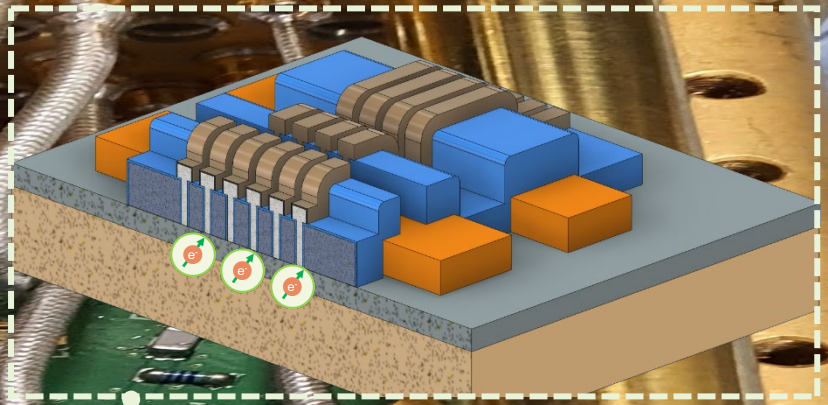
Dec.21.2022

2022 KIAS-SNU wintercamp



Basics of spin-based quantum computing circuits

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Department of Physics & Astronomy, SNU

Outline

Lecture 1. Basics of gate defined quantum dot experiments

- Background *Semiconductor quantum dot quantum computing*
- Approach *Quantum dot: Artificial atoms*
- Measurement of charge *Transport vs. charge sensing*
- Measurement of spin *Spin to charge conversion*

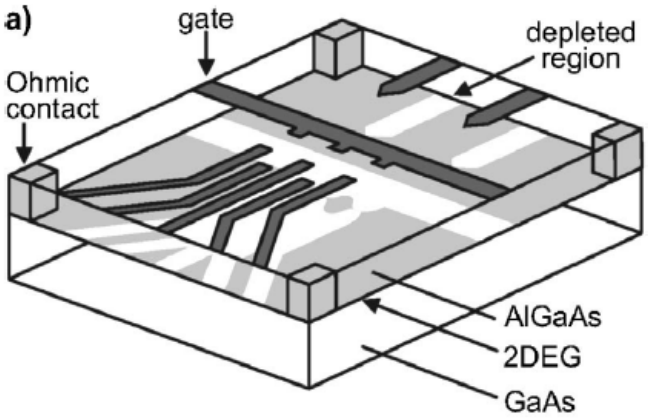
What is a Quantum Dot (Gate-defined) ?

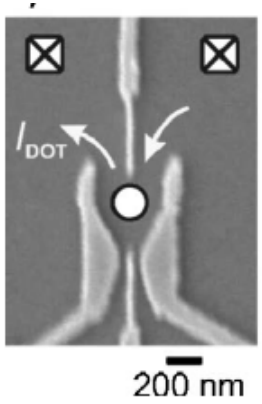
Quantum Dot: artificially confined structure that can be filled with electrons (or holes) → “Artificial atom”

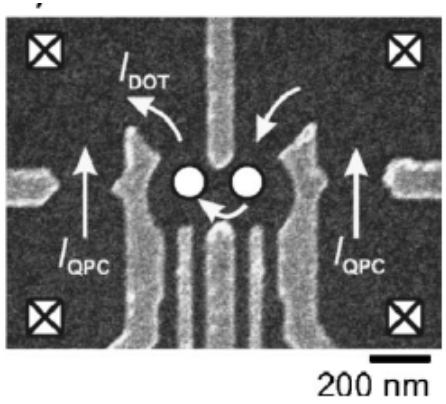
Quantized energy levels, but energy scale is very different from real atoms.

Highly controllable : number of electrons, energy levels, size, shape, material, etc.

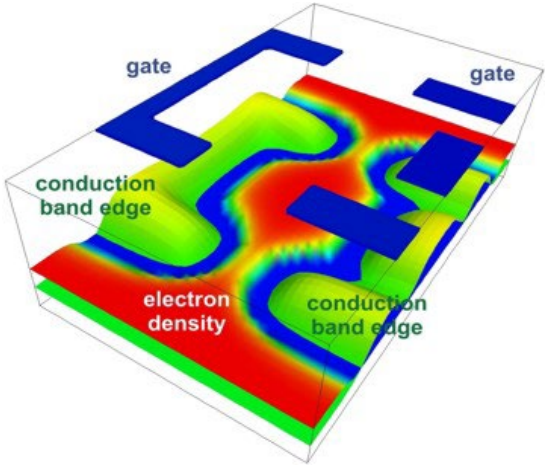
Lateral QD

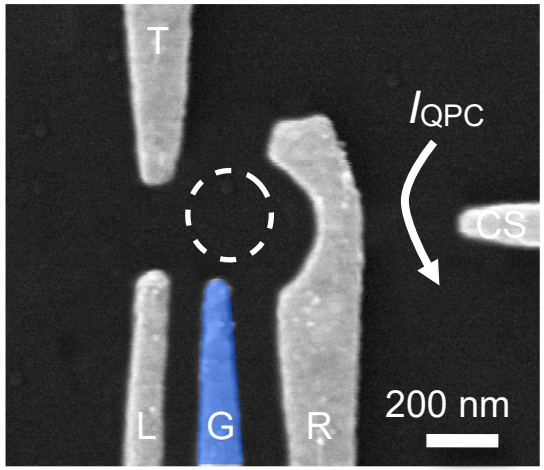


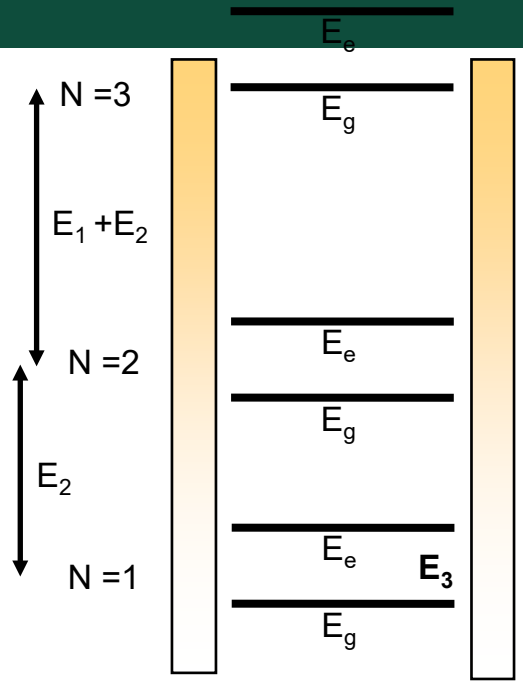




Artificial atom : energy scales







1. Confinement energy spacing : particle in a box
   ~ meV > few K

$$E_1 = \frac{\hbar^2 \pi^2}{2mL} n^2$$

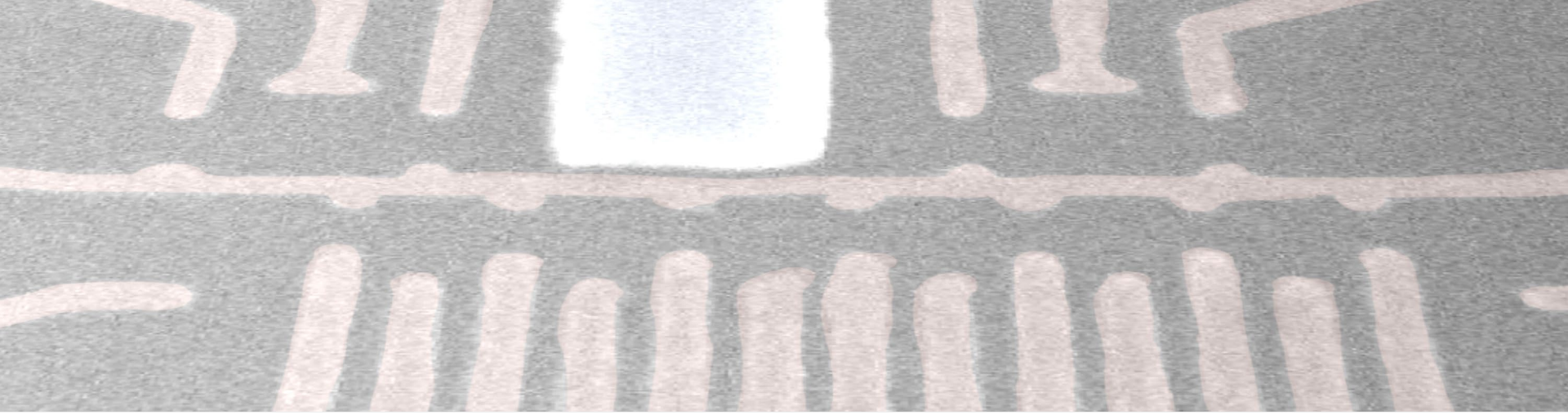
2. Coulomb blockade : charging energy
   ~ meV > few K

