Extracting Information from the Topology of Large Scale Structure

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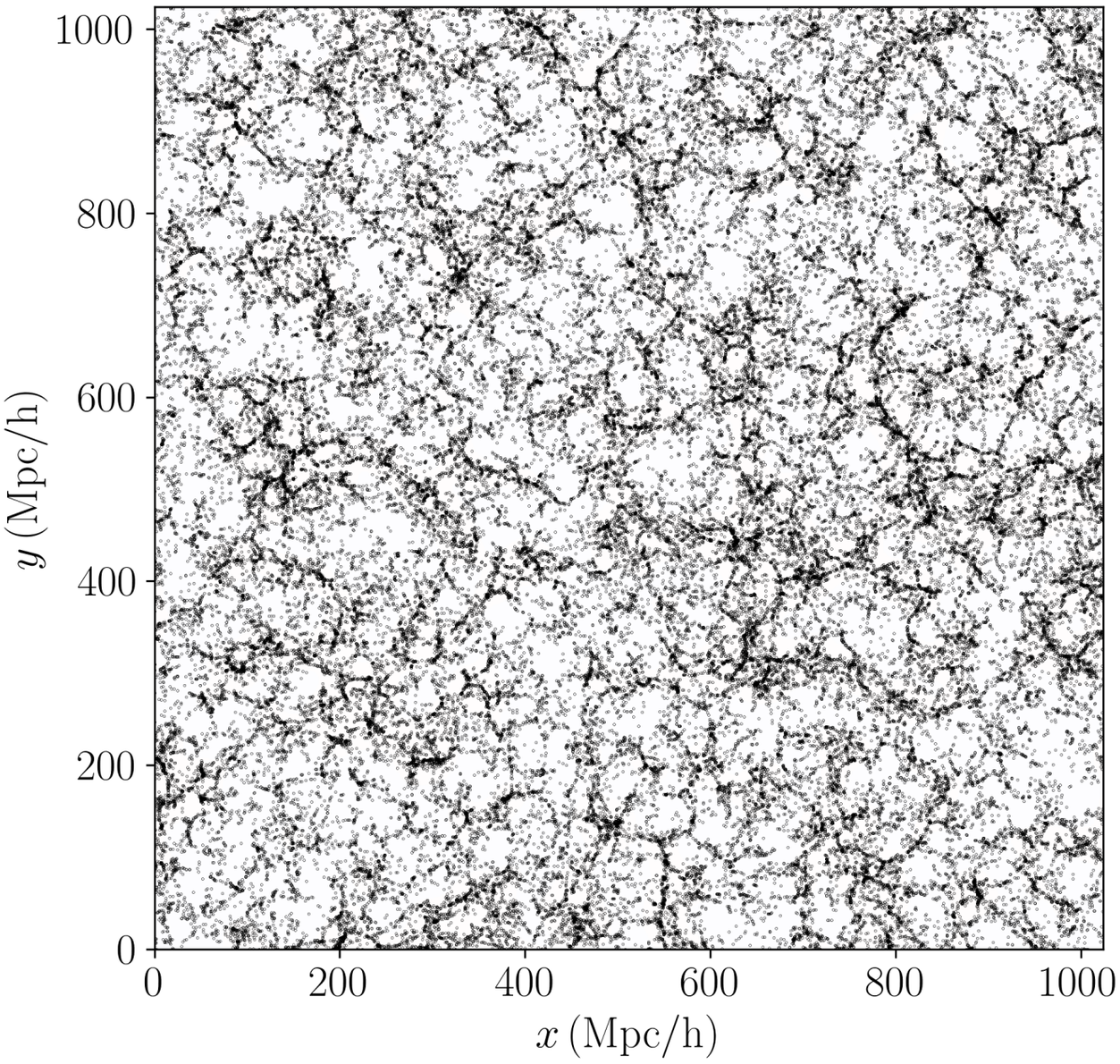
Definition of Genus

- Take a two-dimensional slice (shell) of galaxy data — point distribution.
- Generate a continuous density field by smoothing.
- Create a boundary of constant field value (in two dimensions, this is a one-dimensional perimeter)
- Count the number of connected regions, and the number of holes.



Genus = number of connected regions - number of holes

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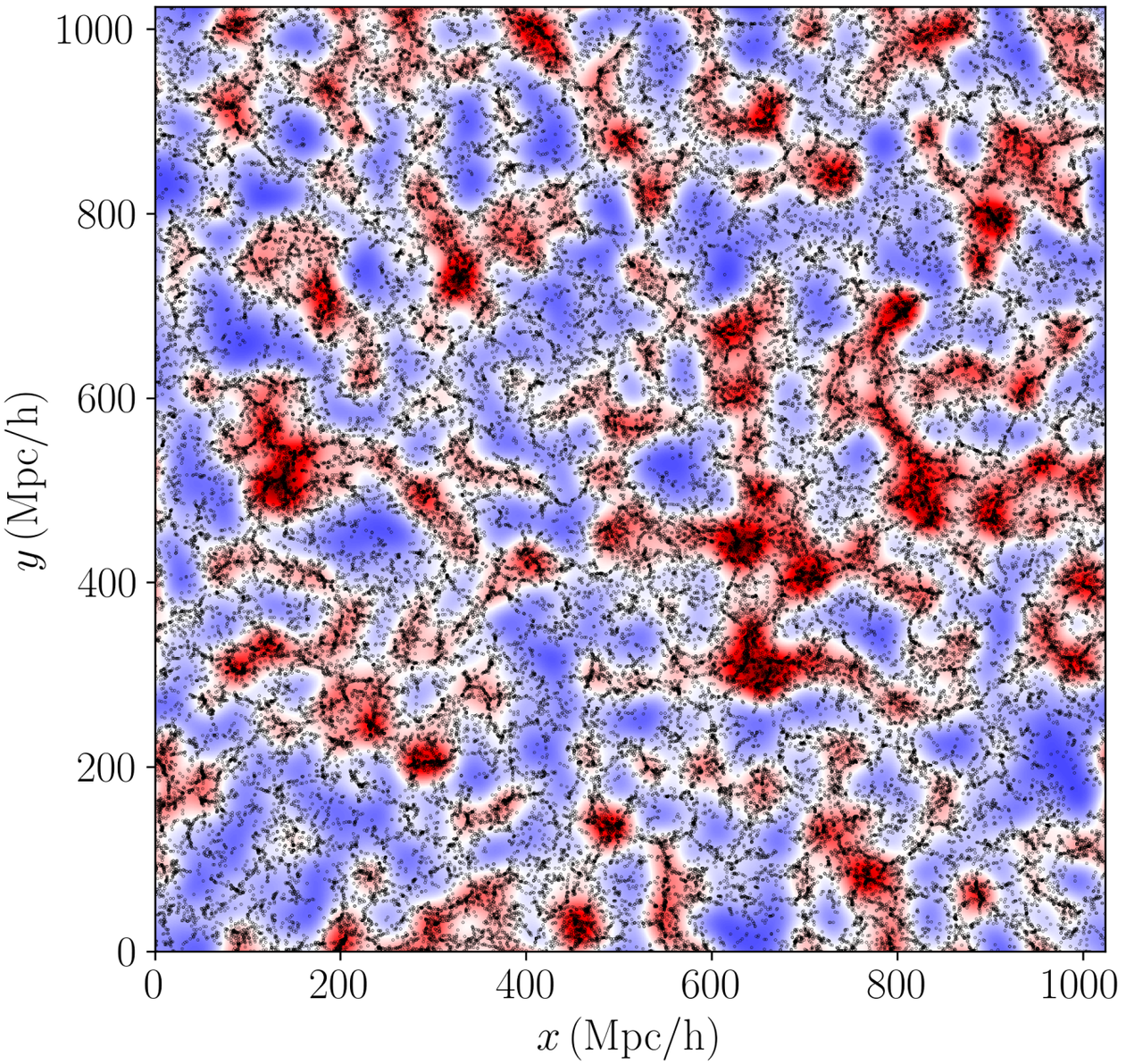

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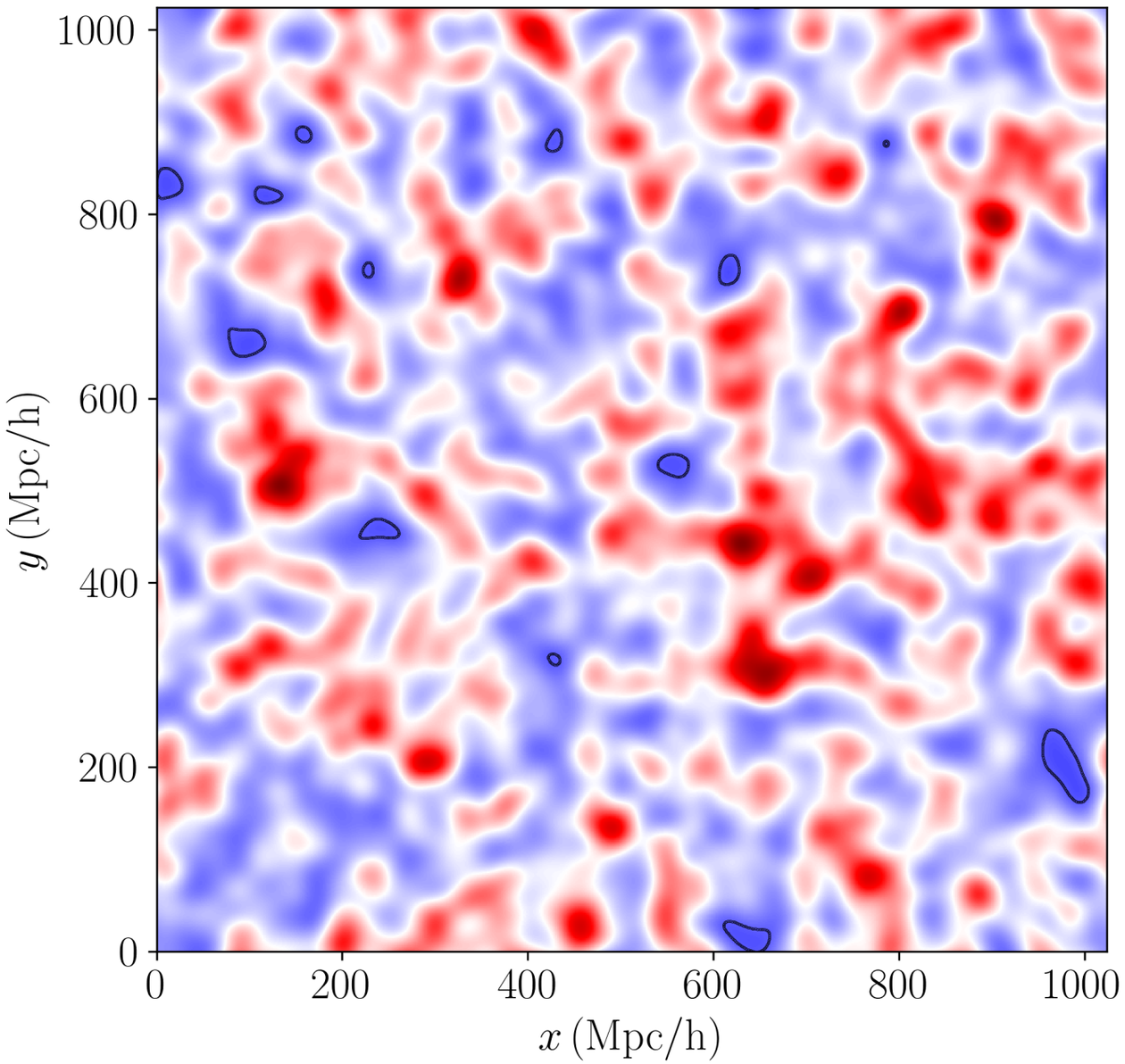

