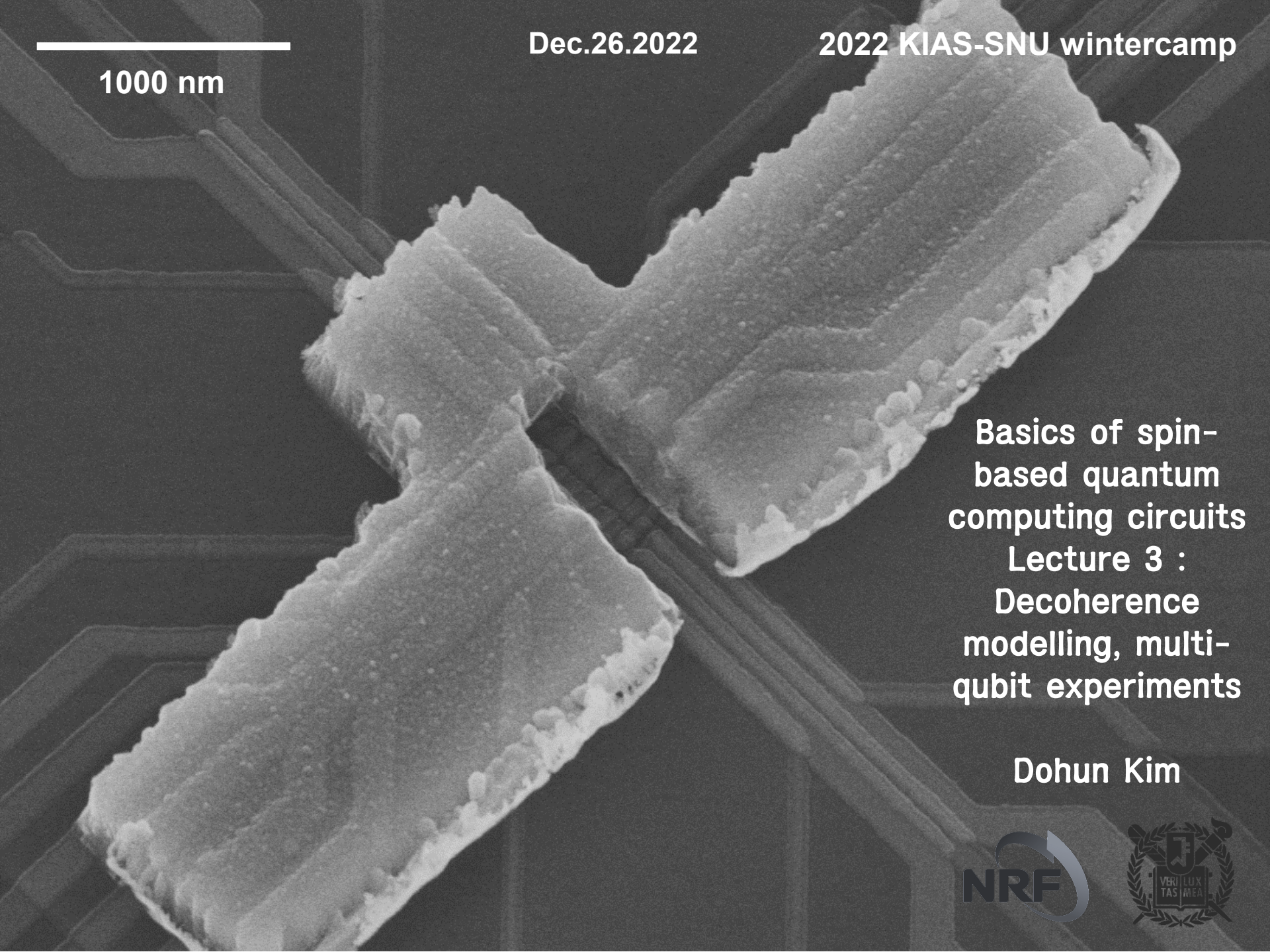


Dec.26.2022

2022 KIAS-SNU wintercamp

Basics of spin-based quantum computing circuits Lecture 3 : Decoherence modelling, multi-qubit experiments

Dohun Kim

Outline

- Background I *Quantum error*
- Background II *Recall EST & PSB, non-demolition measurements*
- Approach *Multi qubit initialization, manipulation, measurement*
- Some recent achievements *Shuttling-based long-distance connection, High-temperature operations*

Open quantum system : 팀프로젝트 문제에 필요한 배경지식

From closed to open quantum system

	Closed	Open
1. State	Ket vector $\|\Psi\rangle$	Density Matrix $\hat{\rho}$
2. Dynamics	Schrodinger	Master eq.
3. Measurement	Projective	Generalized (weak) Measurement

Density matrix

$$\hat{\rho} = \sum_i p_i \left|\Psi_i\right\rangle\left\langle\Psi_i\right|$$

p_i : probability to be in *i* th quantum state.

Properties

$$\hat{\rho}^\dagger = \hat{\rho}$$

$$Tr(\rho) = 1$$

$Tr(\rho^2)$ is called *purity*

$$\rho^2 = \rho$$ iff pure.

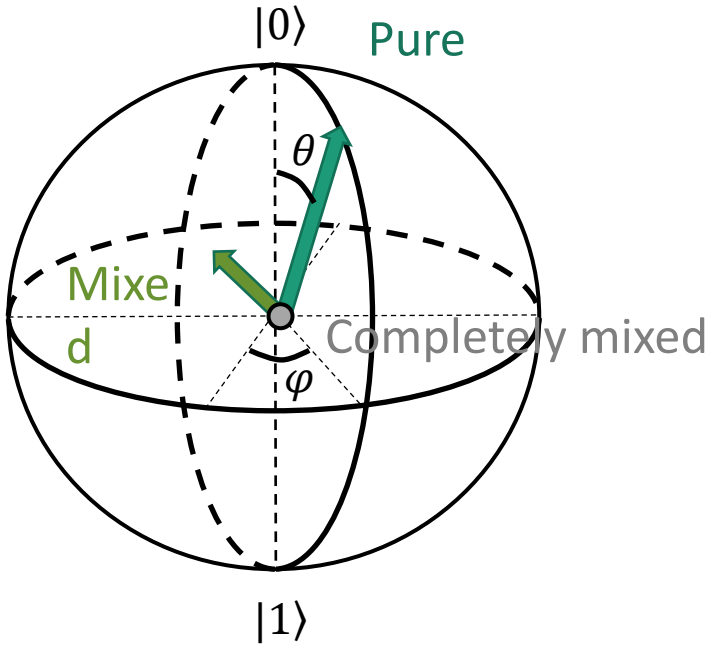
$$Tr(\rho^2) = 1$$ iff pure.

Open quantum system: Two level system

Pauli representation

$$\hat{\rho} = \frac{1}{2}\left(I + \sum_i m_i \hat{\sigma}_i \right)$$

$\hat{\sigma}_i$: Pauli matrix spans space of 2x2 matrices



Purity in Pauli rep.

$$Tr(\rho^2) = \frac{1}{4} Tr(I + 2\sum m_i \hat{\sigma}_i + \sum m_i m_j \hat{\sigma}_i \hat{\sigma}_j)$$

$$= \frac{1}{2}(1 + \sum m_i^{\,2}) \qquad \text{: By orthogonality}$$

$$\frac{1}{2} \le purity \le 1 \Rightarrow |\vec{m}| \le 1 \qquad \text{Bloch Sphere rep.}$$

Open quantum system

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Closed : Unitary evolution

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{\mathrm{H}}, \hat{\rho}]$$

: Liouville von-Neumann equation

Open: System + Environment 의 전체 time evolution 중 system의 상태만 보면 어떻게 변화하는가?

$$\frac{d\hat{\rho}_S}{dt} = -\frac{i}{\hbar}[\hat{\mathrm{H}}, \hat{\rho}_S(t)] + \sum_{\mu}\left(-\frac{1}{2}\hat{L}_\mu \hat{L}_\mu{}^{\dagger}\hat{\rho}_S - \frac{1}{2}\hat{\rho}_S \hat{L}_\mu \hat{L}_\mu{}^{\dagger} + \hat{L}_\mu \hat{\rho}_S \hat{L}_\mu{}^{\dagger}\right)$$

: The master equation in the Lindblad form

Examples of Quantum channel

Application of L form to two-level system

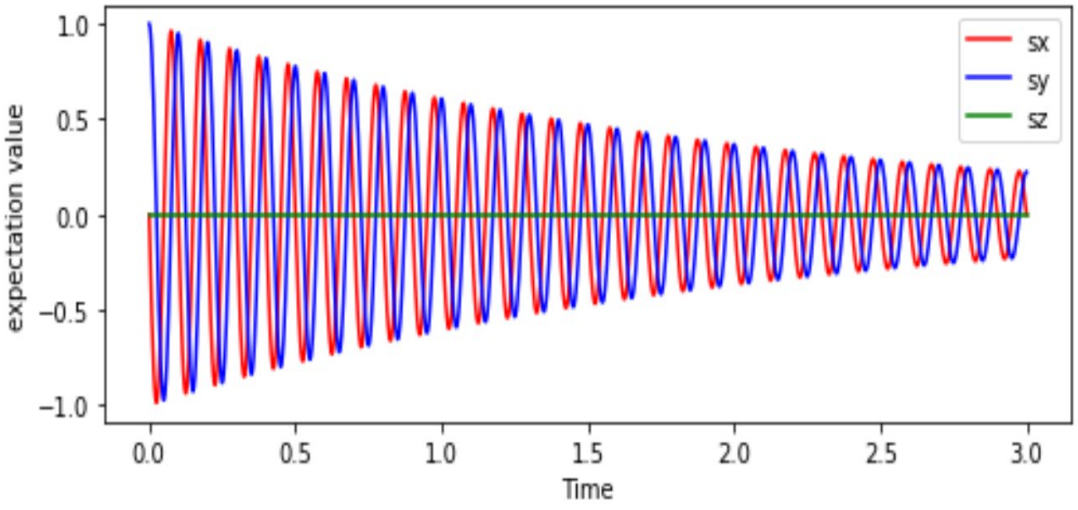
Pure dephasing channel

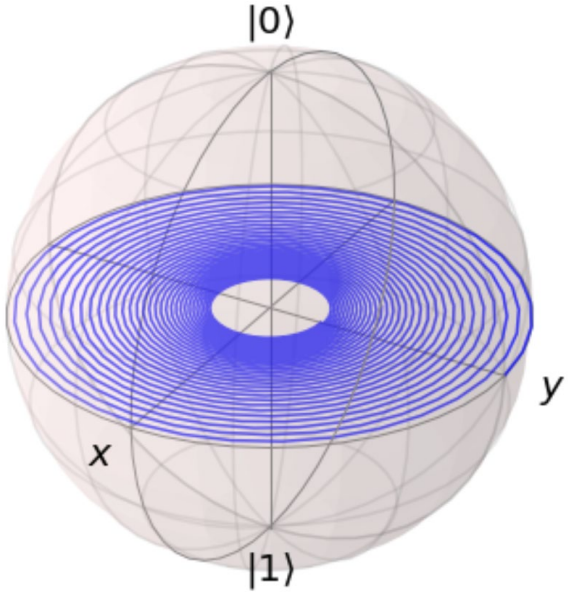
$$H_S = \begin{pmatrix} E_0 & 0 \\ 0 & E_1 \end{pmatrix} \qquad L_1 = L_1^{\dagger} = \sqrt{\gamma}\sigma_+\sigma_- = \sqrt{\gamma}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Fluctuating energy levels

This time scale is called, T_ϕ pure dephasing time

In the Lab frame,





Examples of Quantum channel

Application of L form to two-level system

Amplitude damping (relaxation) channel

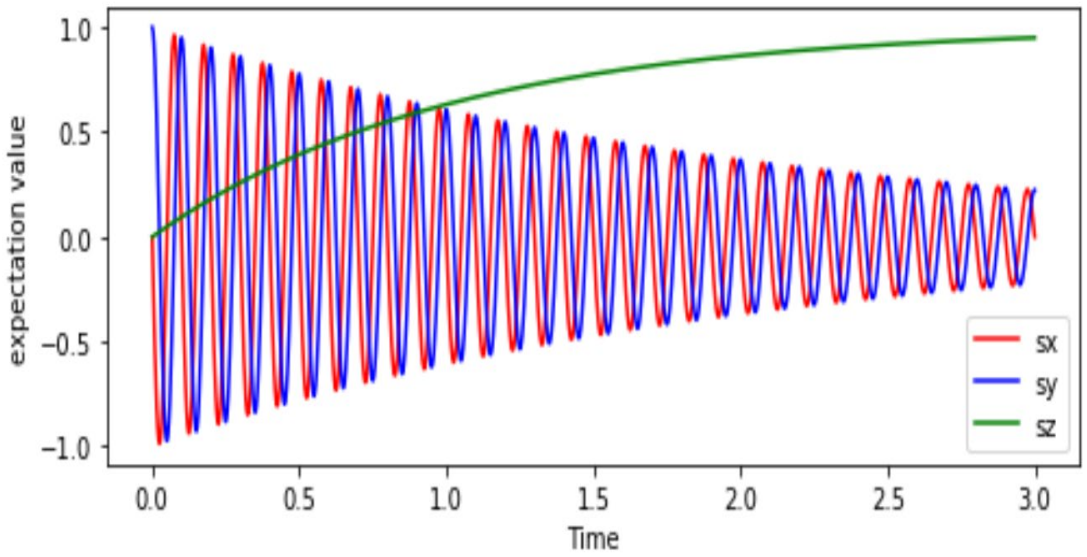
$$L_2 = \sqrt{\gamma}\sigma_- = \sqrt{\gamma}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

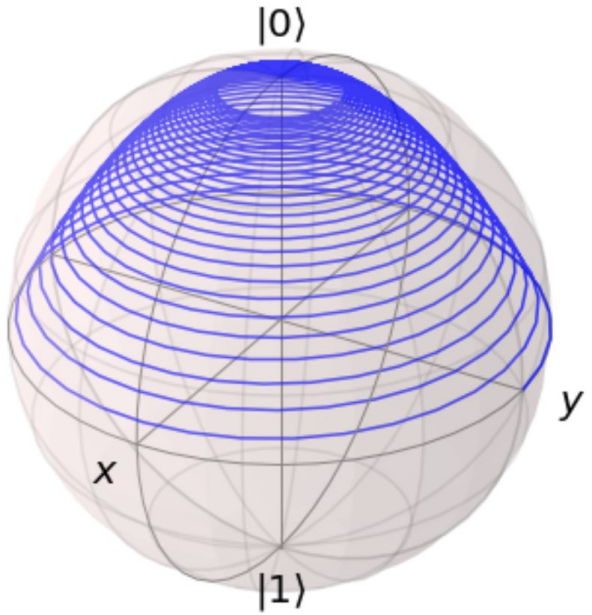
Energy relaxation

Total decoherence rate set by,

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}$$

This time scale is called, T_1 relaxation time





Also, depolarizing channel.. Etc.

