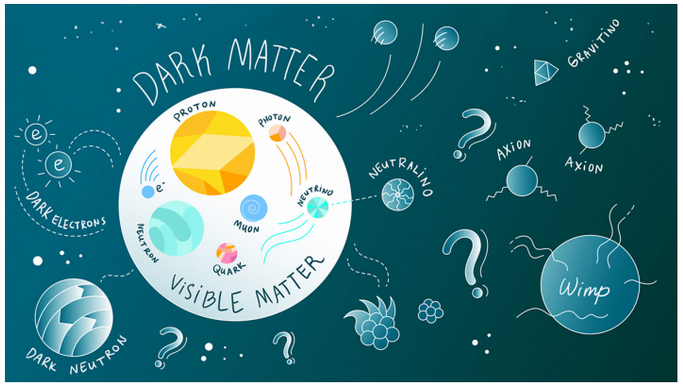


KIAS 입자물리 여름학교 2023

Dark Matter

전북대학교
JEONBUK NATIONAL UNIVERSITY

Seodong Shin

Department of Physics

What is Dark Matter?

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Cold dust (fluid) with some perturbation

What is Dark Matter?

Cold dust (fluid) with some perturbation



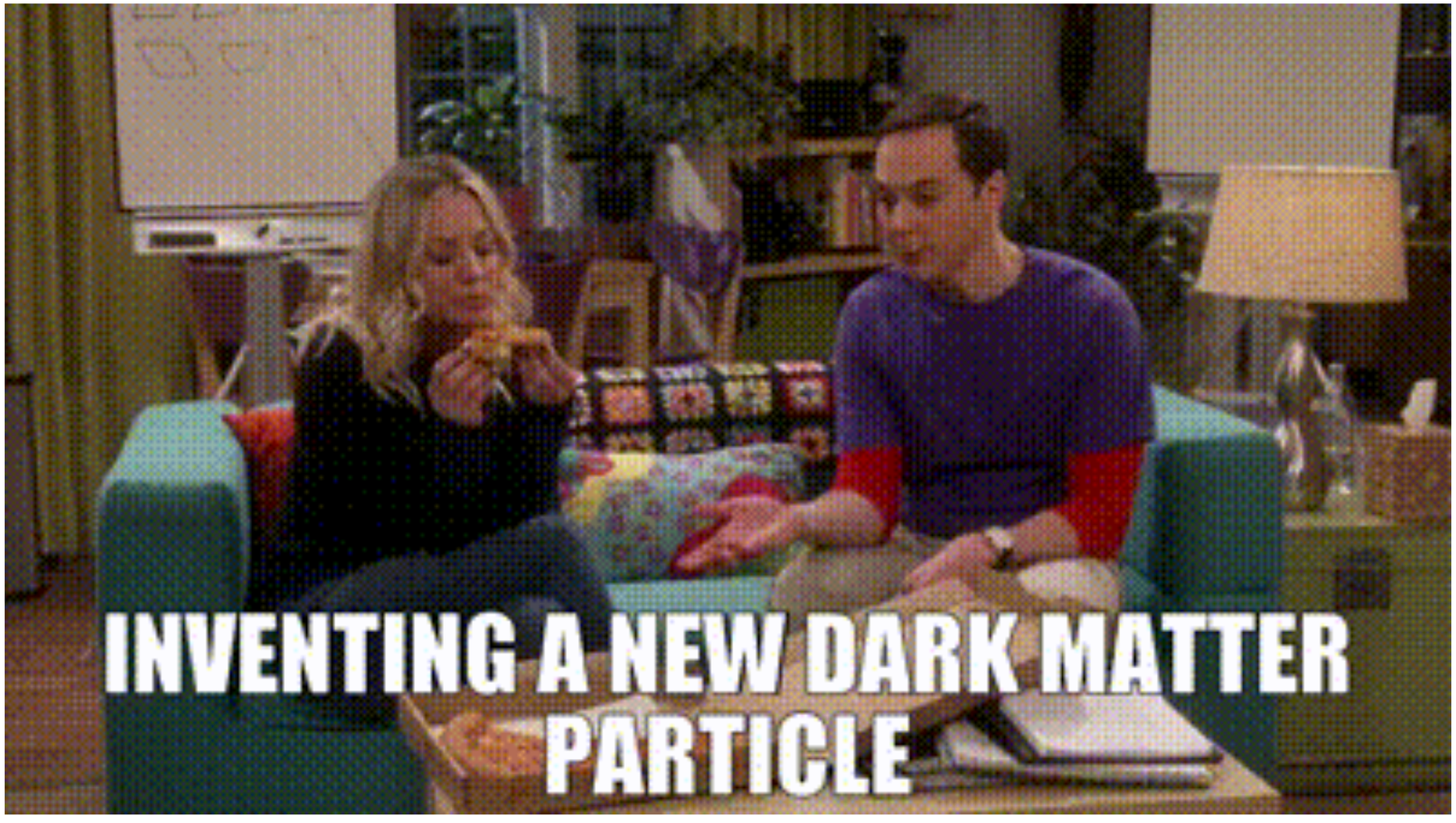
What is Dark Matter?

Cold dust (fluid) with some perturbation

- Mass?
- Spin?
- Interaction?
- Fundamental particle?
- …….

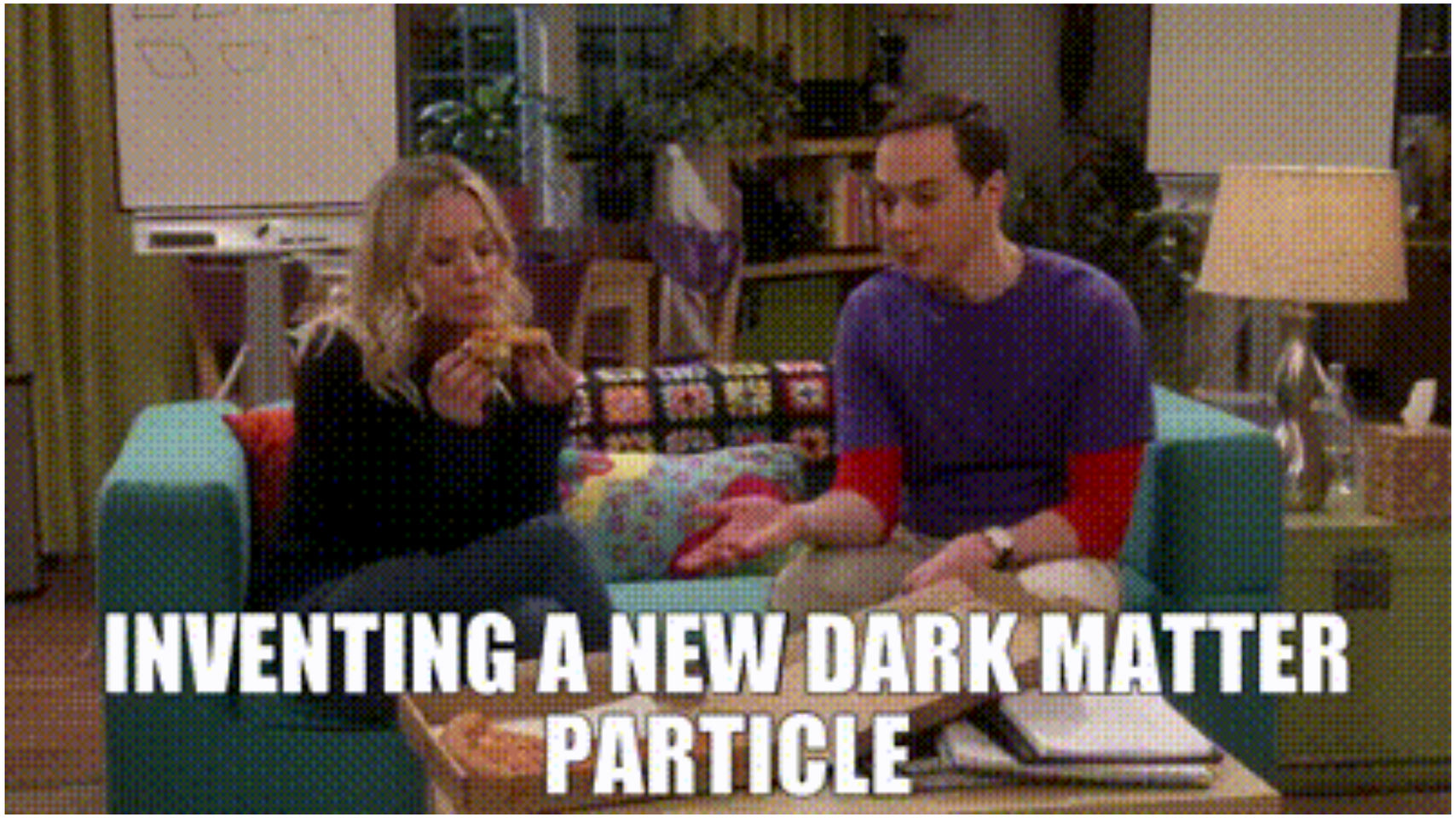


Particle physics properties



Big bang theory series

Particle physics properties



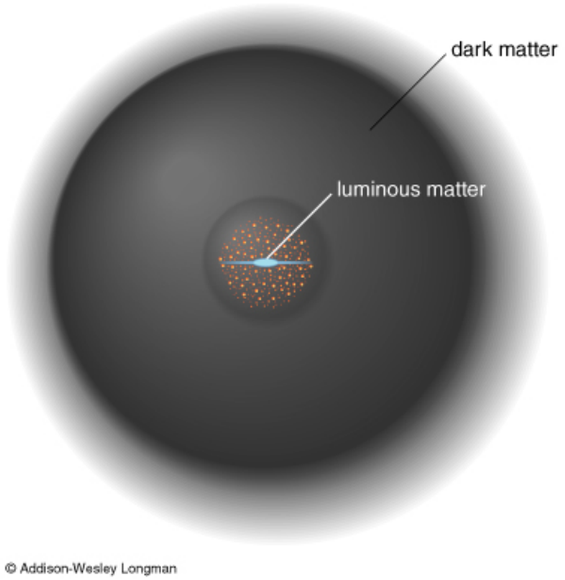
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Particle Physics Properties

Mass range?

Particle Physics Properties

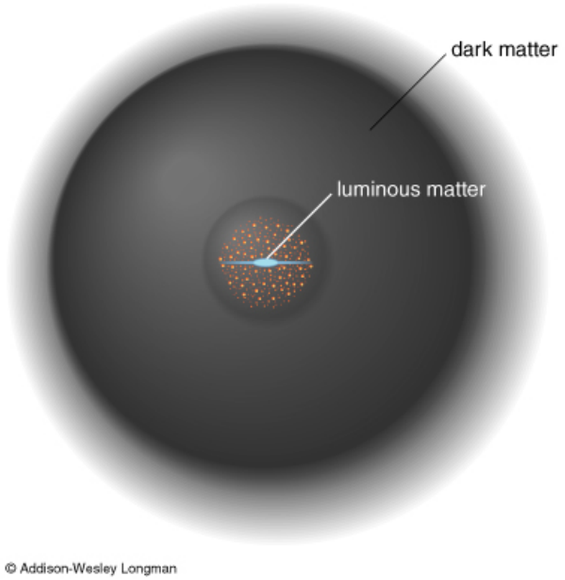
Mass range?



© Addison-Wesley Longman

Particle Physics Properties

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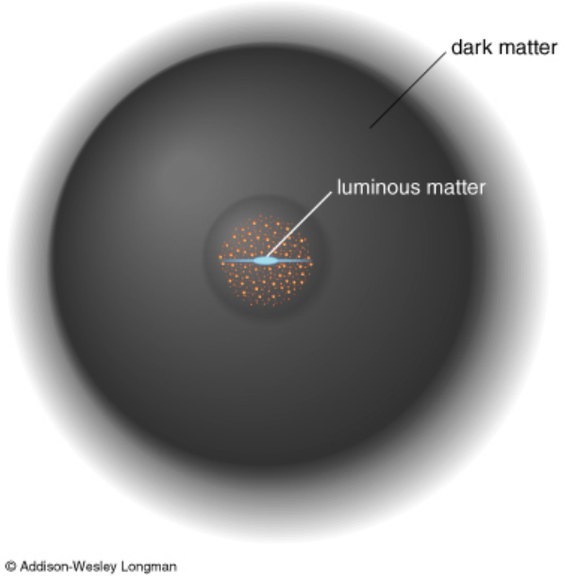


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- Dark halos must be stable!

Particle Physics Properties

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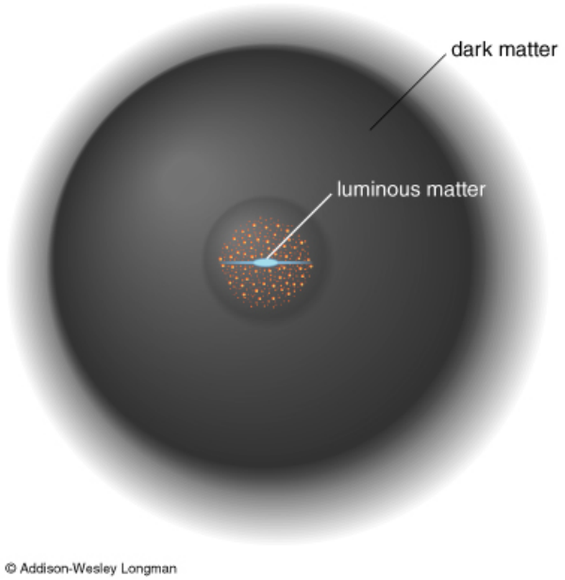


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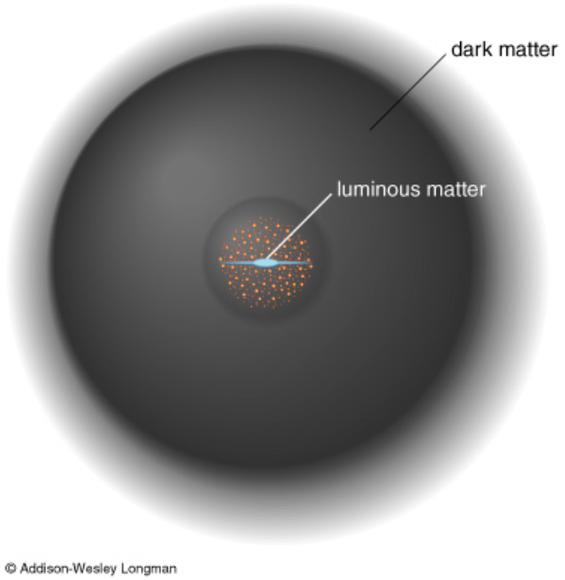
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Bosonic dark matter

Particle Physics Properties

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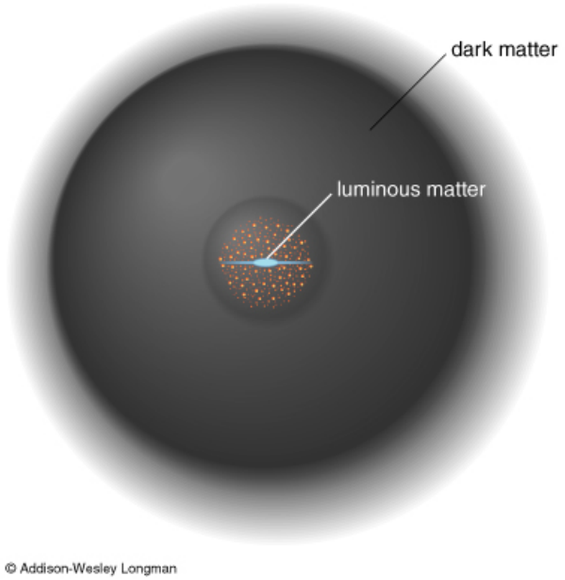
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- Infinite number of particles accumulated: effectively treated as a classical field.

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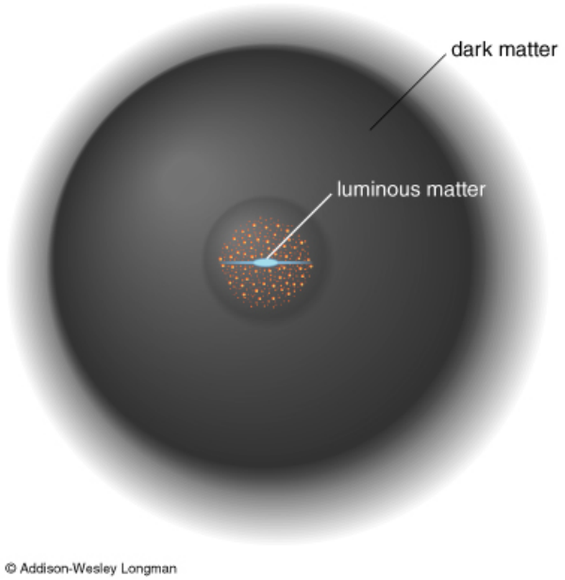
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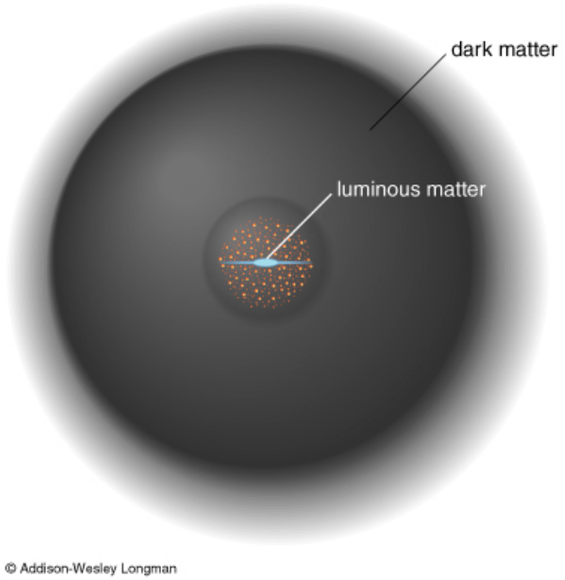
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$$\Delta p \sim m_\chi v$$
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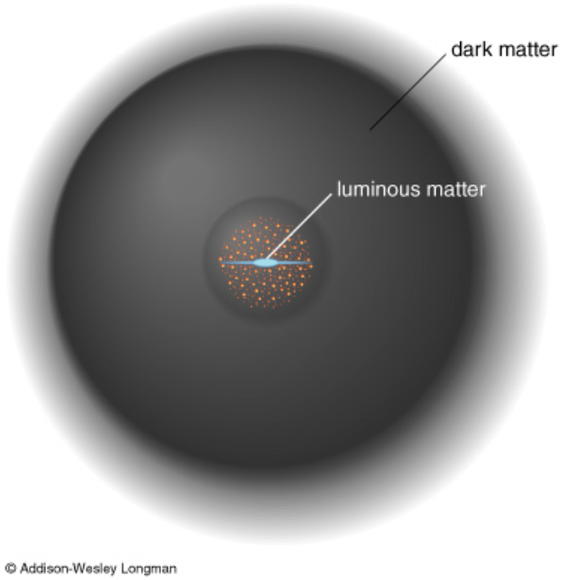
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$$m_\chi \gtrsim 10^{-22}\,{\rm eV} \quad \text{for} \quad R_{\rm halo} \sim {\rm kpc}$$

Fuzzy CDM, Hu et al., PRL 2000

Particle Physics Properties

Bosonic dark matter with mass ≲ eV ⟶ Coherent waves

$$\lambda_{\rm dB} = \frac{2\pi}{m_\chi v_{\rm vir}} \sim {\rm kpc} \left(\frac{10^{-22}\,{\rm eV}}{m_\chi} \right)$$

$$\mathbf{p}_\chi^3 \ll n_\chi$$



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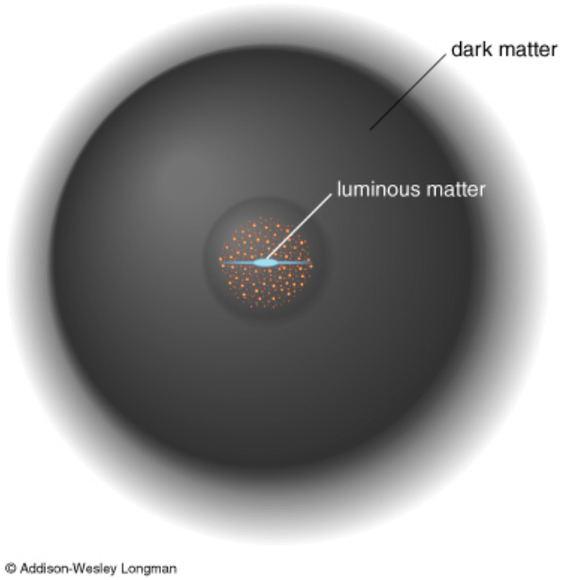
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Ramon Andrade 3DCiencia/
Science Source

Particle Physics Properties

Mass range?