$$E_2 = \frac{e^2}{C}$$

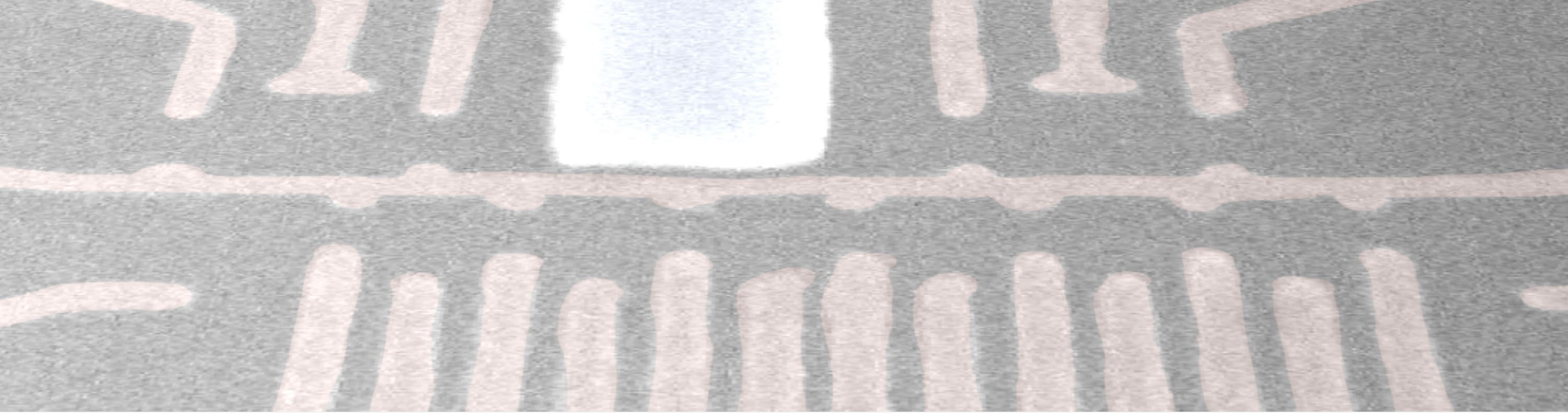
~ tens of μeV

3. Additionally – spin splitting (ex. Zeeman)

$$E_3 = -g \mu_B B$$

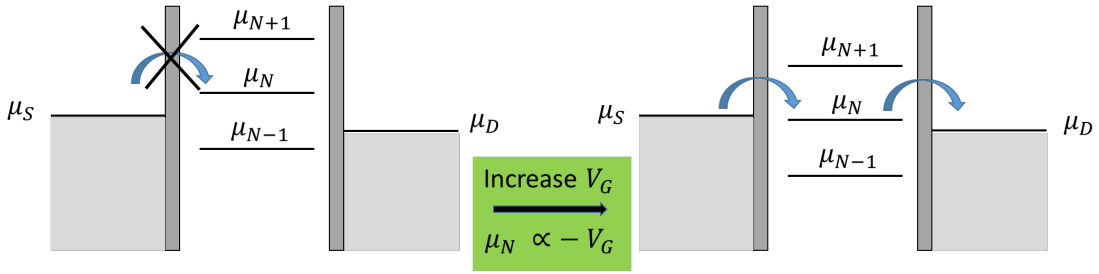


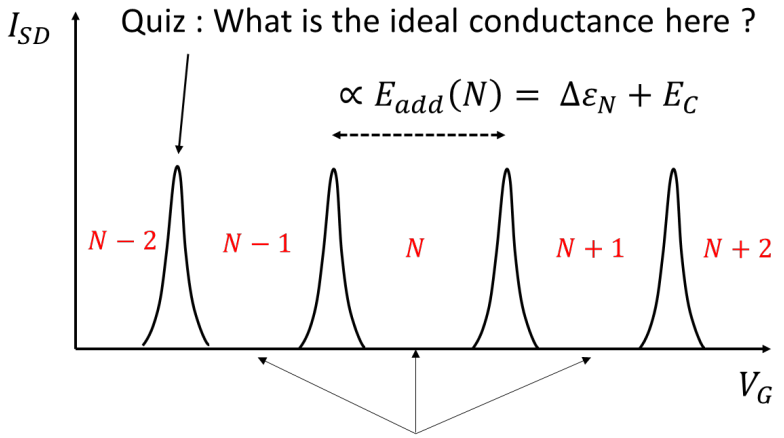
note



note

$\mu_D \leq \mu_N \leq \mu_S$ (Bias window)





No current here --> Coulomb blockade

Conductance quantization

$$\therefore I = I_1 = -I_2 = \frac{q}{\hbar}\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\left[f_1(\varepsilon) - f_2(\varepsilon)\right]$$

↳ No current flow if $f_1(\varepsilon) = f_2(\varepsilon)$

- At low temperature limit $f_1(\varepsilon) \sim 1$ $f_2(\varepsilon) \sim 0$

$$I = \frac{q}{\hbar}\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} = \frac{q\gamma_1}{2\hbar} \quad \text{if } \gamma_1 = \gamma_2$$

← Can we pump unlimited current through this one-level device by increasing γ_1 ?

↳ No, there is maximum limit

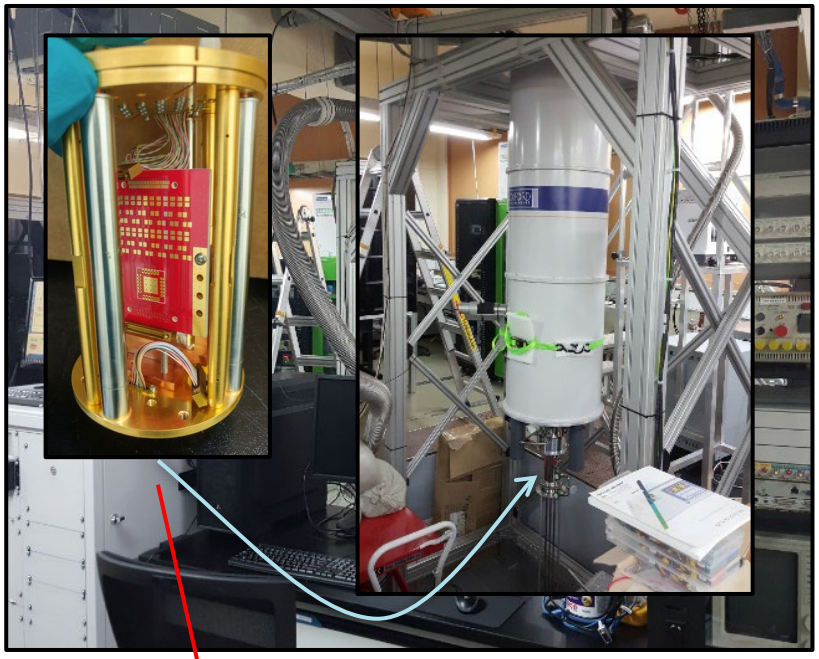
Because of energy broadening ← ⇒ Quantum of conductance

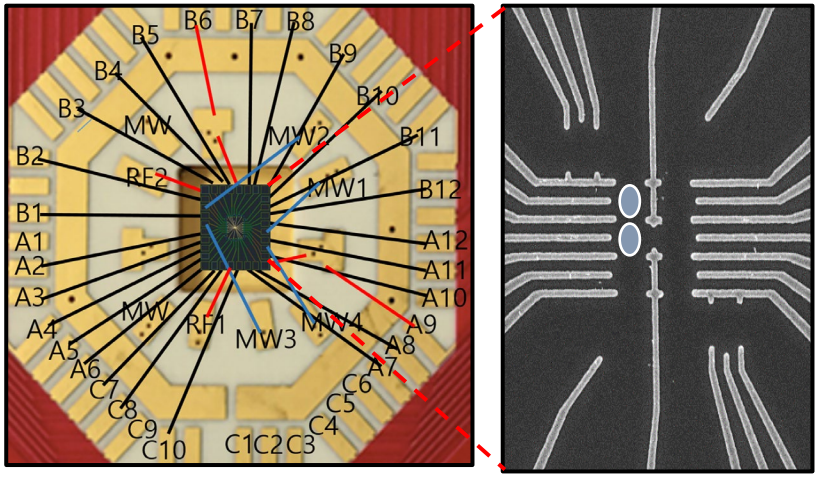


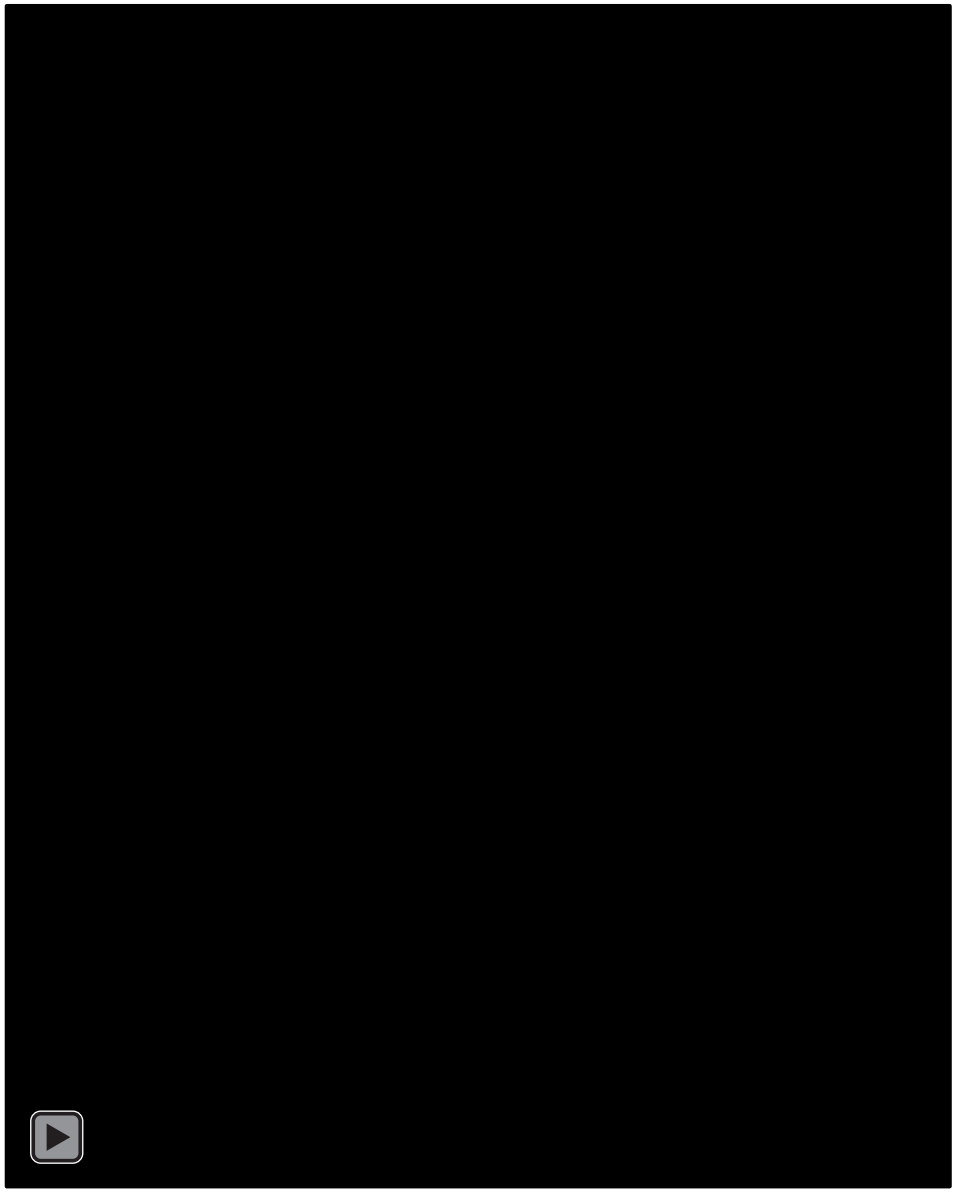
broadening : $(\mu_1 - \mu_2)/C\gamma_1$ ← Fundamentally, this is from uncertainty principle

$$I = \frac{q\gamma_1}{2\hbar}\frac{qV_D}{C\gamma_1} \Rightarrow G = \frac{I}{V_D} = \frac{q^2}{C\hbar}$$