Open quantum system

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$$\frac{d\hat{\rho}_S}{dt} = -\frac{i}{\hbar}[\hat{\mathrm{H}}, \hat{\rho}_S(t)] + \sum_{\mu}\left(-\frac{1}{2}\hat{L}_\mu \hat{L}_\mu{}^{\dagger}\hat{\rho}_S - \frac{1}{2}\hat{\rho}_S \hat{L}_\mu \hat{L}_\mu{}^{\dagger} + \hat{L}_\mu \hat{\rho}_S \hat{L}_\mu{}^{\dagger}\right)$$

: The master equation in the Lindblad form

Dynamics of open quantum system

Time evolution: Kraus operators

$$H_{SE} = H_S \otimes H_E$$

Unitary evolution: $\hat{U}_{SE}$

Environment orthonormal basis: $\left\{ \left| \mu \right\rangle_E \right\}$

Special proposition: $\hat{\rho}_{SE}(t) = \hat{\rho}_S(0) \otimes \hat{\rho}_E(0) = \hat{\rho}_S(0) \otimes \left| 0 \right\rangle_E \left\langle 0 \right|_E$

$$\hat{\rho}_{SE}(t) = \hat{U}_{SE}(t) \hat{\rho}_{SE}(0) \hat{U}^{\dagger}_{SE}(t)$$

$$\hat{\rho}_S(t) = Tr_E(\hat{U}_{SE}(t) \hat{\rho}_{SE}(0) \hat{U}^{\dagger}_{SE}(t))$$

$$= \sum_{\mu} \left\langle \mu \right|_E \hat{U}_{SE}(t) \left| 0 \right\rangle_E \hat{\rho}_S(0) \left\langle 0 \right|_E \hat{U}^{\dagger}_{SE}(t) \left| \mu \right\rangle_E$$

$$= \sum_{\mu} \hat{M}_{\mu}(t) \hat{\rho}_S(0) \hat{M}^{\dagger}_{\mu}(t)$$

$\hat{M}_{\mu}(t)$: Kraus operators

Dynamics of open quantum system

Operator-sum representation

낙서

$$\hat{\rho}_s(t) = \sum_\mu \hat{M}_\mu(t)\hat{\rho}_S(0)\hat{M}^\dagger{}_\mu(t)$$
$$= a[\hat{\rho}_S(0)]$$

Unitary evolution?
in general no.

Special case : pure state

$$\hat{\rho}_s(t) = \sum_\mu \hat{M}_\mu(t)\left|\psi_s(0)\right\rangle\left\langle\psi_s(0)\right|\hat{M}^\dagger{}_\mu(t)$$
$$= \sum_\mu \left|\tilde{\psi}_s(t)\right\rangle\left\langle\tilde{\psi}_s(t)\right|$$
$$= \sum_\mu p_\mu(t)\left|\psi_s(t)\right\rangle\left\langle\psi_s(t)\right|$$

$$\left|\tilde{\psi}_s(t)\right\rangle \equiv \hat{M}_\mu(t)\left|\psi_s(0)\right\rangle$$

$$\left|\psi_s(t)\right\rangle \equiv \frac{\left|\tilde{\psi}_s(t)\right\rangle}{\left\|\left|\tilde{\psi}_s(t)\right\rangle\right\|}$$

Not unitary operator
Not diagonal rep.
Generally mixed state

action of unitary operator on quantum system in general create entanglement

General properties of quantum map

1. Linearity

$$a[\lambda\hat{\rho}_1 + (1-\lambda)\hat{\rho}_2] = \lambda a[\hat{\rho}_1] + (1-\lambda)a[\hat{\rho}_2]$$

2. Completely Positive (CP condition)

$\hat{\rho}_{out} = a[\hat{\rho}_{in}]$: Physical state $\qquad \hat{\rho}_{out} = \hat{\rho}^{\dagger}_{out}$

$$\Rightarrow \hat{\rho}_{out} \geq 0 \quad (\ \hat{\rho}^{\dagger}_{out}\hat{\rho}_{out} \text{non-negative eigenvalue})$$

3. Trace preserving (TP condition)

$$Tr_S[a[\hat{p}_S]] = Tr_S(\sum_{\mu} \hat{M}_{\mu}(t)\hat{\rho}_S(0)\hat{M}^{\dagger}_{\mu}(t))$$

$$= Tr_S(\sum_{\mu} (\hat{M}_{\mu}(t)\hat{M}^{\dagger}_{\mu}(t))\hat{\rho}_S(0))$$

$$\sum_{\mu} \hat{M}_{\mu}(t)\hat{M}^{\dagger}_{\mu}(t) = \sum_{\mu} \langle 0|_E \hat{U}^{\dagger}_{SE}(t)(|\mu\rangle_E \langle\mu|_E)\hat{U}_{SE}(t)|0\rangle_E$$

$$= \langle 0|_E \hat{U}^{\dagger}_{SE}(t)\hat{U}_{SE}(t)|0\rangle_E = \hat{I}_S$$

Dynamics of open quantum system

Closed quantum system

$$\hat{\rho}(t+dt) = \hat{U}(t+dt,t)\hat{\rho}(t)\hat{U}^{\dagger}(t+dt,t)$$: generator of time-translation

$$\hat{U}(t+dt,t) = I - \frac{i}{\hbar}\hat{H}(t)dt$$

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}]$$: Liouville von-Neumann equation

Open quantum system

what is generator of time-translation?

$$\hat{\rho}_S(t_2) = a[\hat{\rho}_S(t_1)]$$

$$\hat{\rho}_S(t+dt) = a(t+dt,t)[\hat{\rho}_S(t)]$$: Markov approx.

$$= \sum_{\mu} \hat{M}_{\mu}(t+dt,t)\hat{\rho}_S(t)\hat{M}^{\dagger}_{\mu}(t+dt,t)$$

Dynamics of open quantum system

Lindblad operator

$$\widehat{M}_{\mu}(t+dt,t) \equiv \hat{L}_{\mu}(t)\sqrt{dt}$$

$$\widehat{M}_{0}(t+dt,t) \equiv \hat{I} + \widehat{G}(t)dt$$

CPTP condition

$$\sum_{\mu} \widehat{M}_{\mu}\widehat{M}^{\dagger}{}_{\mu} = \hat{I}$$

$$\Rightarrow \widehat{M}_{0}\widehat{M}^{\dagger}{}_{0} + \sum_{\mu\neq 0} \widehat{M}_{\mu}\widehat{M}^{\dagger}{}_{\mu} = \hat{I} + (\widehat{G} + \widehat{G}^{\dagger})dt + \sum_{\mu} \hat{L}_{\mu}\hat{L}_{\mu}{}^{\dagger}dt = \hat{I}$$

$$\Rightarrow \widehat{G} + \widehat{G}^{\dagger} = -\sum_{\mu} \hat{L}_{\mu}\hat{L}_{\mu}{}^{\dagger}$$

$$\Rightarrow \widehat{G} \equiv \widehat{K} - \frac{i}{\hbar}\widehat{H} = -\frac{i}{\hbar}\widehat{H} - \frac{1}{2}\sum_{\mu} \hat{L}_{\mu}\hat{L}_{\mu}{}^{\dagger} \equiv -\frac{i}{\hbar}H_{eff}$$

Dynamics of open quantum system

$$\widehat{\rho_S}(t+dt) = \widehat{M}_0 \hat{\rho}_S(t) \widehat{M}^{\dagger}{}_0 + \sum_{\mu \neq 0} \widehat{M}_\mu \hat{\rho}_S(t) \widehat{M}^{\dagger}{}_\mu$$

$$= \left(\hat{I} - \frac{i}{\hbar} \widehat{H}_{eff} dt \right) \hat{\rho}_S(t) \left(\hat{I} - \frac{i}{\hbar} \widehat{H}_{eff} dt \right) + \sum_\mu \hat{L}_\mu \hat{\rho}_S(t) \hat{L}^{\dagger}{}_\mu dt$$

$$\Rightarrow \frac{d\widehat{\rho}_S}{dt} = -\frac{i}{\hbar} [\widehat{H}_{eff}, \hat{\rho}_S(t)] + \sum_\mu \hat{L}_\mu \hat{\rho}_S(t) \hat{L}^{\dagger}{}_\mu$$

expanding effective Hamiltonian

$$\frac{d\widehat{\rho}_S}{dt} = -\frac{i}{\hbar} [\widehat{H}, \hat{\rho}_S(t)] + \sum_\mu \left(-\frac{1}{2} \hat{L}_\mu \hat{L}_\mu{}^{\dagger} \hat{\rho}_S - \frac{1}{2} \hat{\rho}_S \hat{L}_\mu \hat{L}_\mu{}^{\dagger} + \hat{L}_\mu \hat{\rho}_S \hat{L}_\mu{}^{\dagger} \right)$$

: The master equation in the Lindblad form

The differential eqn. whose integral is CPTP map has to be this 'Lindblad form'

Composite systems

Hilbert Space of composite system

$$H = H_A \otimes H_B$$

Two observable algebra A, B

Product State of composite systems

$$|\phi_A\rangle \otimes |\phi_B\rangle,$$

$$where, |\phi_A\rangle \in H_A, |\phi_B\rangle \in H_B$$

Pure product states have density operators

$$\hat{\rho} = |\phi_A\rangle\langle\phi_A| \otimes |\phi_B\rangle\langle\phi_B| = \hat{\rho}_A \otimes \hat{\rho}_B$$

Q. All states are product state?

States which aren't products are correlated

$$\hat{\rho} = p|00\rangle\langle 00| + (1-p)|11\rangle\langle 11|$$

Q. How to check if the state is product state or not?

Schmidt decomposition for pure state

$$|\psi\rangle = \sum_{n=1}^{d} \sqrt{p_i}\,|e_i\rangle|f_j\rangle$$

$$then, \rho_A = \sum_{n=1}^{d} p_j |e_j\rangle\langle e_i|$$

Composite systems: Entanglement

Partial Trace

$$\rho_{AB} = \sum_i p_i \left|\Psi_i\right\rangle_{AB} \left\langle\Psi_i\right|_{AB}$$

'Trace out' environment -> information loss?

$$\rho_A = Tr_B \rho_{AB} = \sum_j \left\langle\psi_i\right|_B p_i \left|\Psi_i\right\rangle_{AB} \left\langle\Psi_i\right|_{AB} \left|\psi_j\right\rangle_B$$

Ex) For singlet state

$$\hat{\rho} = \frac{1}{2}(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle)(\left\langle\uparrow\downarrow\right| - \left\langle\downarrow\uparrow\right|)$$

$$\hat{\rho}_A = \frac{1}{2}\left\langle\uparrow\right|_B (\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle)(\left\langle\uparrow\downarrow\right| - \left\langle\downarrow\uparrow\right|)\left|\uparrow\right\rangle_B$$

$$+\frac{1}{2}\left\langle\downarrow\right|_B (\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle)(\left\langle\uparrow\downarrow\right| - \left\langle\downarrow\uparrow\right|)\left|\downarrow\right\rangle_B$$

$$\Rightarrow \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

Completely Mixed state

Entropy: How much entangled?

$$S = -\sum_\mu p_\mu \log p_\mu \qquad p_\mu \quad : \text{Schmidt coefficient}$$

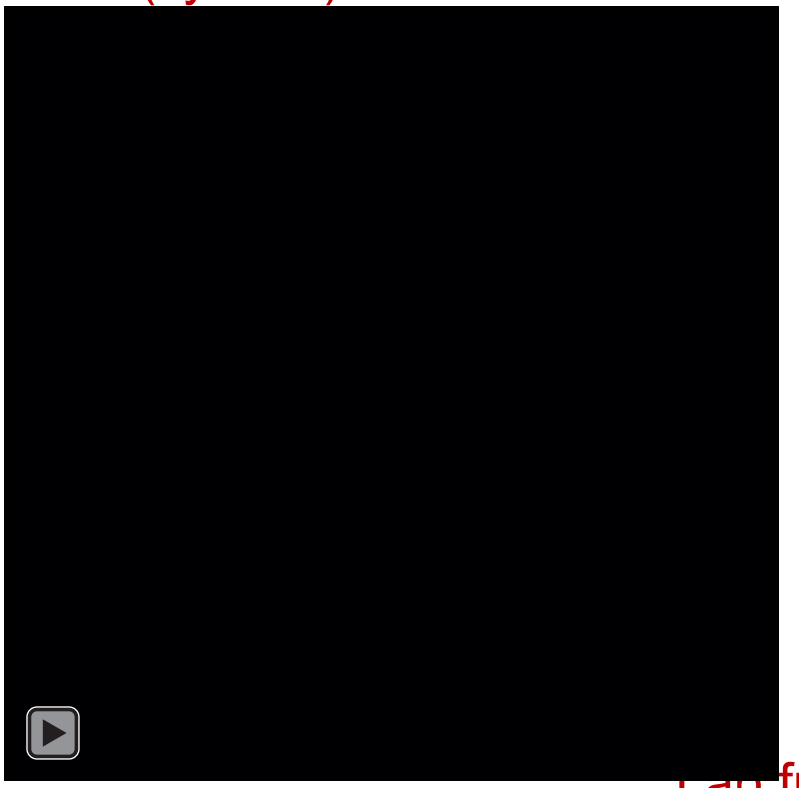
More on entanglement

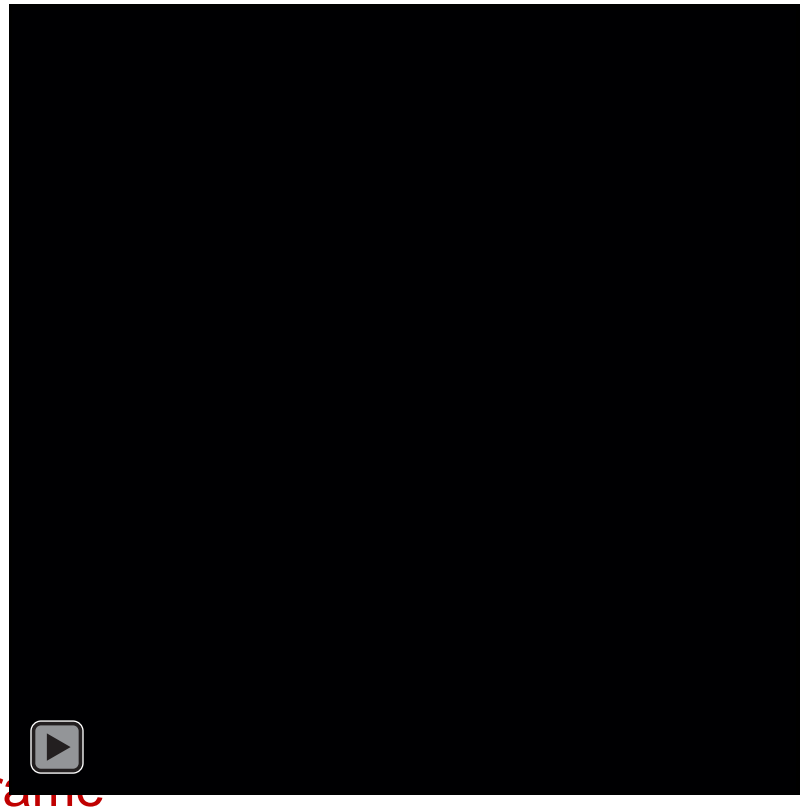
$$\hat{H} = \frac{J_1}{2}(\hat{\sigma}_{z1} \otimes I) + \frac{J_2}{2}(I \otimes \hat{\sigma}_{z2}) + \frac{J_{12}}{2}((\hat{\sigma}_{z1} - I) \otimes (\hat{\sigma}_{z2} - I))$$

Initial state : $|++\rangle$ Note : no decoherence as a whole (no L)

Qubit 1 (system)

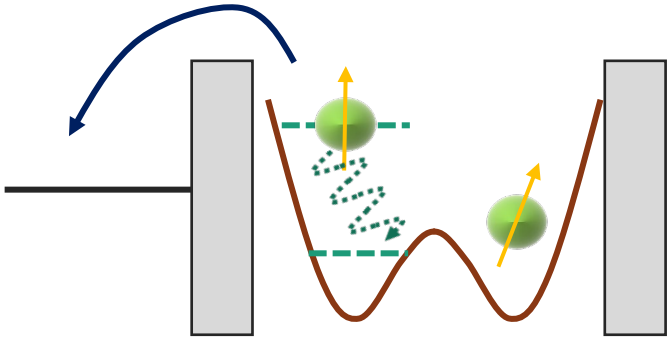
Qubit 2 (Env.)



