Dark Matter beyond WIMP

2023 Cosmology Workshop
on the Crossroad of Astrophysics and
Particle Physics: dark matter
Sono Belle vivali park, June 30, 2023

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Chung-Ang University

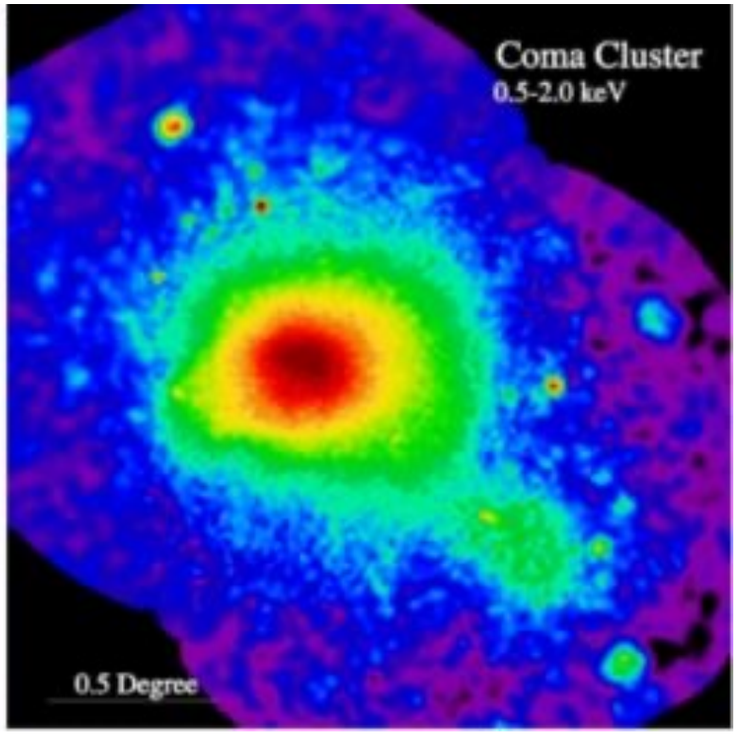
Outline

- What we know about dark matter
- Dark matter at galaxies and detection
- Dark matter with contact interactions
- Dark matter with long range forces

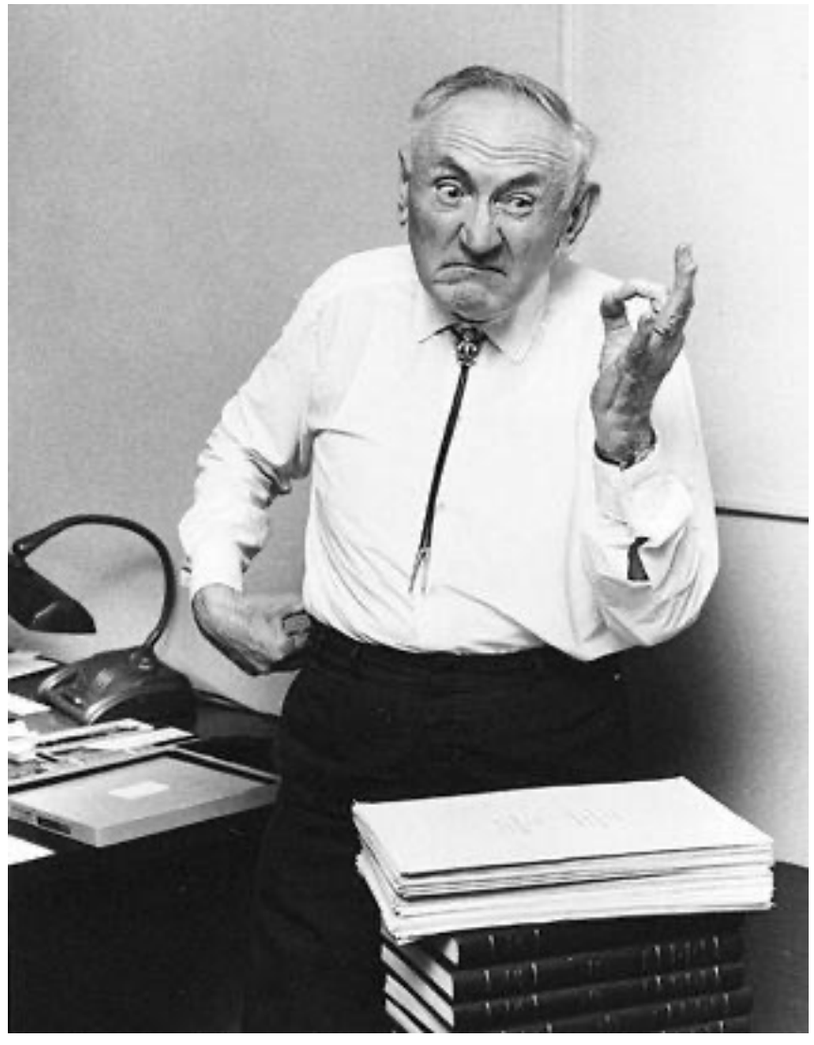
What we know about dark matter

Dunkle Materie





1000 galaxies in Coma Cluster
cannot be held on by visible mass only.

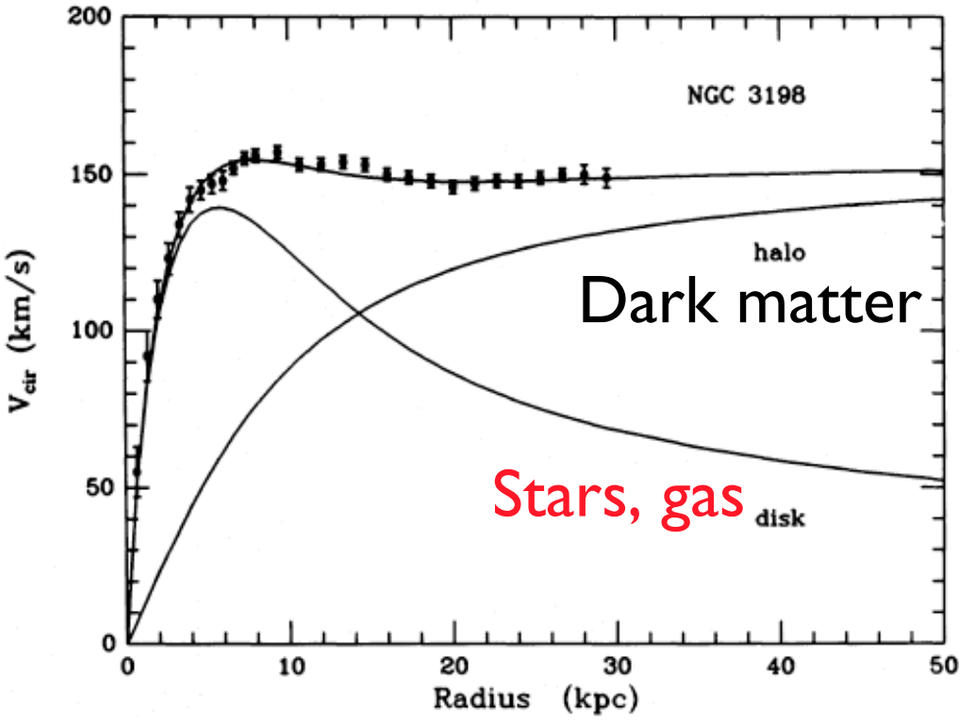


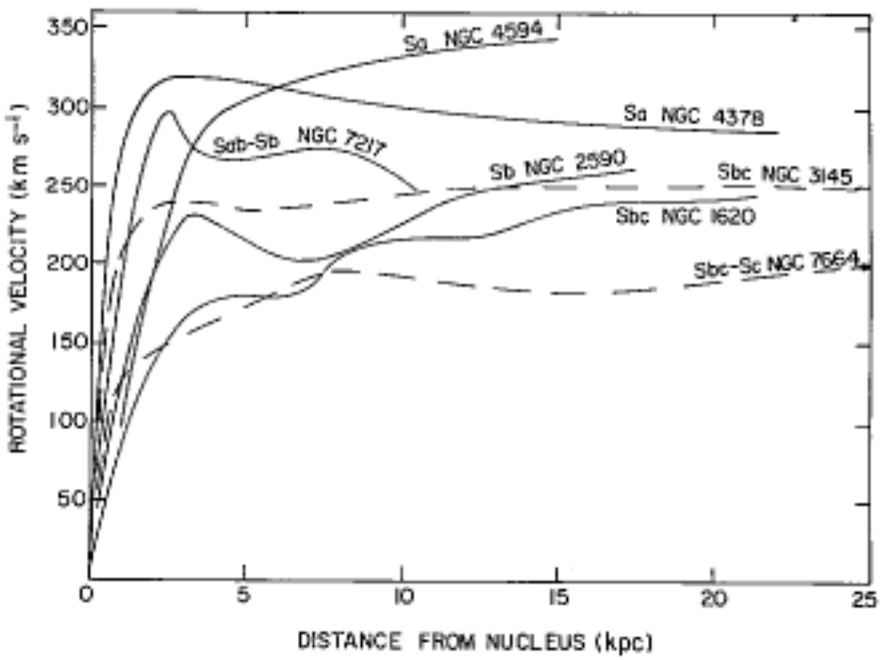
Fritz Zwicky(1933)

Virial theorem: $\langle v^2 \rangle \sim \frac{GM_v}{R_v}$, $\langle M/L \rangle \sim 500$ (corrected, $\langle M/L \rangle \sim 8$)

→ "Dunkle Materie" Advent of Dark Matter history

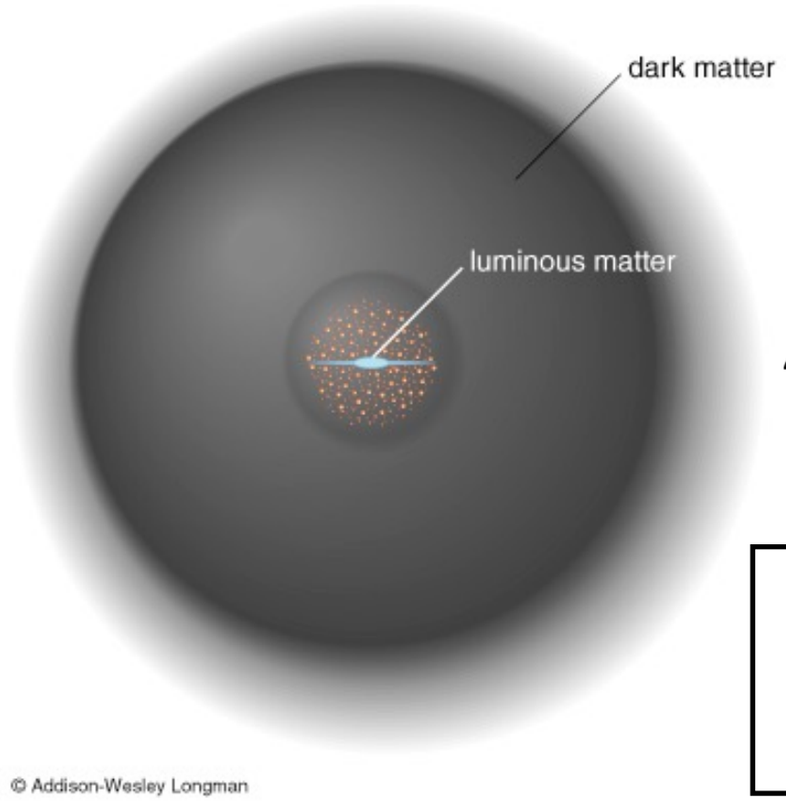
Dark matter in galaxies







V. Rubin (1978)

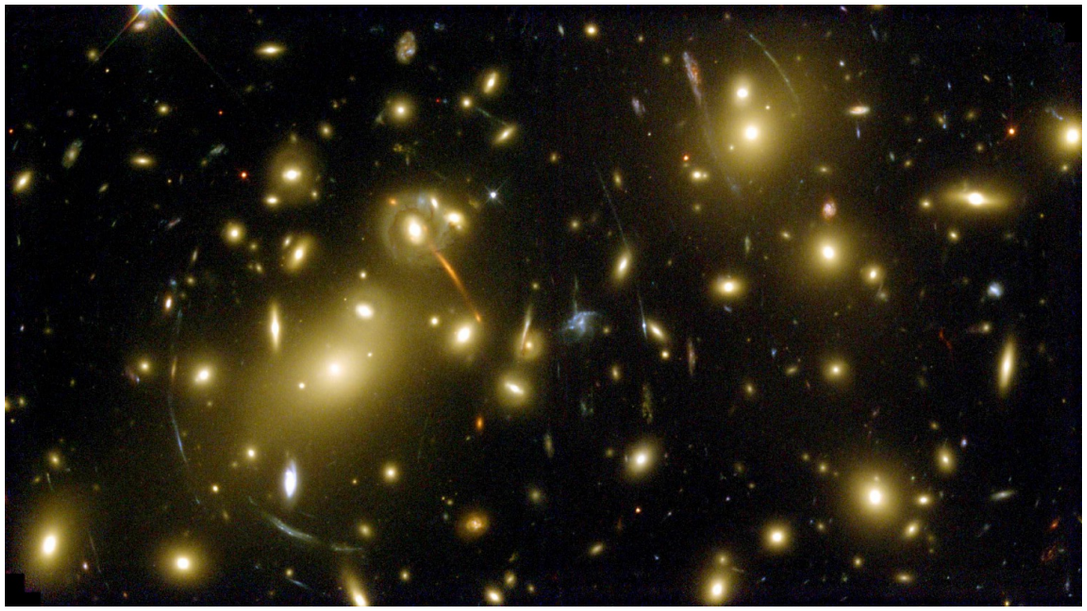


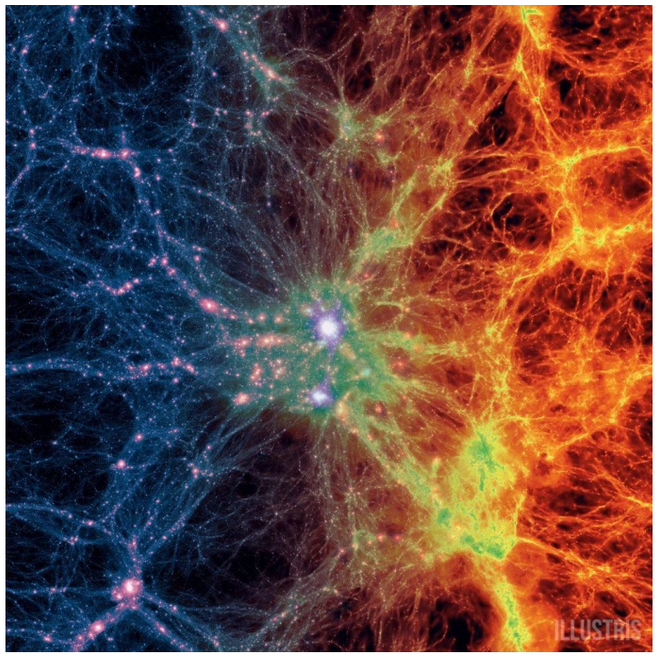
Stars in spiral galaxies
have saturated orbital velocities.

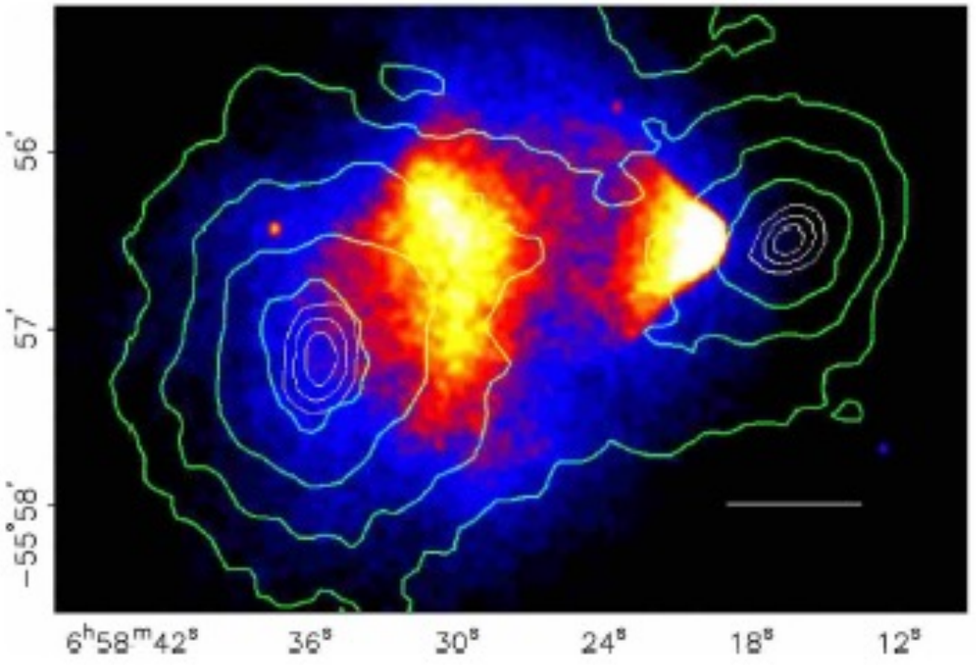
$$v = \sqrt{\frac{GM}{r}} = \text{const} \Rightarrow \begin{array}{l} M \propto r \\ \text{or } \rho(r) \propto \frac{1}{r^2} \end{array}$$

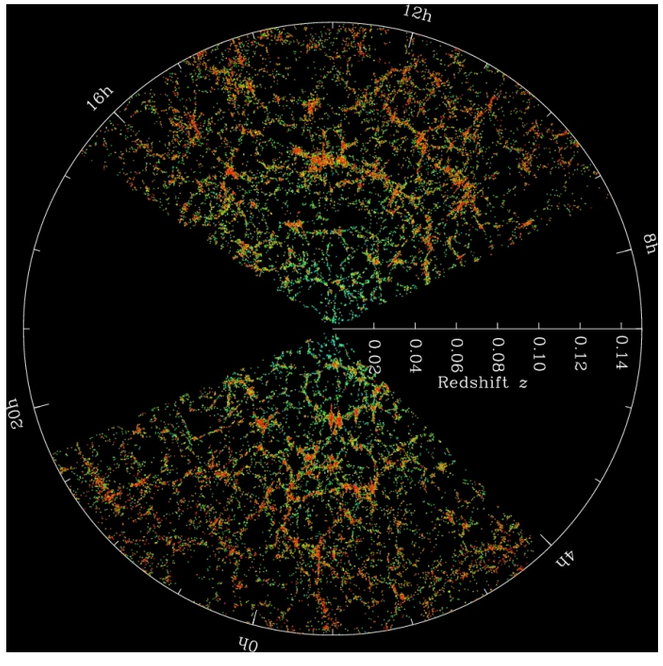
Dark Matter Halo pervades within galaxies:
5-10 times visible matter

Evidences for dark matter



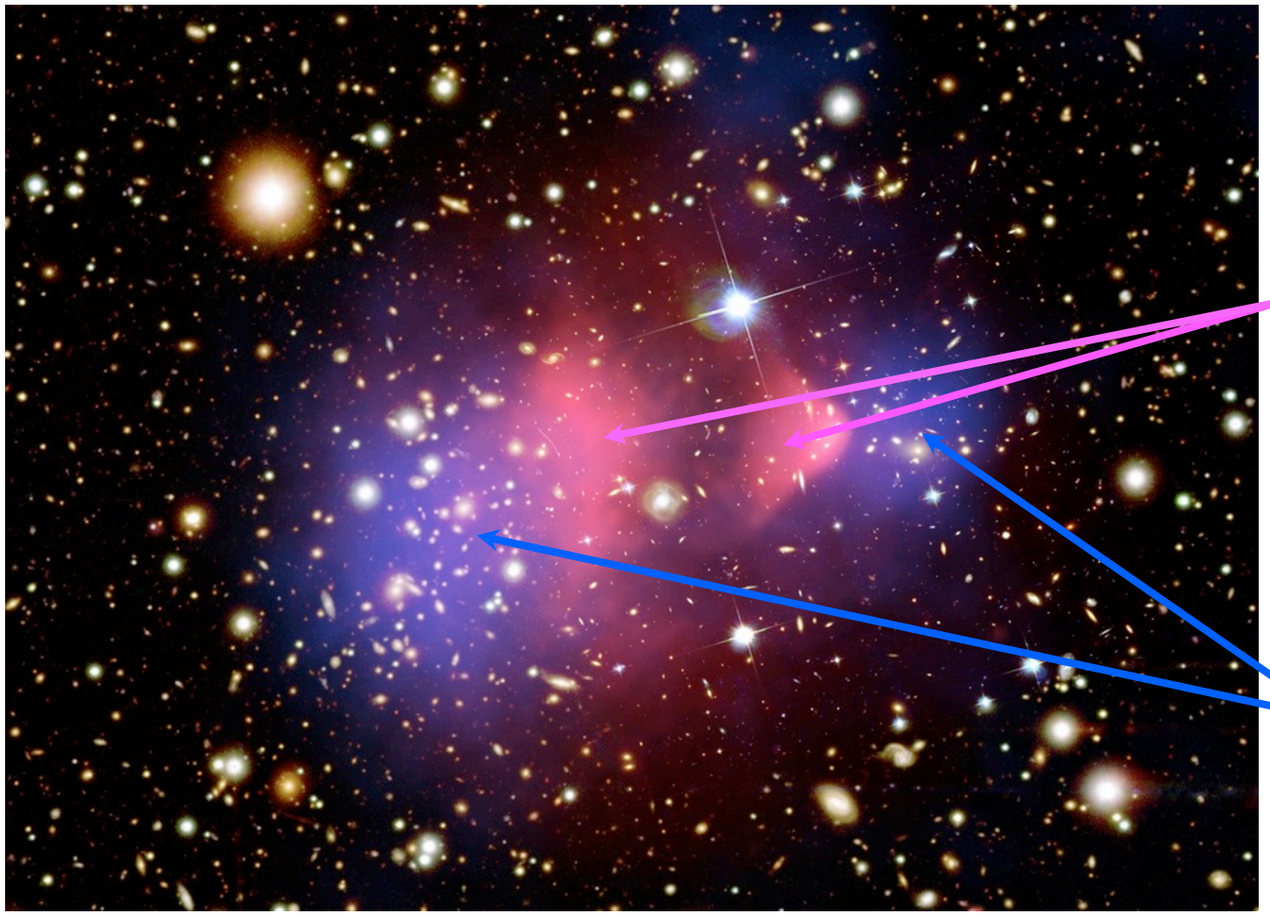






- Strong or weak gravity lensing, Bullet cluster, large scale structure, etc, maps out DM composition.

Bullet Cluster



[Chandra X-ray (2006)]

Gas (X-ray)

Galaxies (visible light)

Dark matter (gravity lensing)

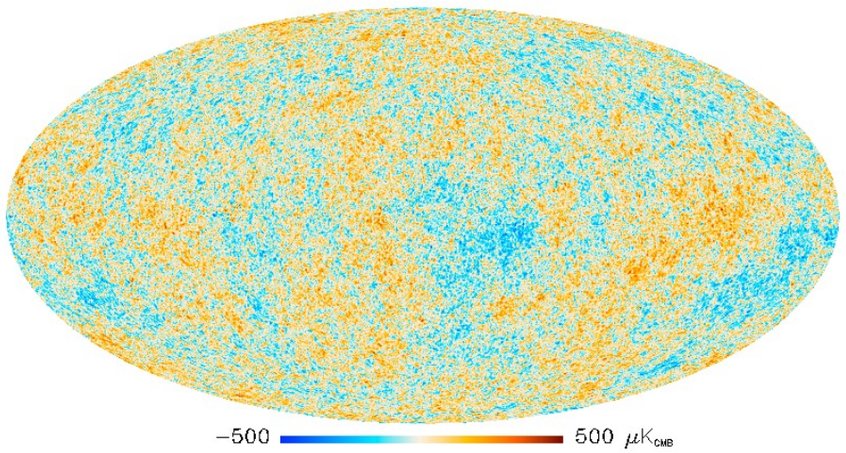
- Hot gas (X-ray) leaves the bow shock due to EM.
- Weak lensing shows that 80% of total mass is dark.

⇒ Strong evidence for (collisionless) dark matter.

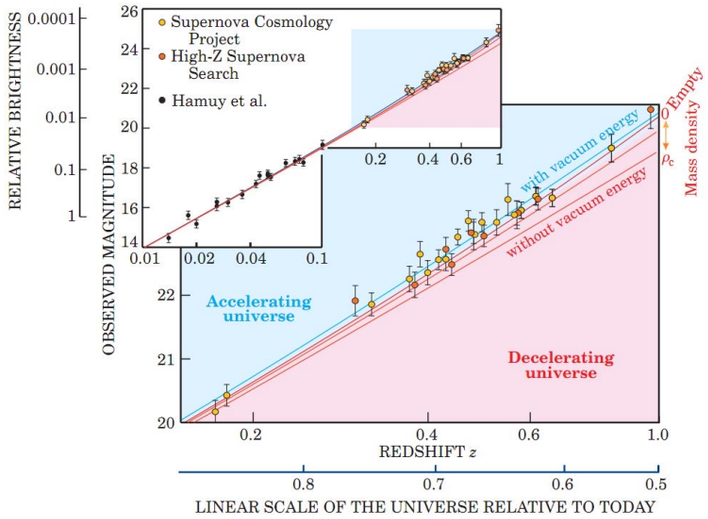
cf. Modified Newtonian Dynamics (MOND) is not possible.