Quantum dot experiment platform



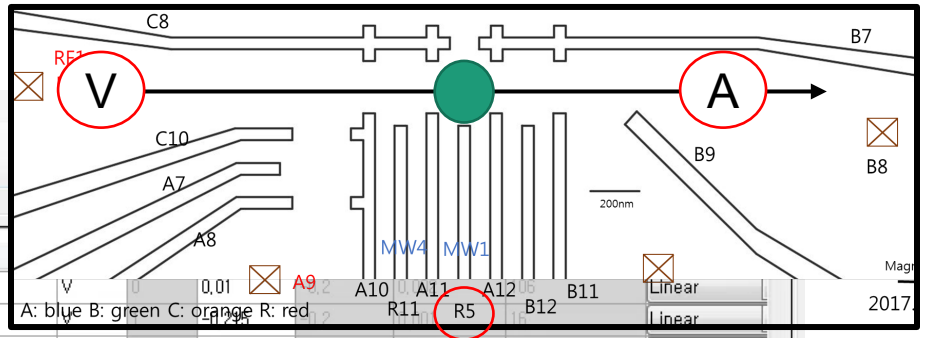




2

Transport experiment

Coulomb blockade measurement



File Instruments Measurement Tools

Start Stop Pause Current Plot 2D Plot Open Empty Plot Export Measurement Settings

Name	Current Value	Units
R9,V	-0,257	V
R5,V	-0,2	V
C2,V	0	V
C8,V	-0,215	V
R11_Fine,V	0	V
b8Vsd2,V	0	V
c7Vsd,V	0	V
TestInstrument28,Output1	0	None
drain,r1	3,6768E-10	A
drain,Vref1	-0,001339	V
drain,Vref2	-0,0011881	V
Timer30,T	98350	s
C4,V	-0,2	V
LIR7,Amplitude	0,2	Vrms
LIR7,Frequency	5387	Hz
LIR7,X	0,0002053	Vrms

Output Settings

Data Name

bt3po91

Data Folder

C:₩Data

Notes

Status

Current Status: Error

Data name already exists

Measurement Setup

Output Instruments

Enabled	Sweep	Name						
☐	☐	B5,V	V	0	0,01			Linear
☐	☐	B7,V	V					Linear
☐	☐	B9,V	V	1	0	0	0,002	1 Linear
☐	☐	B11,V	V	0	-0,26	-0,3	0,002	21 Linear
☐	☐	B12,V	V	0	-0,1	-0,22	0,002	61 Linear
☐	☐	R11,V	V	0	0	-0,2	0,002	101 Linear
☐	☐	R7,V	V	1	0,052	0,02	0,0005	65 Linear
☐	☐	R9,V	V	1	-0,257	-0,256	0,002	1,5000 Linear
☑	☑	R5,V	V	0	0,1	-0,2	0,001	301 Linear
☐	☐	C2,V	V	0	0	0	0,002	1 Linear
☐	☐	C8,V	V	1	-0,215	-0,2	0,001	16 Linear
☐	☐	R11_Fine,V	V	0	0	-0,01	0,0001	101 Linear
☐	☐	b8Vsd2,V	V	0	0	0,0002	5E-06	41 Linear
☐	☐	c7Vsd,V	V	0	0	0	0	NaN Linear
☐	☐	TestInstrument2,..	None	0	0	0	0	NaN Linear
☐	☐	C4,V	V	0	-0,2	-0,4	0,002	101 Linear
☐	☐	LIR7,Amplitude	Vrms	1	0	0	0,01	1 Linear
☐	☐	LIR7,Frequency	Hz	0	737	0	0	Infinity Linear

Input Instruments

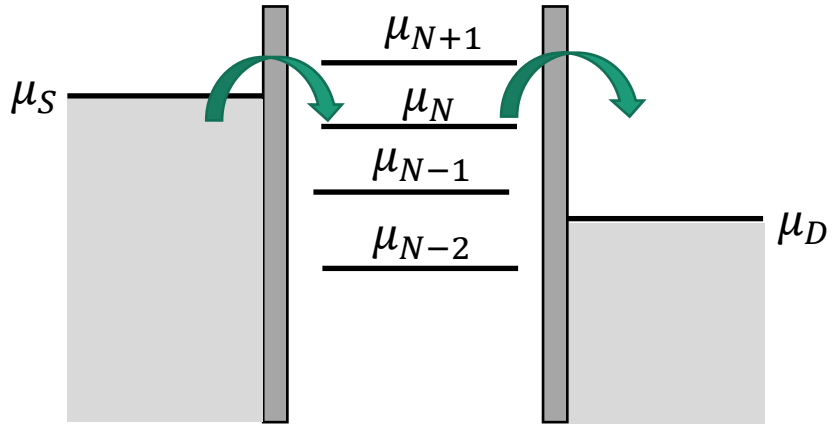
Enabled	Name	Units
☑	drain,r1	A
☐	drain,Vref1	V
☐	drain,Vref2	V
☐	Timer30,T	s
☐	LIR7,X	Vrms

Timing

Settling Time (ms) 50

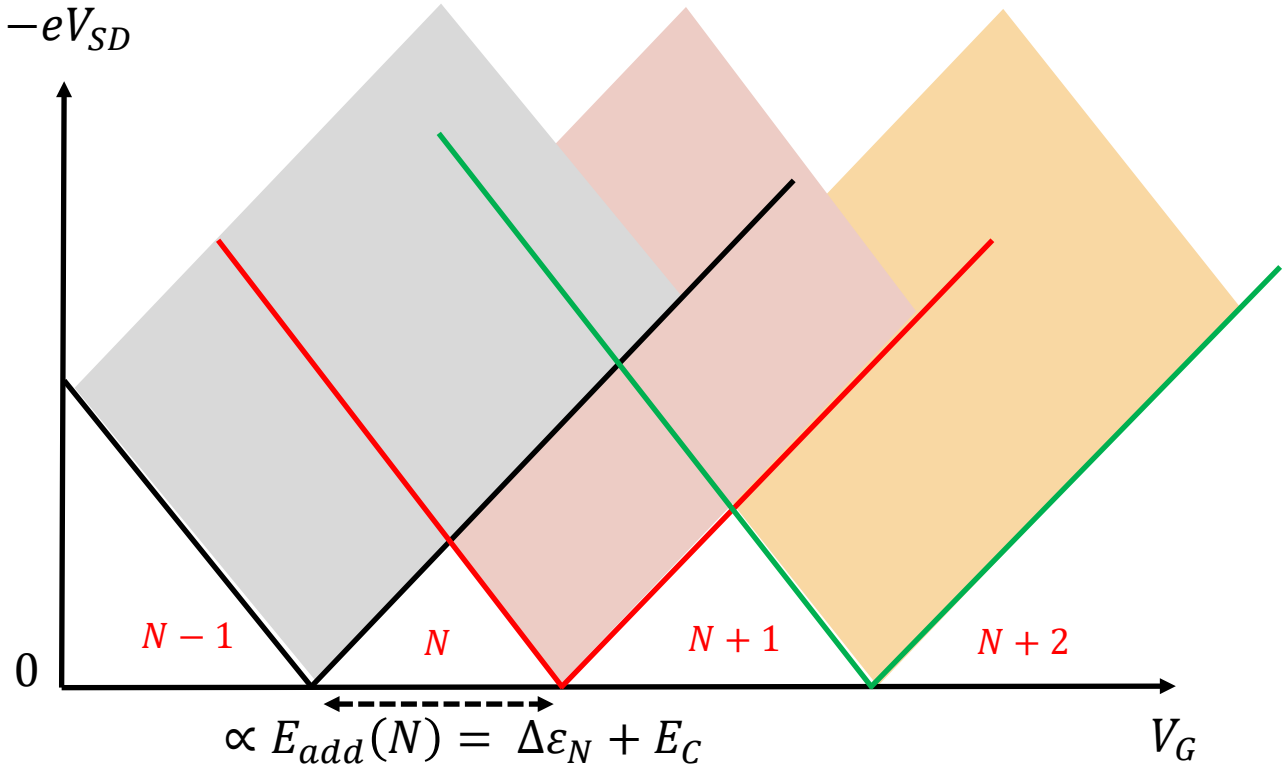
Start Sweep Delay (ms) 3000

Transport through a QD: High Bias



Transport condition:

$\mu_D \leq \mu_N \leq \mu_S$ for at least one N



$\mu_D \leq \mu_N \leq \mu_S$

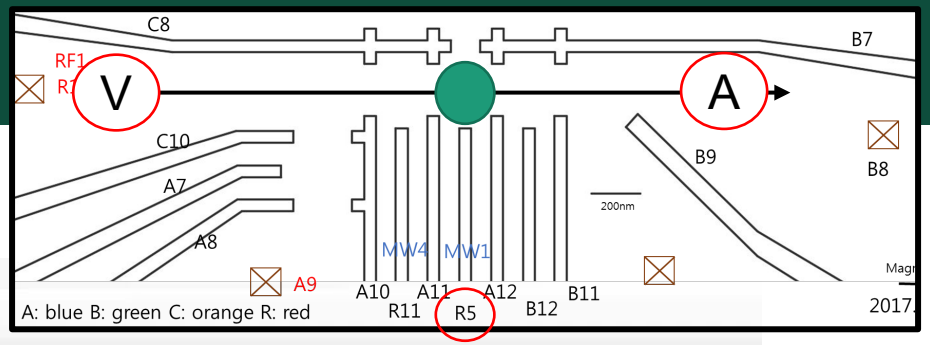
$\mu_D \leq \mu_{N+1} \leq \mu_S$

$\mu_D \leq \mu_{N+2} \leq \mu_S$

2

Transport experiment

Coulomb diamond



File Instruments Measurement Tools

Start Stop Pause Current Plot 2D Plot Open Empty Plot Export Measurement Settings