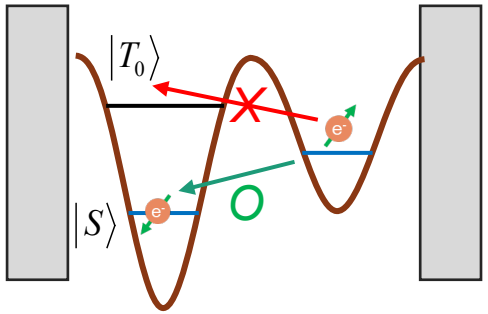


Lab frame

Recall RSB & EST



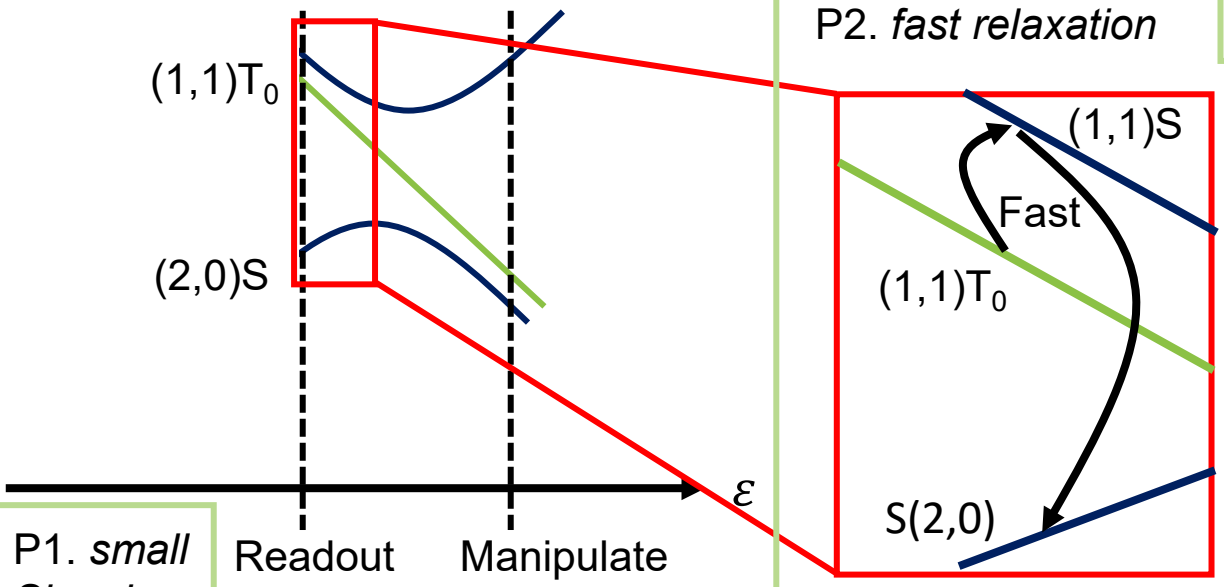
Measure one electron charge change



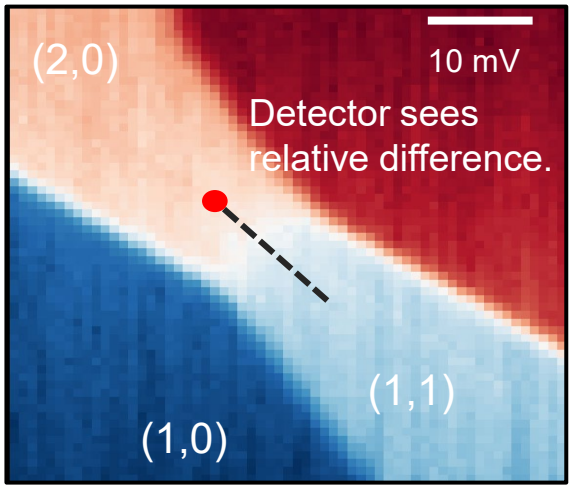
Measure relative position

Typical signal : 그리기

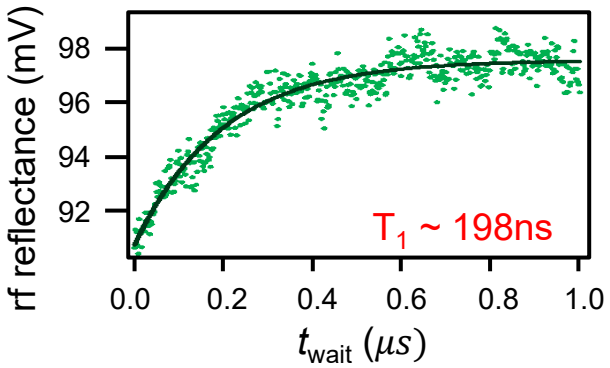
Problem of conventional Read-out of STQ



P1. *small Signal*

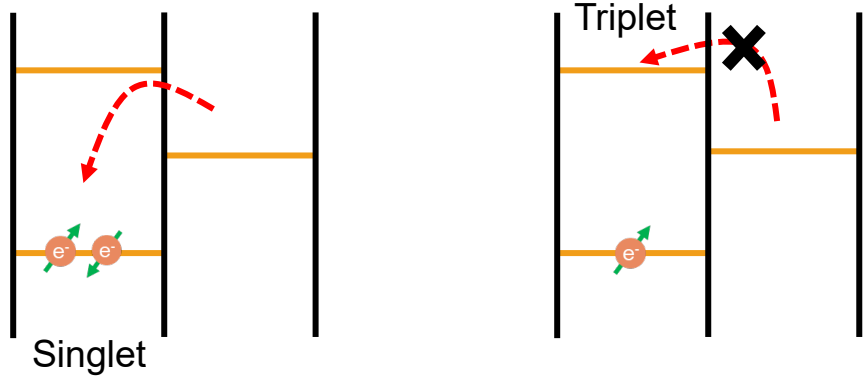


P2. *fast relaxation*

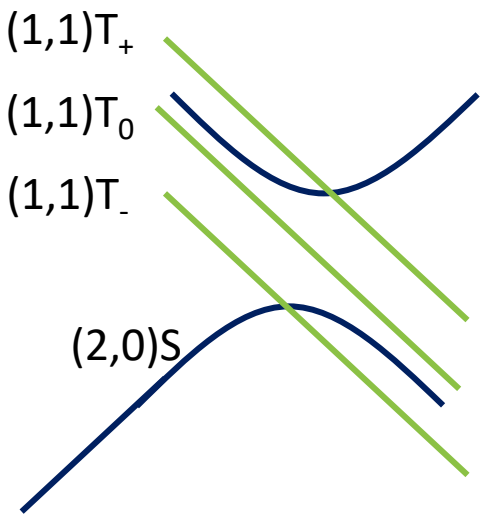


At PSB detuning, ΔB mixes S & T_0: $(1,1)T_0$ relax to S(2,0) – fast at *large* ΔB

At the Readout point : *spin blockade*



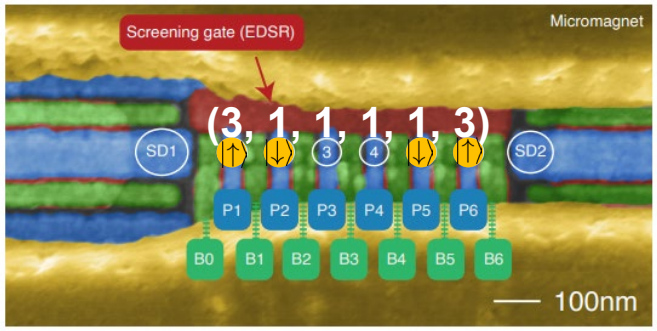
Really a problem ? Parity readout

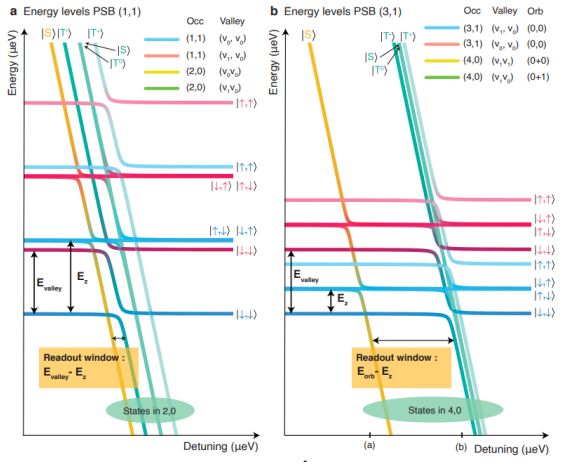


Typical signal : Readout 방법 그리기

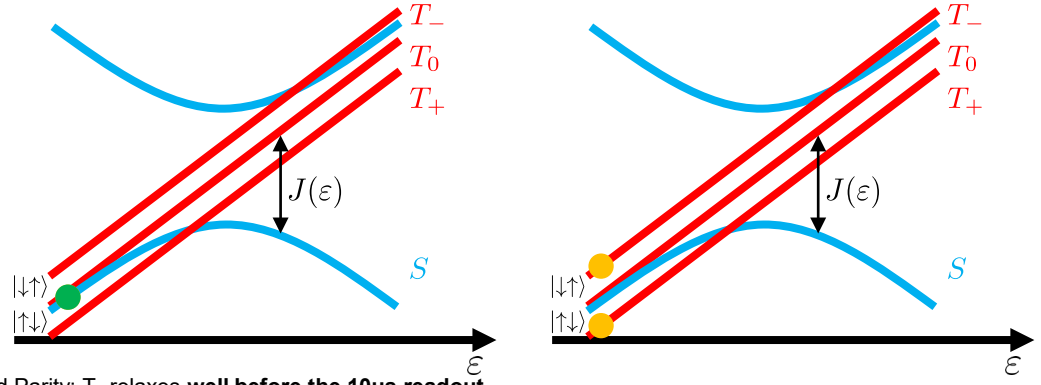
Six qubit operation : initialization

Initialization of the Spin State





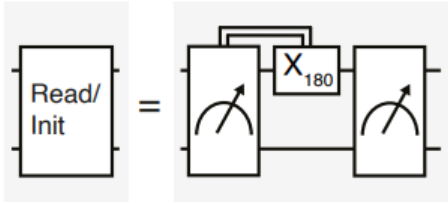
1. Parity based initialization of qubit 1-2 (5-6)



Odd Parity: T_0 relaxes **well before the 10μs readout window** at PSB regime

➡ Readout of state parity using PSB

Real-time feedback initialization



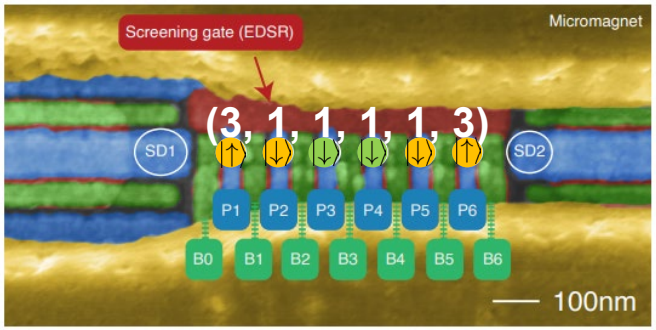
1. First parity measurement
2. Burst a π-pulse on qubit 1 when even parity
3. Second Parity Measurement
4. $|\uparrow\downarrow\rangle$ state is prepared

https://arxiv.org/abs/2202.09252

2 Approach

Six qubit operation : initialization

Initialization of the Spin State



Physical Review A 32.4 (1985): 2287.

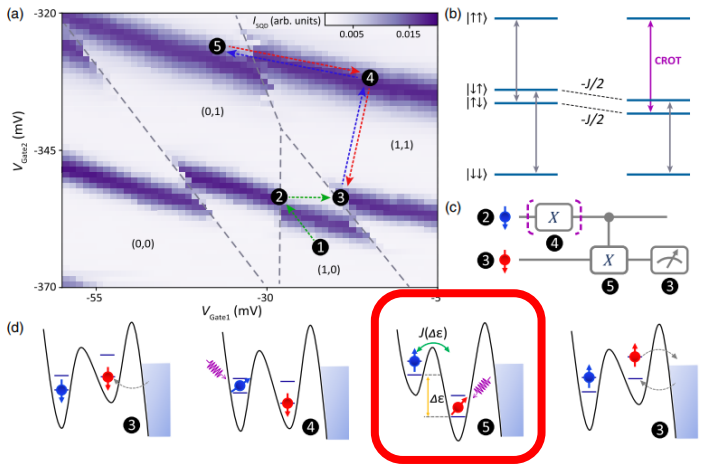
1. The uncertainty introduced by the measurement should not affect the motion of the observable

$$[A_S, H_S] \approx [\sigma_z, \sigma_z] = 0$$

2. The interaction with the ancilla should not affect the motion of the observable

$$[A_S, H_{int}] \approx [\sigma_z, (\sigma_z \otimes \sigma_z - 1)] = 0$$

2 Quantum-Non-Demolition readout of inner spin qubits

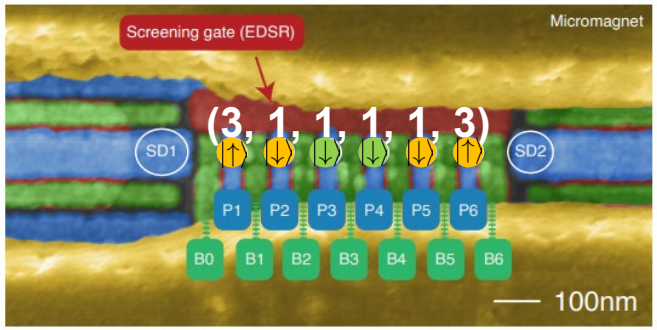


Physical Review X 10.2 (2020): 021006.

용어의 모호함 ?

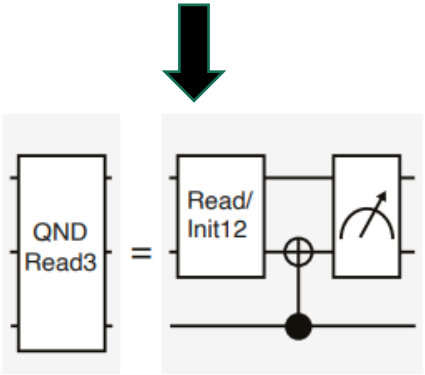
Six qubit operation : initialization

Initialization of the Spin State



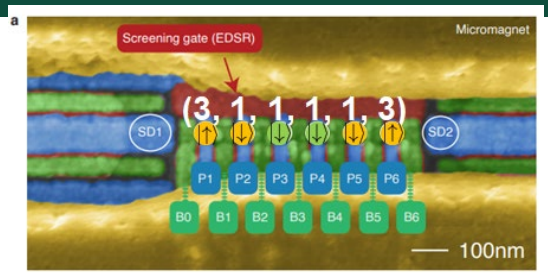
2. Quantum-Non-Demolition readout of inner spin qubits

1. Initialize the ancilla qubit to the spin-down state
2. Turn on J_{int} by pulsing a virtual barrier gate and perform a CNOT gate
3. Pulse to (4,0) and perform the single-shot measurement of the ancilla qubit
4. Parity measurement prepares the desired initial states for qubit 1 and 2
5. If qubit 3 is spin-up, flip it using the π-pulsing microwave