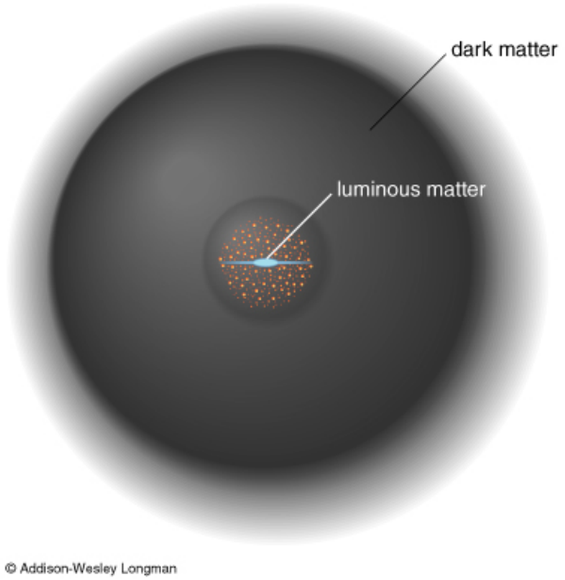
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Fermionic dark matter

Particle Physics Properties

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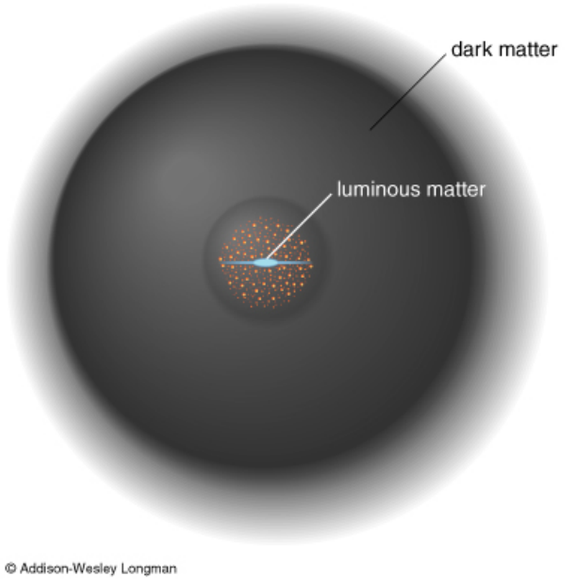
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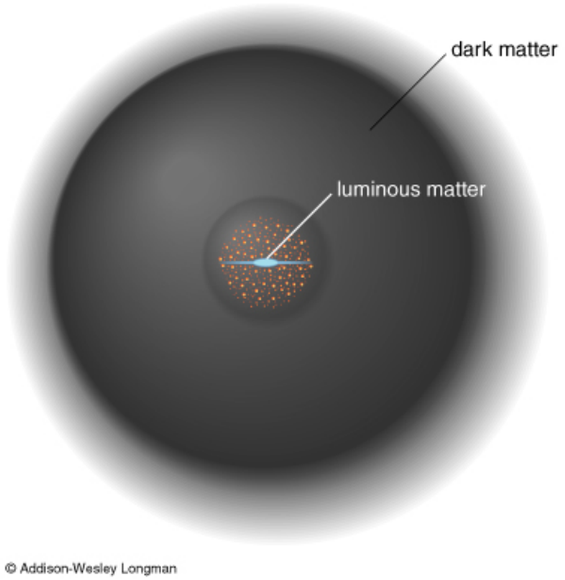
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- Occupation number follows the Pauli exclusion. $M = m_\chi N = m_\chi V n$

$$M_{\rm halo} = m_\chi V \int f(p)\, d^3p \lesssim m_\chi V \int d^3p \sim m_\chi R_{\rm halo}^3 (m_\chi v)^3$$

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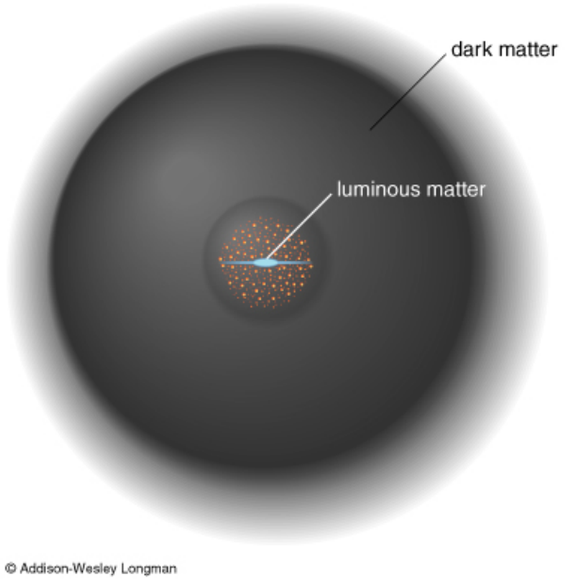
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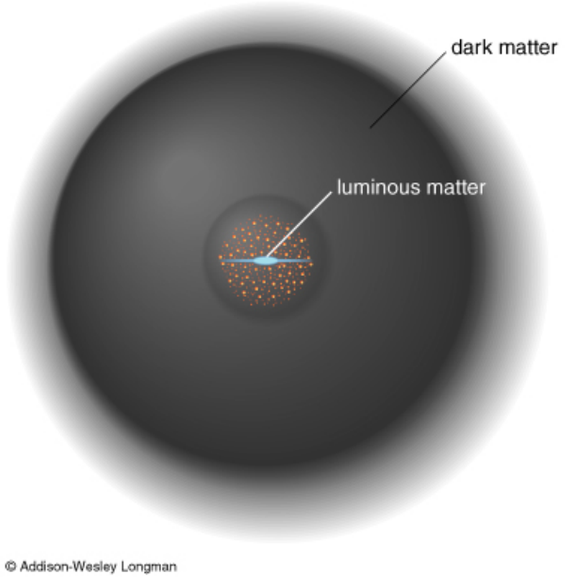
Lisanti, TASI 2016

$$v \sim \sqrt{\frac{G M_{\rm halo}}{R_{\rm halo}}}$$

Phase-space densities of dwarf galaxies ($10^9 M_\odot$) $\quad m_\chi \gtrsim 0.7\,\mathrm{keV}$

Particle Physics Properties

Mass range?



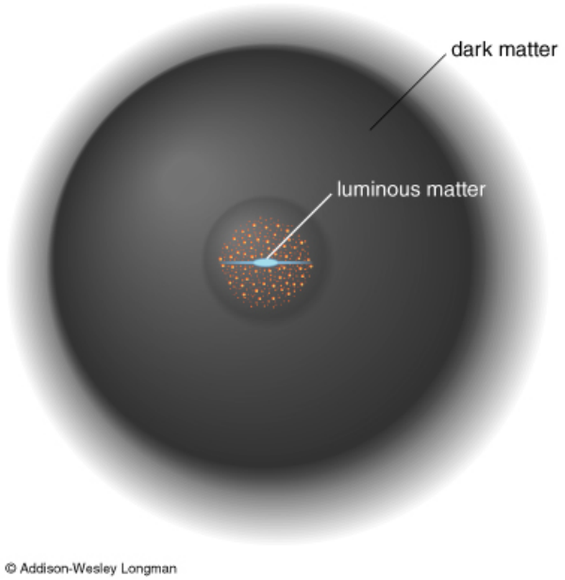
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Upper limit?

Particle Physics Properties

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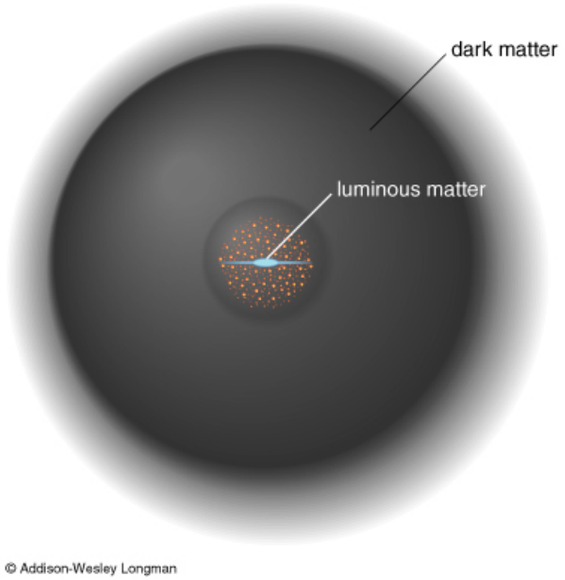
Upper limit?

- Observational bounds on compact object: quite limited by cosmology & lensing

Massive Astrophysical Compact Halo Objects (MACHOs)

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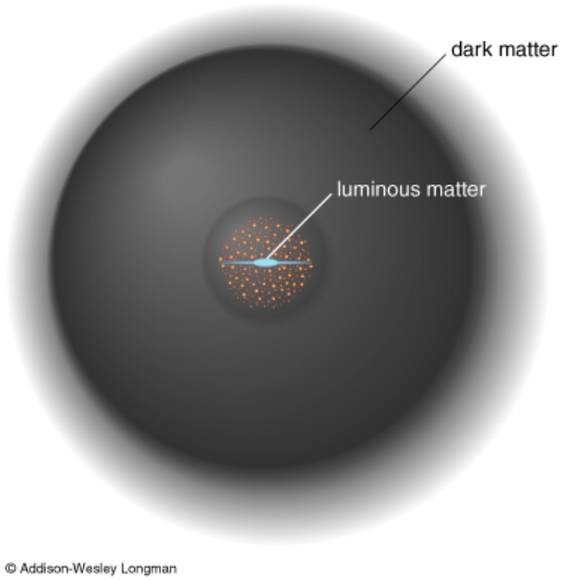
Massive Astrophysical Compact Halo Objects (MACHOs)

$$m_\chi \lesssim 10^{57}\,\mathrm{eV} \text{ or } 10^{79-85}\,\mathrm{eV}$$ for 100% relic

Carl, Kuhnel, 2110.02821

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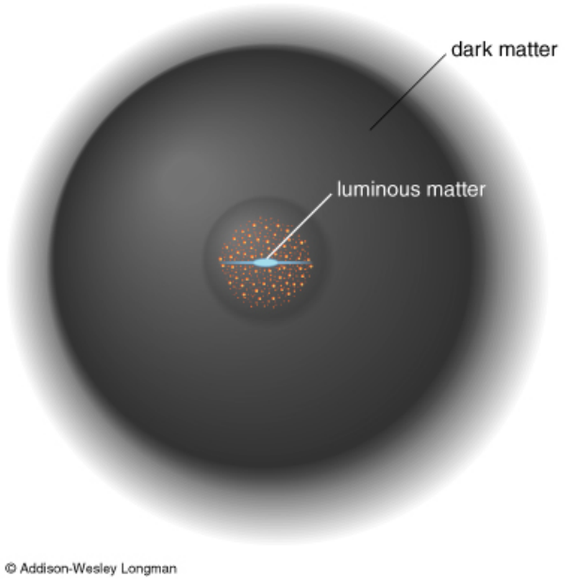
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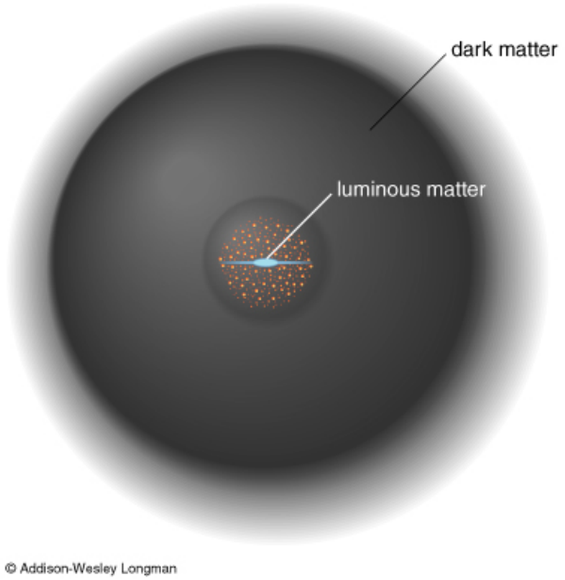
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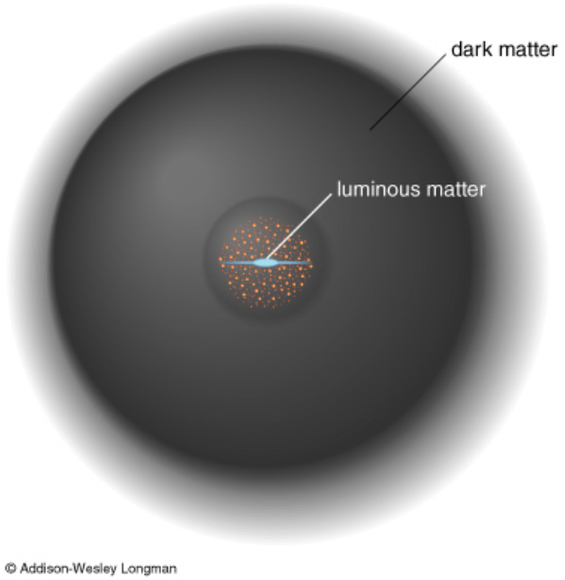
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$$n = \frac{\rho_0}{m} = \frac{81 \times 10^{77}}{(m/\mathrm{eV}) \cdot (100\,\mathrm{kpc})^3} \sim \frac{10^{79}}{(m/\mathrm{eV}) \cdot V_{\mathrm{MW,halo}}}$$

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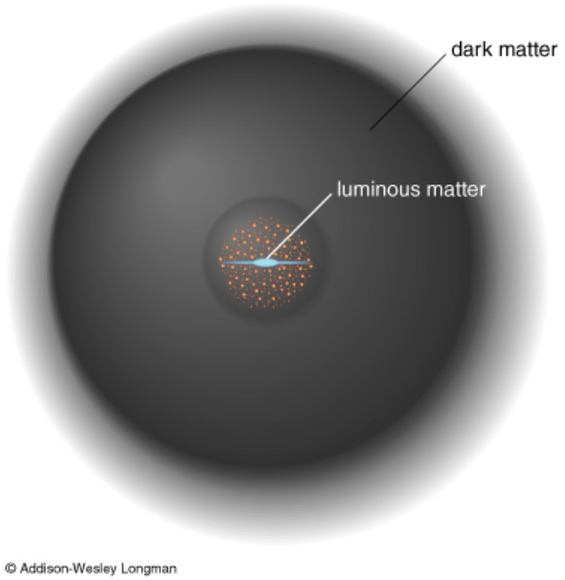
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Particle Dark Matter

$$1\,\mathrm{eV} \lesssim m_\chi \lesssim 10^{19}\,\mathrm{GeV}$$

Dark Matter Interactions

Interactions?

- Gravitational interactions
- Non-gravitational interactions with the SM (no EM or strong)
- Non-gravitational interactions with themselves

Dark Matter Interactions

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- ✔ Gravitational interactions What we have seen so far
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Key role in



New Physics BSM

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Direct probe of DM is possible.

1. Production in accelerators
2. Direct detections
3. Indirect detections

Dark Matter Interactions

Interactions?