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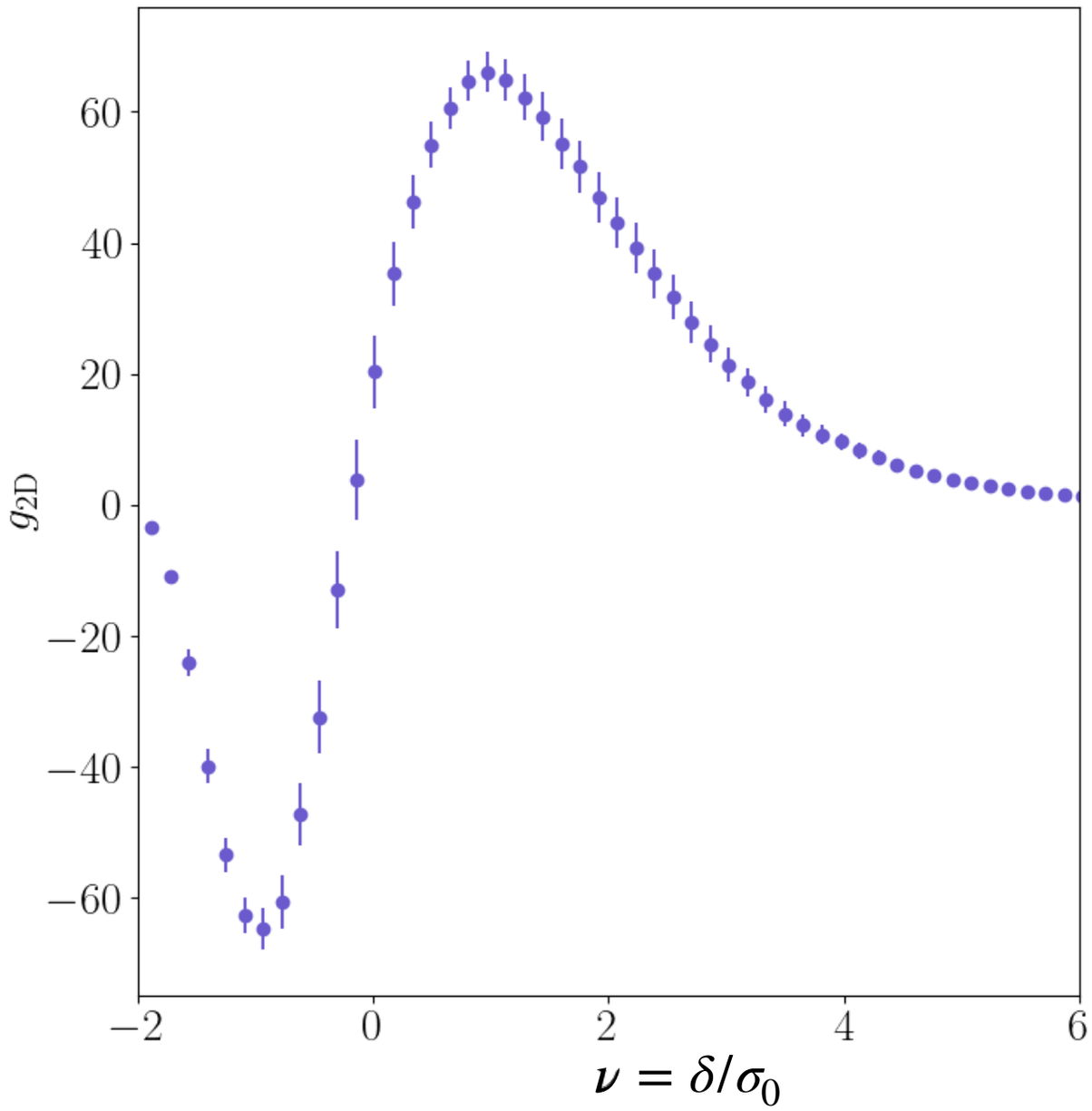


Genus = number of connected regions - number of holes

Theoretical Prediction — Non-Gaussian Field

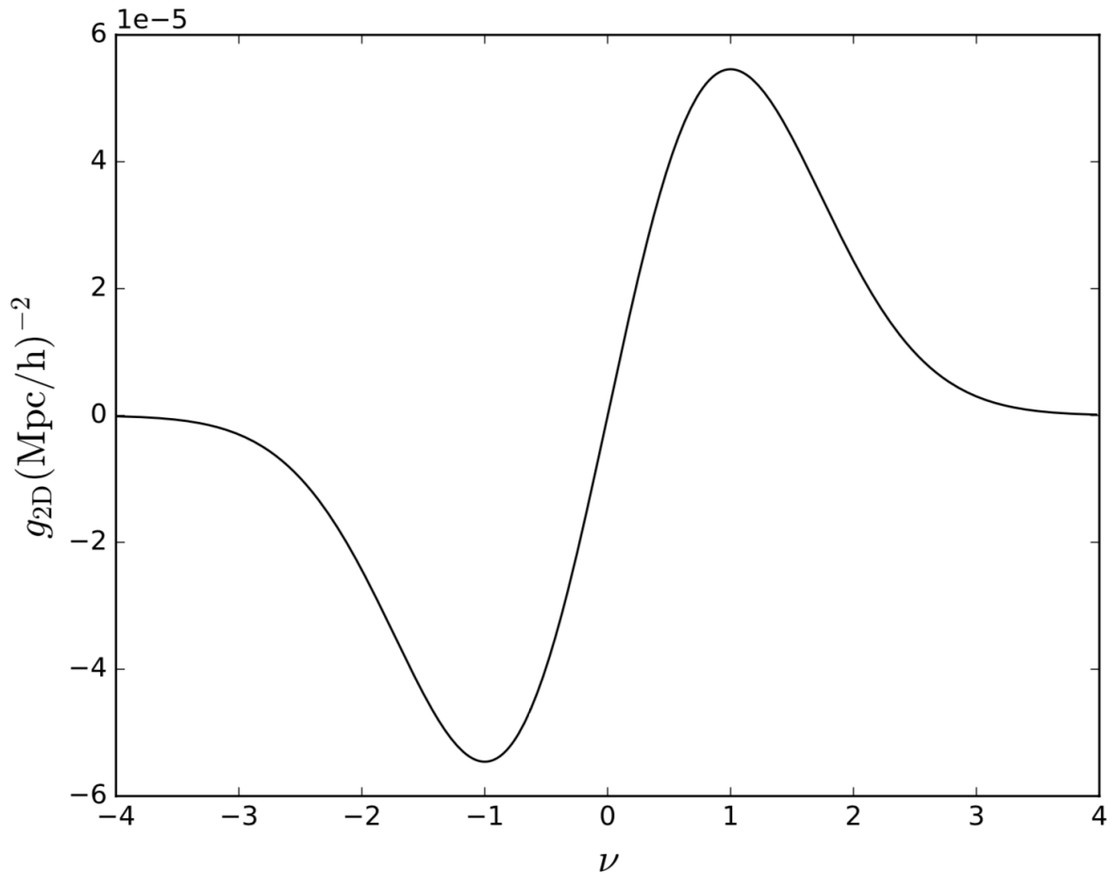
- Non-Gaussian corrections (due to late time gravitational collapse) can modify both the shape and amplitude of the genus

$$g_{2\mathrm{D}}(\nu) = \frac{1}{2(2\pi)^{3/2}} \frac{\sigma_1^2}{\sigma_0^2} e^{-\nu_\mathrm{A}^2/2} \left[H_1(\nu_\mathrm{A}) + \left(\frac{2}{3}[S^{(1)} - S^{(0)}] H_2(\nu_\mathrm{A}) + \frac{1}{3}[S^{(2)} - S^{(0)}] H_0(\nu_\mathrm{A}) \right) \sigma_0 + \mathcal{O}(\sigma_0^2) \right]$$

(Matsubara, 2003)

The genus amplitude is unaffected by non-linear gravitational collapse, to linear order in the σ_0 expansion

For a non-Gaussian field, the shape of the genus curve contains information regarding the shape of the bispectrum (not used here)



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Amplitude

$$\sigma_0^2 = \frac{1}{(2\pi)^2} \int d^2\mathbf{k}_\perp P_{2D}(k_\perp) e^{-k_\perp^2 R_{\mathrm{G}}^2}$$

