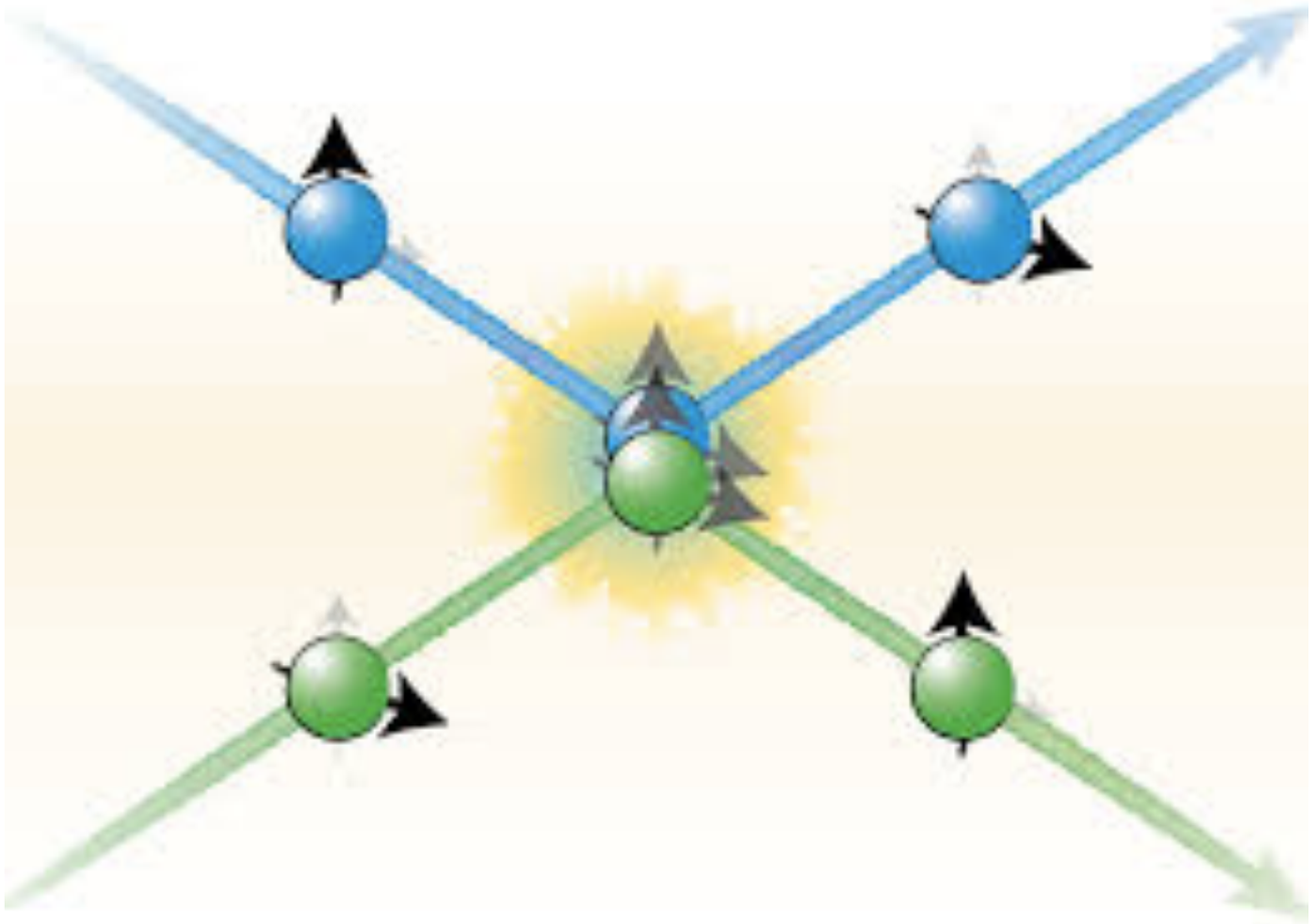
$$m = -j, \cdots, j$$

		3/2	← j
		+3/2	← m
+1	+1/2	1	
m_1	m_2		

		3/2	1/2	← j
		+1/2	+1/2	← m
+1	-1/2	√1/3	√2/3	
0	+1/2	√2/3	-√1/3	
m_1	m_2			

		3/2	1/2	← j
		-1/2	-1/2	← m
0	-1/2	√2/3	√1/3	
-1	+1/2	√1/3	-√2/3	
m_1	m_2			

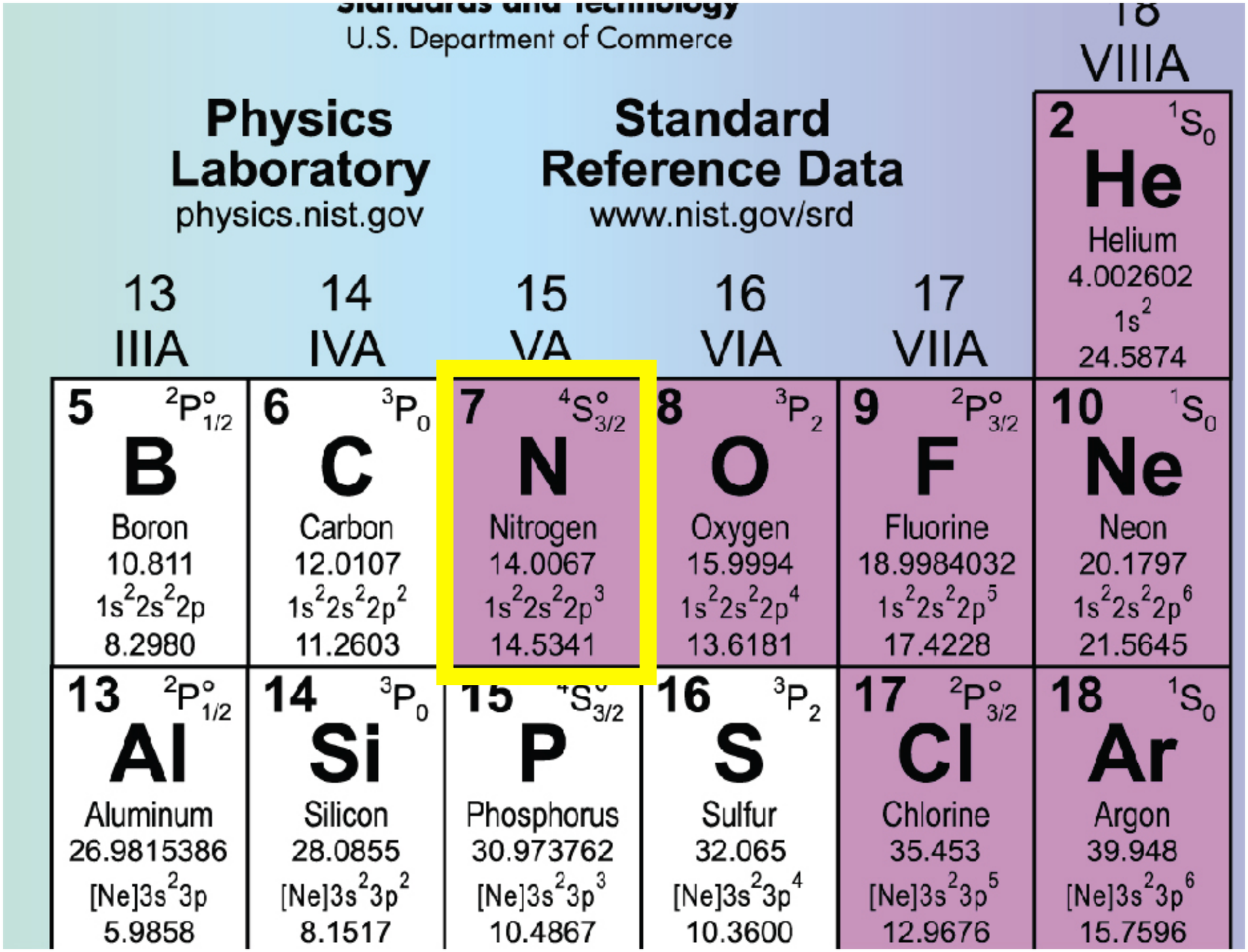
		3/2	← j
		-3/2	← m
-1	-1/2	1	
m_1	m_2		



충돌하는 입자의 스핀에 따라

충돌 후 운동이 다르다

질소


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17 $^2P°_{3/2}$ Cl Chlorine 35.453 [Ne]$3s^23p^5$ 12.9676
18 $^1S_0$ Ar Argon 39.948 [Ne]$3s^23p^6$ 15.7596

질소

- 질량수: 14, 원자 번호: 7

질소

- 질량수: 14, 원자 번호: 7
- **전자를 넣어서 질량수와 전하수를 설명?**

질소

- 질량수: 14, 원자 번호: 7
- 전자를 넣어서 질량수와 전하수를 설명?

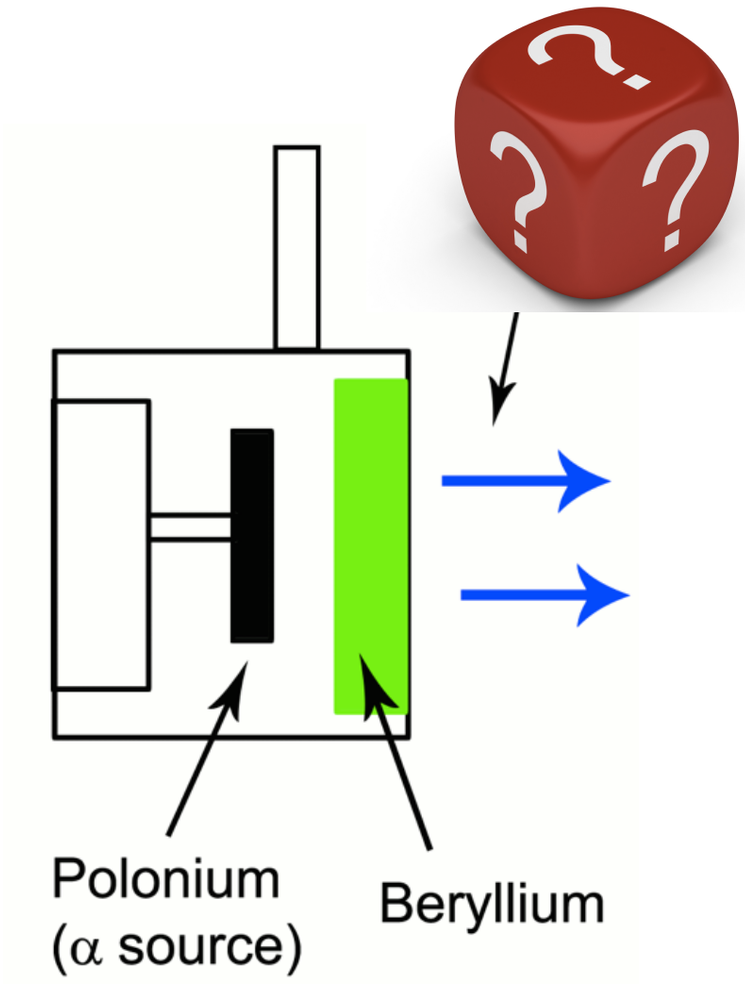
- **14개 양성자 + 7개 전자: 총 21개 (홀수) 스핀-1/2 입자**

질소

- 질량수: 14, 원자 번호: 7
- 전자를 넣어서 질량수와 전하수를 설명?
- 14개 양성자 + 7개 전자: 총 21개 (홀수) 스핀-1/2 입자
- **총 스핀은 반정수**: 충돌 실험의 결과는 스핀-1/2 입자 짝수 개가 필요

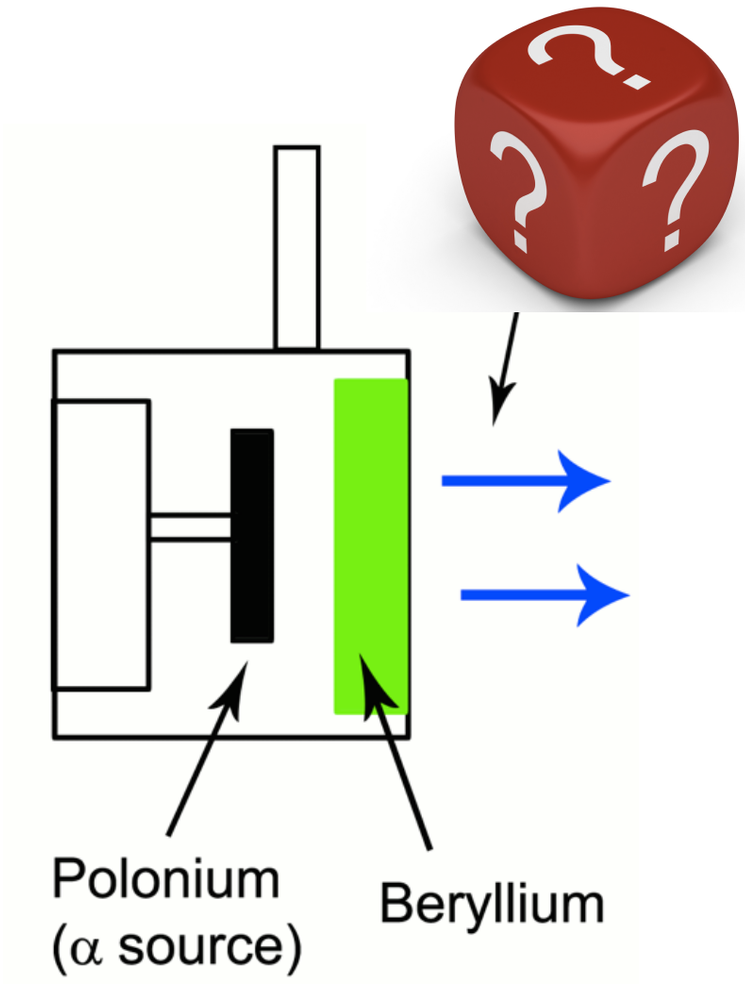
1928년 보테(Bothe)

- 폴로늄에서 나오는 알파선을 베릴륨에 쏘았을 때 나오는 방사선 연구



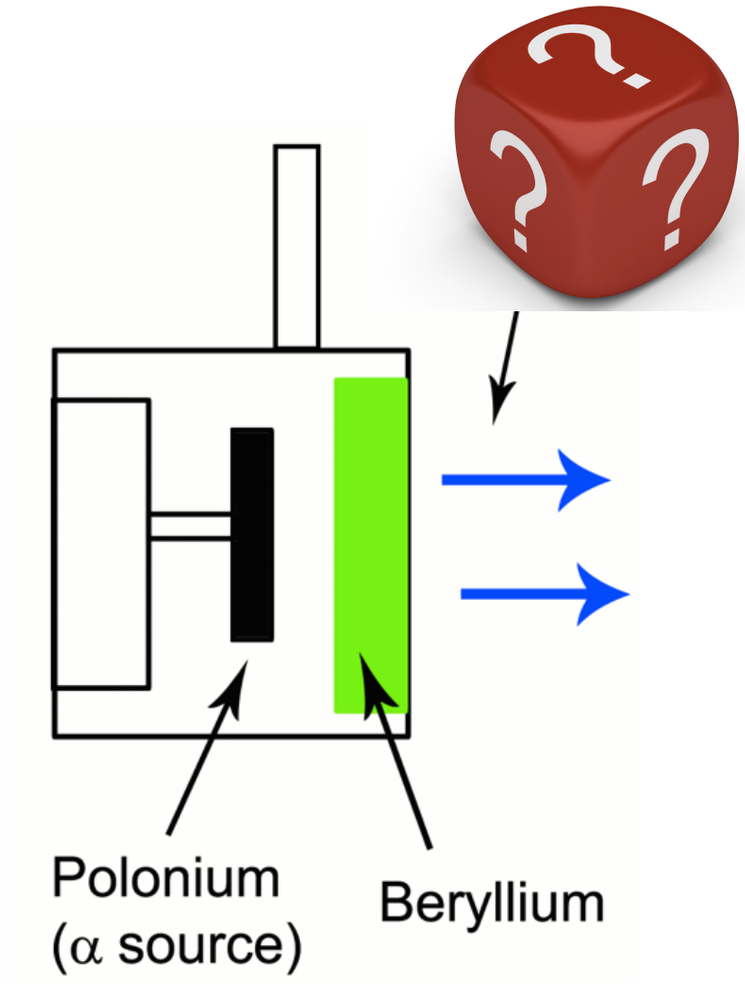
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- 폴로늄에서 나오는 알파선을 베릴륨에 쏘았을 때 나오는 방사선 연구
- 중성



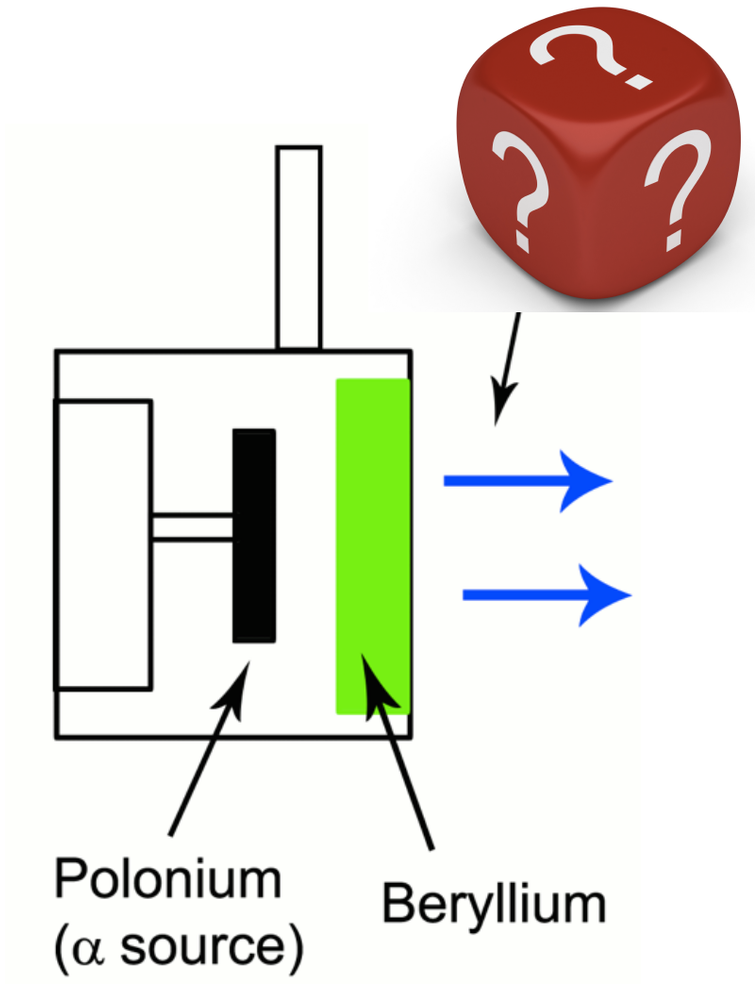
1928년 보테(Bothe)

- 폴로늄에서 나오는 알파선을 베릴륨에 쏘았을 때 나오는 방사선 연구
- 중성
- 감마선?



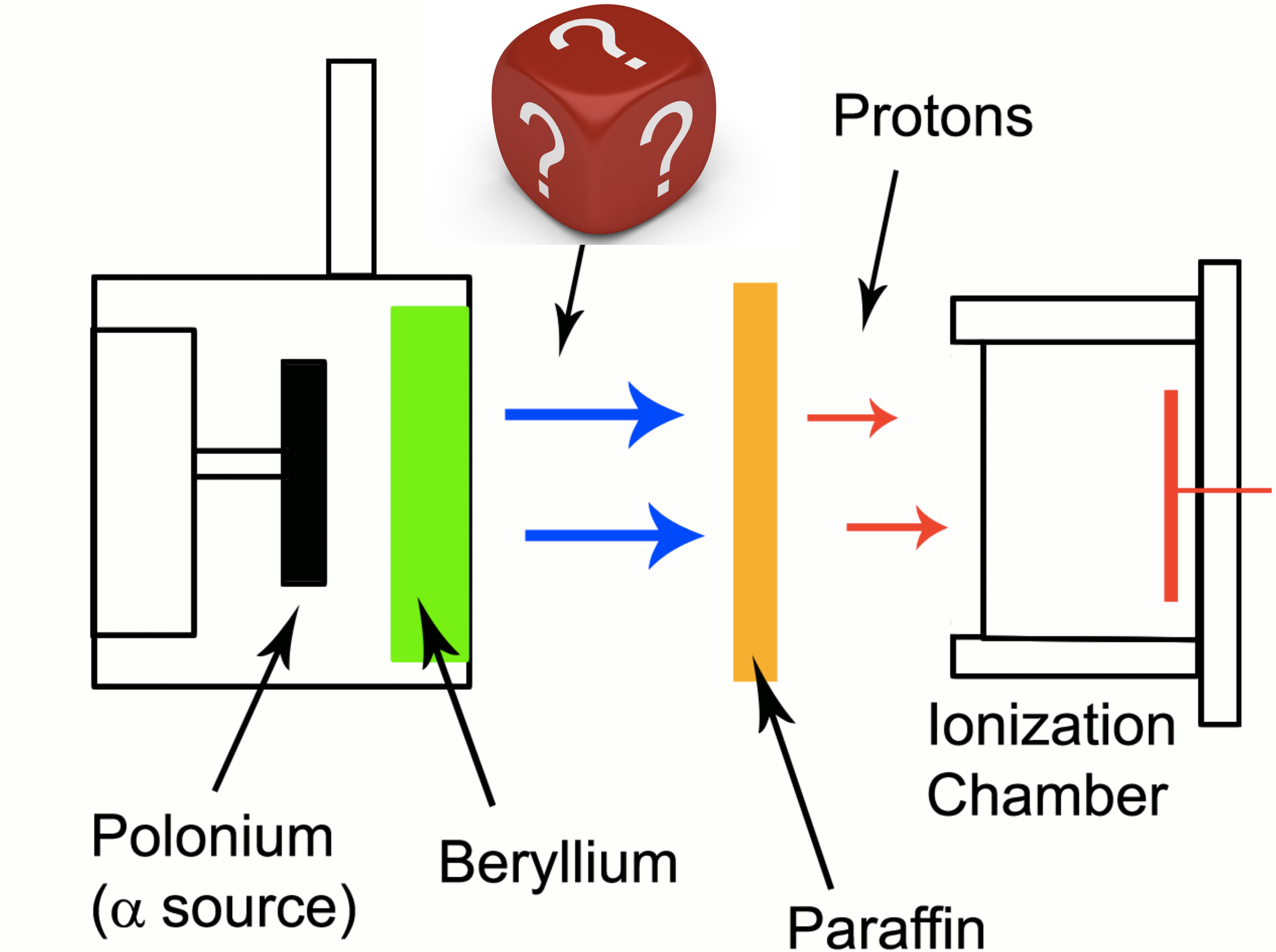
1928년 보테(Bothe)

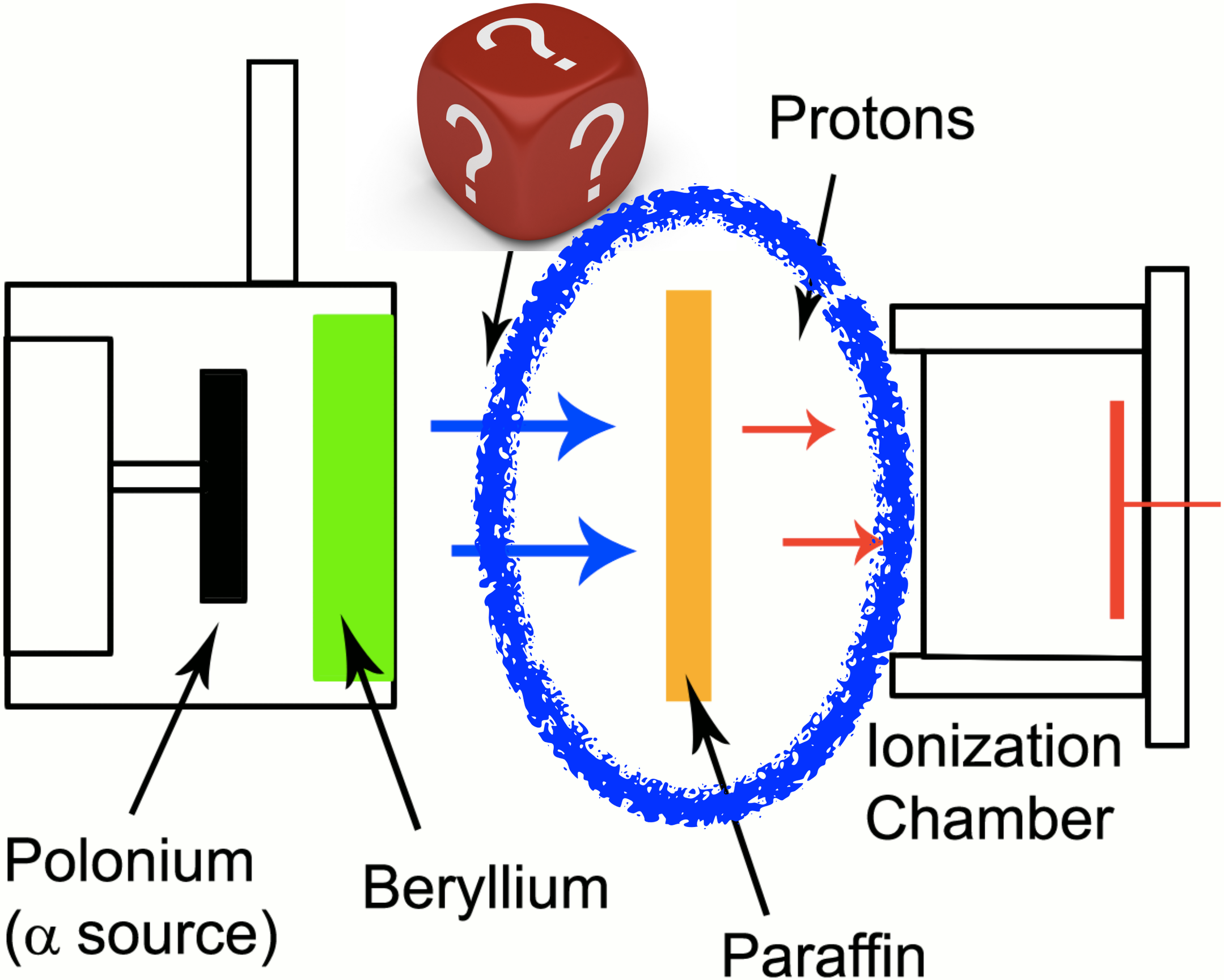
- 폴로늄에서 나오는 알파선을 베릴륨에 쏘았을 때 나오는 방사선 연구
- 중성
- 감마선?
- 처음 쏘았던 알파 입자보다 에너지가 더 큼