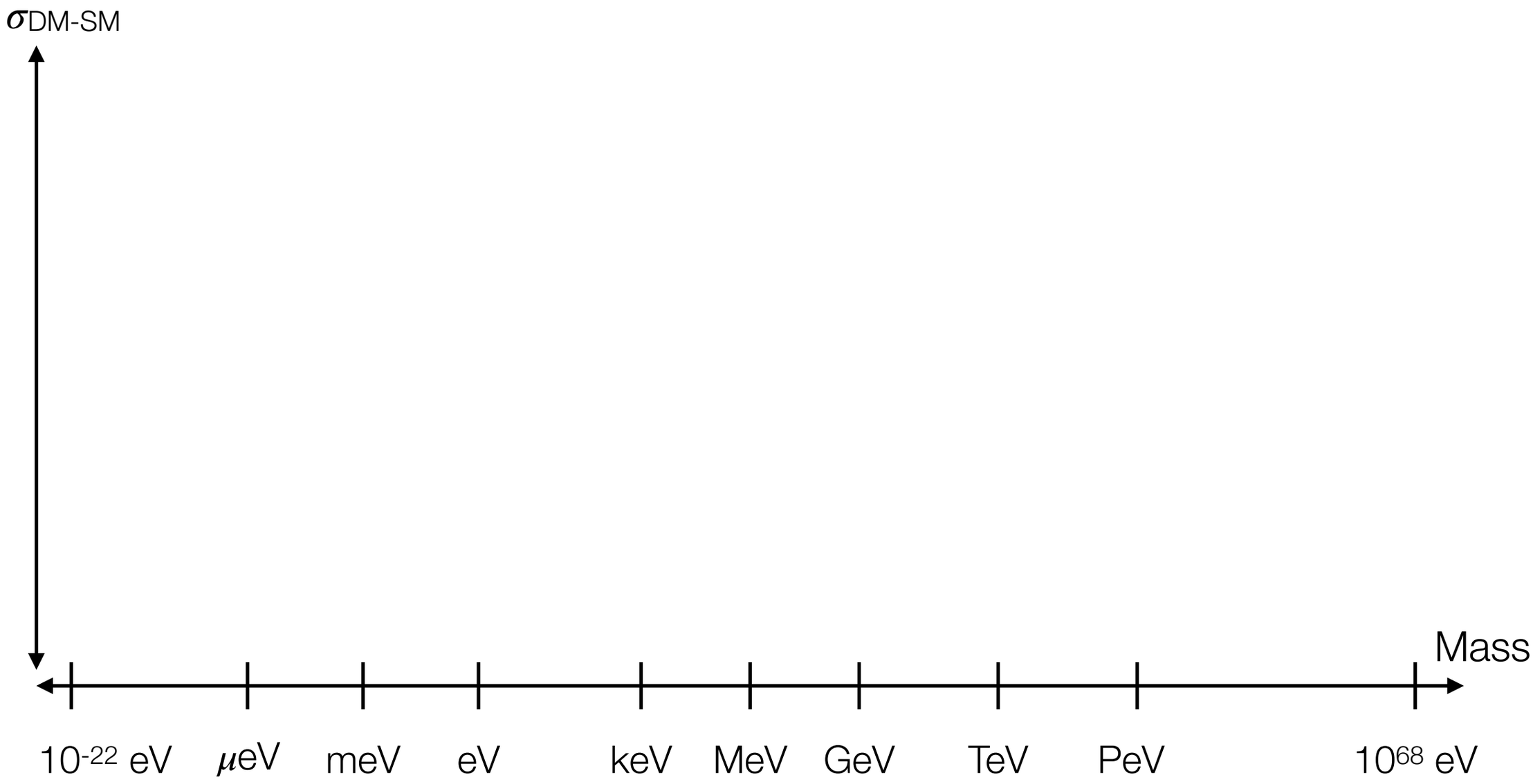
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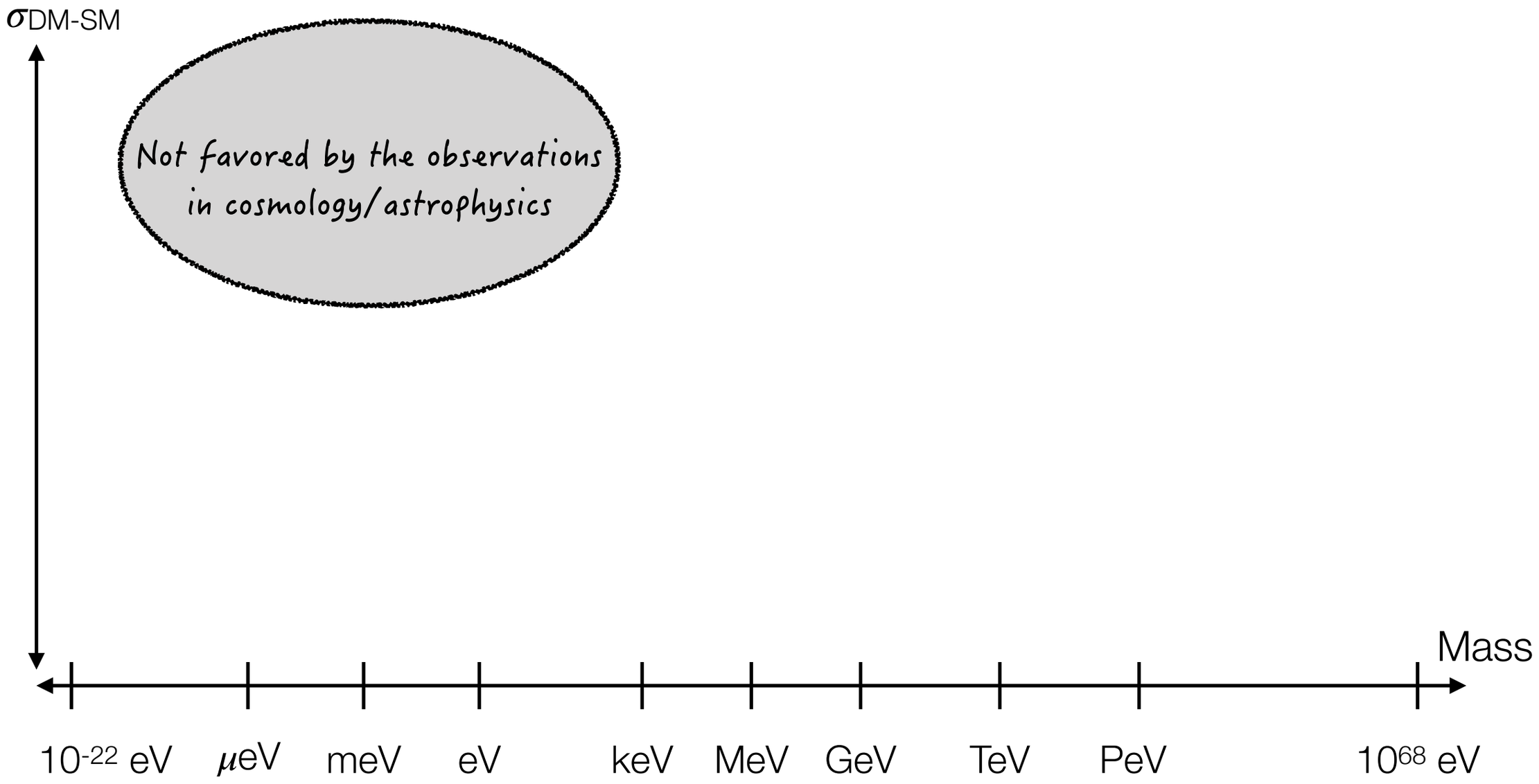
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Amount of DM can be determined.

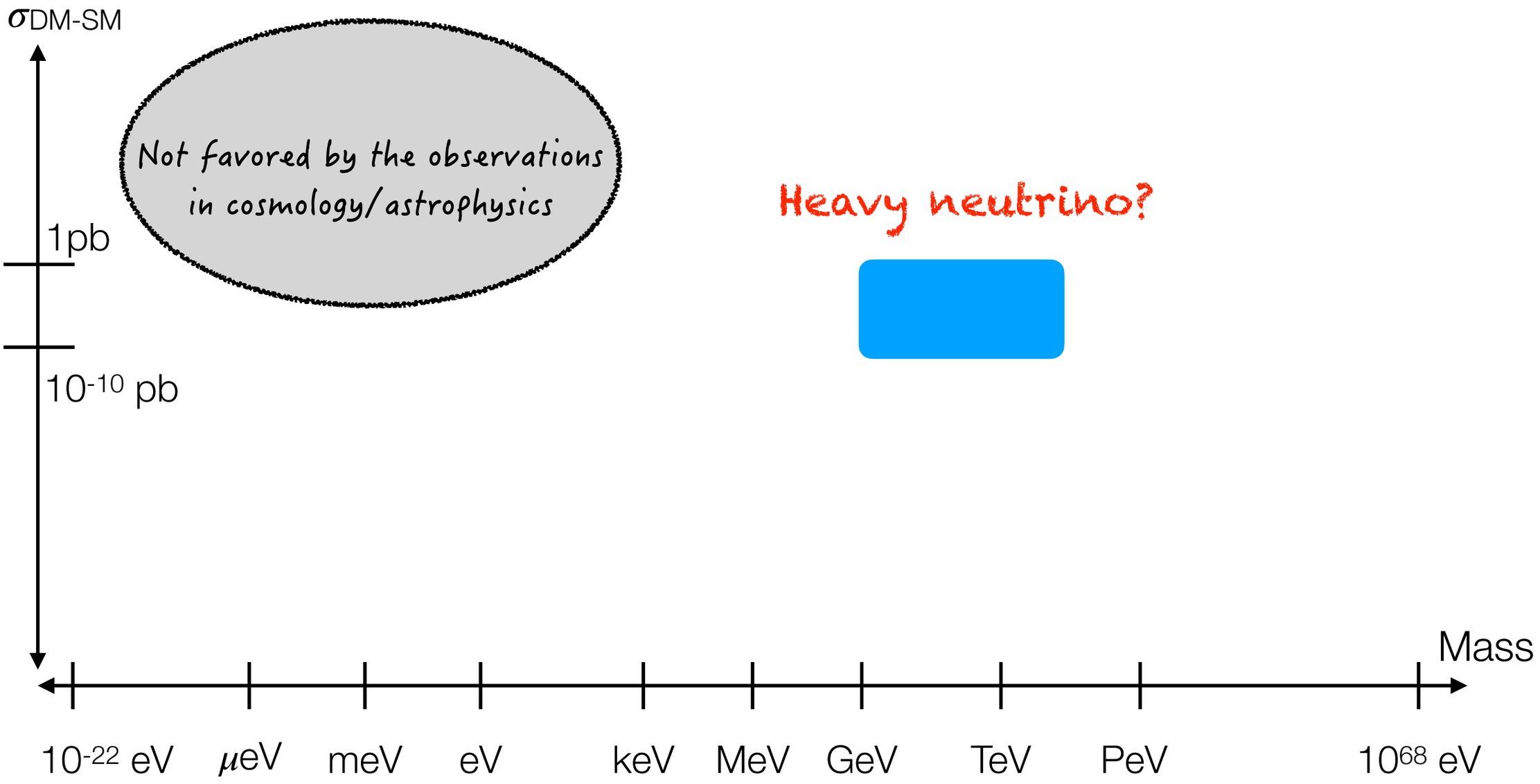
Dark Matter Interactions



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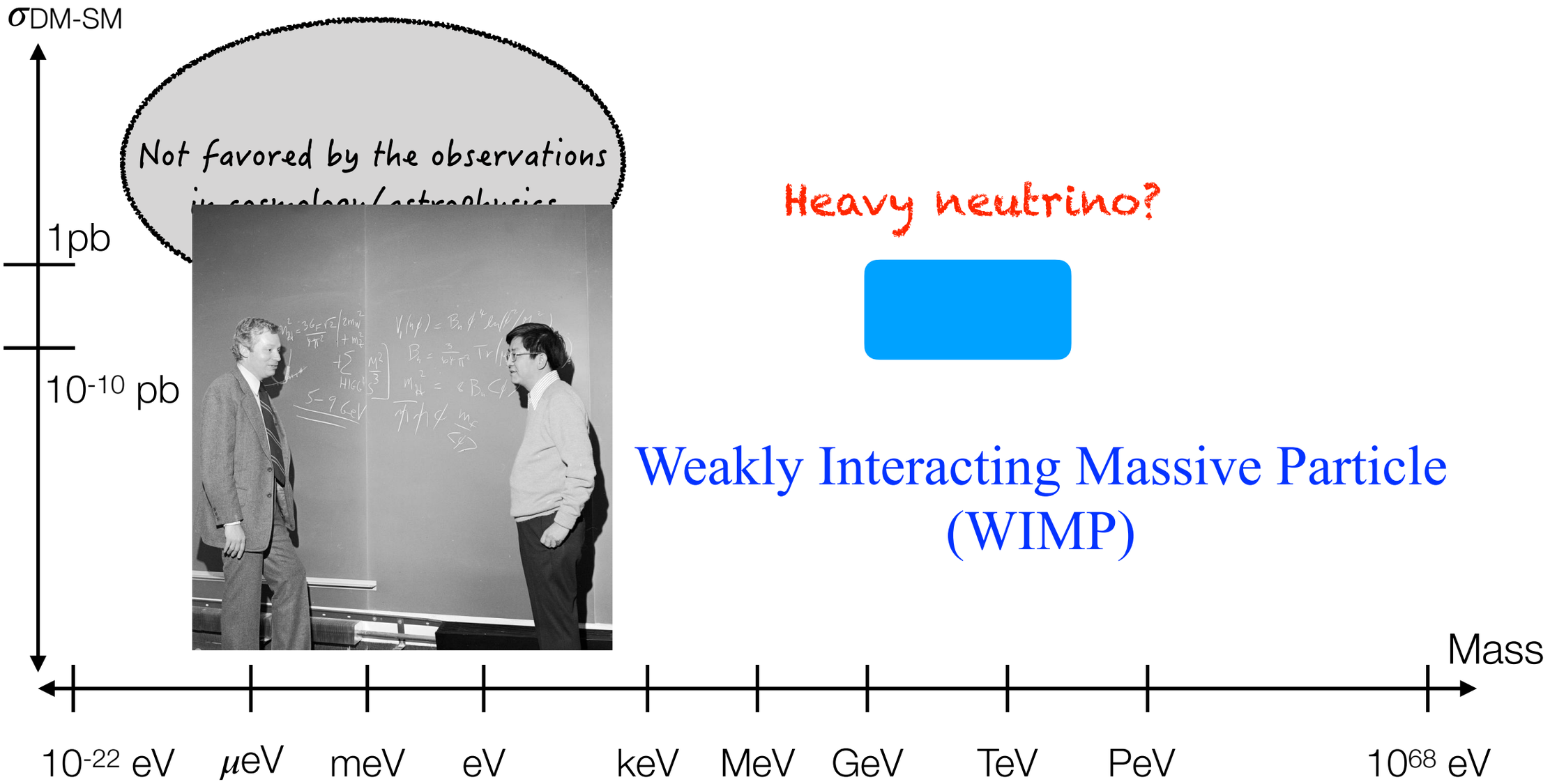


Dark Matter Interactions



Heavy neutrino?

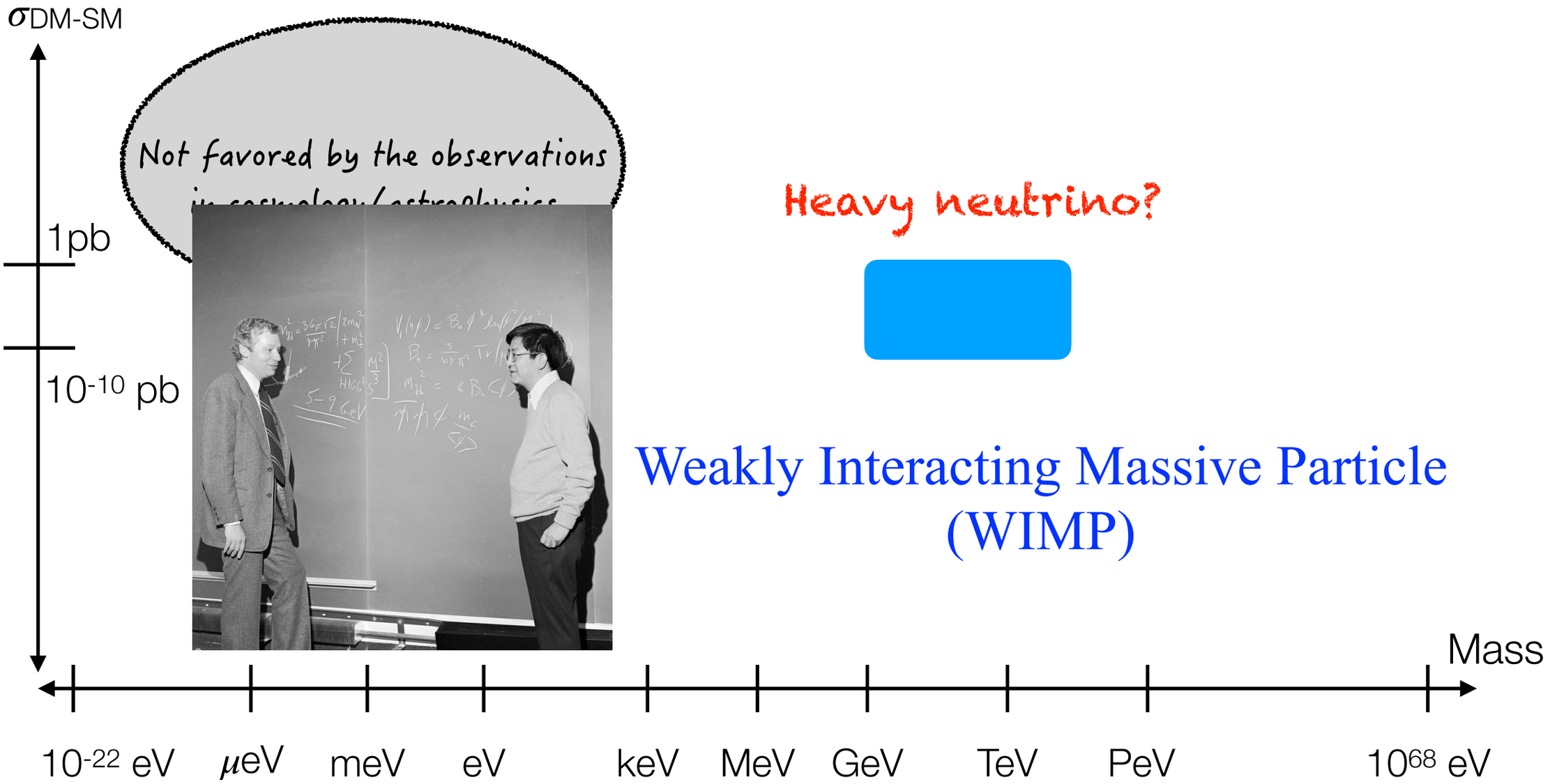
Dark Matter Interactions



Heavy neutrino?

Weakly Interacting Massive Particle (WIMP)

Dark Matter Interactions



Heavy neutrino?