Name	Current Value	Units
R9,V	-0,257	V
R5,V	-0,2	V
C2,V	0	V
C8,V	-0,215	V
R11_Fine,V	0	V
b8Vsd2,V	0	V
c7Vsd,V	0	V
TestInstrument28,Output1	0	None
drain,r1	3,5992E-10	A
drain,Vref1	-0,001417	V
drain,Vref2	-0,0012714	V
Timer30,T	98423	s
C4,V	-0,2	V
LIR7,Amplitude	0,2	Vrms
LIR7,Frequency	5387	Hz
LIR7,X	0,000206	Vrms

Output Settings

Data Name

bt3po92

Data Folder

C:₩Data

Notes

Status

Current Status: Error

Data name already exists

Measurement Setup

Output Instruments

Enabled	Sweep	Name	Units	Loop	Start	Stop	Step	Number of Steps	SweepStyle
☐	☐	B5,V	V	0	0,01	-0,2	0,002	106	Linear
☐	☐	B7,V	V	1	-0,215	-0,2	0,001	16	Linear
☐	☐	B9,V	V	1	0	0	0,002	1	Linear
☐	☐	B11,V	V	0	-0,26	-0,3	0,002	21	Linear
☐	☐	B12,V	V	0	-0,1	-0,22	0,002	61	Linear
☐	☐	R11,V	V	0	0	-0,2	0,002	101	Linear
☐	☐	R7,V	V	1	0,052	0,02	0,0005	65	Linear
☐	☐	R9,V	V	1	-0,257	-0,256	0,002	1,5000	Linear
☑	☑	R5,V	V	1	-0,03	-0,2	0,001	171	Linear
☐	☐	C2,V	V	0	0	0	0,002	1	Linear
☐	☐	C8,V	V	1	-0,215	-0,2	0,001	16	Linear
☐	☐	R11_Fine,V	V	0	0	-0,01	0,0001	101	Linear
☑	☑	b8Vsd2,V	V	0	0,001	-0,001	1E-05	201	Linear
☐	☐	c7Vsd,V	V	0	0	0	0	NaN	Linear
☐	☐	TestInstrument2,..	None	0	0	0	0	NaN	Linear
☐	☐	C4,V	V	0	-0,2	-0,4	0,002	101	Linear
☐	☐	LIR7,Amplitude	Vrms	1	0	0	0,01	1	Linear
☐	☐	LIR7,Frequency	Hz	0	737	0	0	Infinity	Linear

Input Instruments

Enabled	Name	Units
☑	drain,r1	A
☐	drain,Vref1	V
☐	drain,Vref2	V
☐	Timer30,T	s
☐	LIR7,X	Vrms

Timing

Settling Time (ms) 50

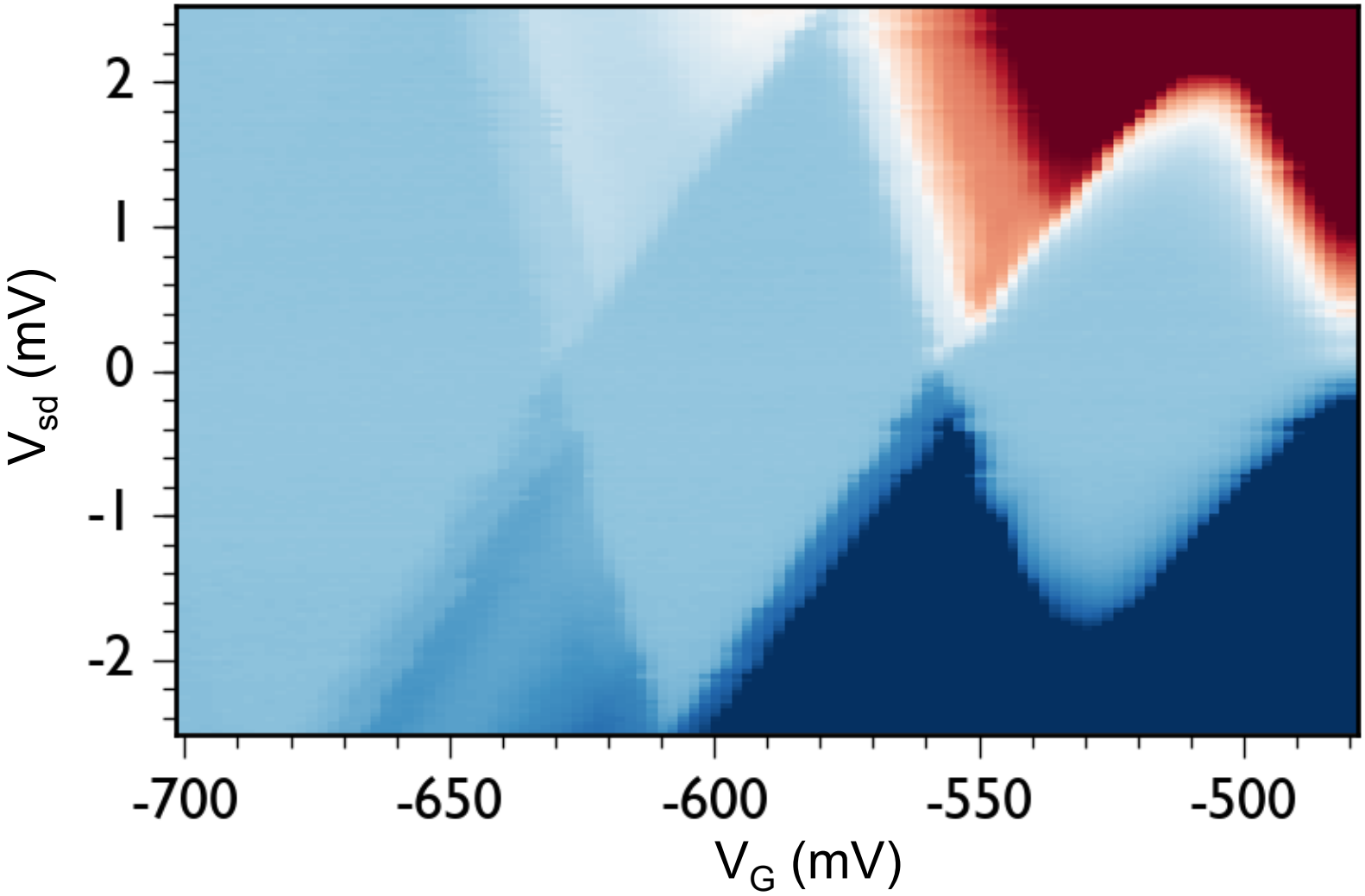
Start Sweep Delay (ms) 3000

Transport experiment

Coulomb diamond

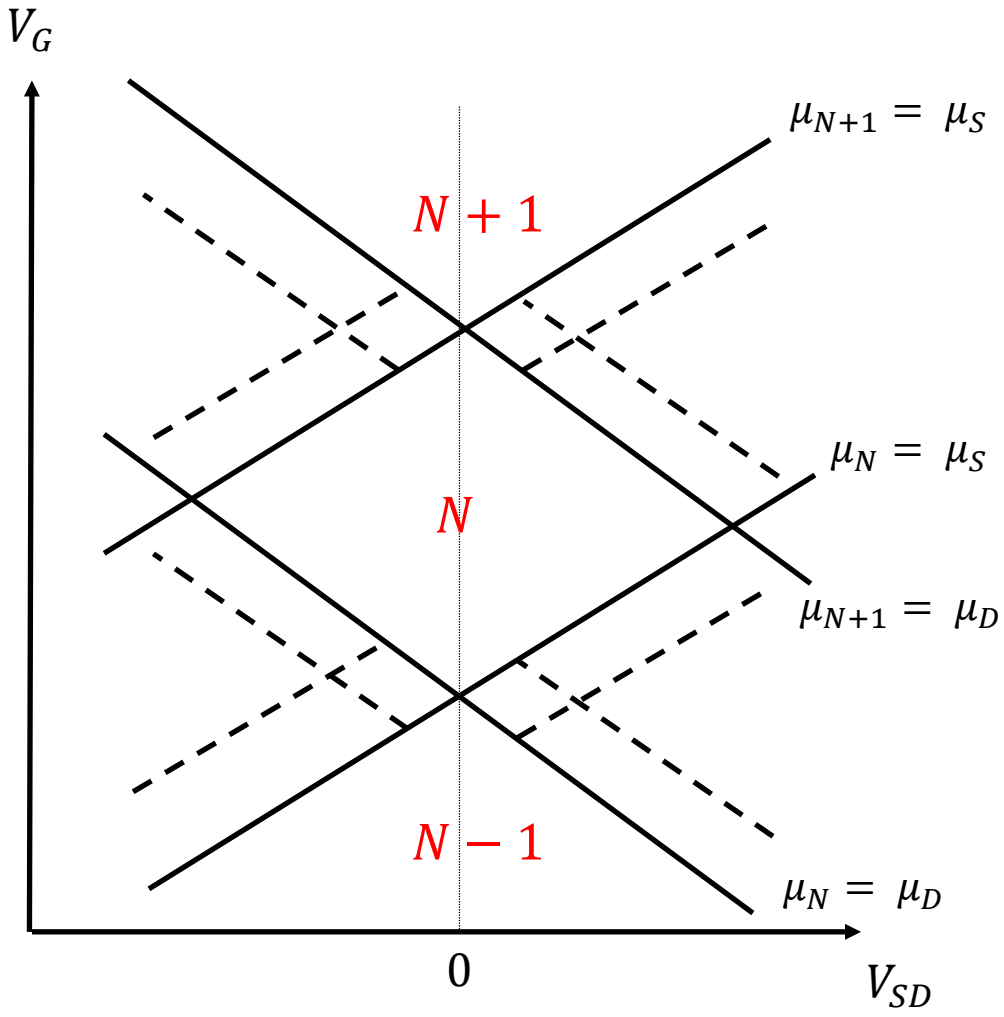
Coulomb diamond tells us essential information of the system