Dark matter in cosmology

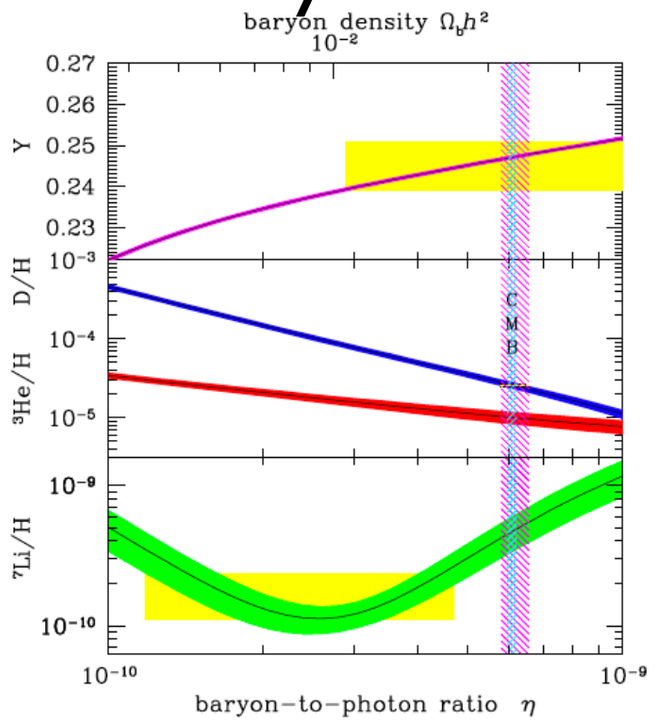
CMB anisotropies



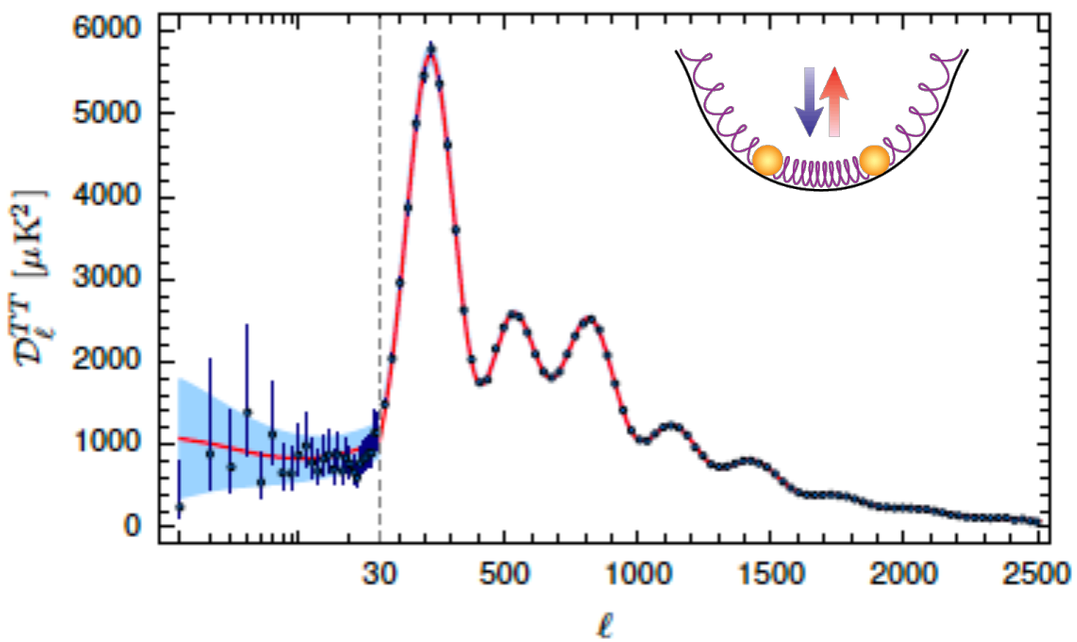
Supernovae(SNIa)



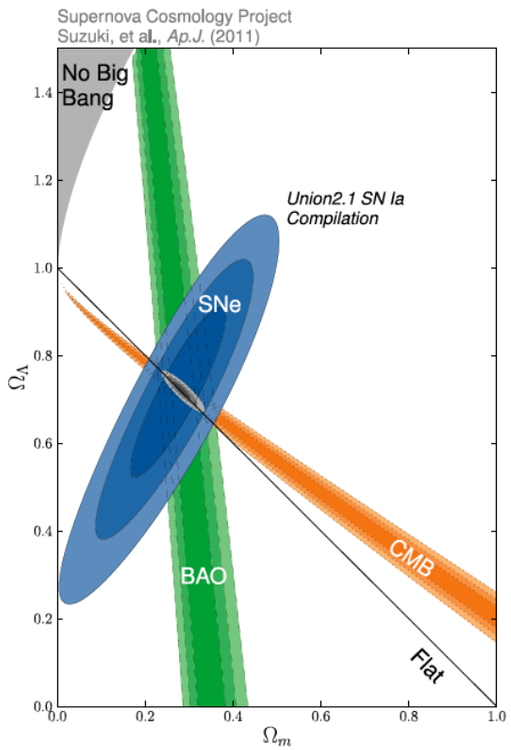
Nucleosynthesis



Gravity pull of DM vs baryon-photon coupling



acceleration: dark energy



baryon abundance

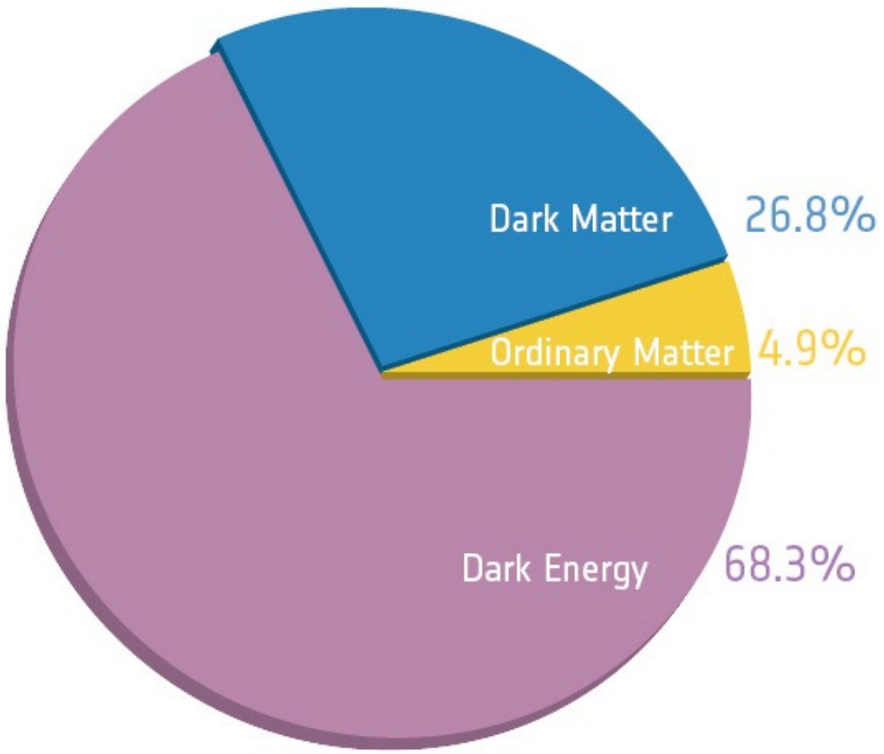
$$\eta_B = (5.8 - 6.6) \times 10^{-10}$$

+ large-scale (BAO)

↓

Precise measurement of dark matter

Cosmic pie and dark matter



CMB (+ flat Universe)
+ SN Ia + BAO

$$\Rightarrow \quad 1 = \Omega = \sum_i \Omega_i,$$

$$\Omega_b = 0.05, \quad \Omega_c = 0.27, \quad \Omega_\Lambda = 0.68.$$

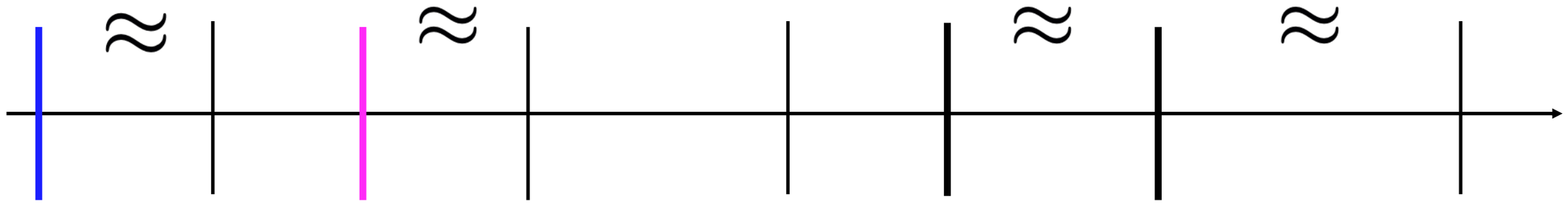
- Gravitational interactions to matter.
- Charge-neutral.
- Occupies 85% of total matter (non-relativistic).
- Not in the Standard Model.

cf. neutrinos: $\Omega_\nu = 0.001(m_\nu/0.2\,\mathrm{eV})$

What is the (dark) matter?

Fermion DM "Pauli exclusion"

$M_{\rm halo} \lesssim m_{\rm DM} V_{\rm halo}\, p_F^3$ Fermi momentum: $p_F \sim m_{\rm DM} v_{\rm DM}$



$10^{-22}\,\mathrm{eV}$ $10^{-6}\,\mathrm{eV}$ keV $10\,\mathrm{GeV}$ $10^{3}\,\mathrm{GeV}$ $10^{5}\,\mathrm{GeV}$ $10^{19}\,\mathrm{GeV}$ $10^{58}\,\mathrm{GeV}$ $(\sim 10\,M_\odot)$

Boson DM "Uncertainty principle"

Planck mass

Black hole

de Broglie wave length < Halo radius

$$\frac{\lambda}{2\pi} = \frac{\hbar}{mv} = 1.92\,\mathrm{kpc}\left(\frac{10^{-22}\,\mathrm{eV}}{m}\right)\left(\frac{10\,\mathrm{km\ s^{-1}}}{v}\right)$$

Dark matter at galaxies and detection

Dark matter halo

Liouville equation

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f - \nabla \Psi \cdot \frac{\partial f}{\partial \vec{v}} = 0 \qquad f : \text{Phase space distribution}$$

$$\longrightarrow \quad f \propto e^{\mathcal{E}/\sigma^2}, \quad \mathcal{E} = \Psi - \frac{1}{2}v^2 \quad \text{Energy per test particle}$$

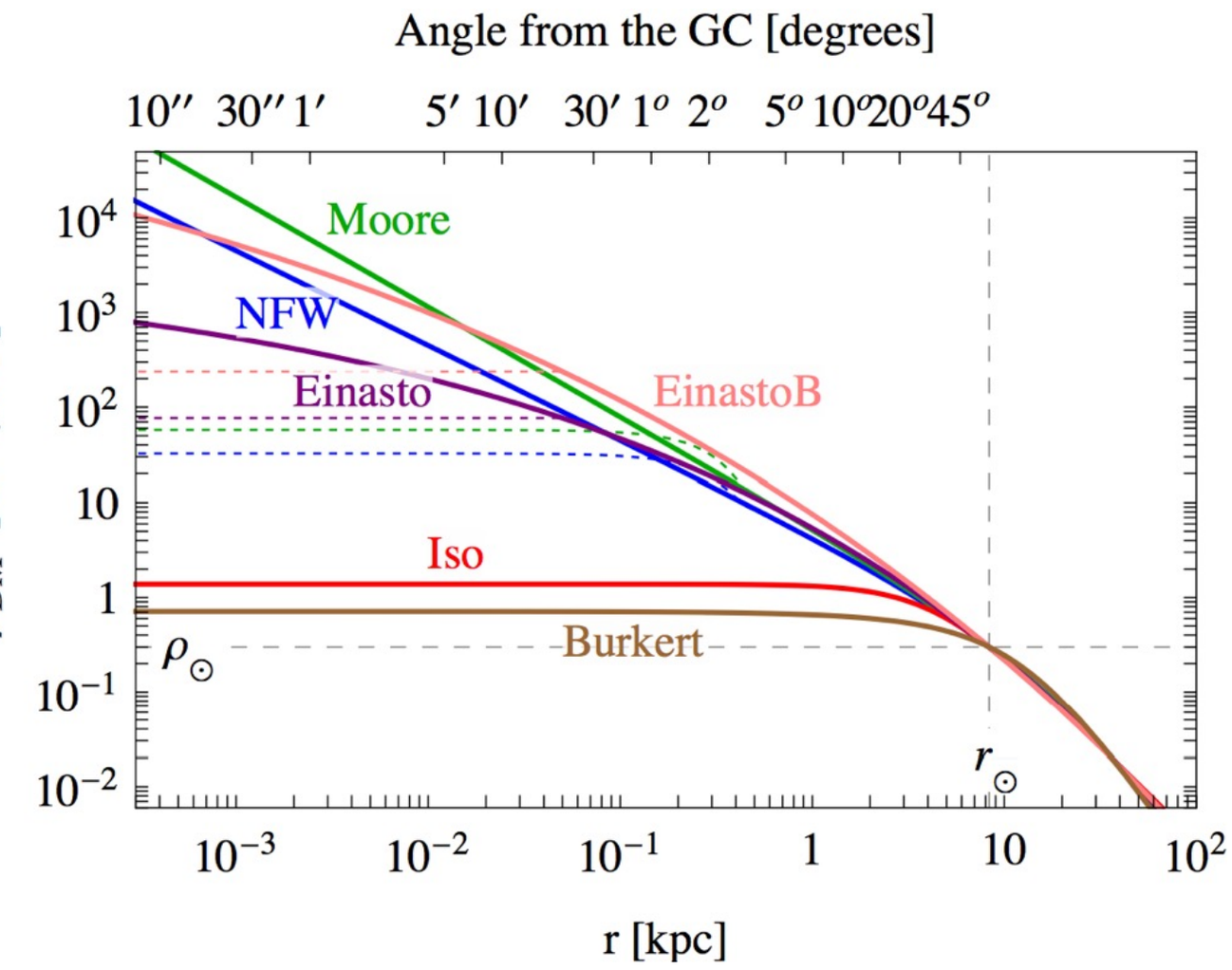
$$\longrightarrow \quad \rho \propto \int_0^\infty dv\, v^2 f(v) \Rightarrow \rho = \rho_c\, e^{\Psi/\sigma^2}$$

Poisson equation $\nabla^2 \Psi = -4\pi G \rho$

$$\longrightarrow \quad \rho(r) = \frac{r_s^2 \rho_s}{r^2}, \quad r_s^2 \rho_s = \frac{\sigma^2}{2\pi G} \quad \text{“Isothermal distribution”}$$

cf. $\rho(r) = \dfrac{\rho_s}{1 + (r/r_s)^2} \to \dfrac{r_s^2 \rho_s}{r^2}, \quad r \gg r_s$

DM density profiles



Navarro-Frenk-White (NFW):
CDM N-body simulation, "cuspy"

Einasto profile: fits well at small scales in galaxies, "mild"

$\rho_{\rm DM} \propto \dfrac{1}{r^2}$: does not fit well with observed galaxies

Dark Matter halo profiles

$$\text{NFW}: \quad \rho_{\rm NFW}(r) = \rho_s \frac{r_s}{r}\left(1+\frac{r}{r_s}\right)^{-2}$$

$$\text{Einasto}: \quad \rho_{\rm Ein}(r) = \rho_s \exp\left\{-\frac{2}{\alpha}\left[\left(\frac{r}{r_s}\right)^{\alpha}-1\right]\right\}$$

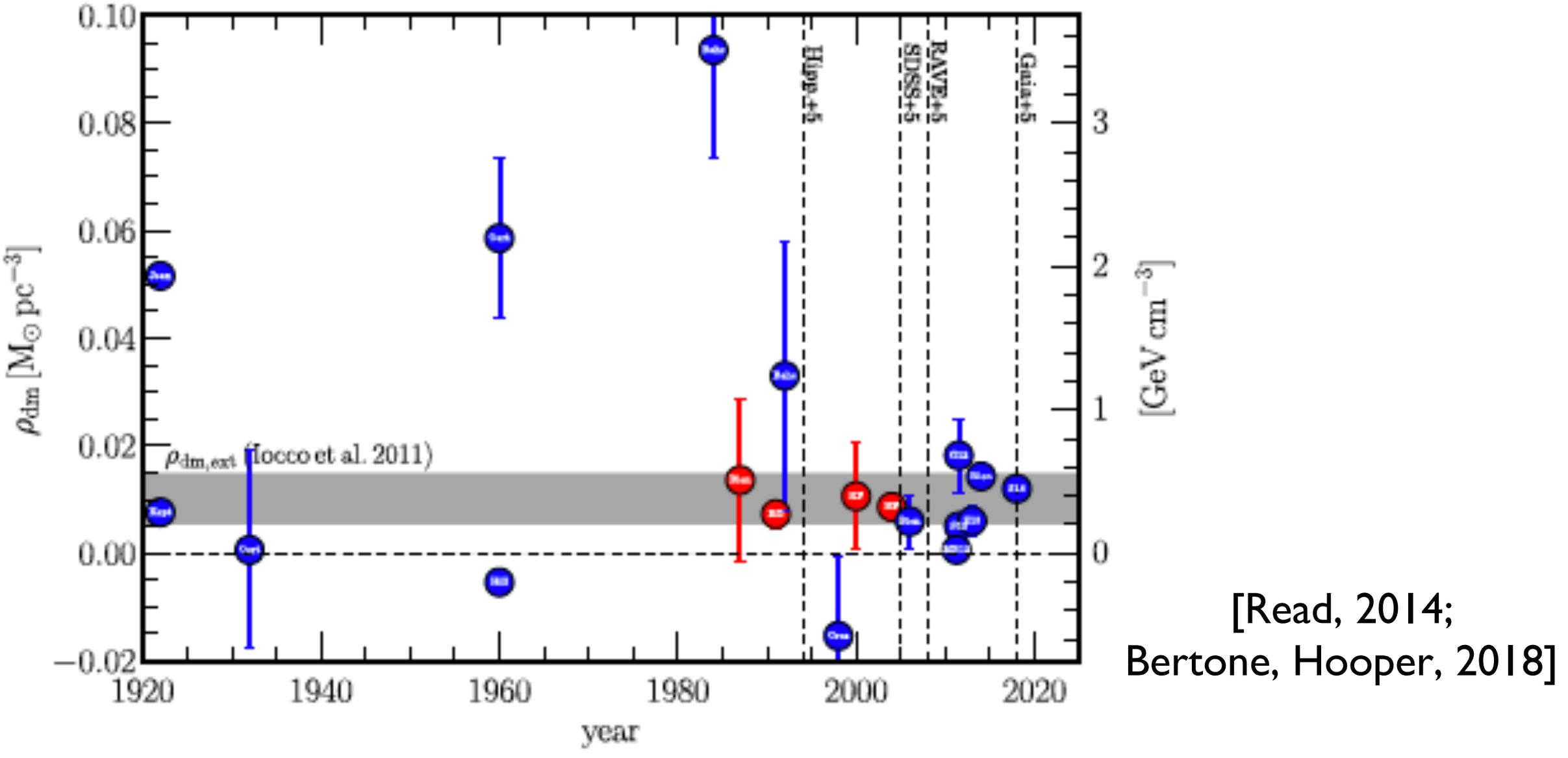
$$\text{Isothermal}: \quad \rho_{\rm Iso}(r) = \frac{\rho_s}{1+(r/r_s)^2}$$

$$\text{Burkert}: \quad \rho_{\rm Bur}(r) = \frac{\rho_s}{(1+r/r_s)(1+(r/r_s)^2)}$$

$$\text{Moore}: \quad \rho_{\rm Moo}(r) = \rho_s \left(\frac{r_s}{r}\right)^{1.16}\left(1+\frac{r}{r_s}\right)^{-1.84}$$

DM halo	α	r_s [kpc]	ρ_s [GeV/cm^3]
NFW	–	24.42	0.184
Einasto	0.17	28.44	0.033
EinastoB	0.11	35.24	0.021
Isothermal	–	4.38	1.387
Burkert	–	12.67	0.712
Moore	–	30.28	0.105

DM local density



[Read, 2014;
Bertone, Hooper, 2018]

Measurements: nearby stars (local) or rotational curves (global).

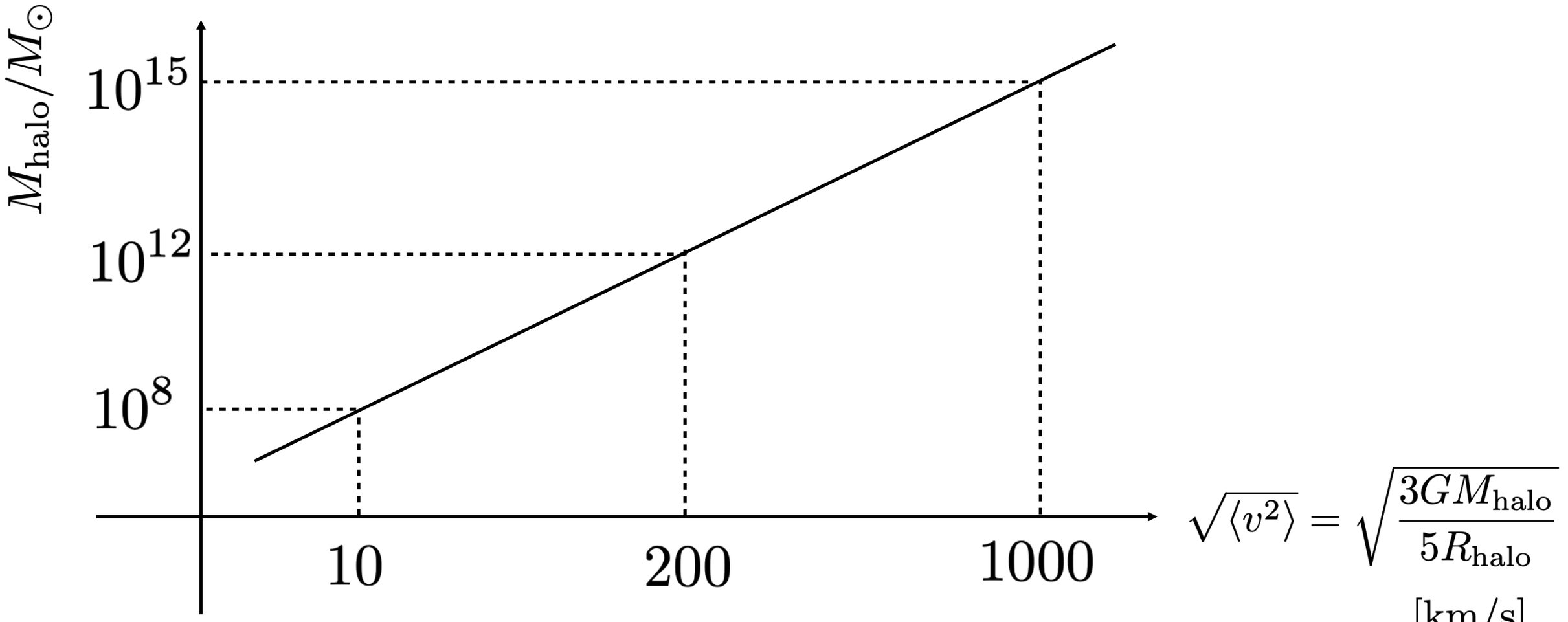
$$\rho_{\rm dm} = \quad [0.3 \pm 0.094]\ {\rm GeV\,cm^{-3}}\,, \quad 0.4 \pm 0.04\ {\rm GeV\,cm^{-3}}$$

Local dark matter density is important for direct detection.