Weakly Interacting Massive Particle (WIMP)

- Weak scale mass: O(1 ~ 100000) × proton mass
- Weak interaction with the SM particles: about < 10^{-12} (in cross section) smaller than EM

Weakly Interacting Massive Particle

PHYSICAL REVIEW
LETTERS

VOLUME 39 25 JULY 1977 NUMBER 4

Cosmological Lower Bound on Heavy-Neutrino Masses

Benjamin W. Lee[a]
Fermi National Accelerator Laboratory,[b] *Batavia, Illinois 60510*

and

Steven Weinberg[c]
Stanford University, Physics Department, Stanford, California 94305
(Received 13 May 1977)

The present cosmic mass density of possible stable neutral heavy leptons is calculated in a standard cosmological model. In order for this density not to exceed the upper limit of 2×10^{-29} g/cm^3, the lepton mass would have to be *greater* than a lower bound of the order of 2 GeV.

Why did people like WIMP?

- Weak scale interaction of heavy neutrino: naturally explain the 27%
- Mechanism of the determining the abundance: insensitive to earlier history
- Byproduct of many BSM theories resolving (or alleviating) the hierarchy problem

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Lightest new stable particle: weak scale interactions

Neutralino in supersymmetric models

Lightest Kaluza-Klein particle in extra-dimensional models

Pseudo Nambu-Goldstone boson in composite Higgs models

.....

Why did people like WIMP?

- How do we obtain the amount of WIMP?
- How can we probe the non-gravitational interactions of WIMP - SM?

Cosmological set-up: Friedmann equation

Prof. K.-Y. Choi's lecture

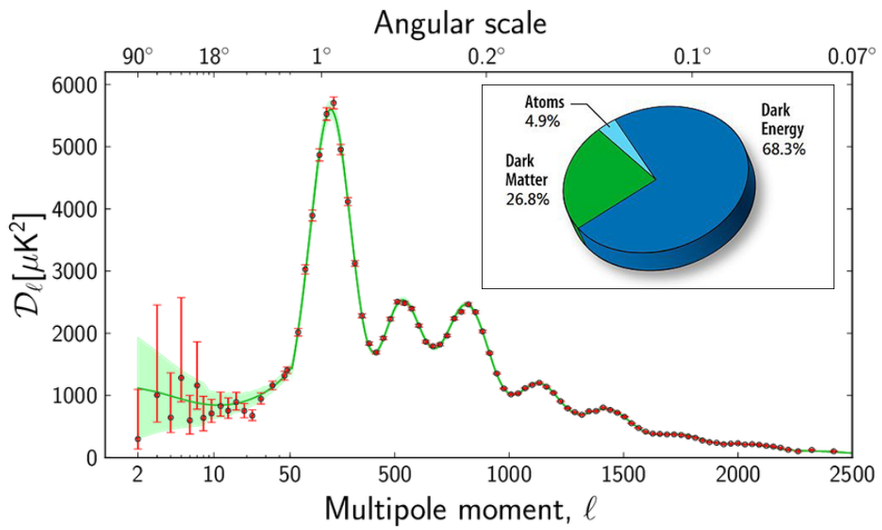
Why did people like WIMP?

- How do we obtain the amount of WIMP? *Relic density*
- How can we probe the non-gravitational interactions of WIMP - SM?

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Prof. K.-Y. Choi's lecture

Relic density of Dark Matter

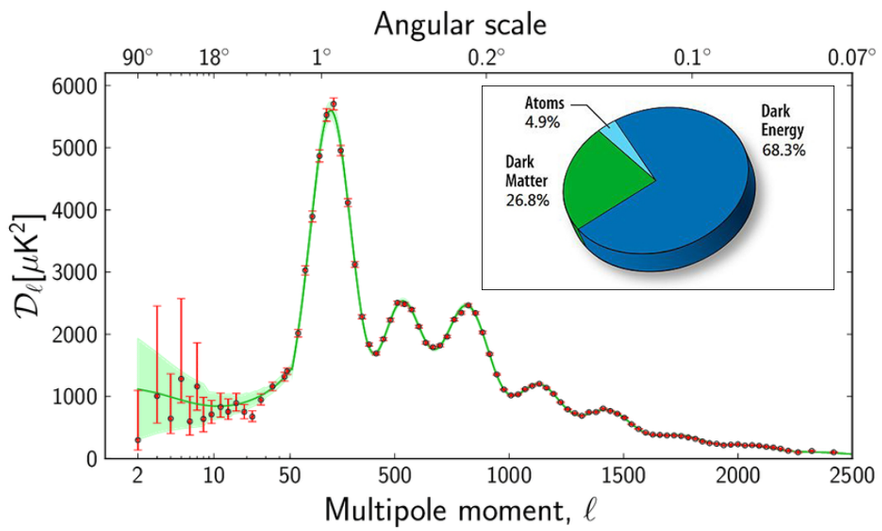


Relic: remnants of early cosmological expansion (present today)





Relic density of Dark Matter



Relic: remnants of early cosmological expansion (present today)



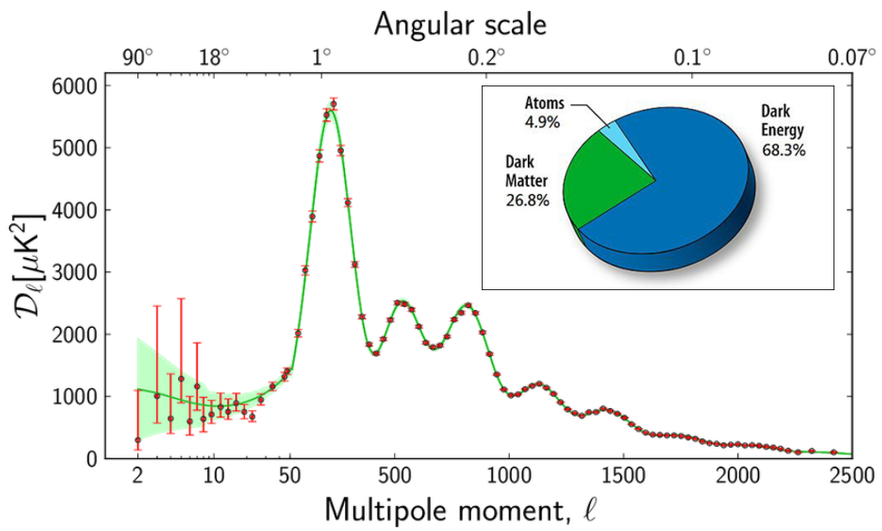


Relic density: energy density

$$\rho = N\frac{E}{V} = N\frac{\sqrt{m^2+p^2}}{V} \simeq N\frac{m}{V} = m \times n$$

for non-relativistic particle

Relic density of Dark Matter



Relic: remnants of early cosmological expansion (present today)





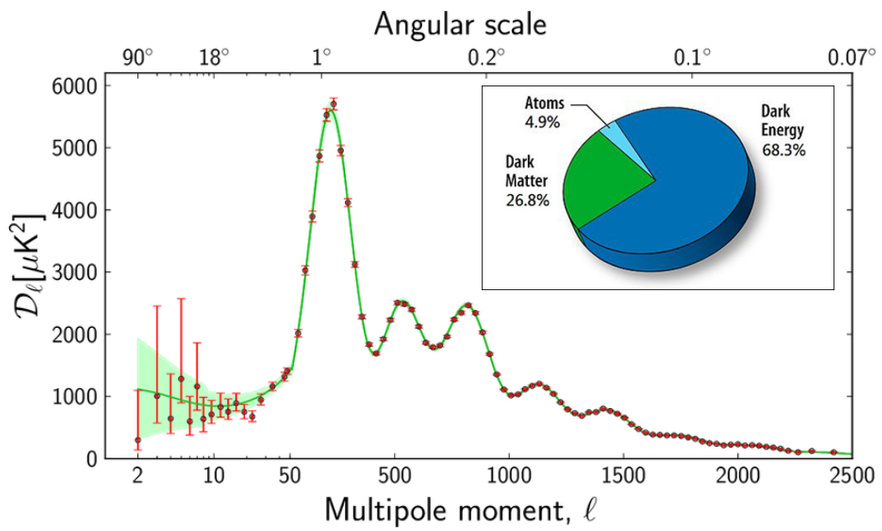
Relic density: energy density

$$\rho = N\frac{E}{V} = N\frac{\sqrt{m^2+p^2}}{V} \simeq N\frac{m}{V} = m \times n$$

for non-relativistic particle

How to determine n?

Relic density of Dark Matter



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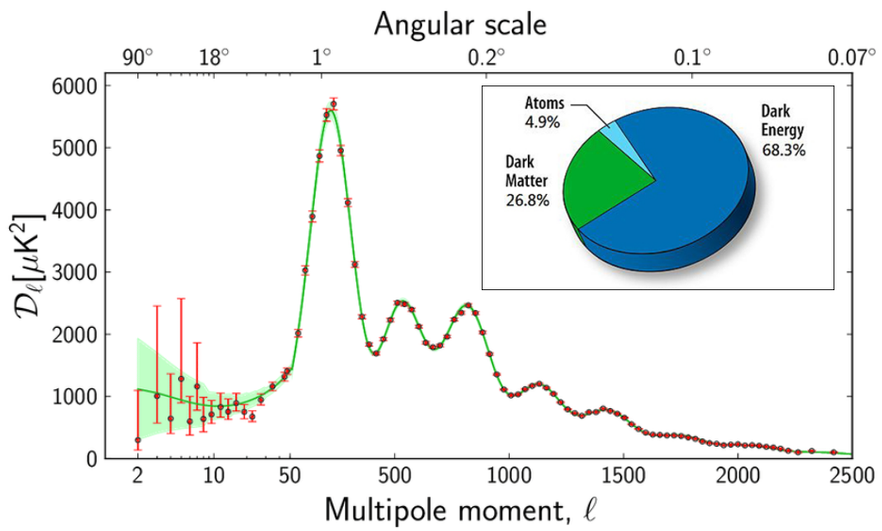
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How to determine n?

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$$\rho = nh\nu = [R^{-4}] \propto T^4$$

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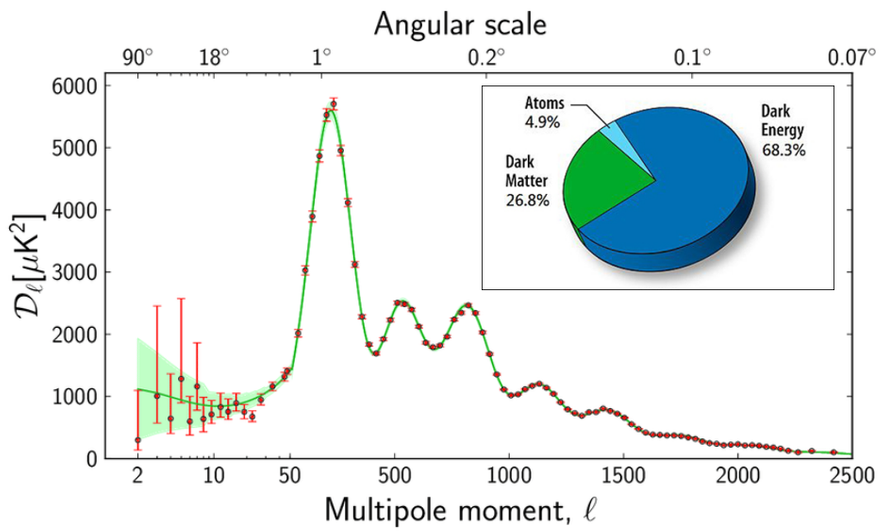
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Relic density of Dark Matter



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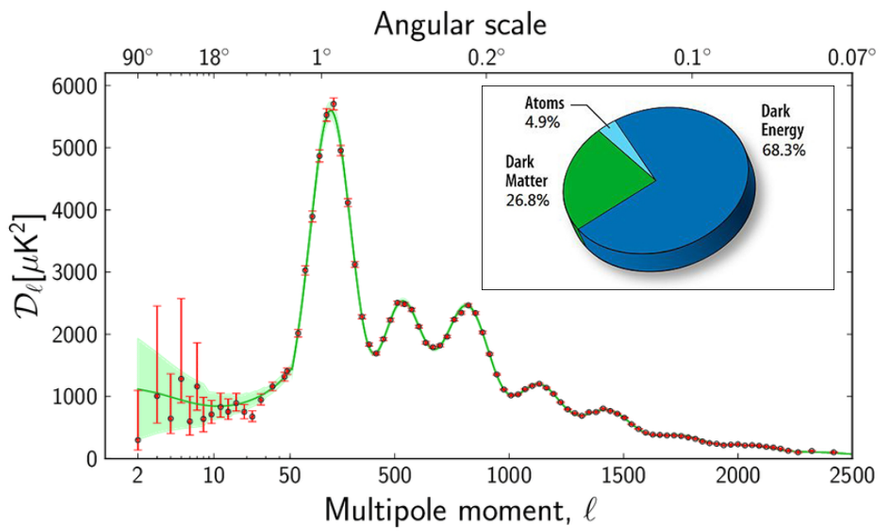
Present day total relic density (critical density)

$$\rho_0 = \frac{3H_0^2}{8\pi G_N} \simeq 10^{-29}\,\mathrm{g/cm^3}$$

10 protons/m^3

Friedmann equations in flat Universe

Relic density of Dark Matter



Relic: remnants of early cosmological expansion (present today)





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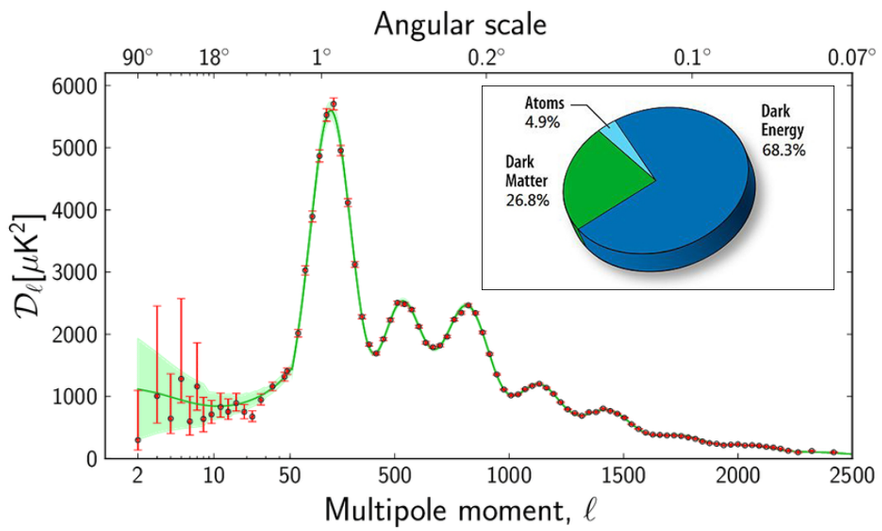
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Friedmann equations in flat Universe

Relic density of Dark Matter at present (0 subscript)

$$\Omega_{\mathrm{DM}} = \frac{\rho_{\mathrm{DM},0}}{\rho_0} \simeq 0.27 \quad \longrightarrow \quad \Omega_{\mathrm{DM}} h^2 \simeq 0.12$$

Relic density of Dark Matter



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$$\rho_{\rm DM,0} \approx 10^{10}\,\frac{M_\odot}{\mathrm{Mpc}^3} \approx 10^{-6}\mathrm{GeV/cm^3}$$