Note : slopes, offset, excited level…

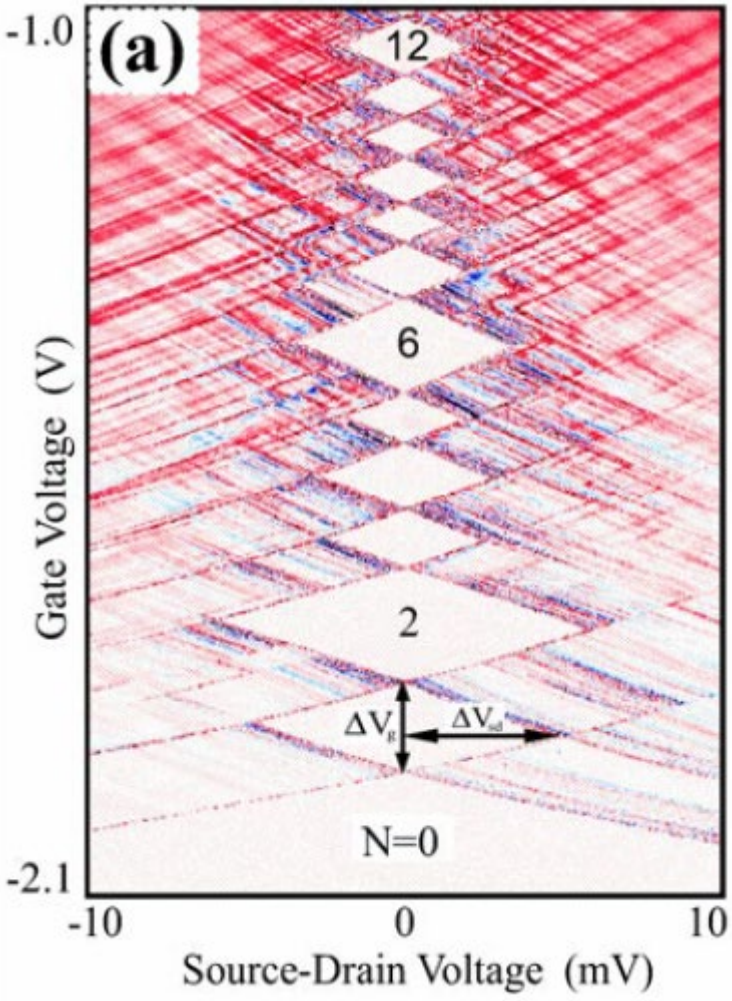


Excited level spectroscopy

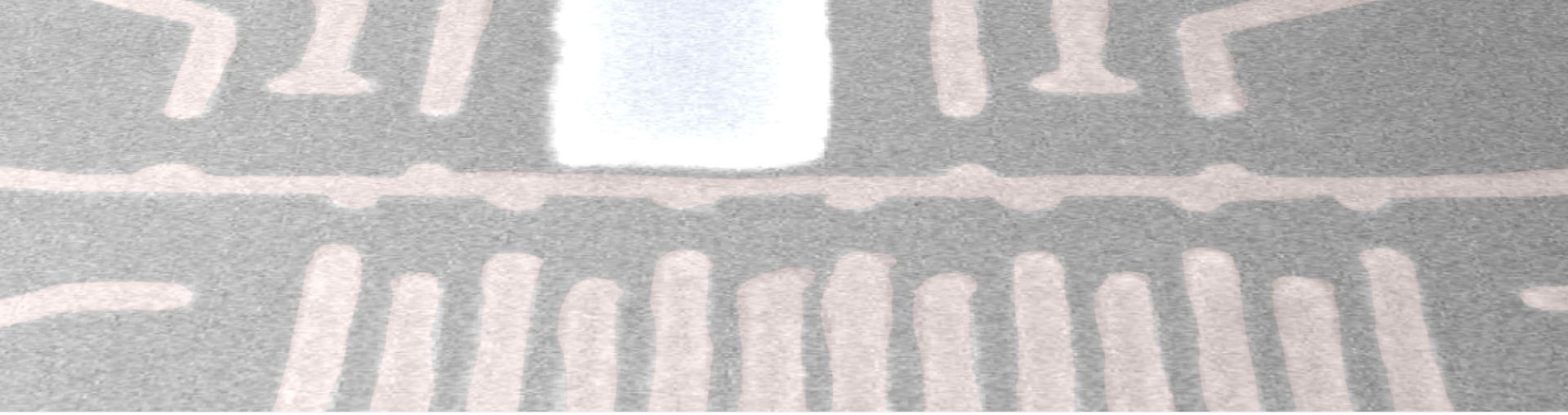
Transport through excited states



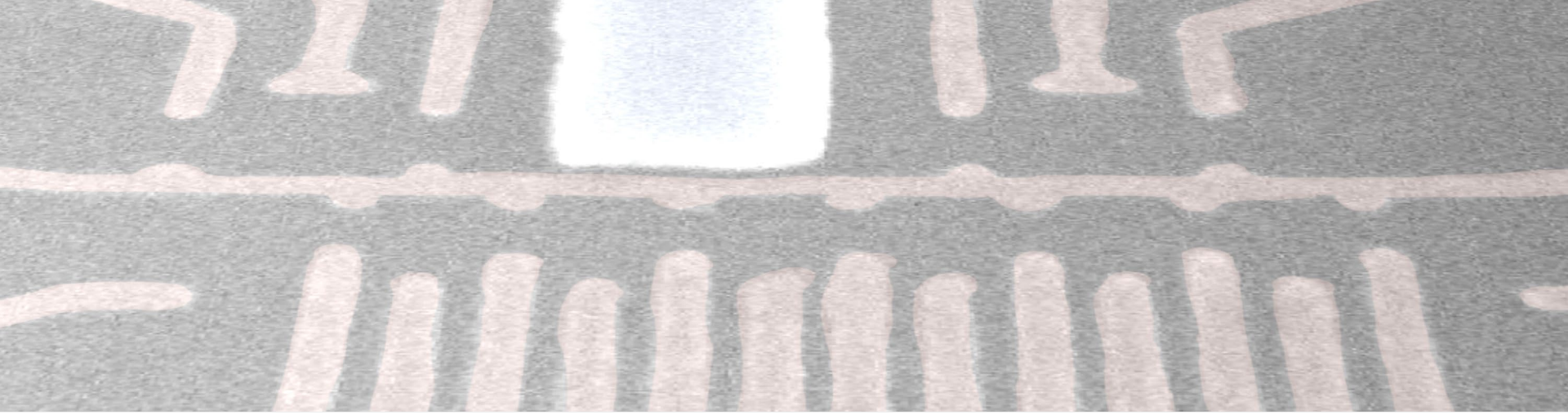
$\frac{\partial I_{SD}}{\partial V_{SD}}$



Picture from Ref. [1]



Note : problem with transport sensing

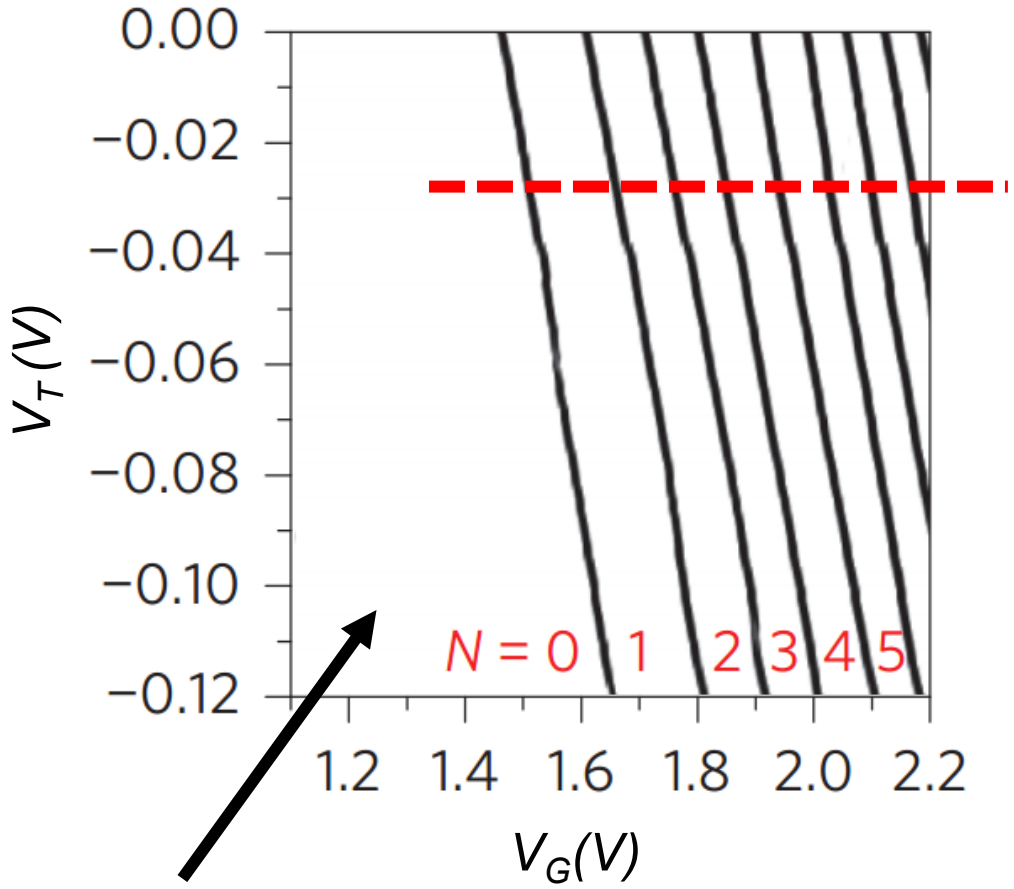


Note : Solution – charge sensing with nearby dot

Transport vs. charge sensing

Single dot charge stability diagram

Note



- Charge sensing enables single electron regime
- Electron number counting
- Also act as qubit state detector