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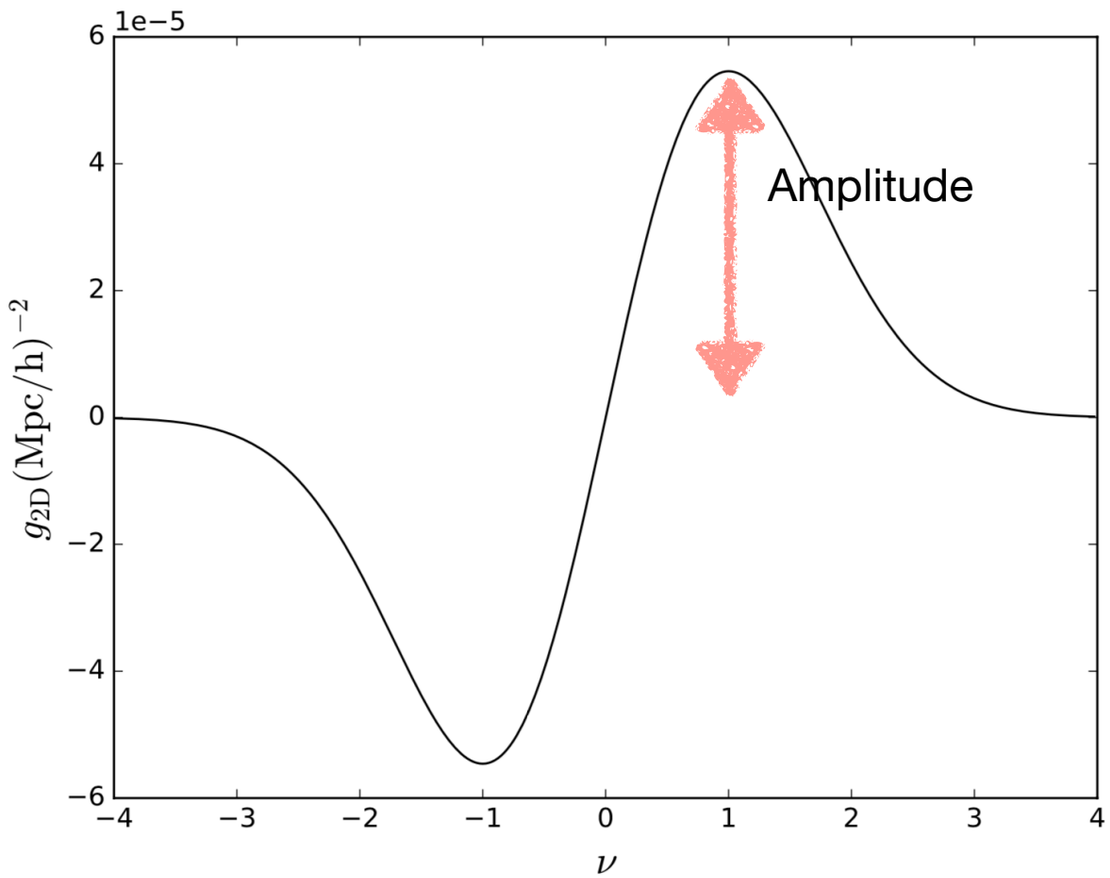

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Shape

$$S^{(0)} = \frac{\langle \delta_{2D}^3 \rangle}{\sigma_0^4}$$

$$S^{(1)} = -\frac{3}{4} \frac{\langle \delta_{2D}^2 \nabla^2 \delta_{2D} \rangle}{\sigma_0^2 \sigma_1^2}$$

$$S^{(2)} = -3 \frac{\langle (\nabla \delta_{2D} \cdot \nabla \delta_{2D}) \nabla^2 \delta_{2D} \rangle}{\sigma_1^4}$$

Amplitude

$$\sigma_0^2 = \frac{1}{(2\pi)^2} \int d^2\mathbf{k}_\perp P_{2D}(k_\perp) e^{-k_\perp^2 R_{\mathrm{G}}^2}$$

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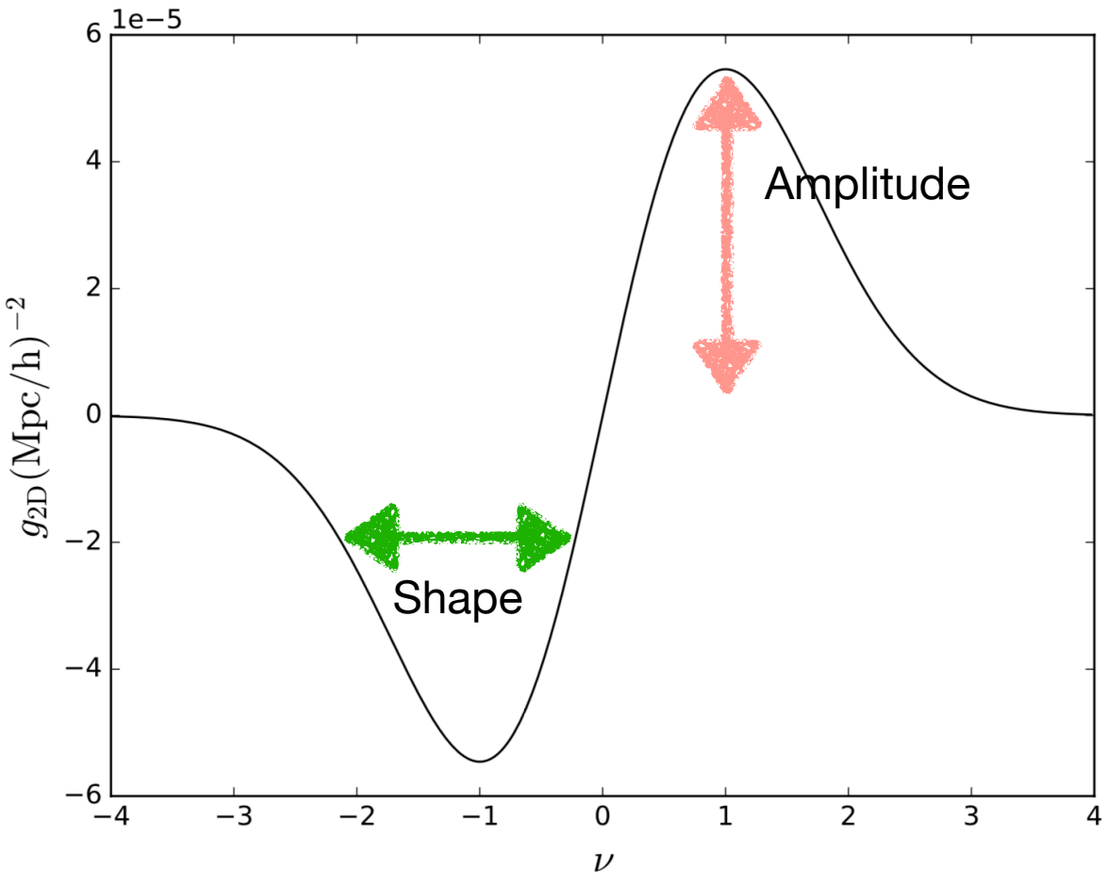