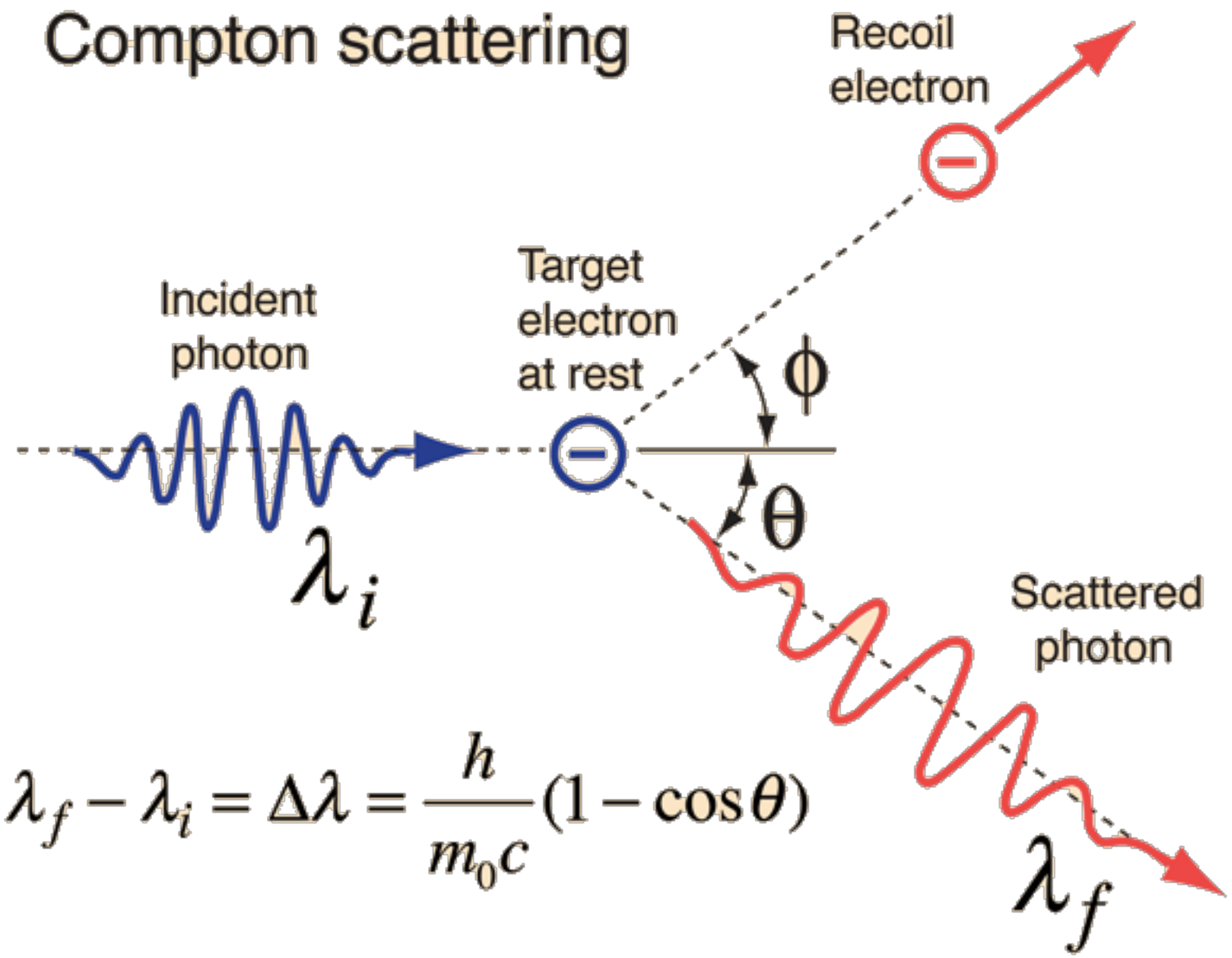
1932년 졸리오-퀴리 부부

- 폴로늄이 가장 많이 확보된 실험실
- 이 방사선을 약화시키기 위하여 파라핀 통과
- 양성자 나옴

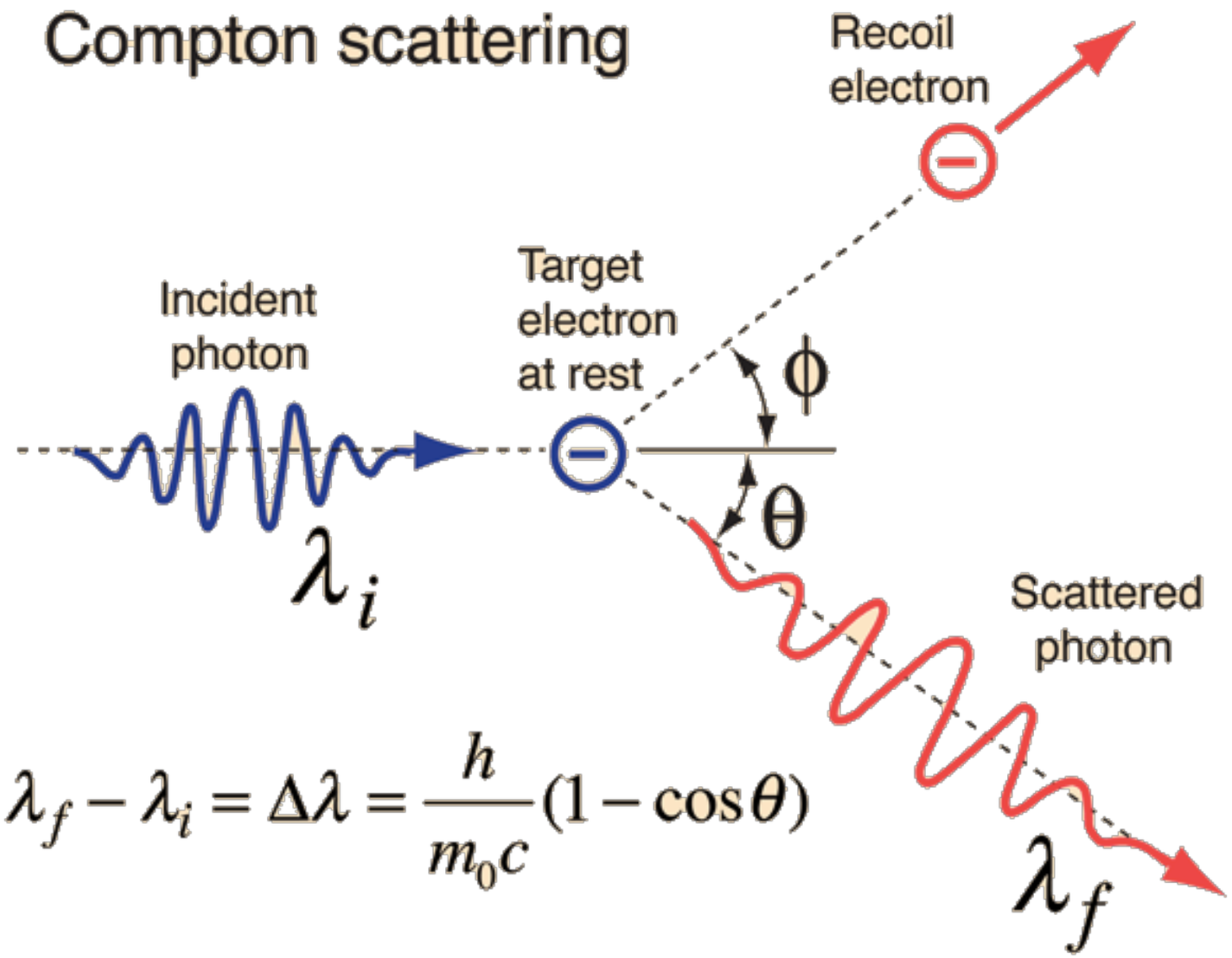

Protons
Polonium
(α source)
Beryllium
Paraffin
Ionization
Chamber


Protons
Polonium
(α source)
Beryllium
Paraffin
Ionization
Chamber

당시 유행?



$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$



Compton scattering of proton?

러더포드의 반격

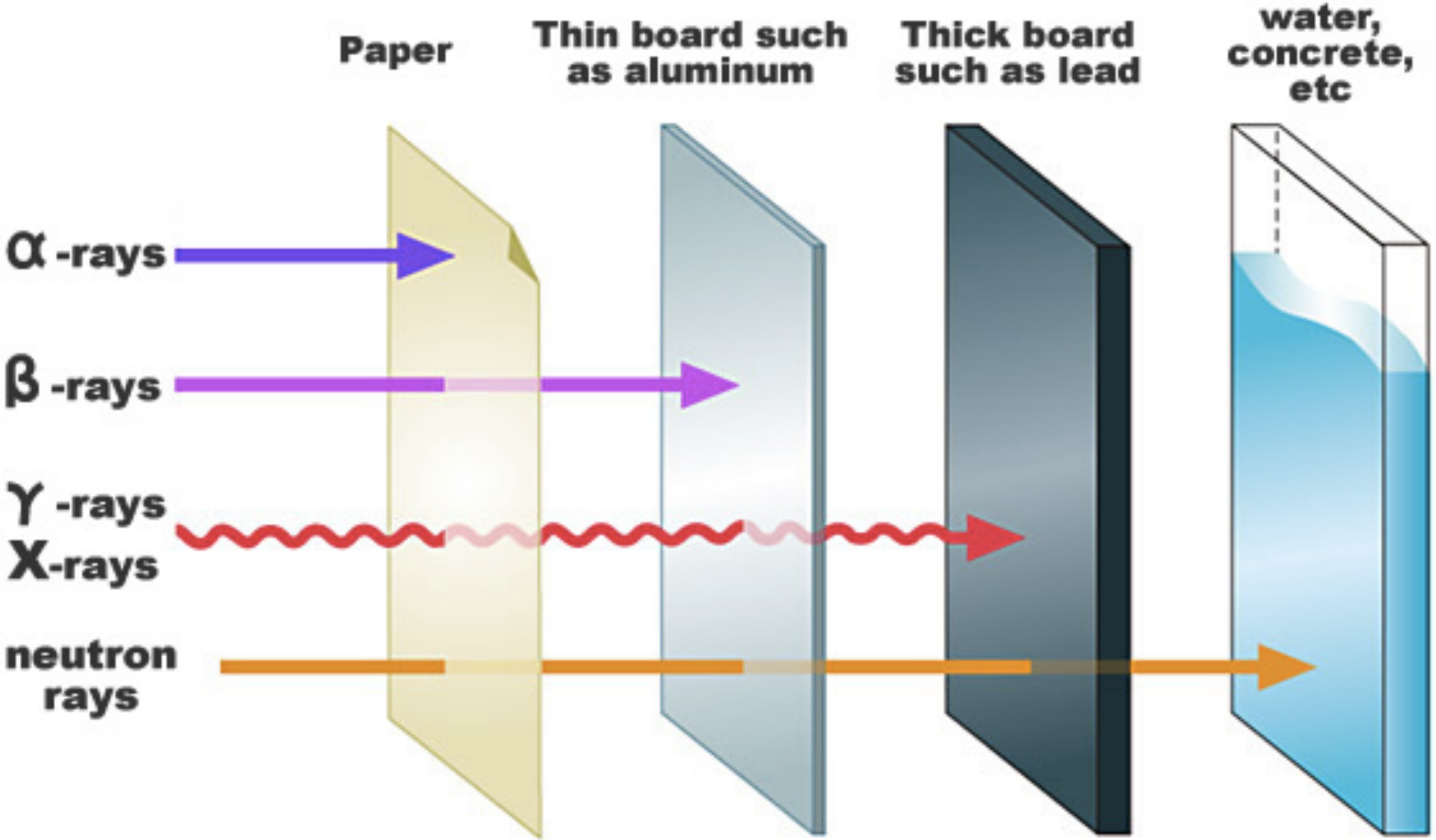
- 전자보다 2000배 무거운 양성자가 Compton scattering?

러더포드의 반격

- 전자보다 2000배 무거운 양성자가 Compton scattering?

- 졸리오부부는 중성자를 발견했지만 그것이 무엇인지 알 지는 못한 것이다

Radiation types and the degree of penetration



1932년 채드윅 (James Chadwick)



- 러더포드 제자

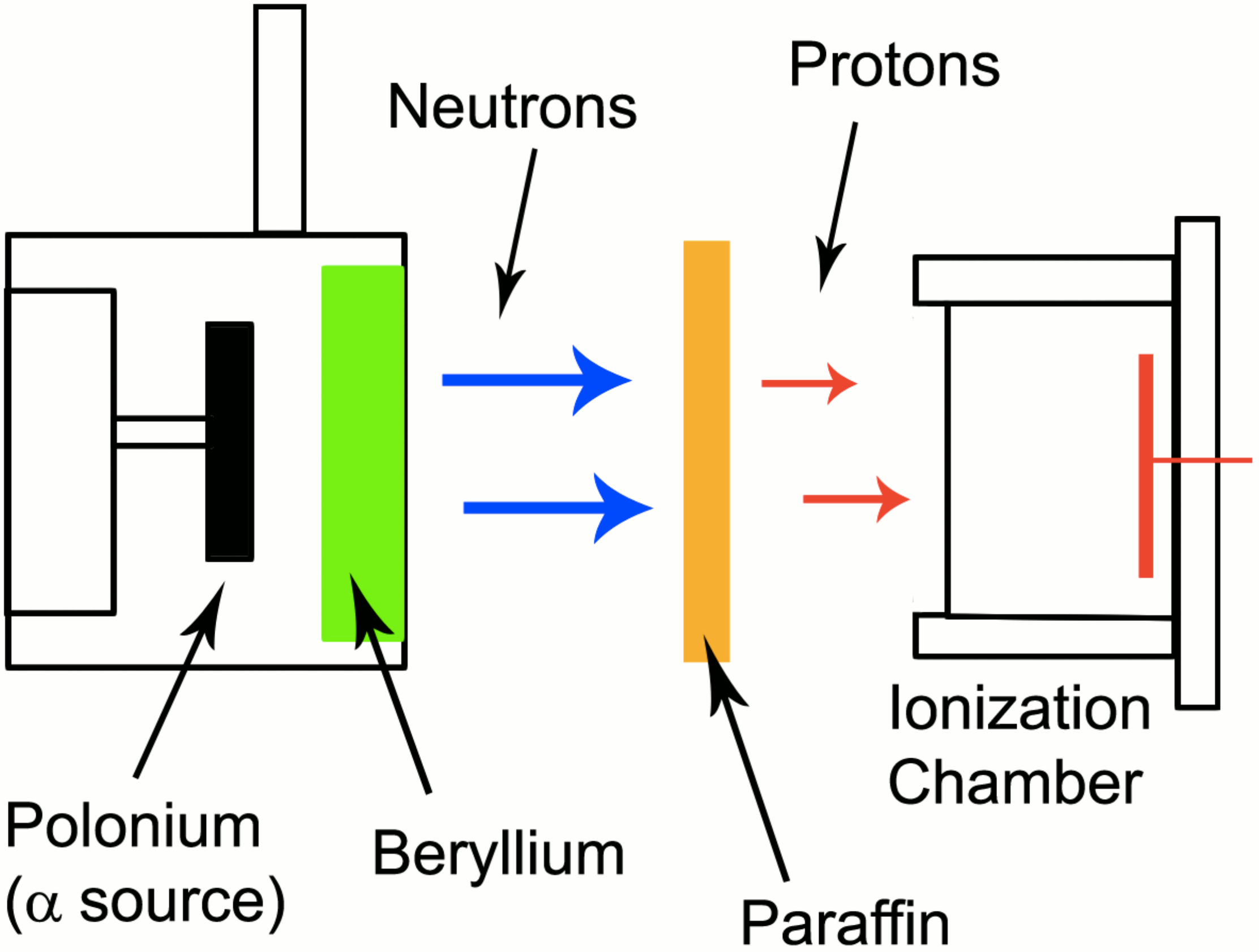
- 원자핵 안에는 전기적으로 중성이면서 양성자와 질량이 거의 비슷한 입자가 있다?

1932년 채드윅 (James Chadwick)



- 붕소로 추가 실험을 한 결과 채드윅은 이와 같은 중성 입자의 존재를 확인

- 그 질량이 양성자보다 약간 더 무겁다


Neutrons
Protons
Polonium
(α source)
Beryllium
Paraffin
Ionization
Chamber

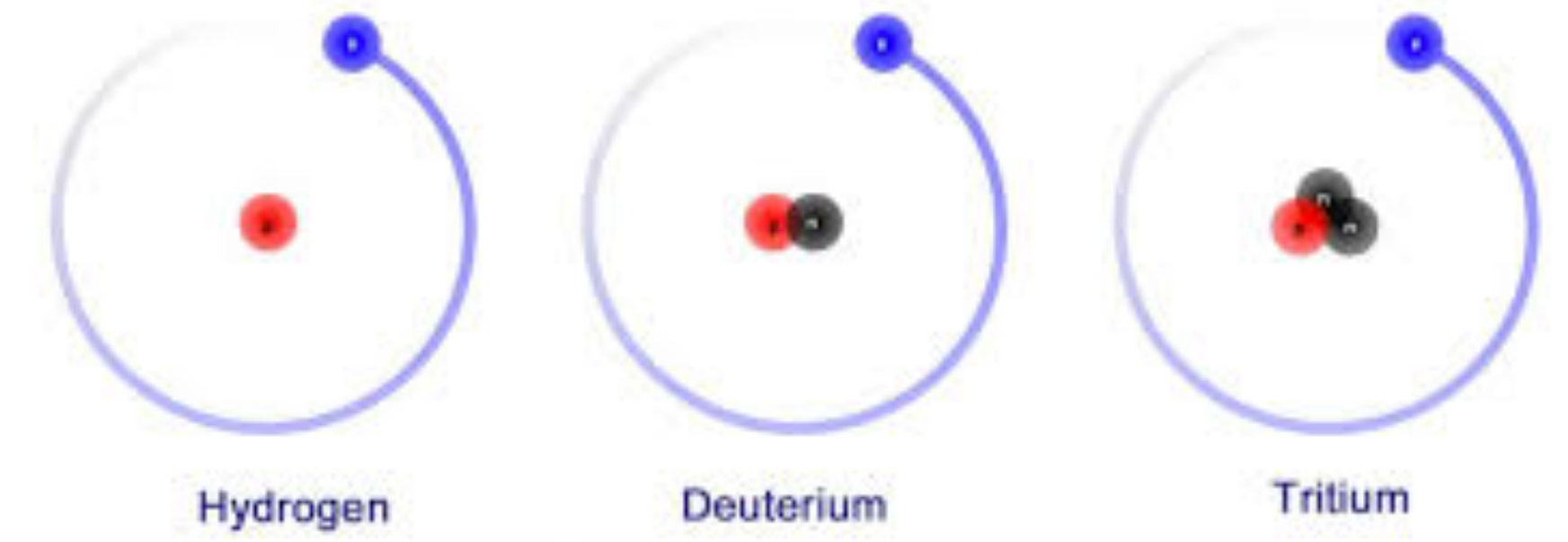
- 1932년 채드윅(Chadwick)은 《네이처》에 〈중성자의 존재 가능성〉을 기고

- 1932년 채드윅은 《네이처》에 〈중성자의 존재 가능성〉을 기고

- 로버트 베이처와 에드워드 콘돈
  - 양성자와 중성자로 이루어져 있는 새로운 원자핵 모형
    →
    질소 원자핵의 스핀 문제를 해결

- 오스트리아에서 나치 독일을 피해 캐번디시 연구소에 방문해 있던 마우리스 골드하버는 중수소(Deuterium)가 감마선에 의해 광붕괴 되면 중성자를 방출할 것이라고 제안

$$ {}^{2}_{1}\mathrm{D} + \gamma \rightarrow {}^{1}_{1}\mathrm{H} + n $$



- 오스트리아에서 나치 독일을 피해 캐번디시 연구소에 와 있던 마우리스 골드하버는 중수소가 감마선에 의해 광붕괴 되면 중성자를 방출할 것이라고 제안

$$ {}_{1}^{2}\mathrm{D} + \gamma \rightarrow {}_{1}^{1}\mathrm{H} + n $$

- 채드윅과 골드하버는 공동으로 연구하여 이를 실험으로 입증하였다.

- 오스트리아에서 나치 독일을 피해 캐번디시 연구소에 와 있던 마우리스 골드하버는 중수소가 감마선에 의해 광붕괴 되면 중성자를 방출할 것이라고 제안

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- 채드윅과 골드하버는 공동으로 연구하여 이를 실험으로 입증하였다.

- 이들은 양성자의 운동에너지가 1.05 MeV임을 측정

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$$ {}^{2}_{1}\mathrm{D} + \gamma \rightarrow {}^{1}_{1}\mathrm{H} + n $$

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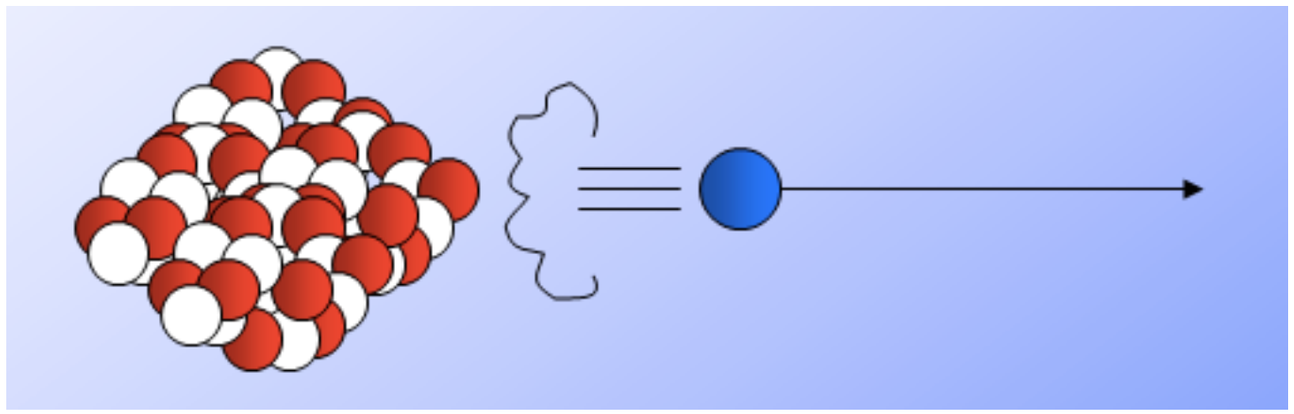
- 이들은 양성자의 운동에너지가 1.05 MeV임을 측정

- 남겨진 질량 → 중성자 질량 → $m_n = 1.00028\, m_p$

Neutrino

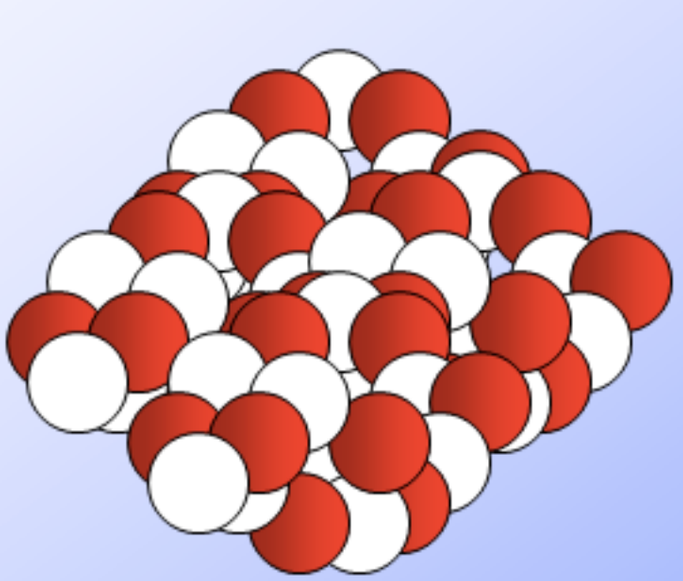
beta decay

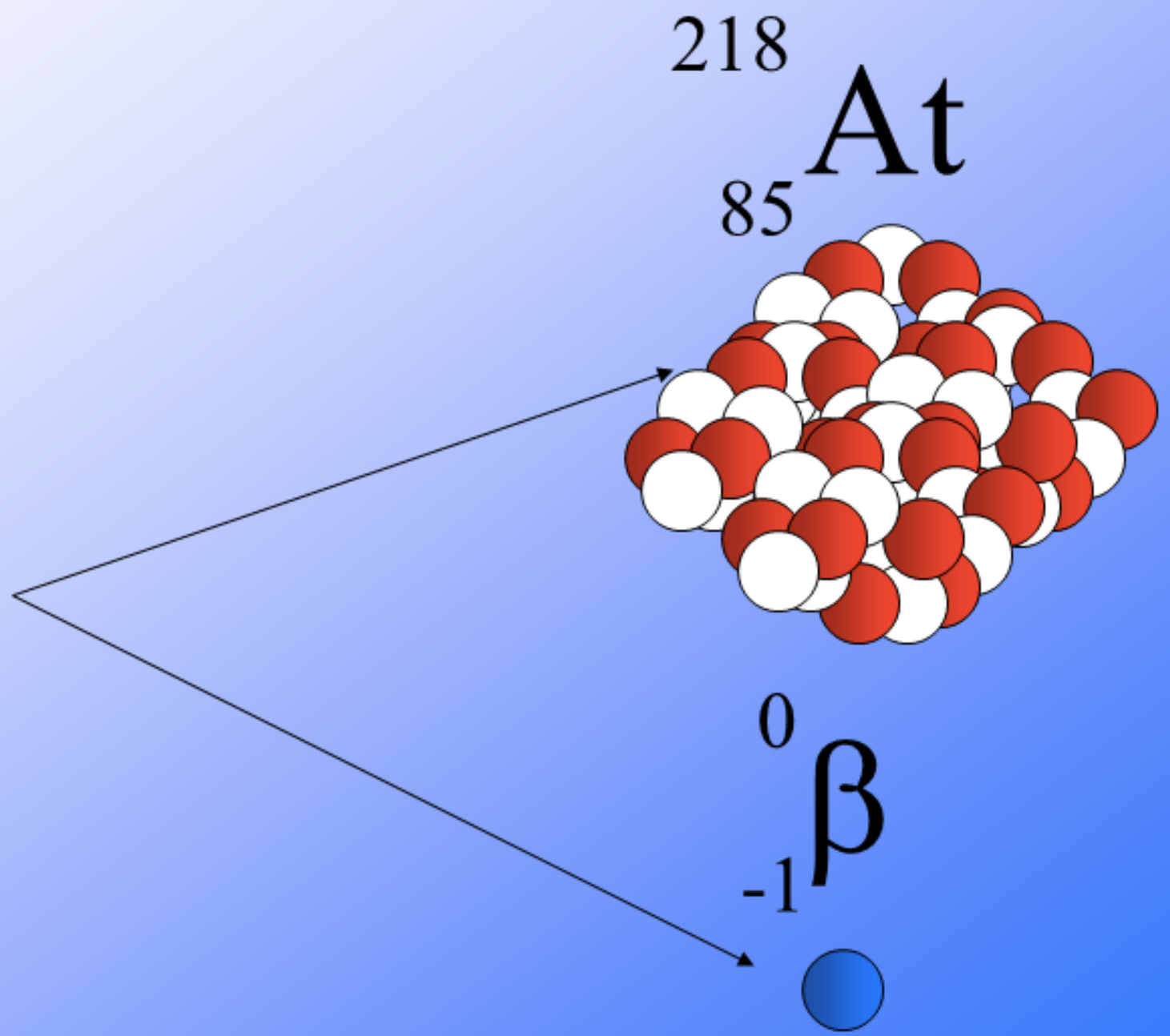
- A beta particle is a fast moving electron which is emitted from the nucleus of an atom undergoing radioactive decay.
- Beta decay occurs when a neutron changes into a proton and an electron.