DM velocity dispersion

DM velocity dispersion:

Virial theorem $\quad 2\langle T\rangle = -\langle U\rangle = \dfrac{3GM_{\rm halo}^2}{5R_{\rm halo}}\,, \quad \langle T\rangle = \dfrac{1}{2}M_{\rm halo}\langle v^2\rangle$

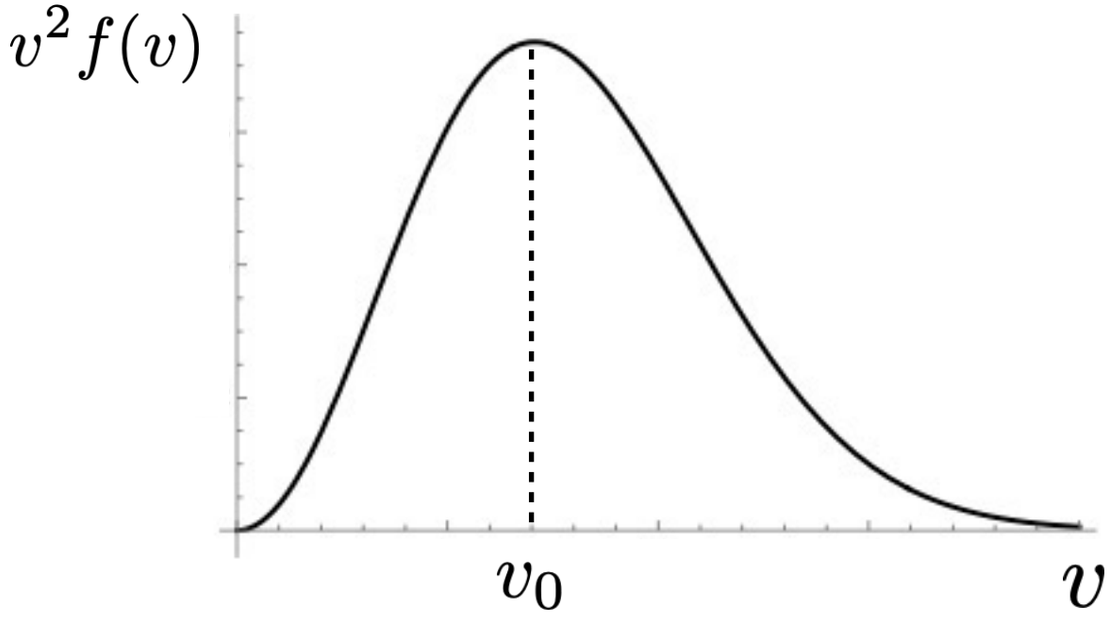


	Dwarf galaxies	MW	Galaxy clusters
$R_{\rm halo}\sim$	$10\,{\rm kpc}$	$100\,{\rm kpc}$	$1\,{\rm Mpc}$

DM velocity distribution

Velocity distribution

$$f \propto e^{(\Psi - \frac{1}{2}v^2)/\sigma^2} \longrightarrow f(v) = \frac{1}{(\pi v_0^2)^{3/2}} e^{-v^2/v_0^2}$$



Maxwell-Boltzmann distribution

MW: $v_0 = 235\,\mathrm{km/s}$

Escape velocity

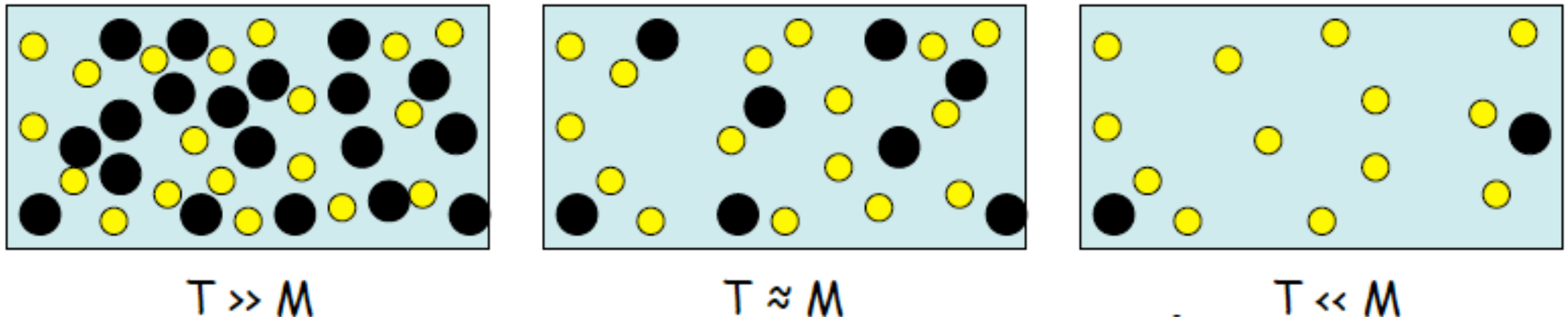
$$E = \frac{1}{2}mv^2 - \frac{GmM_{\rm halo}}{R_{\rm halo}} \geq 0 \longrightarrow v \geq v_{\rm esp} = \sqrt{\frac{2GM_{\rm halo}}{R_{\rm halo}}}$$

Scattering cross section $\langle \sigma v \rangle = 4\pi \int_0^\infty dv\, v^2 f(v) (\sigma v) \theta(v_{\rm esp} - v)$

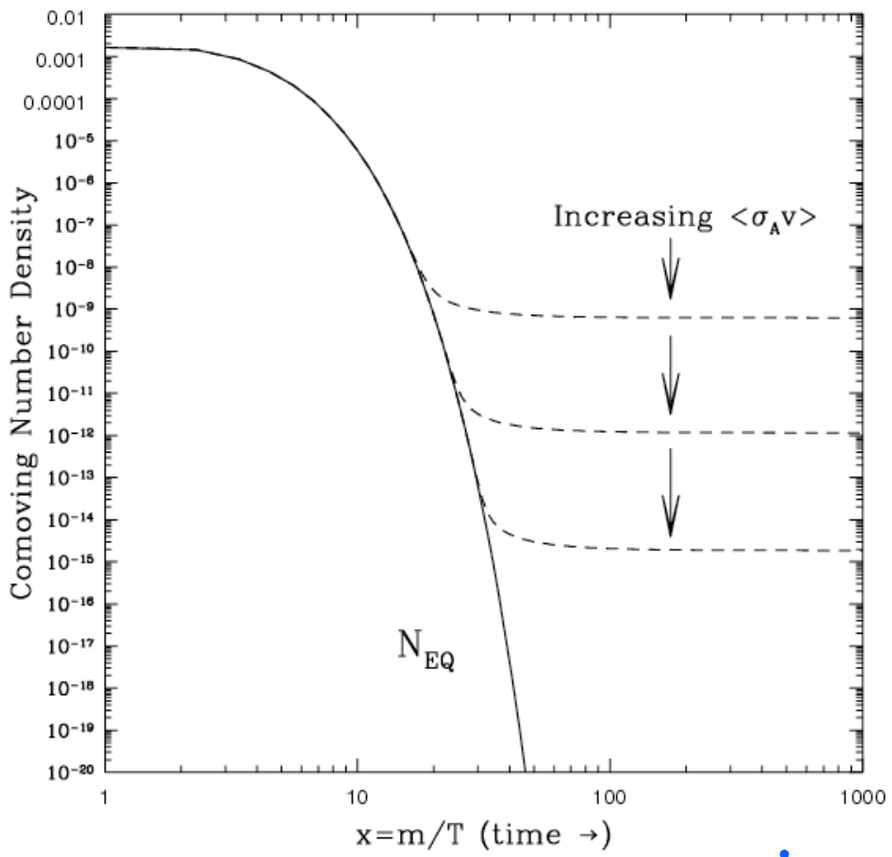
WIMP paradigm

● : SM

● : DM



DM abundance



[Lee,Weinberg(1977)]

Weakly Interacting Massive Particles(WIMP)

Equilibrium: $\chi\chi \leftrightarrow \mathrm{SM\ SM}$

"Freeze-out" condition

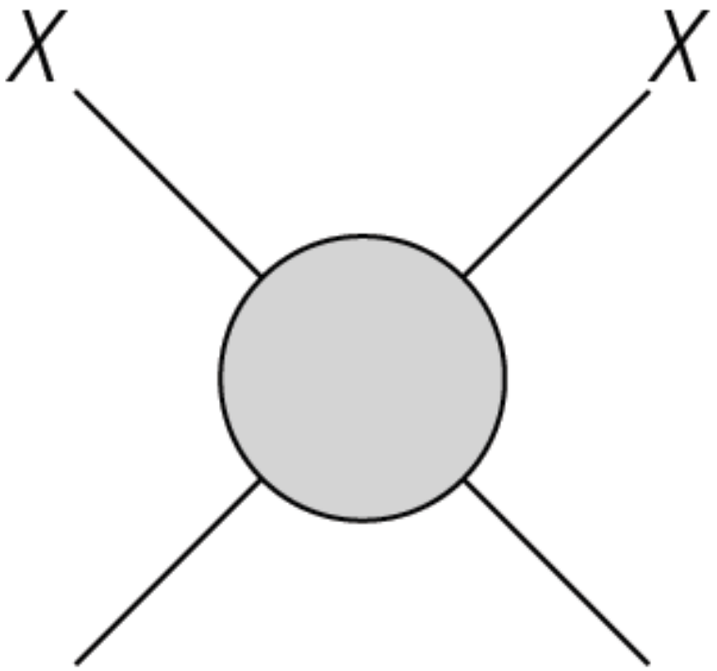
$$t = H^{-1} < t_{\rm int} = (n_{\rm DM}\sigma_A v)^{-1}$$

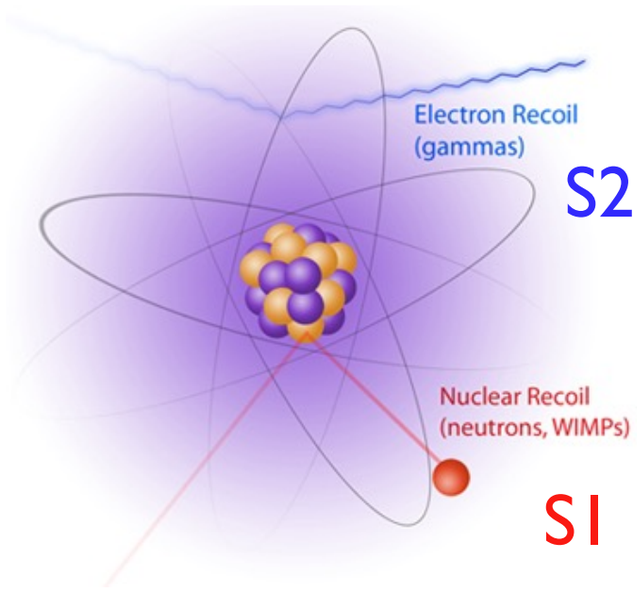
Expansion time Annihilation time

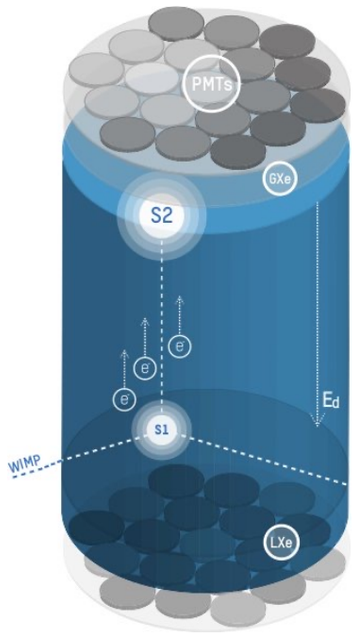
$$\Omega_{\rm DM} = 0.3\left(\frac{3\times10^{-26}{\rm cm}^3/{\rm s}}{\langle\sigma v_A\rangle}\right)$$

: insensitive to initial history.

Dark matter on Earth







e.g. XENON1T

S1: Scintillation (photons)

S2: Ionization (electrons)

WIMP:

$S1/S2 >> (S1/S2)_\gamma$

60~600M DM particles per sec go through our body.

Nucleus recoil E: $E_R = \frac{\vec{q}^2}{2m_N} = \frac{(\mu v)^2}{m_N} \lesssim 50\text{keV}$

Event rate: $\frac{dR}{dE_R} = \frac{\rho_\odot}{m_{\rm DM}} \left\langle \frac{d\sigma}{dE_R} v \right\rangle$ ~1 event/kg/day

Experiment (Nucleus)	Z	A
LUX (Xe)	54	129
XENON1T (Xe)	54	131
PandaX-II (Xe)	54	136
SuperCDMS (Ge)	32	73
CDMSlite (Ge)	32	73
XENON10 (Xe)	54	131
DarkSide-50 (Ar)	18	39

NaI, $CaWO_4$, etc.

^{127}I, ^{29}Si, ^{19}F, ^{23}Na, ^{27}Al

Astrophysics

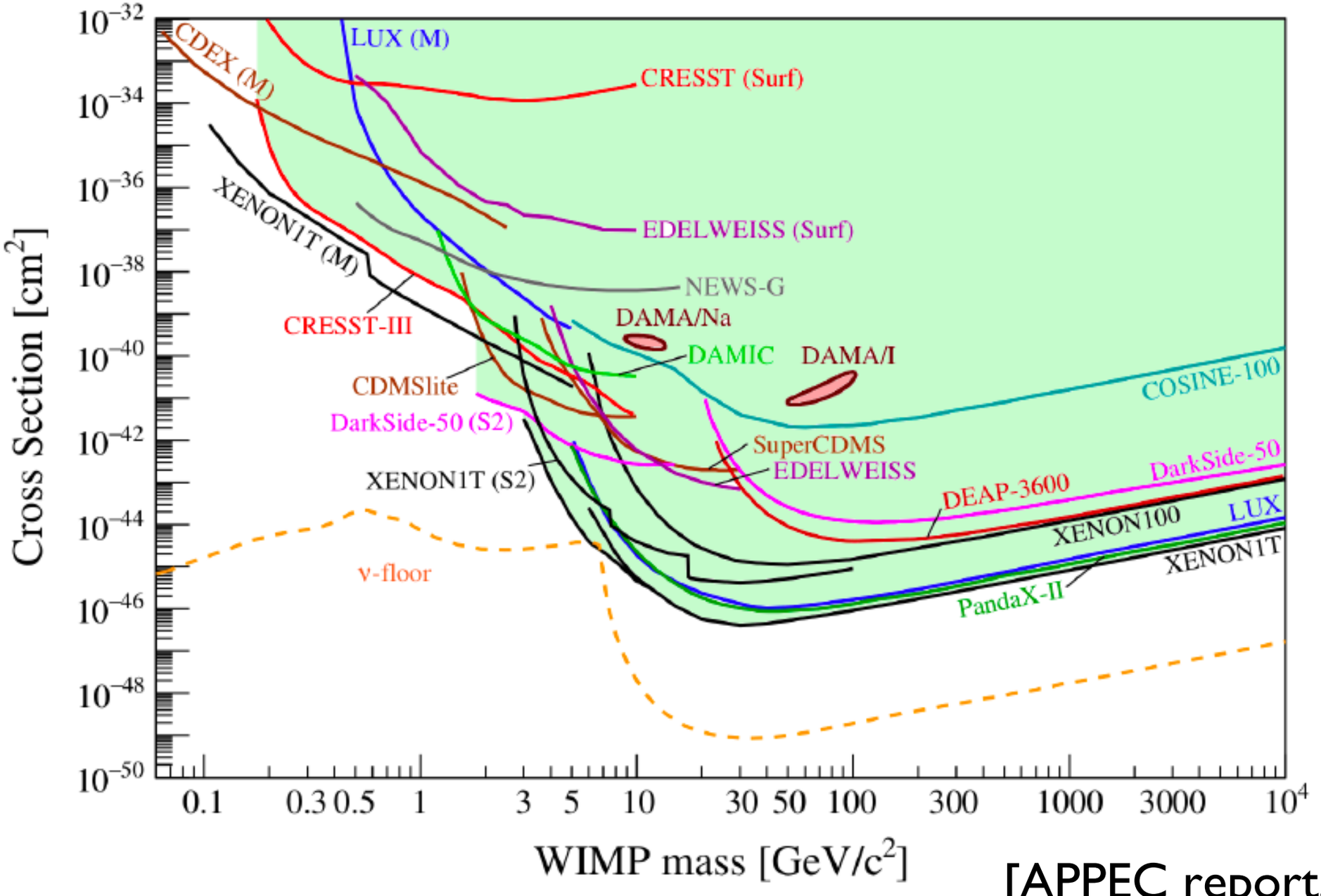
$\rho_\odot = 0.3\,\text{GeV/cm}^3$

$v_{\rm ave} = 220\,\text{km/s}$

Particle Physics

DM spin, mass, interactions

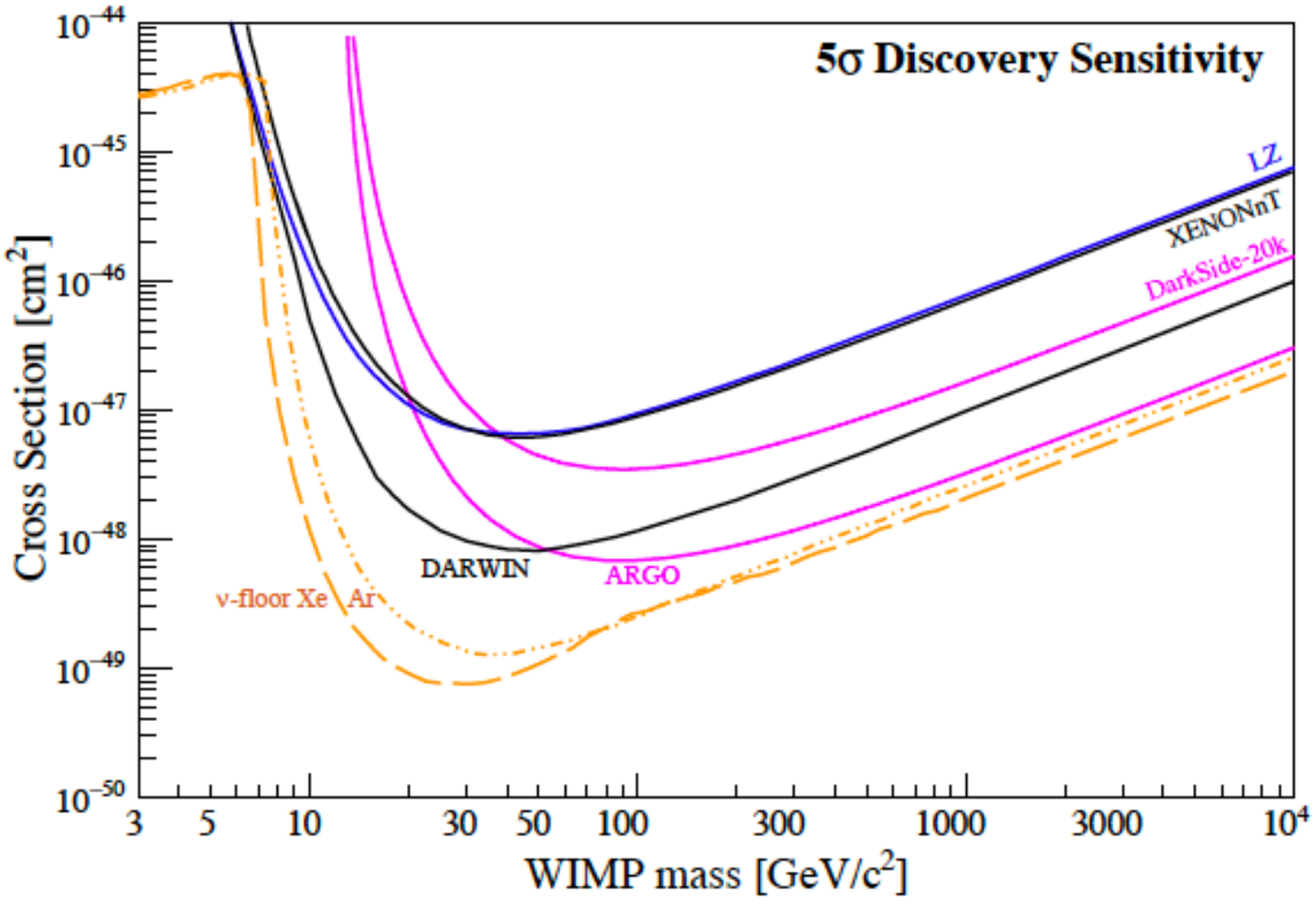
Dark matter bounds



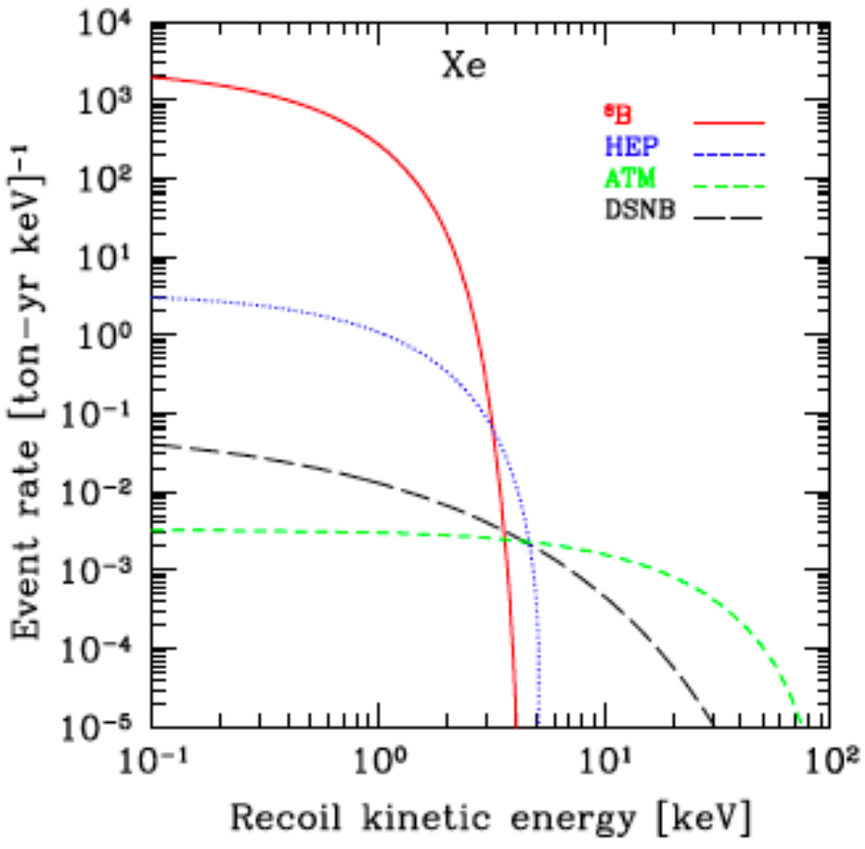
[APPEC report, 2104.07634]

- DM direct detection constrains strongly the interactions between nucleons and dark matter.

Future DM detection



[APPEC report, 2104.07634]

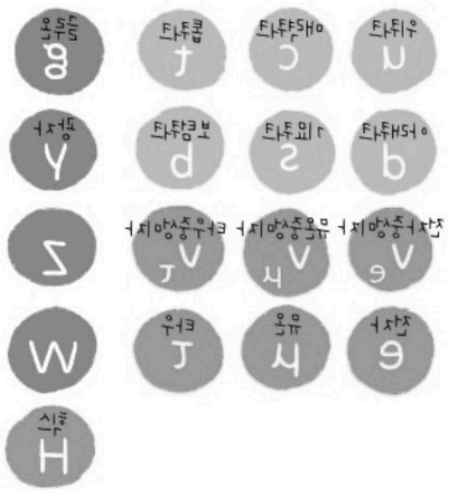


[L. Strigari, 2009]

- Future searches: XENONnT, LZ & DARWIN, etc.
- Challenging to distinguish from the background signals of solar, atmospheric and SN neutrinos.

DM portals

Dark Matter



$$|H|^2\mathcal{O}_{\rm dark}$$

$$B_{\mu\nu}\mathcal{O}^{\mu\nu}_{\rm dark}$$

Standard Model



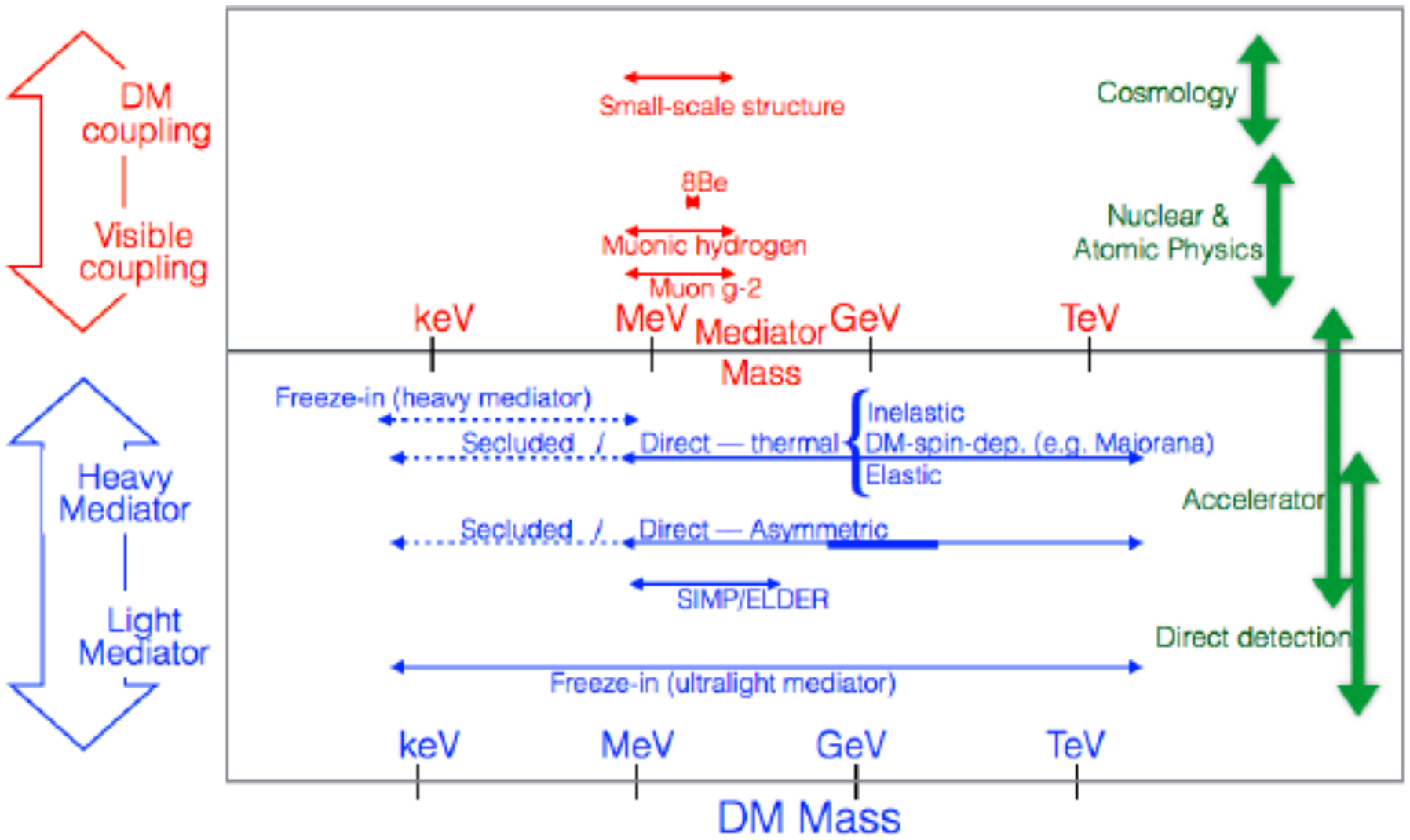
$$N(HL+\mathcal{O}_{\rm dark})$$

$$\frac{a}{f}\left(\mathcal{O}_{\rm dark}+\mathcal{O}_{\rm SM}\right)$$

$$\frac{G^{\mu\nu}}{M}\left(T^{\mu\nu}_{\rm dark}+T^{\mu\nu}_{\rm SM}\right)\cdots$$

- Dark matter are neutral under SM gauge group: DM interactions are governed by mediator particles.
- Higgs portal, Z’ portal, neutrino-portal, axion portal, graviton portal, and new types of portals: leptoquark, four-form, etc.