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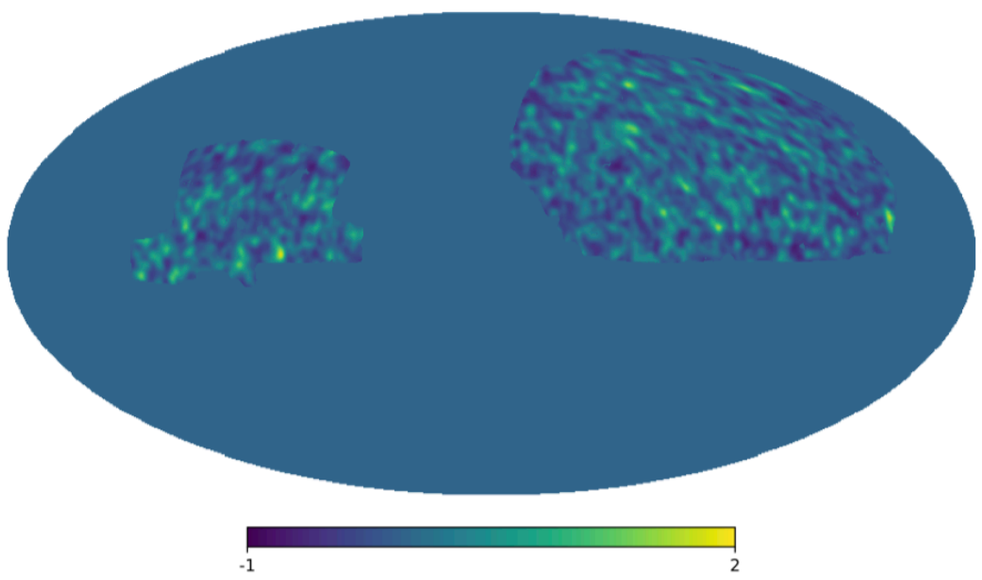
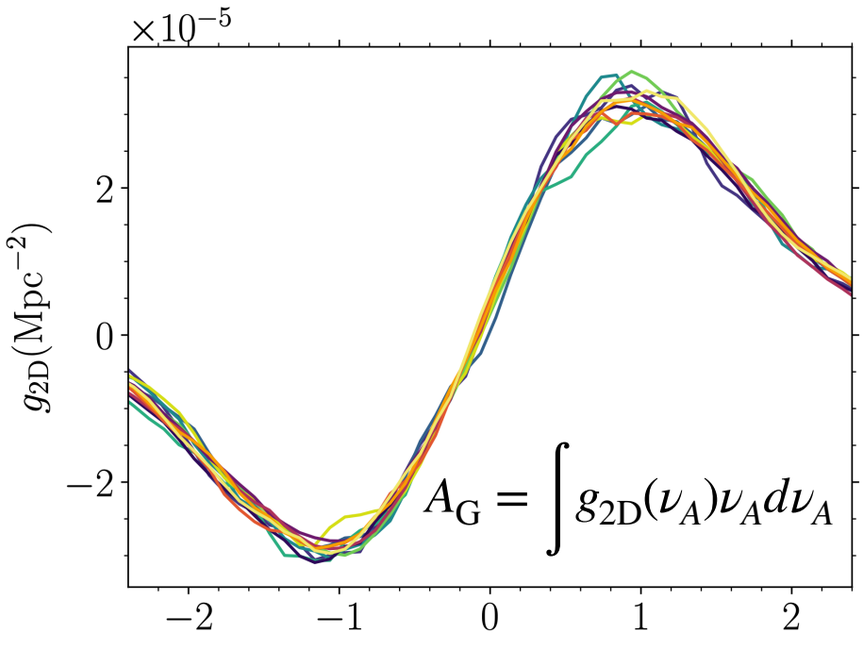
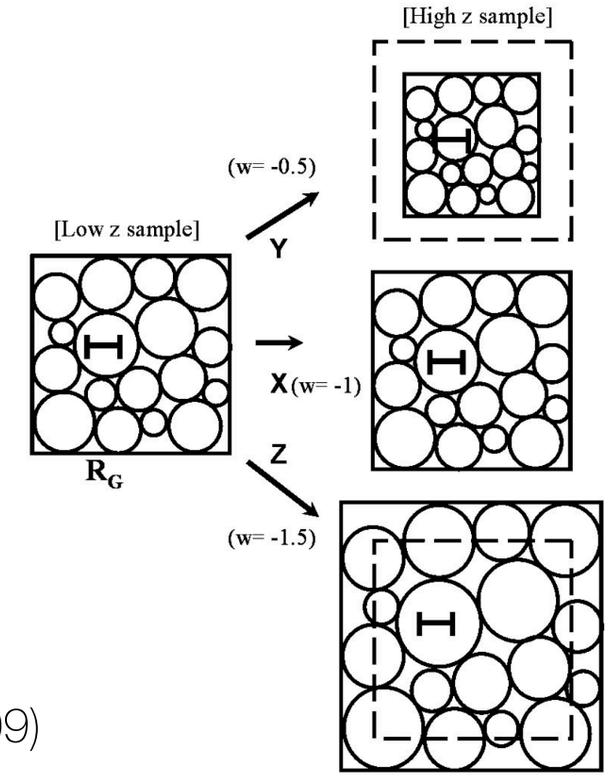
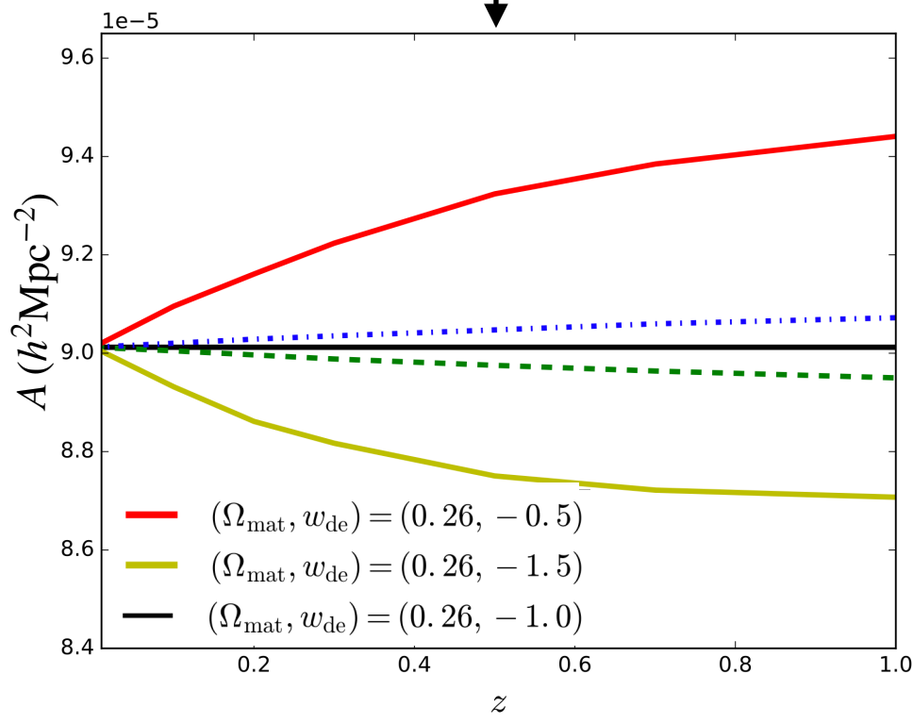
Information Extraction from the genus curve

- The genus amplitude should be conserved with redshift — find the expansion rate that minimizes the redshift dependence of the amplitude







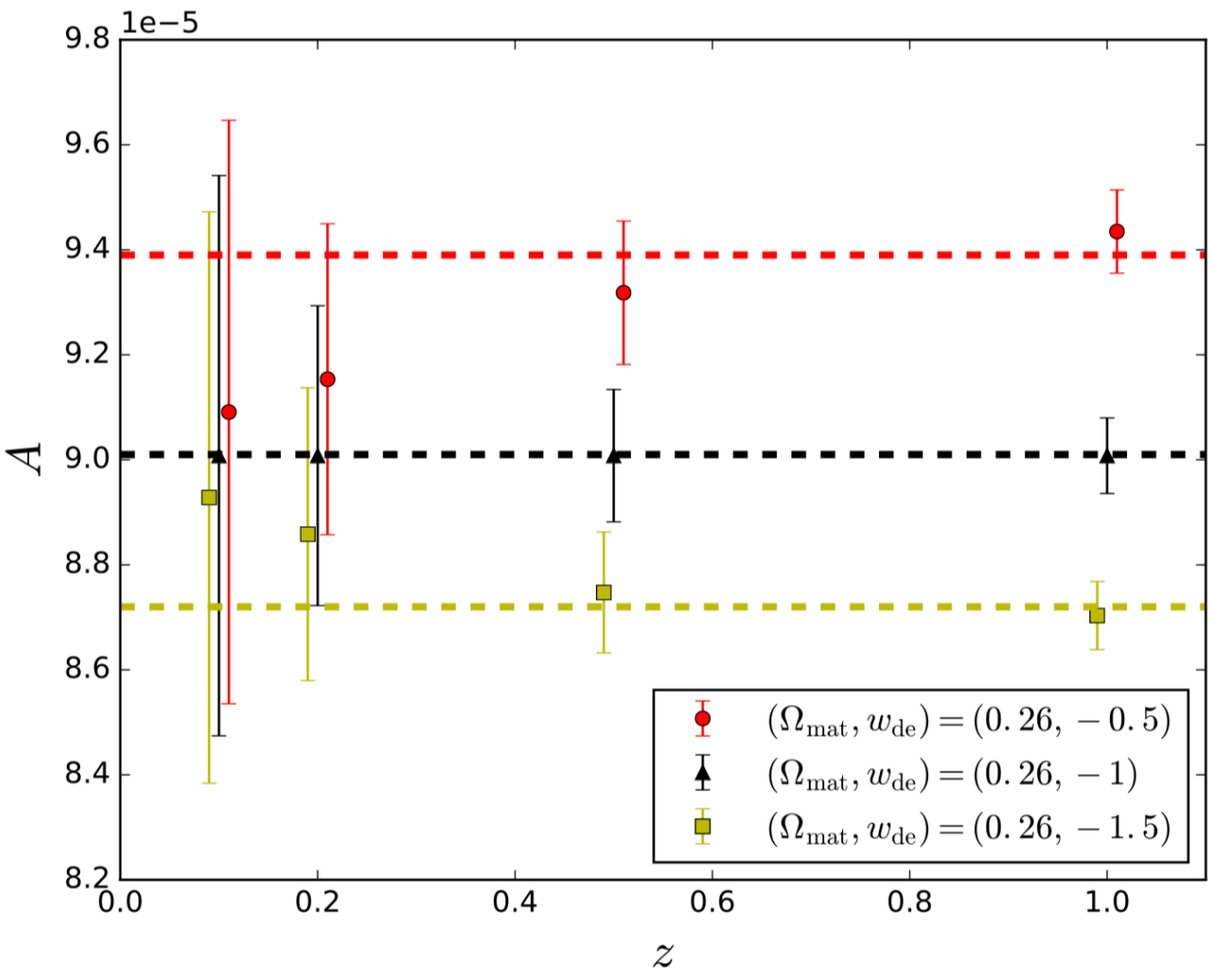


Park & Kim (2009)

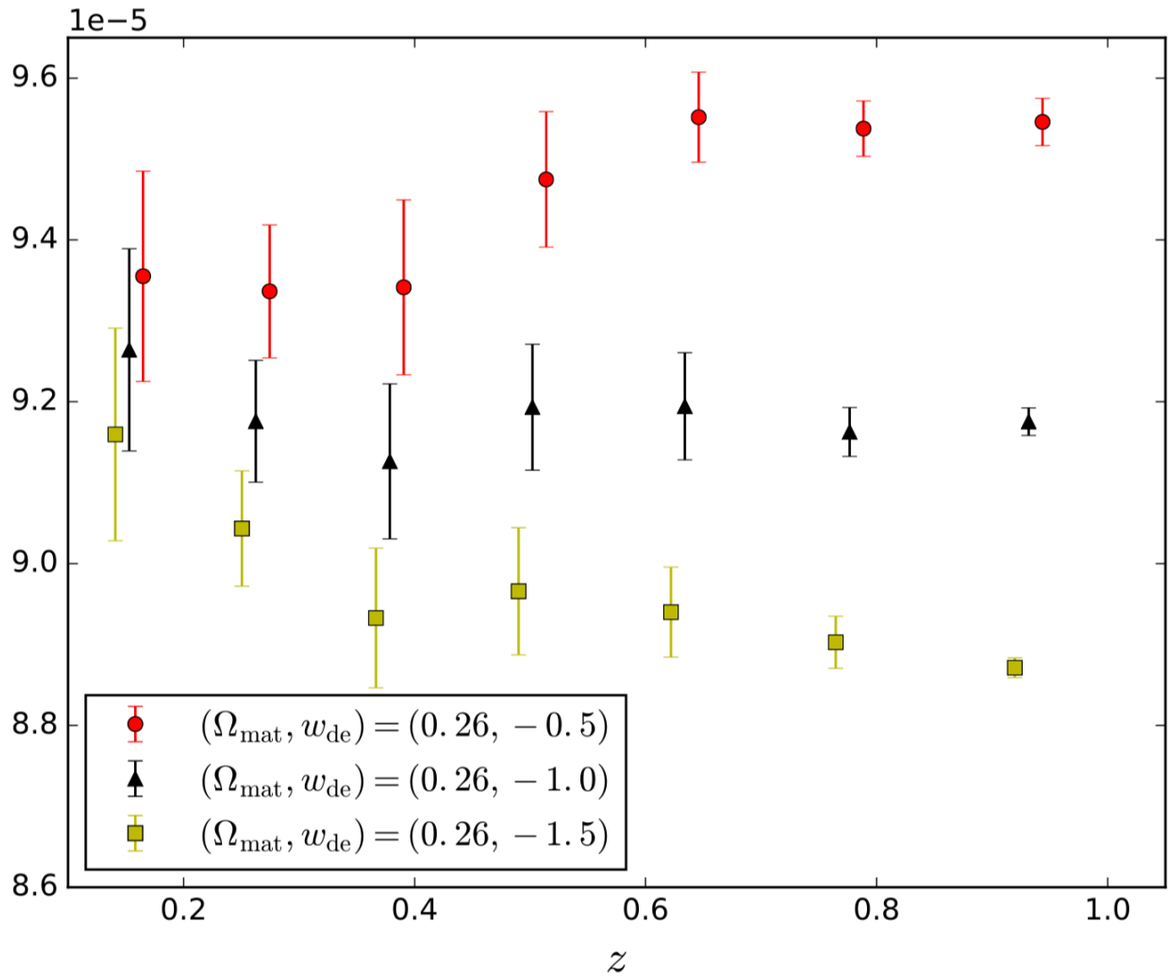
Conservation of Genus with Redshift

- We have measured this effect with Gaussian random fields and Mock galaxy light cone data from simulations (HR4)