Beta Decay

$^{218}_{84}Po$





Energy Conservation

Consider the neutron decay in its rest frame

$$n(P^\mu) \to e^-(k^\mu) + p(q^\mu)$$

Energy Conservation

Consider the neutron decay in its rest frame

$$n(P^\mu) \to e^-(k^\mu) + p(q^\mu)$$

Using the energy-momentum conservation $P^\mu = q^\mu + k^\mu$, the four-momenta are

$$\begin{aligned} P^\mu &= (m_n, 0, 0, 0), \\ k^\mu &= (E_e, 0, 0, p_e), \\ q^\mu &= (E_p, 0, 0, -p_e) \end{aligned}$$

Neglecting the electron mass, the mass relations are

$$\begin{aligned} k \cdot k &= E_e^2 - p_e^2 = 0, \rightarrow E_e = p_e, \\ q \cdot q &= E_p^2 - p_e^2 = m_p^2, \rightarrow E_p = \sqrt{m_p^2 + p_e^2}, \end{aligned}$$

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so that

$$\begin{aligned} P^\mu &= (m_n, 0, 0, 0) \\ q^\mu &= \left(\sqrt{m_p^2 + E_e^2}, 0, 0, E_e\right) \\ k^\mu &= (E_e, 0, 0, -E_e) \end{aligned}$$

Finally

$$m_n = \sqrt{m_p^2 + E_e^2} + E_e, \rightarrow$$

Finally

$$
\begin{aligned}
m_n &= \sqrt{m_p^2 + E_e^2} + E_e, \rightarrow \\
\sqrt{m_p^2 + E_e^2} &= m_n - E_e.
\end{aligned}
$$

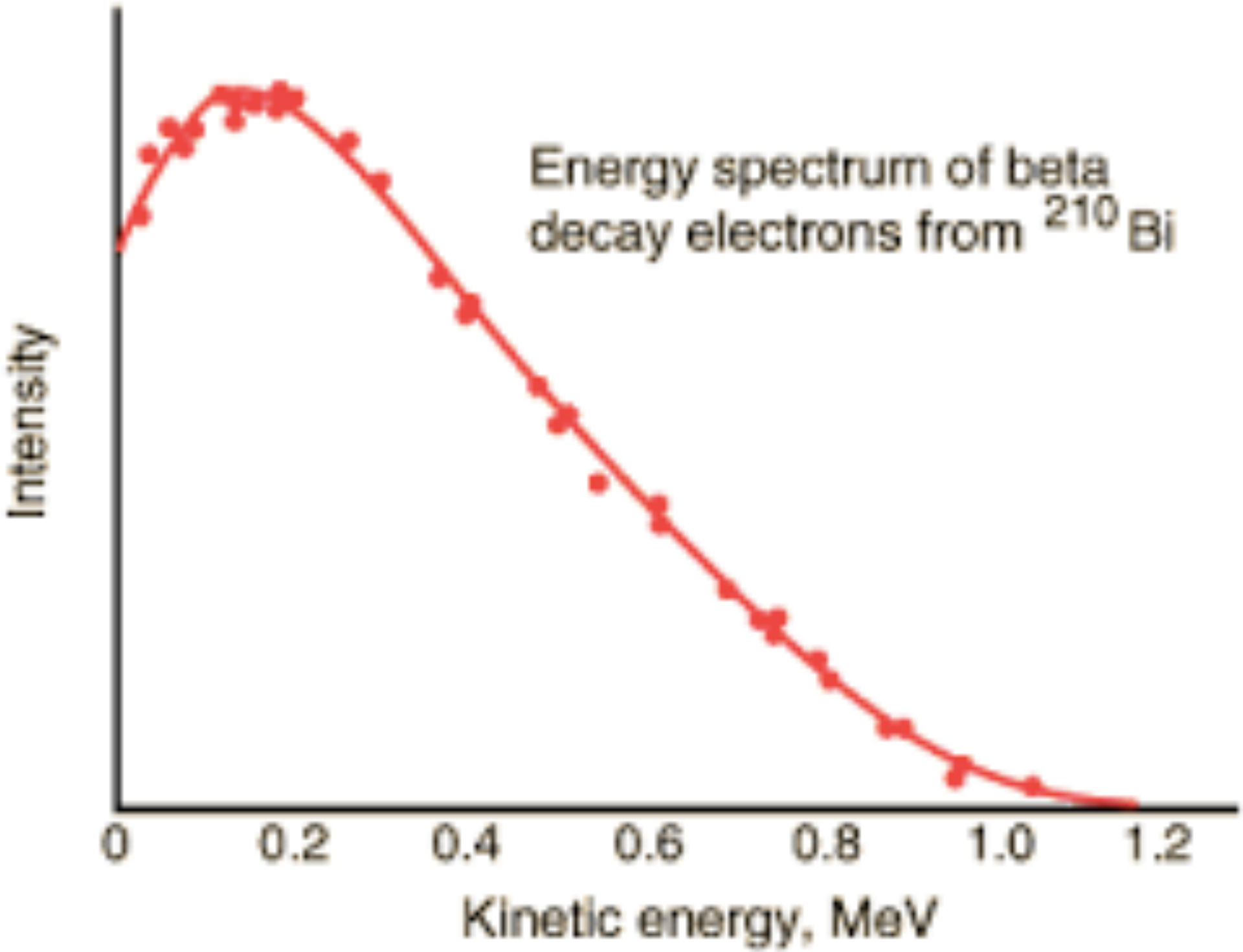
Finally

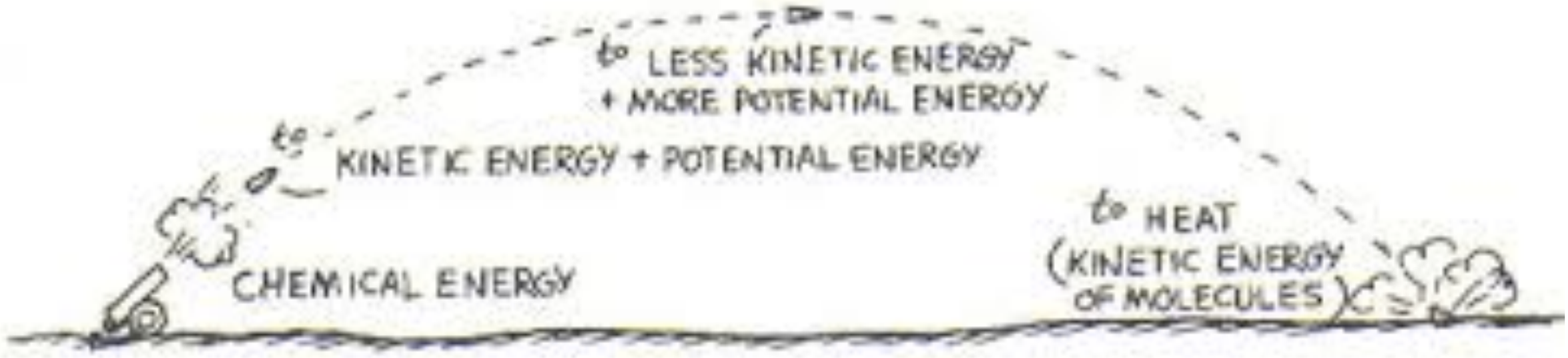
$$\begin{aligned} m_n &= \sqrt{m_p^2 + E_e^2} + E_e, \rightarrow \\ \sqrt{m_p^2 + E_e^2} &= m_n - E_e. \end{aligned}$$

The electron mass is fixed:

$$E_e = \frac{m_n^2 - m_p^2}{2m_n}$$

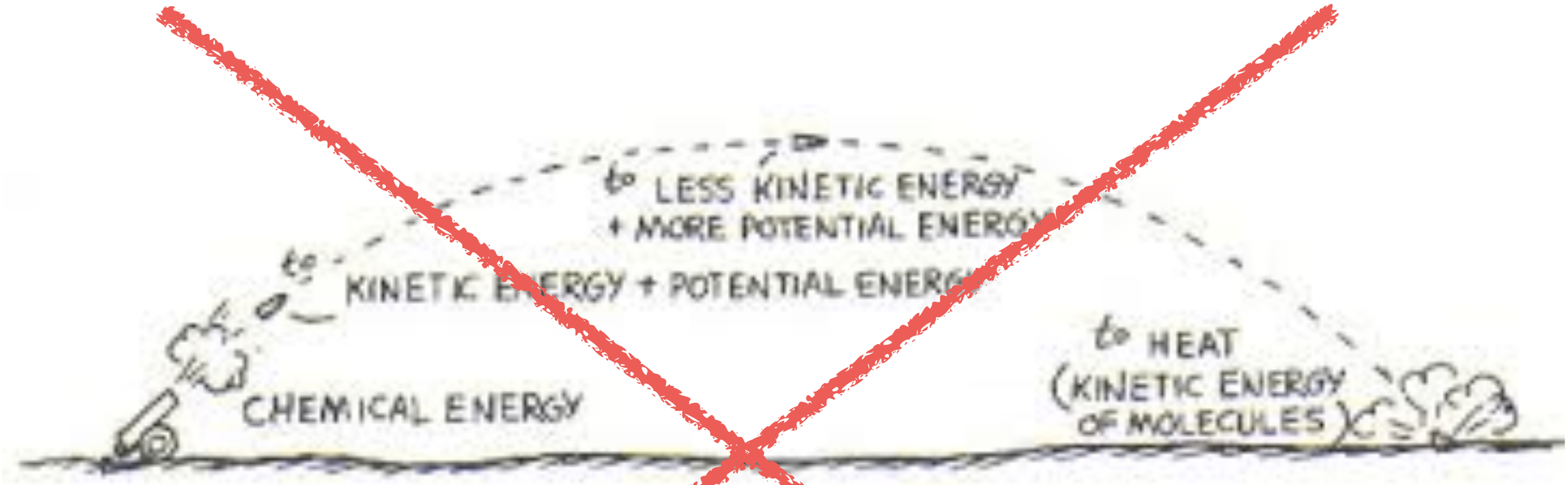
BUT


Energy spectrum of beta decay electrons from ^{210}Bi
Intensity
0
0.2
0.4
0.6
0.8
1.0
1.2
Kinetic energy, MeV



Energy Cannot Be Created or Destroyed

(It just changes forms)



Energy Cannot Be Created or Destroyed

(It just changes forms)

Bohr: Maybe violation of energy conservation?

Pauli: Neutrino hypothesis



Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grössenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

Dear Radioactive Ladies and Gentlemen,

I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that in the nuclei there could exist electrically neutral particles, which I will call neutrons, that have spin 1/2 and obey the exclusion principle

이름 정리

- 1932년에 채드윅이 지금의 중성자를 발견하고 그 입자를 중성자(neutron)

이름 정리

- 1932년에 채드윅이 지금의 중성자를 발견하고 그 입자를 중성자(neutron)
- Fermi가 다음 해인 1933년에 베타 붕괴의 이론을 발표하며, 파울리의 '중성자(neutron)'에 '작다.'라는 이탈리아어의 접미어 '-ino'를 붙여서, '뉴트리노(neutrino)'로 부르게 되었다.

Cowan–Reines neutrino experiment in 1956

- 원자로에서 방출되는 중성미자를 2개의 물탱크로 향하게

Cowan–Reines neutrino experiment in 1956

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Cowan–Reines neutrino experiment in 1956

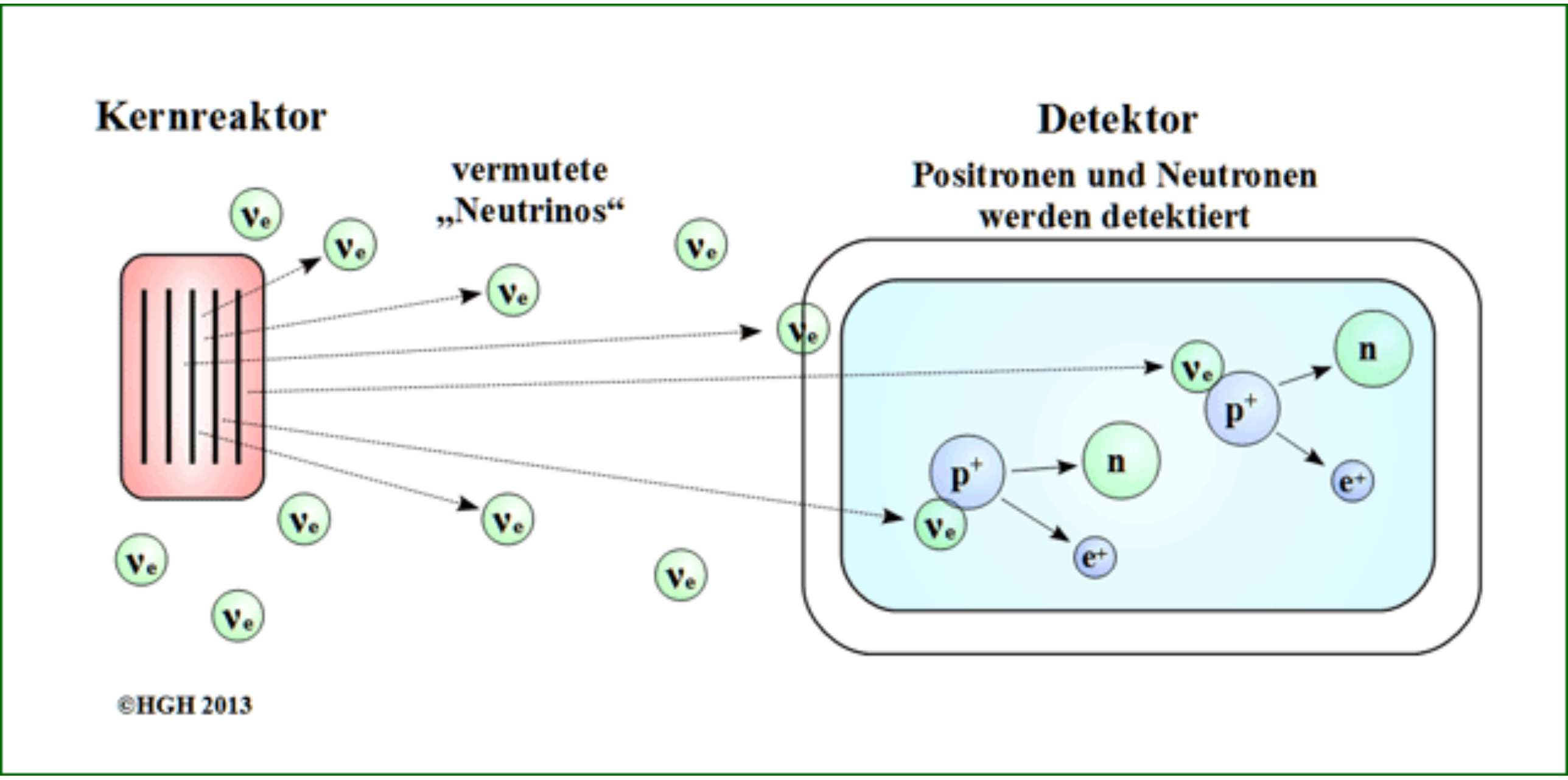
- 원자로에서 방출되는 중성미자를 2개의 물탱크로 향하게
- 역베타붕괴
- 중성미자가 양성자와 반응해 중성자와 양전자가 생성

$$\bar{\nu}_e + p \rightarrow n + e^+$$

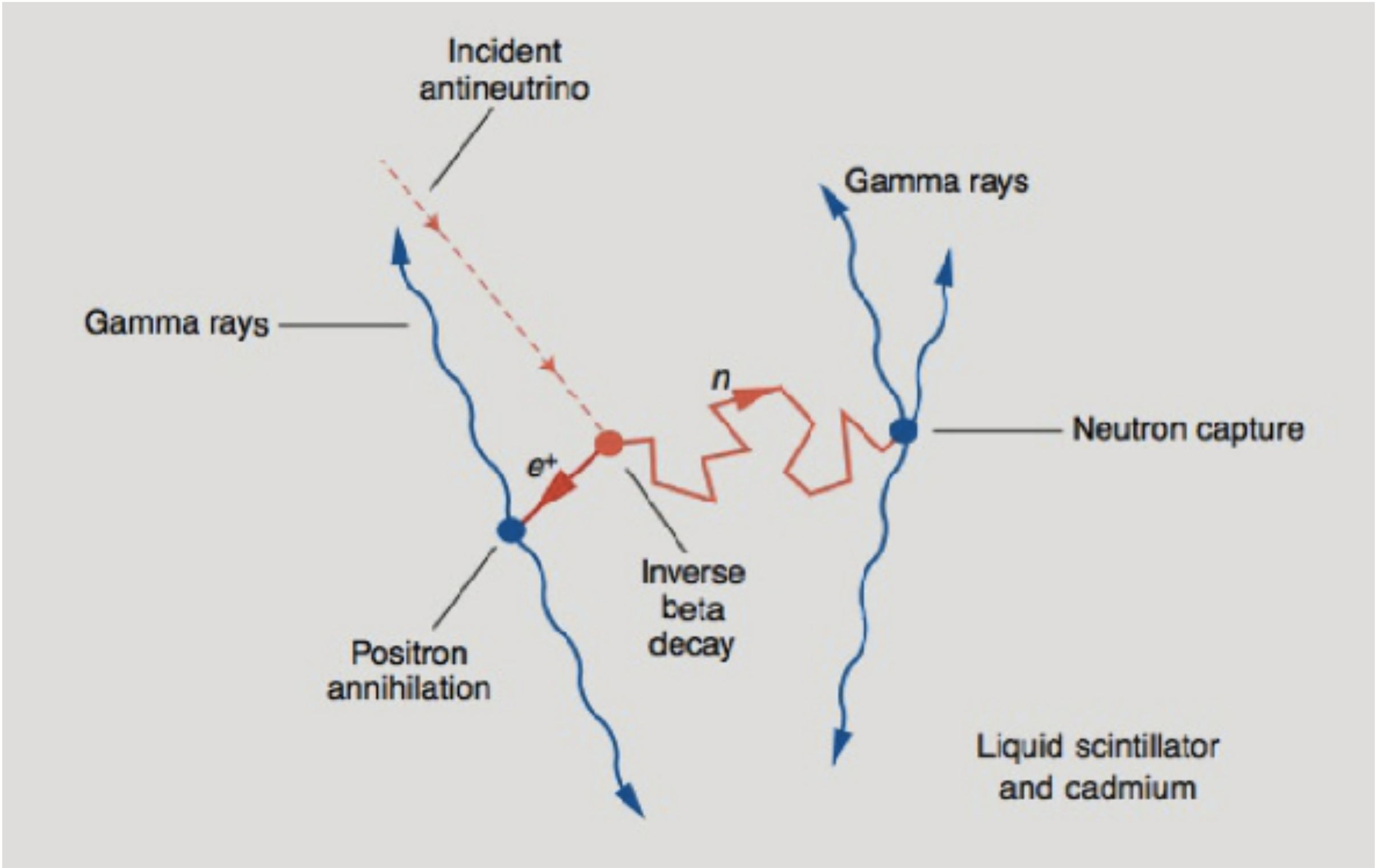
Cowan–Reines neutrino experiment in 1956

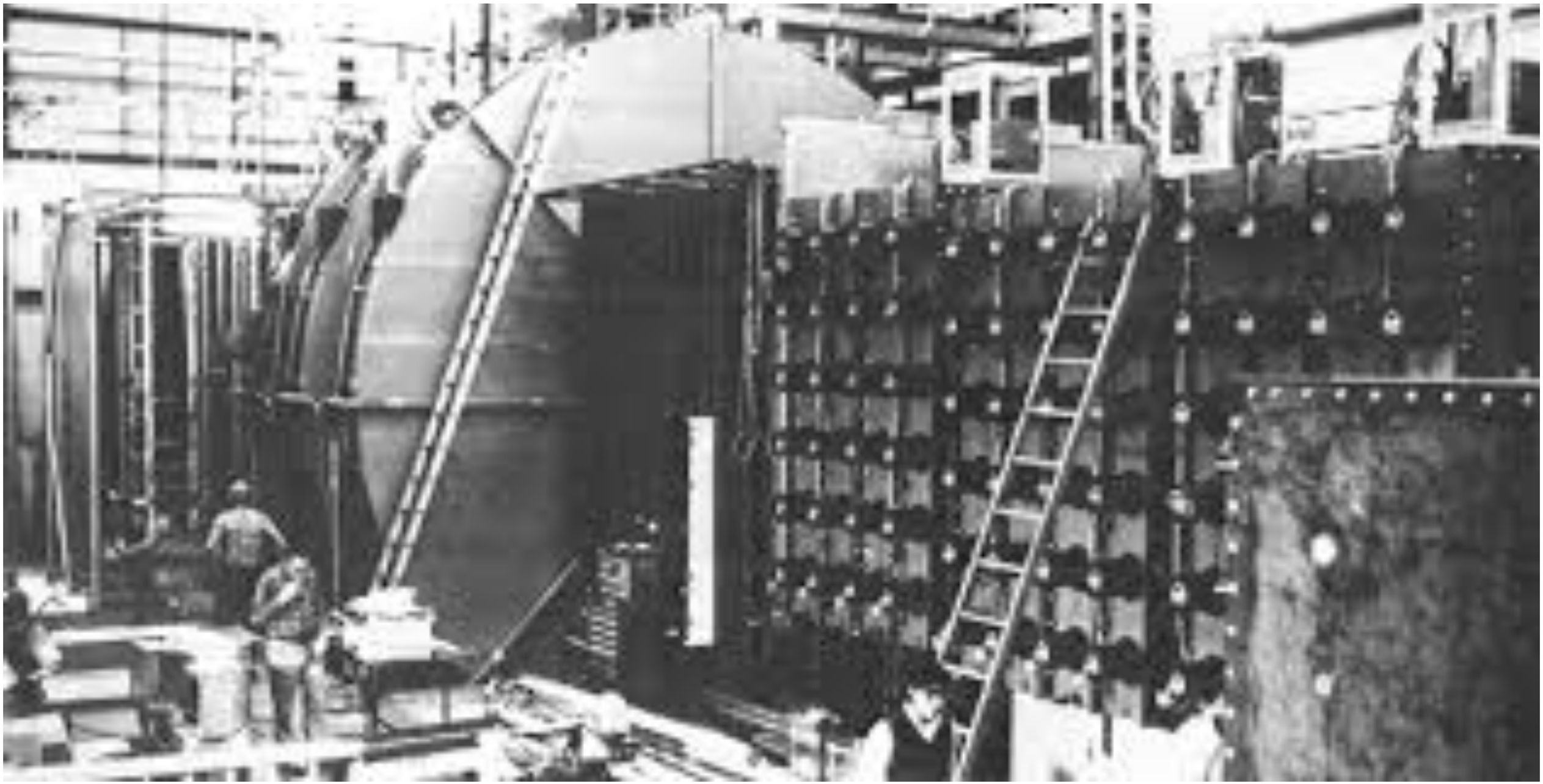
- Cowan은 1976년 사망했으며, Reines는 1995년 중성미자 물리학에 대한 공로로 노벨 물리학상을 수여

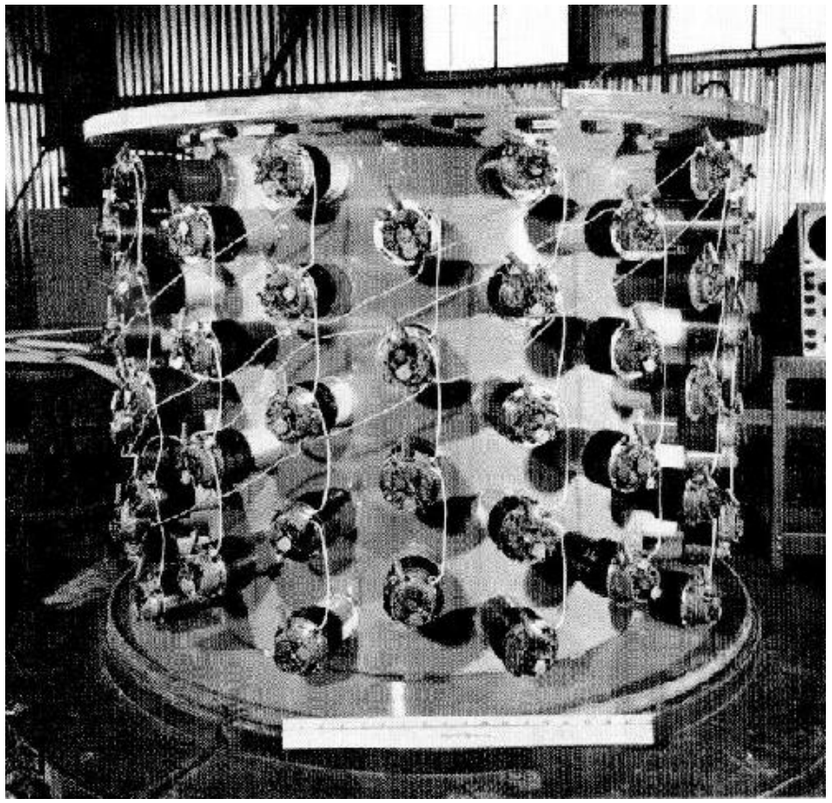
Cowan–Reines neutrino experiment in 1956

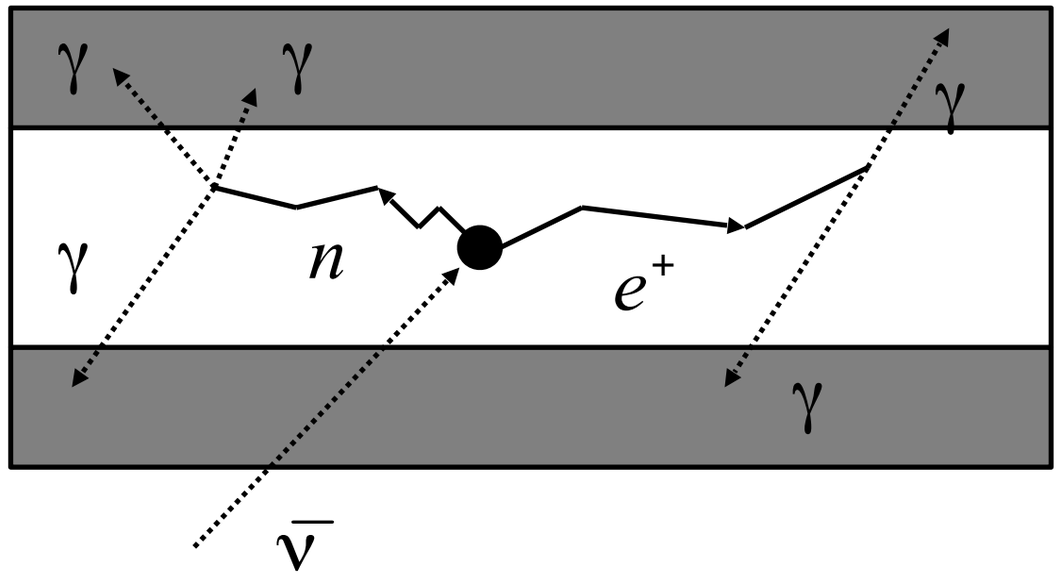


$$\bar{\nu}_e + p \rightarrow n + e^+$$


Incident
antineutrino
Gamma rays
Gamma rays
n
Neutron capture
e+
Inverse
beta
decay
Positron
annihilation
Liquid scintillator
and cadmium







Scintillator

$H_2O + CdCl_2$

Scintillator

Two interesting features of a neutrino:

(1) only left-handed neutrino;
(2) neutrino oscillation

Parity in the weak interaction

➢ Parity Transformation

➢the flip in the sign of one spatial coordinate.

$$\mathbf{P}: \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}.$$





Parity invariance

✓The mirror image of any physical process also represents a perfectly possible physical process.

Parity invariance

- ✓The mirror image of any physical process also represents a perfectly possible physical process.
- ✓Physicist held the mirror symmetry of the laws of nature to be self-evident.

In 1956, Lee and Yang discovered

Parity invariance in strong and electromagnetic processes

No parity invariant confirmation in the weak interactions.

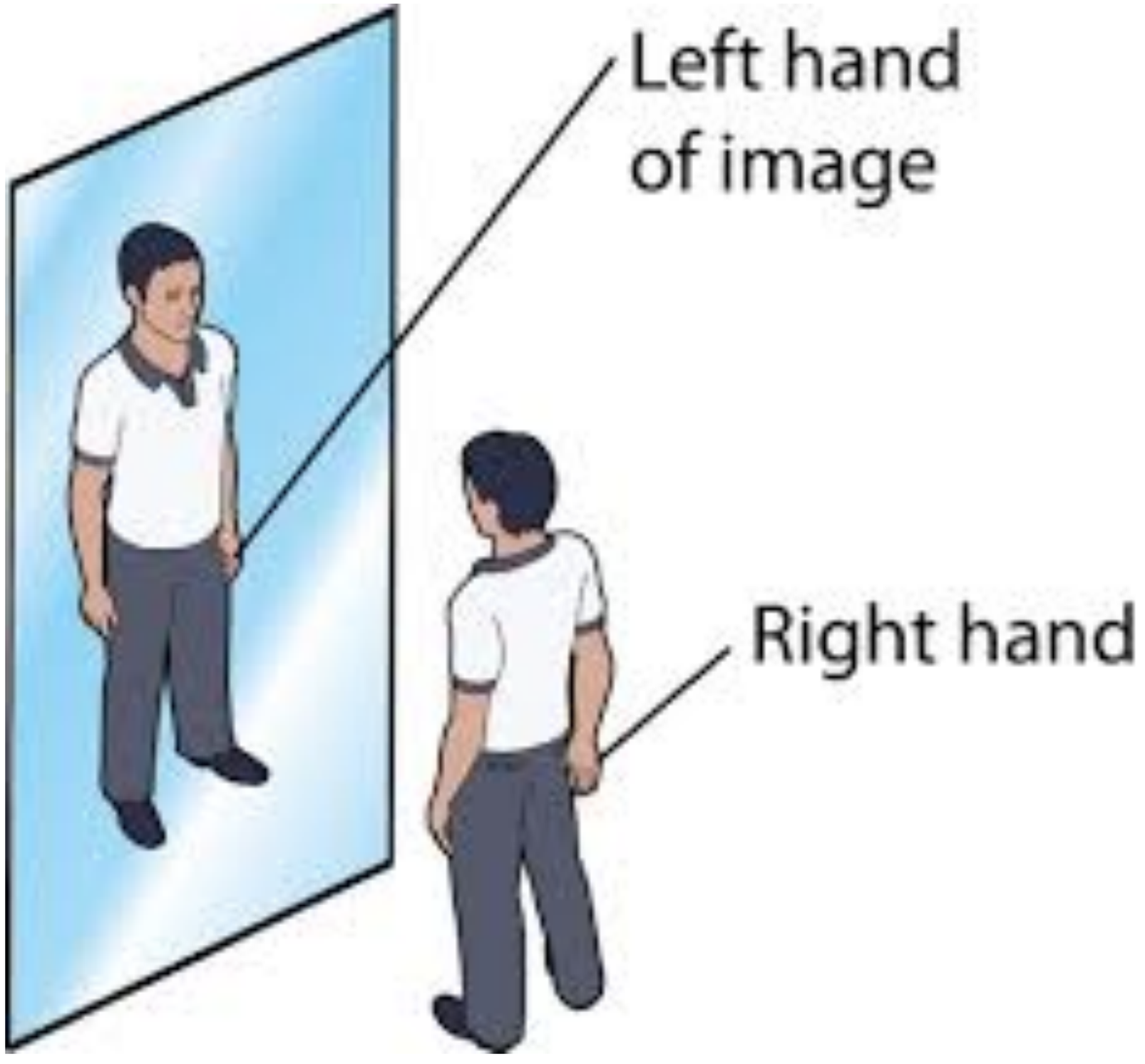


Chen Ning Yang



Tsung-Dao Lee

Lee and Yang proposed a test


Left hand
of image
Right hand

➢ Wu Experiment in 1956



- **Radioactive cobalt 60 nuclei**

$$^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + e^{-} + \bar{\nu}_e + 2\gamma$$