New visions for dark matter



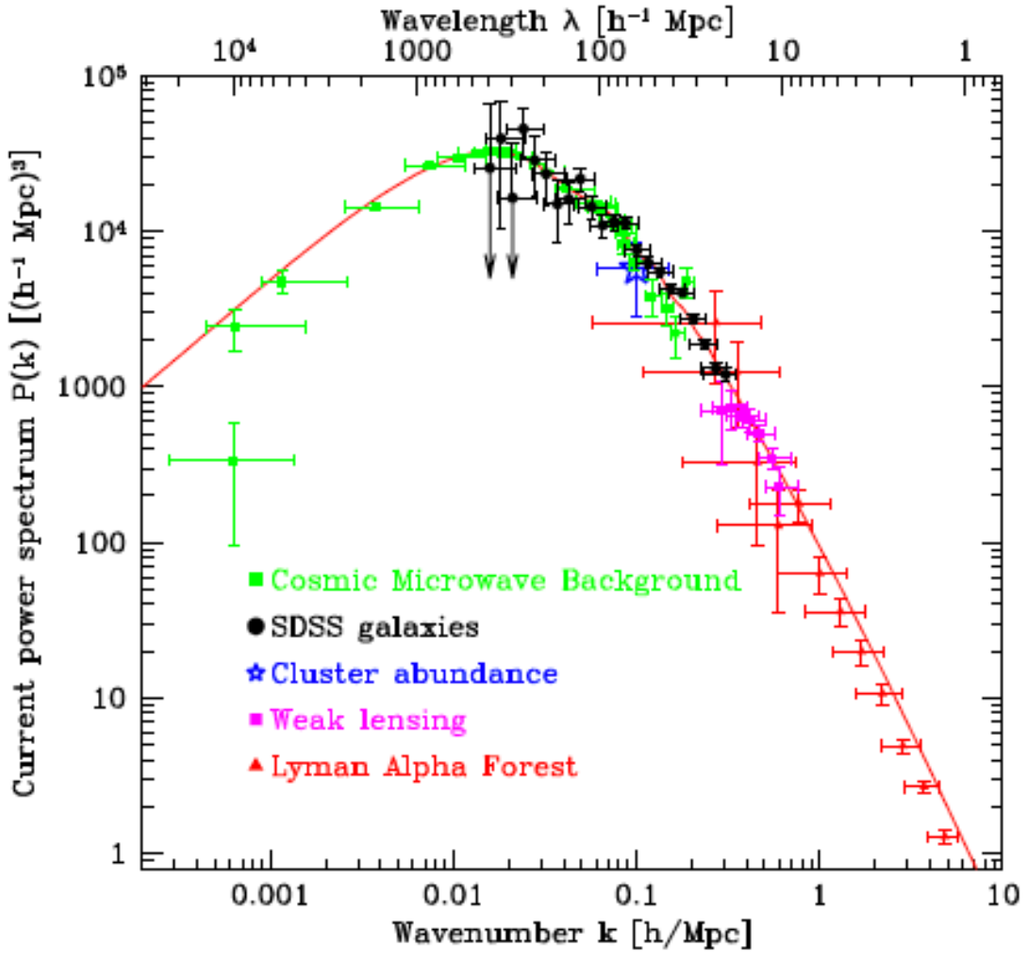
US Cosmic Visions, 1707.04591

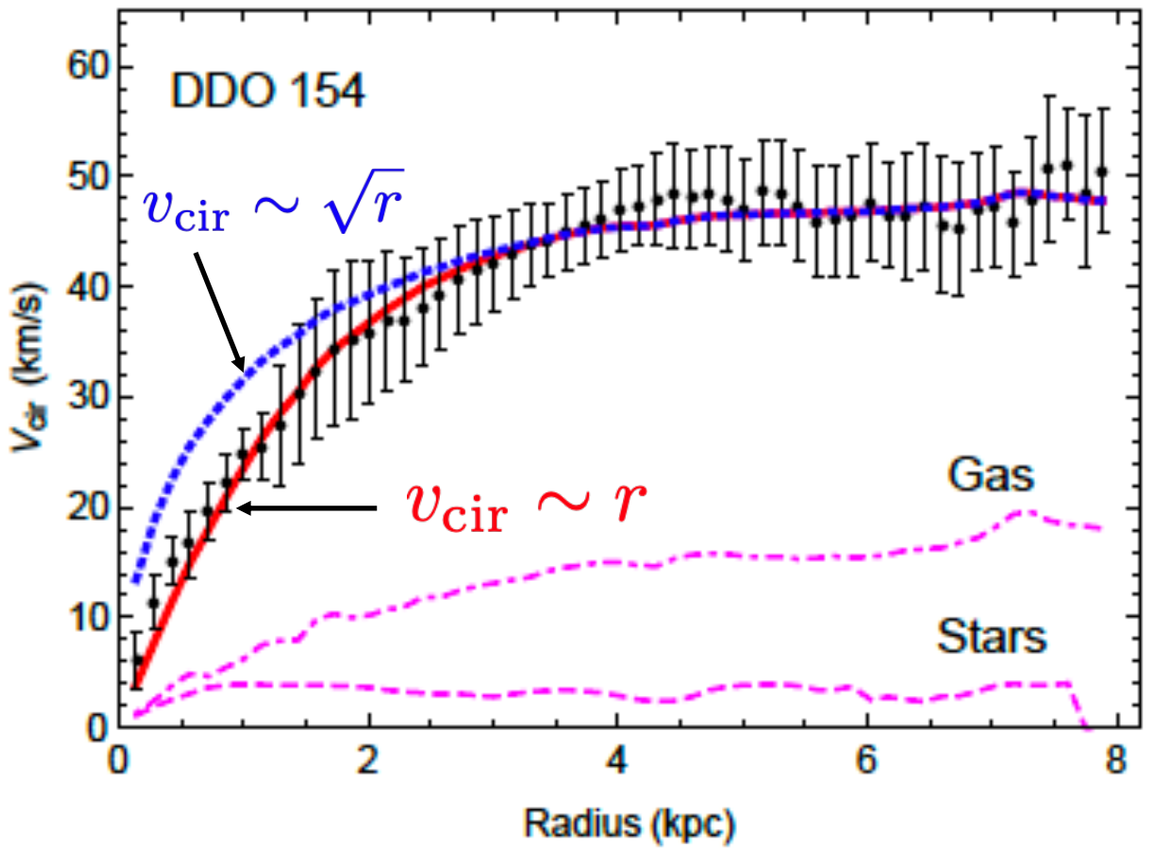
Various production mechanisms beyond WIMP paradigm.

Light and weakly coupled ⟷ New strategies for dark matter searches

Dark matter with contract interactions

Small-scale crisis





WIMP explains large scale structures (>Mpc).

Galaxies at small scales(~kpc) [Spergel, Steinhardt, 2000; Tulin, Yu, 2017]

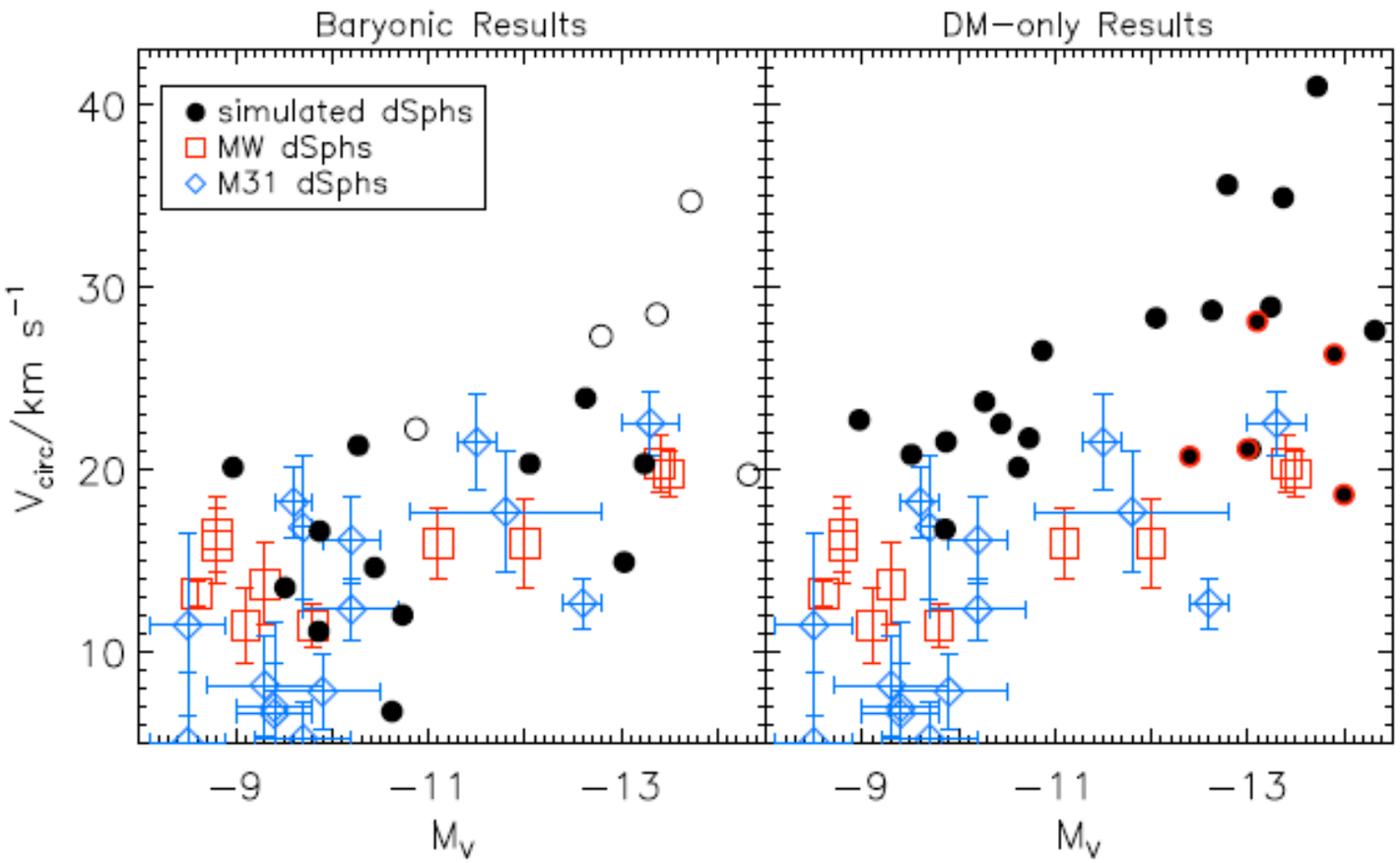
- Core-Cusp problem:

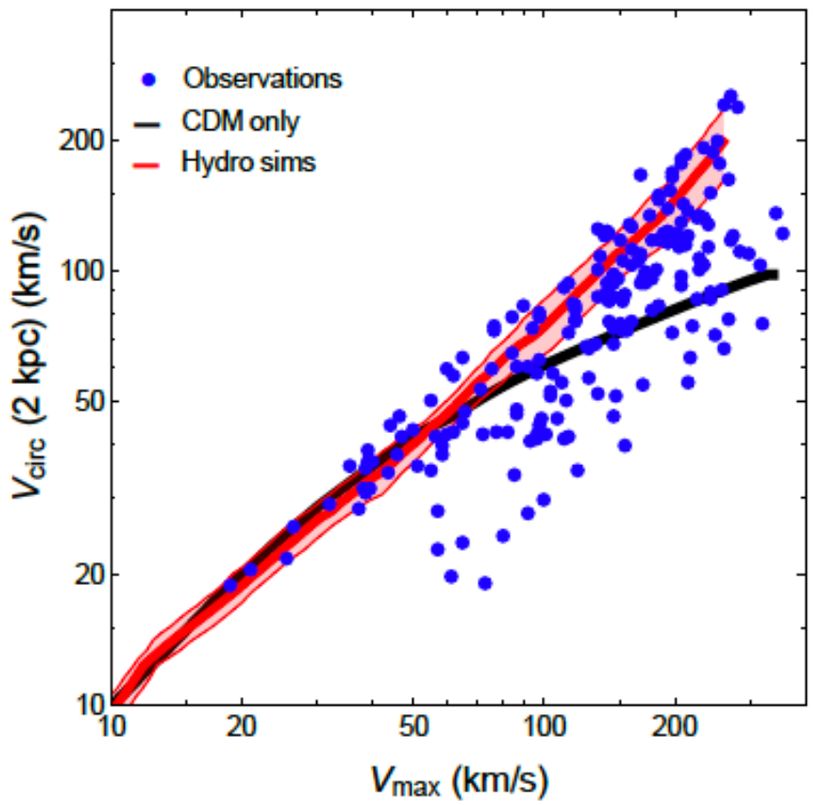
$$v_{\rm cir} = \sqrt{\frac{GM(r)}{r}}$$

WIMP N-body simulation: $v_{\rm cir} \sim \sqrt{r}, \quad \rho_{\rm dm} \sim r^{-1}$ "Cuspy"

Observed galaxies: $v_{\rm cir} \sim r, \quad \rho_{\rm dm} \sim r^{0}$ "Cored"

Too-big-to-fail problem





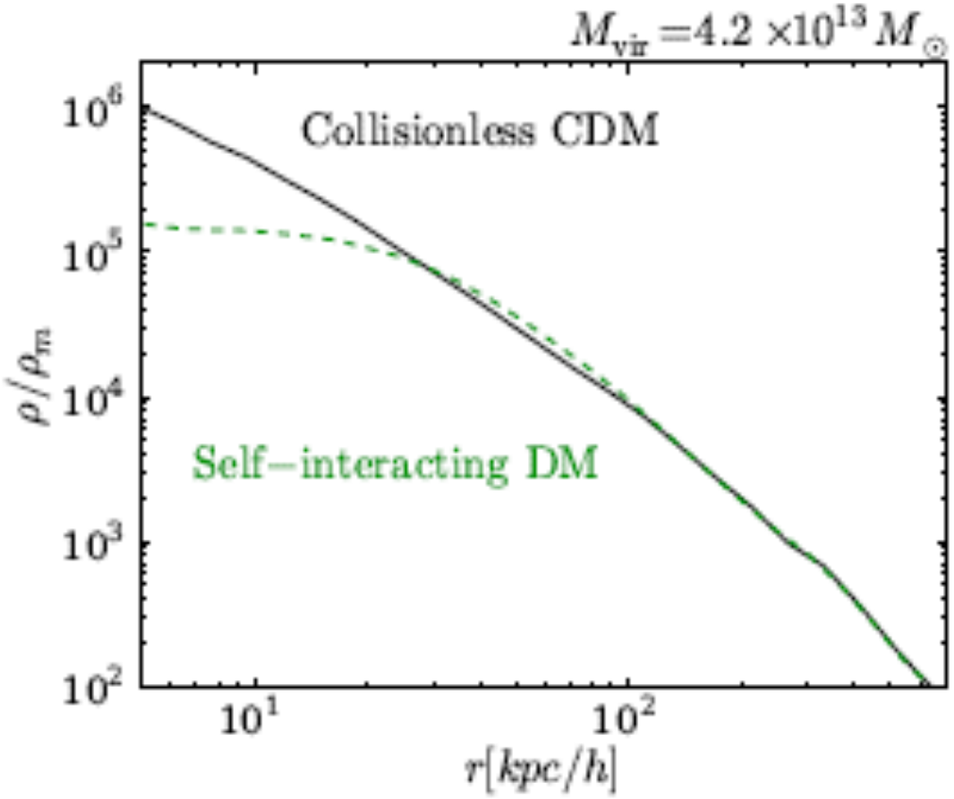
[Brooks & Zolotov, 2012]

Massive subhalos are too dense to host dwarf galaxies.

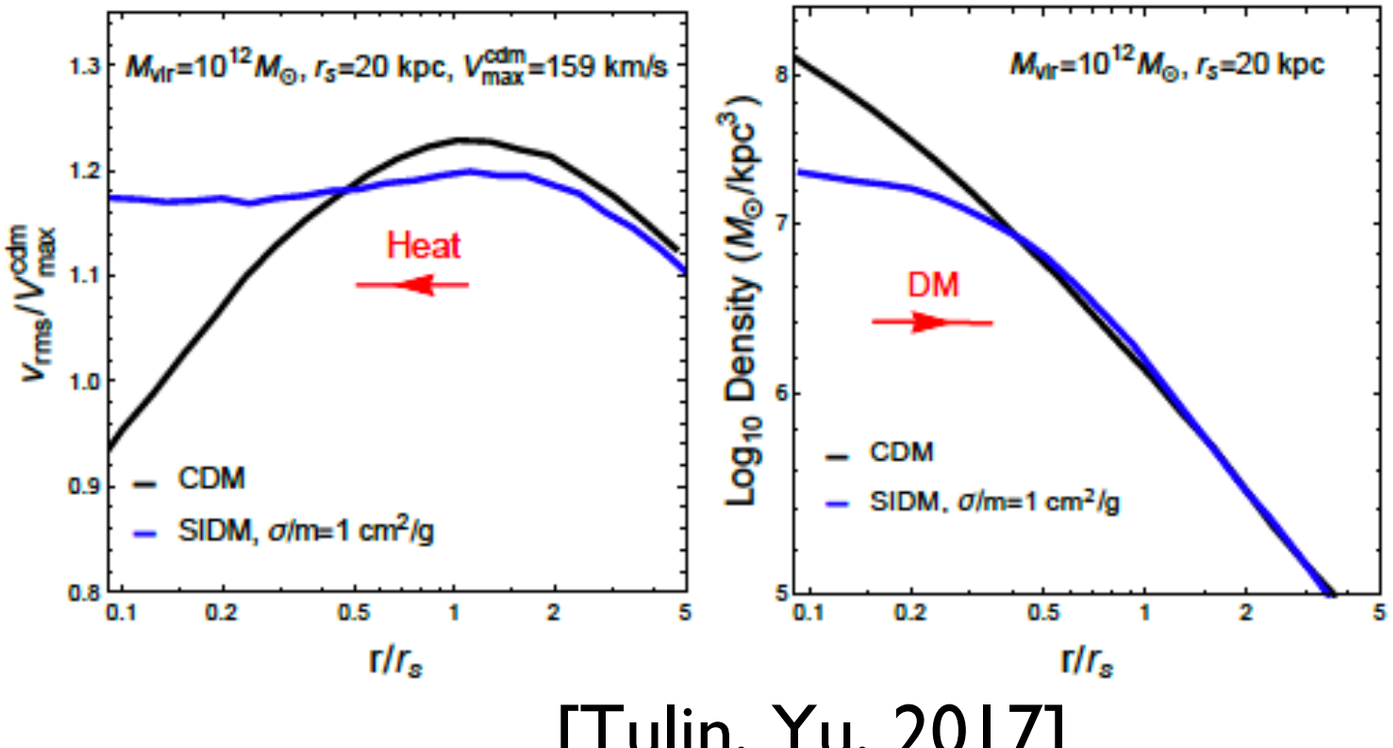
Baryonic effect (SN feedback) can make halo profile shallow.

Diversity problem: large scatter for the same maximum velocity

Self-interacting dark matter



[Weinberg et al, 2013]



[Tulin, Yu, 2017]

DM scattering transport heat from hotter outer to cooler inner of DM halo.

"Cuspy" ➡ "Cored"

"Self-Interacting Dark Matter (SIDM)"

$$\frac{\sigma}{m_{\rm DM}} \sim 1\,{\rm cm}^2/{\rm g} \sim 10^{-24}\,{\rm cm}^2/{\rm GeV}$$

cf. Warm DM (DM velocity), Ultralight DM (de Broglie~halo radius):

➡ incompatible with Lyman-α forest (at large-scales).

Hints for SIDM?

Yes

Positive observations	σ/m	$v_{\rm rel}$	Observation
Cores in spiral galaxies (dwarf/LSB galaxies)	$\gtrsim 1\ {\rm cm}^2/{\rm g}$	$30-200\ {\rm km/s}$	Rotation curves
Too-big-to-fail problem			
Milky Way	$\gtrsim 0.6\ {\rm cm}^2/{\rm g}$	$50\ {\rm km/s}$	Stellar dispersion
Local Group	$\gtrsim 0.5\ {\rm cm}^2/{\rm g}$	$50\ {\rm km/s}$	Stellar dispersion
Cores in clusters	$\sim 0.1\ {\rm cm}^2/{\rm g}$	$1500\ {\rm km/s}$	Stellar dispersion, lensing
Abell 3827 subhalo merger	$\sim 1.5\ {\rm cm}^2/{\rm g}$	$1500\ {\rm km/s}$	DM-galaxy offset
Abell 520 cluster merger	$\sim 1\ {\rm cm}^2/{\rm g}$	$2000-3000\ {\rm km/s}$	DM-galaxy offset
Constraints			
Halo shapes/ellipticity	$\lesssim 1\ {\rm cm}^2/{\rm g}$	$1300\ {\rm km/s}$	Cluster lensing surveys
Substructure mergers	$\lesssim 2\ {\rm cm}^2/{\rm g}$	$\sim 500-4000\ {\rm km/s}$	DM-galaxy offset
Merging clusters	$\lesssim$ few $\ {\rm cm}^2/{\rm g}$	$2000-4000\ {\rm km/s}$	Post-merger halo survival (Scattering depth $\tau < 1$)
Bullet Cluster	$\lesssim 0.7\ {\rm cm}^2/{\rm g}$	$4000\ {\rm km/s}$	Mass-to-light ratio

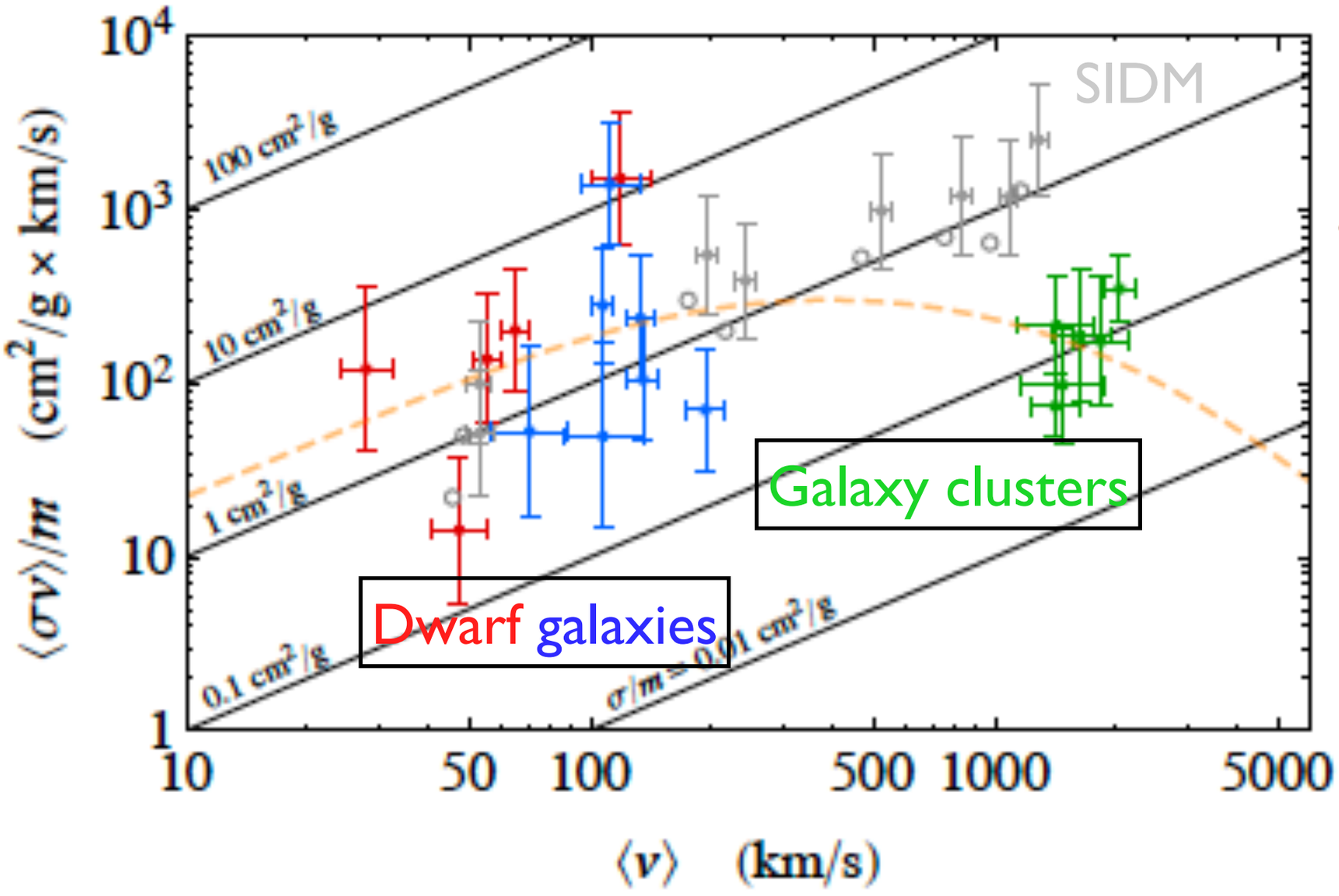
No

small DM velocity

large DM velocity

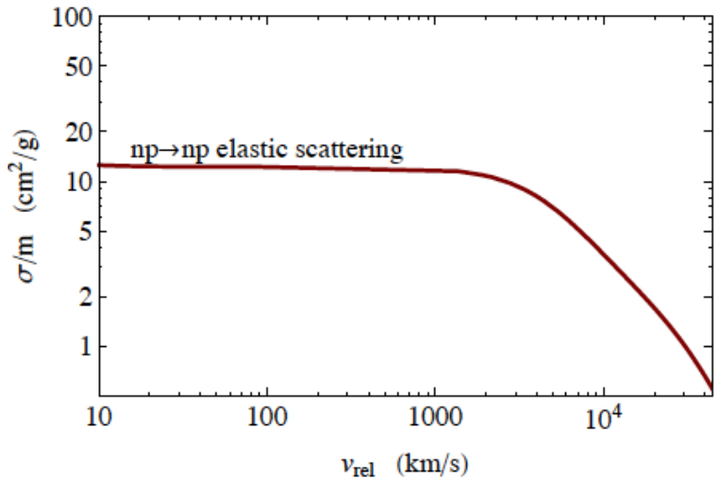
[S. Tulin and H. Yu, Phys. Rept. 730, 1(2018)]

SIDM: velocity matters!