Gaussian Random Fields



Mock Galaxies (HR4)



SA, Park, Hong & Kim (2018)

Theory Corrections — Redshift Space Distortion

- Compare the measured genus to its Gaussian expectation value

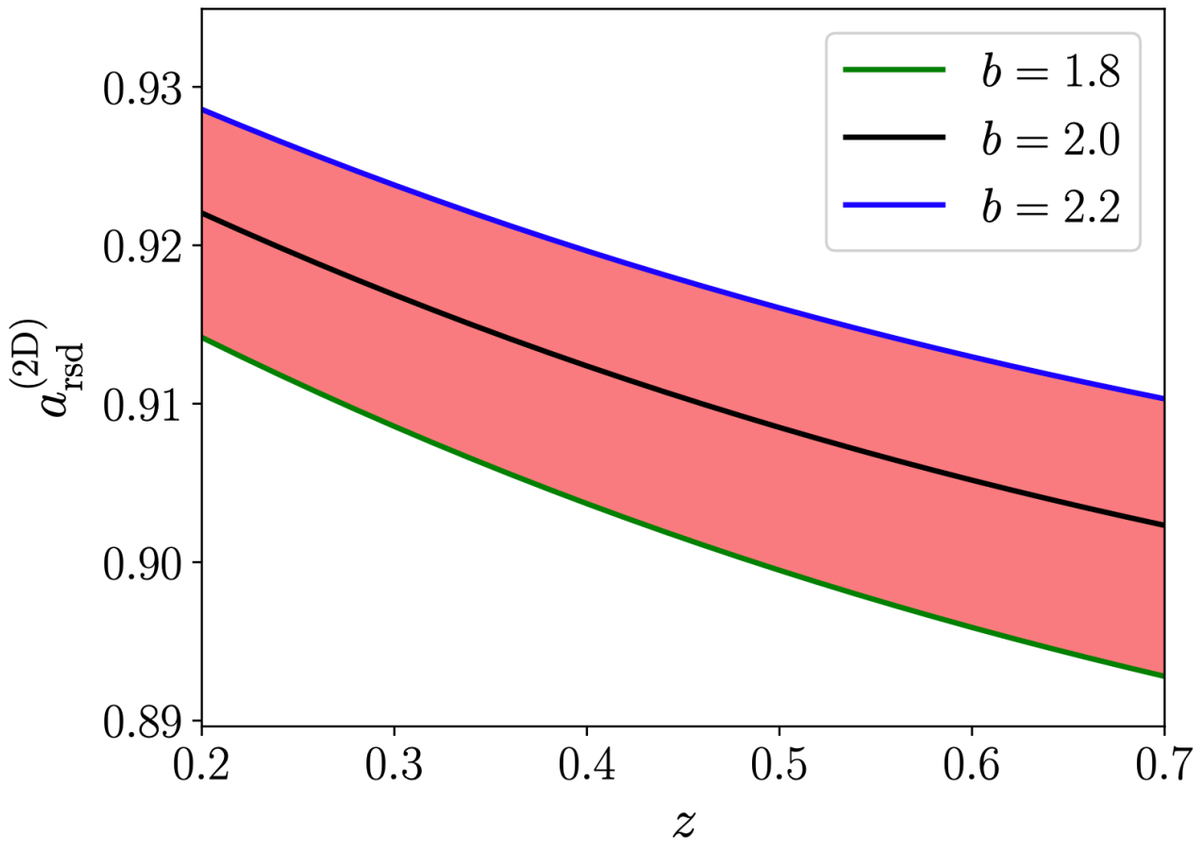
$$A_{\rm G} = \frac{1}{2(2\pi)^{3/2}} \frac{\sigma_1^2}{\sigma_0^2}$$

$$P_{3D}(k) = b^2 \left(1 + \beta \frac{k_\parallel^2}{k^2}\right)^2 P_{\rm m}(k) + P_{\rm SN}$$

Linear RSD

- Redshift space distortion decreases the genus amplitude

- Introduces a weak dependence on the galaxy bias



* $\beta = f/b$ $\quad f = d\ln D / d\ln a$

$\sigma_0^2 = \langle \delta^2 \rangle$ $\quad \sigma_1^2 = \langle |\nabla \delta|^2 \rangle$

Theory Corrections — Shot Noise

- Compare the measured genus to its Gaussian expectation value

$$g_{\rm 2D}(\nu) = \frac{1}{2(2\pi)^{3/2}} \frac{\sigma_1^2}{\sigma_0^2} e^{-\nu_{\rm A}^2/2} \left[H_1(\nu_{\rm A}) + \left(\frac{2}{3}[S^{(1)} - S^{(0)}] H_2(\nu_{\rm A}) + \frac{1}{3}[S^{(2)} - S^{(0)}] H_0(\nu_{\rm A}) \right) \sigma_0 + \mathcal{O}(\sigma_0^2) \right]$$

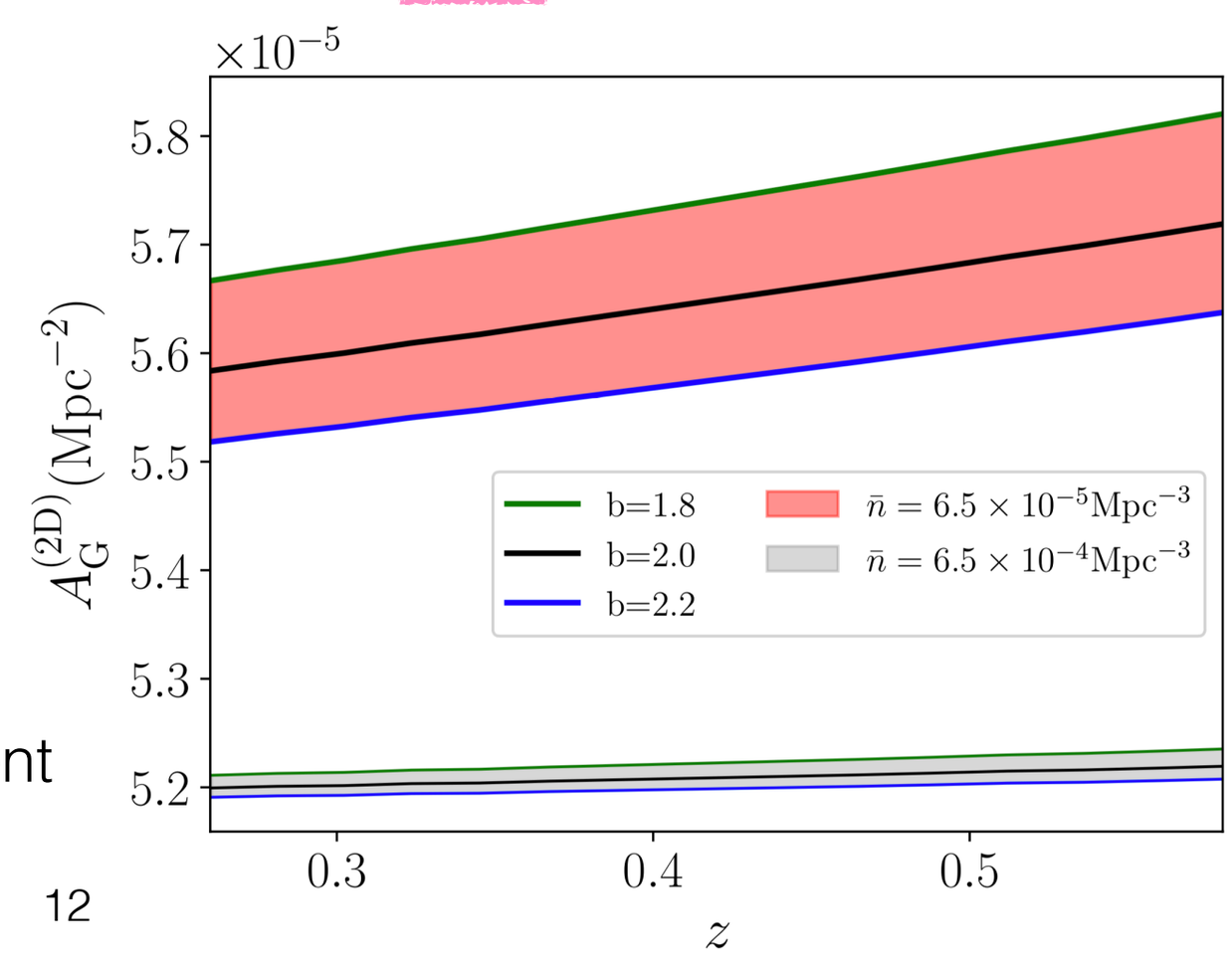
$$P_{3D}(k) = b^2 \left(1 + \beta \frac{k_\parallel^2}{k^2} \right)^2 P_{\rm m}(k) + P_{\rm SN}$$

Shot noise

- Shot noise artificially generates small scale power, increases the genus amplitude

- Creates sensitivity to the amplitude of the power spectrum!