- **Radioactive cobalt 60 nuclei**
  - **cooled to near absolute zero**

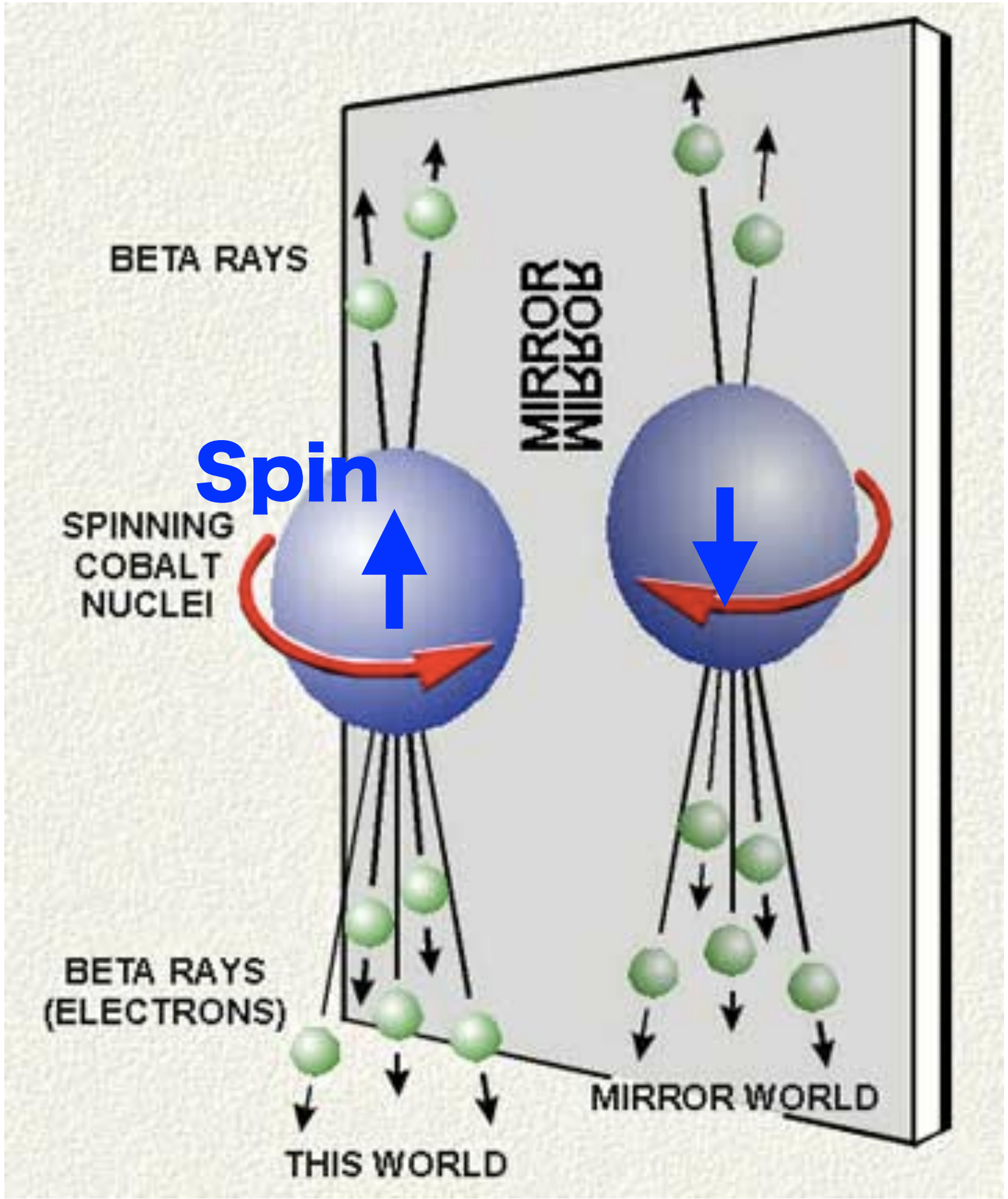
$$^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e + 2\gamma$$

- **Radioactive cobalt 60 nuclei**
  - **cooled to near absolute zero**
  - **aligned in a uniform magnetic field**

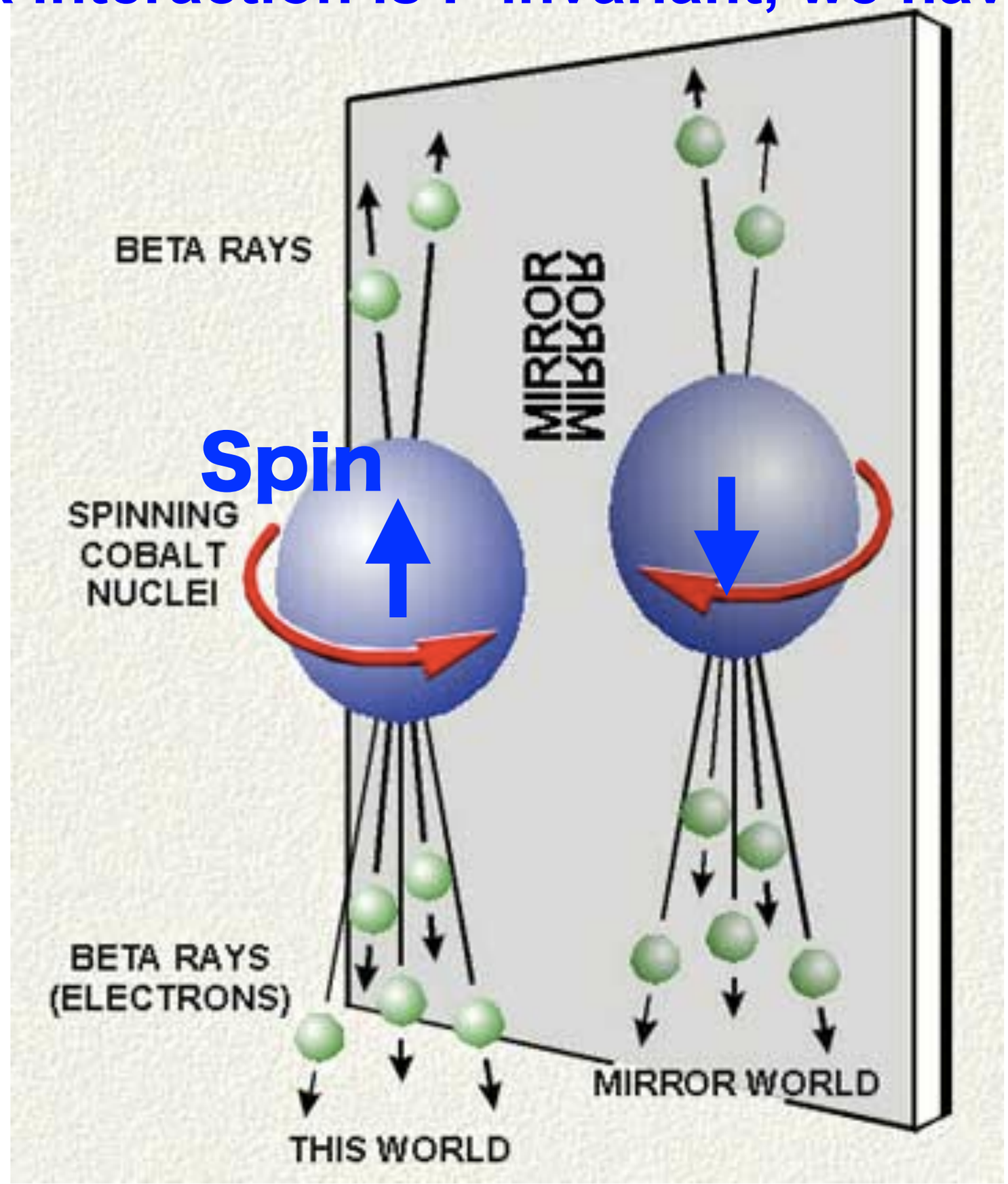
$$^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e + 2\gamma$$

- **Radioactive cobalt 60 nuclei**
  - **cooled to near absolute zero**
  - **aligned in a uniform magnetic field**
  - **their spins aligned in the same direction.**

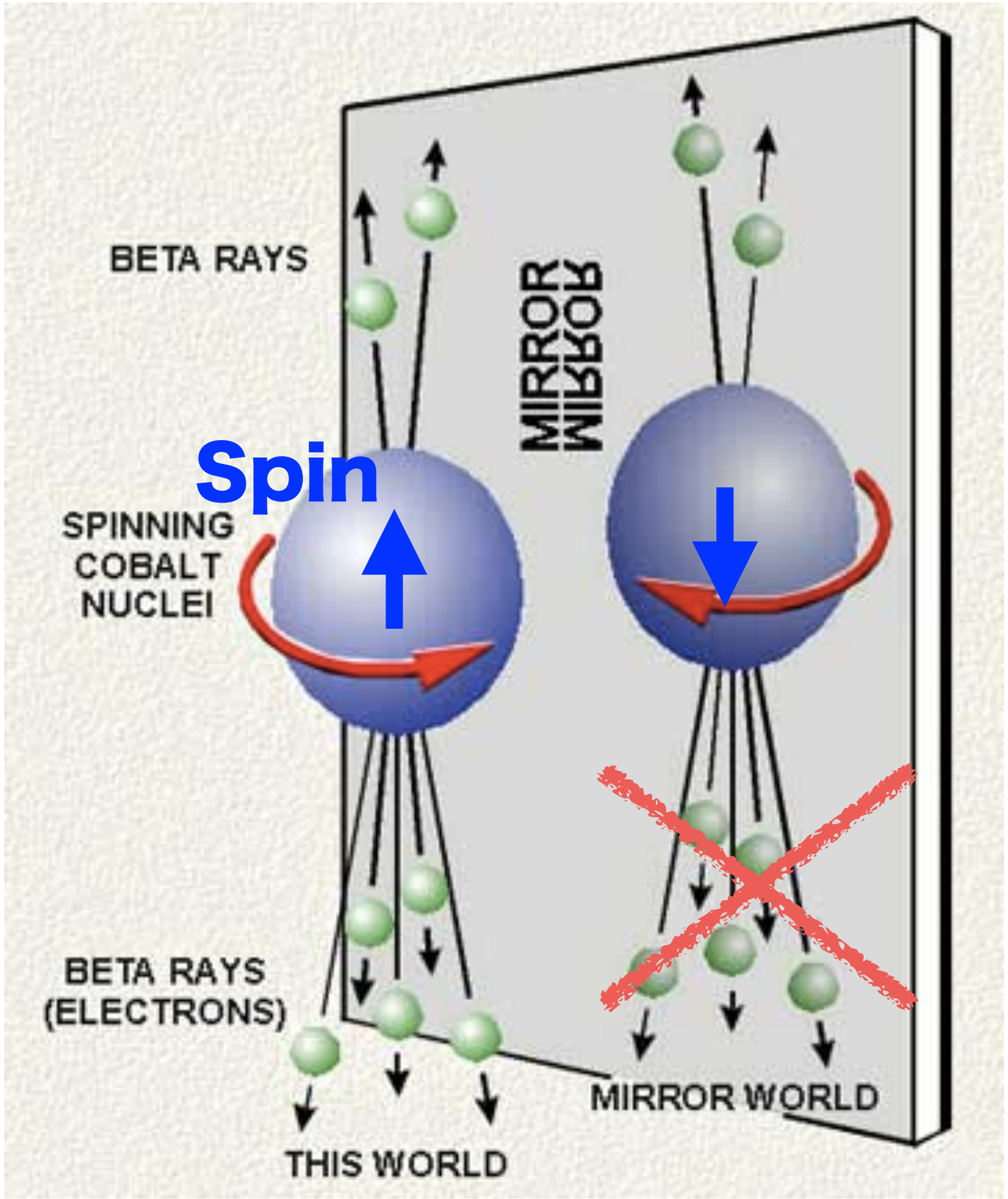
$$^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e + 2\gamma$$


BETA RAYS
MIRROR
MIRROR
Spin
SPINNING
COBALT
NUCLEI
BETA RAYS
(ELECTRONS)
THIS WORLD
MIRROR WORLD

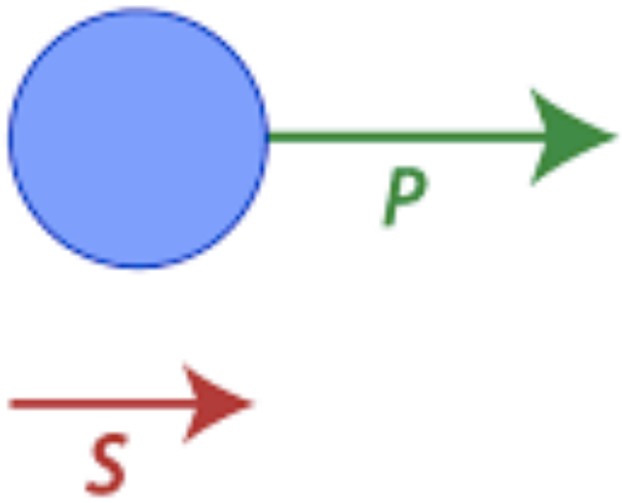
➢ If weak interaction is P invariant, we have to see both.



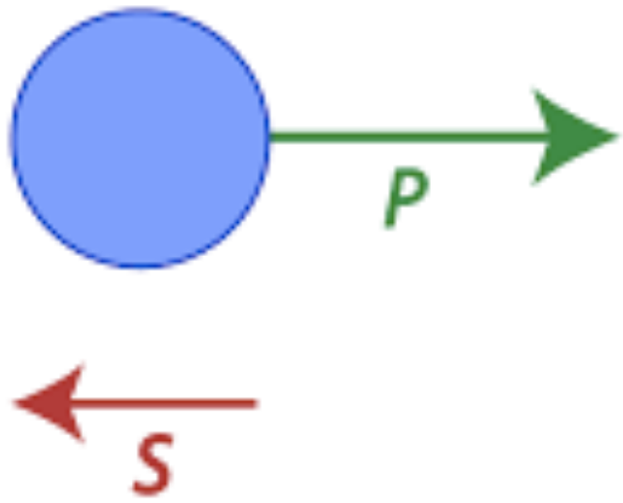
➢ **BUT**

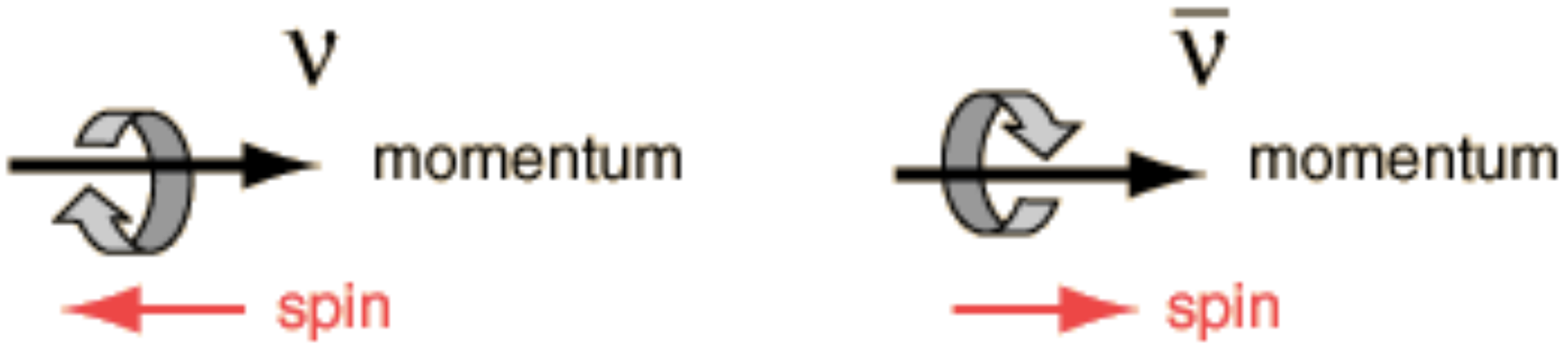

BETA RAYS
MIRROR
Spin
SPINNING
COBALT
NUCLEI
BETA RAYS
(ELECTRONS)
THIS WORLD
MIRROR WORLD

Right-handed:



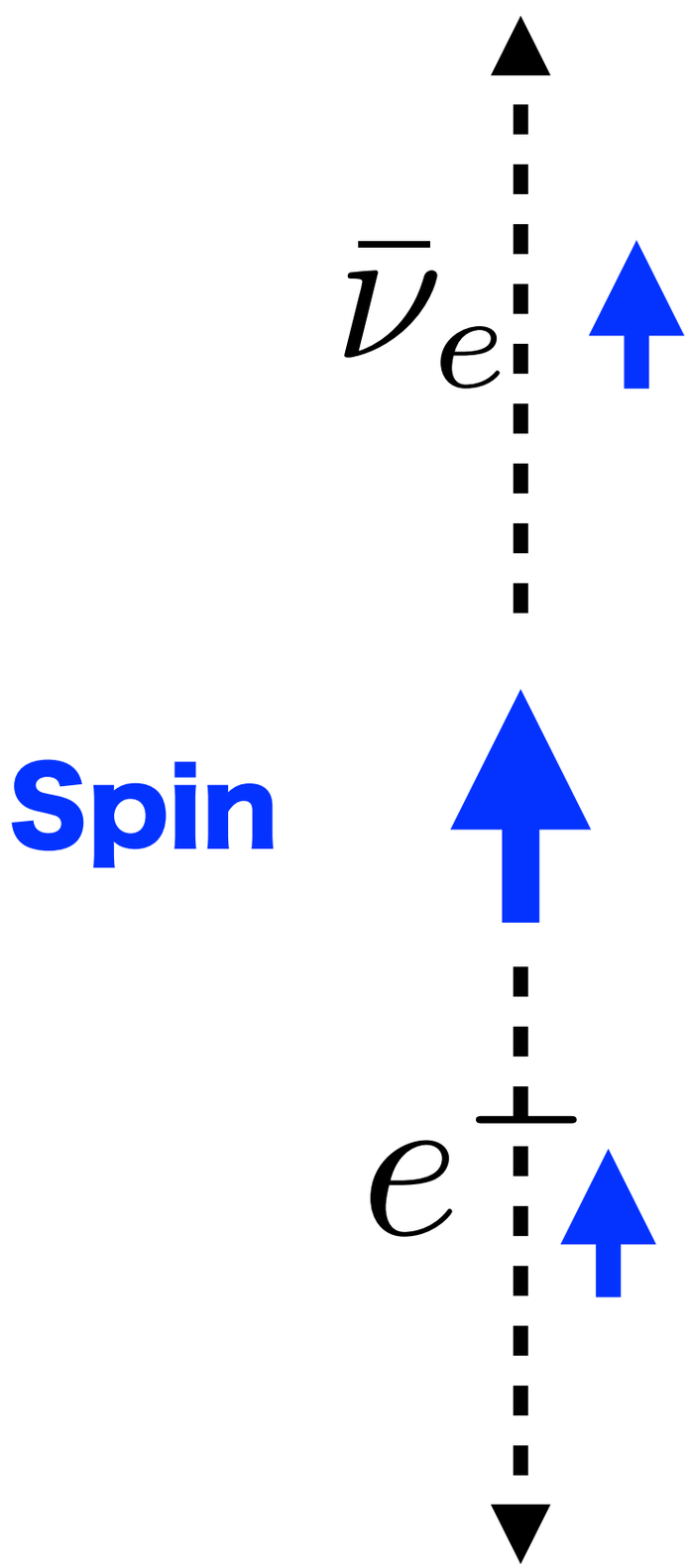
Left-handed:

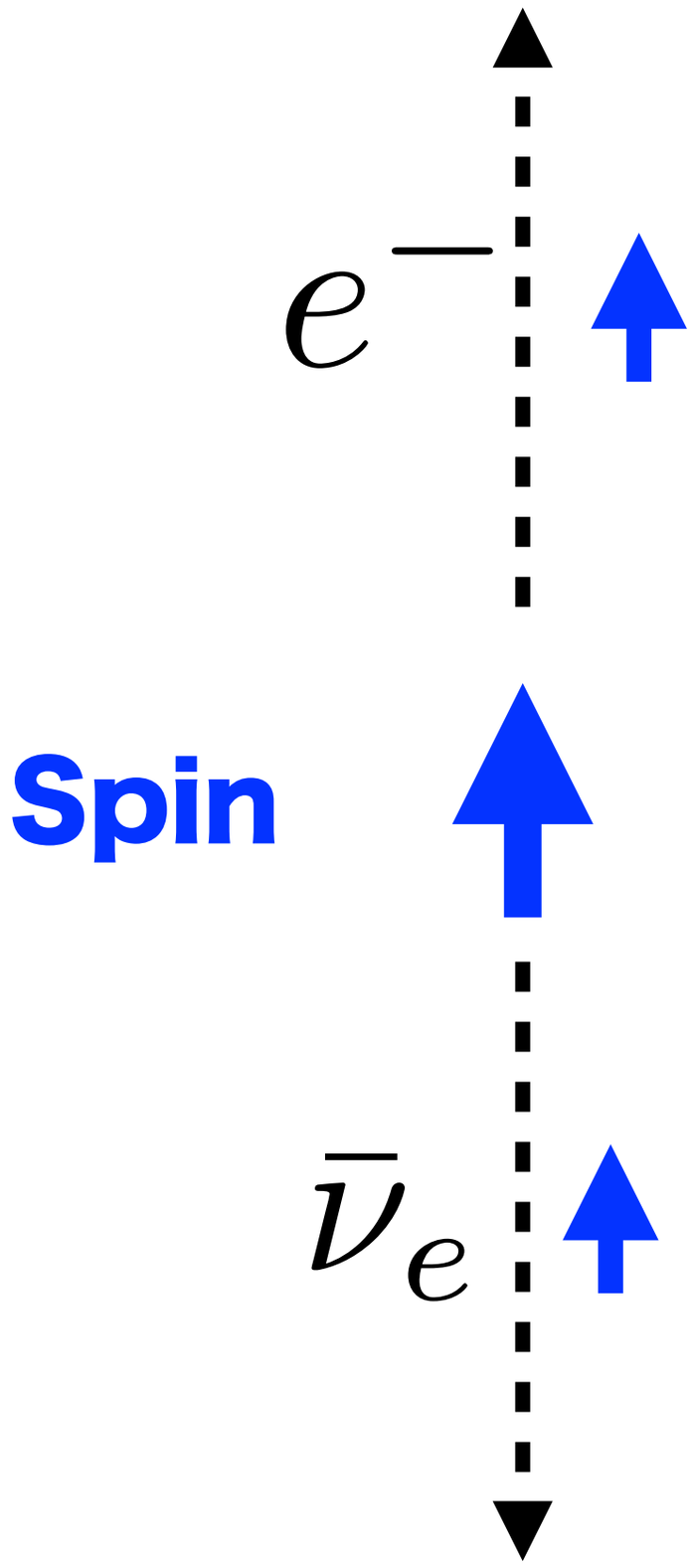


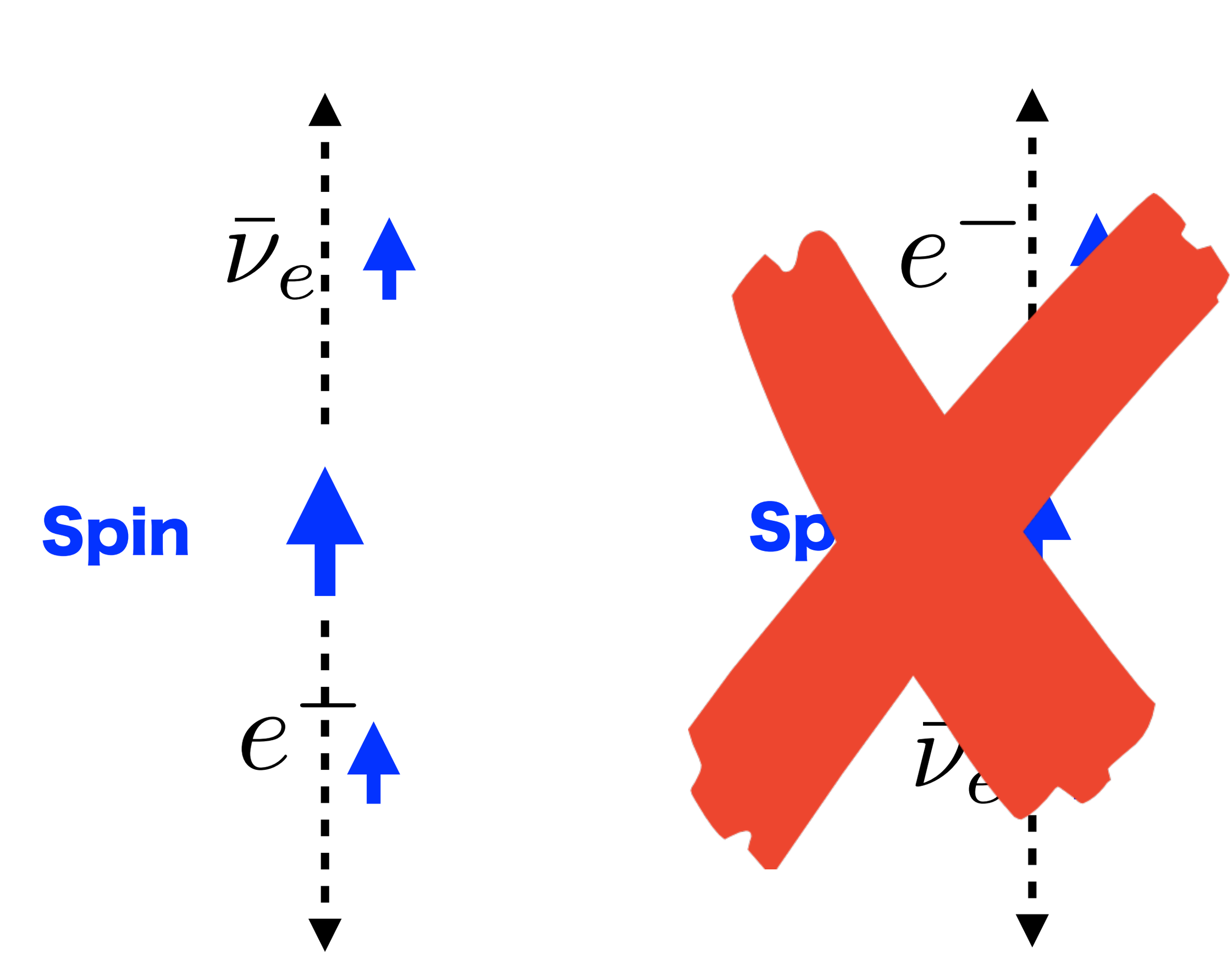


Neutrino
(left-handed)

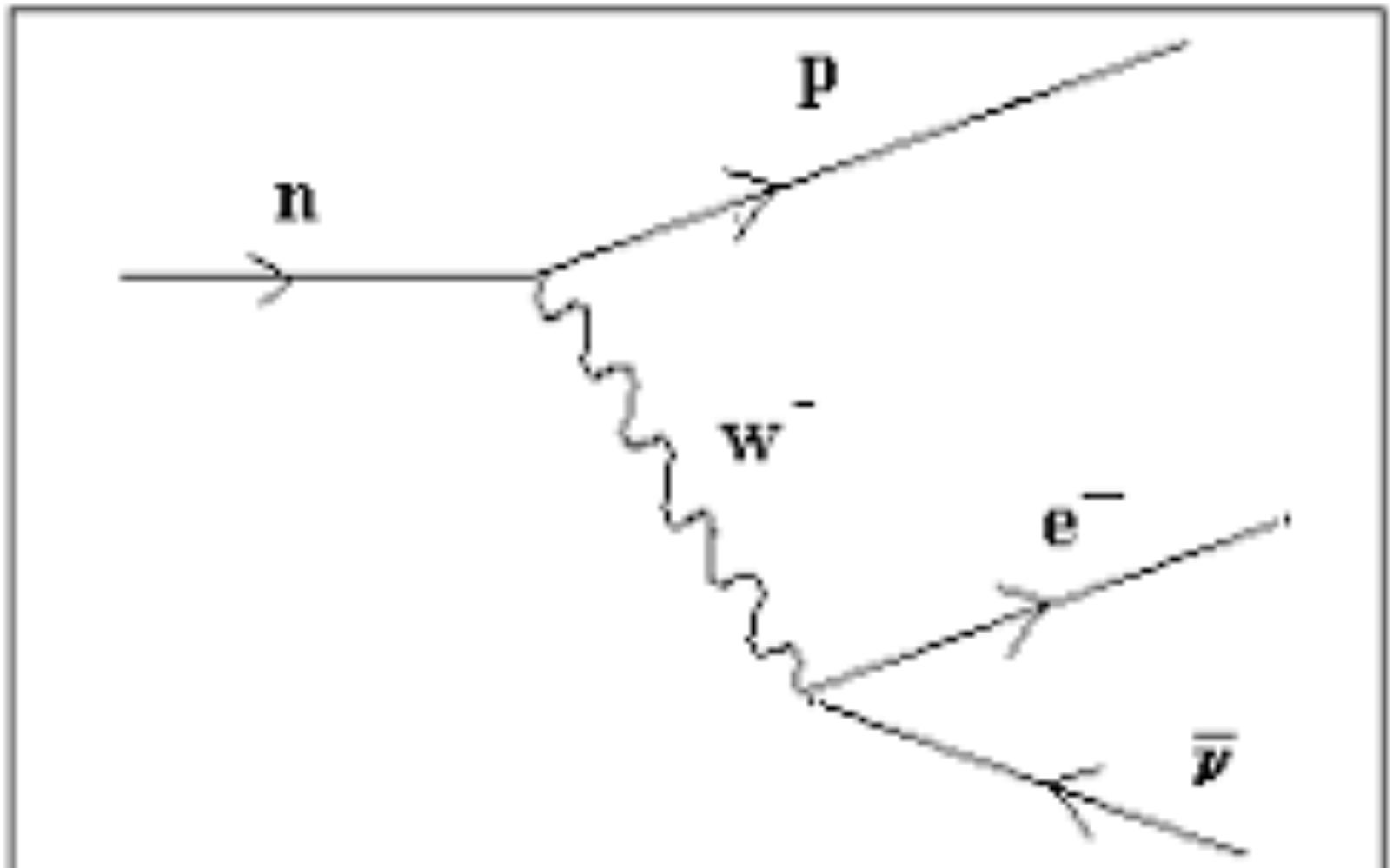
Antineutrino
(right-handed)