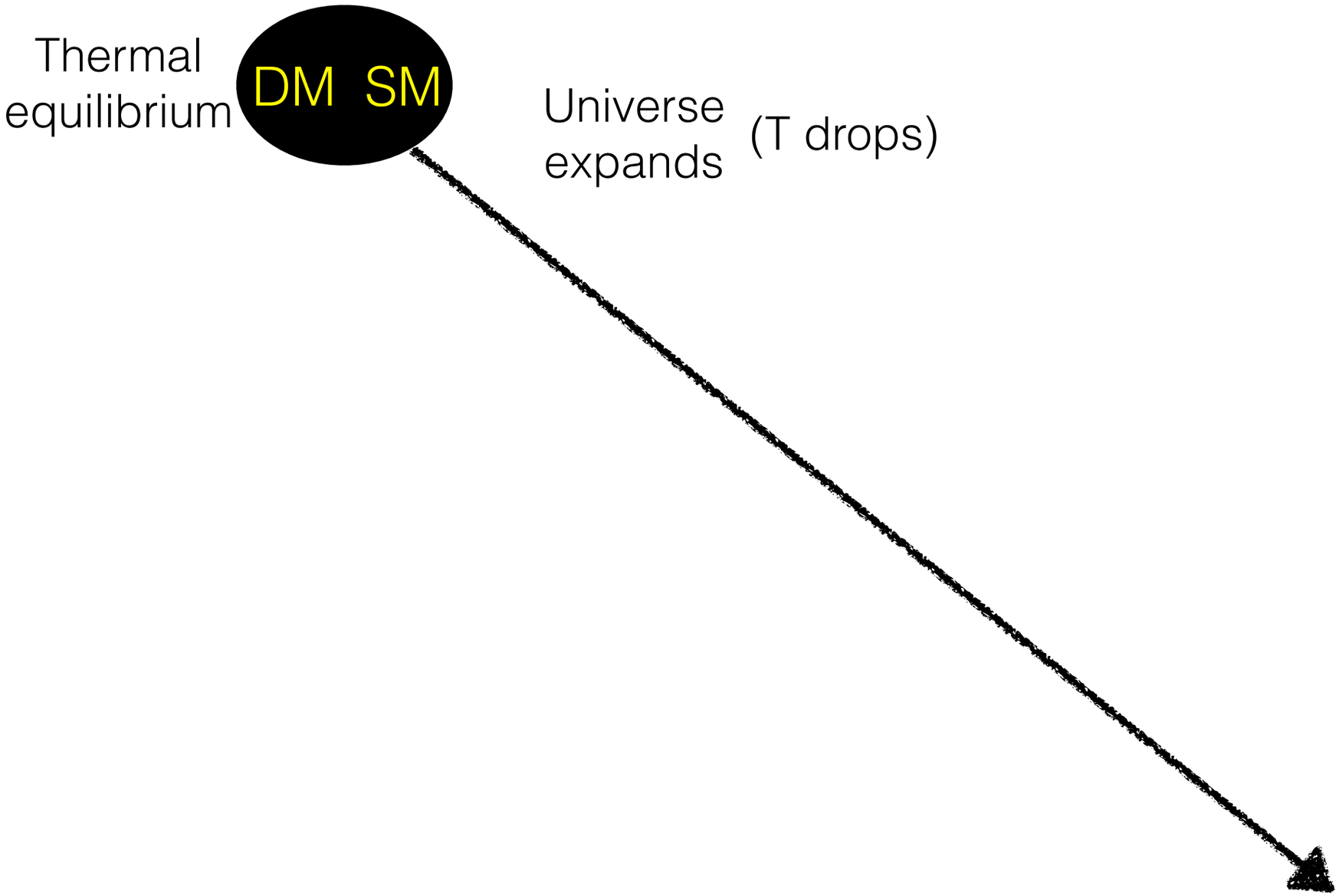
10^{-5} of clusters

10^{-6} of galaxies

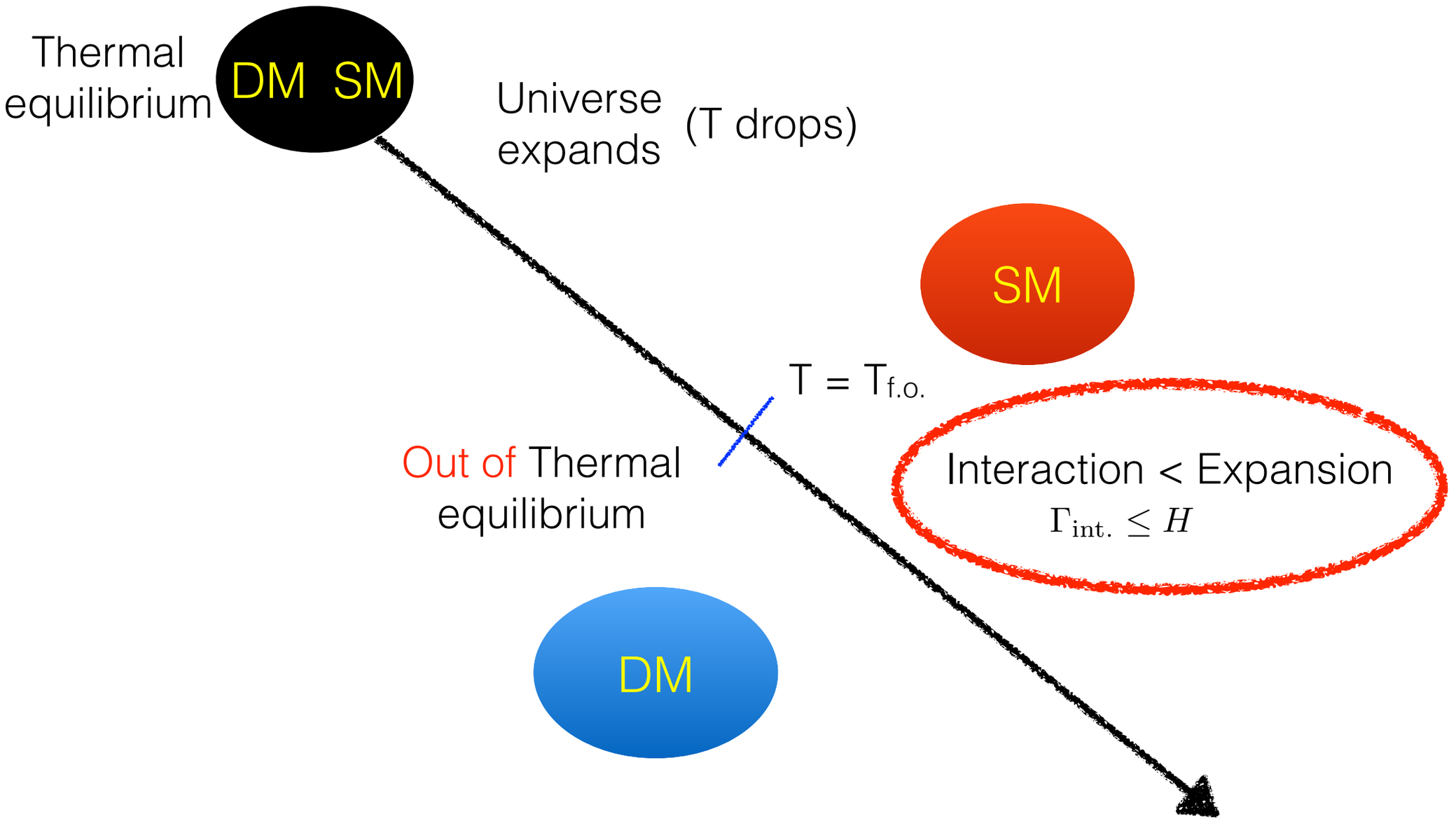
Thermal freeze-out: schematic view



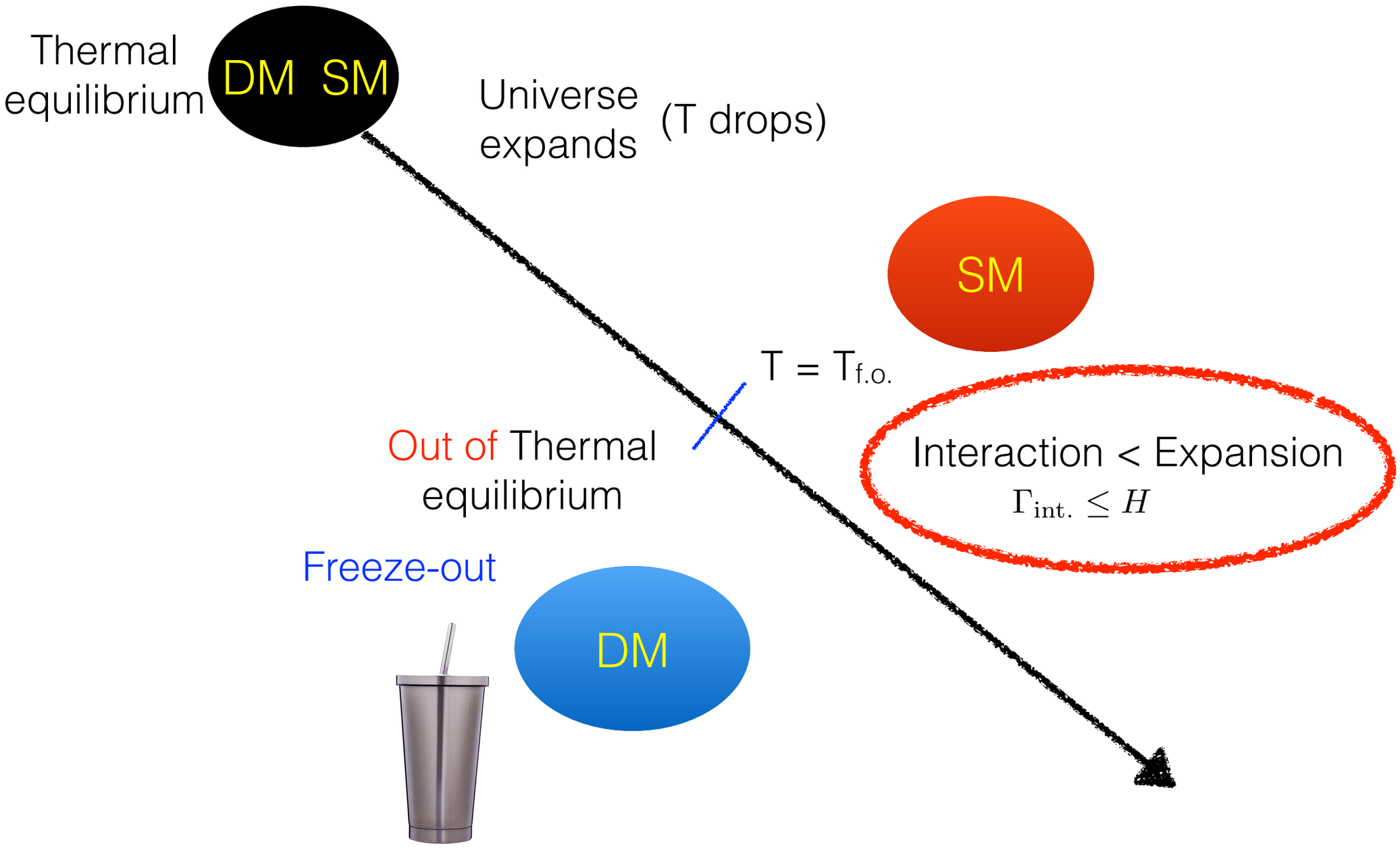
Thermal freeze-out: schematic view



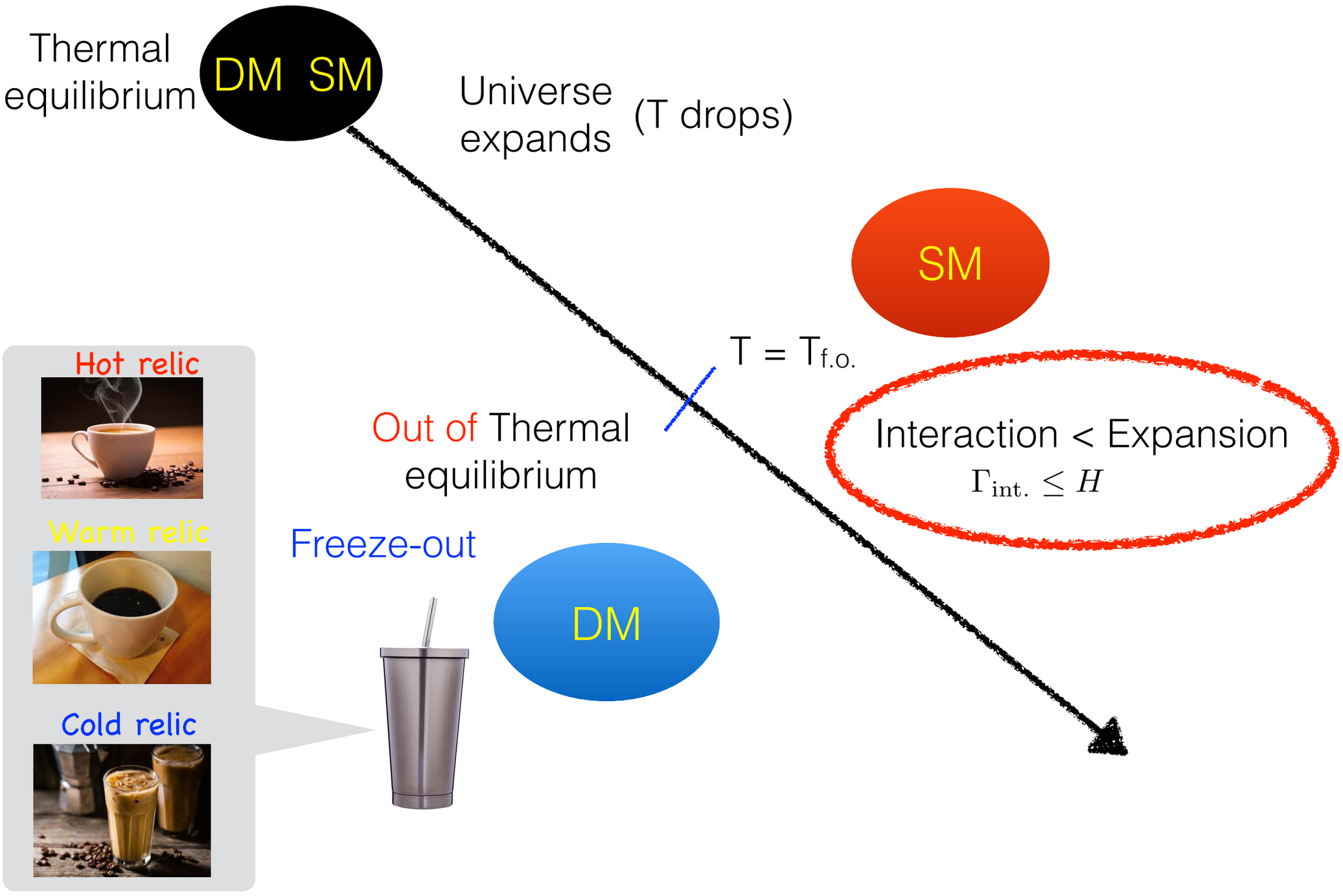
Thermal freeze-out: schematic view



Thermal freeze-out: schematic view



Thermal freeze-out: schematic view



Thermal freeze-out: schematic view



Thermal equilibrium

DM SM

Universe expands (T drops)

SM

$T = T_{f.o.}$

Out of Thermal equilibrium

Interaction < Expansion

$\Gamma_{\rm int.} \leq H$

Freeze-out

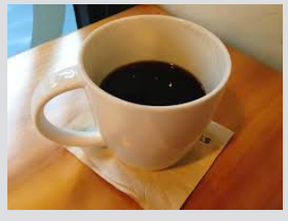
DM



Hot relic



Warm relic



Cold relic

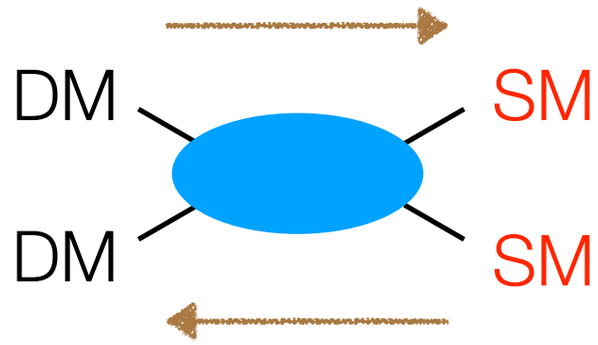


Thermal freeze-out: schematic view

Thermal equilibrium



$T \gg T_{DM}$

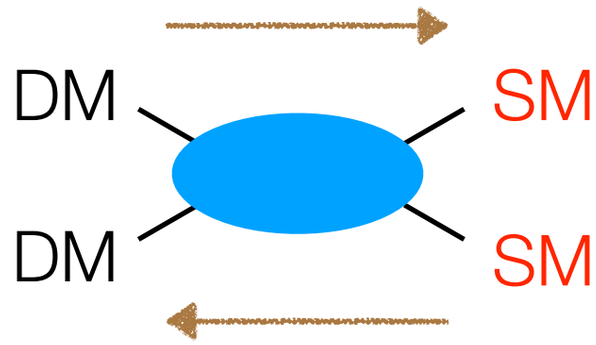


Thermal freeze-out: schematic view

Thermal equilibrium



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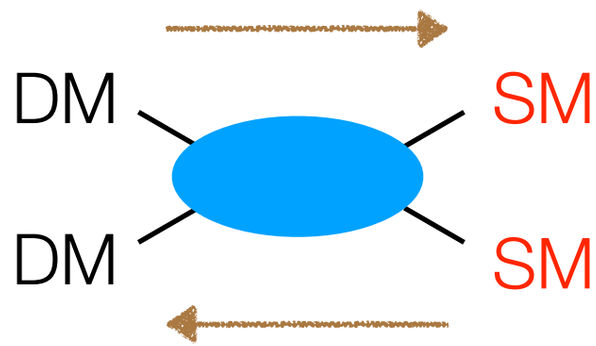
Thermal & Chemical equilibrium

Thermal freeze-out: schematic view

Thermal equilibrium



$T \gg T_{DM}$



Thermal & Chemical equilibrium

$$n_{\rm DM} = n_{\rm eq} \propto T^3$$

Phase space distribution

$$f(\mathbf{p}) = \frac{g}{e^{(E(\mathbf{p})-\mu)/T} \pm 1}$$

\+ : Fermi-Dirac
– : Bose-Einstein
g: # of spin, flavor

$$n_\chi = \int \frac{d^3\mathbf{p}}{(2\pi)^3} f(\mathbf{p})$$

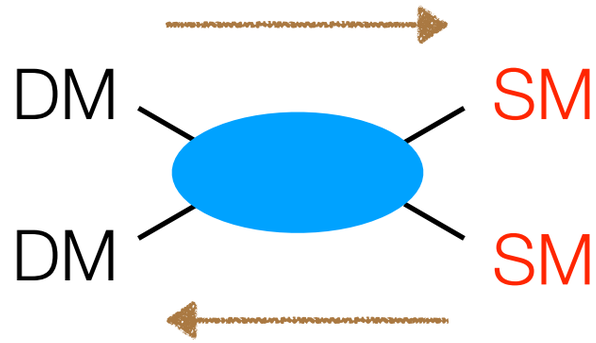
$$n_{\rm eq} = n_{\rm DM} e^{-\mu/T} \quad \text{for} \quad T < E - \mu$$

Thermal freeze-out: schematic view

Thermal equilibrium



$T \gg T_{DM}$



Thermal & Chemical equilibrium

$n_{\rm DM} = n_{\rm eq} \propto T^3$

Constant N_{DM}



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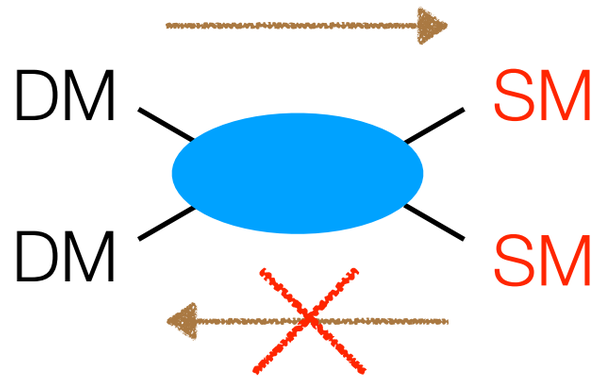
Thermal freeze-out: schematic view

Thermal equilibrium

DM SM

Universe expands (T drops)

T = m_{DM}



Thermal & Chemical equilibrium

if reaction rate per DM > H

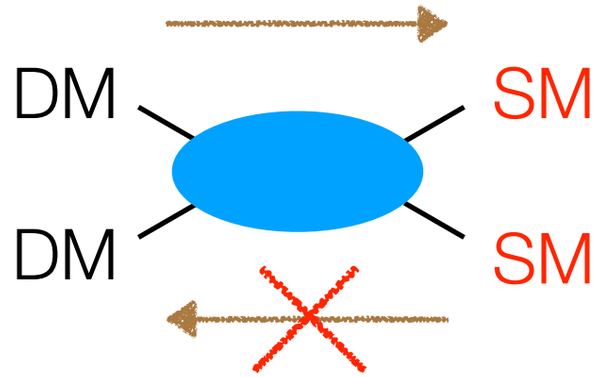
Thermal freeze-out: schematic view

Thermal equilibrium

DM SM

Universe expands (T drops)