THINGS dwarf galaxies (red), LSB galaxies (blue), and clusters (green).

SIDM N-body simulations, with $\sigma/m = 1\ \mathrm{cm}^2/\mathrm{g}$, *(gray)*



$\langle \sigma v \rangle$ =constant

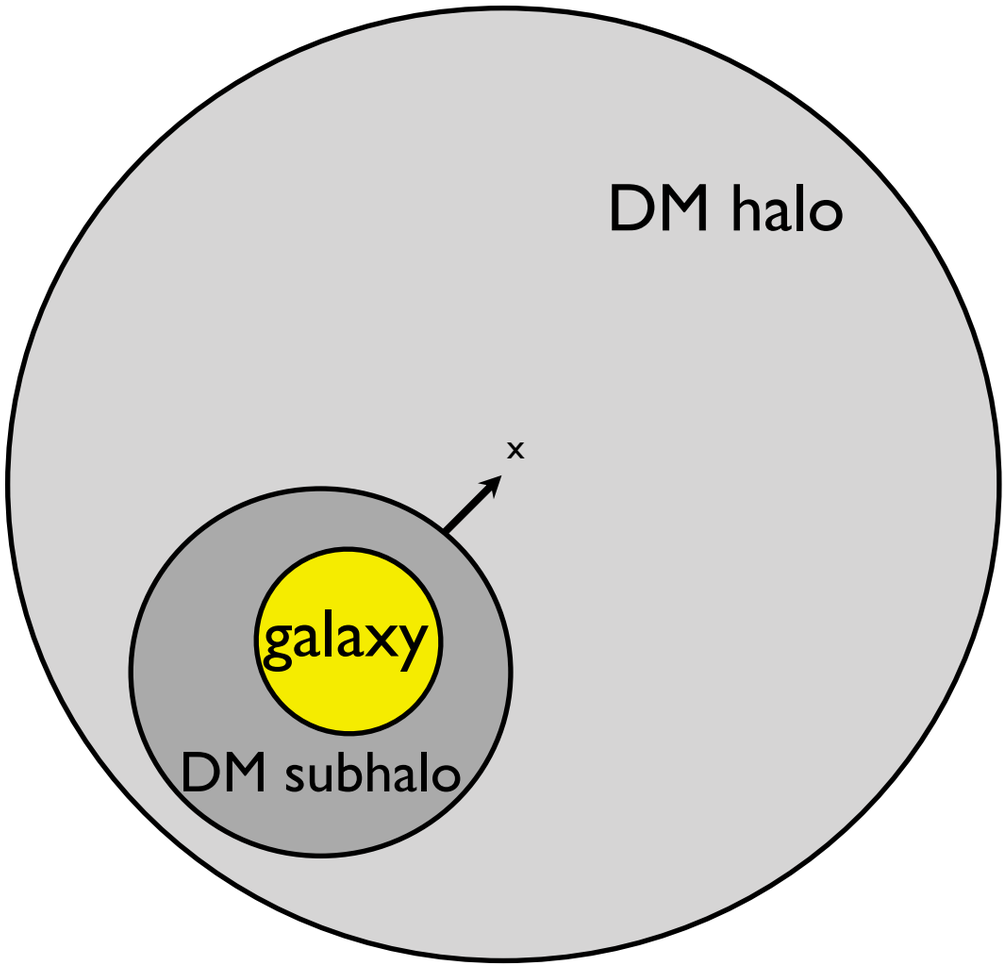
$$\Rightarrow \langle \sigma \rangle \propto \frac{1}{\langle v \rangle}$$

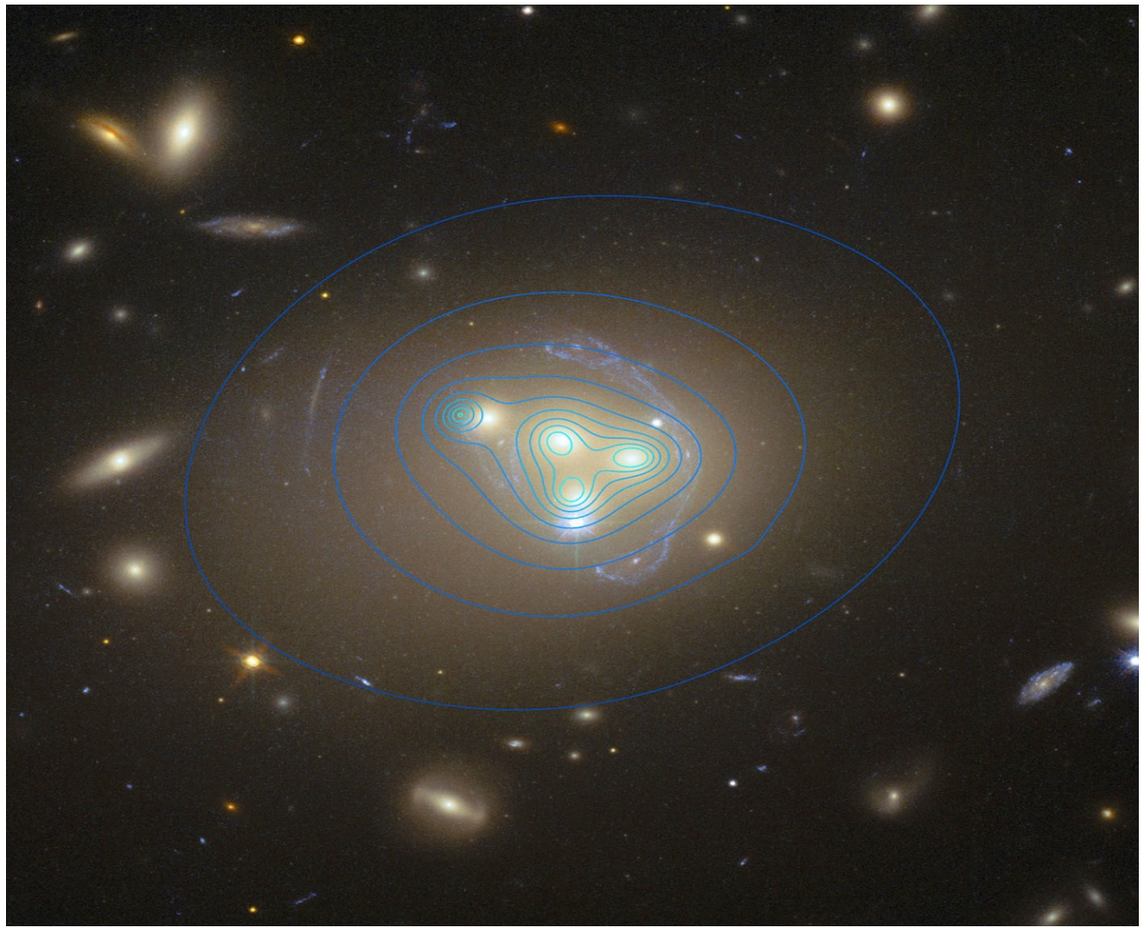
: suppressed at clusters

[M. Kaplinghat et al, 2015]

- Velocity-dependence resolves tension for galaxies & clusters.

SIDM: Halo separation





Friction between halos > Gravity between sub-halo and galaxy

→ Splitting of sub-halo from galaxy

e.g. Abell 3827 cluster

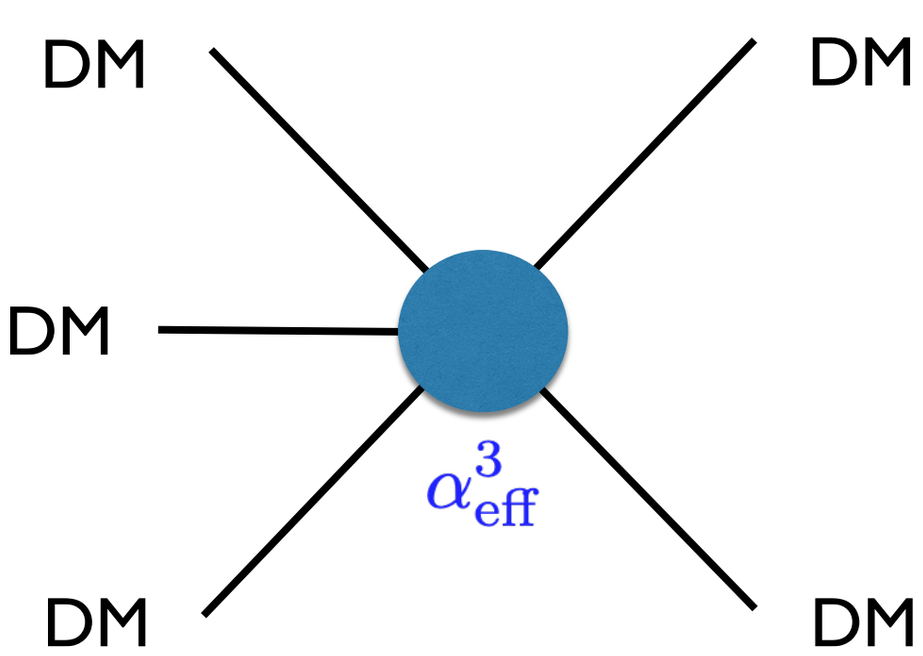
off-set: $\Delta = 1.62^{+0.47}_{-0.49}\,\mathrm{kpc}$

[Massey et al(2015); F. Kalhoefer et al (2015)]

SIMP dark matter

- Strongly Interacting Massive Particles (SIMP):
  DM self-interactions determine the dark matter density.

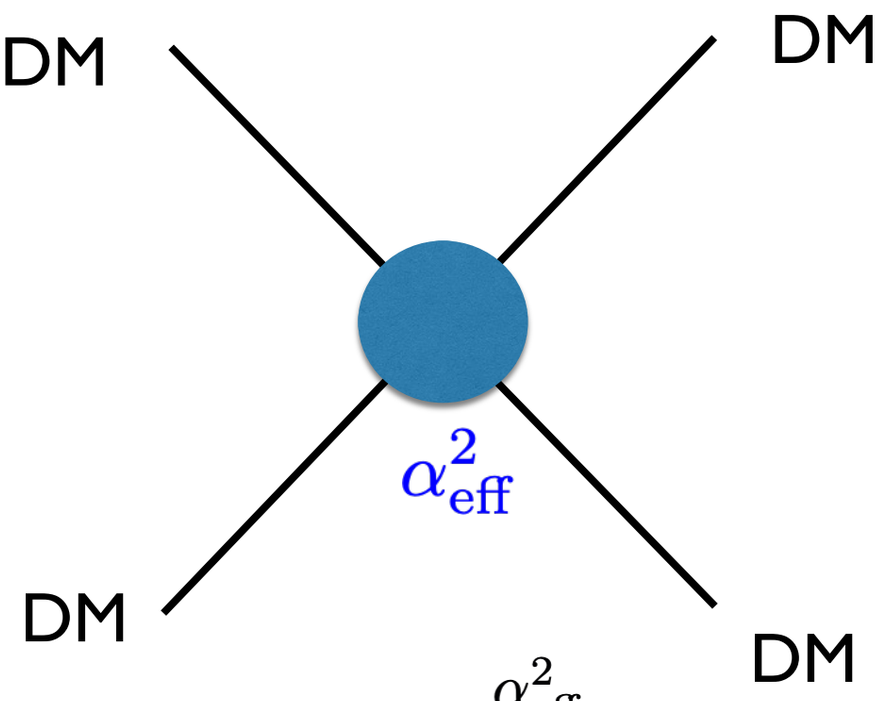
[Hochberg et al (2014); M.S. Seo, HML(2015)]



$$\frac{dn_{\rm DM}}{dt} + 3Hn_{\rm DM} \approx -\langle\sigma v^2\rangle_{3\to 2}(n_{\rm DM}^3 - n_{\rm DM}^2 n_{\rm DM}^{\rm eq})$$

Determines DM abundance

$$\Omega_{\rm DM} = 0.3\left(\frac{m_{\rm DM}/35\,{\rm MeV}}{\alpha_{\rm eff}}\right)^{3/2}$$



$$\sigma_{\rm self} = \frac{\alpha_{\rm eff}^2}{m_{\rm DM}^2}$$

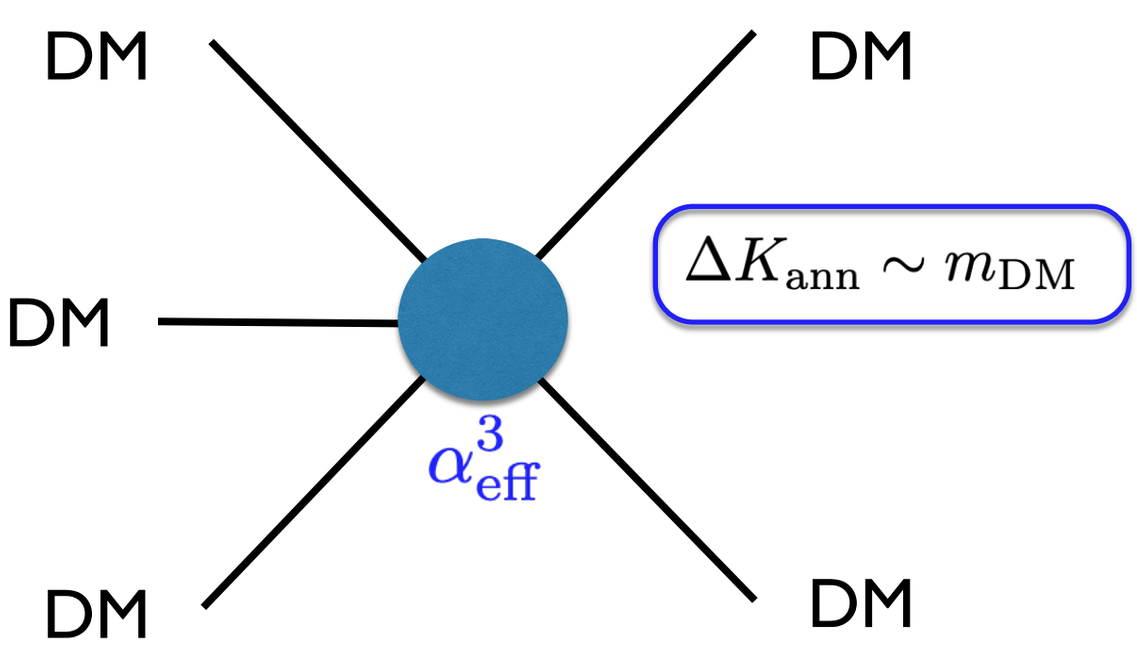
Solves core-cusp problem

$$\frac{\sigma_{\rm self}}{m_{\rm DM}} \simeq \left(\frac{5}{\alpha_{\rm eff}}\right) 1{\rm cm}^2/{\rm g}$$

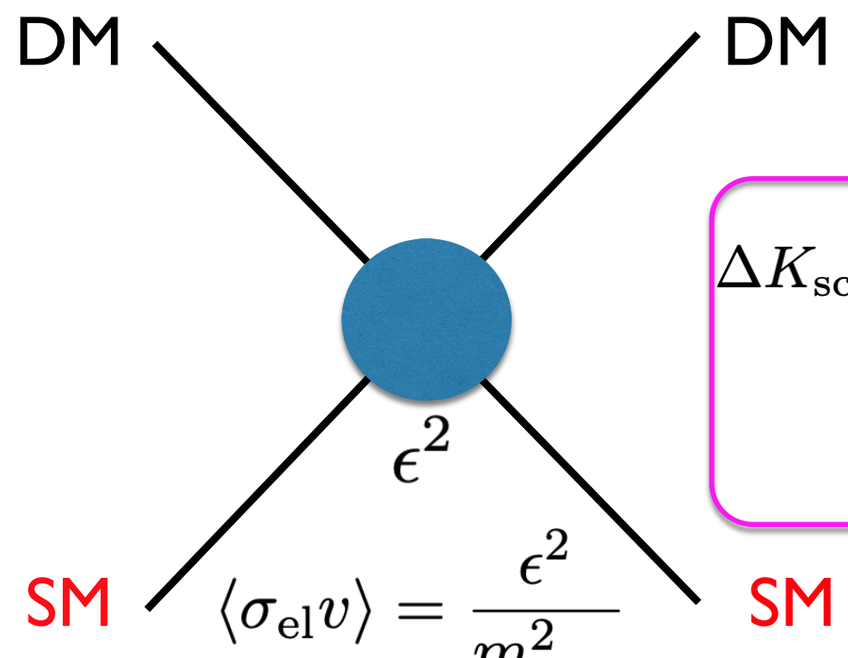
Cooling SIMP

- Equilibriate SIMP kinetic energy by kinetic scattering to avoid overheat for the structure formation.

[de Laix et al, 1995; Murayama, HML et al, 2017]



$\Delta K_{\rm ann} \sim m_{\rm DM}$



$$\Delta K_{\rm scatt} = \frac{q^2}{2m_{\rm DM}} \sim \frac{T^2}{m_{\rm DM}}$$

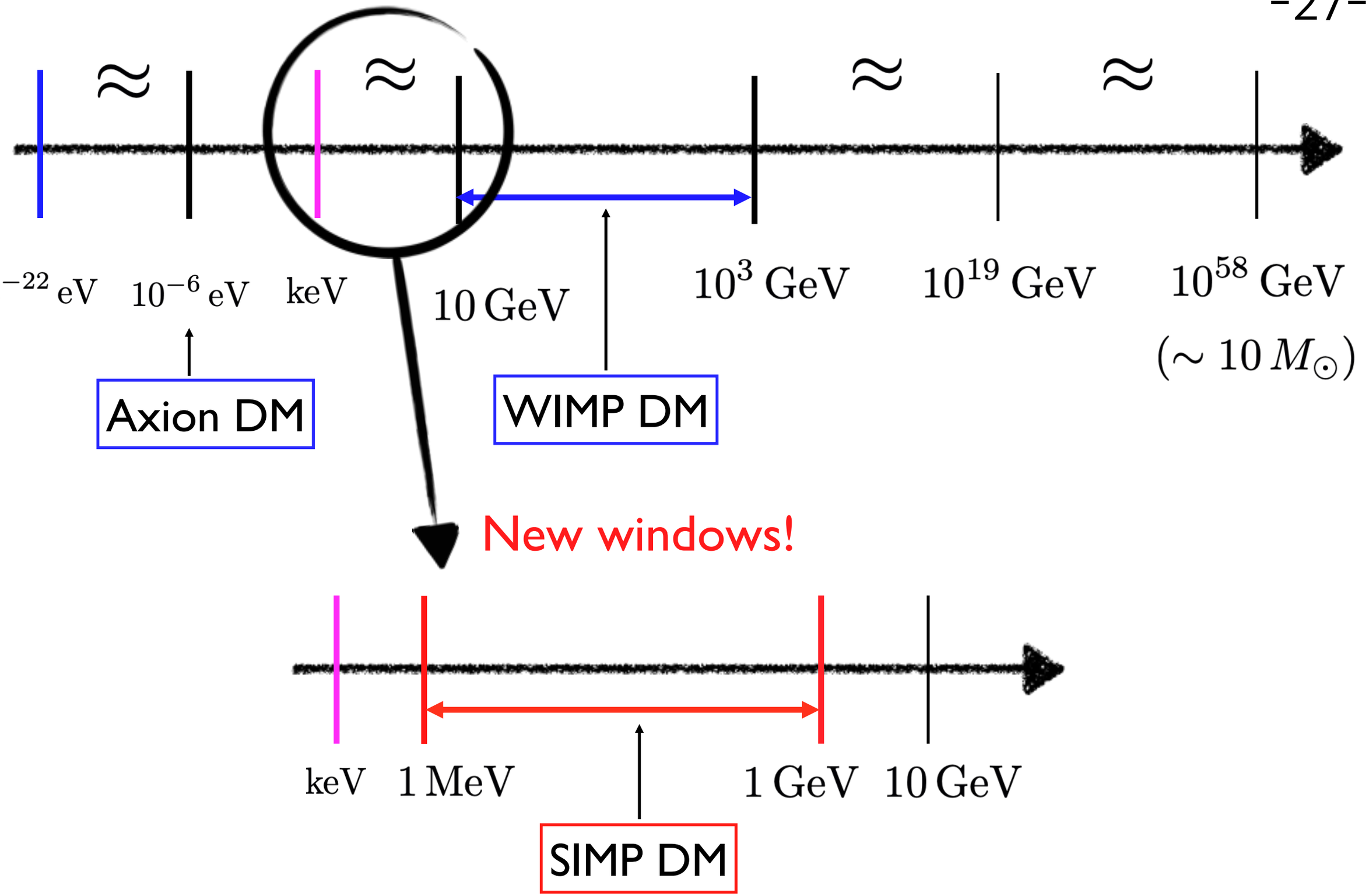
$$\langle \sigma_{\rm el} v \rangle = \frac{\epsilon^2}{m_{\rm DM}^2}$$

$$\dot{K} + 2HK = \underbrace{-m_{\rm DM}^2 H T^{-1}}_{3\to 2} + \underbrace{T\gamma(T)}_{\text{elastic scatt.}}, \qquad \gamma(T) \sim \left\langle n_Y \sigma_{\rm el} v_Y \frac{T}{m_{\rm DM}} \right\rangle.$$

Kinetic equilibrium: $\gamma(T) > H \left(\frac{m_{\rm DM}}{T} \right)^2$ ⇒ $\epsilon > 10^{-8} \alpha_{\rm eff}^{1/2}$

cf. WIMP: $\Delta K_{\rm ann} \sim T \longrightarrow \gamma(T) > H \left(\frac{m_{\rm DM}}{T} \right)$

What is (the) dark matter?

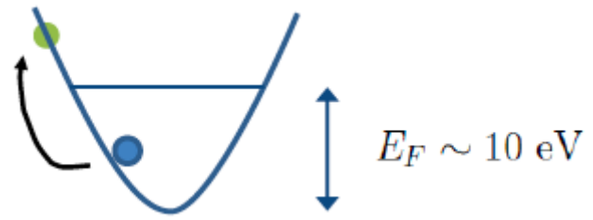


Light DM detection

- DM-electron scattering is sensitive to light dark matter.