$\bar{\nu}_e$
Spin
e^-

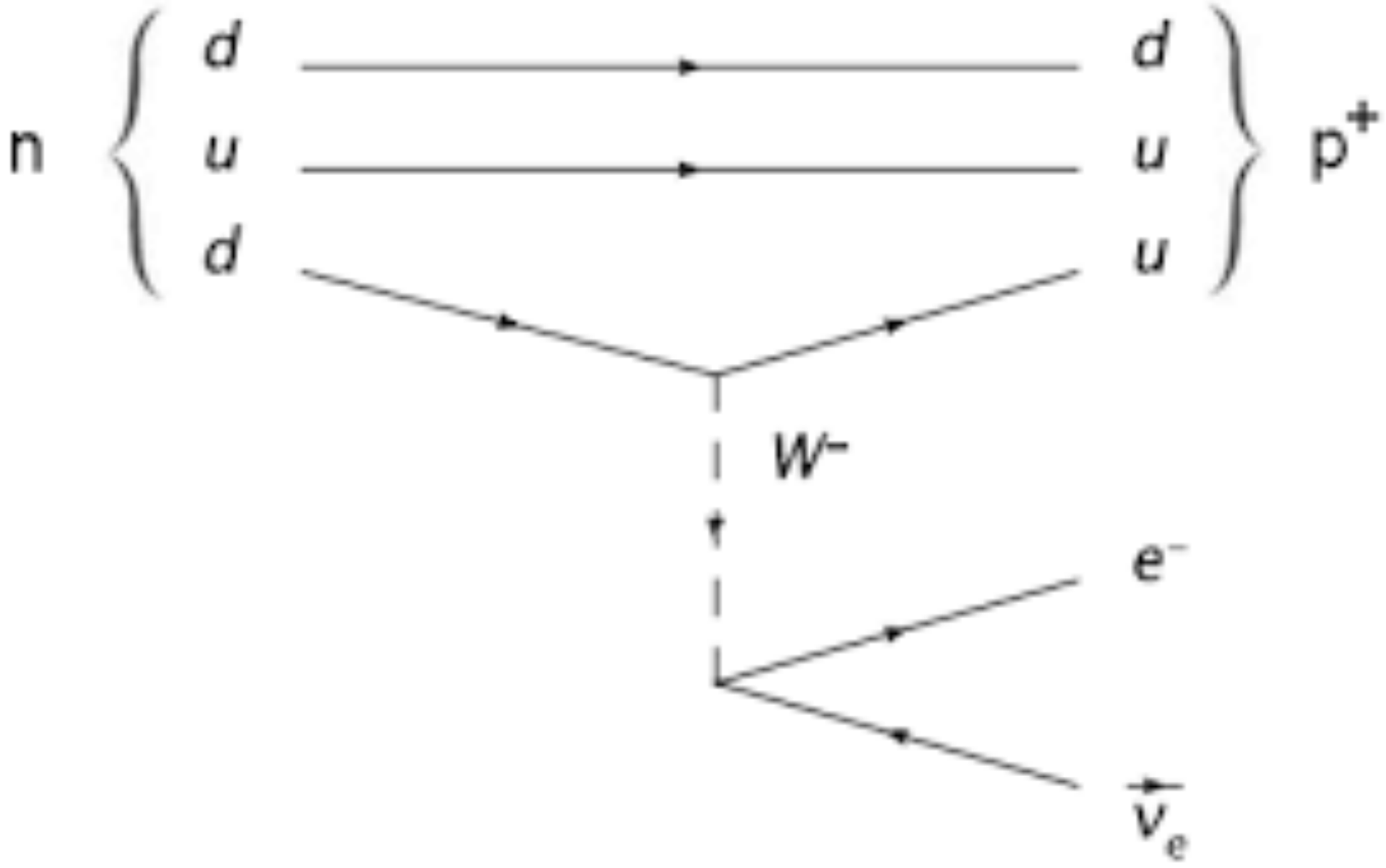

e^-
Spin
$\bar{\nu}_e$


$\bar{\nu}_e$
Spin
e^-
Sp
$\bar{\nu}_e$

Fermi beta decay theory



Feynman Diagram for Beta Decay


n
d
u
d
d
u
u
p+
W−
e−
ν̄e

Only left-handed neutrino

(2) Neutrino oscillation

There are three kinds of neutrinos

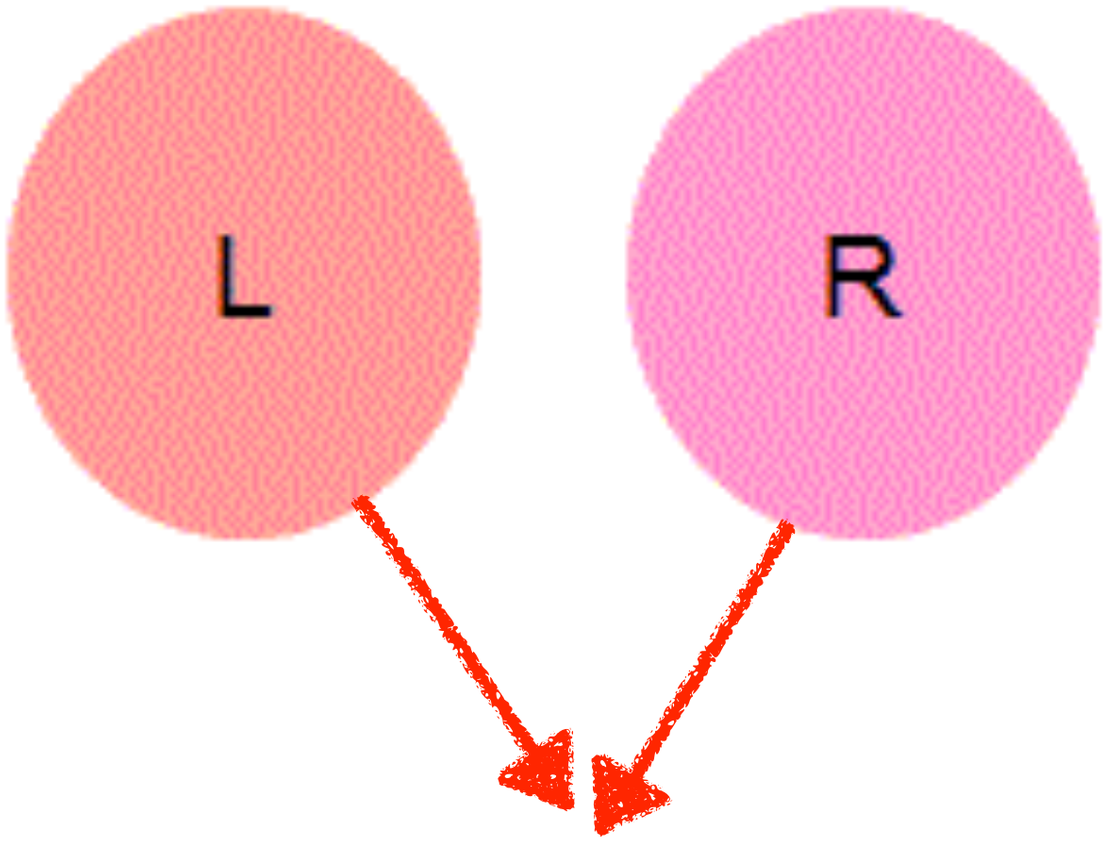
Lepton sector in the SM

There are three generations:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L,$$

$$e^-_R, \quad \mu^-_R, \quad \tau^-_R.$$

➢ **Mass term connect the left-handed fermion and the right-handed fermion**



➢ **There is no right-handed neutrino.**

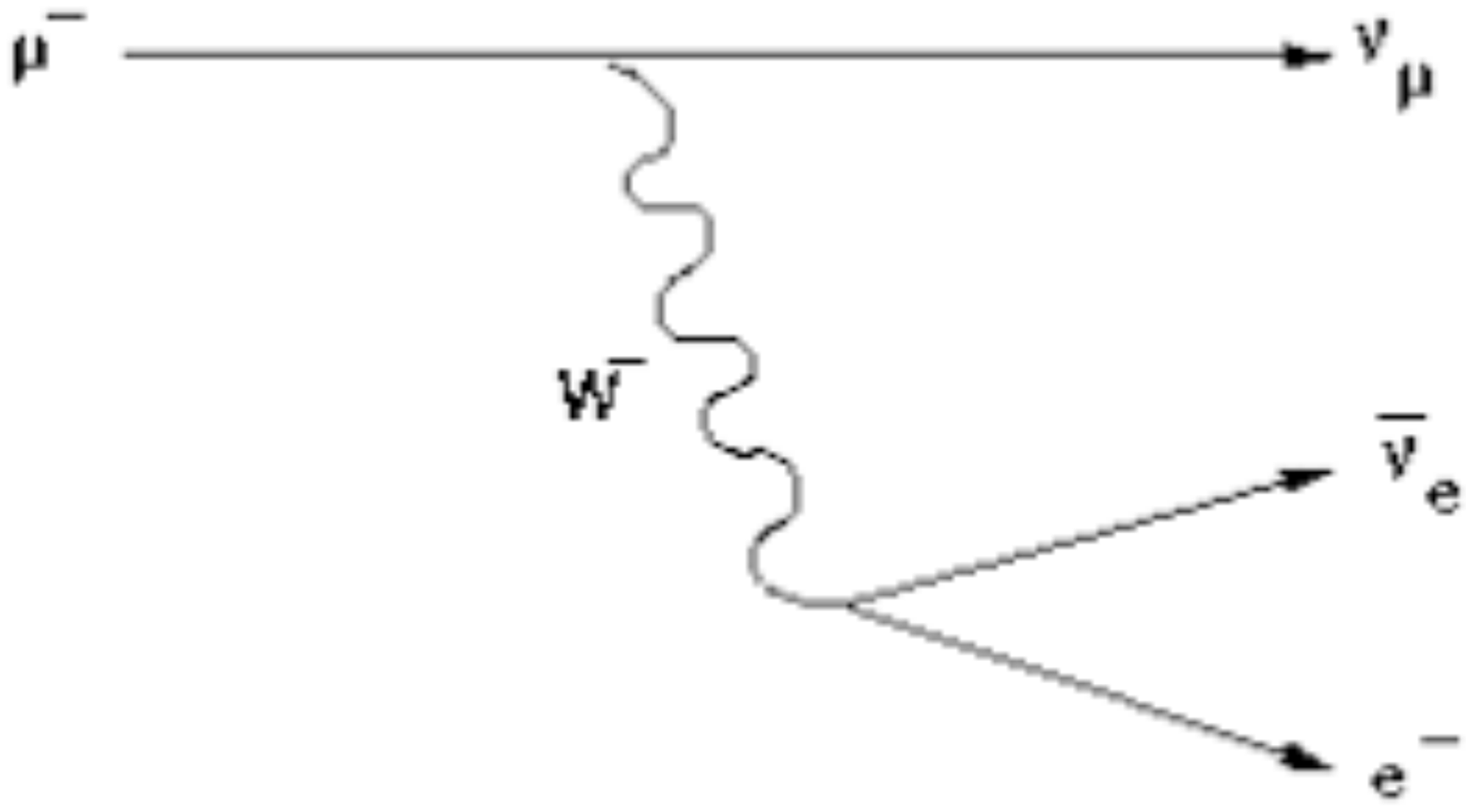
- **There is no right-handed neutrino.**
- **The neutrino is massless in the Standard Model.**

➢ Lepton number conservation

Lepton	Conserved Quantity	Lepton Number
e^-	L_e	+1
ν_e		+1
μ^-	L_μ	+1
ν_μ		+1
τ^-	L_τ	+1
ν_τ		+1

Anti-Lepton	Conserved Quantity	Lepton Number
e^+	L_e	-1
$\overline{\nu}_e$		-1
μ^+	L_μ	-1
$\overline{\nu}_\mu$		-1
τ^+	L_τ	-1
$\overline{\nu}_\tau$		-1

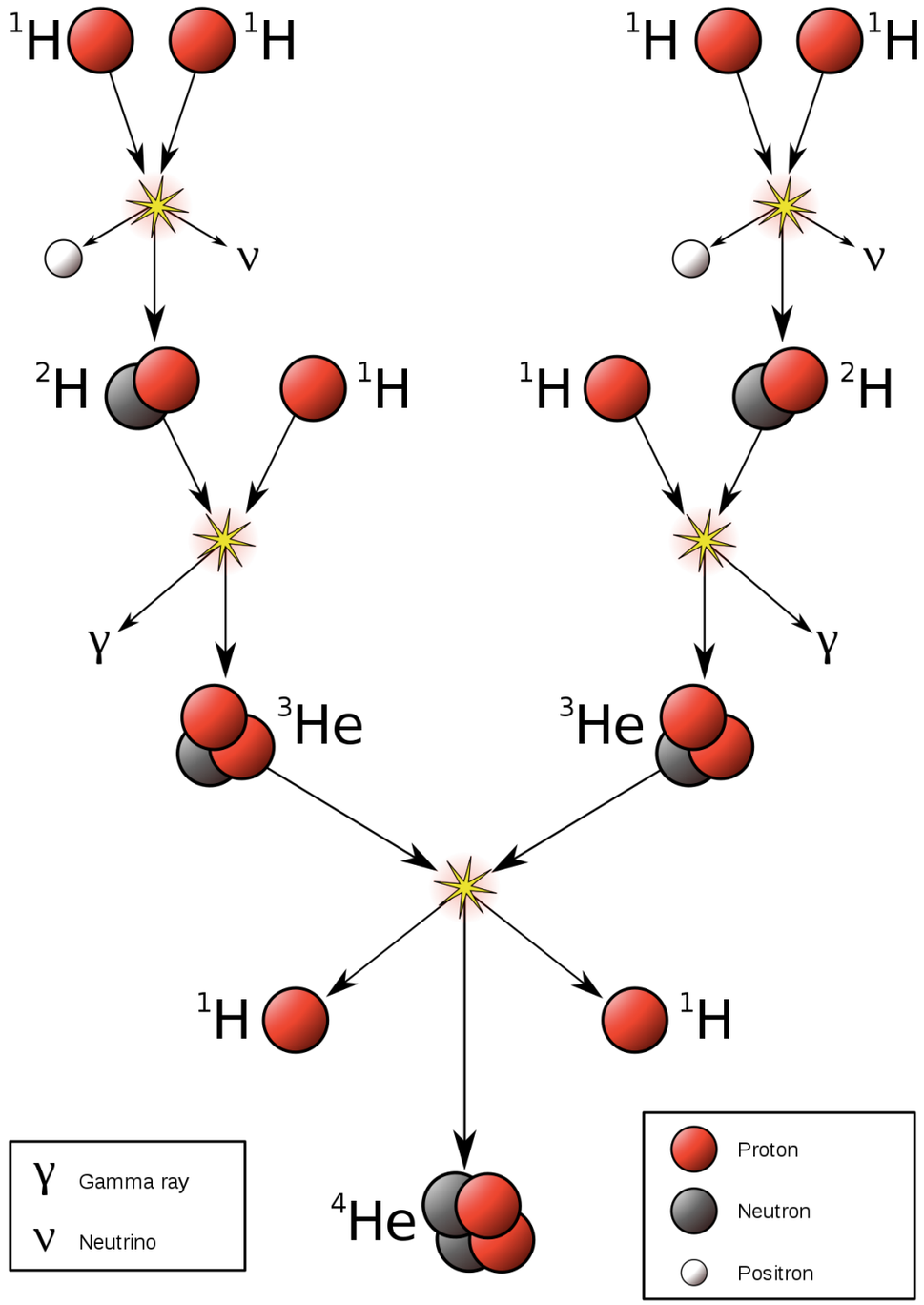
We find that L_e, L_μ and L_τ are each conserved quantities


μ⁻
ν μ
W⁻
ν̄ e
e⁻

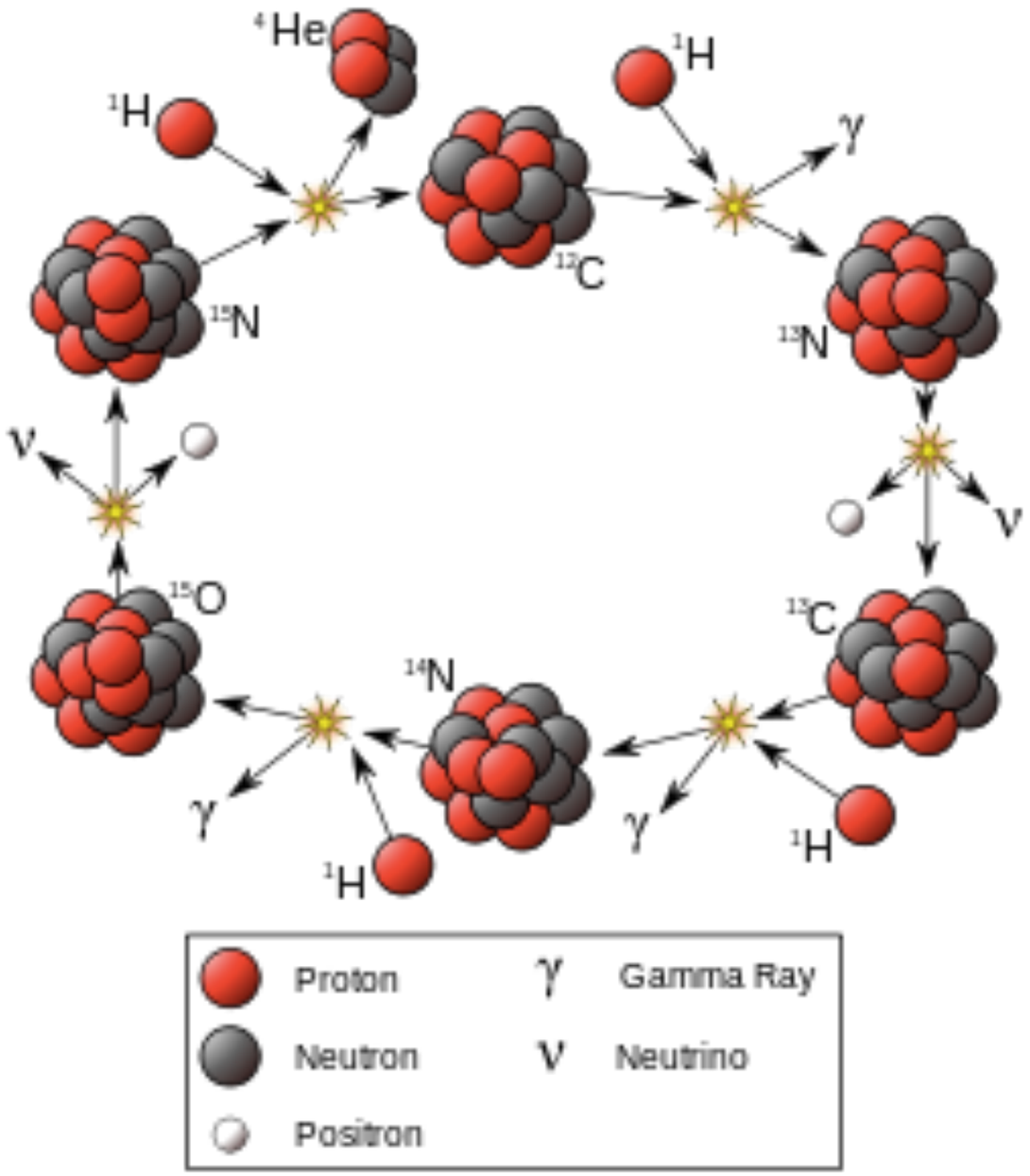
$$
\begin{array}{rccccccc}
 & \mu^- & \rightarrow & e^- & + & \overline{\nu}_e & + & \nu_\mu \\
L: & 1 & = & 1 & - & 1 & + & 1 \\
L_e: & 0 & = & 1 & - & 1 & + & 0 \\
L_\mu: & 1 & = & 0 & + & 0 & + & 1
\end{array}
$$

Why do stars shine?

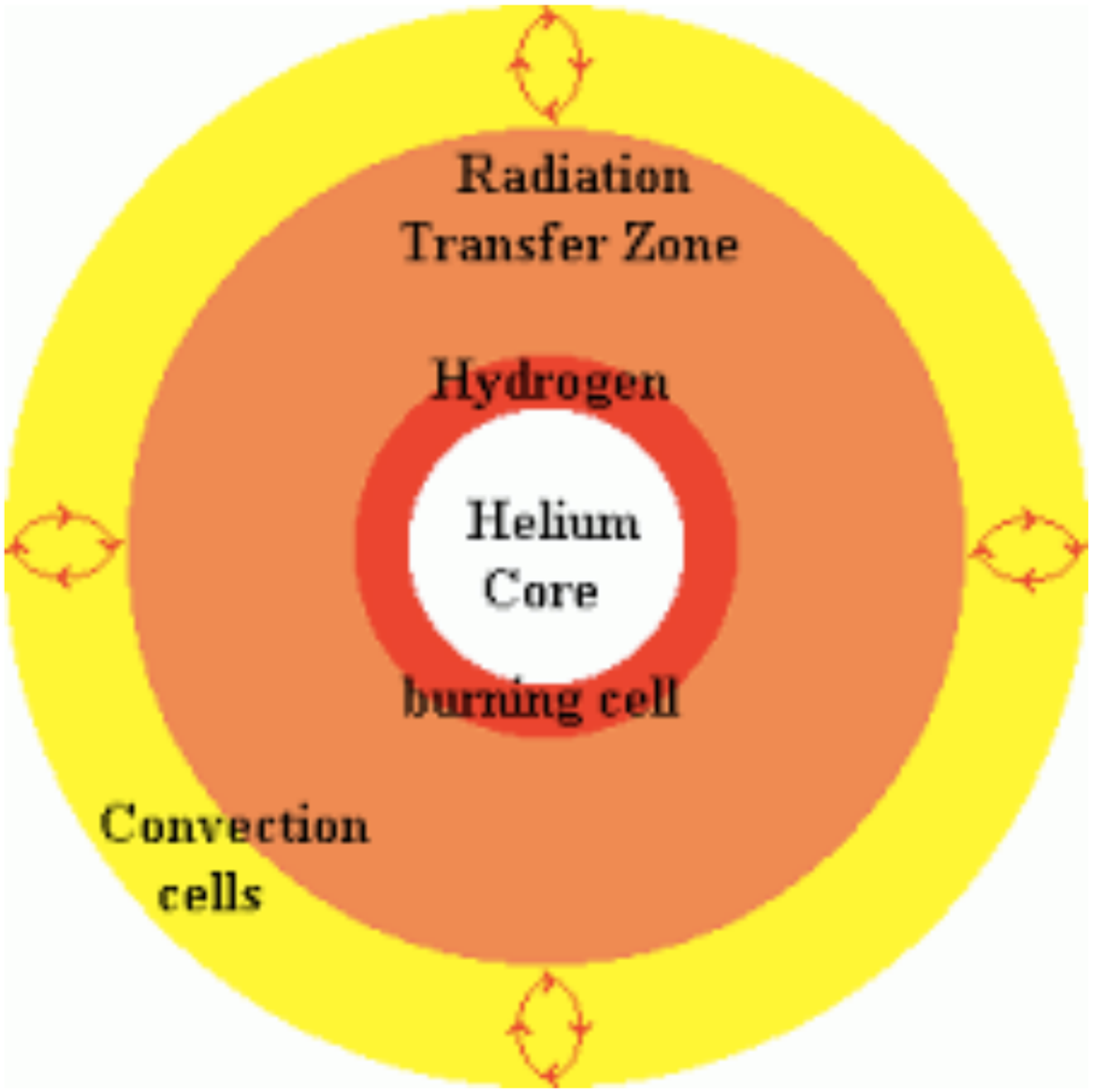
1. pp chain


^{1}H
^{1}H
ν
^{2}H
^{1}H
γ
^{3}He
^{1}H
^{1}H
ν
^{1}H
^{2}H
γ
^{3}He
^{1}H
^{1}H
^{4}He
γ Gamma ray
ν Neutrino
Proton
Neutron
Positron

2. CNO chain


⁴He
¹H
¹⁵N
¹²C
¹H
γ
¹³N
ν
¹⁵O
¹⁴N
¹²C
γ
¹H
γ
¹H
Proton
Neutron
Positron
γ Gamma Ray
ν Neutrino

Standard Solar Model



pp chain: many neutrinos are coming out

Solar neutrino flux

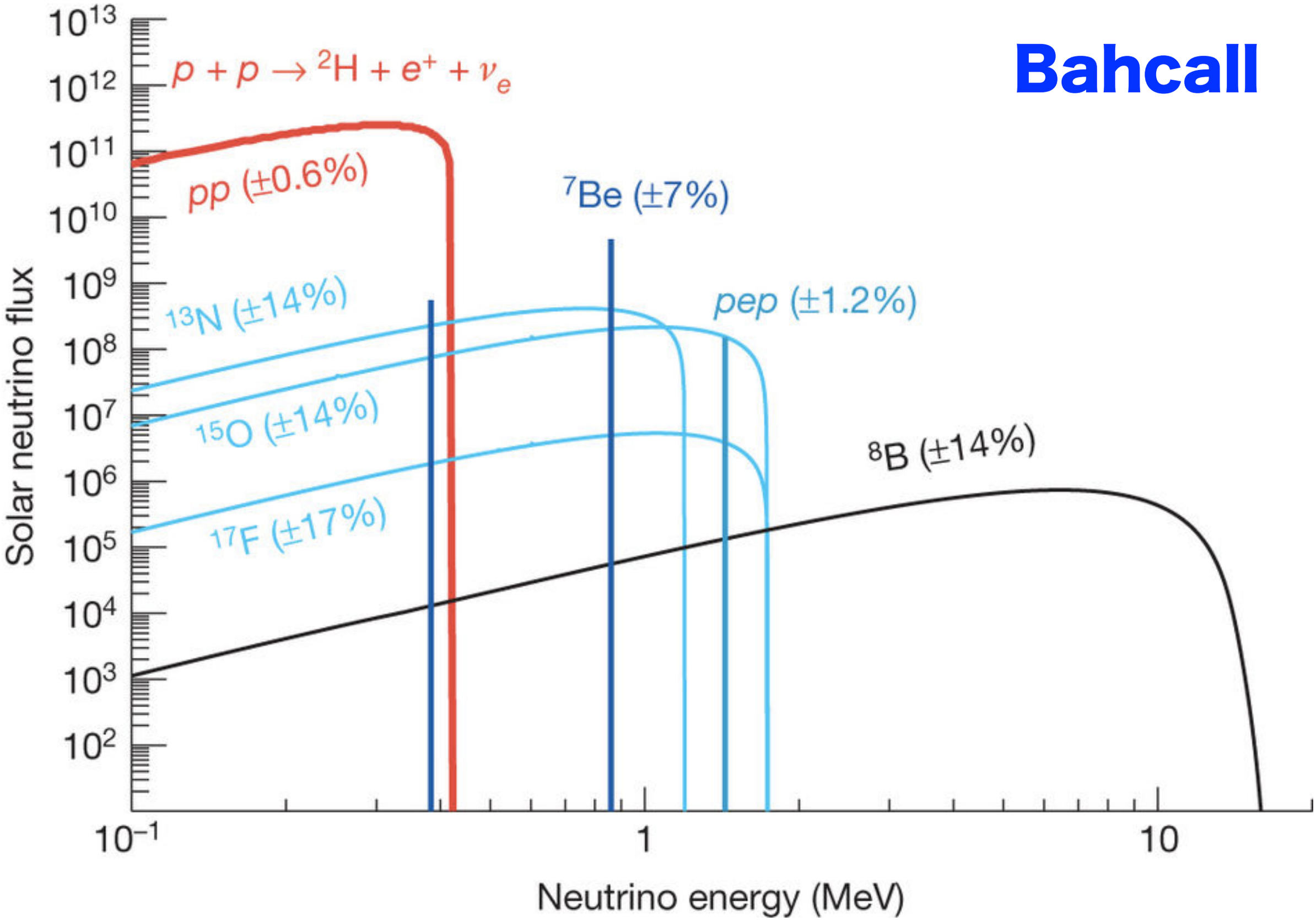
Neutrino flux on Earth: $\Phi(\nu_e) \approx 6.4 \times 10^{10}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$



Bahcall

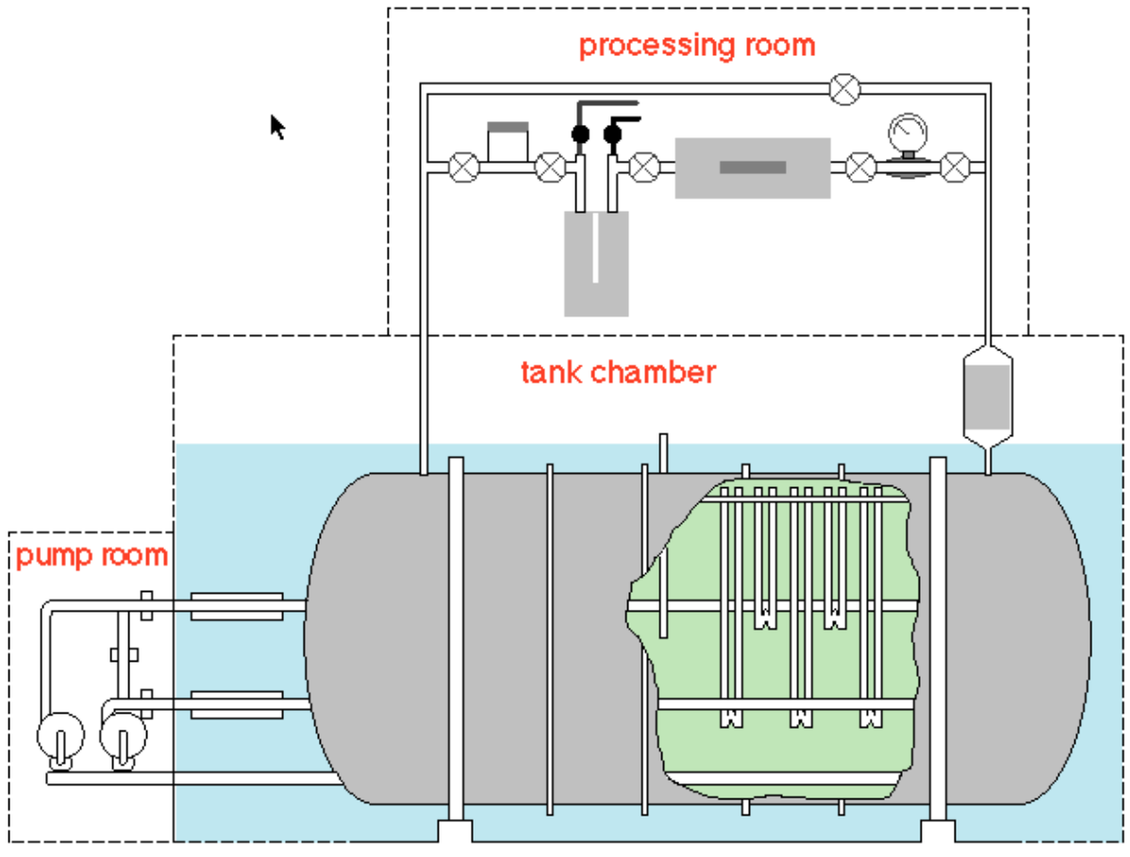
Davis

Bahcall


$p + p \rightarrow {}^2\mathrm{H} + e^+ + \nu_e$
pp (±0.6%)
^{7}Be (±7%)
pep (±1.2%)
^{13}N (±14%)
^{15}O (±14%)
^{17}F (±17%)
^{8}B (±14%)
Solar neutrino flux
10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{-1}
1
10
Neutrino energy (MeV)

The Homestake experiment: Ray Davis

to collect and count neutrinos emitted by nuclear fusion taking place in the Sun.