T = $m_{\rm DM}$



Thermal & Chemical equilibrium

if reaction rate per DM > H

$$n_{\rm eq} \propto (mT)^{3/2} e^{-m/T}$$

$$\mu = 0$$

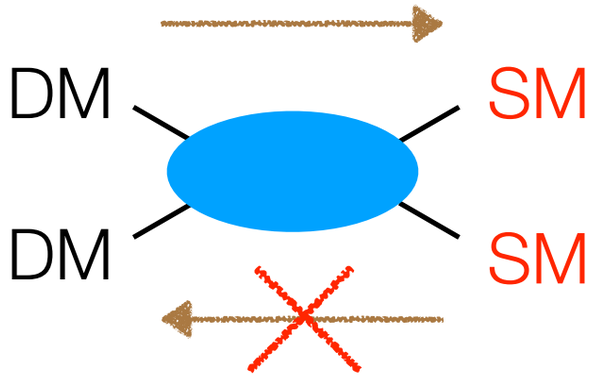
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Thermal & Chemical equilibrium

if reaction rate per DM > H

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Boltzmann suppression

$\mu = 0$

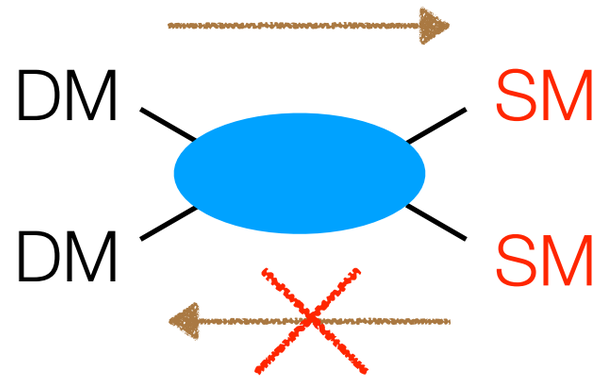
Thermal freeze-out: schematic view

Thermal equilibrium

DM SM

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Thermal & Chemical equilibrium

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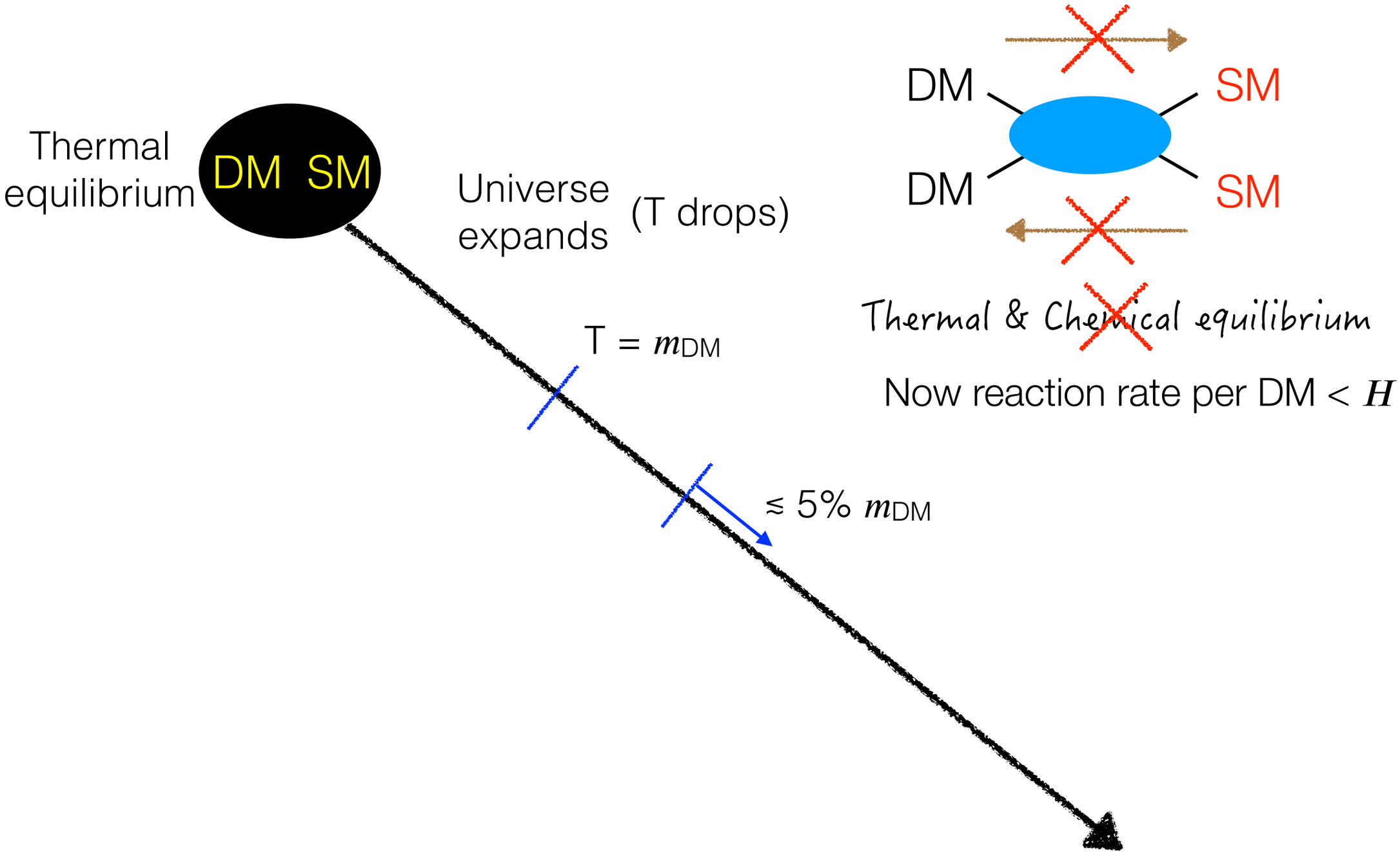
$\mu = 0$

Boltzmann suppression

Decreasing $N_{\rm DM}$



Thermal freeze-out: schematic view



Thermal freeze-out: schematic view

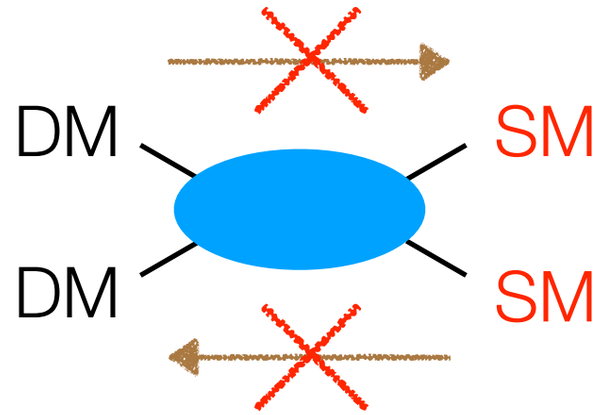
Thermal equilibrium

DM SM

Universe expands (T drops)

T = m_{DM}

≲ 5% m_{DM}



Thermal & Chemical equilibrium

Now reaction rate per DM < H



Thermal freeze-out: schematic view

Thermal equilibrium

DM SM

Universe expands (T drops)

T = m_{DM}

DM SM → DM SM

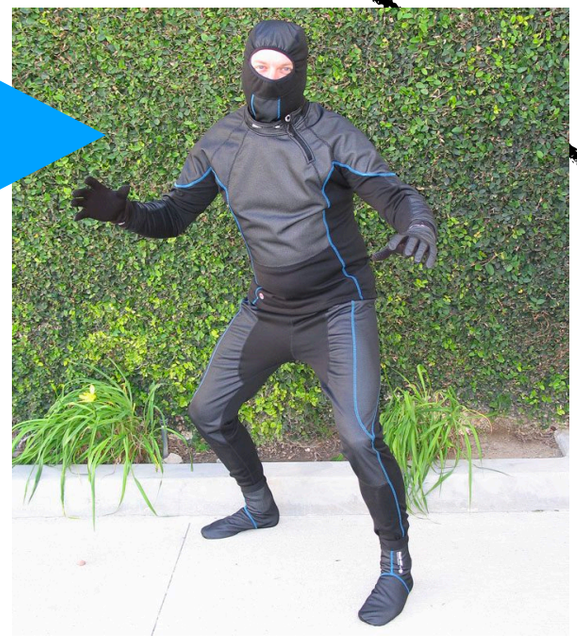
Thermal & Chemical equilibrium

Now reaction rate per DM < H

≲ 5% m_{DM}

Frozen N_{DM}





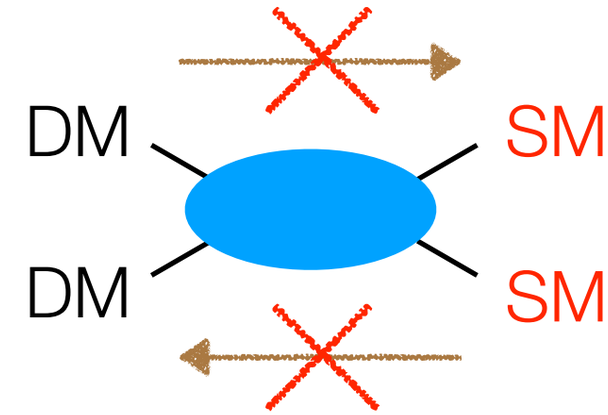
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Thermal & Chemical equilibrium

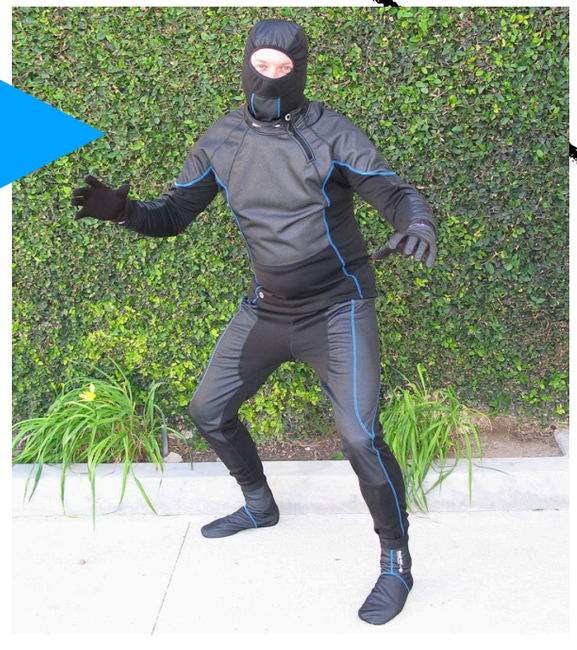
Now reaction rate per DM < H

T = $T_{f.o.}$

$\lesssim$ 5% m_{DM}

Frozen N_{DM}







Cold Dark Matter

Thermal freeze-out: schematic view

Thermal equilibrium

DM SM

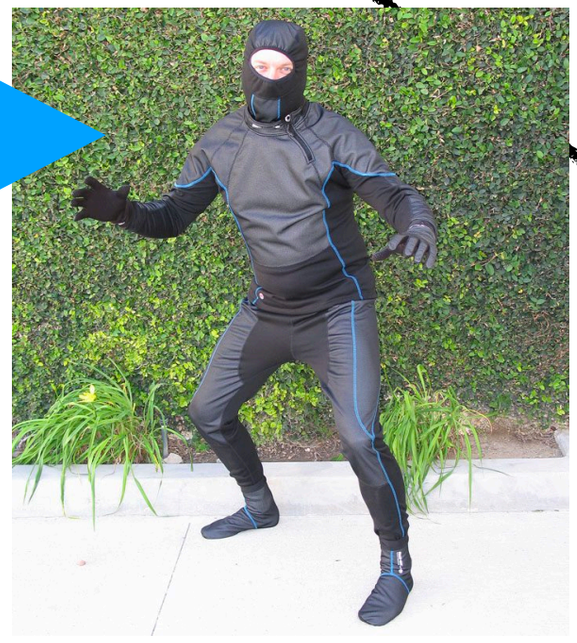
Universe expands (T drops)

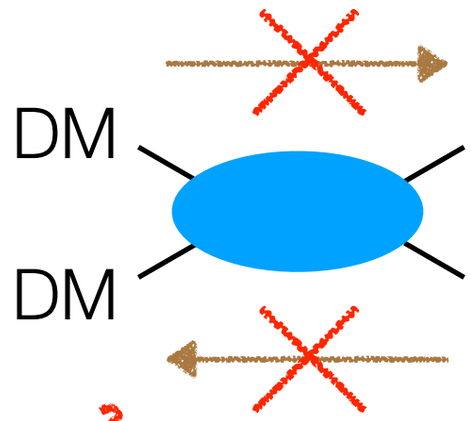
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T = $T_{f.o.}$

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?

Thermal & Chemical equilibrium

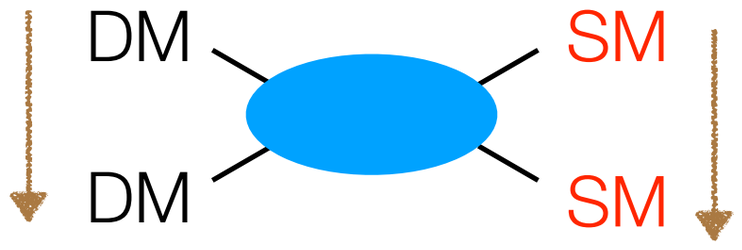
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Frozen N_{DM}



Cold Dark Matter

Thermal freeze-out: schematic view

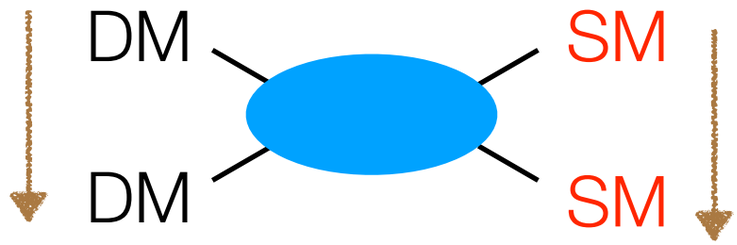


Thermal & Kinetic equilibrium

Thermal freeze-out: schematic view

Elastic (number conserving) scattering occurs in Hubble time even after chemical decoupling of Cold Dark Matter (CDM).

μ is produced.



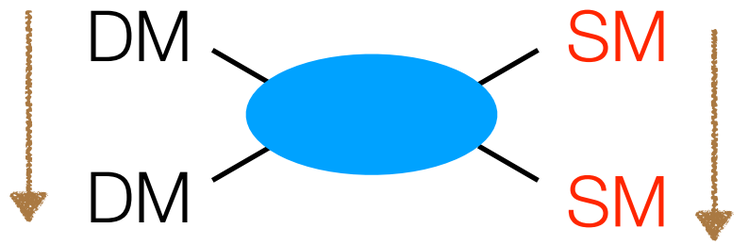
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How???

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Thermal & Kinetic equilibrium

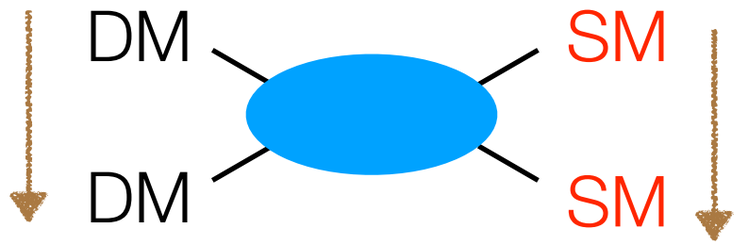
Thermal freeze-out: schematic view

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How???

μ is produced.

Interaction rate?



Thermal & Kinetic equilibrium

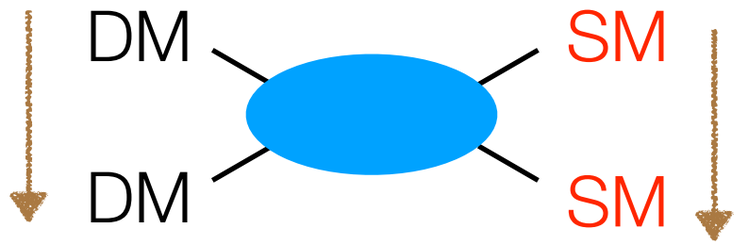
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Thermal & Kinetic equilibrium

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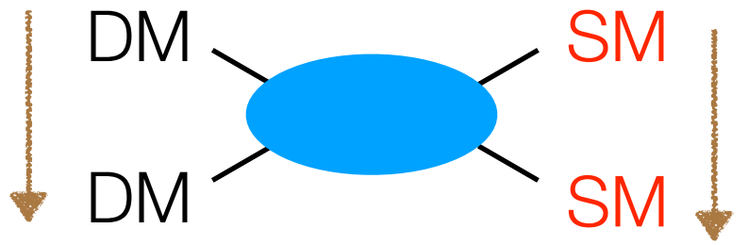
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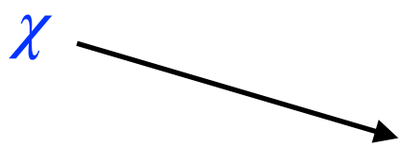
How???

μ is produced.

Interaction rate?



Thermal & Kinetic equilibrium



No interaction rate if free stream

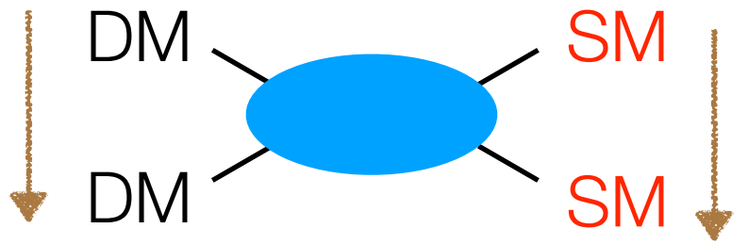
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Elastic (number conserving) scattering occurs in Hubble time even after chemical decoupling of Cold Dark Matter (CDM).

How???

μ is produced.

Interaction rate?



Thermal & Kinetic equilibrium



No interaction rate if free stream

Interaction occurs after meeting A ($\propto n_A$)

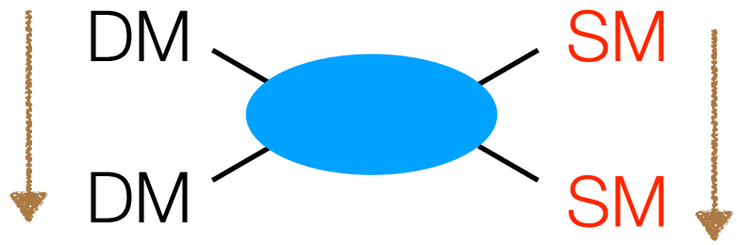
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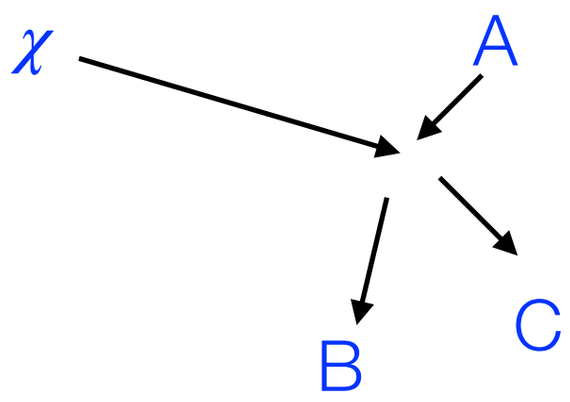
How???