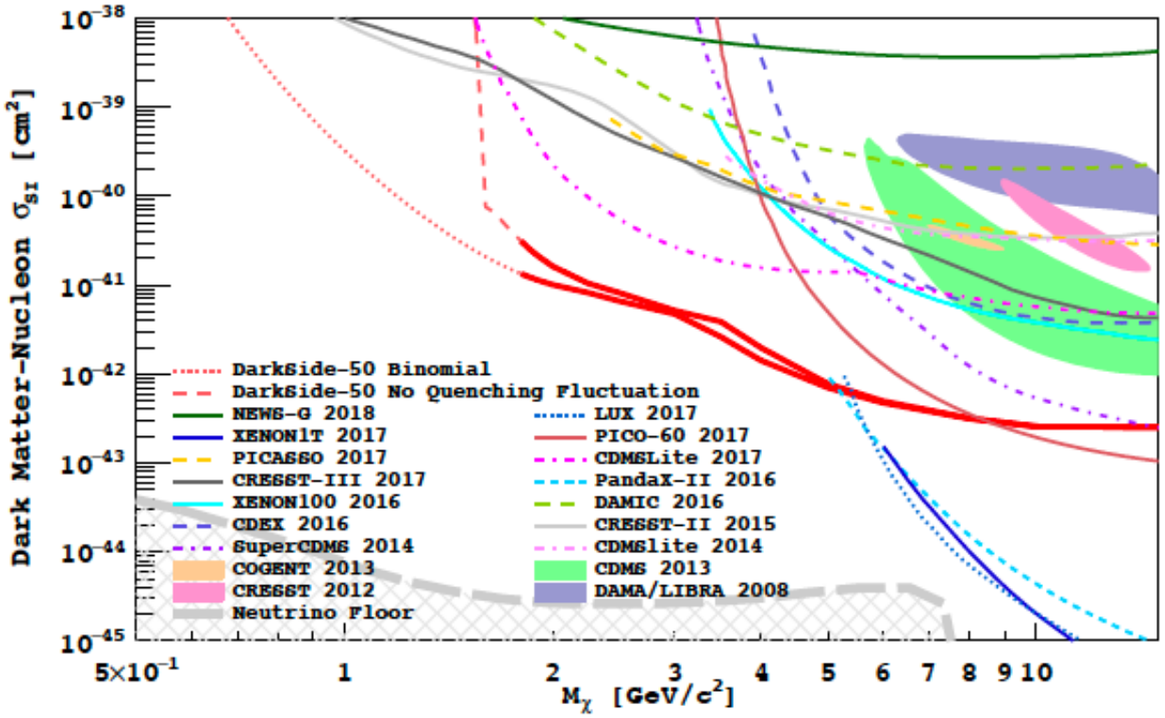
$$E_R \sim \frac{\mu^2 v_{\rm rel}^2}{m_e} \sim m_e v_{\rm rel}^2 \sim 0.3\,{\rm eV} - 20\,{\rm eV}$$

[Hochberg et al (2015)]

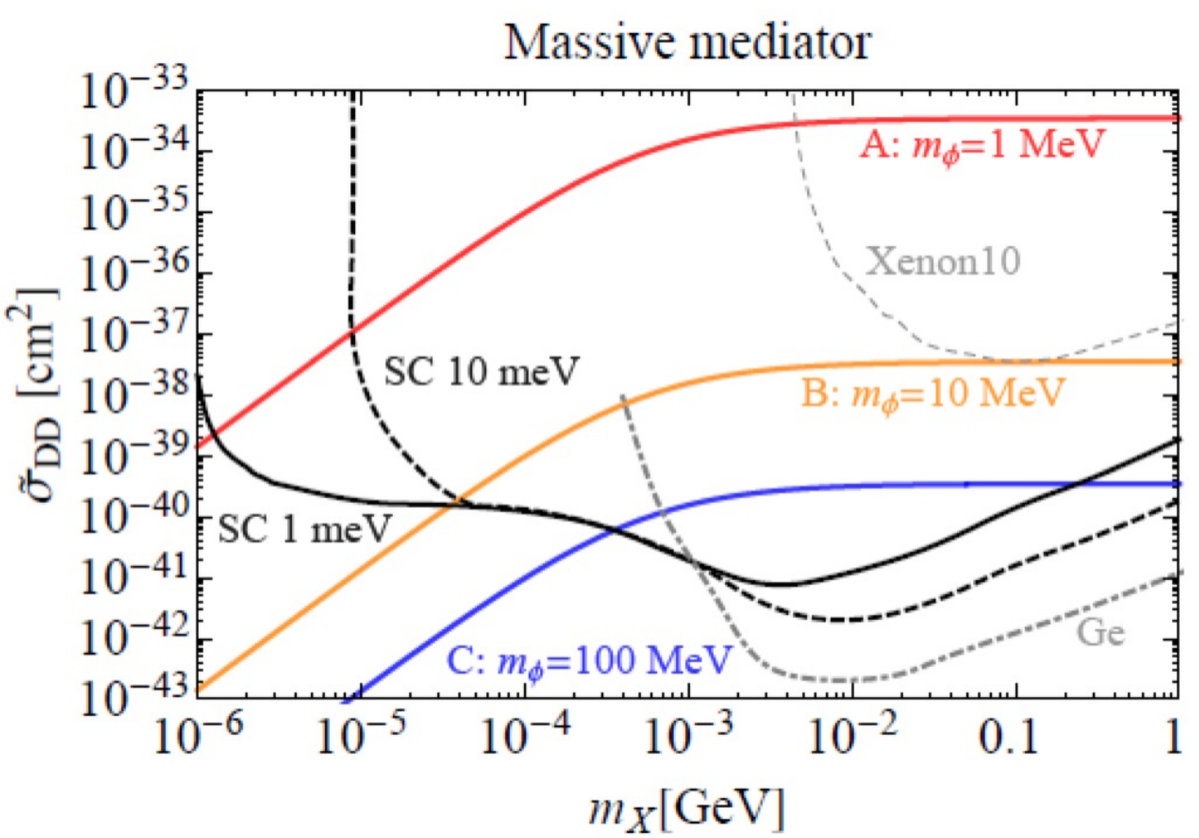


Cooper-pair breaking $m_{\rm DM} v^2 \gtrsim \Delta \sim {\rm meV}$

e.g. Al: $E_F = 11.7\,{\rm eV},\ v_{\rm rel} \sim v_F \sim 10^{-2}$



DM-nucleon scattering



DM-electron scattering

SIMP beyond Z_2

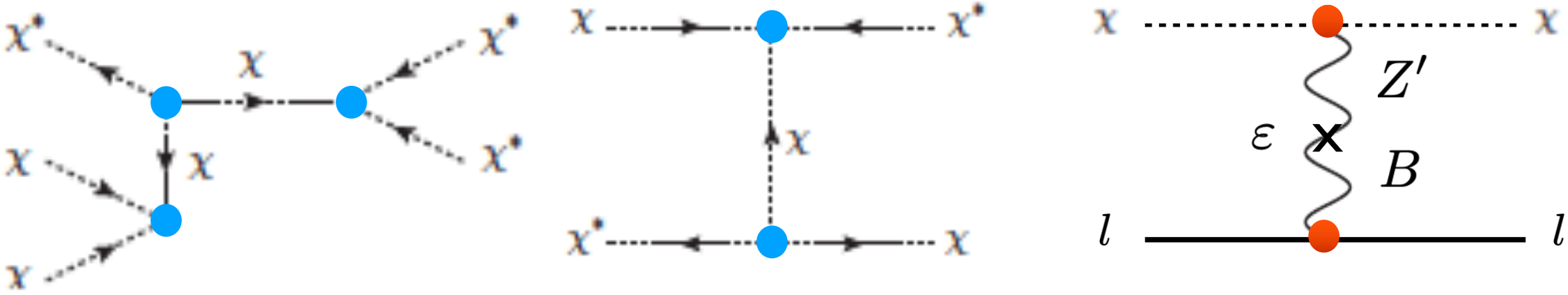
- Z_3 is the minimal discrete symmetry from a dark local U(1) and Z' portal is built-in for kinetic equilibrium.

[S.-M.Choi, HML, 2015]

	ϕ	χ
$U(1)_V$	$+3$	$+1$

Table 1: $U(1)_V$ charges.

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}v' \quad \Rightarrow \quad U(1)_V \to Z_3.$$



- Correlation btw self-scattering and 3→2 annihilation.

$$\sigma_{\rm self} \sim \frac{\kappa^4}{m_\chi}, \quad \langle\sigma v^2\rangle \sim \frac{\kappa^6}{m_\chi^5} \quad \Rightarrow \quad \alpha_{\rm eff} \sim \kappa^2$$

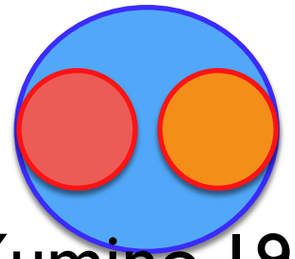
SIMP from Dark mesons

Dark light fermions
$G=SU(N_f)_L \times SU(N_f)_R$



dark condensate

Dark mesons
$H=SU(N_f)_V$ flavor sym.

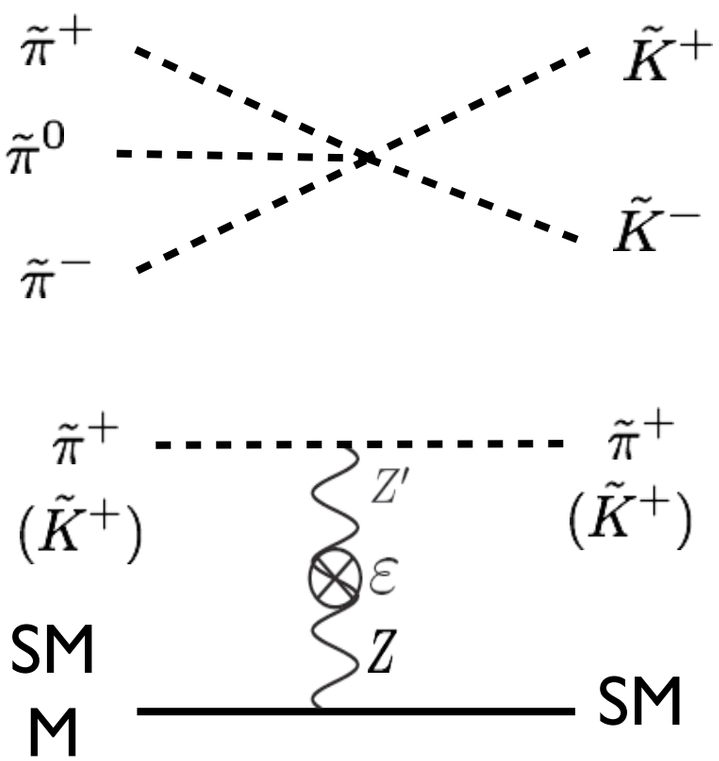


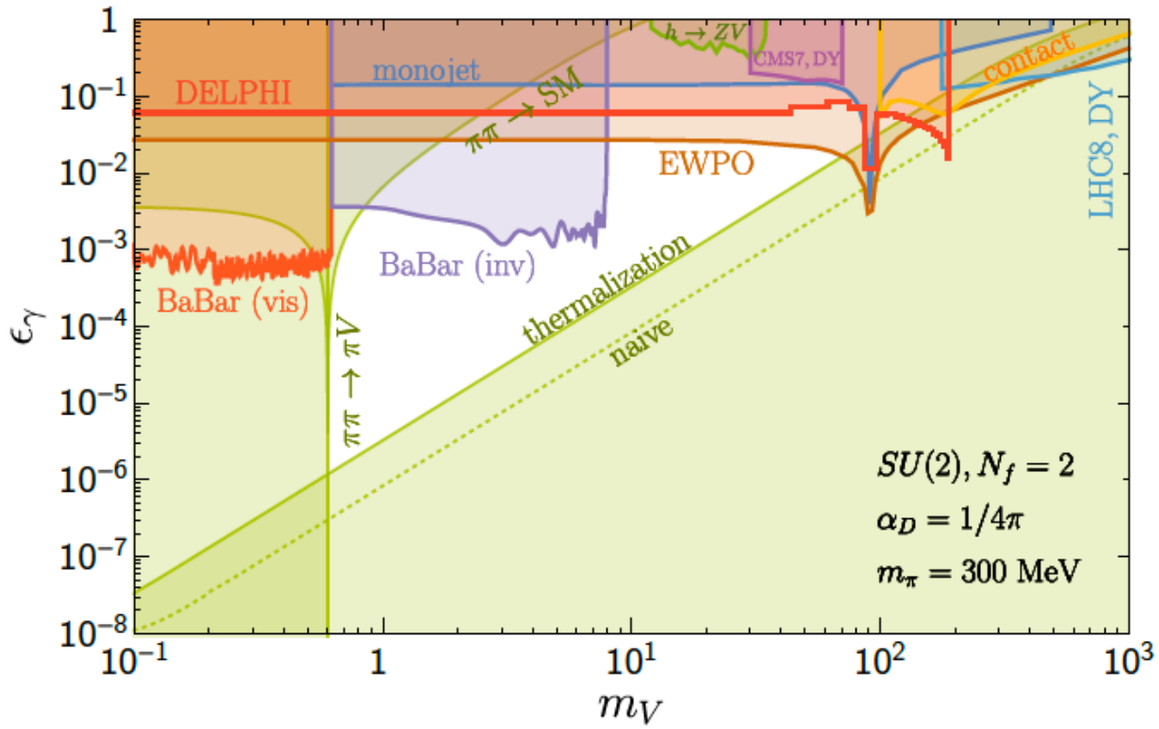
[Wess, Zumino,1971;Witten, 1983]

- Wess-Zumino-Witten term

$$\mathcal{L}_{WZW} = \frac{2N_c}{15\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[\pi\partial_\mu\pi\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi]$$

$\pi_5(G/H)=Z$

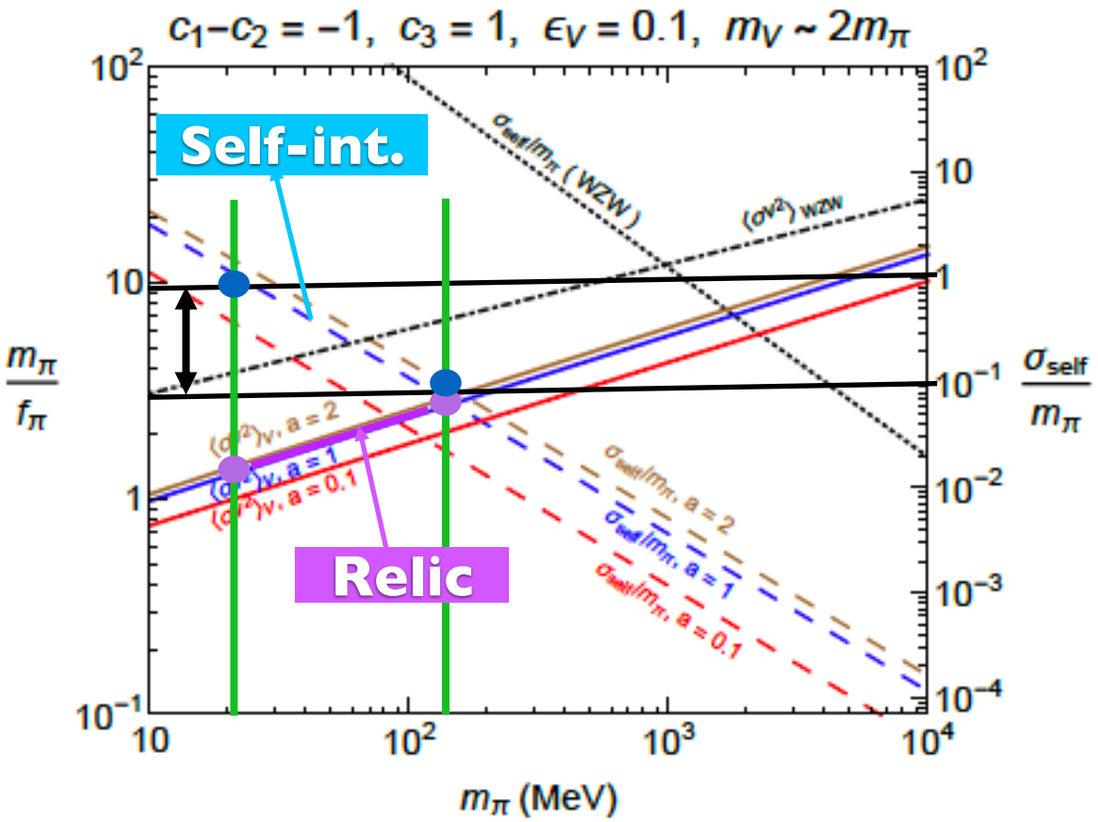




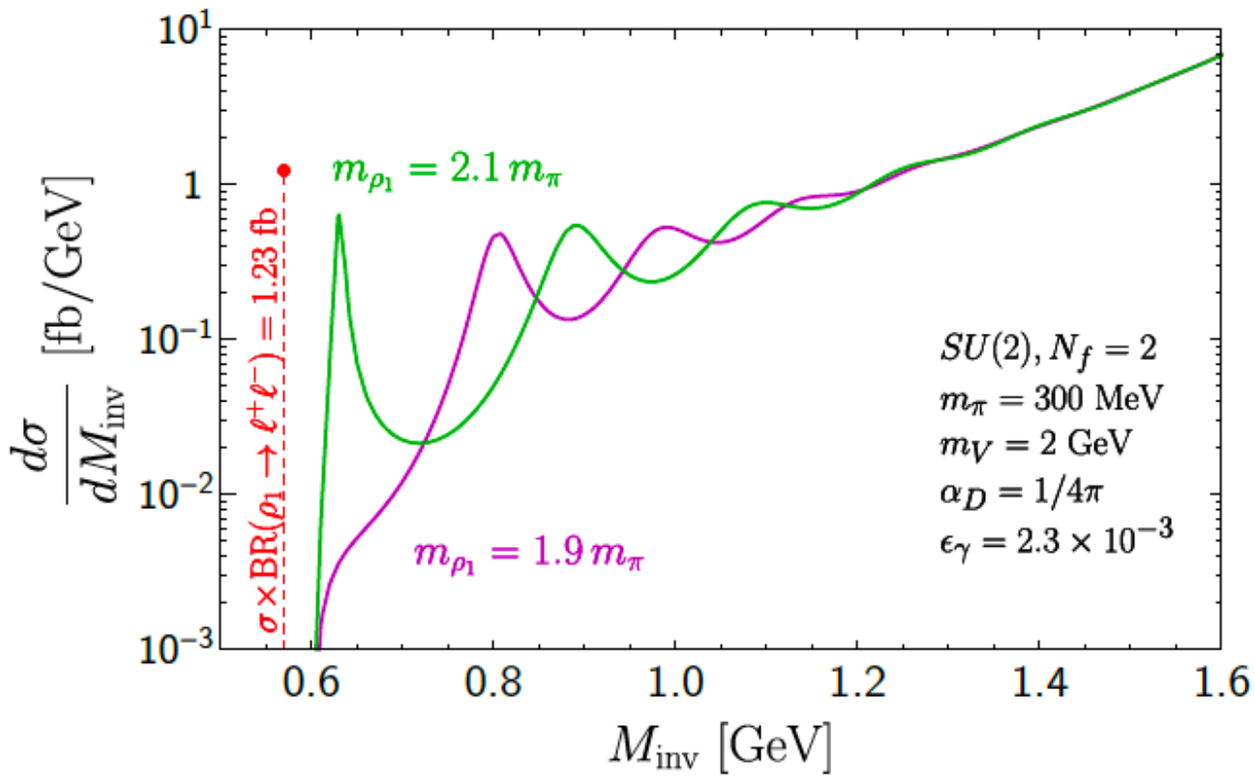
[Seo, HML, 2015; Hochberg et al, 2015]

SIMP Spectroscopy

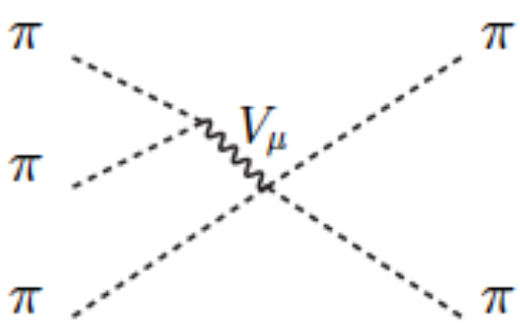
Global $SU(3)_L \times SU(3)_R/SU(3)_V$ + $H_{local}=SU(3)_V$



[S.-M. Choi, Ko, HML, Natal, 2018]



[Hochberg et al, 2015]



Perturbative SIMP couplings are allowed.

Vector mesons: invisible/lepton modes

"SIMP spectroscopy"

Dark matter with long range forces

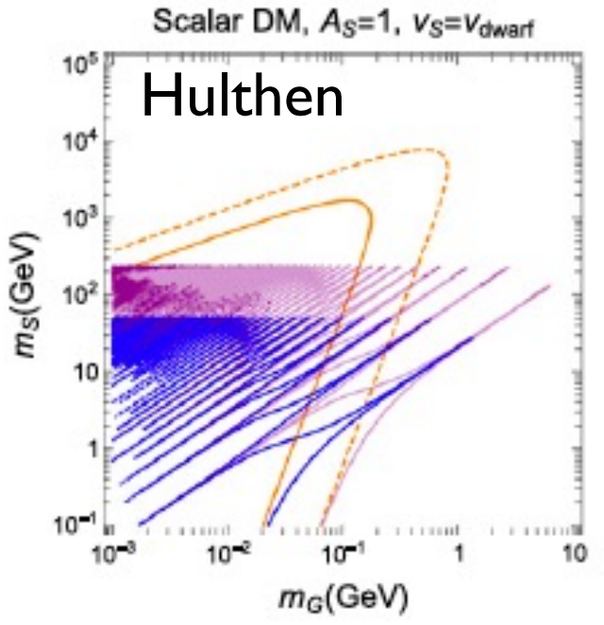
DM long-range forces

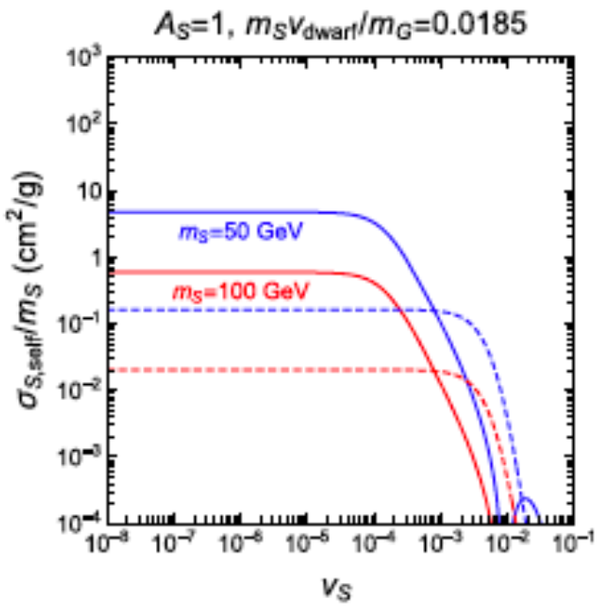
$\times e^{-m_\phi r}$

Mediator	Interaction	Yukawa $1/r$	spin-spin $(\vec{s}_1\cdot\vec{s}_2)/r$	dipole D_{12}/r^3
Scalar	$\lambda_s\bar{\chi}\chi s$	$-\lambda_s^2$	0	0
Pseudoscalar	$i\lambda_a\bar{\chi}\gamma^5\chi a$	0	$\frac{\lambda_a^2 m_a^2}{3m_\chi^2}$	$\frac{\lambda_a^2}{m_\chi^2}h(m_a,r)$
Goldstone	$\frac{1}{f}\bar{\chi}\gamma^\mu\gamma^5\chi\partial_\mu a$	0	$\frac{4}{3}\frac{m_a^2}{f^2}$	$\frac{4}{f^2}h(m_a,r)$
Vector	$g_v\bar{\chi}\gamma^\mu\chi A_\mu$	$\pm g_v^2\left(1+\frac{m_A^2}{4m_\chi^2}\right)$	$\pm\frac{2g_v^2m_A^2}{3m_\chi^2}$	$\mp\frac{g_v^2}{m_\chi^2}h(m_A,r)$
Axial vector	$g_a\bar{\chi}\gamma^\mu\gamma^5\chi A_\mu$	0	$-\frac{8g_a^2}{3}\left(1-\frac{m_A^2}{8m_\chi^2}\right)$	$g_a^2\left(\frac{1}{m_\chi^2}+\frac{4}{m_A^2}\right)h(m_A,r)$
Field strength	$\frac{i}{2\Lambda}\bar{\chi}\sigma^{\mu\nu}\chi F_{\mu\nu}$	0	$\mp\frac{2m_A^2}{3\Lambda^2}$	$\pm\frac{1}{\Lambda^2}h(m_A,r)$
Graviton	$-\frac{1}{\Lambda}T^{\chi,\mu\nu}G_{\mu\nu}$	$-\frac{2m_\chi^2}{3\Lambda^2}\left(1-\frac{3}{2}\frac{m_G^2}{m_S^2}\right)$	$-\frac{m_G^2}{3\Lambda^2}$	$\frac{1}{2\Lambda^2}h(m_G,r)$
Graviton	$-\frac{1}{\Lambda}T^{S,\mu\nu}G_{\mu\nu}$	$-\frac{2}{3}\frac{m_S^2}{\Lambda^2}\left(1-\frac{1}{2}\frac{m_G^2}{m_S^2}\right)$	0	0
Graviton	$-\frac{1}{\Lambda}T^{X,\mu\nu}G_{\mu\nu}$	$-\frac{2}{3}\frac{m_X^2}{\Lambda^2}\left(1+\frac{1}{6}\frac{m_G^2}{m_X^2}\right)$	$-\frac{m_G^2}{3\Lambda^2}$	$-\frac{1}{2\Lambda^2}h(m_G,r)$

[Kang, HML, 2020]

$$h(m,r)=1+mr+\tfrac{1}{3}(mr)^2$$





Effective DM potential with light mediators:

$$V_{\chi,\text{eff}}(r)=-\frac{1}{4m_\chi^2}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\,\mathrm{e}^{i\vec{q}\cdot\vec{r}}\mathcal{L}_{\chi,\text{eff}}$$

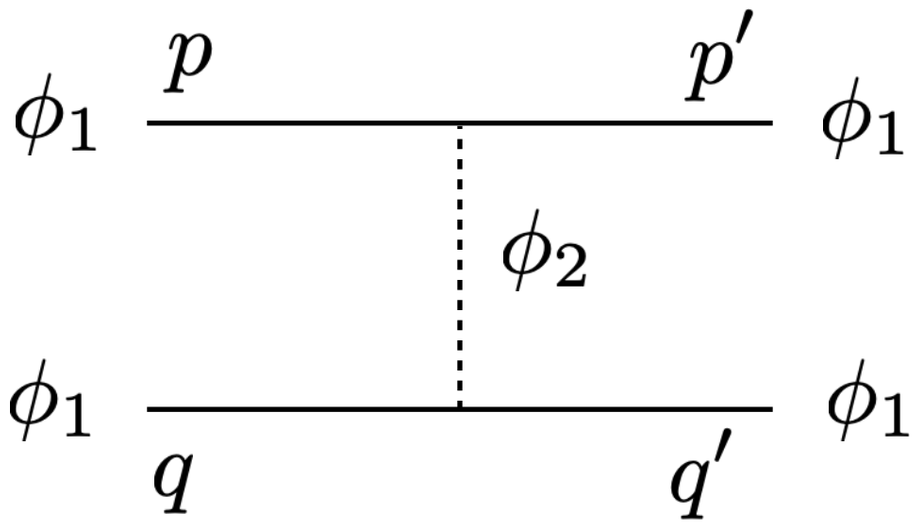
$$=\frac{1}{4\pi r}\left\{c_1(r)+c_2(r)(\vec{s}_1\cdot\vec{s}_2)+\frac{c_3(r)}{m_\chi^2r^2}\left[3(\vec{s}_1\cdot\hat{r})(\vec{s}_2\cdot\hat{r})-\vec{s}_1\cdot\vec{s}_2\right]+\frac{c_7(r)}{m_\chi r}(\vec{s}_1+\vec{s}_2)\cdot(\hat{r}\times\vec{v})\right\}\times e^{-m_\phi r}$$

t-channel forces

- Consider dark matter and mediator with masses, m_1 & m_2.

Triple coupling for dark matter: $\mathcal{L}_{\rm int} = -g\, m_1 \phi_2 \phi_1^2$

⇒ Elastic 2→2 scattering in t-channel



$$\tilde{\Gamma}_t(p,q;p',q') = \frac{4g^2 m_1^2}{|\vec{p}-\vec{p}'|^2 + m_2^2 - \omega^2}$$

$w = p_0 - p_0' \simeq 0$: "Instantaneous" interaction

⟶ Small mass m_2 for enhanced Coulomb force!