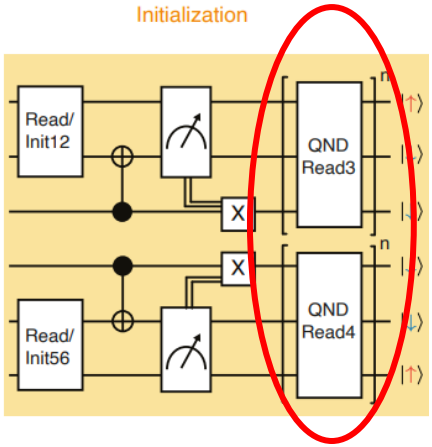
2 Approach

Six qubit operation : initialization

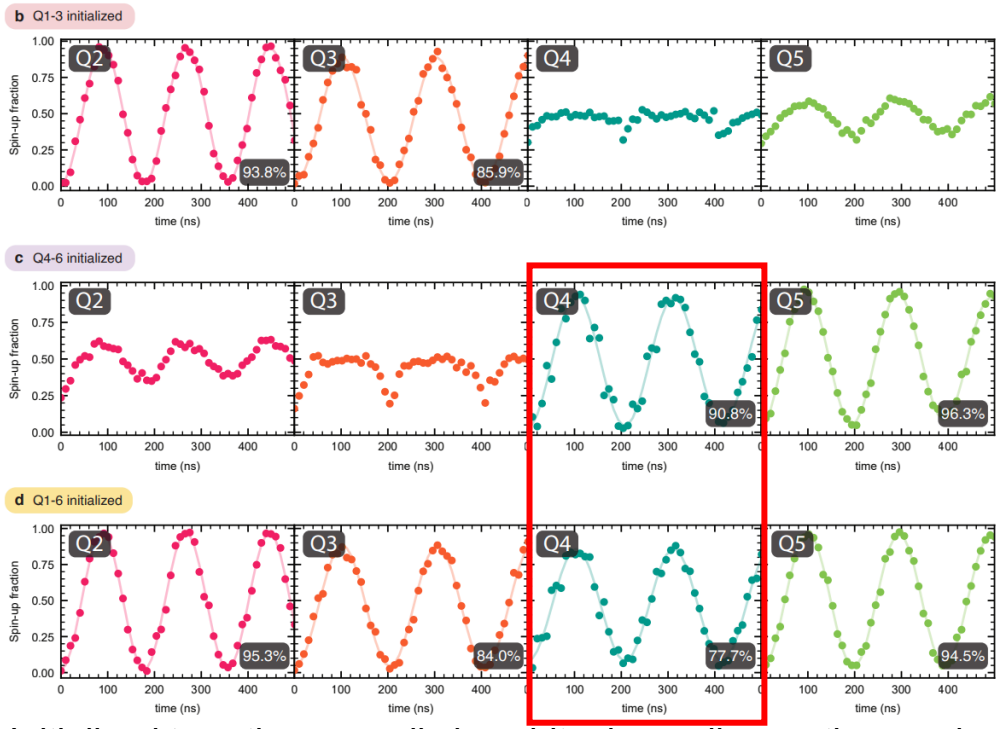


Initialization of the Spin State

1. Parity based initialization of qubit 1-2 (5-6)
2. Quantum-Non-Demolition readout of inner spin qubits



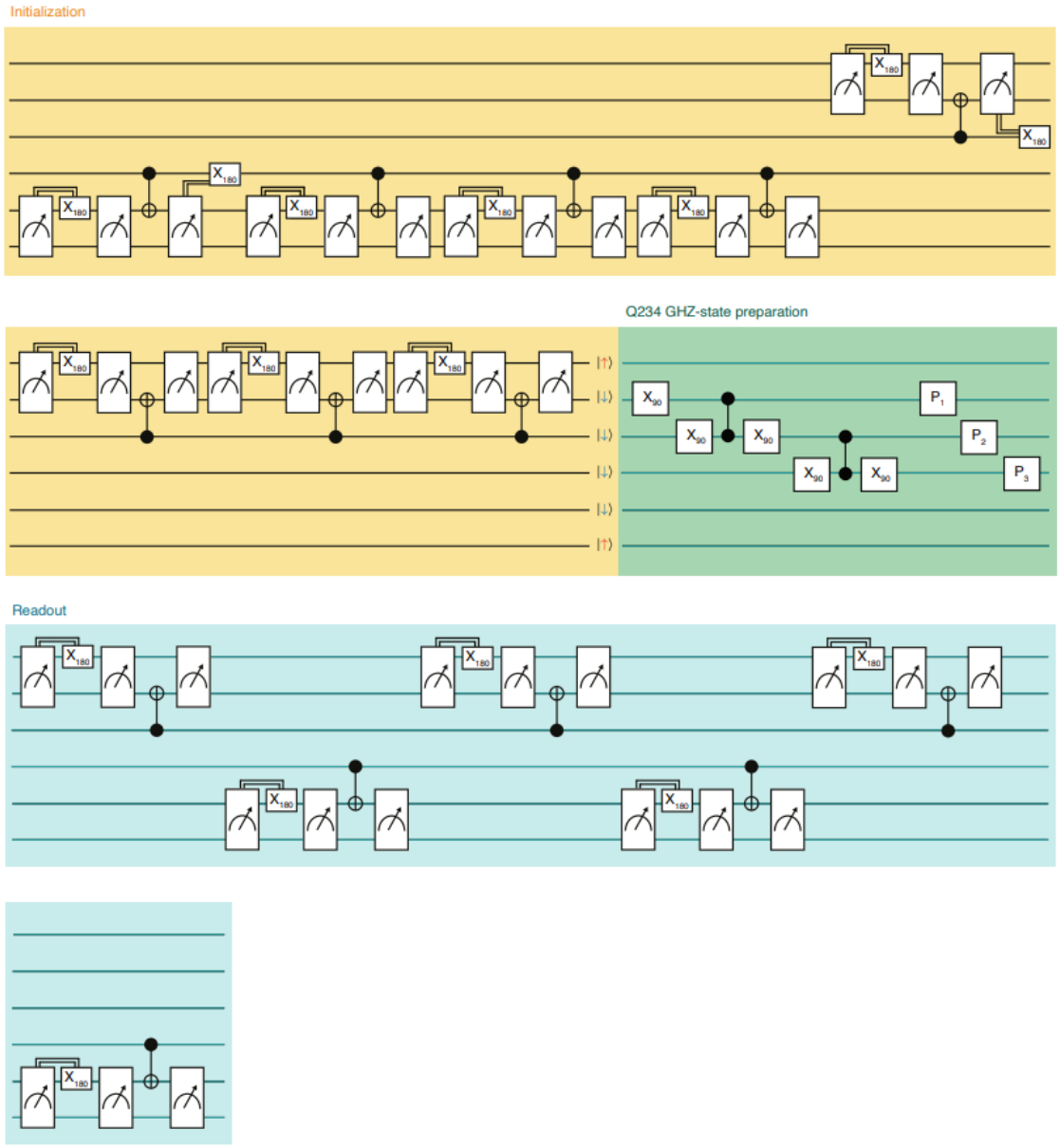
Run QND measurement three times and **post-select** runs with three identical QND readout outcome
(except for GHZ state preparation and tomography, where the **majority vote** is used)



Initialized two, three, or all six qubits depending on the requirement of the specific quantum circuit

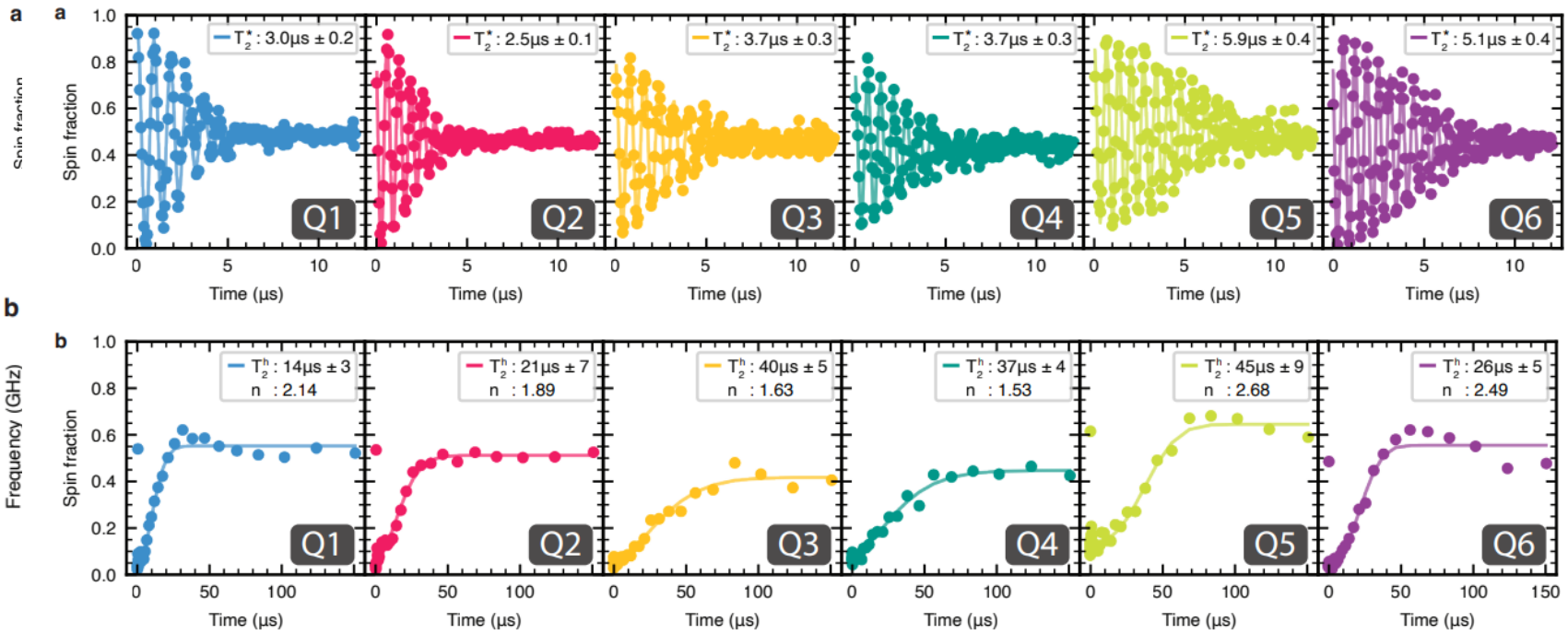
Approach

Six qubit operation : readout



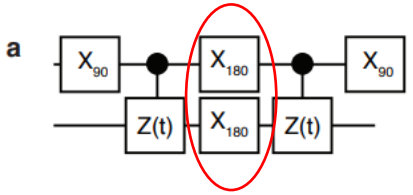
Six qubit operation : operation

1. EDSR Based Rabi Oscillation

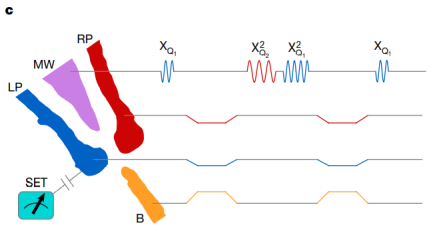


Six qubit operation : operation

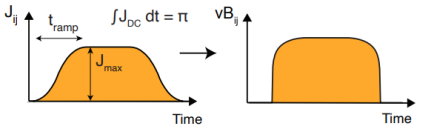
2. Two Qubit Gate (C-Phase) by Virtual Barrier Gate Pulse



Apply π to both Q1 and Q2, which offsets the oscillation by J_s and J_A yet preserves the oscillation by J_{int}



Nature 601.7893 (2022): 343-347.



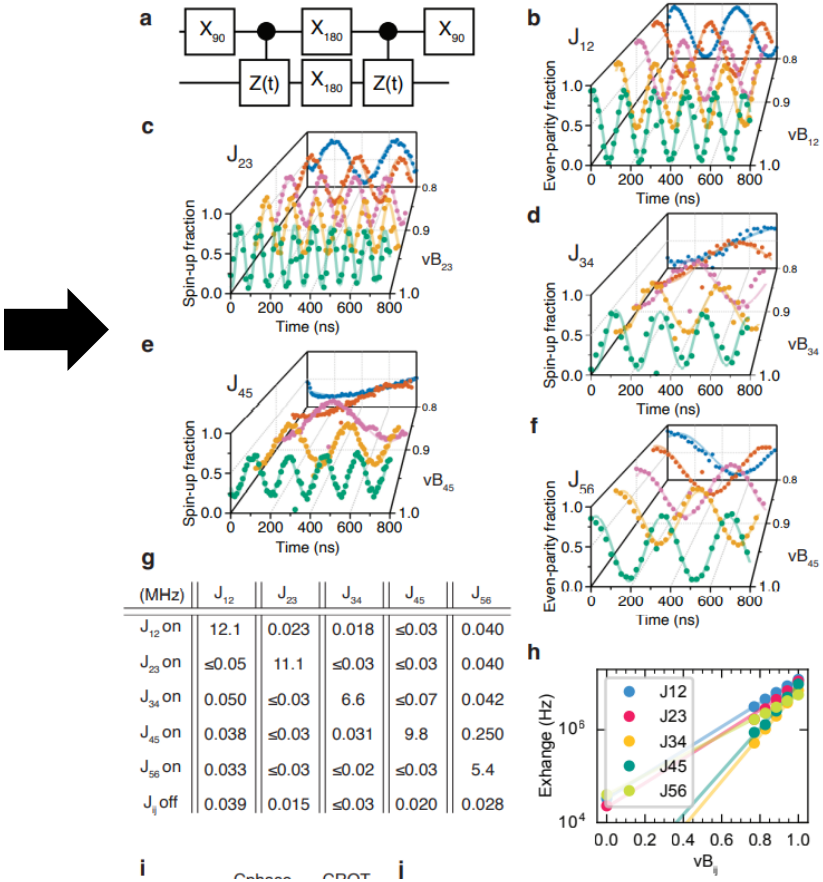
$$W(t,r)=\begin{cases}\frac{1}{2}\left[1-\cos\left(\frac{2\pi t}{rt_p}\right)\right] & 0\le t\le\frac{rt_p}{2}\\ 1 & \frac{rt_p}{2}<t<t_p-\frac{rt_p}{2},\\ \frac{1}{2}\left[1-\cos\left(\frac{2\pi(t_p-t)}{rt_p}\right)\right] & t_p-\frac{rt_p}{2}\le t\le t_p\end{cases}$$

Nature 601.7893 (2022): 343-347.

Tukey window with a ramp time of

$$t_{ramp}=\frac{3}{\sqrt{\delta B^2+J_{max}^2}}$$

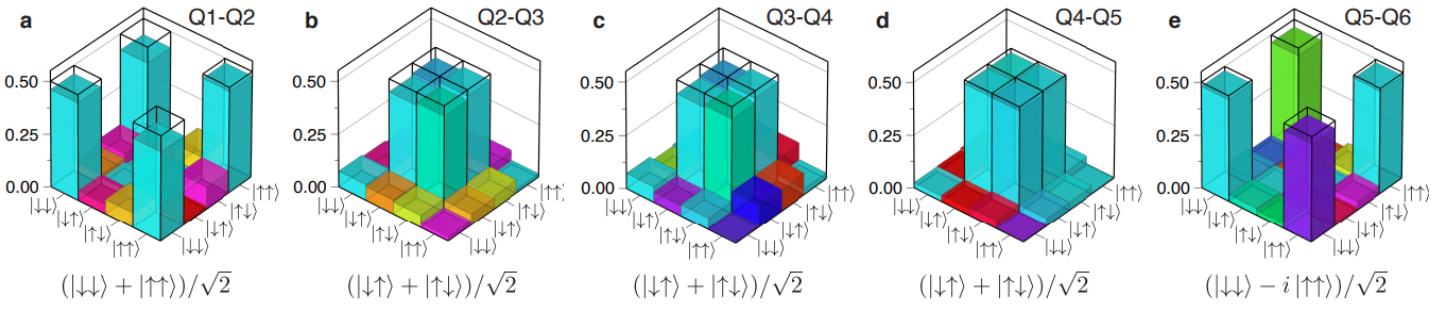
Physical Review A 90.2 (2014): 022307.