Expect about 1.5 Ar atoms/day

- **The experiment operated continuously from 1970 until 1994.**

$$\nu_{\mathrm{e}} + {}^{37}\mathrm{Cl} \longrightarrow {}^{37}\mathrm{Ar} + \mathrm{e}^{-}.$$

- **The experiment operated continuously from 1970 until 1994.**

$$\nu_e + {}^{37}\mathrm{Cl} \longrightarrow {}^{37}\mathrm{Ar} + e^- .$$

- **Davis placed 1,478 meters (4,850 feet) underground a 380 cubic meter (100,000 gallon) tank of perchloroethylene, a common dry-cleaning fluid.**

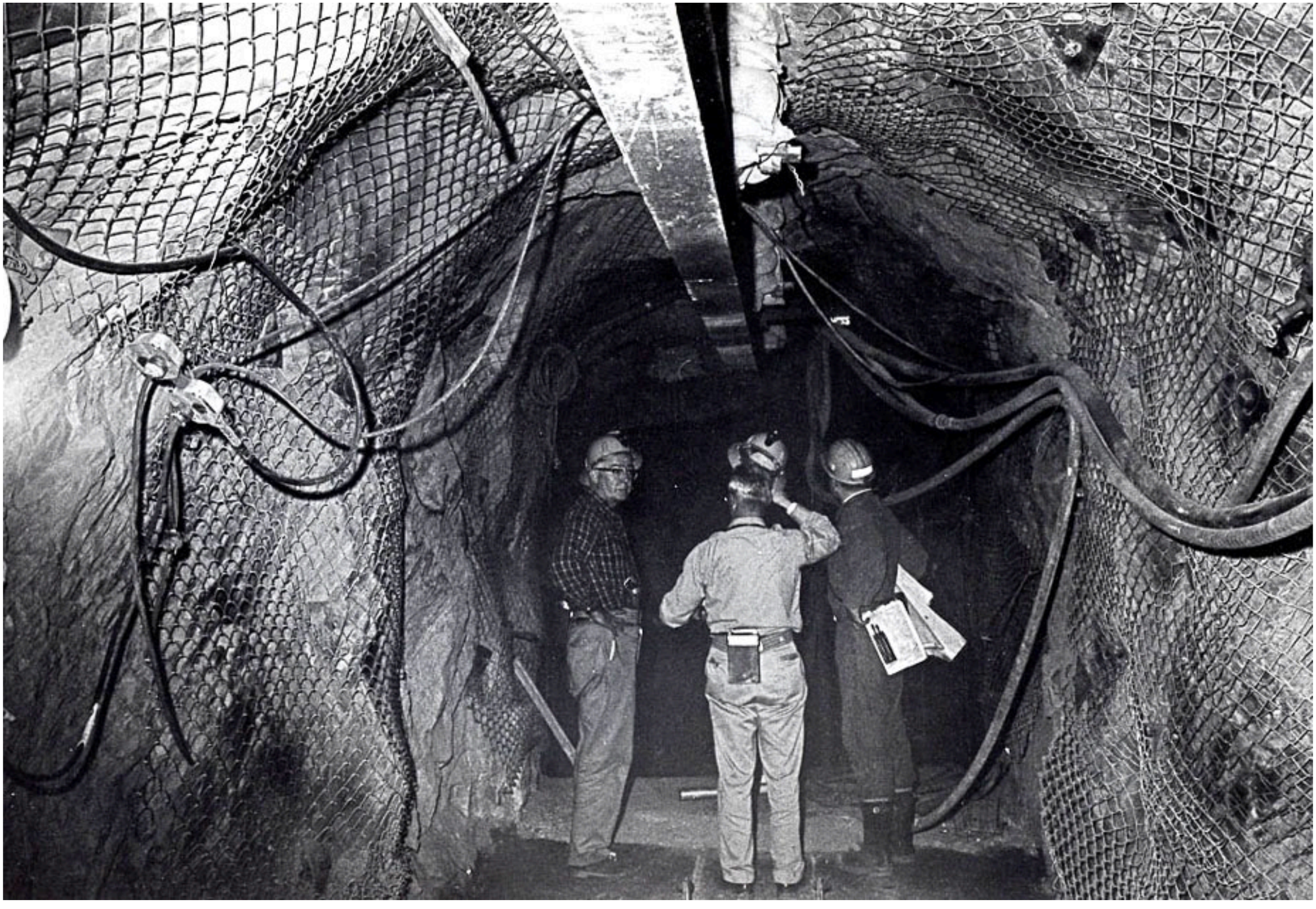
- **The experiment operated continuously from 1970 until 1994.**

$$\nu_{\mathrm{e}} + {}^{37}\mathrm{Cl} \longrightarrow {}^{37}\mathrm{Ar} + \mathrm{e}^{-}.$$

- **Davis placed 1,478 meters (4,850 feet) underground a 380 cubic meter (100,000 gallon) tank of perchloroethylene, a common dry-cleaning fluid.**

- **Deep underground was needed to prevent interference from cosmic rays**



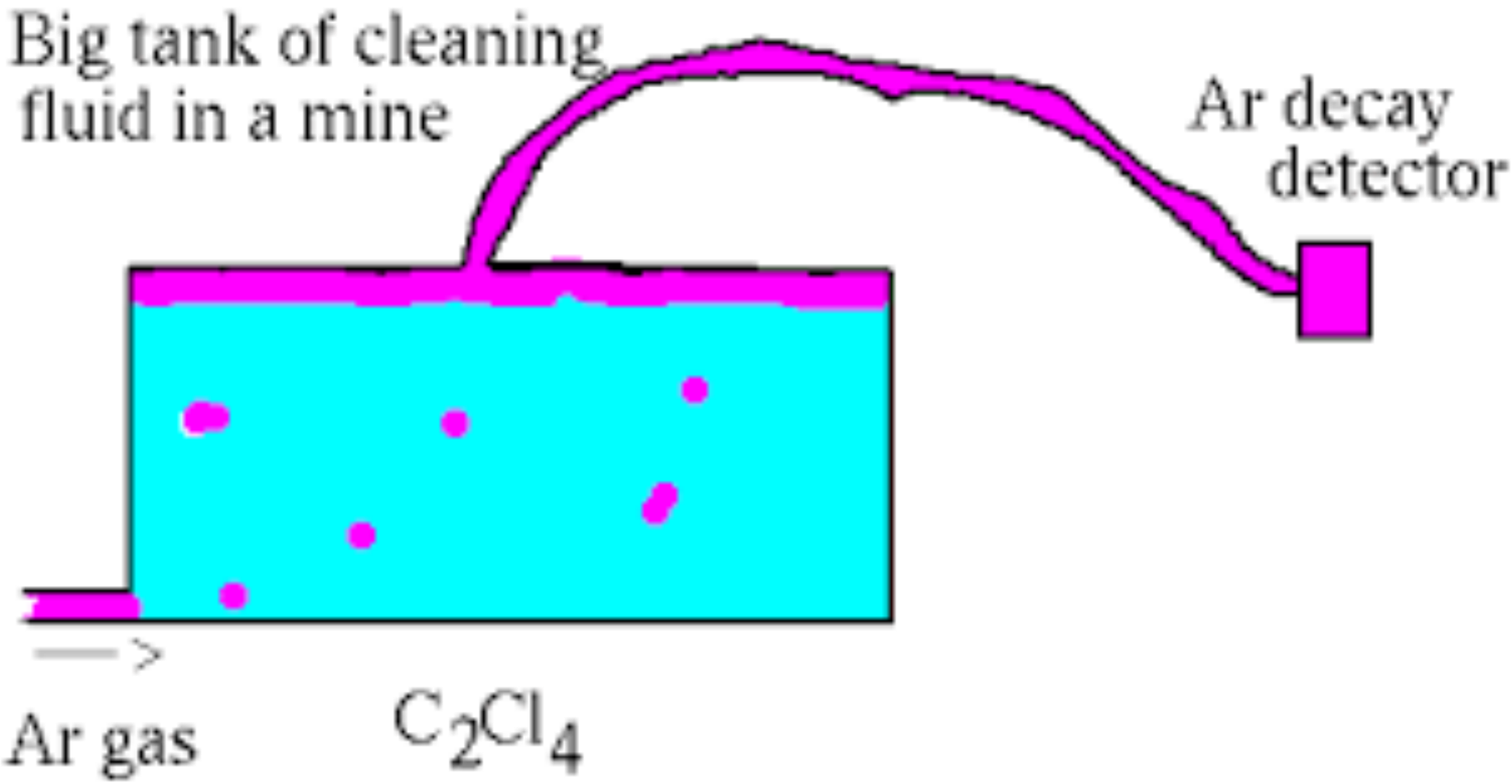




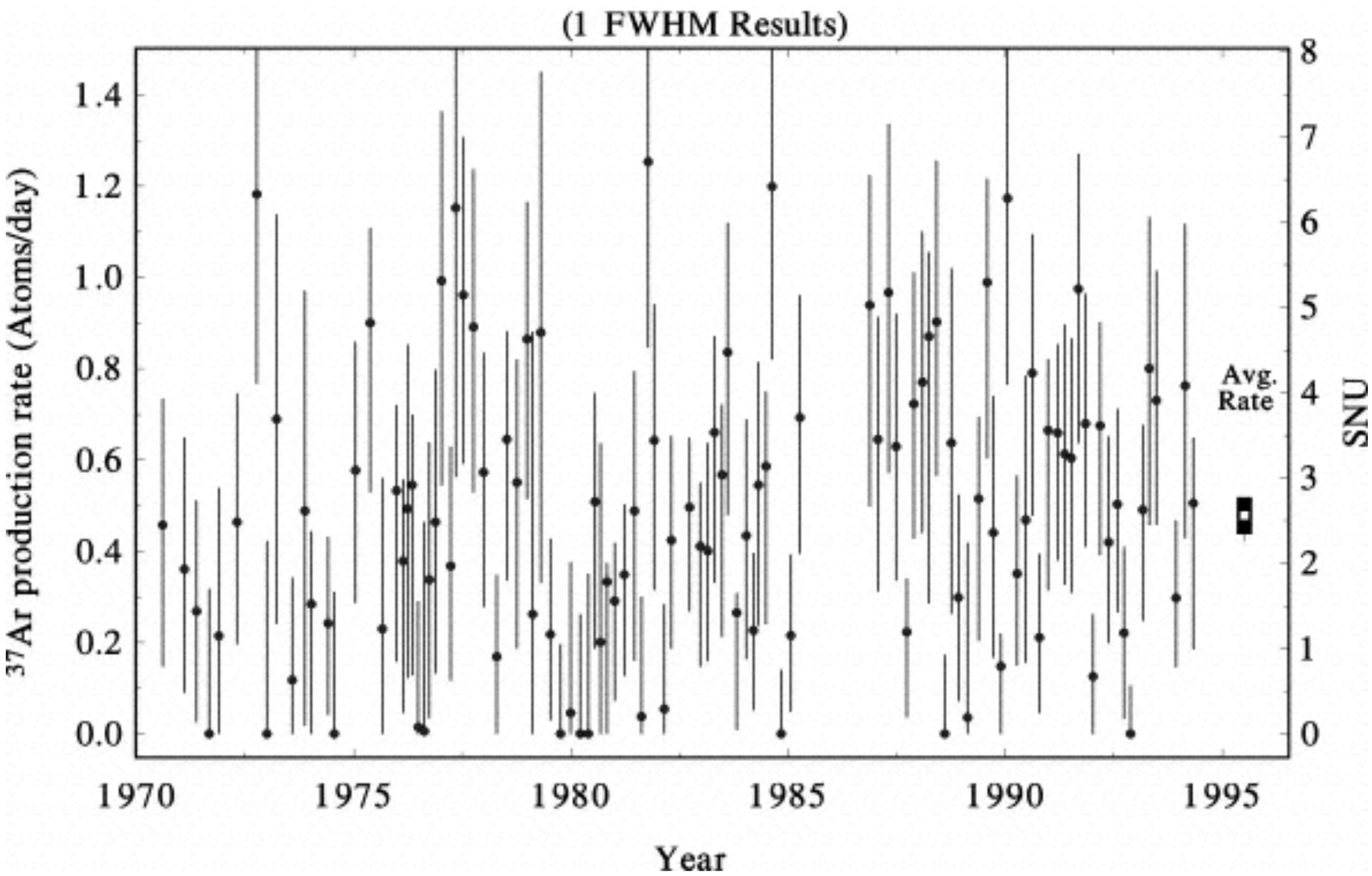
- **Every few weeks, Davis bubbled helium through the tank to collect the argon that had formed.**

- **Solar Model Expectation = 7.7±1.3 SNU (1.5 atoms/day)**

1 SNU = 1 interaction per 10^{36} target atoms per second


Big tank of cleaning fluid in a mine
Ar decay detector
Ar gas
C_2Cl_4



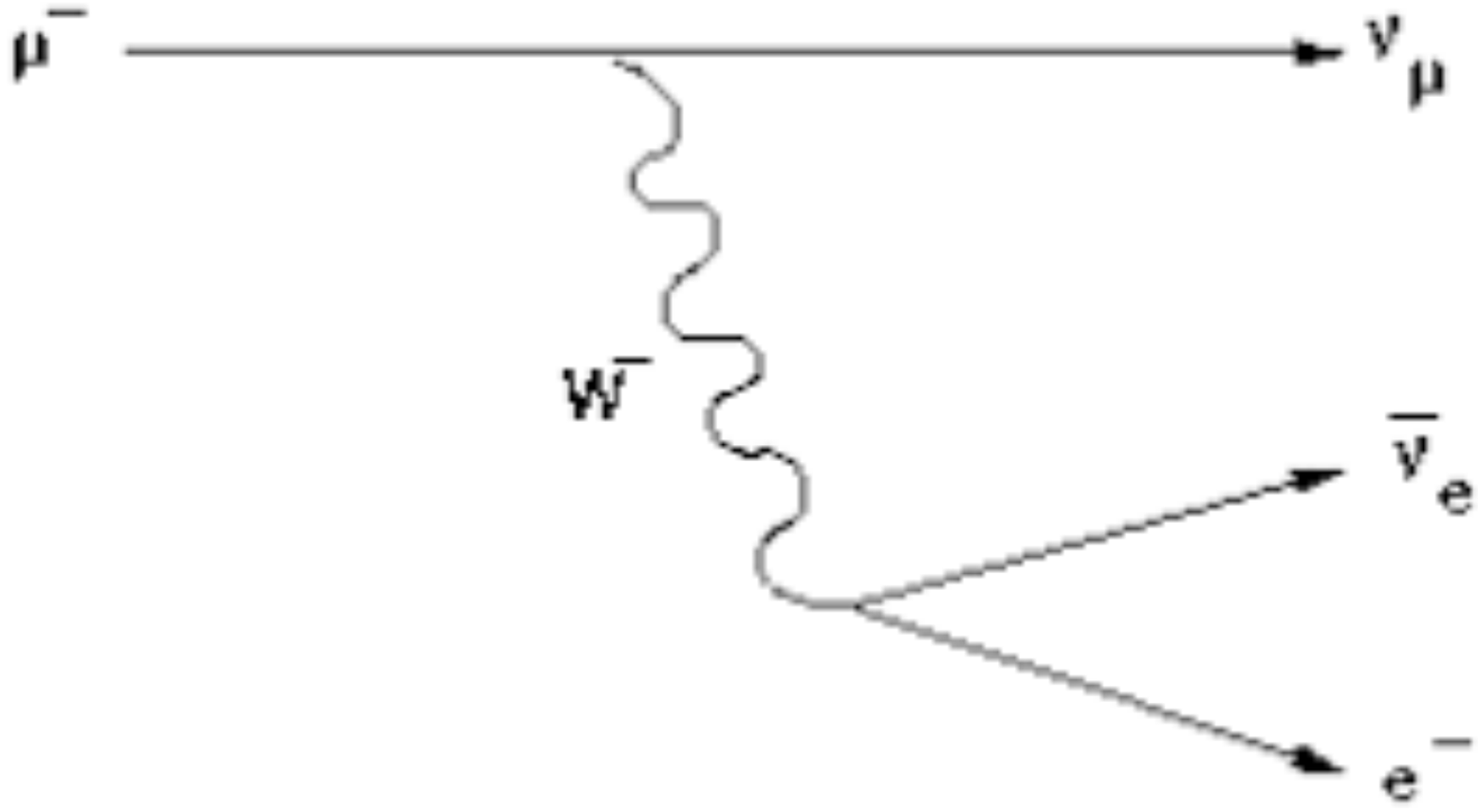

(1 FWHM Results)
^{37}Ar production rate (Atoms/day)
SNU
Avg. Rate
Year
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0
1
2
3
4
5
6
7
8
1970
1975
1980
1985
1990
1995

How to explain this?

Neutrino oscillation

A weak eigenstate is NOT a mass eigenstate!

➢ Weak eigenstates



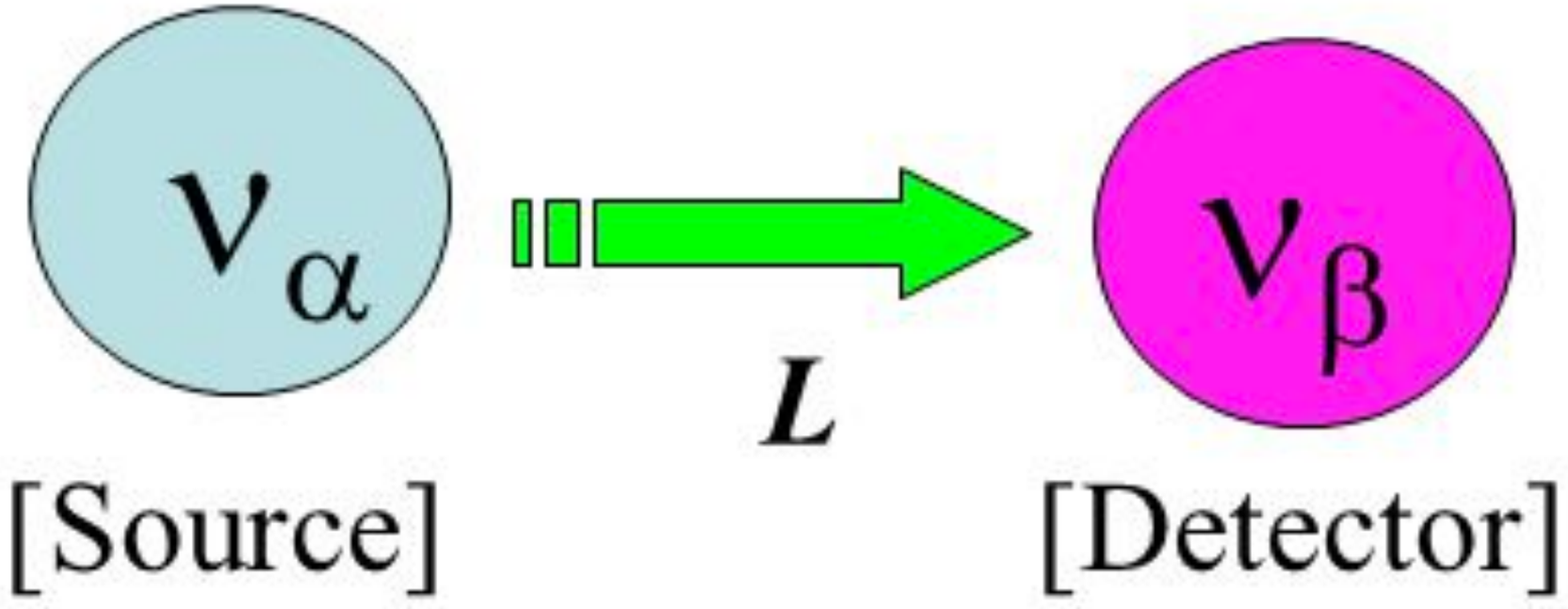
➢ Lepton number conservation

Lepton	Conserved Quantity	Lepton Number
e^-	L_e	+1
ν_e		+1
μ^-	L_μ	+1
ν_μ		+1
τ^-	L_τ	+1
ν_τ		+1

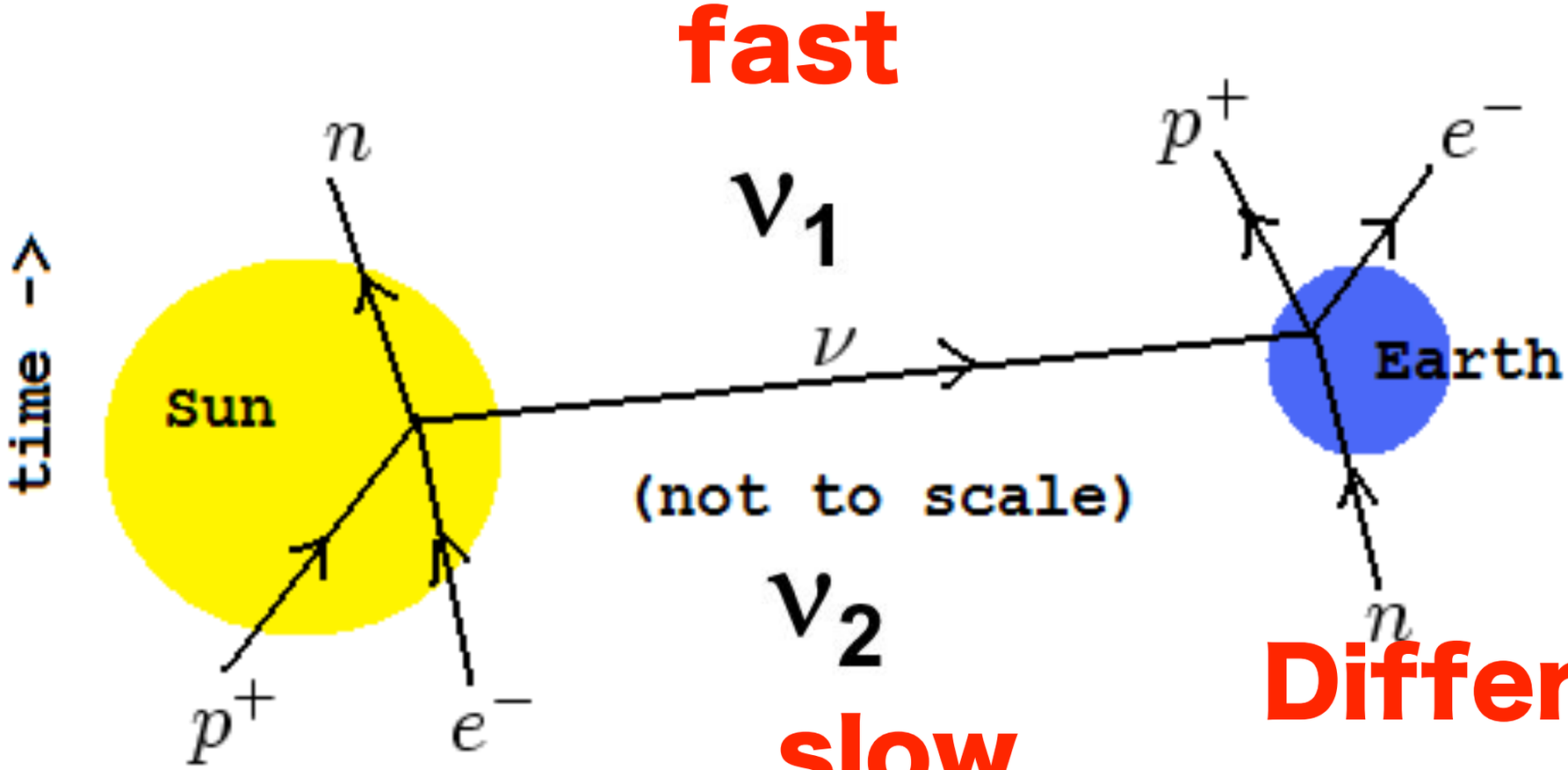
Anti-Lepton	Conserved Quantity	Lepton Number
e^+	L_e	-1
$\overline{\nu}_e$		-1
μ^+	L_μ	-1
$\overline{\nu}_\mu$		-1
τ^+	L_τ	-1
$\overline{\nu}_\tau$		-1

We find that L_e, L_μ and L_τ are each conserved quantities

➢ Weak eigenstates: both source and detector



➢ Mass eigenstates



Consider two neutrinos.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \begin{matrix} \boxed{\text{mass } m_1} \\ \\ \boxed{\text{mass } m_2} \end{matrix}$$

Consider ν_μ at t=0

$$|\nu_\mu(t=0)\rangle = \nu_\mu = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

Definite energy state → time evolution

Let $\hbar = c = 1$.

$$|\nu_\mu(t)\rangle = -\sin\theta|\nu_1\rangle e^{-iE_1 t} + \cos\theta|\nu_2\rangle e^{-iE_2 t}$$

where $E_i = \sqrt{p^2 + m_i^2}$.