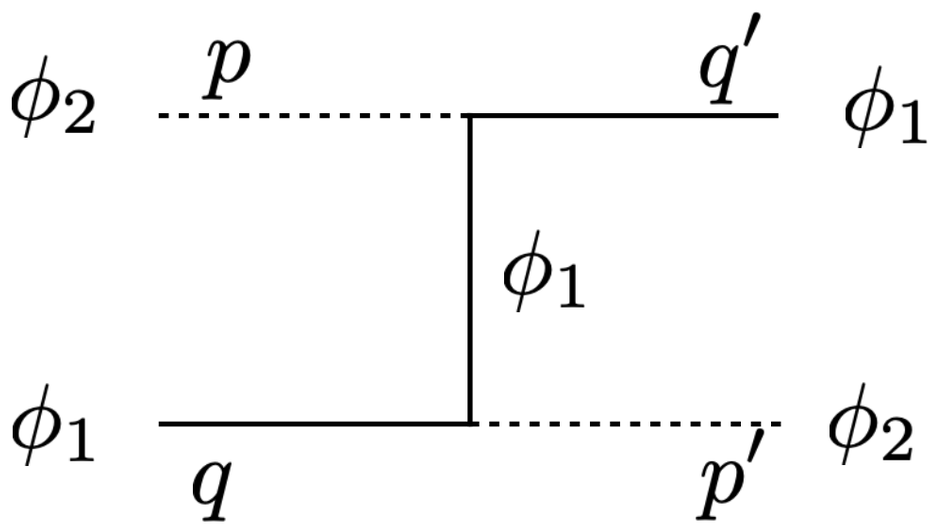
Resummation of ladder diagrams: Sommerfeld factor

u-channel forces

- Take two component dark matter with masses, m_1 & m_2

  Triple coupling between dark matter: $\mathcal{L}_{\rm int} = -g\, m_1 \phi_2 \phi_1^2$

  ⇒ Elastic 2→2 scattering in u-channel



$$\tilde{\Gamma}_u(p,q;p',q') = \frac{4g^2 m_1^2}{|\vec{p}-\vec{q}'|^2 + m_1^2 - \omega^2}$$

[S. Kim, HML, B. Zhu, 2021]

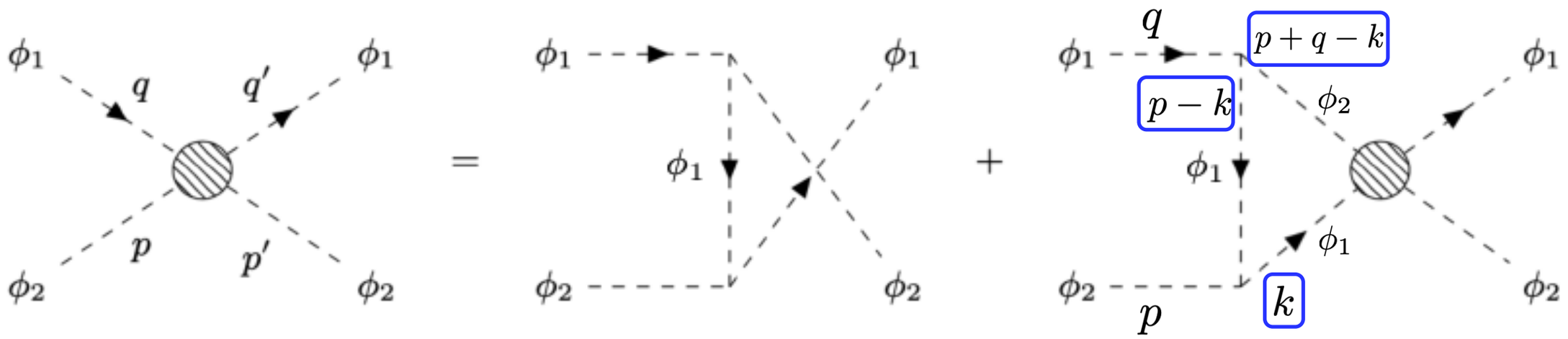
$\omega = p_0 - q_0' \approx m_2 - m_1 \neq 0$ "Delayed" interaction

$m_2 = 2m_1$ ⟶ "Effectively massless" dark matter

Resummation of ladder diagrams is also needed.

Non-perturbative scattering

Non-perturbative scattering in u-channel



Bethe-Salpeter wave function: $\chi(p,q;p',q') \equiv G_2(p)G_1(q)\Gamma(p,q;p',q')$

$$i\chi(p,q) = -G_2(p)G_1(q)\int \frac{d^4k}{(2\pi)^4}\widetilde{\Gamma}(p,q;p+q-k,k)\,\chi(p+q-k,k)$$

$$\left(-\frac{1}{2\mu}\nabla^2 - E\right)\psi_{BS}(\vec{x}) \;=\; -V(\vec{x})\psi_{BS}\left(-\frac{m_2}{m_1}\vec{x}\right)$$

$$\begin{cases} V(\vec{x}) \;= -\dfrac{\alpha}{r}\,e^{-Mr} \\ \alpha \equiv \frac{g^2}{4\pi}, \quad M \equiv m_2\sqrt{2-\dfrac{m_2}{m_1}} \end{cases}$$

[S. Kim, HML, B. Zhu, 2021, 2022]

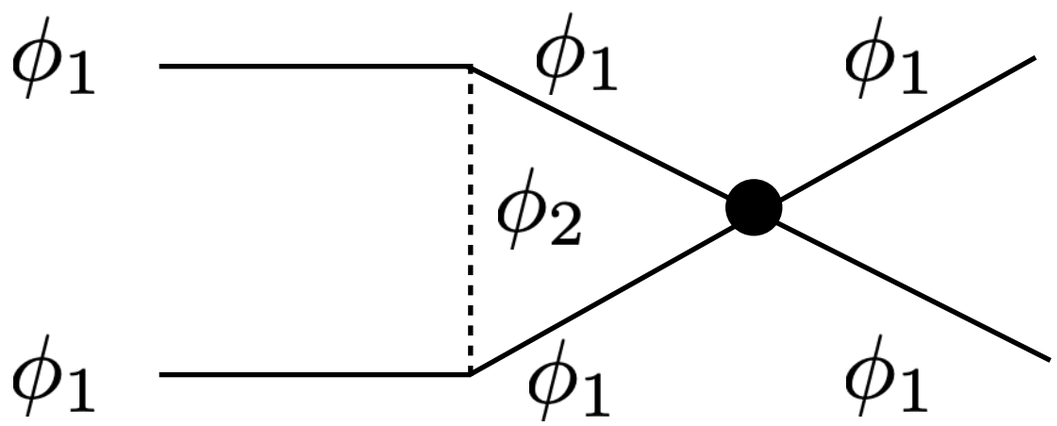
Schroedinger-like equation with DM mediator:

$$m_2 \lesssim 2m_1 : \quad M \ll m_2$$

t-channel vs u-channel

t-channel:

$$\tilde{\Gamma}_t = \frac{4g^2 m_1^2}{|\vec{p} - \vec{p}'|^2 + m_2^2}$$



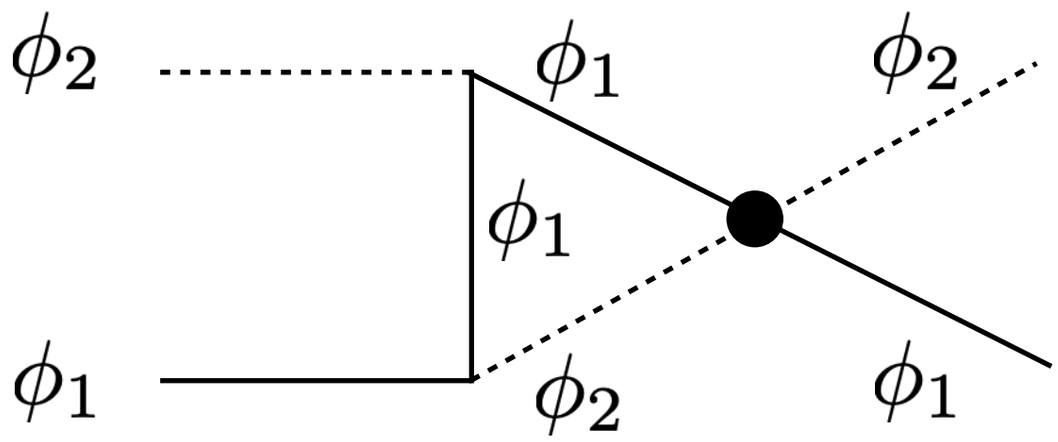
$$\chi(1,1') \sim \tilde{\Gamma}_t \chi(1,1')$$

$$\left(\frac{\nabla^2}{2\mu} + E\right)\psi(\vec{x}) = V(\vec{x})\psi(\vec{x})$$

No DM exchange in loops =>
No flip for BS wave-function

u-channel:

$$\tilde{\Gamma}_u = \frac{4g^2 m_1^2}{\left|\sqrt{\frac{m_1}{m_2}}\vec{p} - \sqrt{\frac{m_2}{m_1}}\vec{q}'\right|^2 + m_2(2m_1 - m_2)}$$



$$\chi(1,2) \sim \tilde{\Gamma}_u \chi(2,1)$$

$$\left(\frac{\nabla^2}{2\mu} + E\right)\psi(\vec{x}) = V(\vec{x})\psi\left(-\frac{m_2}{m_1}\vec{x}\right)$$

DM exchange in loops =>
BS wave-function flips!

Partial wave expansions

BS w.f. in spherical coordinates:

$$\psi_{\rm BS}(\vec{x}) = R_l(r)Y_l^m(\theta,\phi) \longrightarrow \psi_{\rm BS}\Big(-\frac{m_2}{m_1}\vec{x}\Big) = \boxed{(-1)^l} R_l\Big(\frac{m_2}{m_1}r\Big)Y_l^m(\theta,\phi)$$

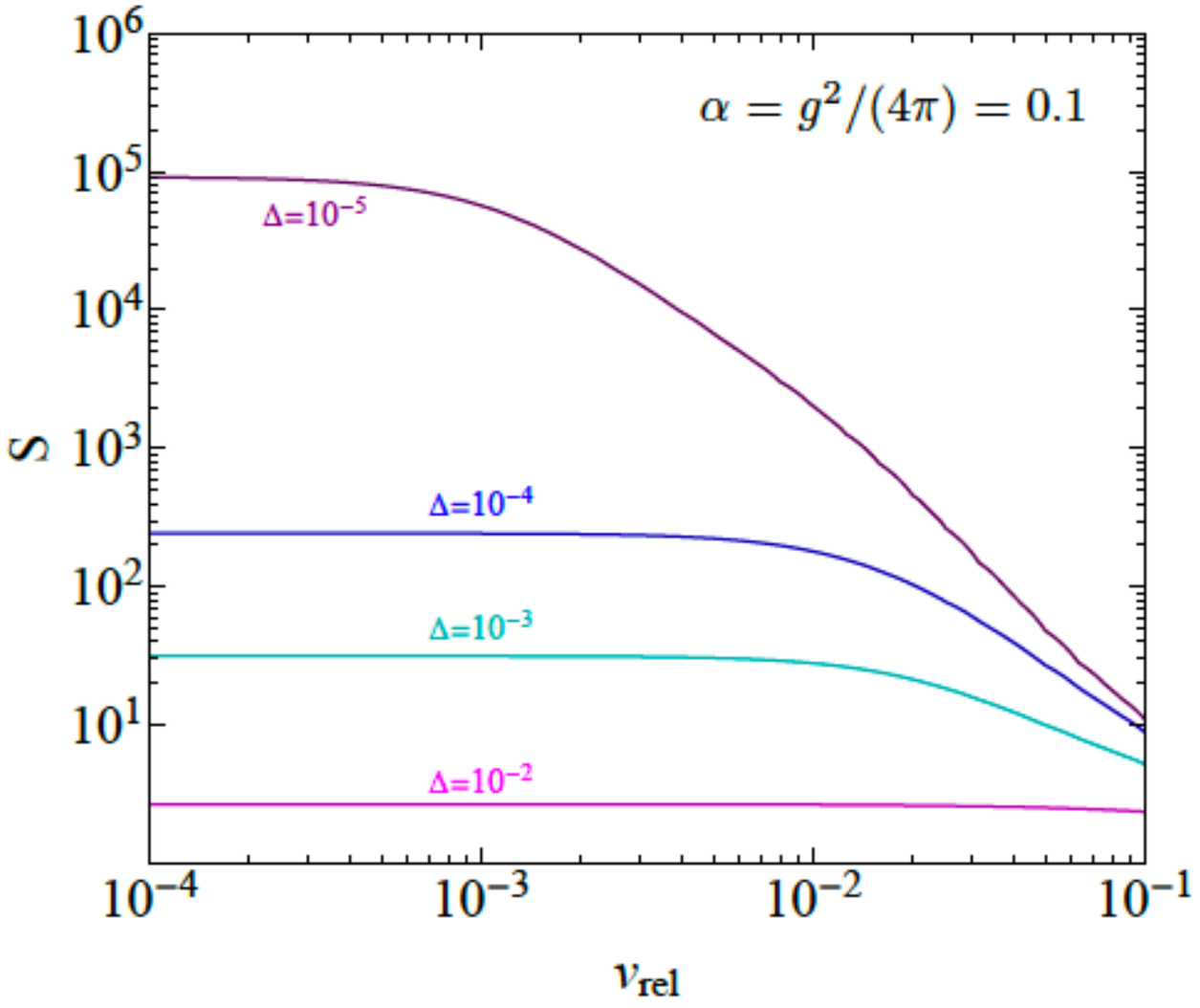
Radial equation : $R_l(x) = u_l(x)/x\,, \quad a = \frac{2v_{\rm rel}}{\alpha},\ b = \frac{m_2}{m_1}$ and $c = \frac{2M}{\mu\alpha}$

$$\Rightarrow \quad \left(\frac{d^2}{dx^2} - \frac{l(l+1)}{x^2}\right)u_l(x) + \frac{4e^{-cx}}{b\,x}(-1)^l\,u_l(bx) + a^2\,u_l(x) = 0$$

- Potential flips: attractive for l=even ; repulsive for l=odd.
- Effective mediator mass: $M \equiv m_2\sqrt{2-\frac{m_2}{m_1}} \to 0, \quad m_2 \to 2m_1.$
- Delay differential eq:

$$x = e^{-\rho} \longrightarrow \tilde{u}_0''(\rho) + \tilde{u}_0'(\rho) + 2e^{-\rho}\,\tilde{u}_0(\underbrace{\rho - \ln 2}_{\text{delay term}}) + a^2e^{-2\rho}\,\tilde{u}_0(\rho) = 0$$

Sommerfeld factor



Sommerfeld factor (s-wave):

$$S = \frac{|\psi_{\rm BS}(0)|^2}{|\psi_{\rm pert}(0)|^2} = A^2$$

Effective mediator mass

$$\longleftrightarrow \quad \Delta = 1 - \frac{m_2}{2m_1} \geq 0$$

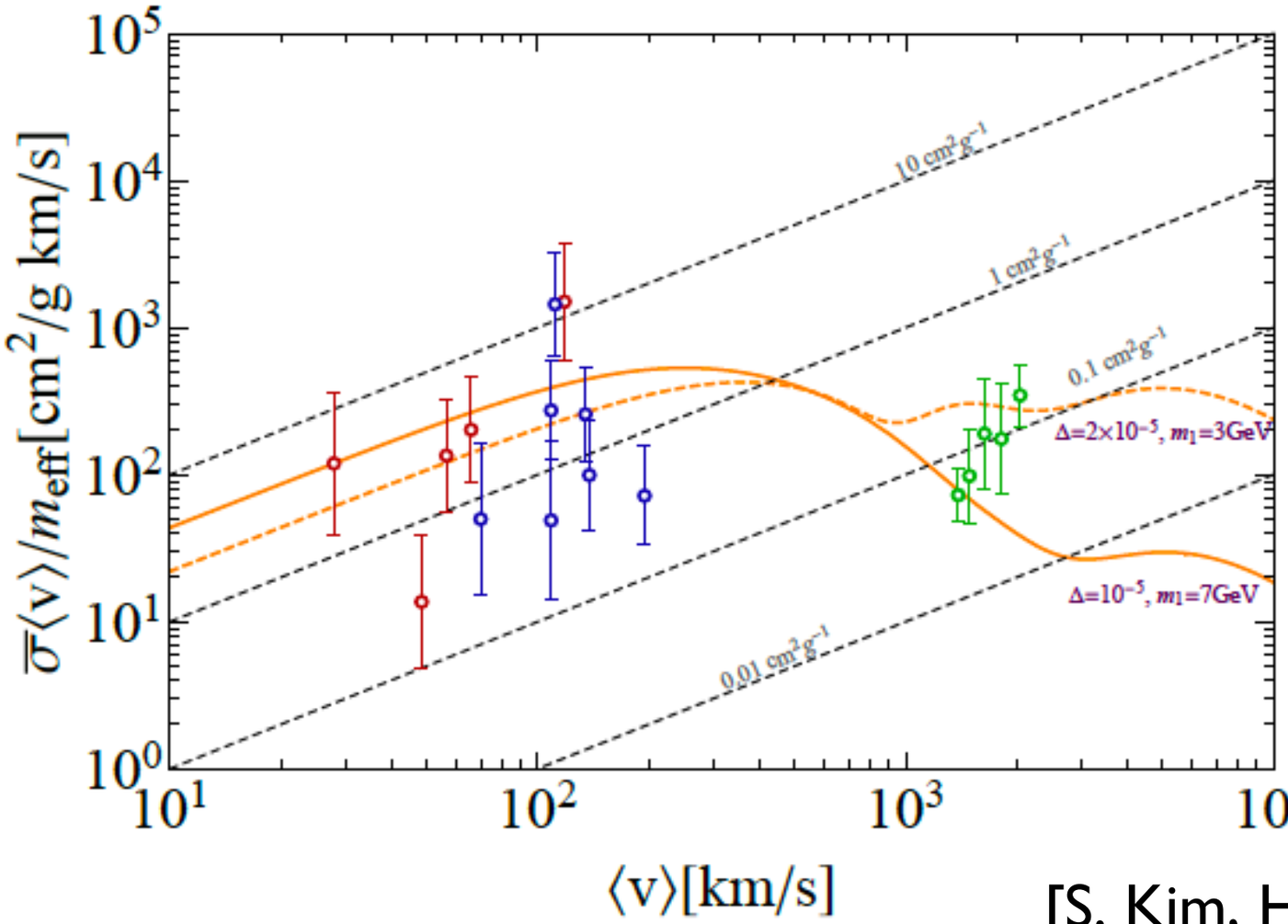
[S. Kim, HML, B. Zhu, 2021]

Boundary conditions (s-wave):

$$\tilde{u}_0(\rho) \longrightarrow \frac{1}{a}\sin(a\,e^{-\rho} + \delta_0), \qquad \rho \to -\infty, \qquad \text{“plane-wave”}$$

$$\tilde{u}_0(\rho) \longrightarrow A\,e^{-\rho}, \qquad \rho \to +\infty \qquad \text{“constant R”}$$

SIDM from co-scattering



$\alpha = g^2/(4\pi) = 0.1$

[S. Kim, HML, B. Zhu, 2021]

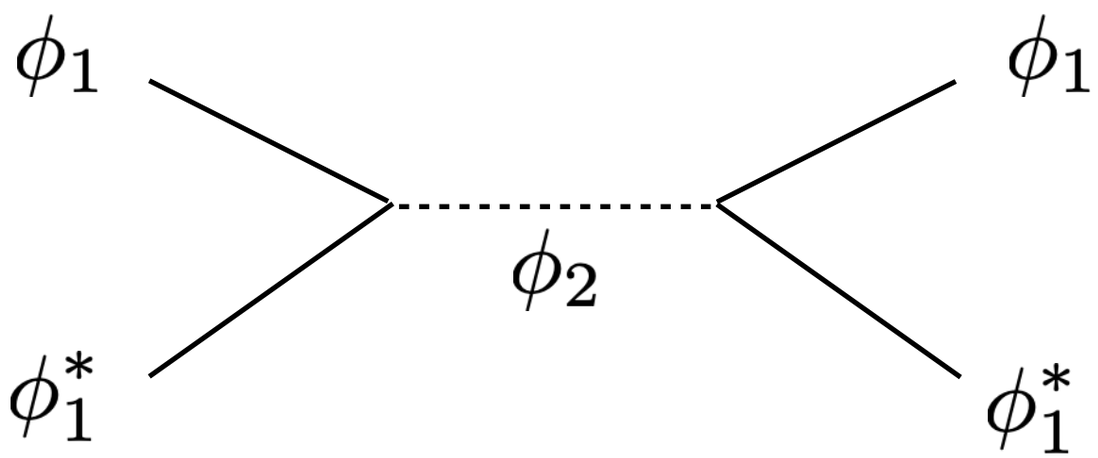
$\phi_1\phi_2 \to \phi_1\phi_2$: $\sigma = \frac{4\pi}{k^2}\sin^2\delta_0$, total cross section

$\bar{\sigma} = \langle\sigma v_{\rm rel}^3\rangle/(24/\sqrt{\pi}v_0^3)$: energy-transfer average

⟹ Self-scattering for SRDM is velocity-dependent.

Other channels for SIDM

s-channel



$$\sigma^s = \frac{g^4 m_1^2}{4\pi(m_1^2(4+v_{\rm rel}^2)-m_2^2)^2}$$

$$\Delta = 1 - \frac{m_2}{2m_1} \geq 0$$

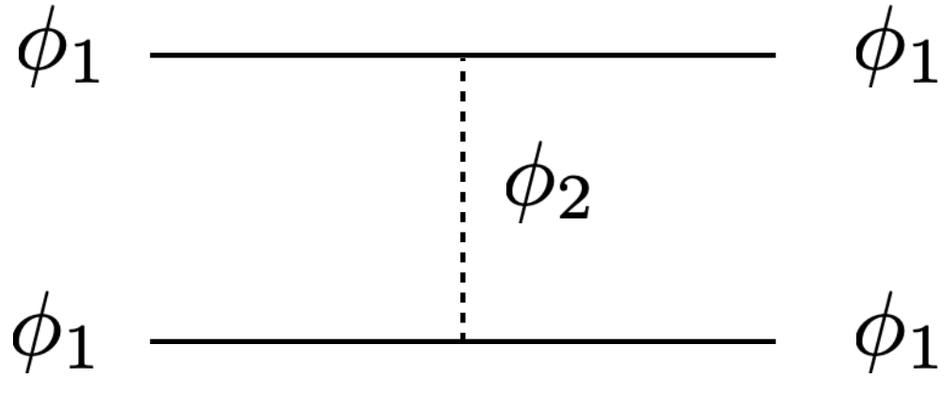
$$\longrightarrow \quad \sigma^s \simeq \frac{g^4}{4\pi m_1^2(v_{\rm rel}^2+8\Delta)^2}$$

Additional enhancement $\text{for } v_{\rm rel}, \Delta \ll 1$

s-channel contribution is comparable for $v_{\rm rel}^2 \sim \alpha \gtrsim 8\Delta.$

[S. Kim, HML, B. Zhu, in progress]

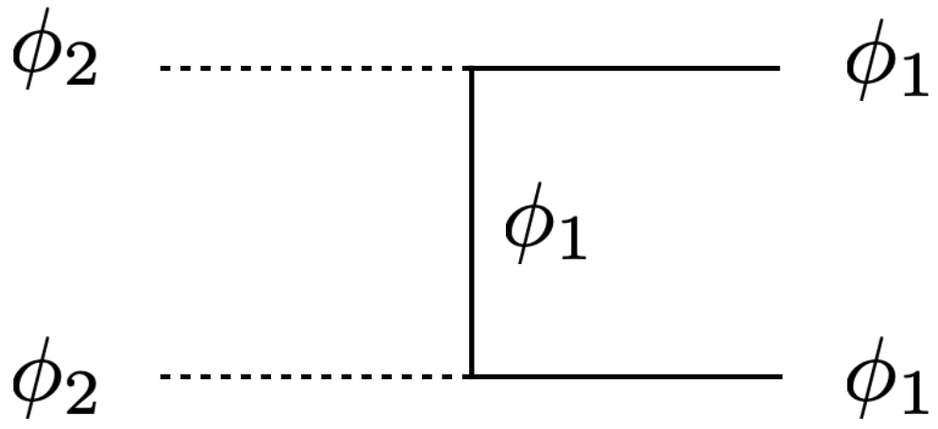
t-channel



$m_2 \approx 2m_1$

No light mediator:
no enhancement

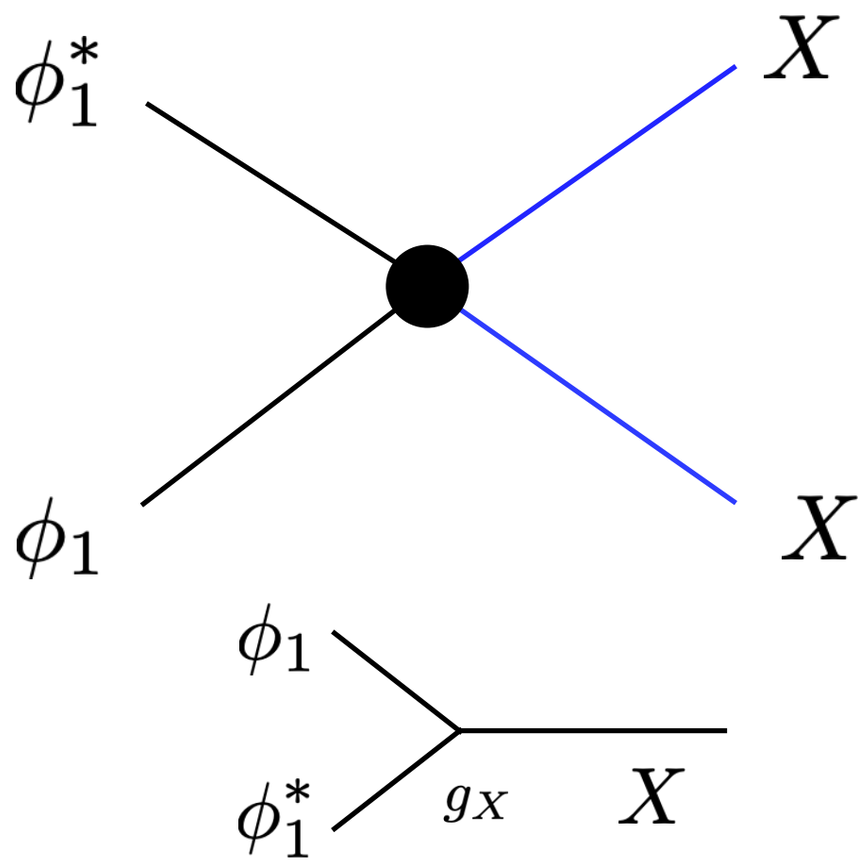
DM annihilation



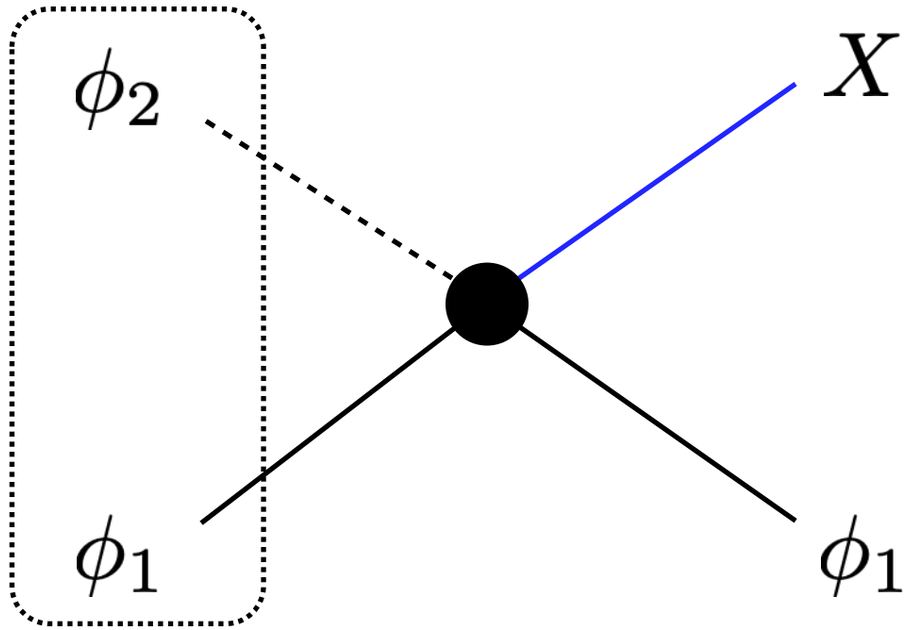
Heavier DM always annihilates into lighter DM, but no freeze-out.