μ is produced.

Interaction rate?



Thermal & Kinetic equilibrium



No interaction rate if free stream

Interaction occurs after meeting A ($\propto n_A$) also $\propto \langle \sigma v \rangle_{\chi A \to BC}$

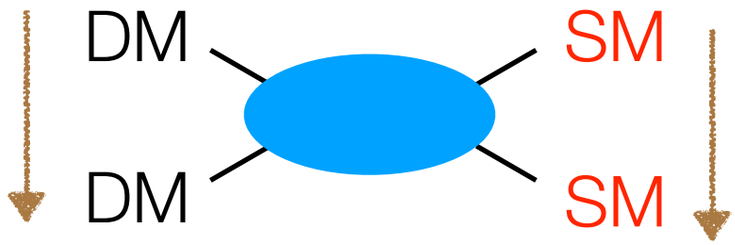
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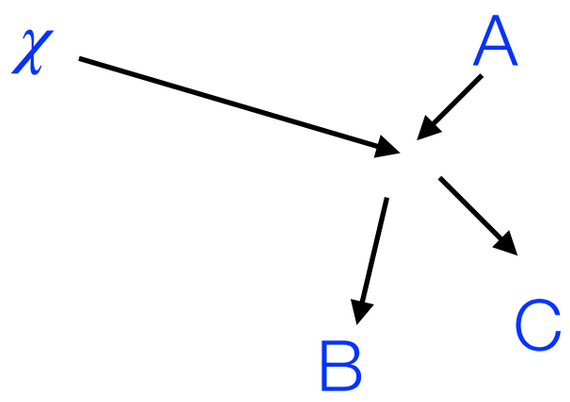
How???

μ is produced.

Interaction rate?



Thermal & Kinetic equilibrium



$$\Gamma_{\text{int.}} = n_A \langle \sigma v \rangle_{\chi A \to BC}$$

No interaction rate if free stream

Interaction occurs after meeting A ($\propto n_A$) also $\propto \langle \sigma v \rangle_{\chi A \to BC}$

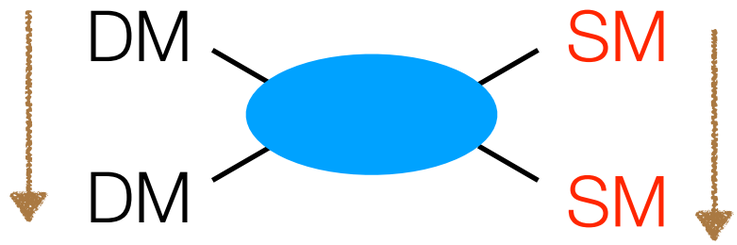
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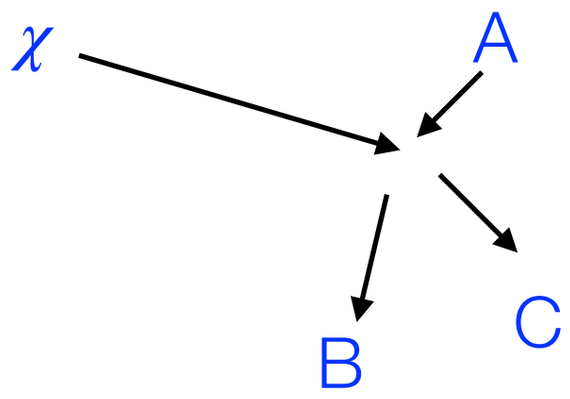
How???

μ is produced.

Interaction rate?



Thermal & Kinetic equilibrium



$$\Gamma_{\text{int.}} = n_A \langle \sigma v \rangle_{\chi A \to BC}$$

v: Møler velocity

(from the phase space integration)

$$(v)_{ij} = \frac{\sqrt{(E_i E_j - \vec{p}_i \cdot \vec{p}_j)^2 - (m_i m_j)^2}}{E_i E_j}$$

No interaction rate if free stream

$(v)_{ij} n_i n_j$ Lorentz invariant

Interaction occurs after meeting A ($\propto n_A$) also $\propto \langle \sigma v \rangle_{\chi A \to BC}$

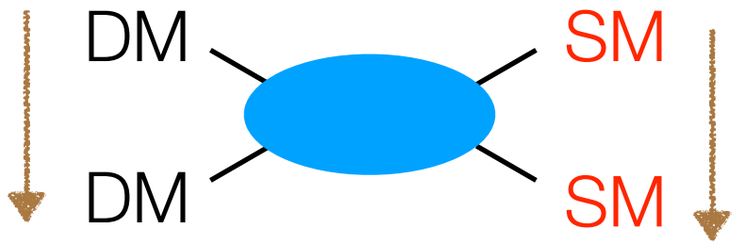
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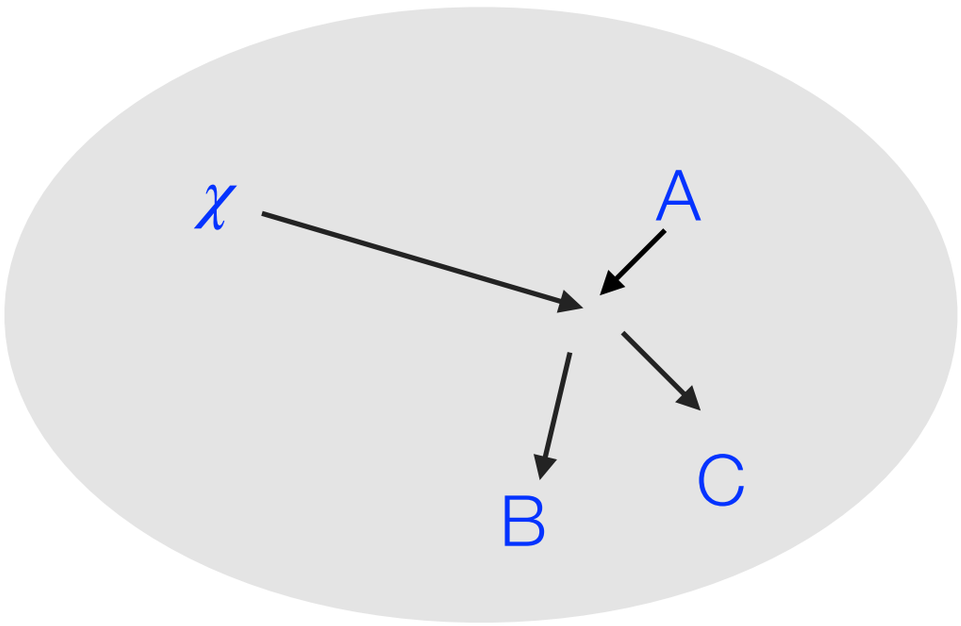
How???

μ is produced.

Interaction rate?



Thermal & Kinetic equilibrium



$$\Gamma_{\text{int.}} = n_A \langle \sigma v \rangle_{\chi A \to BC}$$

No interaction rate if free stream

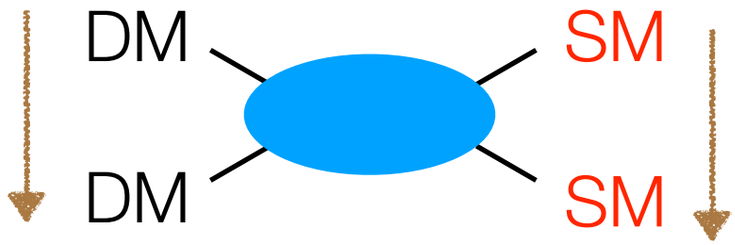
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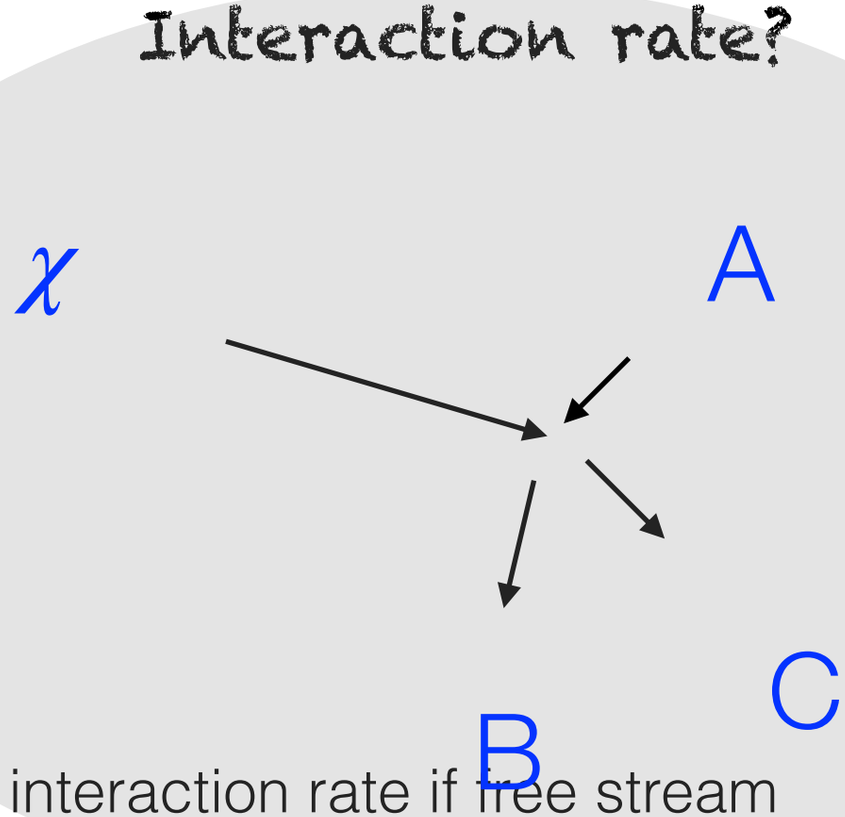
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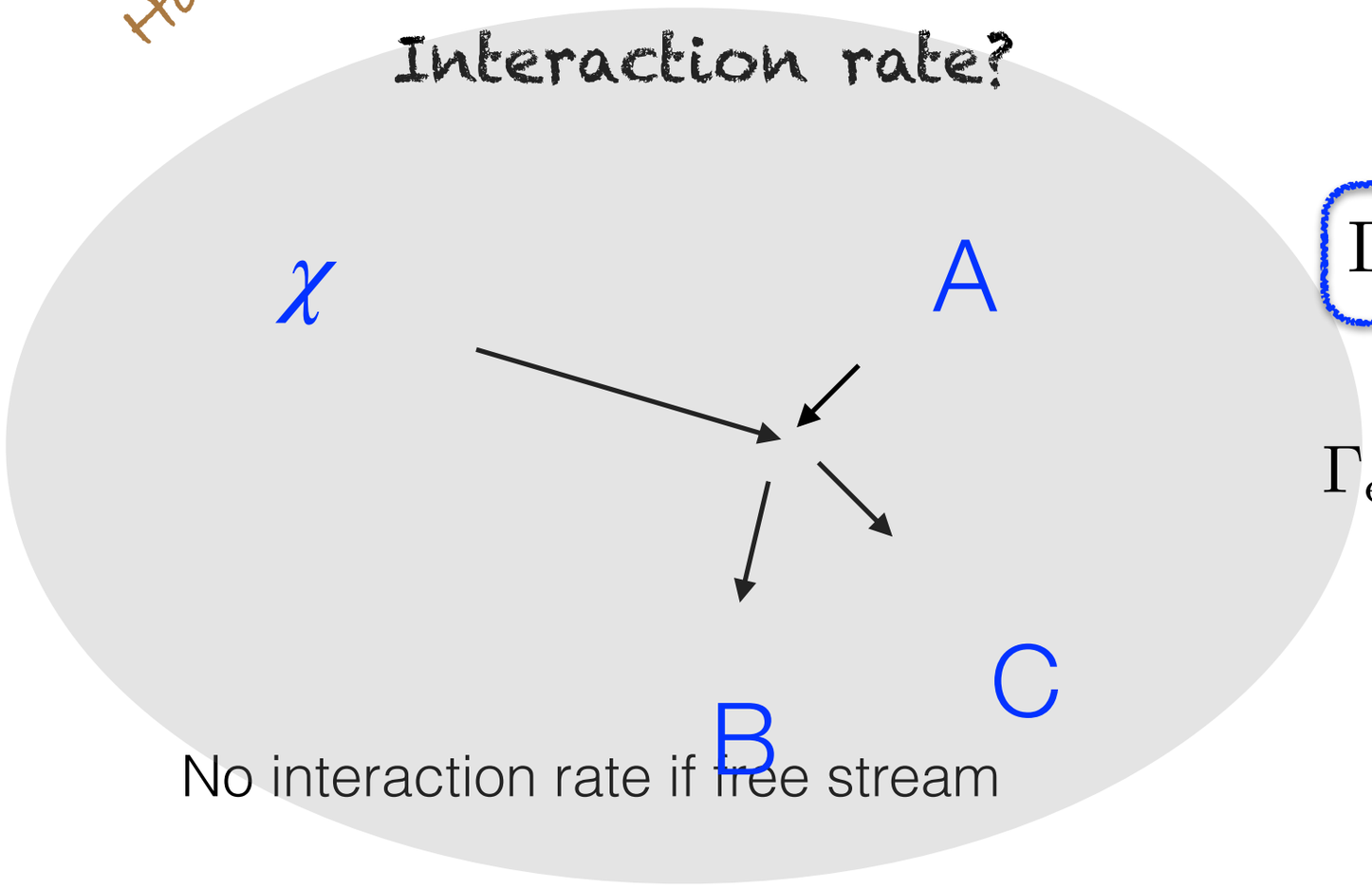
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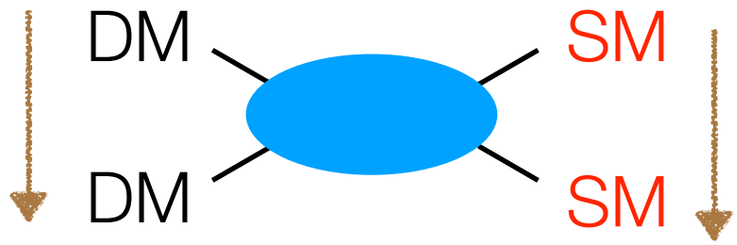
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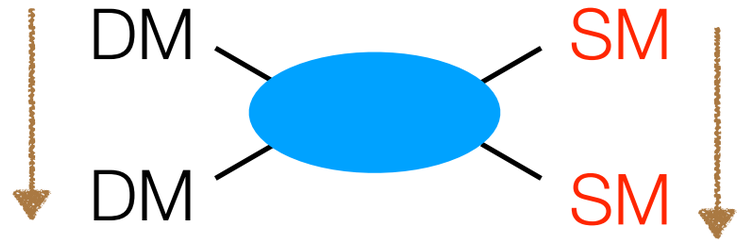
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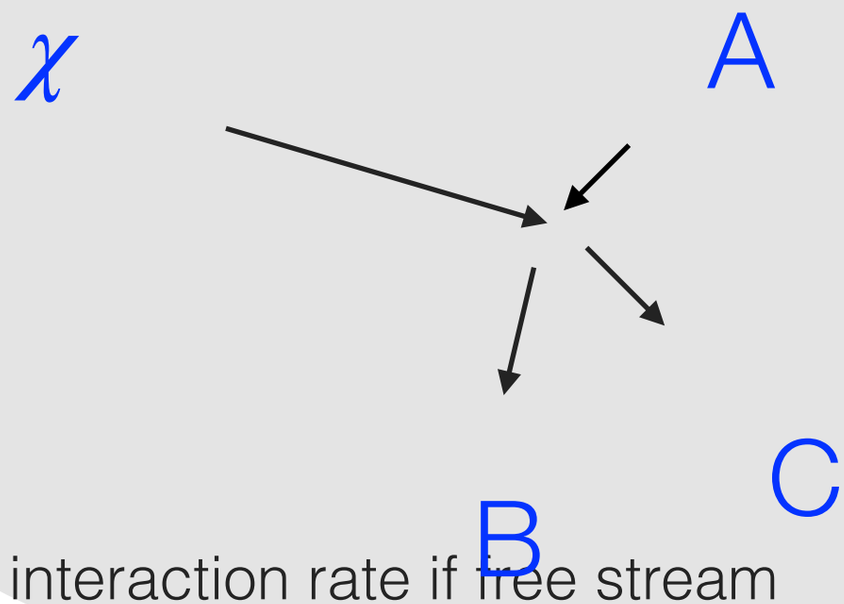
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Chemical equilibrium: $n_A = n_\chi = n_\chi^{\text{eq}}$

Kinetic equilibrium: $n_A = n_{\text{SM}} \propto T^3$

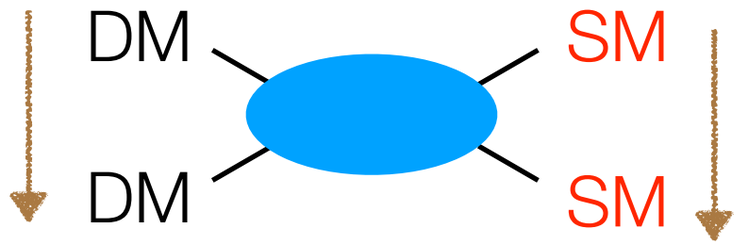
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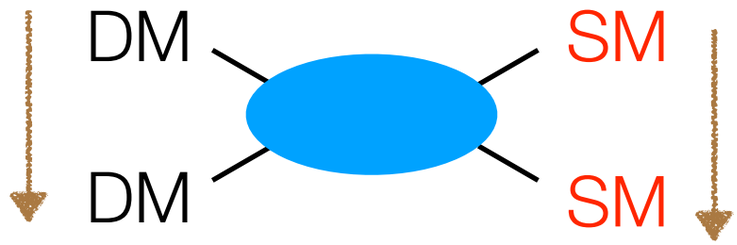
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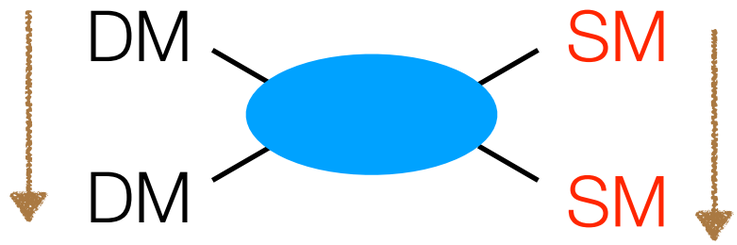
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Out of thermal equilibrium after the kinetic equilibrium of CDM

Depends on DM scenarios

Early kinetic decoupling scenarios do exist!
(Hot DM, forbidden channel, …)

Estimation: hot relic

Thermal relic: produced via interactions with the thermal bath (SM particles).