Assume that neutrinos are relativistic.

$$E \simeq p + \frac{1}{2}\frac{m^2}{p}, \quad \text{if } p \gg m.$$

$$|\nu_\mu(t)\rangle = e^{-i\{p+m_1^2/(2p)\}t}\left[-\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle e^{i(m_1^2-m_2^2)t/(2p)}\right]$$

Assume that neutrinos are relativistic.

$$E \simeq p + \frac{1}{2}\frac{m^2}{p}, \quad \text{if } p \gg m.$$

$$|\nu_\mu(t)\rangle = e^{-i\{p+m_1^2/(2p)\}t}\left[-\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle e^{i(m_1^2-m_2^2)t/(2p)}\right]$$

For simplicity, $\boxed{\Delta m^2 = m_1^2 - m_2^2, \quad e^{-iz} = e^{-i\{p+m_1^2/(2p)\}t}}$

$$|\nu_\mu(t)\rangle = e^{-iz}\left[-\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle e^{i(\Delta m^2/2p)t}\right]$$

Probability for $\nu_\mu \to \nu_e$?

$$\boxed{P\left(\nu_\mu \to \nu_e\right) \;=\; \left|\left\langle \nu_e | \nu_\mu(t) \right\rangle\right|^2}$$

$$\left\langle \nu_e \right| \;=\; \cos\theta \;\left\langle \nu_1 \right| \;+\; \sin\theta \left\langle \nu_2 \right|$$

Probability for $\nu_\mu \to \nu_e$?

$$\boxed{P\left(\nu_\mu \to \nu_e\right) \;=\; \left|\left\langle \nu_e | \nu_\mu(t) \right\rangle\right|^2}$$

$$\left\langle \nu_e \right| \;=\; \cos\theta \;\left\langle \nu_1 \right| \;+\; \sin\theta \left\langle \nu_2 \right|$$

$$\boxed{\left\langle \nu_i | \nu_j \right\rangle = \delta_{ij}.}$$

$$\left\langle \nu_e | \nu_\mu(t) \right\rangle = e^{-iz} \left(-\sin\theta\cos\theta + \sin\theta\cos\theta \; e^{\frac{i}{2}\frac{\Delta m^2}{p}x} \right)$$

$$P\left(\nu_\mu \to \nu_e\right) \;=\; \left|\langle \nu_e | \nu_\mu(t)\rangle\right|^2$$

$$= \;\frac{1}{2}\sin^2 2\theta\left(1-\cos\left(\frac{\Delta m^2}{2}\frac{L}{E_\nu}\right)\right)$$

$$p = E_\nu$$

$$x = L$$

$$\boxed{(1-\cos 2\theta)/2 = \sin^2\theta}$$

$$P\left(\nu_\mu \to \nu_e\right) \;=\; \sin^2 2\theta \;\sin^2\left(\frac{\Delta m^2}{4}\frac{L}{E_\nu}\right)$$

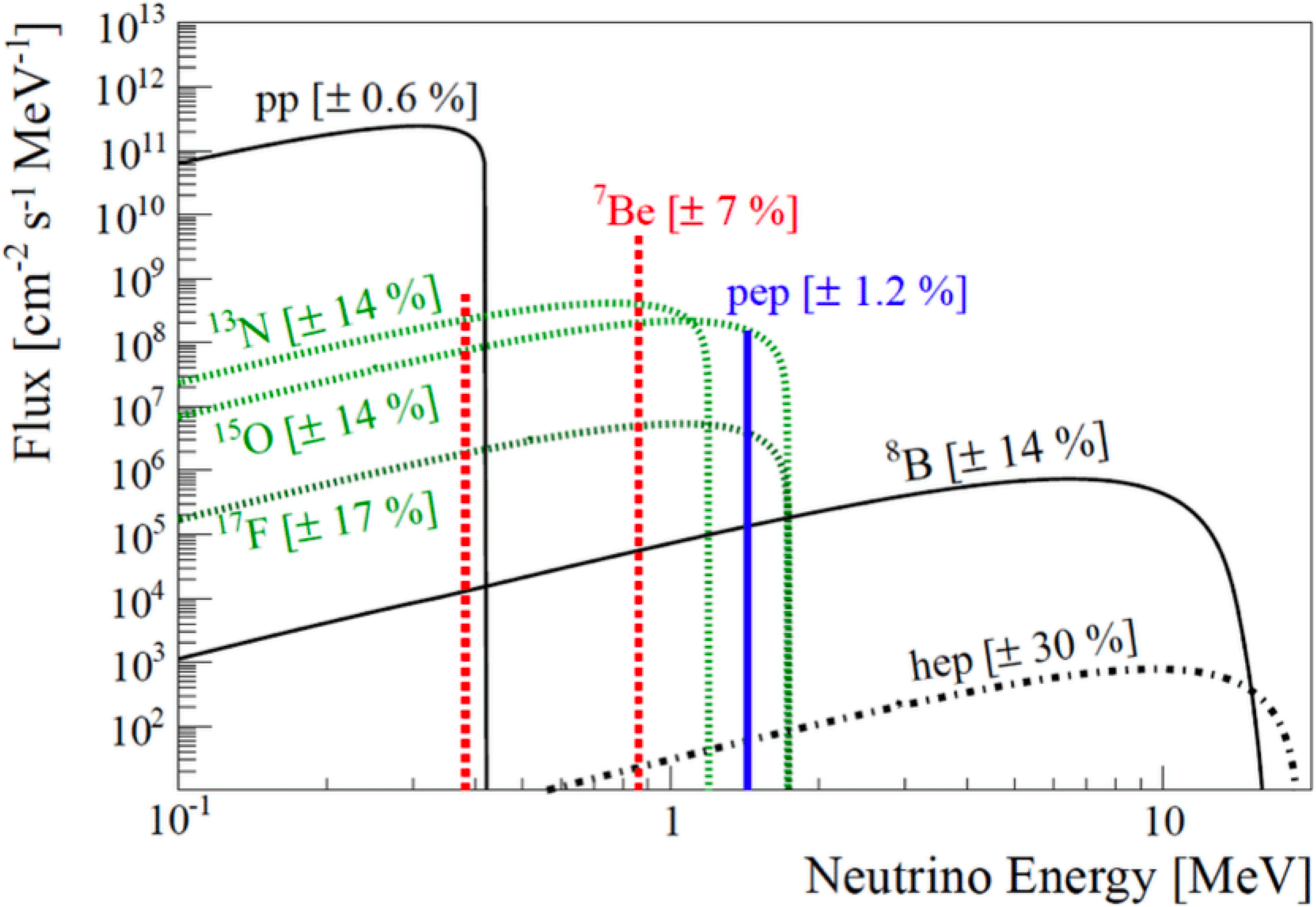
$$\boxed{P_{\nu_\mu \to \nu_e}(L, E) \;=\; \sin^2 2\theta \;\sin^2\left(1.27\;\Delta m^2 \frac{L}{E_\nu}\right)}$$

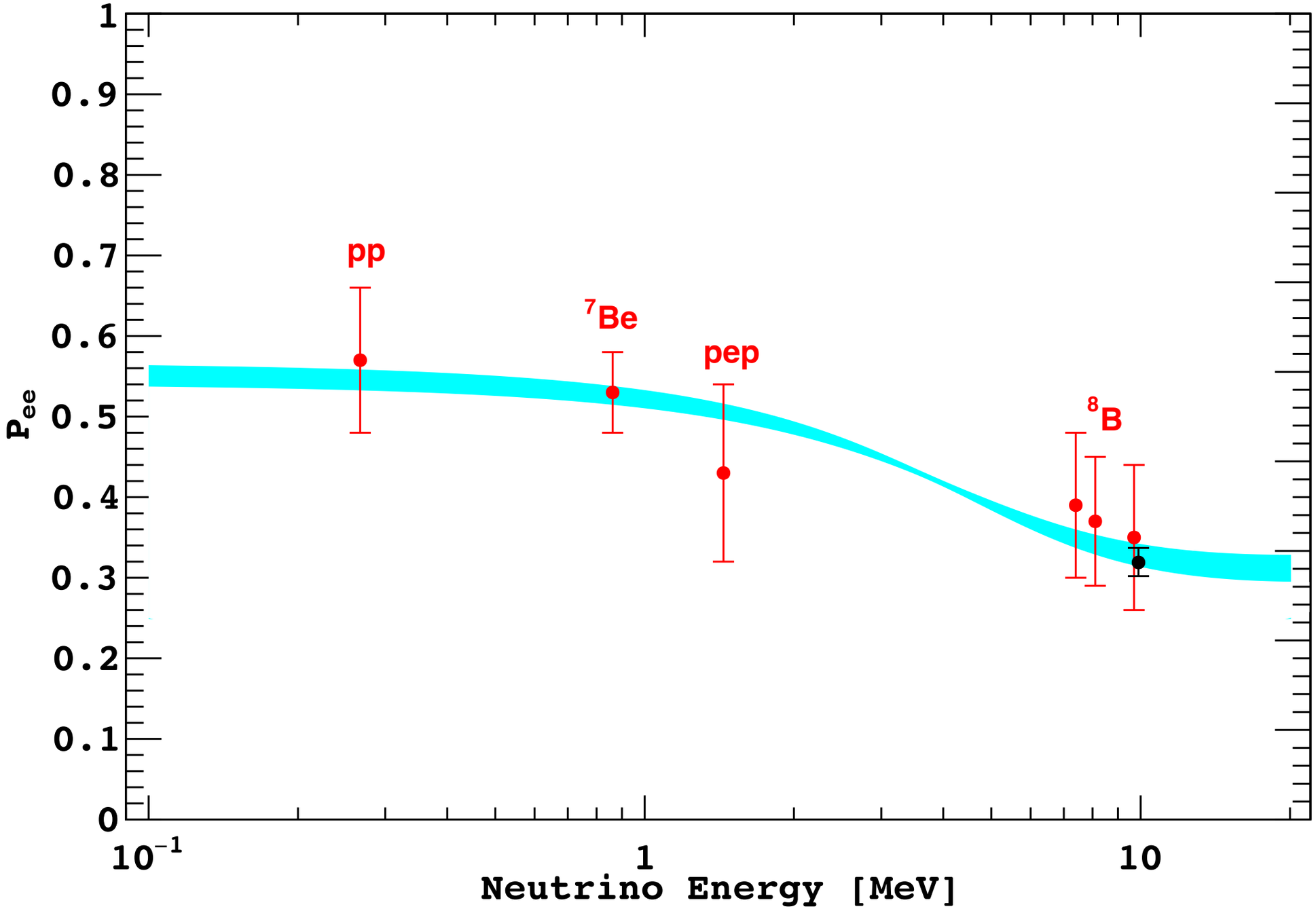
Δm^2 in eV^2, L in meters, and E in MeV.

Δm^2 in eV^2, L in km, and E in GeV.

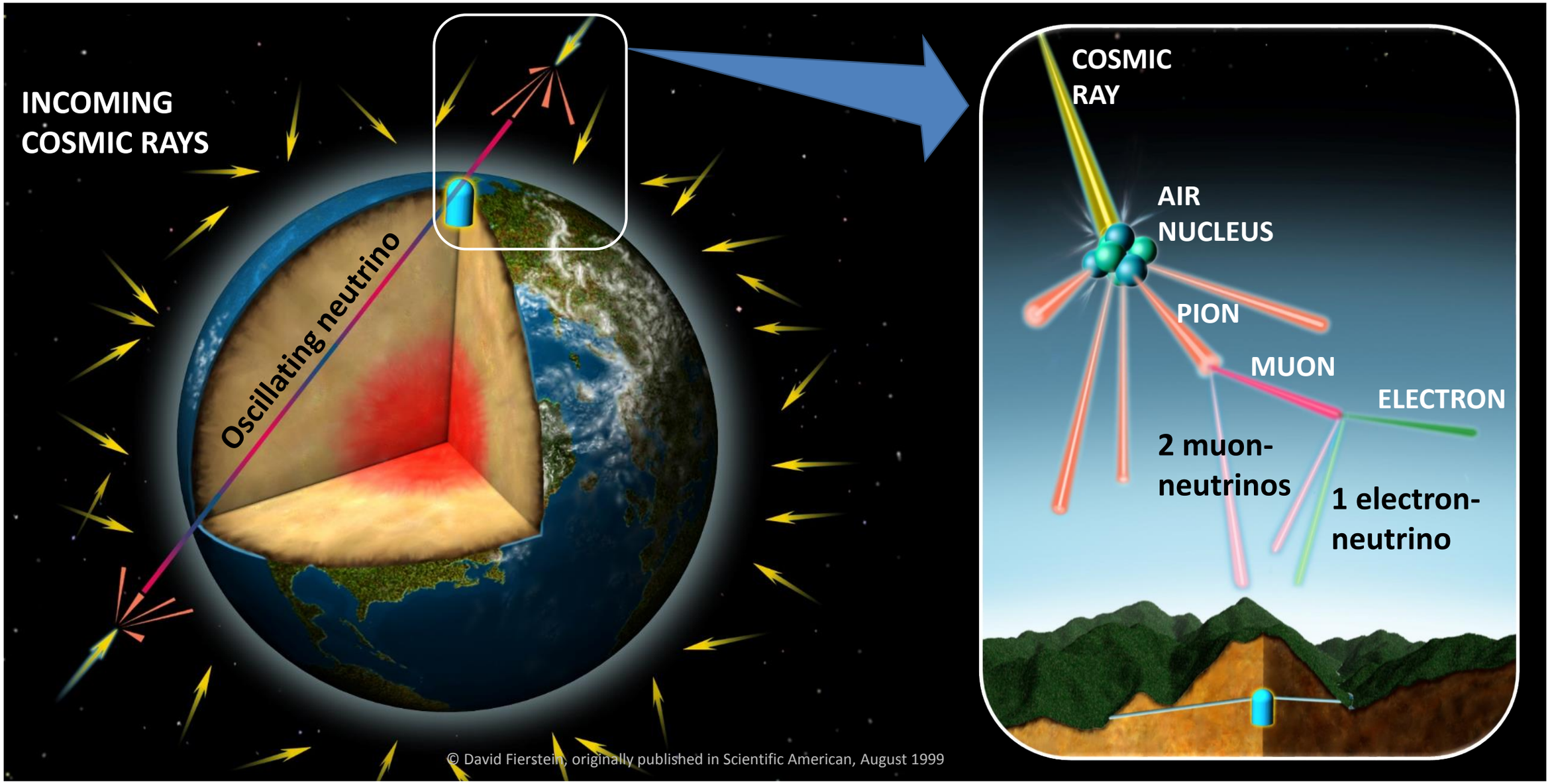
Neutrino oscillation happens only when neutrinos are massive.

Solar neutrino energy

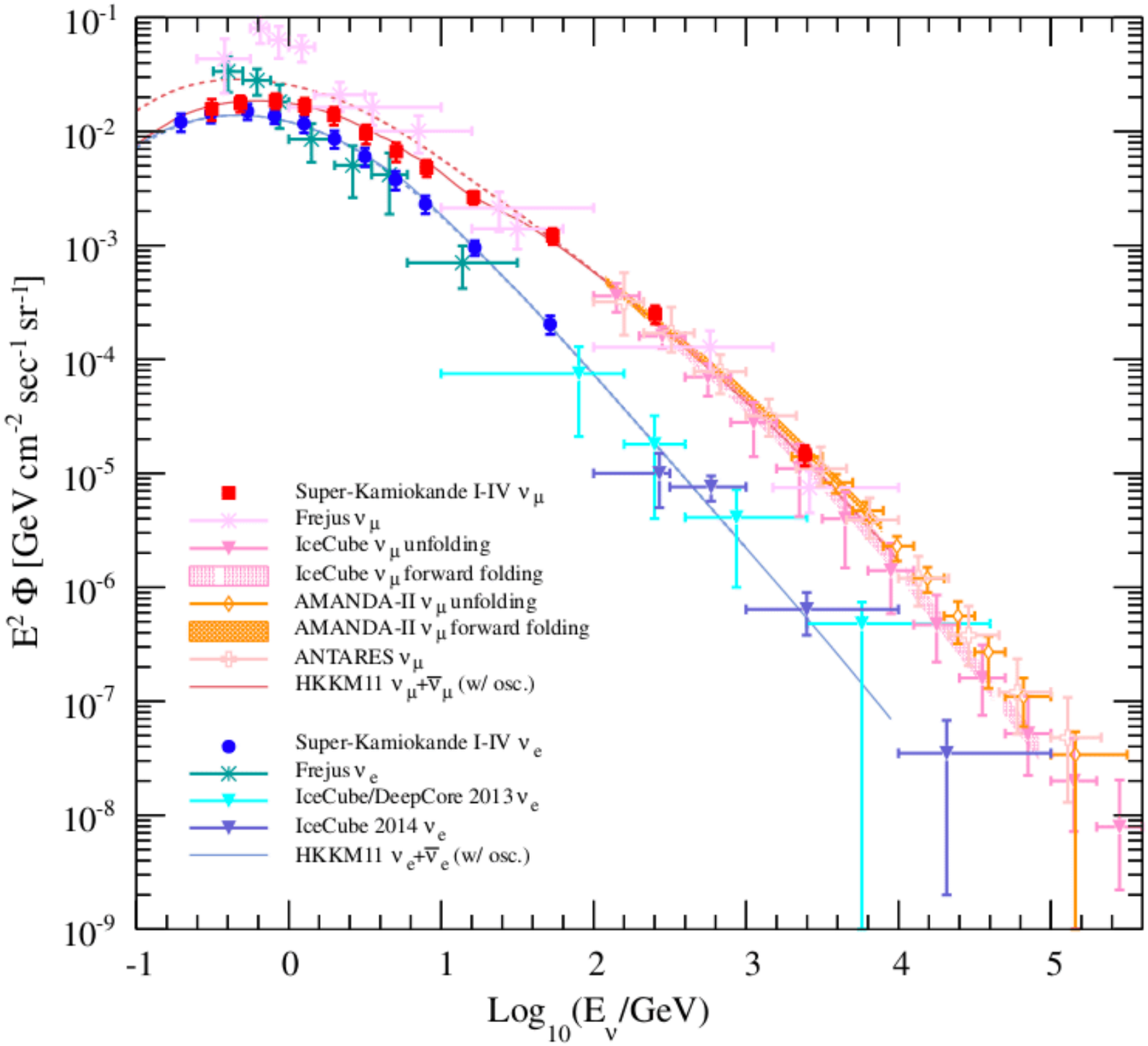



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
P_{ee}
pp
^{7}Be
pep
^{8}B
10^{-1}
1
10
Neutrino Energy [MeV]

Atmospheric neutrino



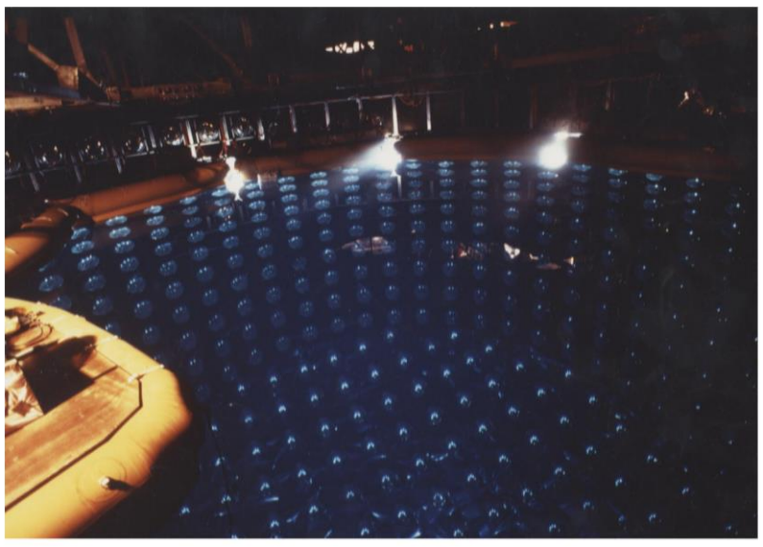
Atmospheric neutrino energy



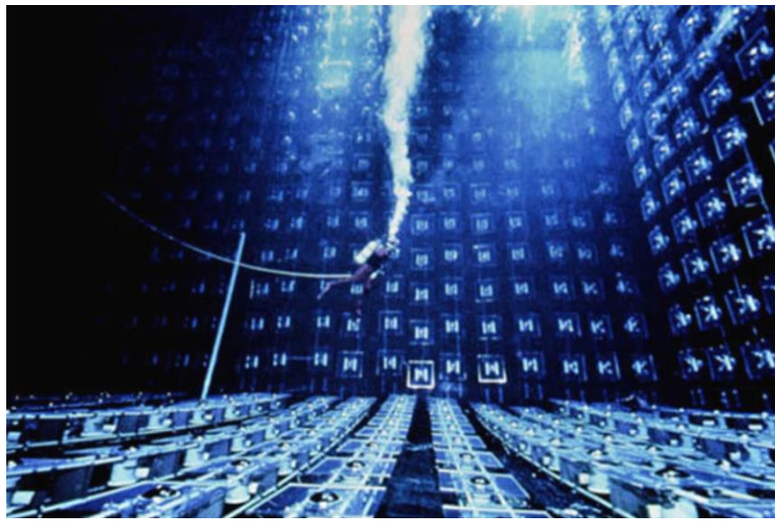
Atmospheric neutrinos as backgrounds for the proton decay experiments

Grand Unified Theories
(in the 1970's)
➔ $\tau_p = 10^{30\pm2}$ years

Kamiokande (1000 ton)

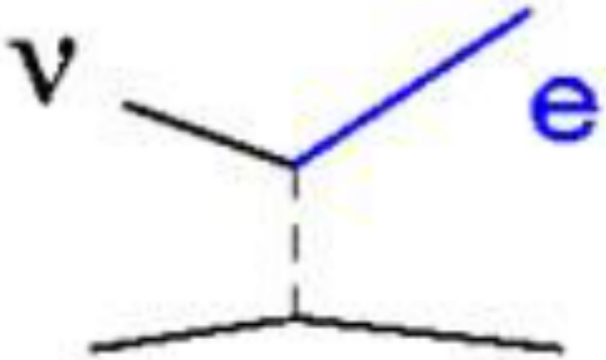


IMB (3300 ton)



They observed many atmospheric neutrino events (background for proton decay).

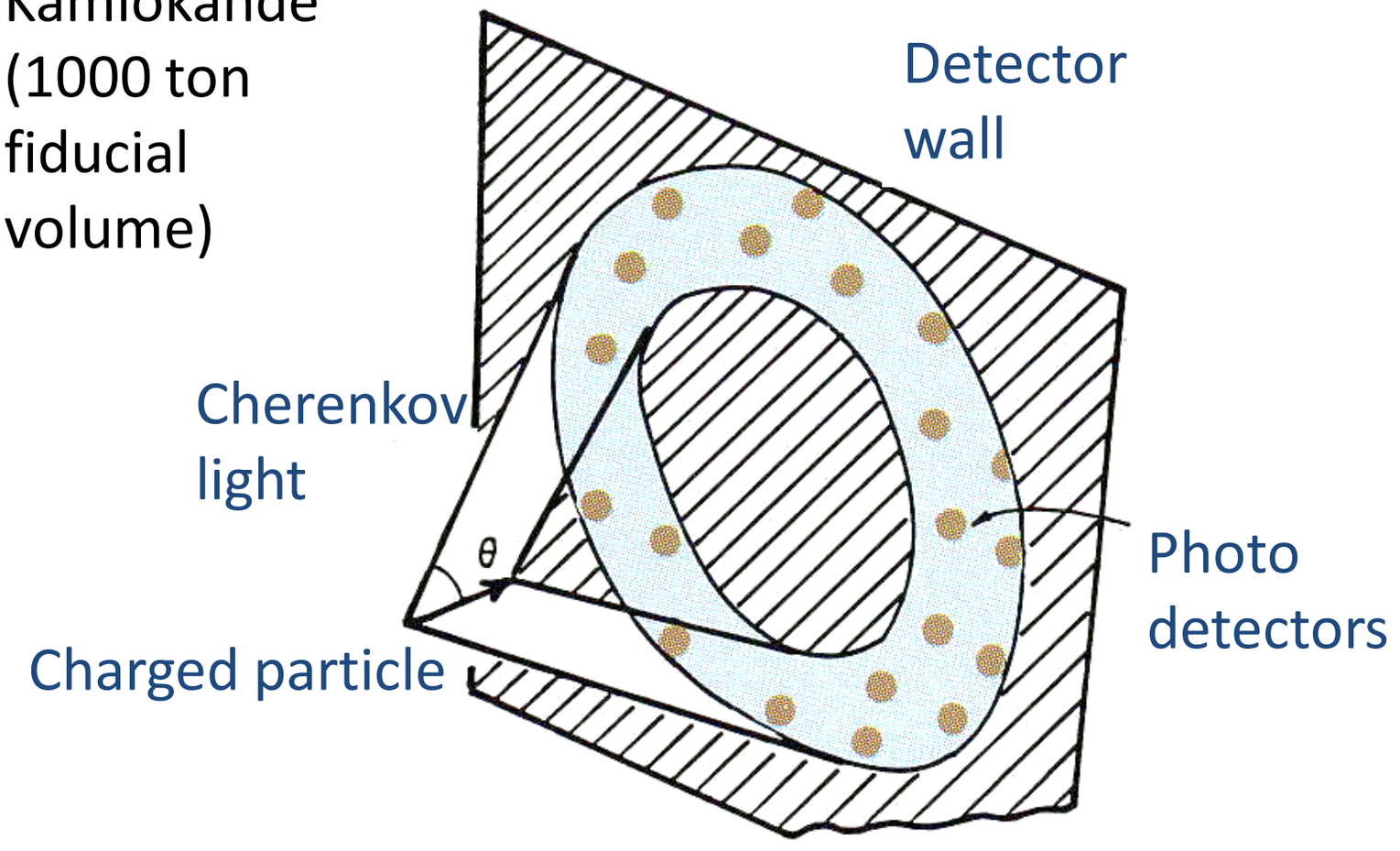
Electron neutrino



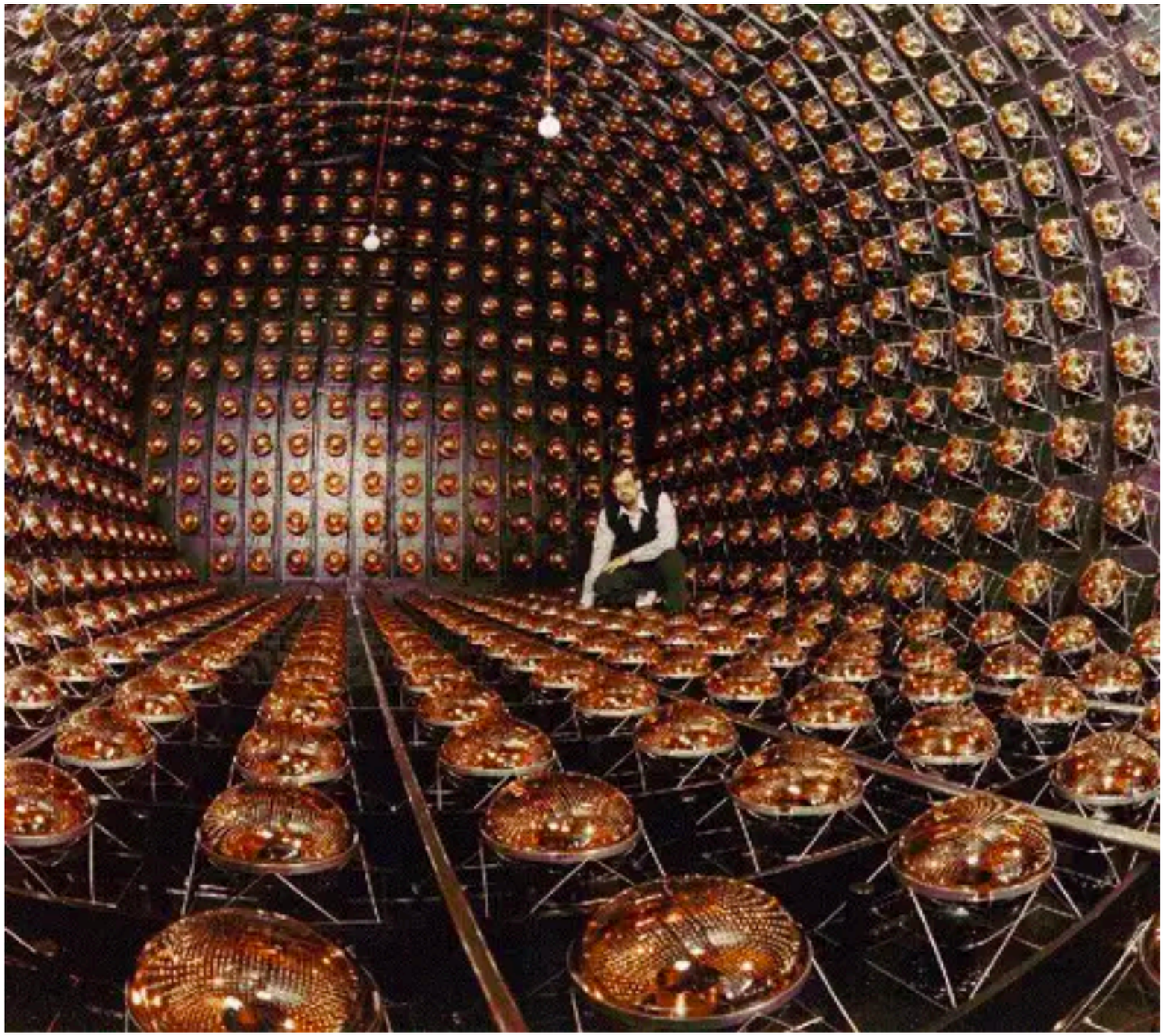
Muon neutrino



Kamiokande (1000 ton fiducial volume)