⟶ extra 2→2 annihilation

e.g. dark photon portal

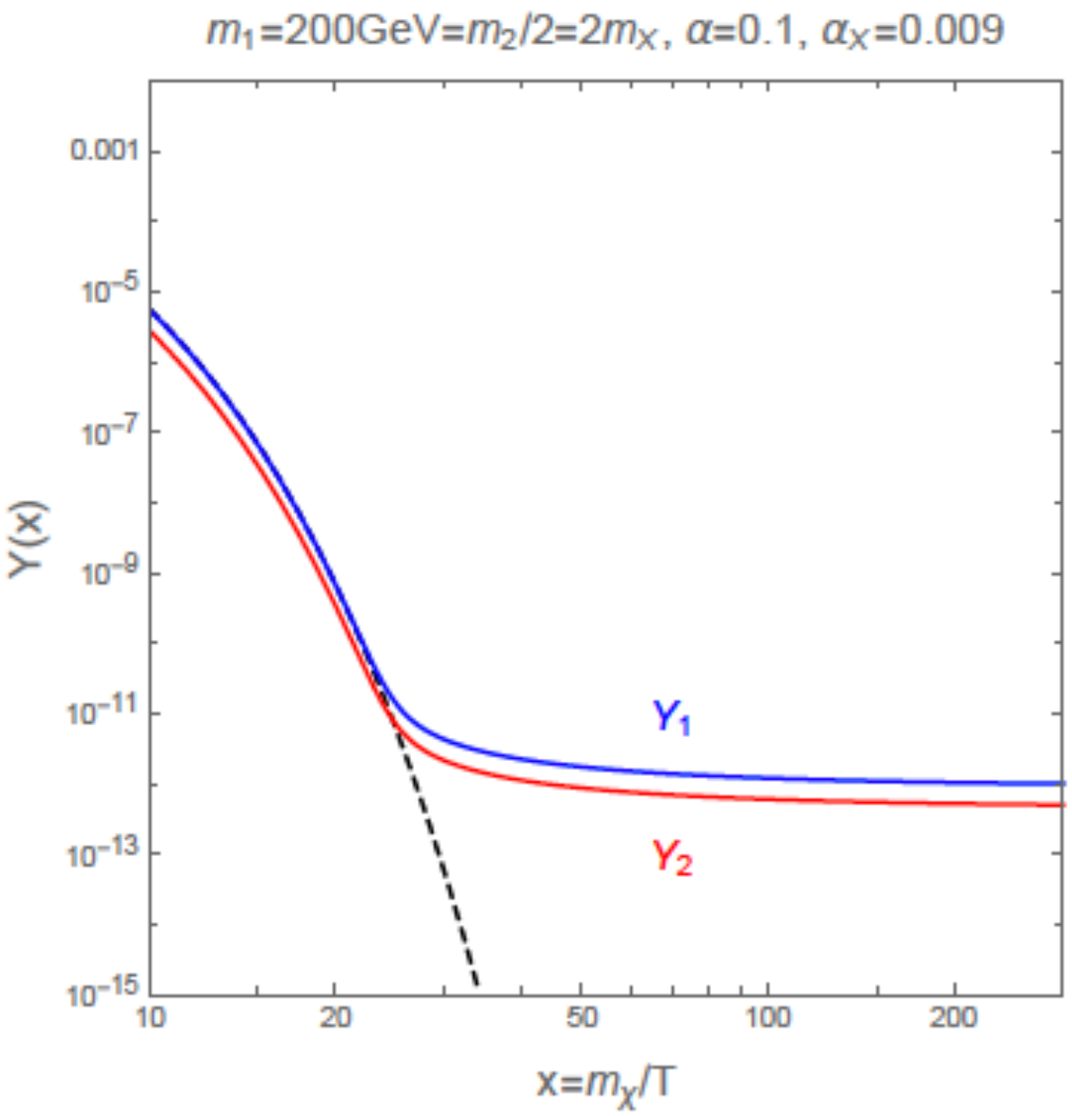


BS w.f.



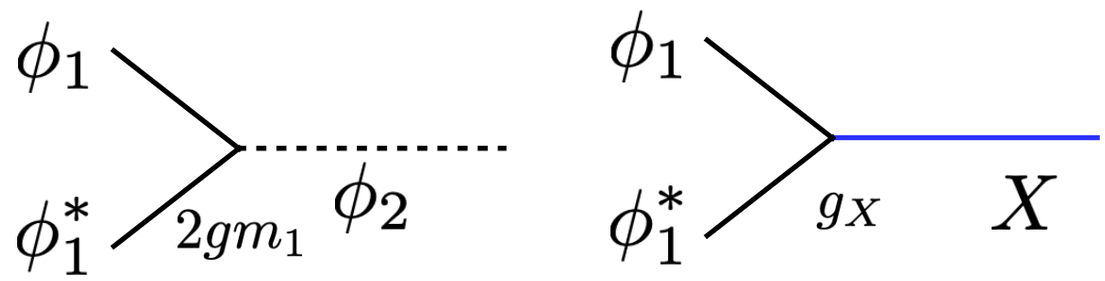
u-channel
Sommerfeld enhancement

DM relic density



$$\frac{dY_1}{dx} = \lambda_1\, x^{-2}\left(4Y_2^2 - Y_1^2\right) - \lambda_2\, x^{-2}\left(Y_1^2 - (Y_1^{\rm eq})^2\right)$$
$$- \rho_1 x^{-5} Y_1 \left(Y_1^2 - \frac{(Y_1^{\rm eq})^2}{Y_2^{\rm eq}} Y_2\right) - \rho_2 x^{-5} Y_1 \left(Y_1^2 - (Y_1^{\rm eq})^2\right),$$
$$\frac{dY_2}{dx} = -\lambda_1\, x^{-2}\left(4Y_2^2 - Y_1^2\right) - \lambda_3\, x^{-2} Y_1 \left(Y_2 - Y_2^{\rm eq}\right)$$
$$+ \frac{1}{4}\rho_1 x^{-5} Y_1 \left(Y_1^2 - \frac{(Y_1^{\rm eq})^2}{Y_2^{\rm eq}} Y_2\right)$$

Dark photon portal:



Dark photon in thermal equilibrium with SM plasma

$\alpha \gg \alpha_X$: large self-coupling

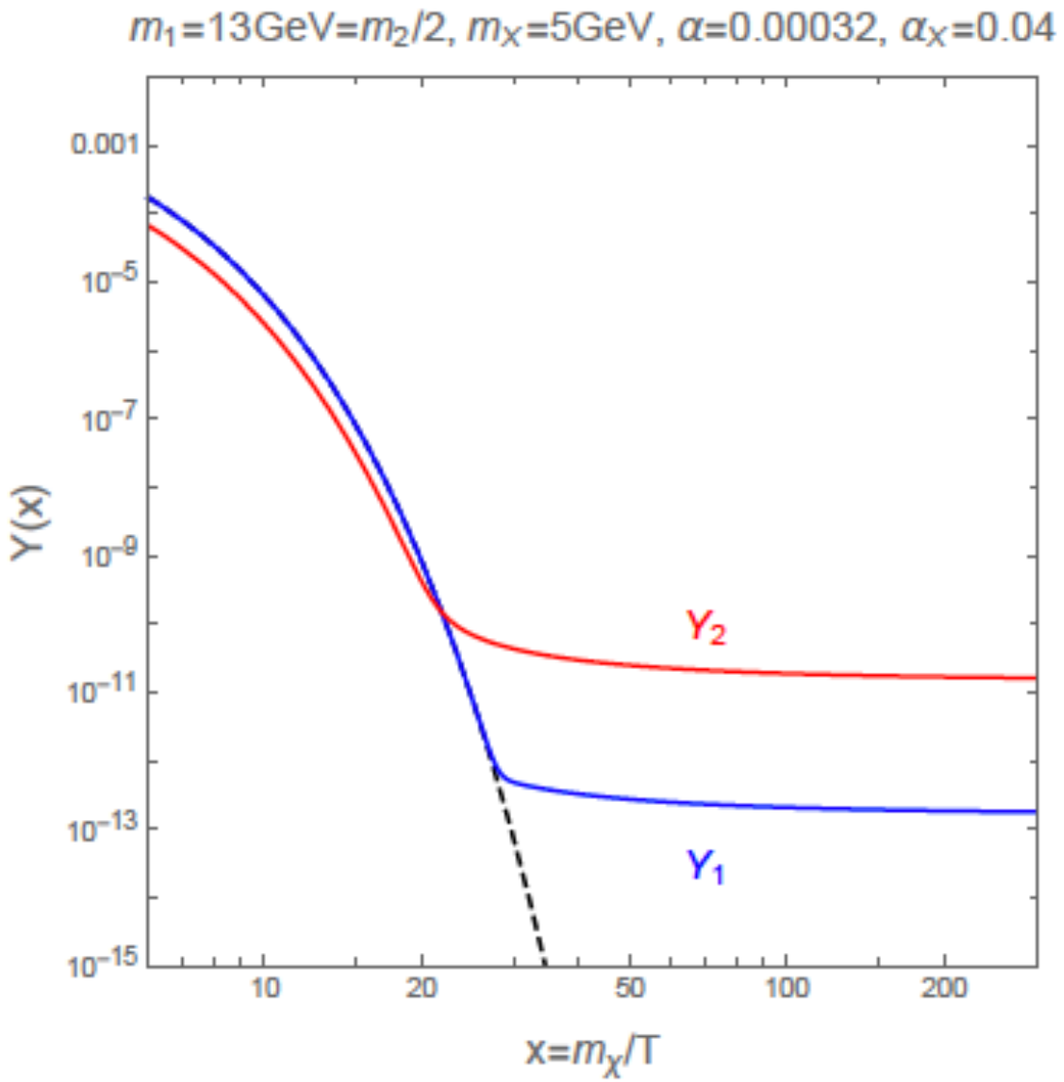
$\Rightarrow \langle\sigma v\rangle_{\phi_2\phi_2\to\phi_1\phi_1^*} \gg \langle\sigma v\rangle_{\phi_1\phi_1^*\to XX}$

$Y_1 \simeq 2Y_2$: equal abundances

$m_2 \simeq 2m_1$: strong u-channel resonance

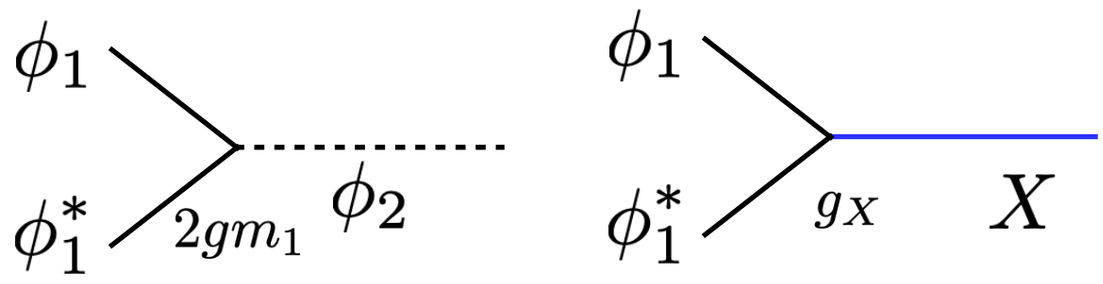
[S. Kim, HML, B. Zhu, 2022]

DM relic density



$$\frac{dY_1}{dx} = \lambda_1\, x^{-2}\left(4Y_2^2 - Y_1^2\right) - \lambda_2\, x^{-2}\left(Y_1^2 - (Y_1^{\rm eq})^2\right)$$
$$- \rho_1 x^{-5} Y_1\left(Y_1^2 - \frac{(Y_1^{\rm eq})^2}{Y_2^{\rm eq}}\, Y_2\right) - \rho_2 x^{-5} Y_1\left(Y_1^2 - (Y_1^{\rm eq})^2\right),$$
$$\frac{dY_2}{dx} = -\lambda_1\, x^{-2}\left(4Y_2^2 - Y_1^2\right) - \lambda_3\, x^{-2} Y_1\left(Y_2 - Y_2^{\rm eq}\right)$$
$$+ \frac{1}{4}\rho_1 x^{-5} Y_1\left(Y_1^2 - \frac{(Y_1^{\rm eq})^2}{Y_2^{\rm eq}}\, Y_2\right)$$

Dark photon portal:



Dark photon in thermal equilibrium with SM plasma

$\alpha \ll \alpha_X$: small self-coupling

$\Rightarrow \langle\sigma v\rangle_{\phi_2\phi_2\to\phi_1\phi_1^*} \ll \langle\sigma v\rangle_{\phi_1\phi_1^*\to XX}$

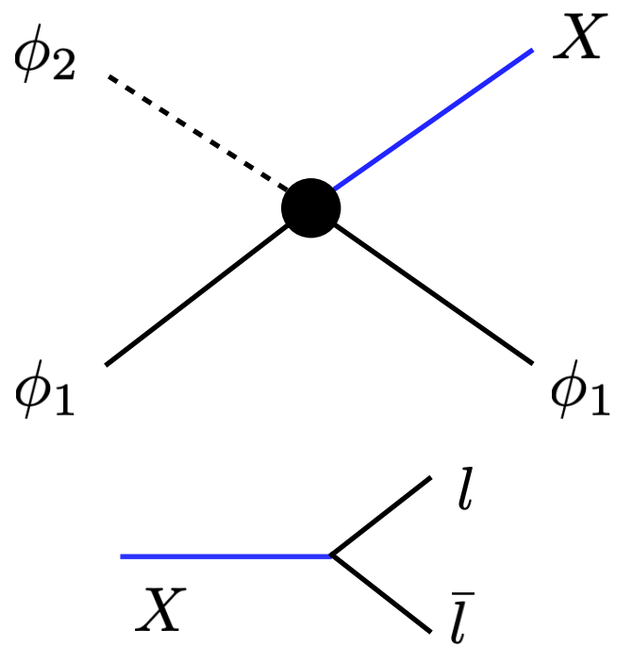
$Y_1 \ll Y_2$:hierarchical abundances

$m_2 \simeq 2m_1$: mild u-channel resonance

[S. Kim, HML, B. Zhu, 2022]

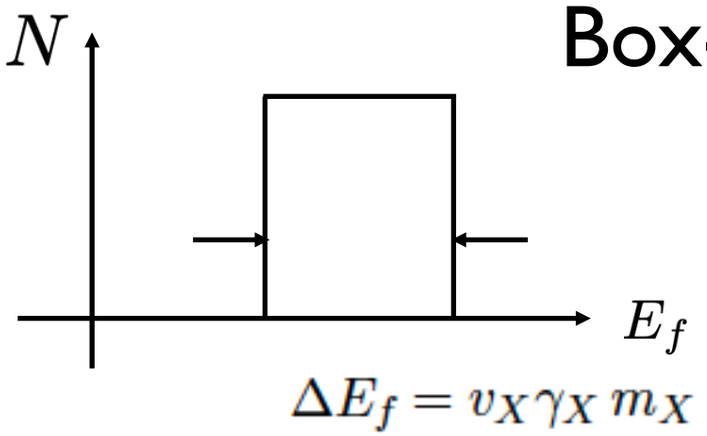
Indirect detection

Cosmic ray (from semi-annihilation):



Leptons boosted in galactic center frame

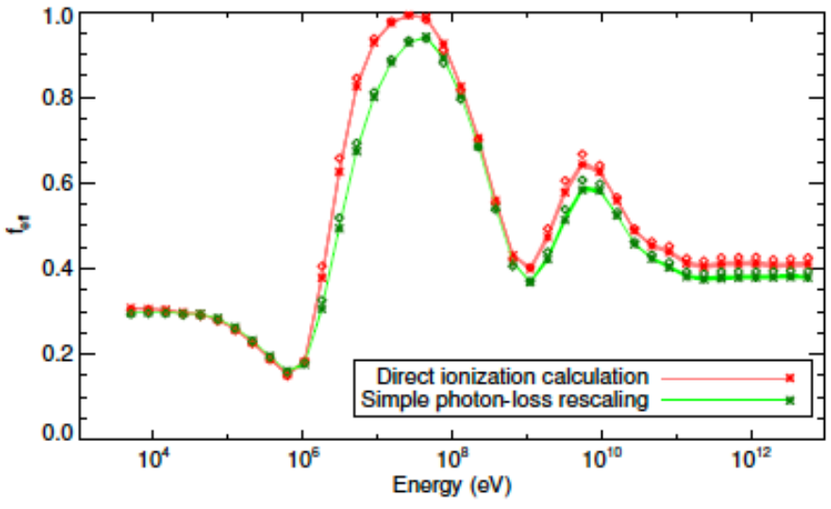
$$E_f = \frac{1}{\gamma_X}\bar{E}_f\,(1 - v_X\,\cos\theta)^{-1}, \qquad \bar{E}_f = \frac{m_X}{2}$$



Box-shaped positron energy
Fermi-LAT & AMD-02

cf. gamma-ray from
Fermi-LAT, HESS, etc.

CMB (from semi-annihilation): [S. Kim, HML, B. Zhu, 2022]



[T. Slatyer, 2015]

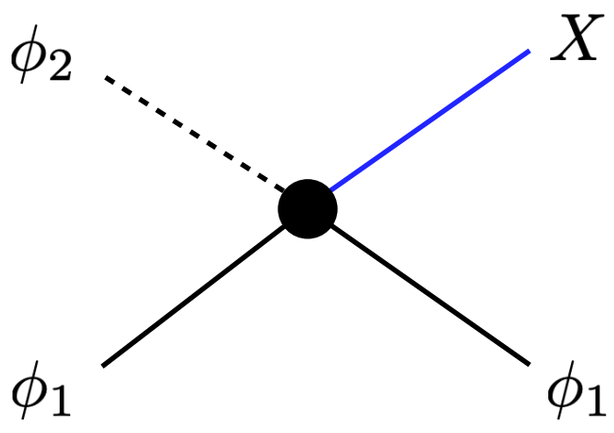
Leptons injects energy to CMB photons

$$\langle\sigma v\rangle_{\phi_1\phi_2\to\phi_1 X} < 4\times10^{-25}\,\mathrm{cm}^3/\mathrm{s}\left(\frac{f_{\rm eff}}{0.1}\right)^{-1}\cdot\frac{1}{r_1(1-r_1)}\cdot\left(\frac{m_2}{100\,\mathrm{GeV}}\right)$$

Efficient factor: $f_{\rm eff}(m_2) = \dfrac{\int_0^{m_2/2} dE_e\, E_e\, 2 f_{\rm eff}^{e^+e^-}\,\frac{dN_e}{dE_e}}{m_2}$, $\quad r_1 = \Omega_1/\Omega_{\rm DM}$

Direct detection

Boosted DM from semi-annihilation: [S. Kim, HML, B. Zhu, 2022]

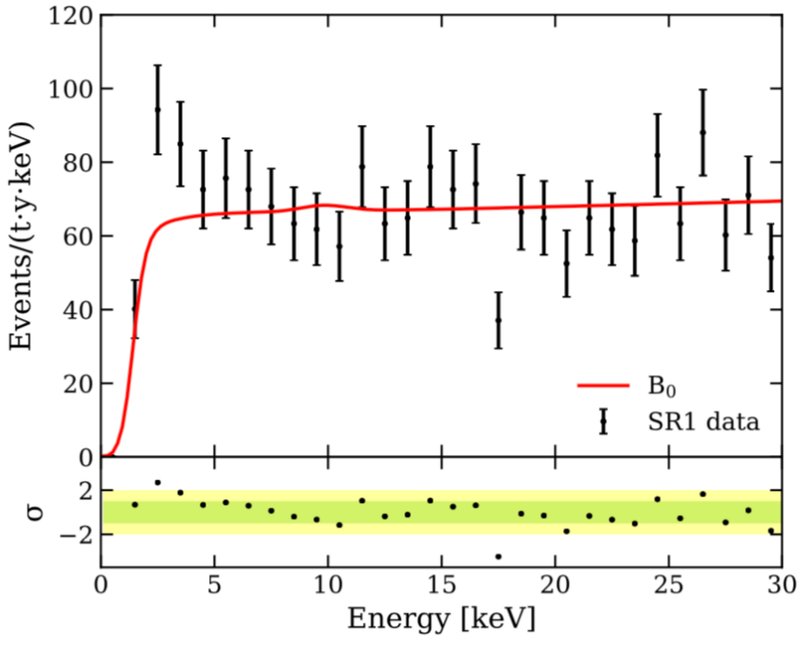


Gamma factor for boosted DM

$$\gamma_1 = (10m_1^2 - m_X^2)/(6m_1^2) \qquad m_2 \simeq 2m_1$$

DM flux from galactic center (NFW)

$$\Phi_1^{\rm G.C.} = 1.6\times 10^{-4}{\rm cm}^{-2}{\rm s}^{-1}\left(\frac{\langle\sigma v\rangle_{\phi_1\phi_2\to\phi_1 X}}{5\times 10^{-26}{\rm cm}^3/{\rm s}}\right)\left(\frac{(1\,{\rm GeV})^2}{m_1 m_2}\right) r_1(1-r_1)$$



XENON1T (old) excess

DM-electron scattering in XENON

$$\sigma_e = 10^{-33}\,{\rm cm}^2\left(\frac{10^{-1}{\rm cm}^{-2}{\rm s}^{-1}}{\Phi_1^{\rm G.C.}}\right)\left(\frac{N_{\rm sig}}{10}\right)$$

$$E_e = 2m_e v_1^2 = 3.6\,{\rm keV}, \quad m_1 = 2m_2 \sim 1\,{\rm GeV} \text{ and } r_1 \simeq \tfrac{1}{2}$$

$$\longrightarrow \quad m_X = 1.99729 m_1$$

XENON sensitive to u-channel resonance

EFT for SRDM

Candidates for co-scattering dark matter:

N

dark matter	scalar	pseudo-scalar	fermion	vector	axial-vector
scalar(ϕ)	$+4g^2m_\phi^2$	$+4g^2m_\phi^2$	$+2y_\chi^2 m_\chi(2m_\chi - m_\phi)$	NA	NA
pseudo-scalar(a)	–	–	$-2\lambda_\chi^2 m_\chi m_a$	NA	NA
fermion(χ)	–	–	NA	$-2g_{Z'}^2 m_\chi m_{Z'}$	$+2g_{A'}^2 m_\chi(2m_\chi - m_{A'})$
vector(Z')	NA	NA	–	$-6g_X^2 m_X(2m_X - m_{X_3})$	NA
axial-vector(A')	NA	NA	–	–	NA

$$\tilde{\Gamma}_u(p,q;p',q') = \frac{N}{\left(\sqrt{\frac{m_1}{m_2}}\vec{p} - \sqrt{\frac{m_2}{m_1}}\vec{q'}\right)^2 + m_2(2m_1 - m_2)}$$

[S. Kim, HML, B. Zhu, 2022]

Scalar & (pseudo)scalar DM: s-wave

fermion & pseudoscalar(or vector) DM: p-wave

⟶ Self-resonant dark matter with different spins/parities