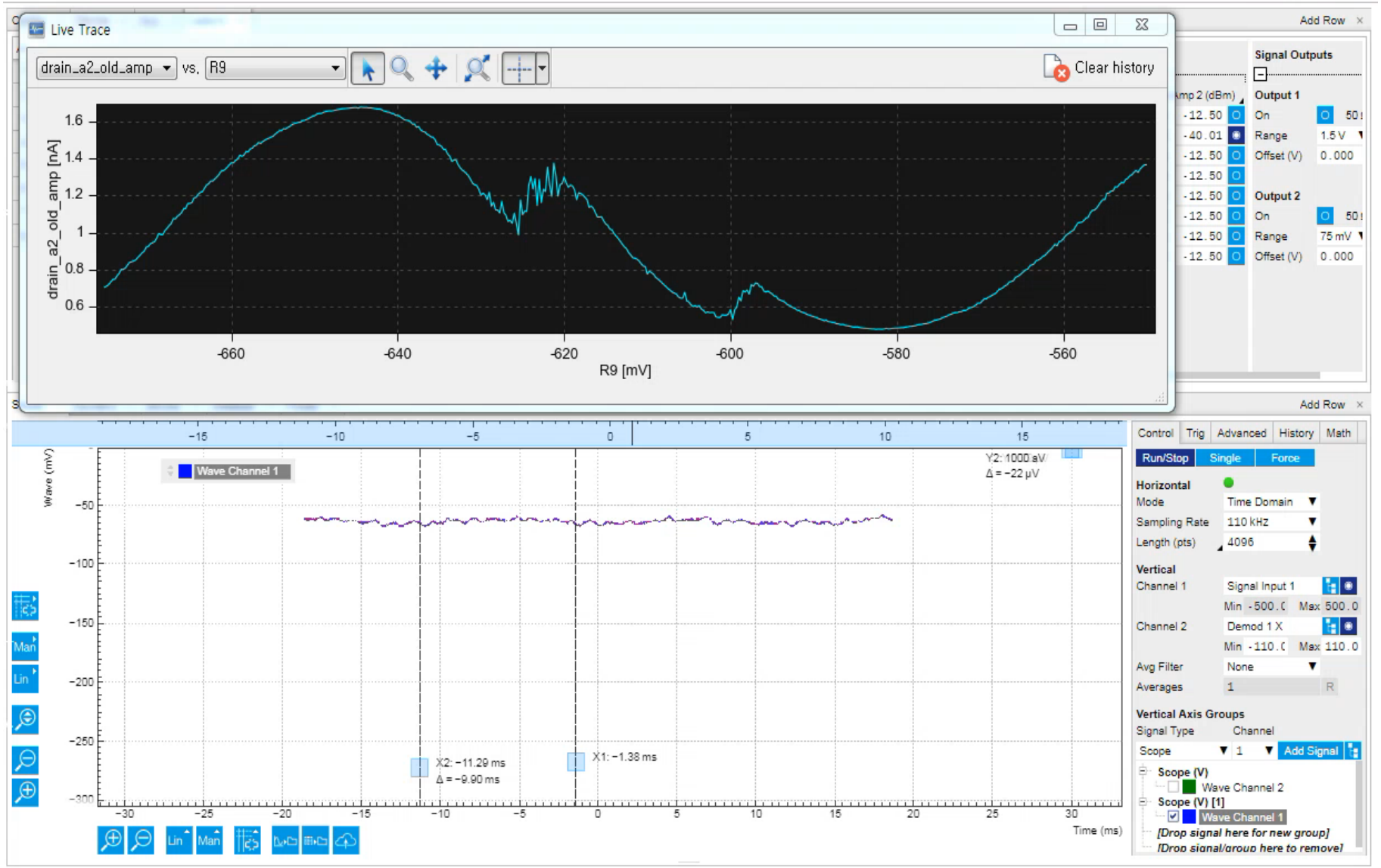
No electron in the dot

M. Veldhorst, et al., *Nature Nanotech.* **9** 981 (2014)

3

Measurement of charge

Observing single electron tunneling events



Beyond constant-interaction model

$$\hat{H} = \sum_{i,\sigma} \varepsilon_i \, c_{i\sigma}^{+} c_{i\sigma} + \frac{1}{2} \sum_{\substack{i,j,k,l \\ \sigma,\sigma'}} \langle ij|\hat{V}|kl\rangle \, c_{i\sigma}^{+} c_{j\sigma'}^{+} c_{l\sigma'} c_{k\sigma}$$

$$\approx \sum_{i,\sigma} \varepsilon_i \, c_{i\sigma}^{+} c_{i\sigma} + \sum_{i,j} U_{ij} n_i n_j - \sum_{\substack{i,j \\ \sigma}} U_{ij}^{ex} n_{i\sigma} n_{j\sigma}$$

$$n_{i\sigma} = c_{i\sigma}^{+} c_{i\sigma}$$

$$n_i = n_{i\uparrow} + n_{i\downarrow}$$

Orbital energy — Direct Coulomb int. — Exchange int.

$$U_{ij} = \langle ij|\hat{V}|ij\rangle = \int d\mathbf{r}\, d\mathbf{r}' \; \phi_i(\mathbf{r})\phi_j(\mathbf{r}') \frac{e^2}{|\mathbf{r}-\mathbf{r}'|} \phi_i(\mathbf{r})\phi_j(\mathbf{r}')$$

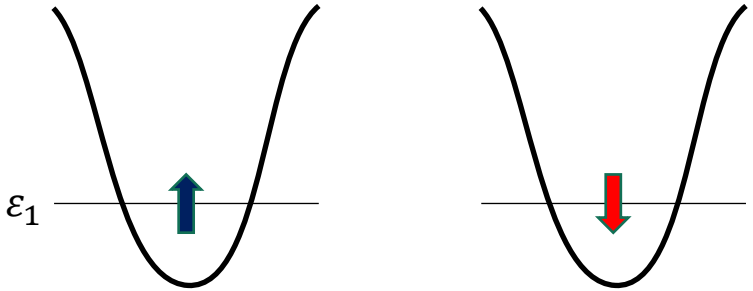
$$U_{ij}^{ex} = \langle ij|\hat{V}|ji\rangle = \int d\mathbf{r}\, d\mathbf{r}' \; \phi_i(\mathbf{r})\phi_j(\mathbf{r}') \frac{e^2}{|\mathbf{r}-\mathbf{r}'|} \phi_j(\mathbf{r})\phi_i(\mathbf{r}')$$

Constant-interaction model assumes $U_{ij} = U$ and neglect the exchange terms.
Exchange term is important when considering spin structure of electrons in QD.

Two-electron singlet-triplet splitting

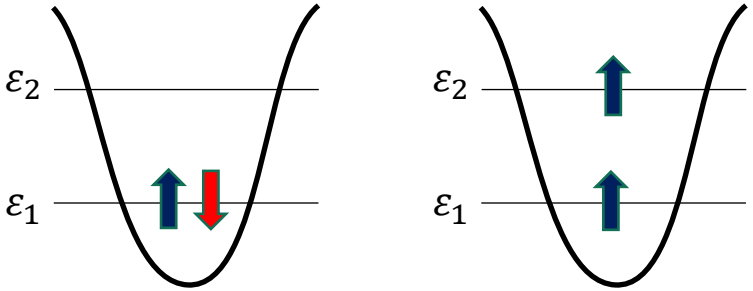
Spin states of two electrons in a QD

$N = 1$



$$E_\uparrow = E_\downarrow = \varepsilon_1$$

$N = 2$



$$E_S = 2\,\varepsilon_1 + U$$

$$E_T = \varepsilon_1 + \varepsilon_2 + U - U_{ex}$$

$E_T - E_S = \Delta\varepsilon - U_{ex} < 0$ Singlet is the ground state.

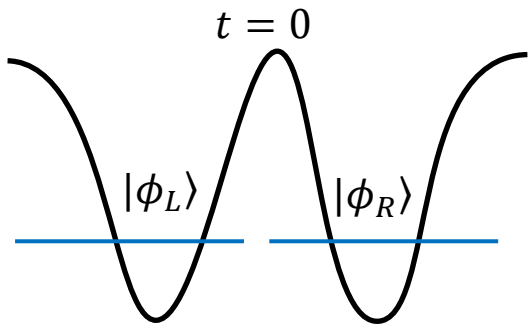
$$|S\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\}$$

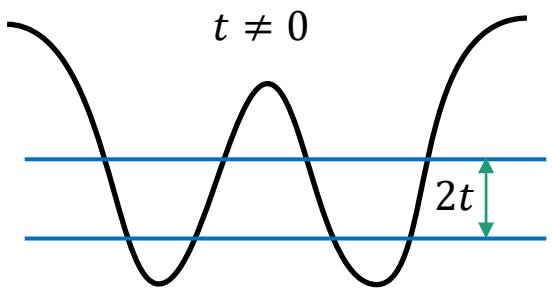
$$|T_0\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\} \qquad |T_+\rangle = |\uparrow\uparrow\rangle \qquad |T_-\rangle = |\downarrow\downarrow\rangle$$

Two-electron singlet-triplet splitting

Spin states of two electrons in a DQD

Single electron in DQD





$$\phi_A(\mathbf{r}) = \frac{1}{\sqrt{2}}\left[\phi_L(\mathbf{r}) - \phi_R(\mathbf{r})\right]$$

$$\phi_S(\mathbf{r}) = \frac{1}{\sqrt{2}}\left[\phi_L(\mathbf{r}) + \phi_R(\mathbf{r})\right]$$

Two electron in DQD



$$\Psi_S(\mathbf{r}_1,\mathbf{r}_2) = \phi_S(\mathbf{r}_1)\phi_S(\mathbf{r}_2) = \frac{1}{2}\left[\phi_L(\mathbf{r}_1)\phi_R(\mathbf{r}_2) + \phi_R(\mathbf{r}_1)\phi_L(\mathbf{r}_2) + \phi_L(\mathbf{r}_1)\phi_L(\mathbf{r}_2) + \phi_R(\mathbf{r}_1)\phi_R(\mathbf{r}_2)\right]$$