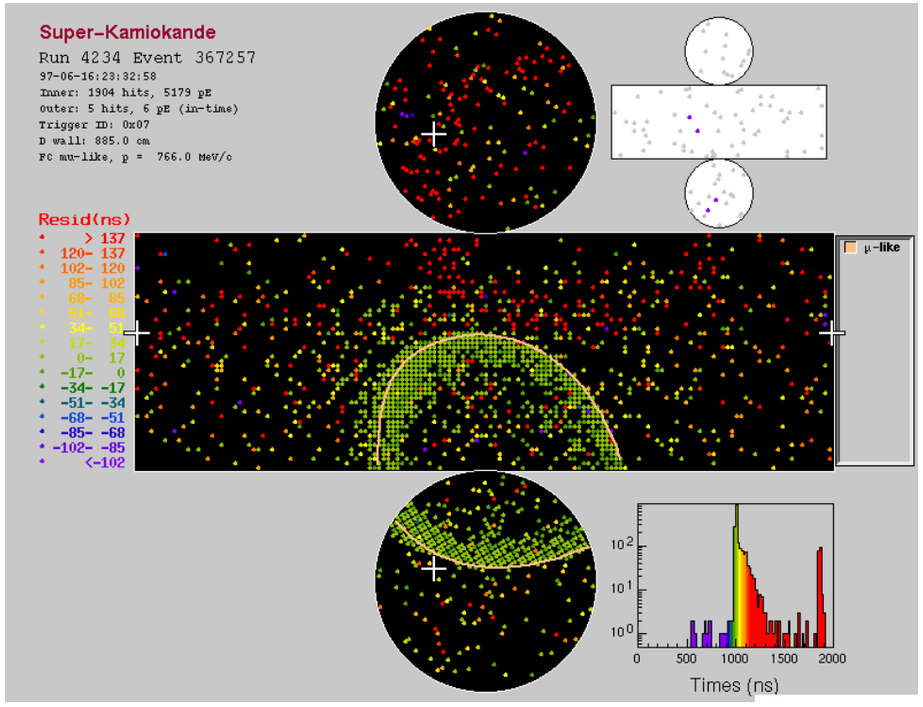


Measuring photons from Cherenkov radiation

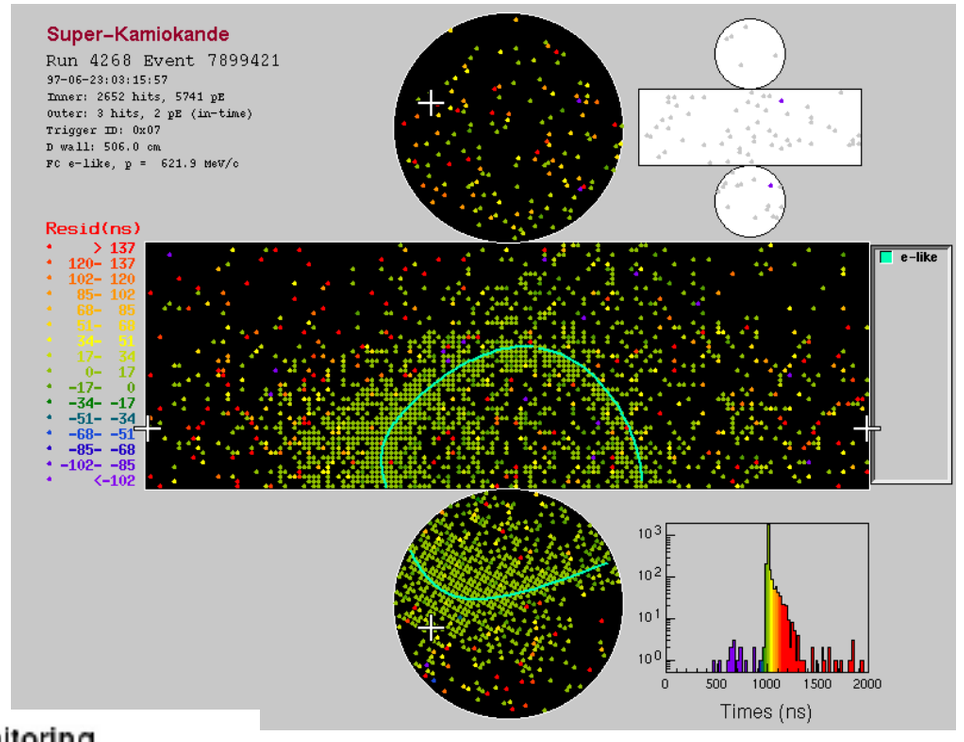




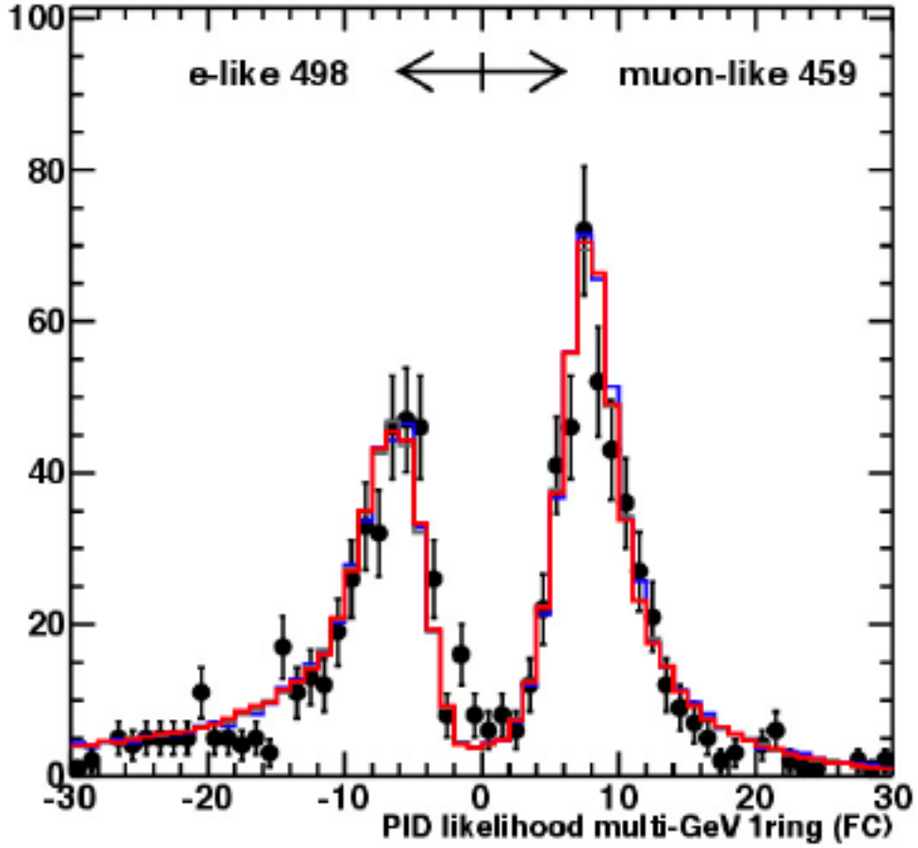
Muon neutrino



Electron neutrino



Super Kamiokande IV 960.6 days : Monitoring



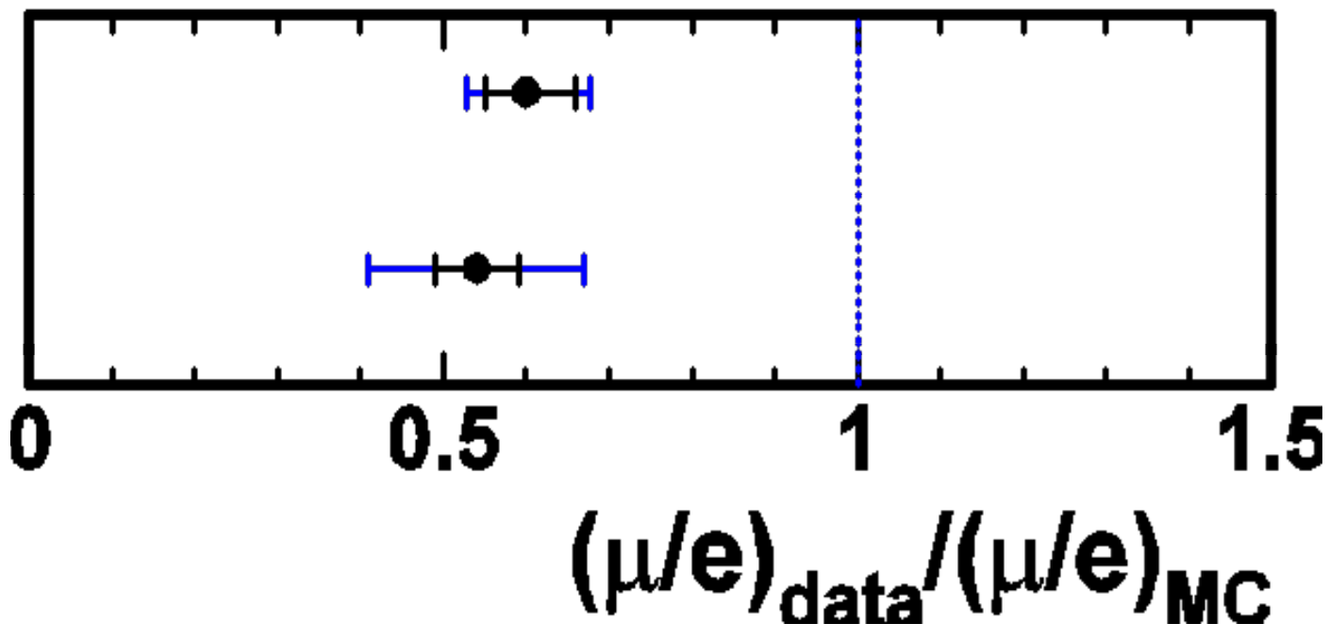
A significant deficit of atmospheric muon-neutrino events was found

K. Hirata et al, Phys.Lett.B 205 (1988) 416.

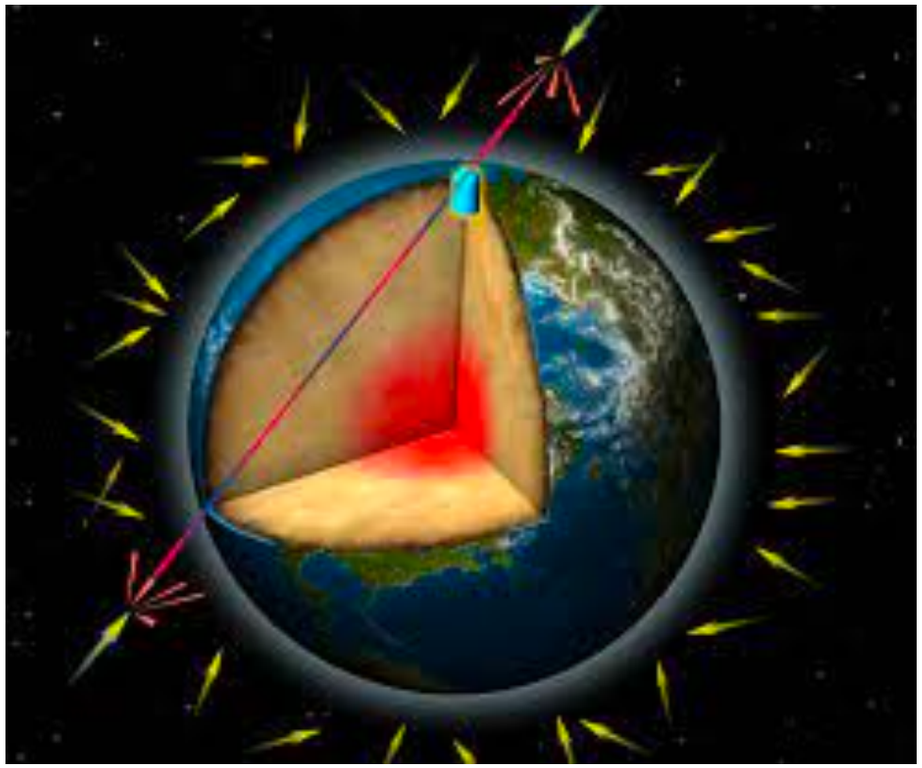
	Data	Prediction
ν_e events	93	88.5
ν_μ events	85	144.0

Kamiokande (1988, 92, 94)

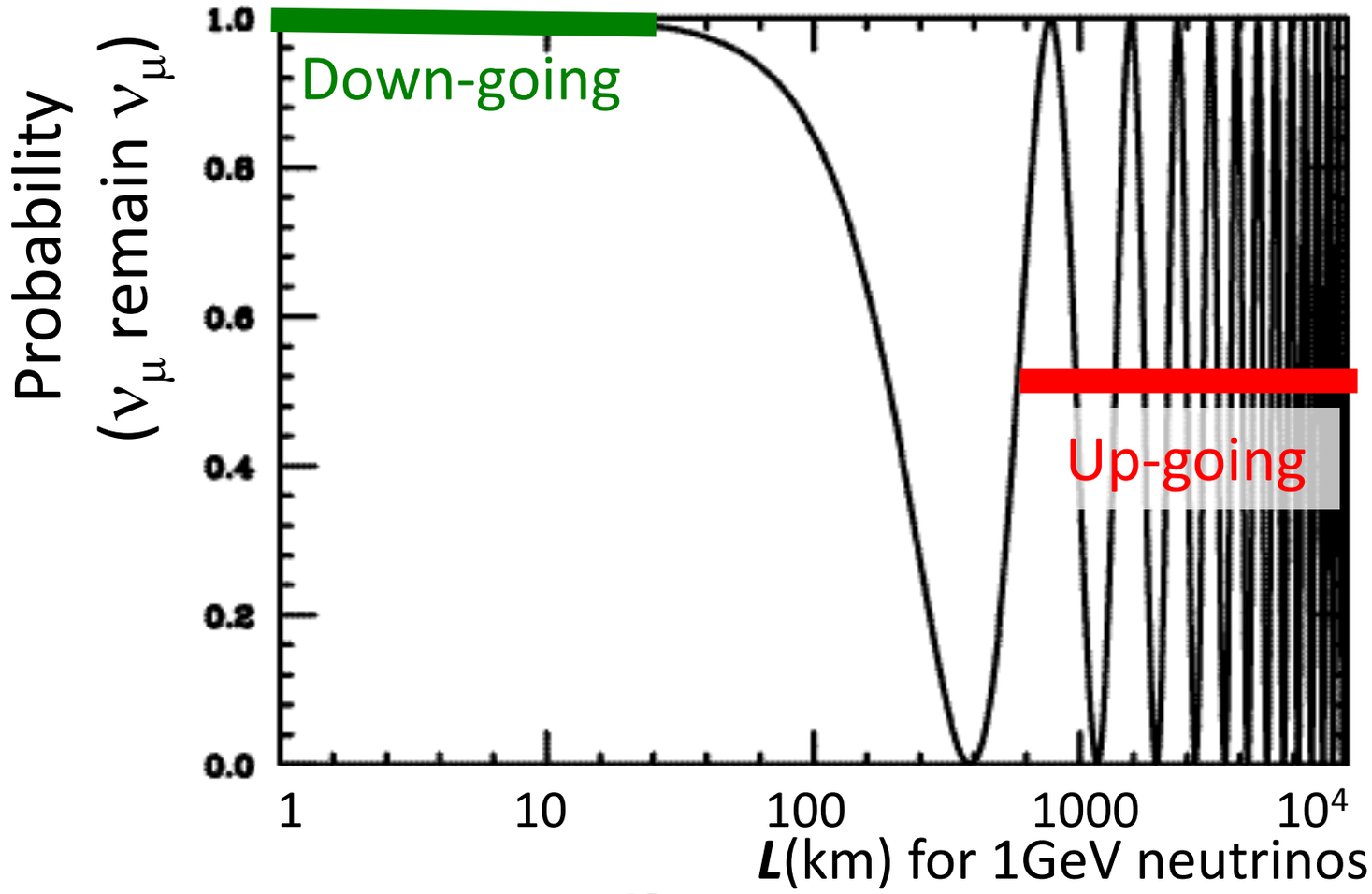
IMB (1991, 92)



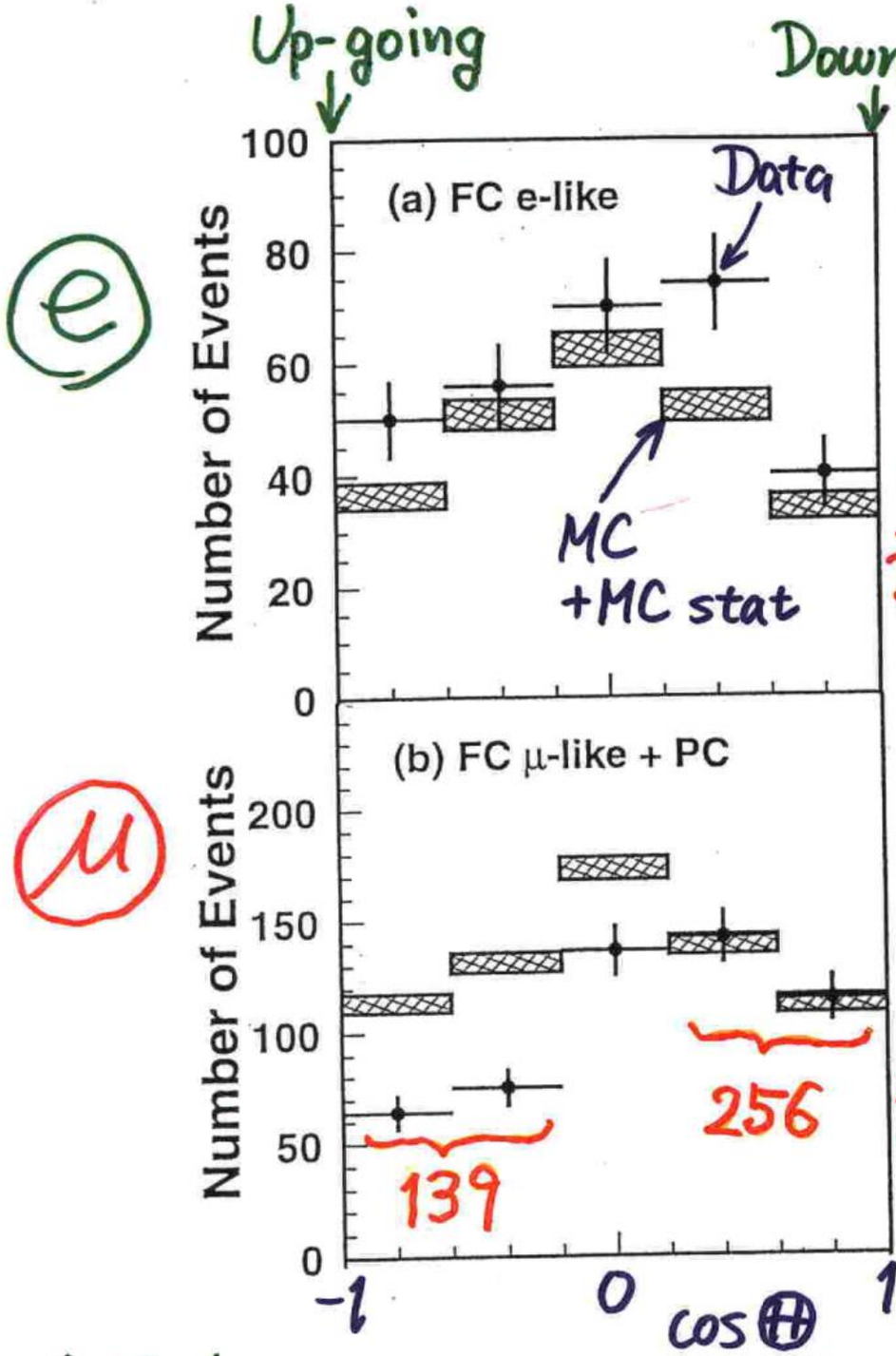
Down: not long enough to oscillate



Up: long enough to oscillate



Zenith angle dependence (Multi-GeV)



e

χ^2(shape) = 2.8/4 dof

$\frac{Up}{Down} = 0.93^{+0.13}_{-0.12}$

μ

χ^2(shape) = 30/4 dof

$\frac{Up}{Down} = 0.54^{+0.06}_{-0.05}$

(6.2σ !!)

T. Kajita: Nobel Laureate

Back to three neutrino scheme

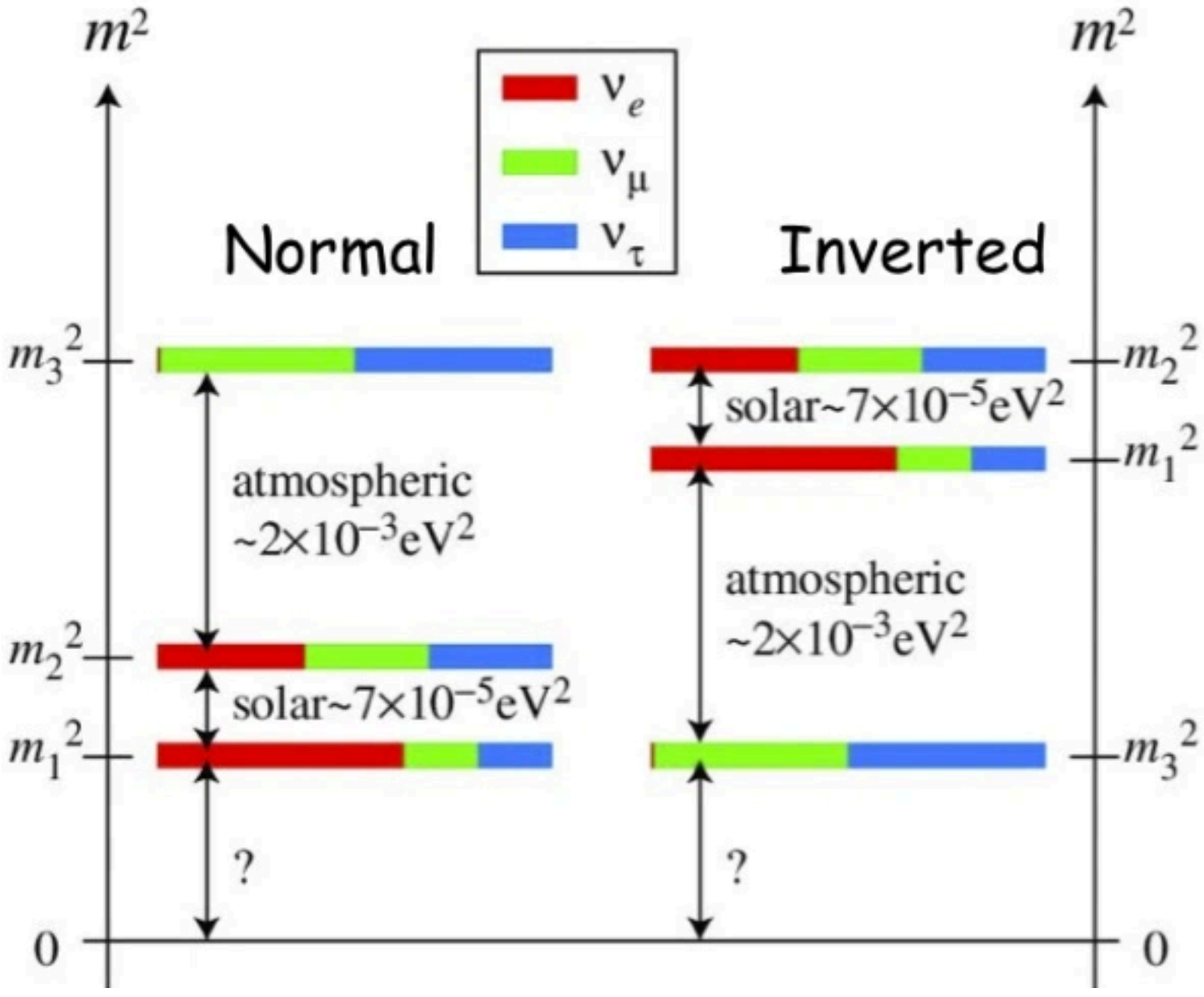
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{\mathrm{CP}}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{\mathrm{CP}}} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}\, c_{23} - c_{12}\, s_{13}\, s_{23}\, e^{i\delta_{\mathrm{CP}}} & c_{12}\, c_{23} - s_{12}\, s_{13}\, s_{23}\, e^{i\delta_{\mathrm{CP}}} & c_{13}\, s_{23} \\ s_{12}\, s_{23} - c_{12}\, s_{13}\, c_{23}\, e^{i\delta_{\mathrm{CP}}} & -c_{12}\, s_{23} - s_{12}\, s_{13}\, c_{23}\, e^{i\delta_{\mathrm{CP}}} & c_{13}\, c_{23} \end{pmatrix}$$

Normal hierarchy: $m_1 \ll m_2 < m_3$

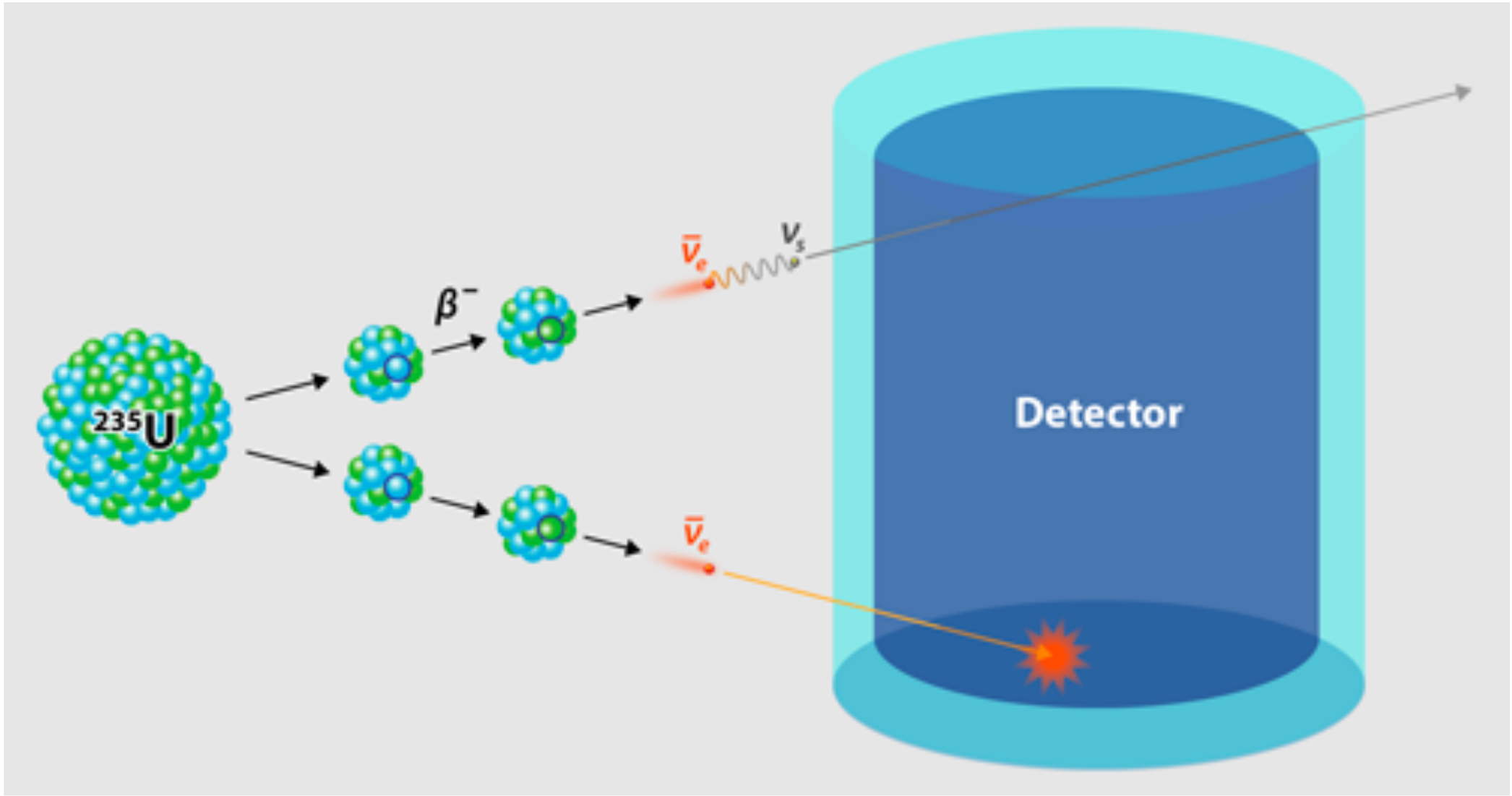
Inverted hierarchy: $m_3 \ll m_1 < m_2$


m^2
ν_e
ν_μ
ν_τ
Normal
Inverted
m^2
m_3^2
m_2^2
solar~7×10^{-5}eV^2
m_1^2
atmospheric
~2×10^{-3}eV^2
atmospheric
~2×10^{-3}eV^2
m_2^2
solar~7×10^{-5}eV^2
m_1^2
m_3^2
?
?
0
0

Full neutrino experiments

Experiment		L (m)	E (MeV)	$\lvert\Delta m^2\rvert$ (eV^2)
Solar		10^{10}	1	10^{-10}
Atmospheric		$10^4 - 10^7$	10^2–10^5	$10^{-1} - 10^{-4}$
Reactor	SBL	$10^2 - 10^3$	1	$10^{-2} - 10^{-3}$
	LBL	$10^4 - 10^5$		$10^{-4} - 10^{-5}$
Accelerator	SBL	10^2	10^3–10^4	> 0.1
	LBL	$10^5 - 10^6$	$10^3 - 10^4$	$10^{-2} - 10^{-3}$

Reactor neutrino experiments



SM cannot explain the neutrino mass

Fermions	ν_{Li}	e_{Li}	e_{Ri}	u_{Li}	d_{Li}	u_{Ri}	d_{Ri}
Y	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$
I_{3L}	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0