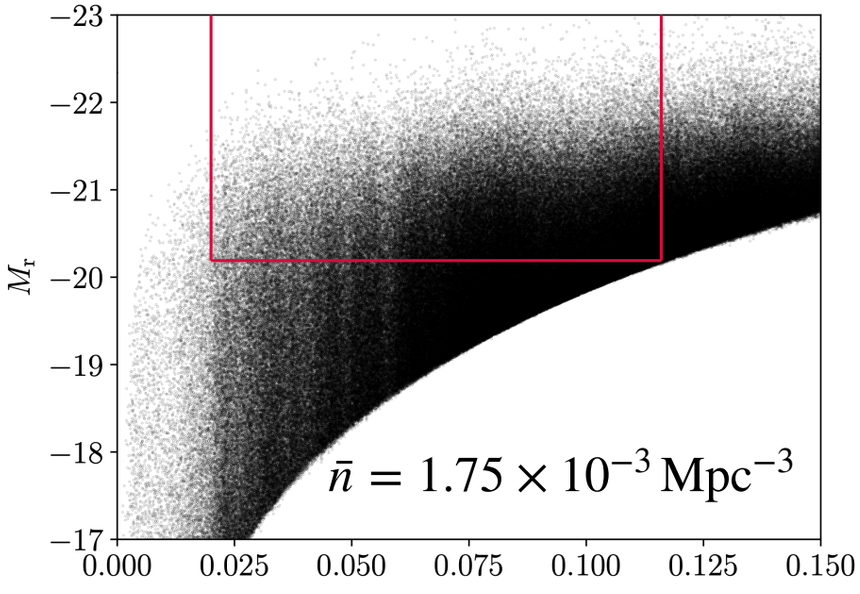
- Shot noise is Poisson distributed, hence is drawn from a different moment generating functional.

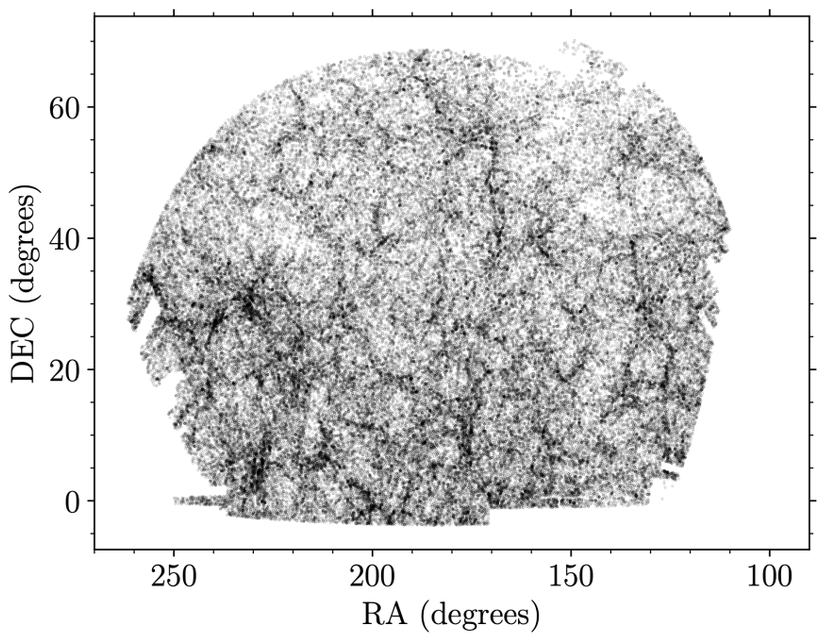


Data

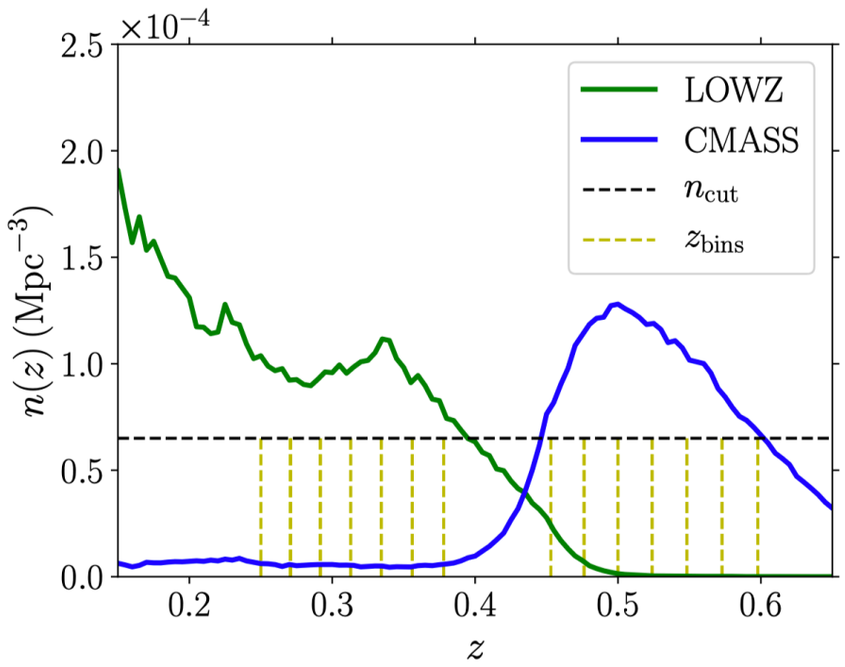
- We measure the genus amplitude from low- and high-redshift catalogs, compare their values.