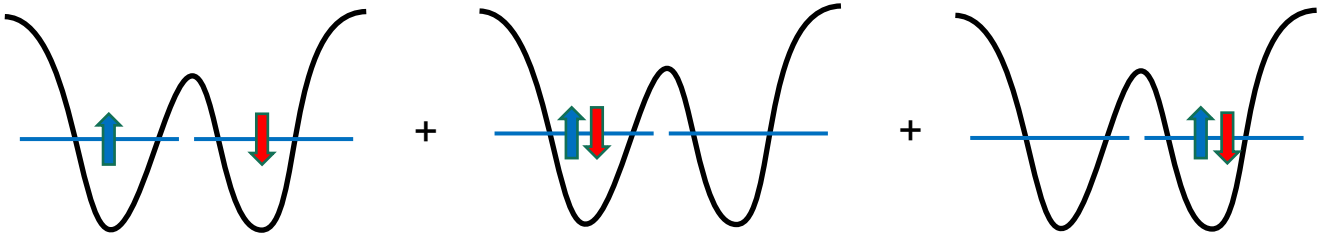
$$\Psi_T(\mathbf{r}_1,\mathbf{r}_2) = \frac{1}{\sqrt{2}}\left[\phi_S(\mathbf{r}_1)\phi_A(\mathbf{r}_2) - \phi_A(\mathbf{r}_1)\phi_S(\mathbf{r}_2)\right] = \frac{1}{\sqrt{2}}\left[\phi_R(\mathbf{r}_1)\phi_L(\mathbf{r}_2) - \phi_L(\mathbf{r}_1)\phi_R(\mathbf{r}_2)\right]$$

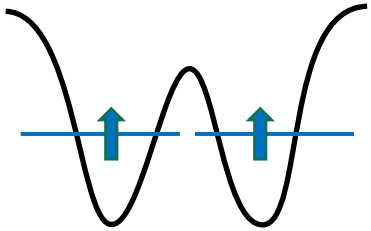
Two-electron singlet-triplet splitting

Spin states of two electrons in a DQD

$$\Psi_S(\mathbf{r}_1,\mathbf{r}_2) = \phi_S(\mathbf{r}_1)\phi_S(\mathbf{r}_2) = \frac{1}{2}\left[\phi_L(\mathbf{r}_1)\phi_R(\mathbf{r}_2) + \phi_R(\mathbf{r}_1)\phi_L(\mathbf{r}_2) + \phi_L(\mathbf{r}_1)\phi_L(\mathbf{r}_2) + \phi_R(\mathbf{r}_1)\phi_R(\mathbf{r}_2)\right]$$

$$\Psi_T(\mathbf{r}_1,\mathbf{r}_2) = \frac{1}{\sqrt{2}}\left[\phi_S(\mathbf{r}_1)\phi_A(\mathbf{r}_2) - \phi_A(\mathbf{r}_1)\phi_S(\mathbf{r}_2)\right] = \frac{1}{\sqrt{2}}\left[\phi_R(\mathbf{r}_1)\phi_L(\mathbf{r}_2) - \phi_L(\mathbf{r}_1)\phi_R(\mathbf{r}_2)\right]$$

$\Psi_S =$  +

$\Psi_T =$ 

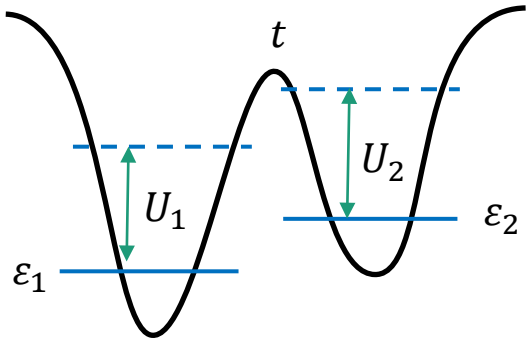
$$E_S = \langle \Psi_S | \hat{H} | \Psi_S \rangle \qquad E_T = \langle \Psi_T | \hat{H} | \Psi_T \rangle$$

Single is the Ground State. Exchange splitting = $E_T - E_S > 0$

Two-electron singlet-triplet splitting

Spin states of two electrons in a DQD

HM in the limit of $\Delta\varepsilon \gg U_i\,,\ t$ → Hubbard model



$$\hat{H} = \sum_{\sigma}(\varepsilon_1\, c_{1\sigma}^{+}\, c_{1\sigma} + \varepsilon_2\, c_{2\sigma}^{+}\, c_{2\sigma}) + \sum_{\sigma} t\,(c_{1\sigma}^{+}\, c_{2\sigma} + c_{2\sigma}^{+}\, c_{1\sigma})$$

$$+U_1 n_{1\uparrow} n_{1\downarrow} + U_2 n_{2\uparrow} n_{2\downarrow} + V\, n_1 n_2$$

$[\hat{H}, \hat{S}_{tot}^2] = [\hat{H}, \hat{S}_{tot}^z] = 0$ → S_{tot} and S_{tot}^z are good quantum numbers.

$S_{tot} = 1$ (Triplet)

$$|T_+\rangle = |\uparrow_1 \uparrow_2\rangle \quad (S_{tot}^z = 1)$$

$$|T_0\rangle = \frac{1}{\sqrt{2}}\,[|\uparrow_1 \downarrow_2\rangle + |\downarrow_1 \uparrow_2\rangle] \quad (S_{tot}^z = 0)$$

$$|T_-\rangle = |\downarrow_1 \downarrow_2\rangle \quad (S_{tot}^z = -1)$$

$$E_T = \varepsilon_1 + \varepsilon_2 + V$$

Two-electron singlet-triplet splitting

Spin states of two electrons in a DQD

$S_{tot} = 0$ (Singlet) : 3-dimensional subspace

For the basis states $|S_1\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_1 \downarrow_2\rangle - |\downarrow_1 \uparrow_2\rangle\right]$ $|S_2\rangle = |\uparrow_1 \downarrow_1\rangle$ $|S_3\rangle = |\uparrow_2 \downarrow_2\rangle$

$$H_S = \begin{pmatrix} \varepsilon_1 + \varepsilon_2 + V & \sqrt{2}t & \sqrt{2}t \\ \sqrt{2}t & 2\varepsilon_1 + U_1 & 0 \\ \sqrt{2}t & 0 & 2\varepsilon_2 + U_2 \end{pmatrix}$$

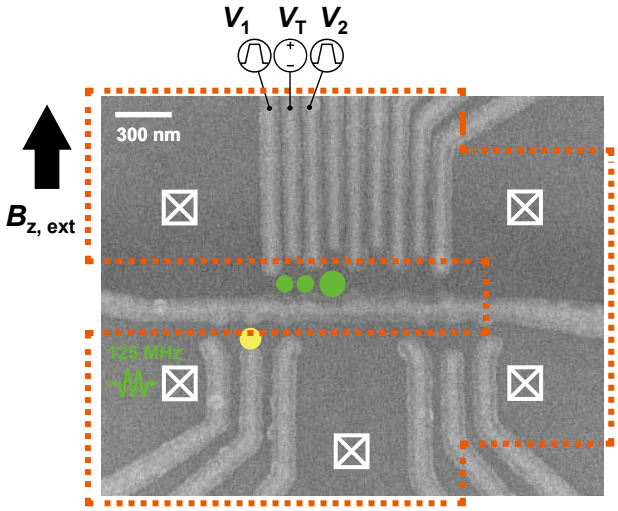
For $U_i \gg V, t$, we can get the singlet ground state energy by perturbation theory

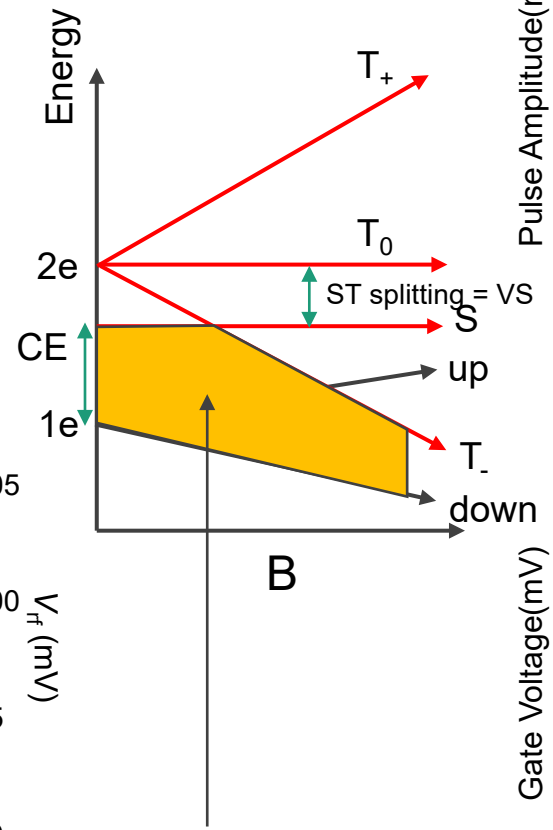
$$E_S = \varepsilon_1 + \varepsilon_2 + V + 2t^2 \left(\frac{1}{U_1 - V + \varepsilon_1 - \varepsilon_2} + \frac{1}{U_2 - V - \varepsilon_1 + \varepsilon_2} \right)$$