(MHz)	J_{12}	J_{23}	J_{34}	J_{45}	J_{56}
J_{12} on	12.1	0.023	0.018	≤0.03	0.040
J_{23} on	≤0.05	11.1	≤0.03	≤0.03	0.040
J_{34} on	0.050	≤0.03	6.6	≤0.07	0.042
J_{45} on	0.038	≤0.03	0.031	9.8	0.250
J_{56} on	0.033	≤0.03	≤0.02	≤0.03	5.4
J_{ij} off	0.039	0.015	≤0.03	0.020	0.028

Six qubit operation : operation

3. Bell State Tomography/GHZ State Tomography

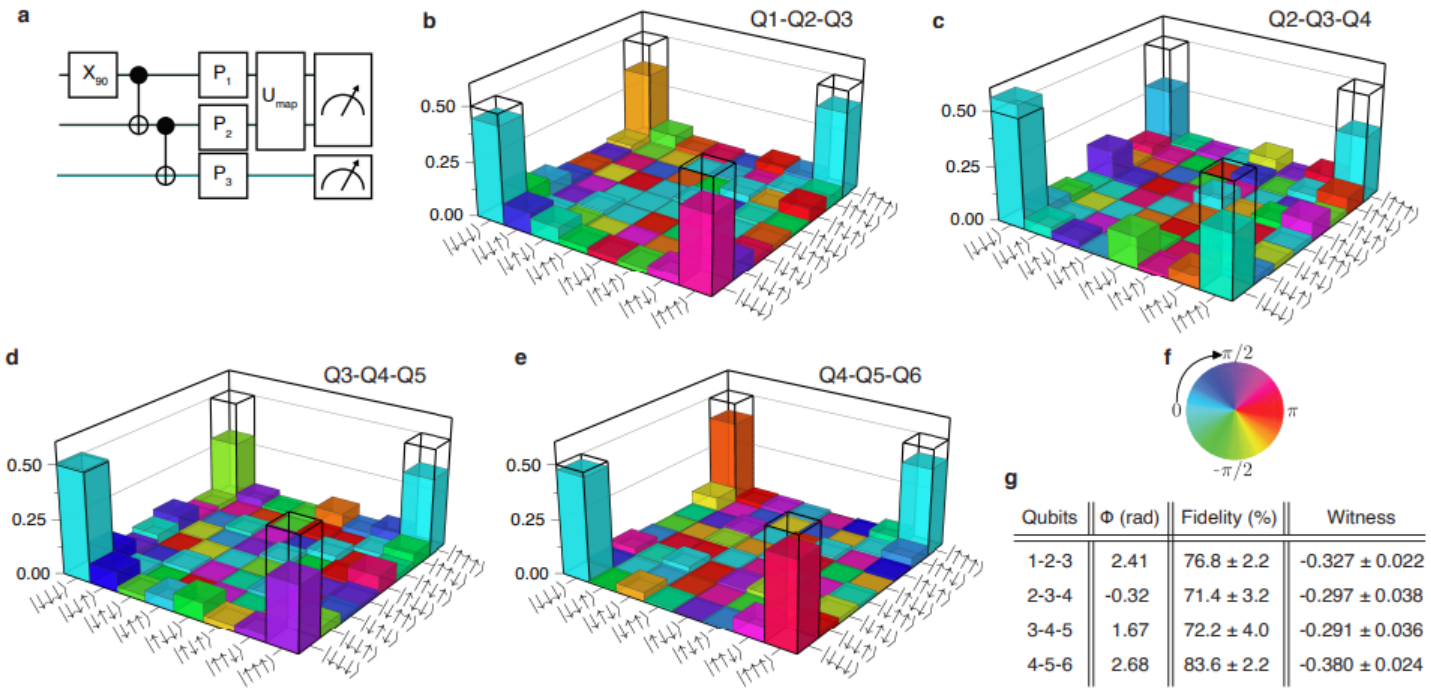


h

Qubits	Fidelity (%)	Concurence (%)
1-2	89.2 ± 2.2	86.7 ± 3.2
2-3	90.1 ± 2.2	83.9 ± 3.8
3-4	88.3 ± 3.6	87.9 ± 5.0
4-5	95.6 ± 2.0	94.9 ± 3.2
5-6	94.1 ± 1.4	90.6 ± 3.6

Six qubit operation : operation

4. GHZ State Tomography



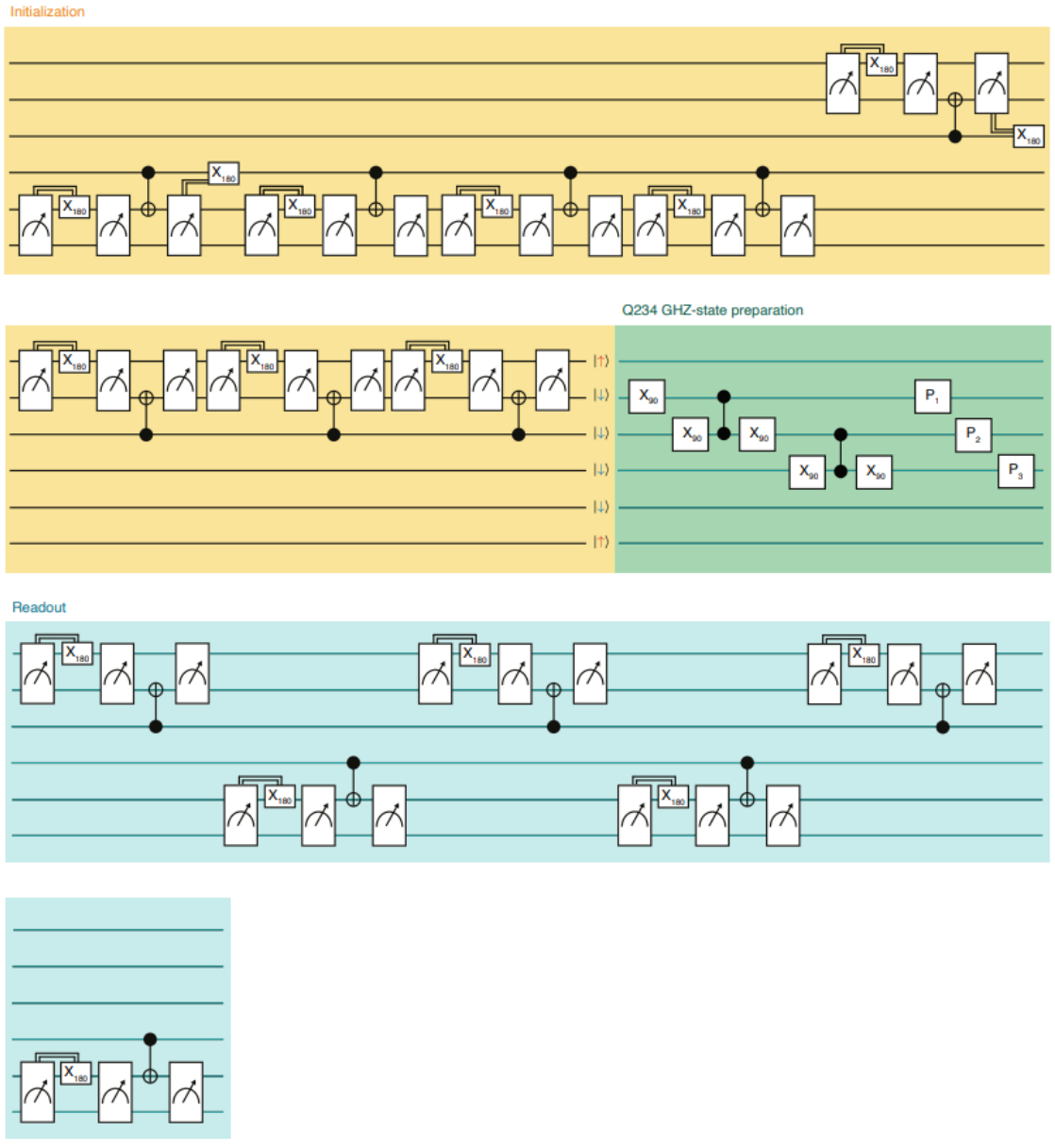
g

Qubits	Φ (rad)	Fidelity (%)	Witness
1-2-3	2.41	76.8 ± 2.2	-0.327 ± 0.022
2-3-4	-0.32	71.4 ± 3.2	-0.297 ± 0.038
3-4-5	1.67	72.2 ± 4.0	-0.291 ± 0.036
4-5-6	2.68	83.6 ± 2.2	-0.380 ± 0.024

2

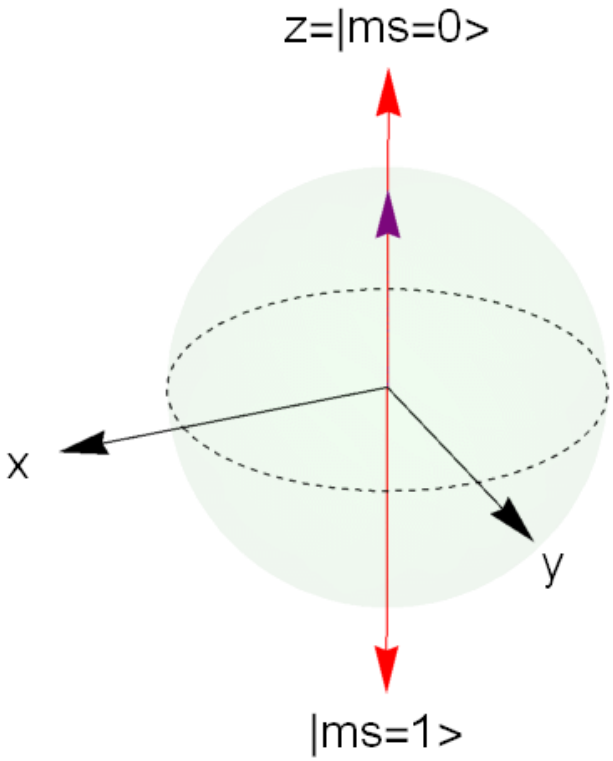
Approach

Six qubit operation : 전체회로 recall



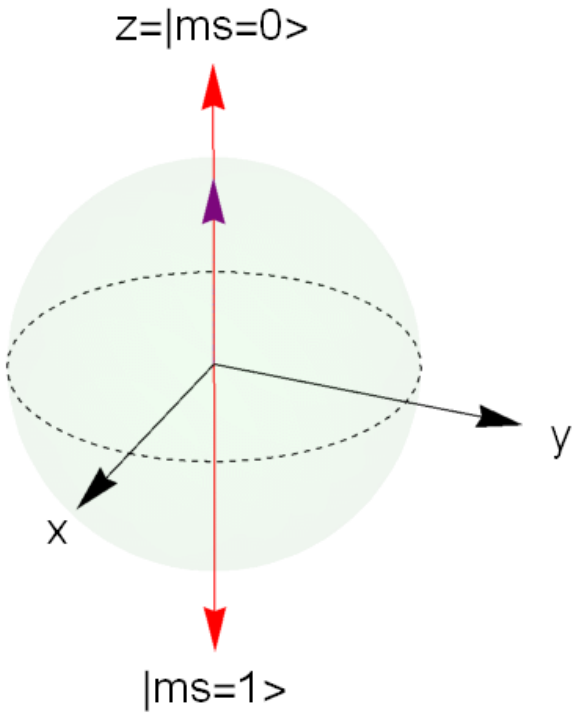
One more technique : Spin-echo

Ramsey sequence

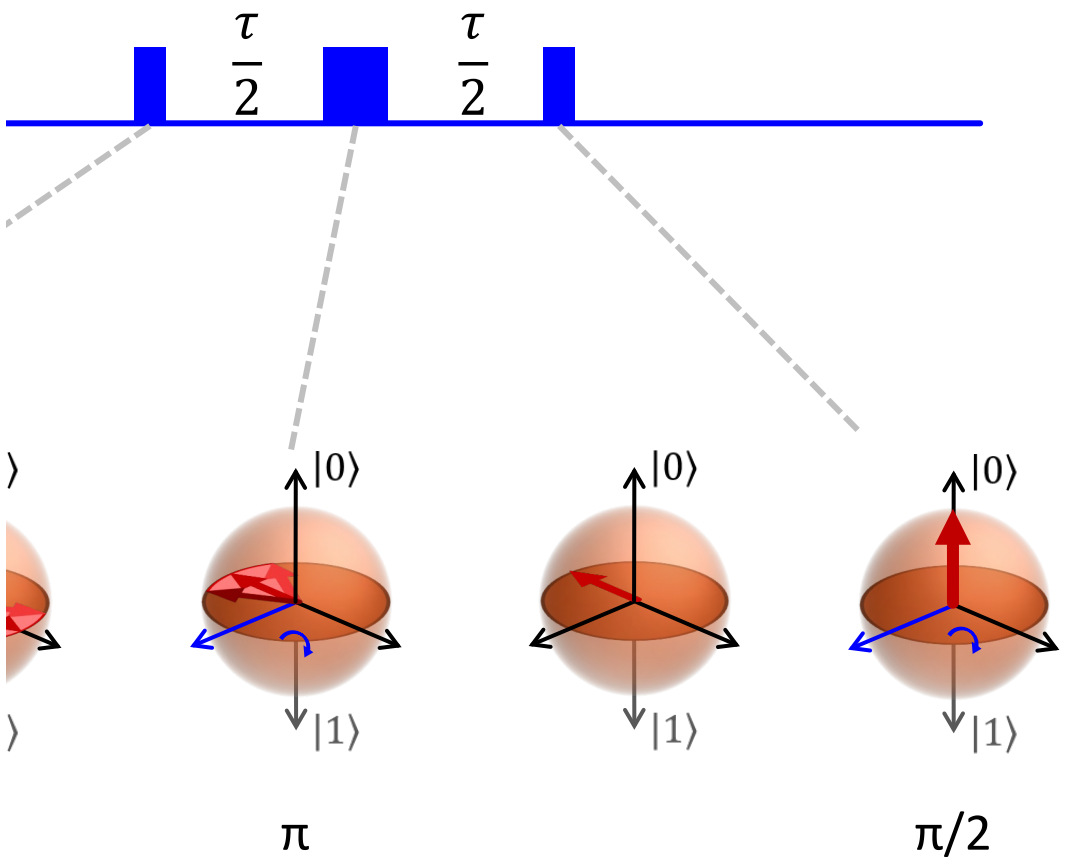


One more technique : Spin-echo

Spin echo



o pulse sequence



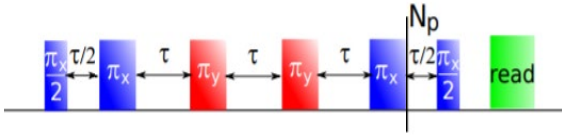
ː를 π 펄스를 이용하여 상쇄시켜줌

Slide : courtesy of J.Lee

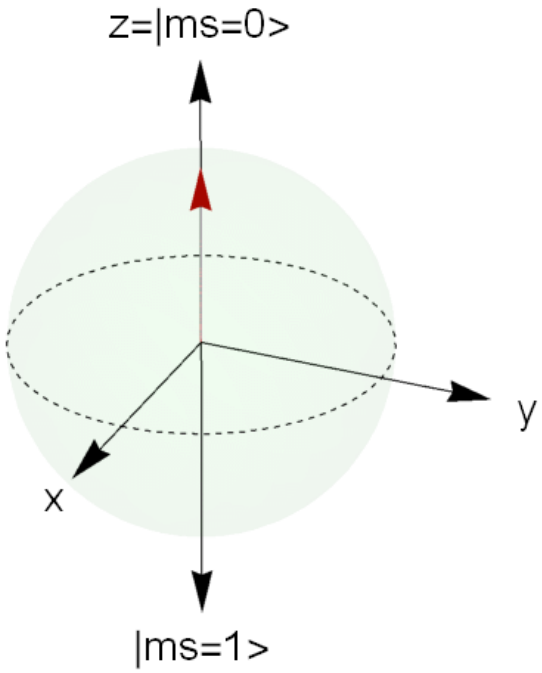
One more technique : Spin-echo

Ex: CPMG (Carr-Purcell-Mieboom-Gill) pulse

Cpmg sequence



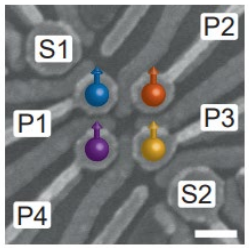
CPMG XY4

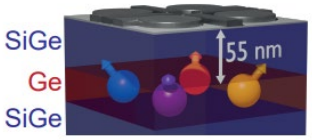


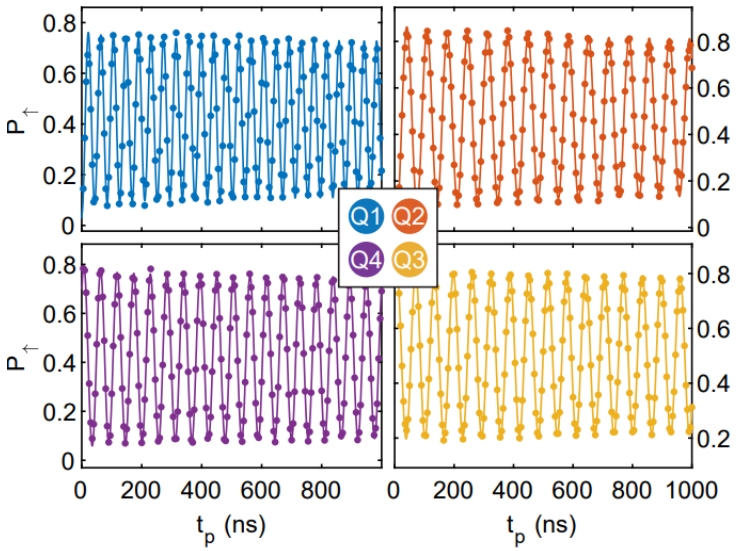
Dynamic decoupling 의 핵심 : dynamic decoupling pulse sequence 는 frequency filter, 이를 이용하면 environment 의 noise spectrum 측정 가능 – 좀 더 advanced course 에서…