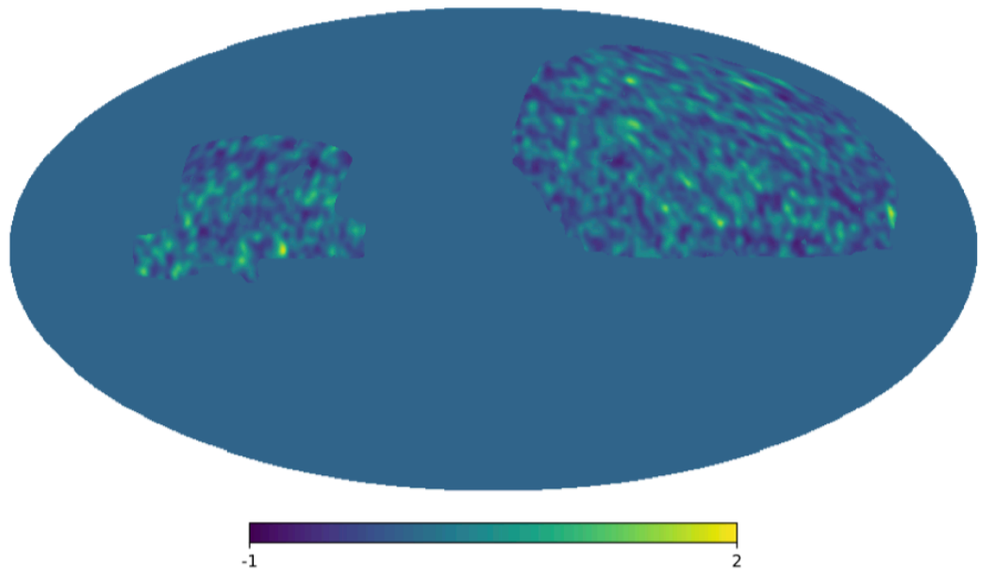
Low Redshift





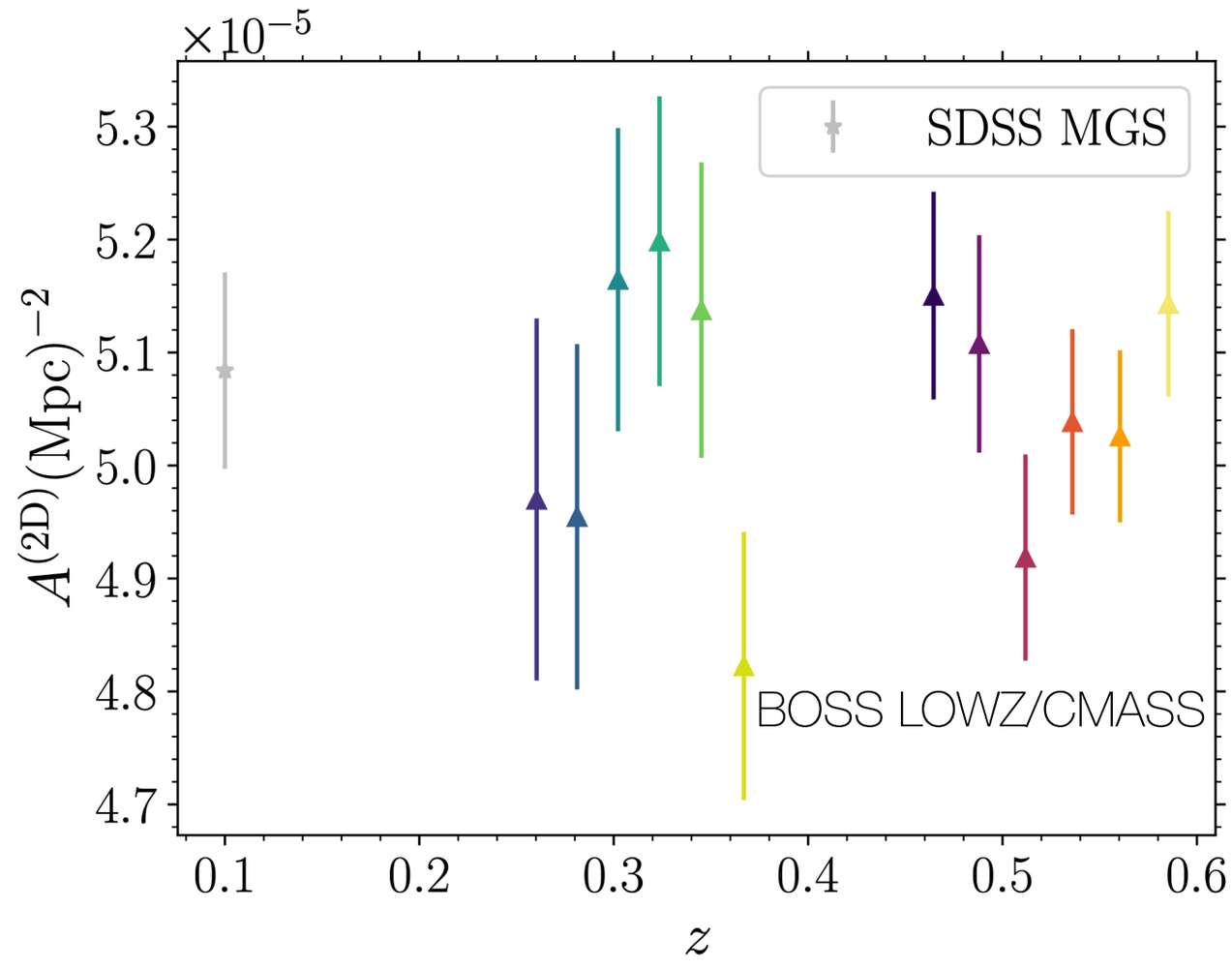
High Redshift

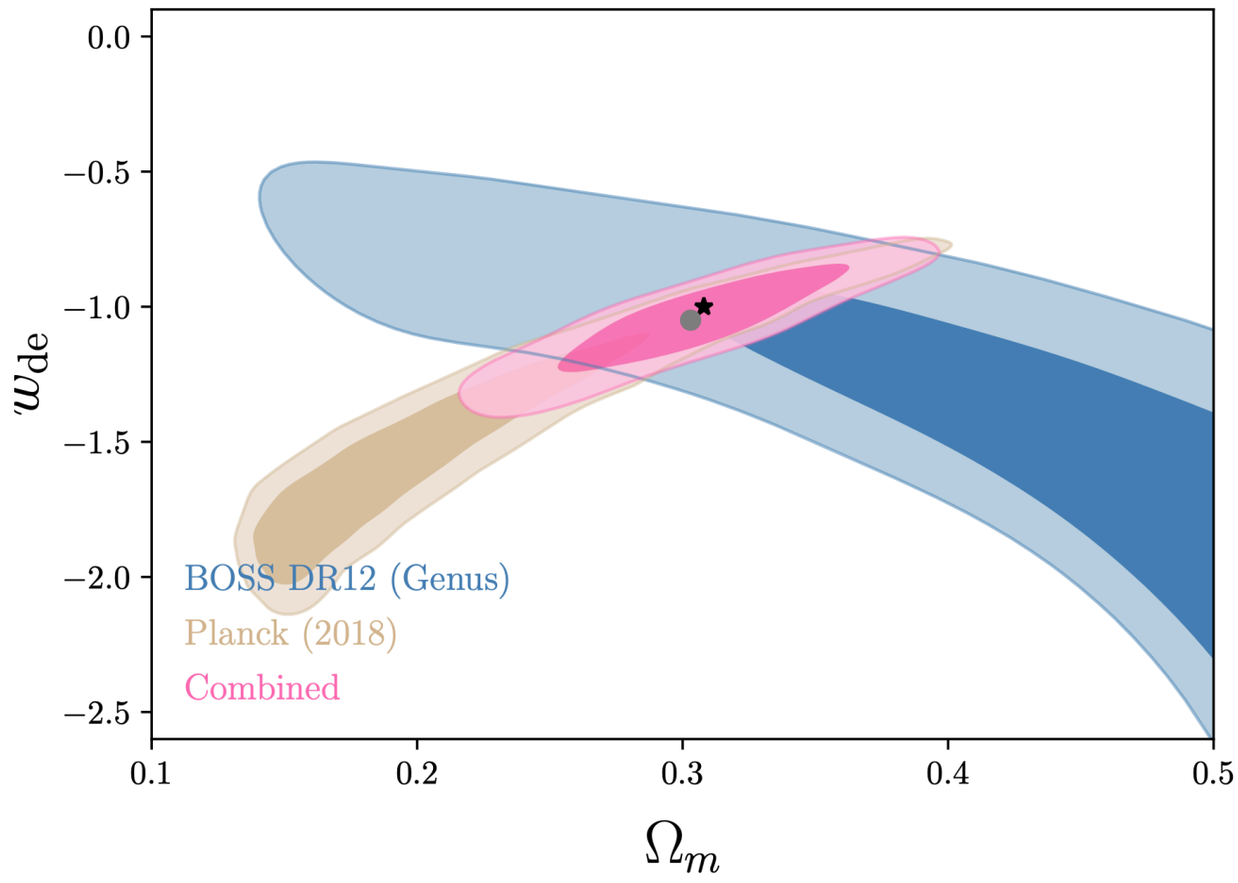




Results

- Combining BOSS DR12 + SDSS MGS to constrain the expansion history —



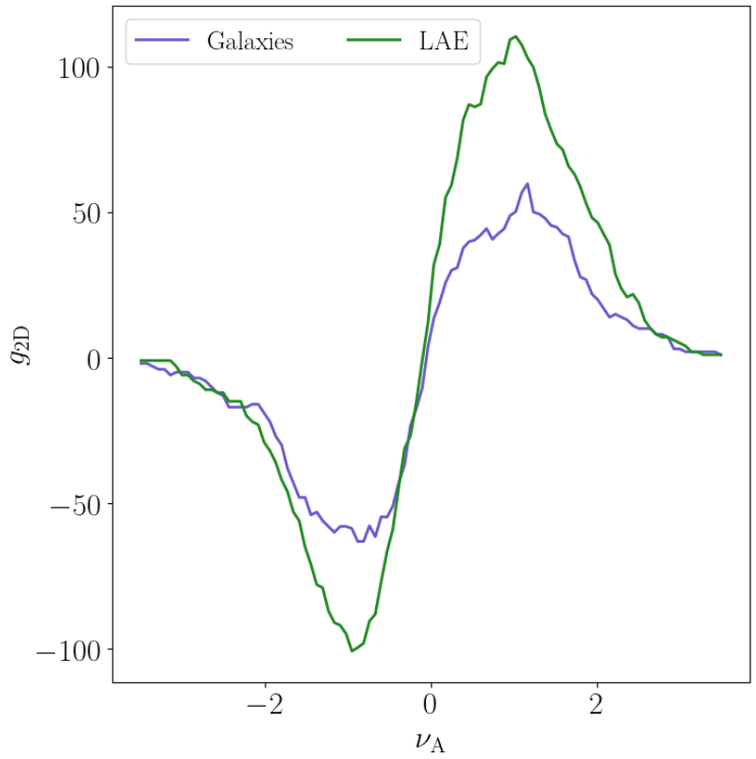


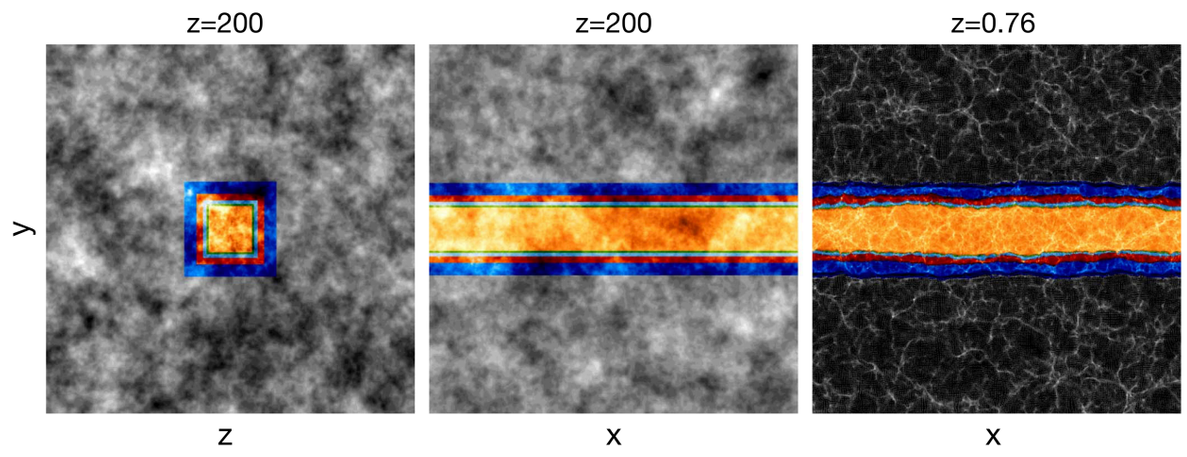
Data	Ω_{m}	w_{de}
Genus (BOSS+MGS)	$0.507^{+0.104}_{-0.126}$	$-2.24^{+1.07}_{-1.14}$
Genus (BOSS+MGS) + Planck (2018)	0.303 ± 0.036	$-1.05^{+0.13}_{-0.12}$

Refining the Methodology I — Data

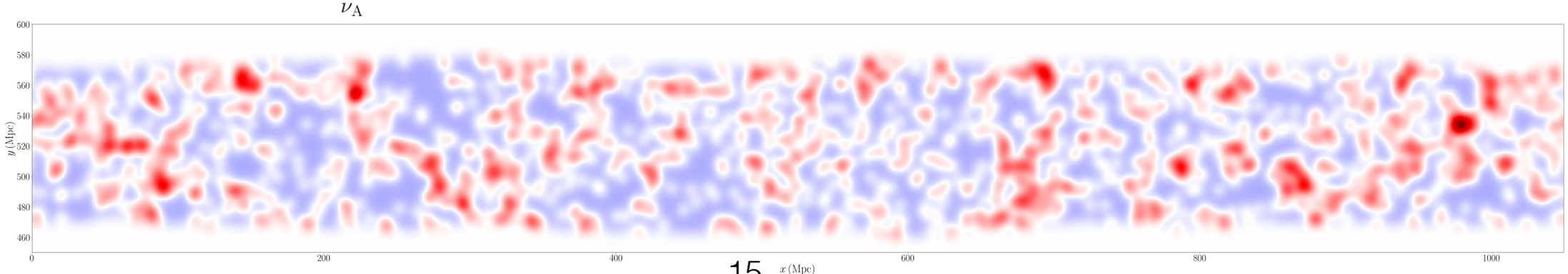
- More data, at both high- and low-redshift.
- DESI : $\sim 10^7$ LRGs, $0.4 < z < 0.8$, $\bar{n} \sim 1.5 \times 10^{-4}\,\mathrm{Mpc}^{-3}$
- ODIN : Lyman-alpha emitters observed at cosmic time slices $z = 2.4, 3.1, 4.5$ $\quad V = 0.24\,\mathrm{Gpc}^3$