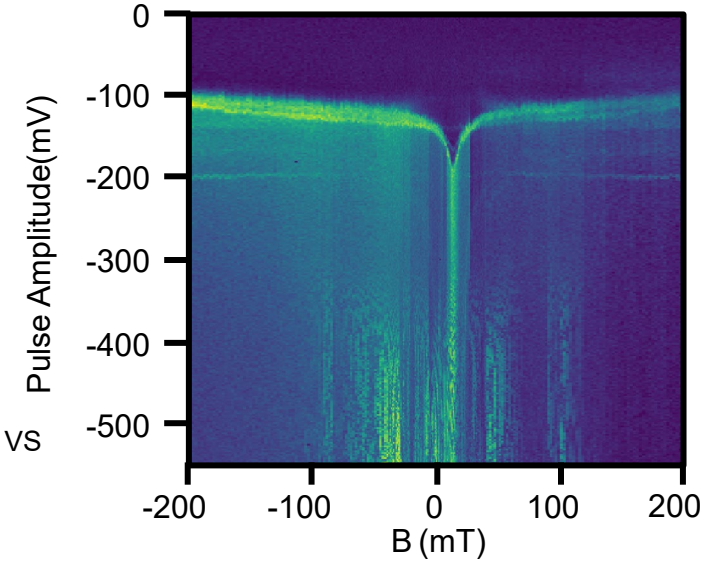
Exchange splitting $E_T - E_S = 2t^2 \left(\frac{1}{U_1 - V + \varepsilon_1 - \varepsilon_2} + \frac{1}{U_2 - V - \varepsilon_1 + \varepsilon_2} \right)$

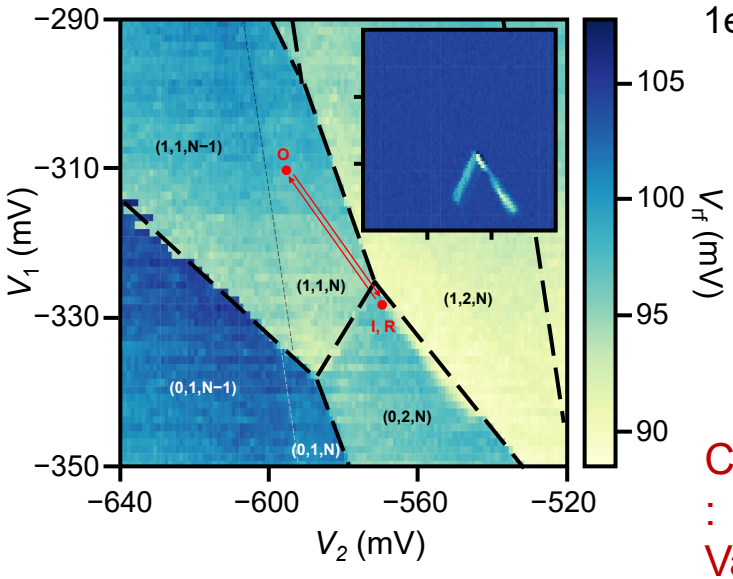
$$= \frac{4t^2}{U - V} \text{ for } U_1 = U_2 = U \text{ and} \varepsilon_1 = \varepsilon_2$$

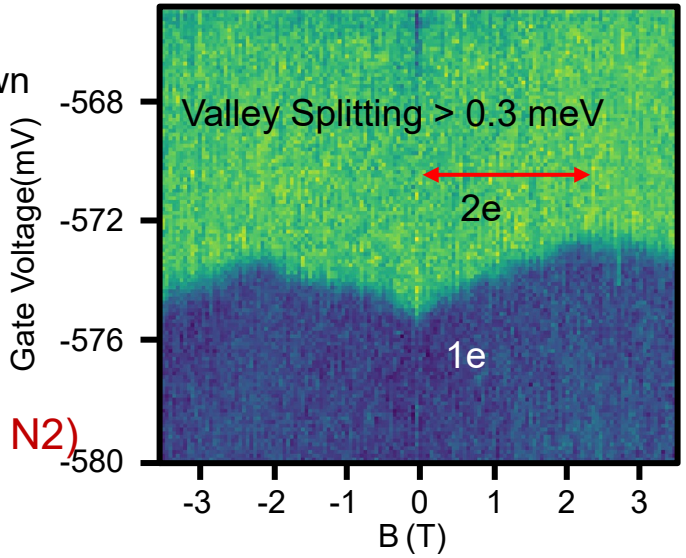
Measurement of excited level with charge sensing





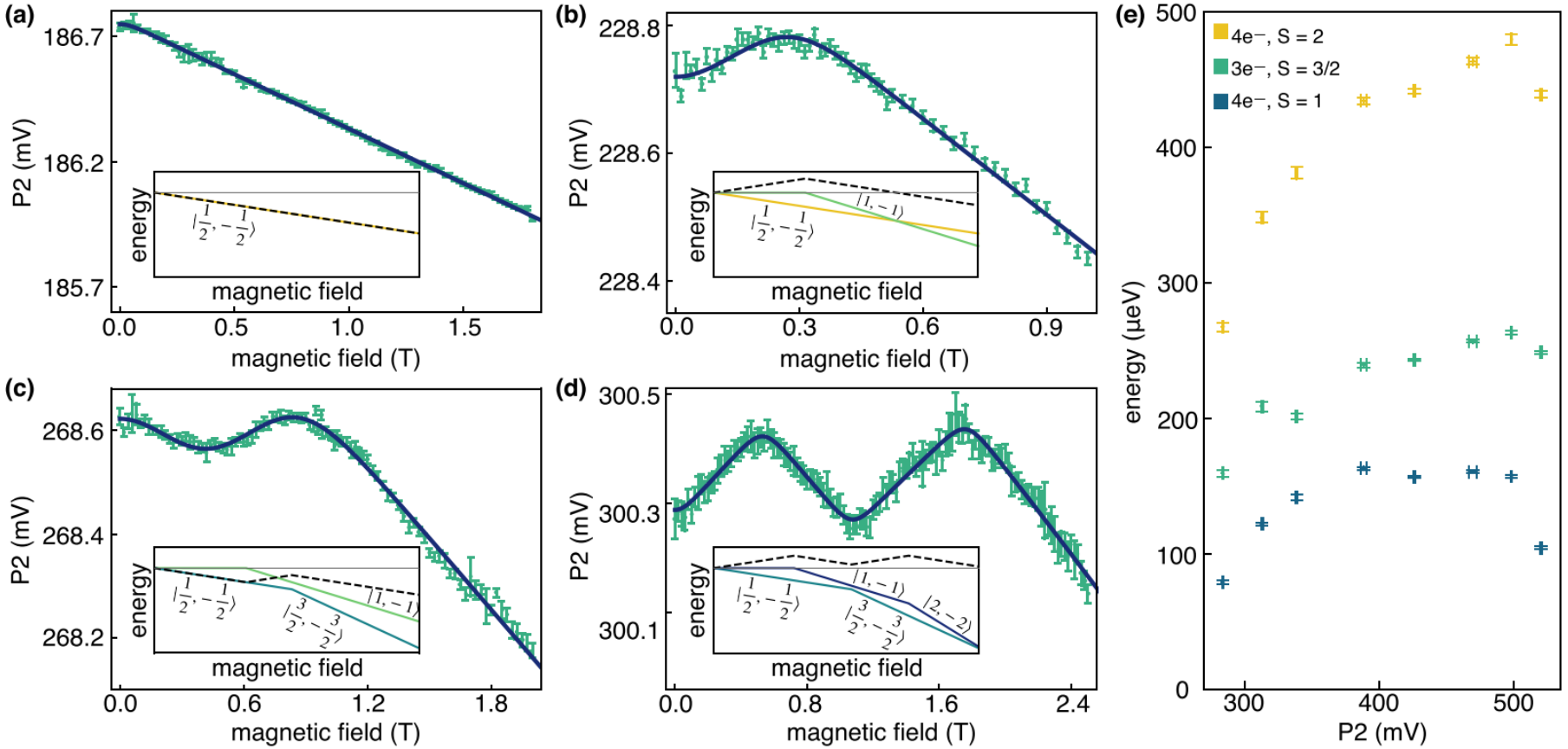






Chemical potential (N1 – N2)
: 변곡점이 ST splitting = Valley splitting

Measurement of excited level with charge sensing



T. McJunkin, et al., *Phys. Rev. B* **104** 085406 (2021)

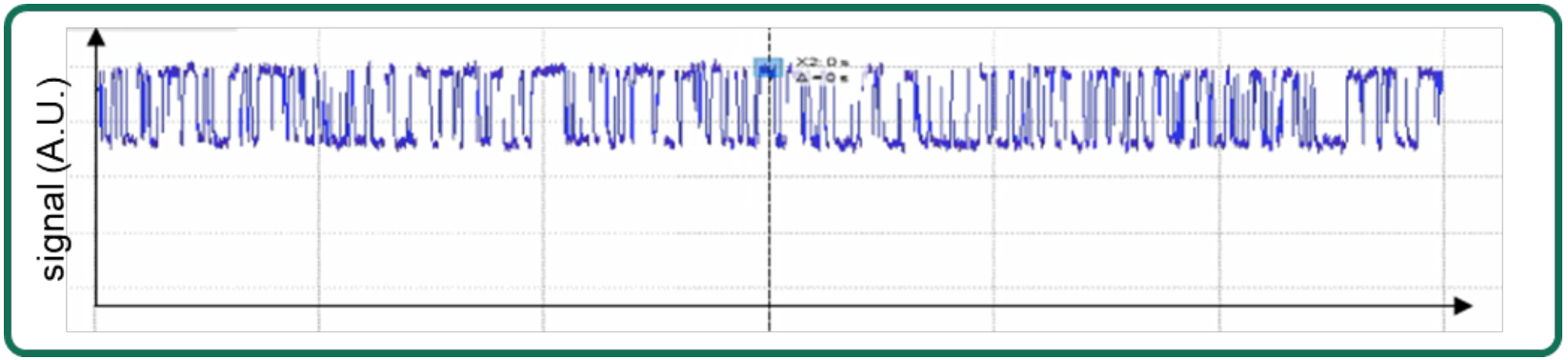
Summary of Lecture 1

Quantum dot basics

Exchange splitting $E_T - E_S = 2t^2 \left(\frac{1}{U_1 - V + \varepsilon_1 - \varepsilon_2} + \frac{1}{U_2 - V - \varepsilon_1 + \varepsilon_2}\right)$

$= \frac{4t^2}{U - V}$ for $U_1 = U_2 = U$ and $\varepsilon_1 = \varepsilon_2$

Keywords: CB, Exchange energy, Charge sensing…



다음시간: 이런 Basic 을 기반으로 하는 큐비트 실험 : Single-spin qubit, Singlet-triplet qubit..