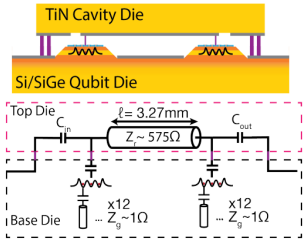
Example: Si, GaAs, Ge.. Boosting up results

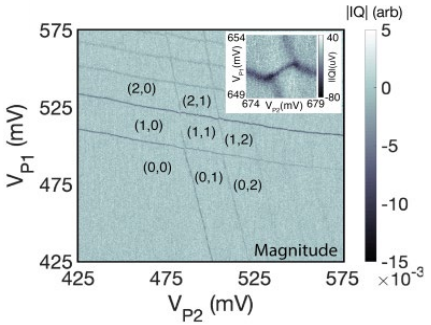
Most recent developments : Germanium 4 qubit processing & 3D integration

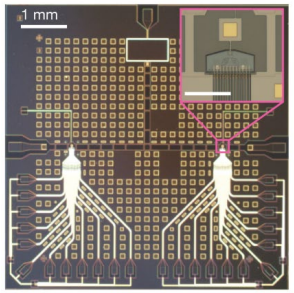


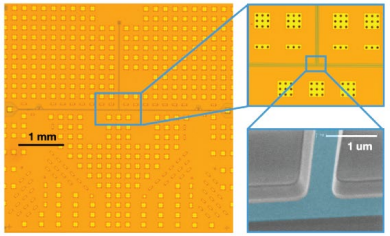


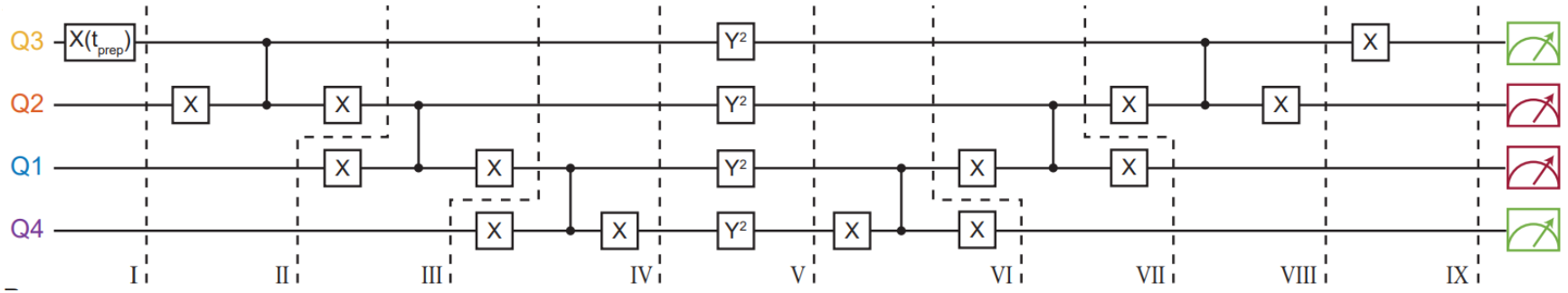








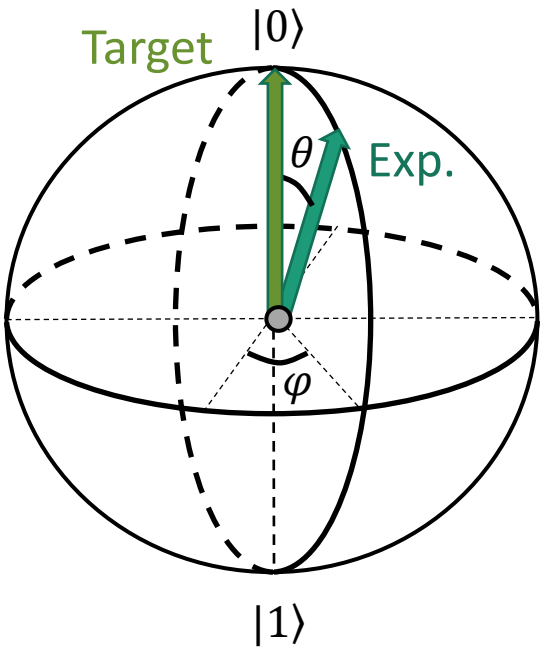


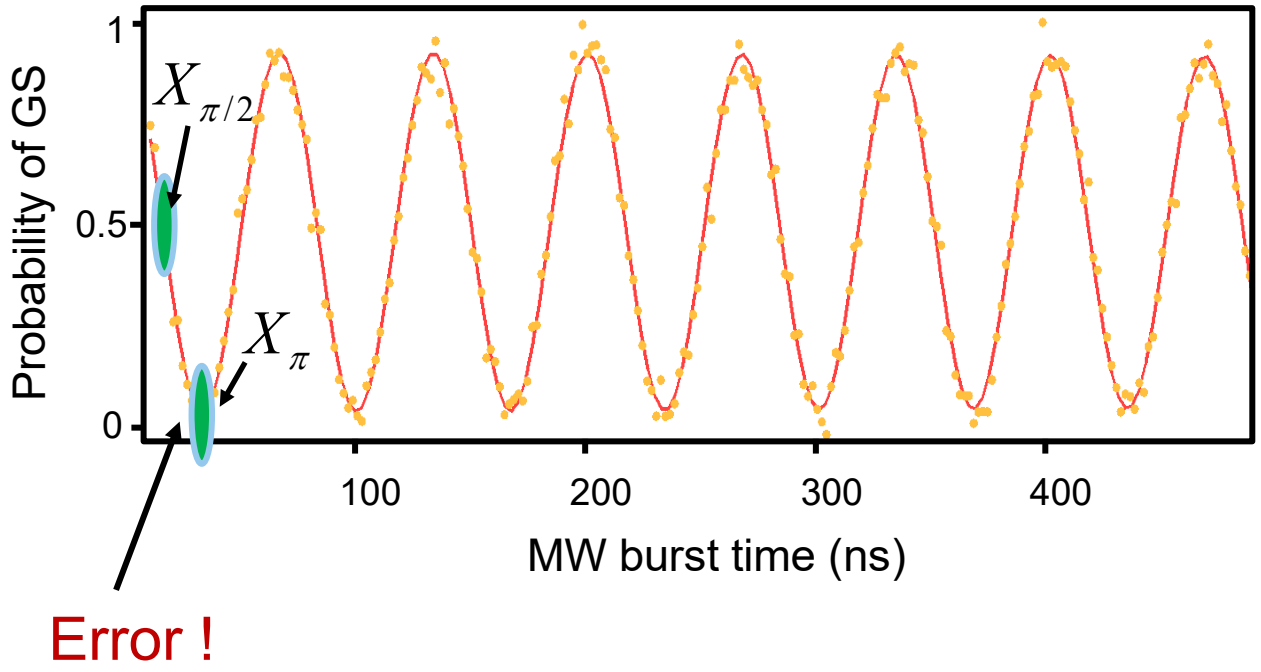


quTech, Delft Sep. 2020
M.Eriksson group UW Nov. 2020

Quantum Errors

Quantum gate set is discrete, but quantum control generating a quantum gate is inherently analog.





Quantum Errors

A general quantum error is a superoperator (Kraus operator):

$$\rho \to \sum A_k \rho A_k^{\dagger}$$

Examples of single-qubit errors:

Bit Flip X: $X|0\rangle = |1\rangle,\ X|1\rangle = |0\rangle$

Phase Flip Z: $Z|0\rangle = |0\rangle,\ Z|1\rangle = -|1\rangle$

Complete dephasing: $\rho \to 1/2(\rho + Z\rho Z^{\dagger})$
(decoherence)

Rotation: $R_0|0\rangle = |0\rangle,\ R_0|1\rangle = e^{i\theta}|1\rangle$

Classical Repetition Code

To correct a single bit-flip error for classical data,

we can use the repetition code:

$$0 \rightarrow 000$$

$$1 \rightarrow 111$$

If there is a single bit flip error, we can correct the state by choosing the majority of the three bits, e.g. $010 \rightarrow 0$.

When errors are rare, one error is more likely than two.

Barriers to Quantum Error Correction

1. Measurement of error destroys superpositions.
2. No-cloning theorem prevents repetition.
3. Must correct multiple types of errors (e.g., bit flip and phase errors).
4. How can we correct continuous errors and decoherence?

Measurement Destroys Superpositions?

Let us apply the classical repetition code to a quantum state to try to correct a bit flip error:

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$$

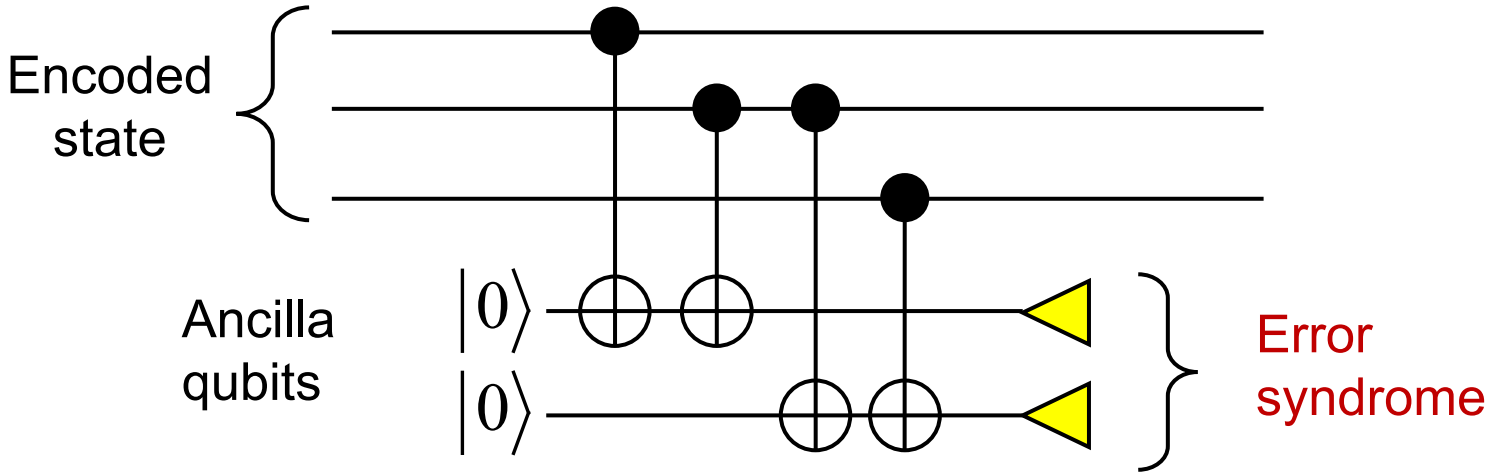
Bit flip error (X) on 2nd qubit:

$$\alpha|010\rangle + \beta|101\rangle$$

2nd qubit is now different from 1st and 3rd. We wish to measure that it is different without finding its actual value.

Measure the Error, Not the Data

Use this circuit:



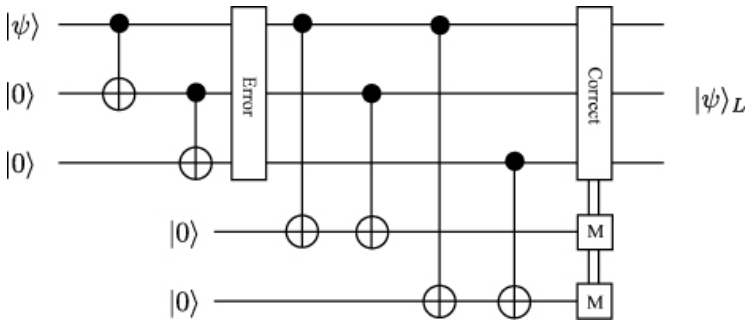
1st bit of error syndrome says whether the first two bits of the state are the same or different.

2nd bit of error syndrome says whether the second two bits of the state are the same or different.

Redundancy, Not Repetition

This encoding does not violate the no-cloning theorem:

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$$
$$\neq (\alpha|0\rangle + \beta|1\rangle)^{\otimes 3}$$



We have repeated the state only in the computational basis; superposition states are spread out (redundant encoding), but not repeated (which would violate no-cloning).

Update on the Problems

✓ 1. Measurement of error destroys superpositions.

✓ 2. No-cloning theorem prevents repetition.

3. Must correct multiple types of errors (e.g., bit flip and phase errors).

4. How can we correct continuous errors and decoherence?

Correcting Just Phase Errors

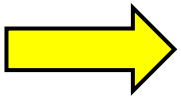
Hadamard transform H exchanges bit flip and phase errors:

$$H(\alpha|0\rangle + \beta|1\rangle) = \alpha|+\rangle + \beta|-\rangle$$

$$X|+\rangle = |+\rangle, X|-\rangle = -|-\rangle \quad \text{(acts like phase flip)}$$

$$Z|+\rangle = |-\rangle, Z|-\rangle = |+\rangle \quad \text{(acts like bit flip)}$$

Repetition code corrects a bit flip error



Repetition code in Hadamard basis corrects a phase error.

$$\alpha|+\rangle + \beta|-\rangle \rightarrow \alpha|+++\rangle + \beta|---\rangle$$

Nine-Qubit Code

To correct both bit flips and phase flips, use both codes at once:

$$\alpha|0\rangle+\beta|1\rangle\rightarrow$$
$$\alpha(|000\rangle+|111\rangle)^{\otimes 3}+\beta(|000\rangle-|111\rangle)^{\otimes 3}$$

Repetition 000, 111 corrects a bit flip error,

repetition of phase +++, --- corrects a phase error

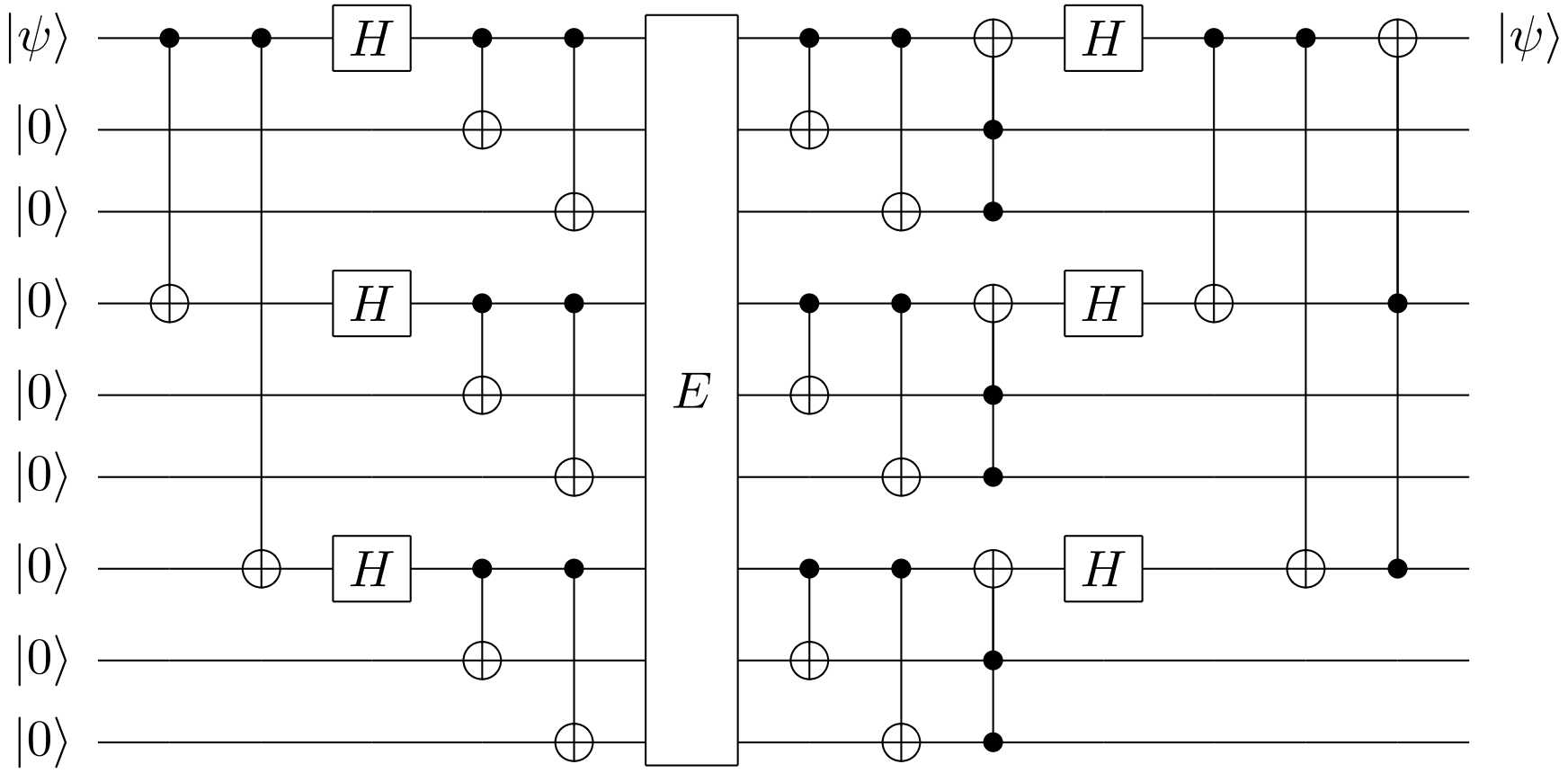
Actually, this code corrects a bit flip and a phase, so it also corrects a Y error:

$$Y=iXZ;\ Y|0\rangle=i|1\rangle,\ Y|1\rangle=-i|0\rangle$$

(global phase irrelevant)

Nine-Qubit Code

Circuit for the nine-qubit code



Update on the Problems

✓ 1. Measurement of error destroys superpositions.