$N \sim 65{,}000$



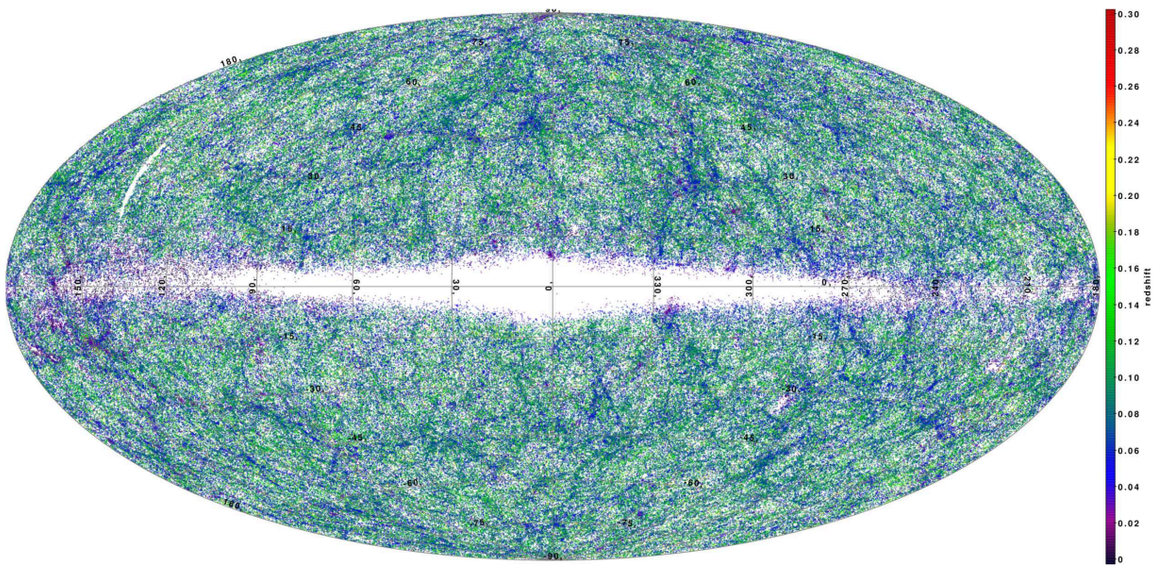


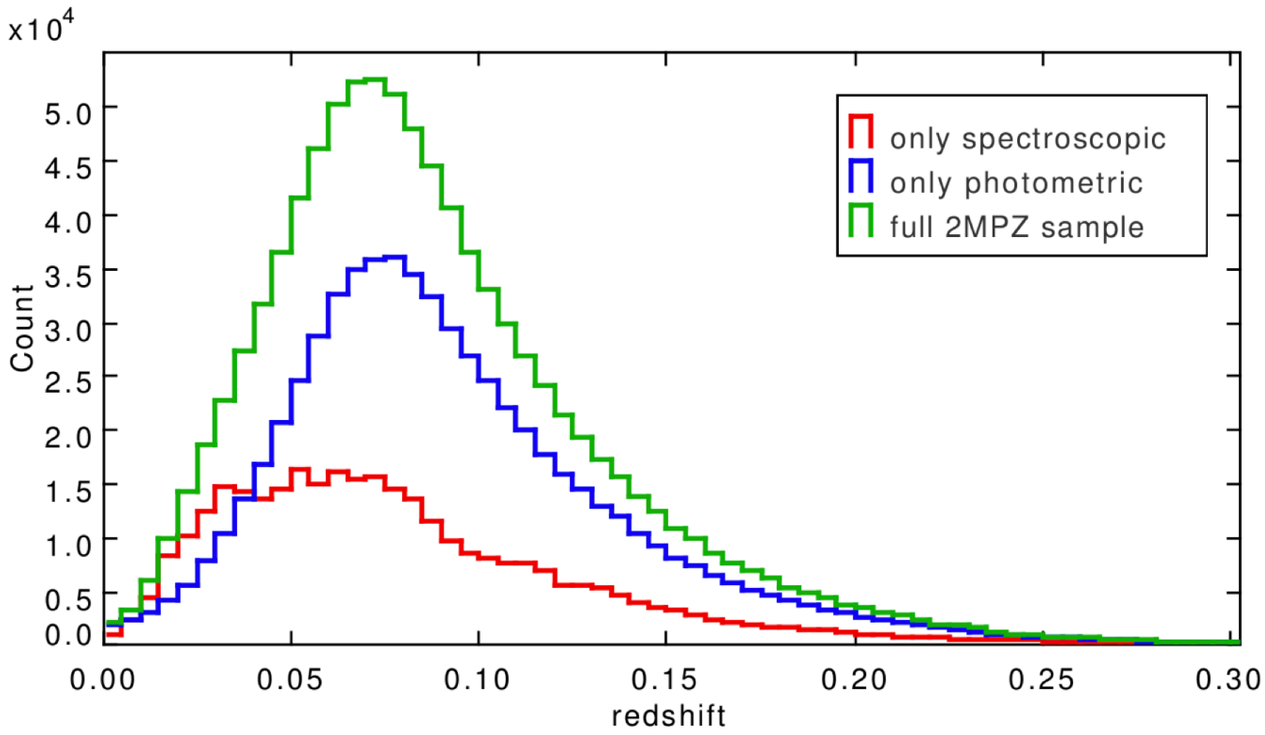
Lee et al. ApJ 908:11 (2021)



Refining the Methodology I — Data

- More data, at both high- and low-redshift.
- 2MASS photometric redshift catalog : $\sim 10^6$ galaxies, $z_{\rm med} \sim 0.1$, $\Delta z \sim 12\,\%$

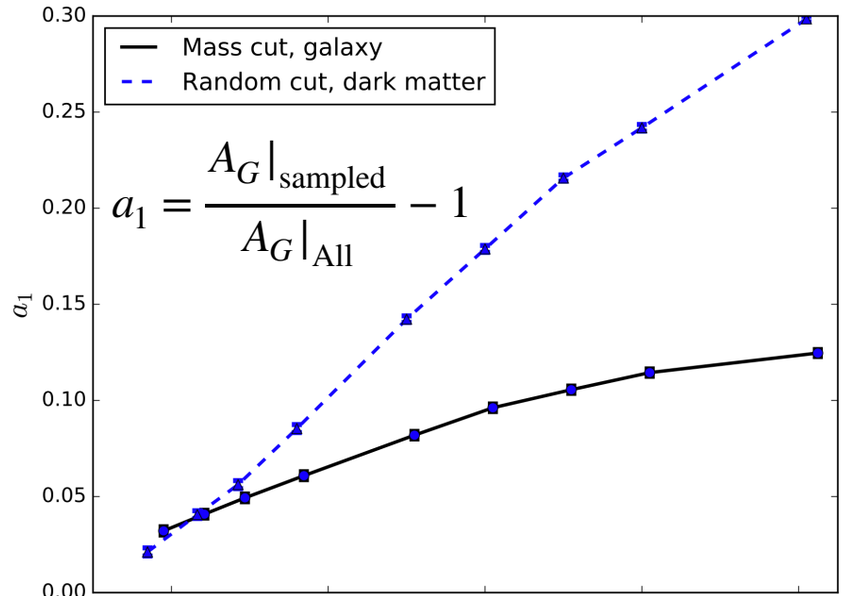


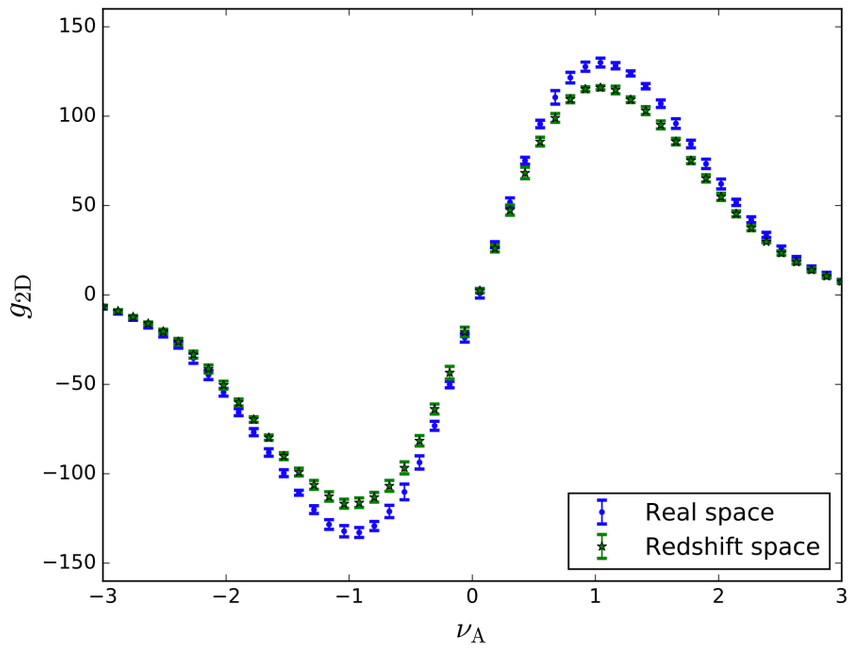


Bilicki et al. Astrophys.J.Suppl. 210 (2014) 9

Refining the Methodology II — Theory

- Improvements to theoretical modeling.
- High redshift galaxy samples are typically sparse — strongly affected by shot noise.
- The standard lore in topological analyses — we must smooth on at least the mean particle separation of the sample $R_{\rm G} \gtrsim \bar{n}^{-1/3}$.
- Multi-scale smoothing allows us to extract more information. What is the ensemble average when the smoothing is inhomogeneous?
- Redshift space distortion — not well described by perturbation theory, even on relatively large scales.
- ‘Gravitational smoothing’ : Higher order effect, the amplitude of the genus decreases at second order in perturbation theory.
- The north and south sky genus show a $\gtrsim 2\sigma$ discrepancy in amplitude. Statistical fluctuation? We are looking for a 1% effect!





Conclusions

- Two-dimensional slices of the matter density field can be used to constrain the expansion history of the Universe, using the genus amplitude as a standard population.

- Current constraints are modest, restricted by data volume and theoretical limitations.

- Theoretically shot noise and redshift space distortion are pre-eminent issues. Shot noise is a non-Gaussian sampling effect, redshift space distortion is not well described using linear perturbation theory.

- All sky data at low redshift, and more dense high redshift data, is required to test the standard model of cosmology.