✓ 2. No-cloning theorem prevents repetition.

✓ 3. Must correct multiple types of errors (e.g., bit flip and phase errors).

4. How can we correct continuous errors and decoherence?

Correcting Continuous Rotation

Let us rewrite continuous rotation

$$R_\theta |0\rangle = |0\rangle, R_\theta |1\rangle = e^{i\theta} |1\rangle$$

$$R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta/2} \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

$$= \cos(\theta/2)\, I - i \sin(\theta/2)\, Z$$

$$R_\theta^{(k)} |\psi\rangle = \cos(\theta/2) |\psi\rangle - i \sin(\theta/2) Z^{(k)} |\psi\rangle$$

($R_\theta^{(k)}$ is R_θ acting on the kth qubit.)

Correcting Continuous Rotations

How does error correction affect a state with a continuous rotation on it?

$$R_\theta^{(k)} \left|\psi\right\rangle = \cos(\theta/2)\left|\psi\right\rangle - i\sin(\theta/2) Z^{(k)} \left|\psi\right\rangle$$

$$\longrightarrow \quad \cos(\theta/2)\left|\psi\right\rangle\left|I\right\rangle - i\sin(\theta/2) Z^{(k)} \left|\psi\right\rangle\left|Z^{(k)}\right\rangle$$

Error syndrome

Measuring the error syndrome collapses the state:

$$\text{Prob. } \cos^2(\theta/2): \ \left|\psi\right\rangle \text{ (no correction needed)}$$

$$\text{Prob. } \sin^2(\theta/2): \ Z^{(k)}\left|\psi\right\rangle \text{ (corrected with } Z^{(k)}\text{)}$$

Correcting All Single-Qubit Errors

Theorem: If a quantum error-correcting code (QECC) corrects errors A and B, it also corrects $\alpha A + \beta B$.

Any 2x2 matrix can be written as $\alpha I + \beta X + \gamma Y + \delta Z$.

A general single-qubit error $\rho \to \sum A_k \rho A_k^{\dagger}$ acts like a mixture of $|\psi\rangle \to A_k |\psi\rangle$, and A_k is a 2x2 matrix.

Any QECC that corrects the single-qubit errors X, Y, and Z (plus I) corrects every single-qubit error.

Correcting all t-qubit X, Y, Z on t qubits (plus I) corrects all t-qubit errors.

Small Error on Every Qubit

Suppose we have a small error U_ε on every qubit in the QECC,

where $U_\varepsilon = I + \varepsilon E.$

Then

$$U_\varepsilon^{\otimes n} |\psi\rangle = |\psi\rangle + \varepsilon(E^{(1)} + \ldots + E^{(n)})|\psi\rangle + O(\varepsilon^2).$$

If the code corrects one-qubit errors, it corrects the sum of the $E^{(i)}$s. Therefore it corrects the $O(\varepsilon)$ term, and the state remains correct to order ε^2.

A code correcting t errors keeps the state correct to order ε^{t+1}.

QECC is Possible

✓ 1. Measurement of error destroys superpositions.

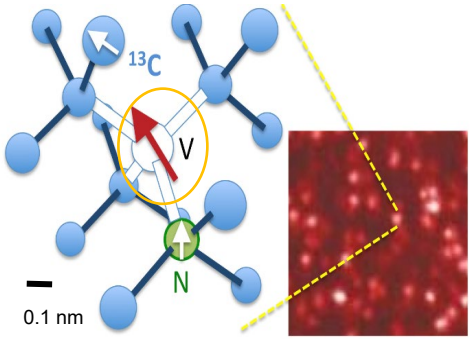
✓ 2. No-cloning theorem prevents repetition.

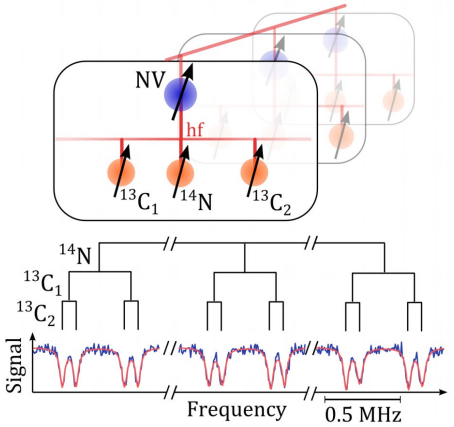
✓ 3. Must correct multiple types of errors (e.g., bit flip and phase errors).

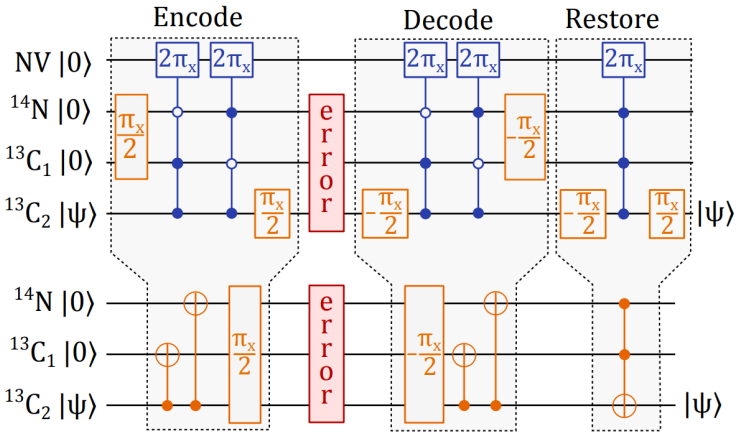
✓ 4. How can we correct continuous errors and decoherence?

Formally, stabilizer formalism for constructing & designing error correction codes

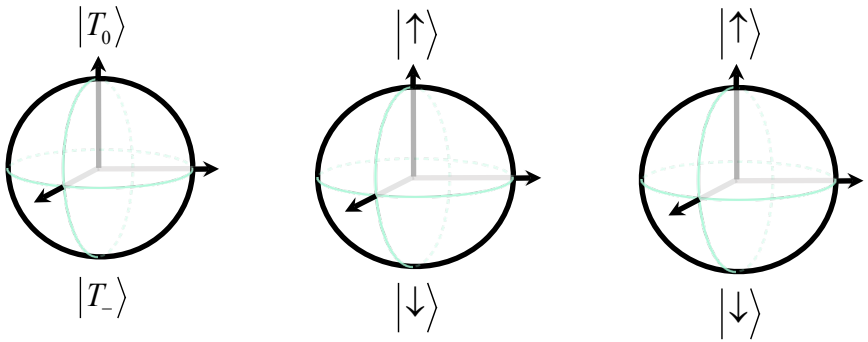
Error correction example : Diamond NV center

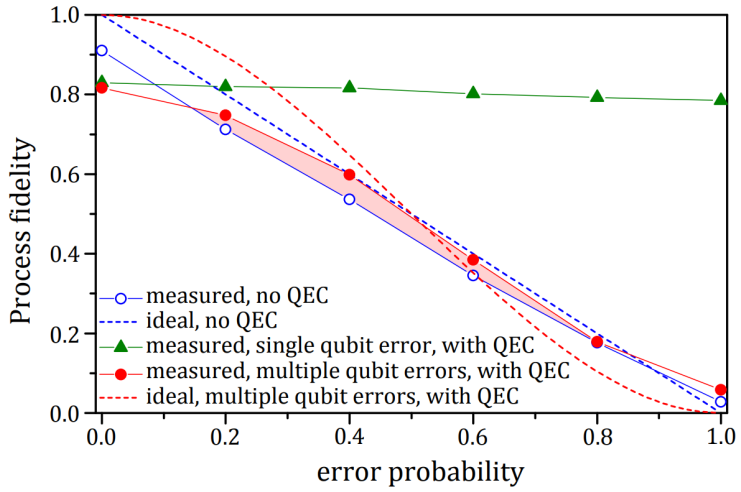






전자 스핀큐빗 질소 핵스핀큐빗 ^{13}C 핵스핀큐빗

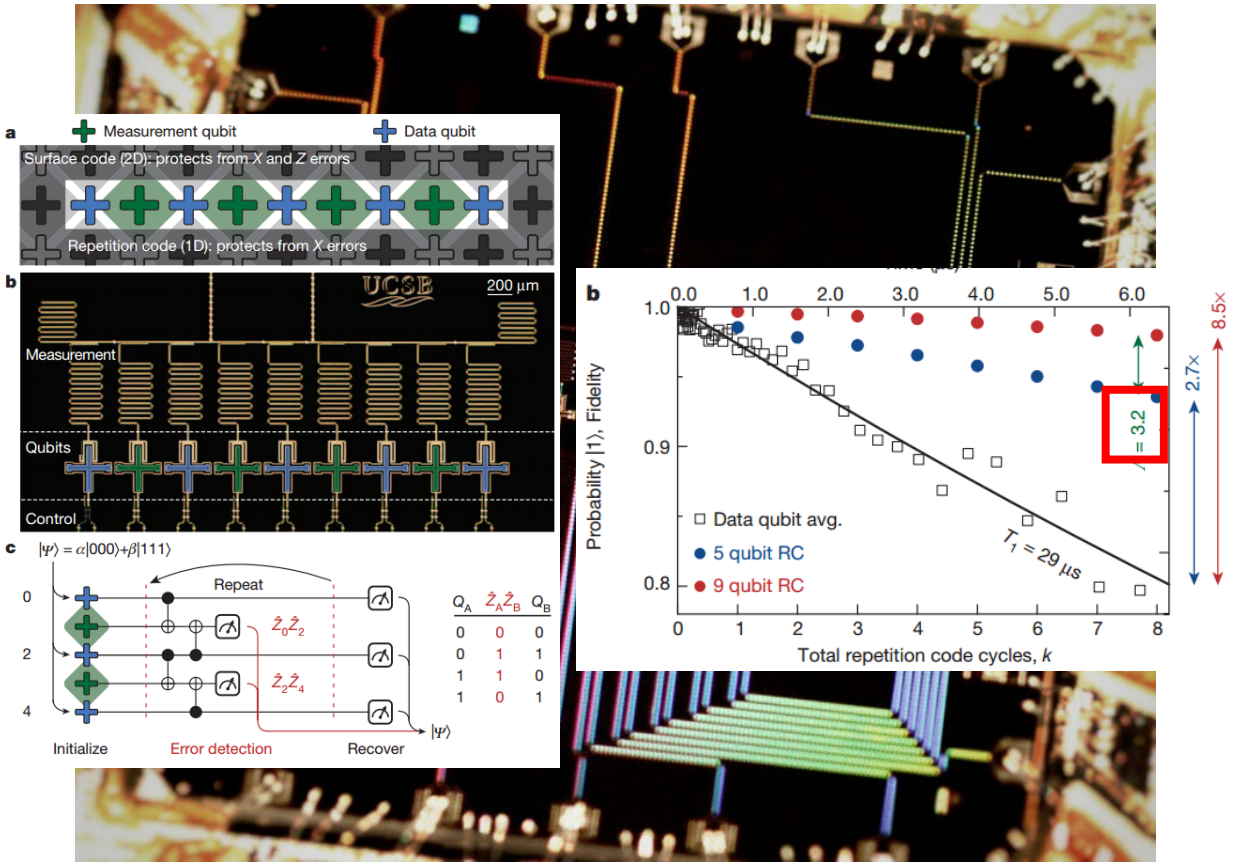




No syndrome measurement, but proof-of-principle

관련문제 : 6

Error correction example : Transmon

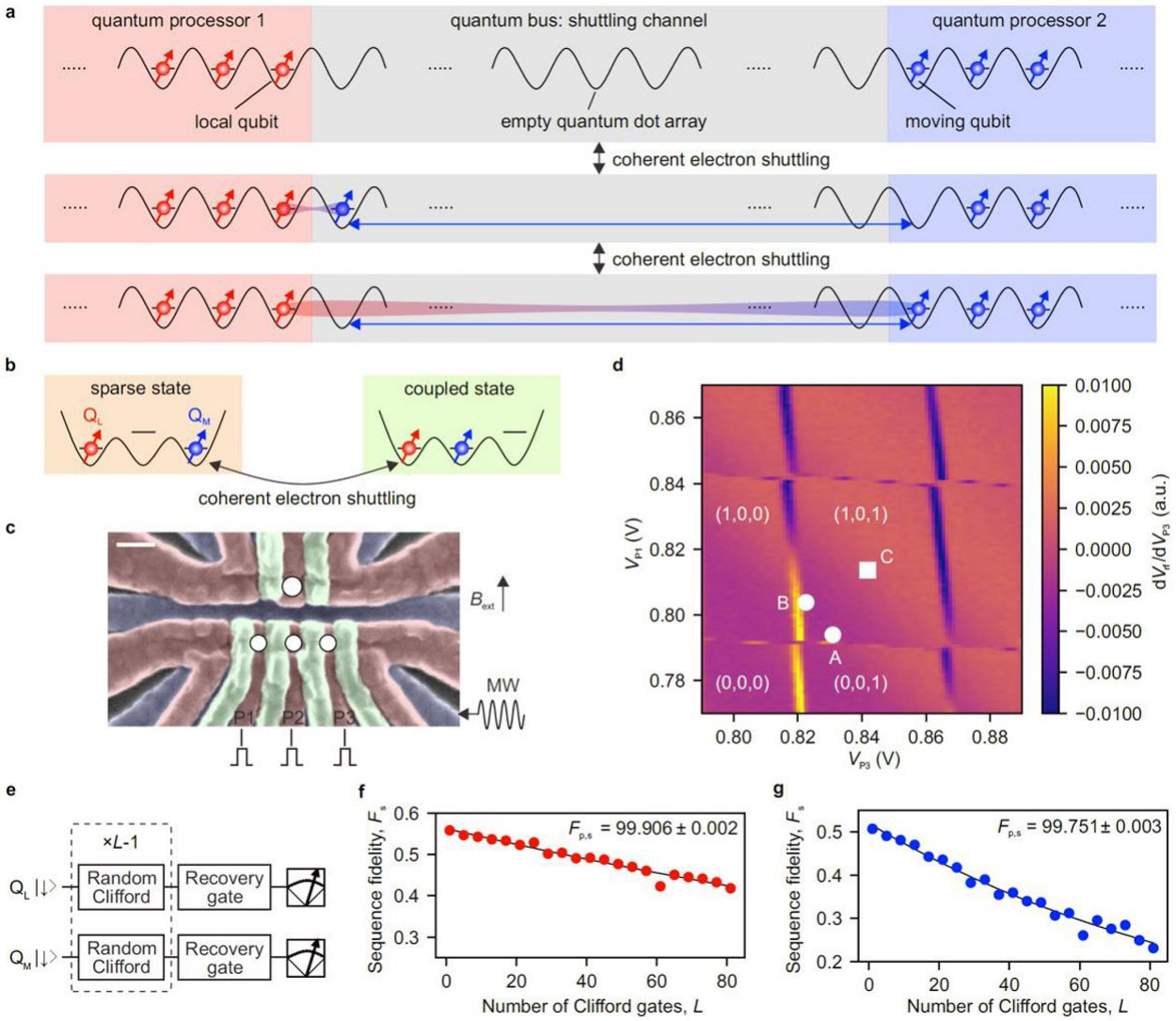


State preservation by repetitive measurements

- John Martinis – UCSB, Google
- 9 coupled **linear** transmon array
- Surface code error correction demonstrated (**X flip** error)
- X,Z gate time ~ 10ns
- CPHASE gate time ~ 100ns
- T_2~ 10μs
- 1Q gate error < 0.2%
- 2Q gate error < 1%
- 2Q gate enabled by direct capacitive coupling
- Frequency multiplexed single-shot dispersive readout

Recent developments

Coherent Shuttling

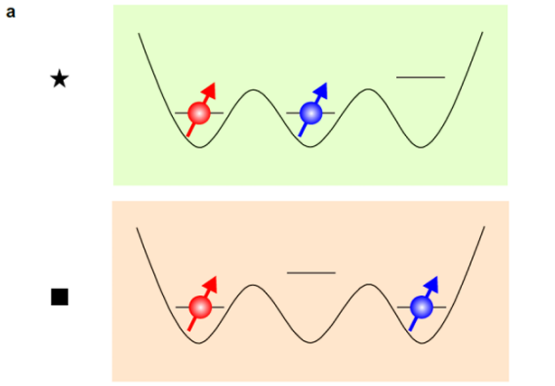


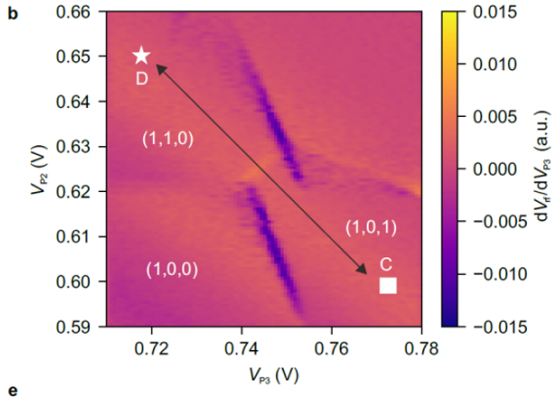
https://arxiv.org/abs/2202.01357

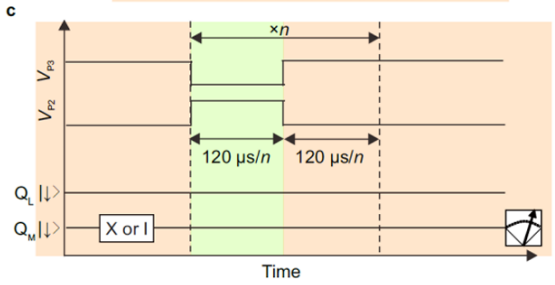
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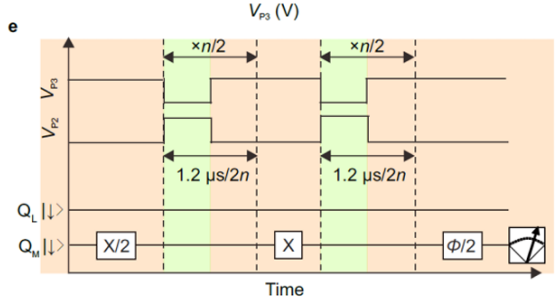
Recent developments

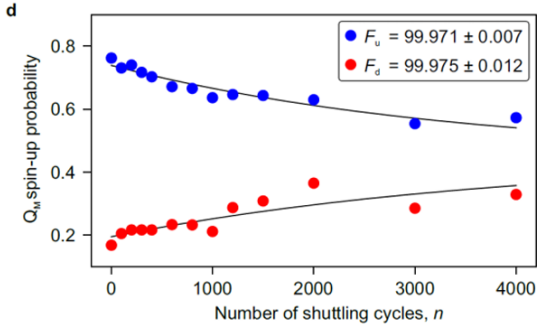
Coherent Shuttling

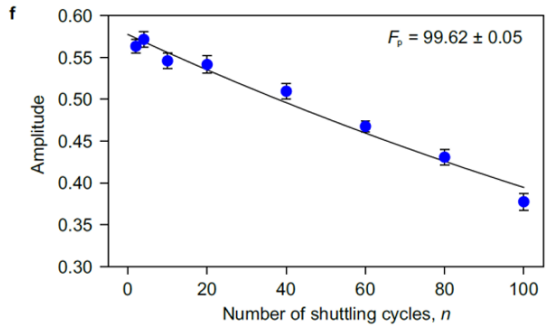


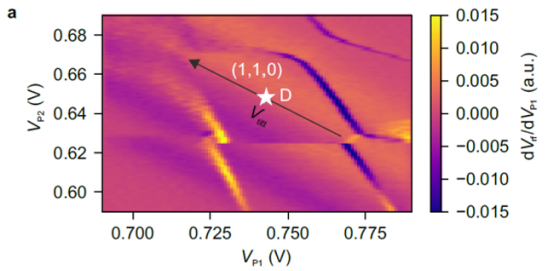


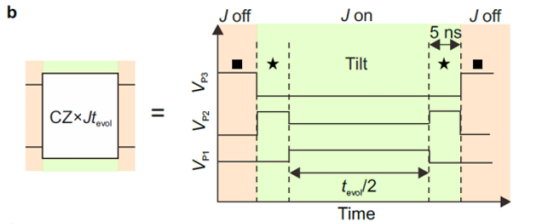


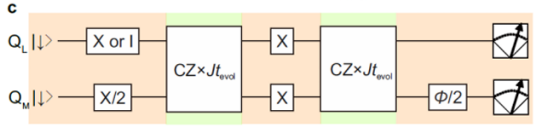


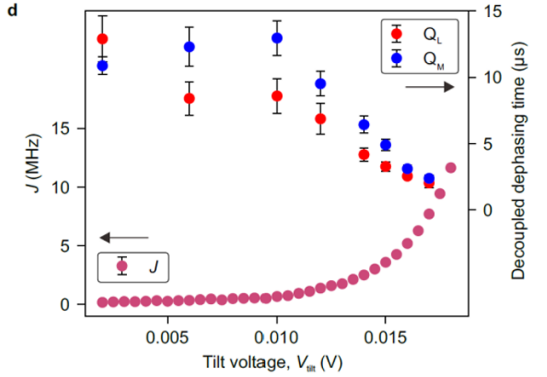






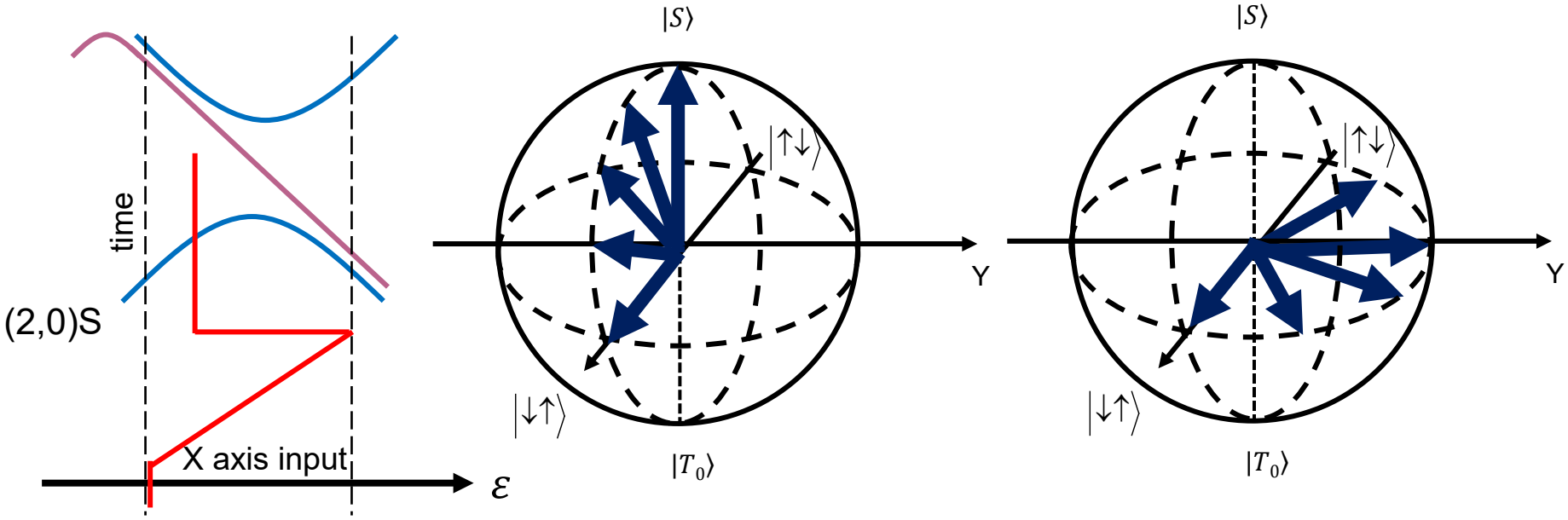






https://arxiv.org/abs/2202.01357

Other than shuttling : spin exchange without moving

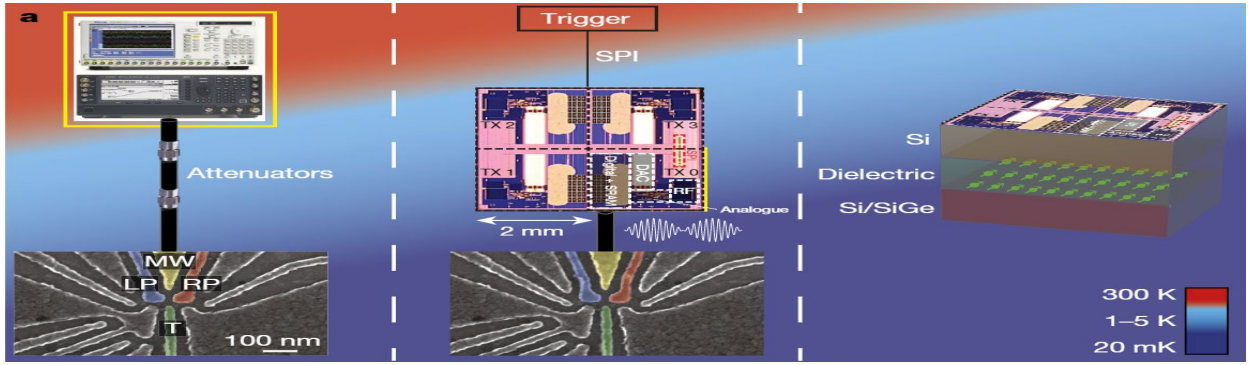


few K operation = Integration with Classical CMOS

Article

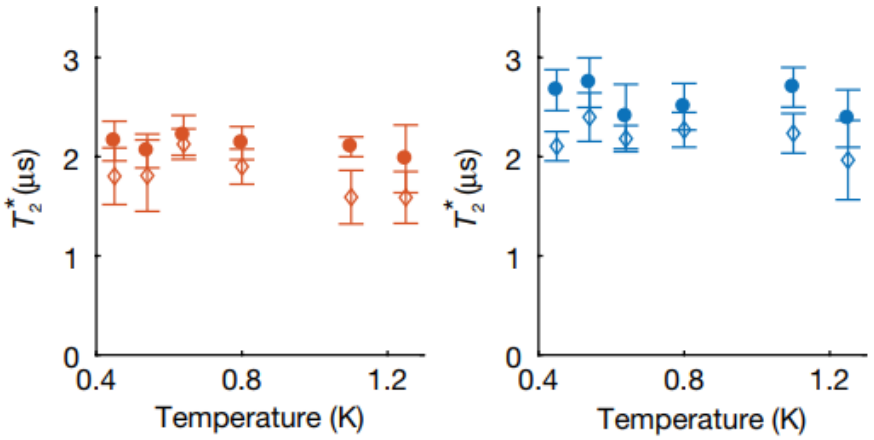
Universal quantum logic in hot silicon qubits

L. Petit et al, *Nature* **580**, 355 (2020)



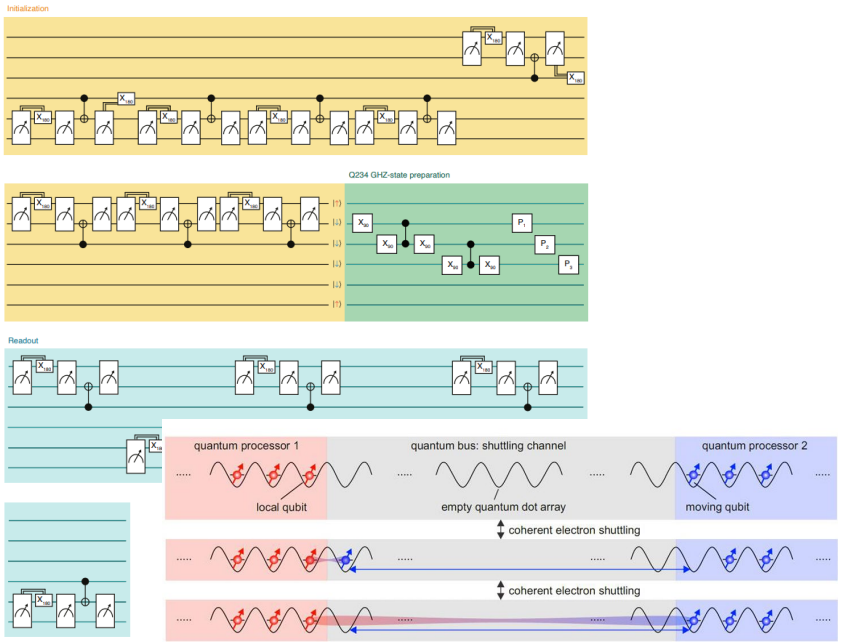
QuTech / Intel.



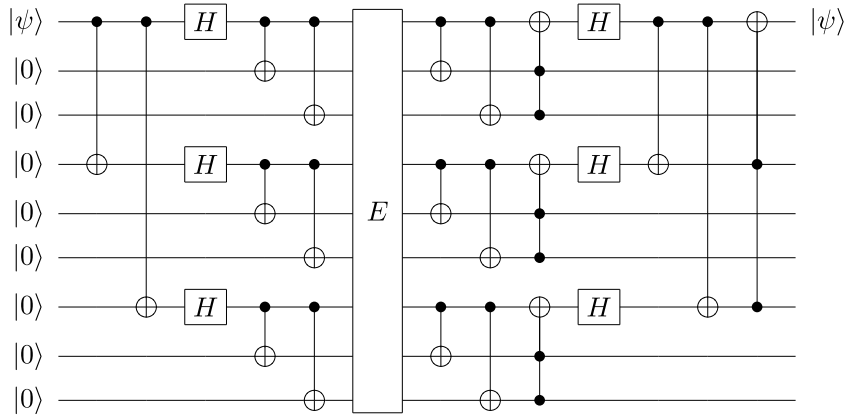


Summary of Lecture 3

Multi qubit operation : 종합선물세트



Decoherence : open system



We actually need many, many qubits for practical QC.

결론: 반도체 양자컴퓨팅 – 어렵지만 promising developments