Estimation: hot relic

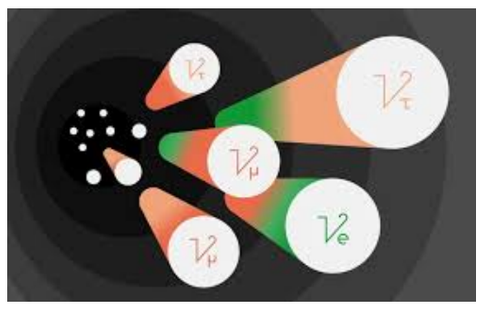
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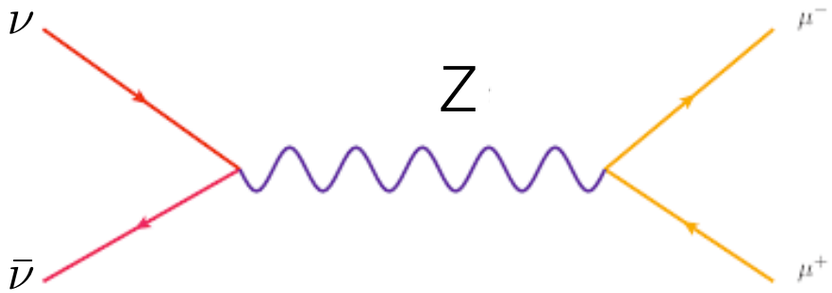


Neutrinos → Cosmic $\boldsymbol{\nu}$ Background

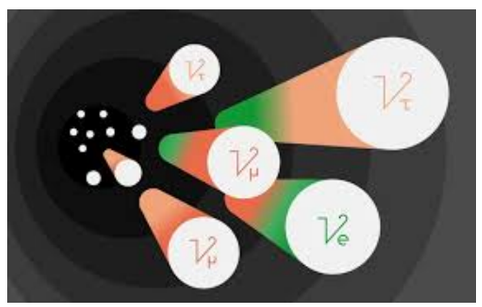
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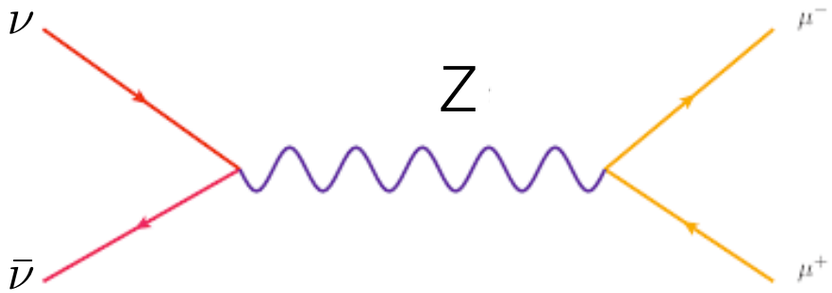


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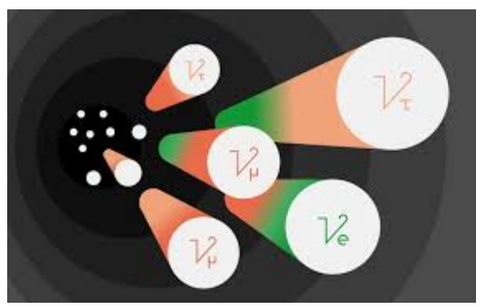
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Freeze-out condition



$$T_{\nu,\mathrm{f.o.}} \simeq \left(G_F^2 M_P\right)^{-1/3} \sim 1\,\mathrm{MeV}$$

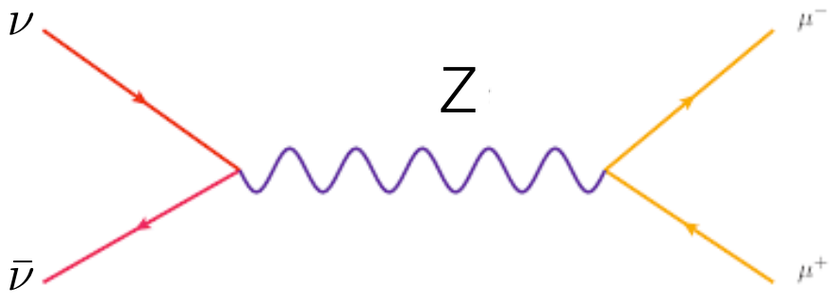
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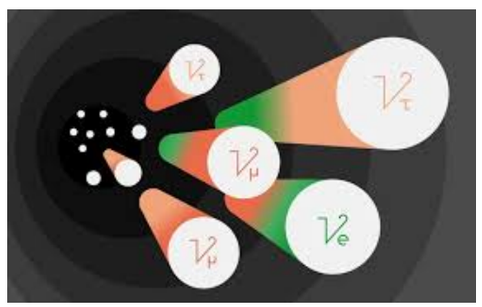
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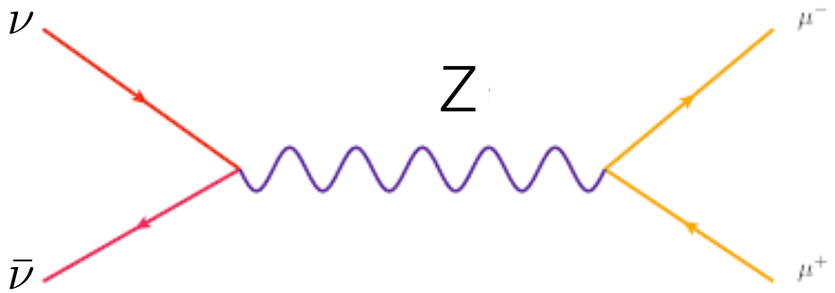
$\Omega_\nu h^2 \approx \dfrac{\sum m_\nu}{94\,\mathrm{eV}}$ ≤ 0.0028, too small!

ν oscillations: $\sum m_\nu \lesssim 0.26\,\mathrm{eV}\,(95\%\mathrm{C.L.})$

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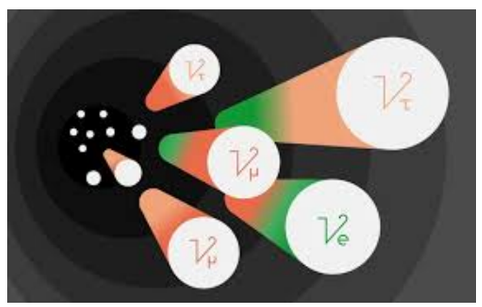
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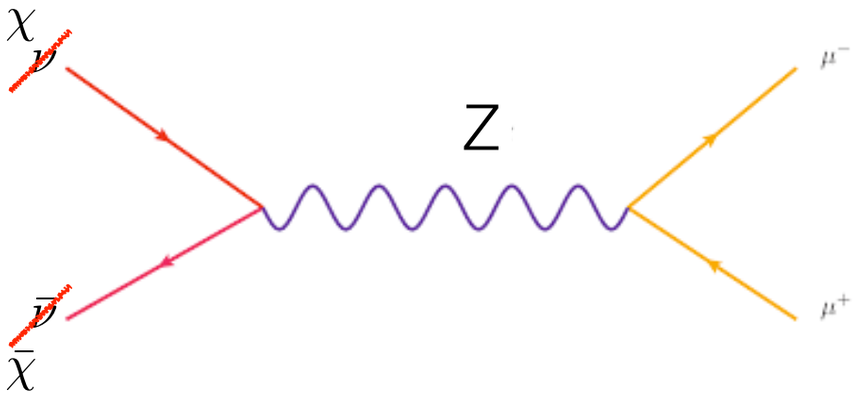
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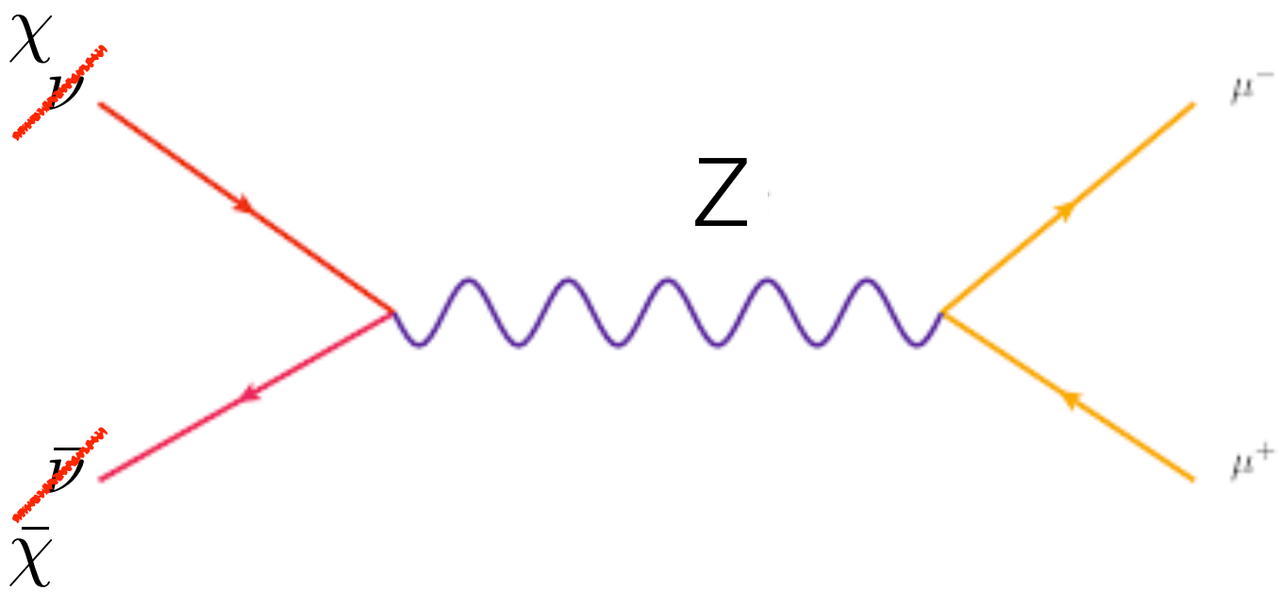
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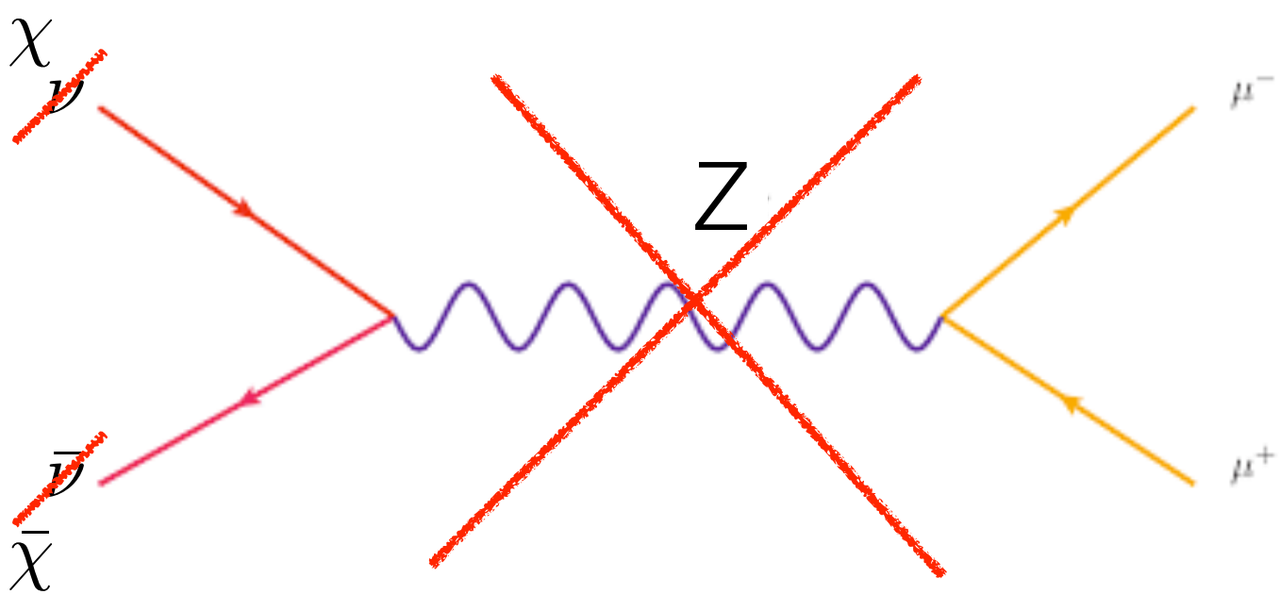
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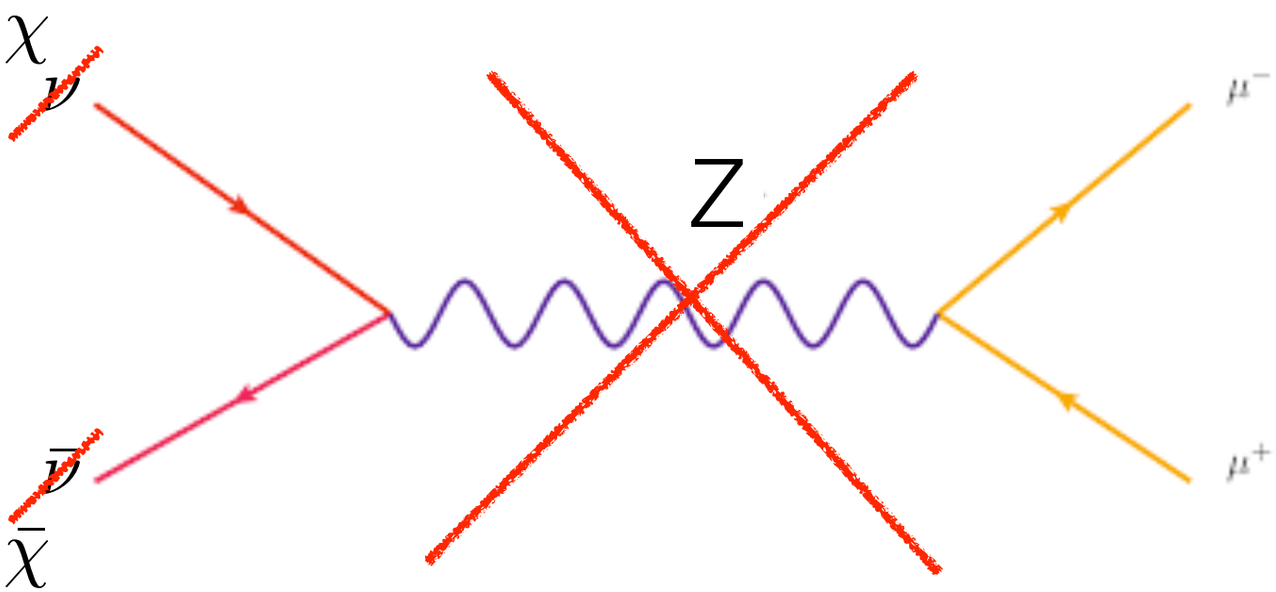
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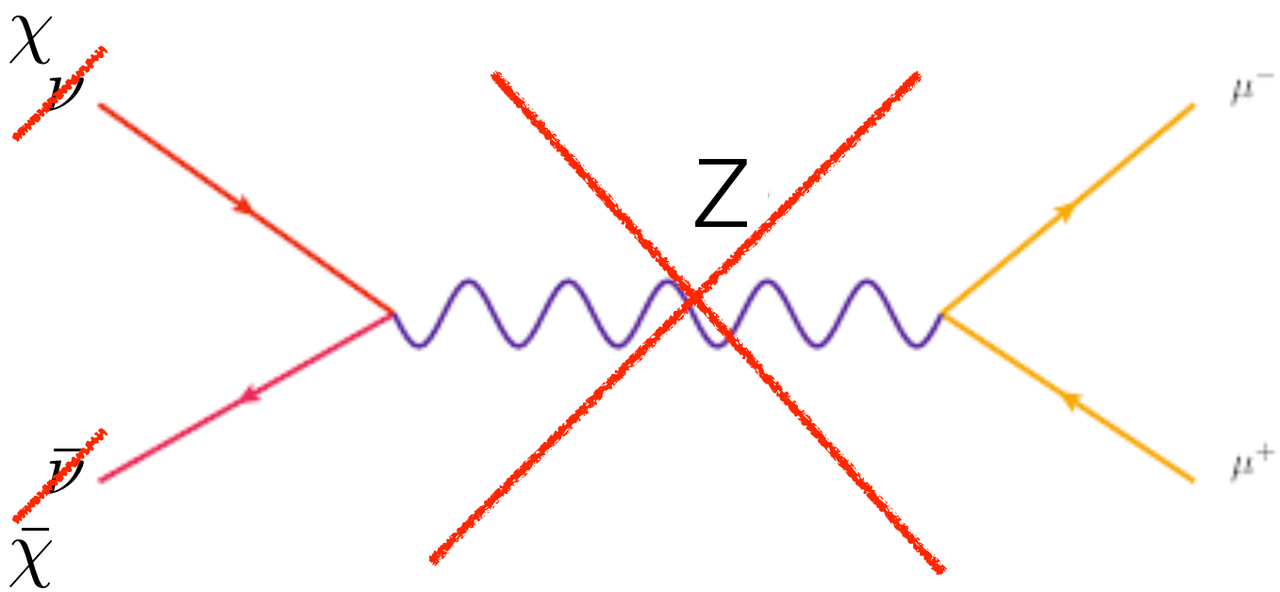
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$$M_P = \sqrt{\frac{\hbar c}{8\pi G_N}} \approx 2.4 \times 10^{18}\,{\rm GeV}$$

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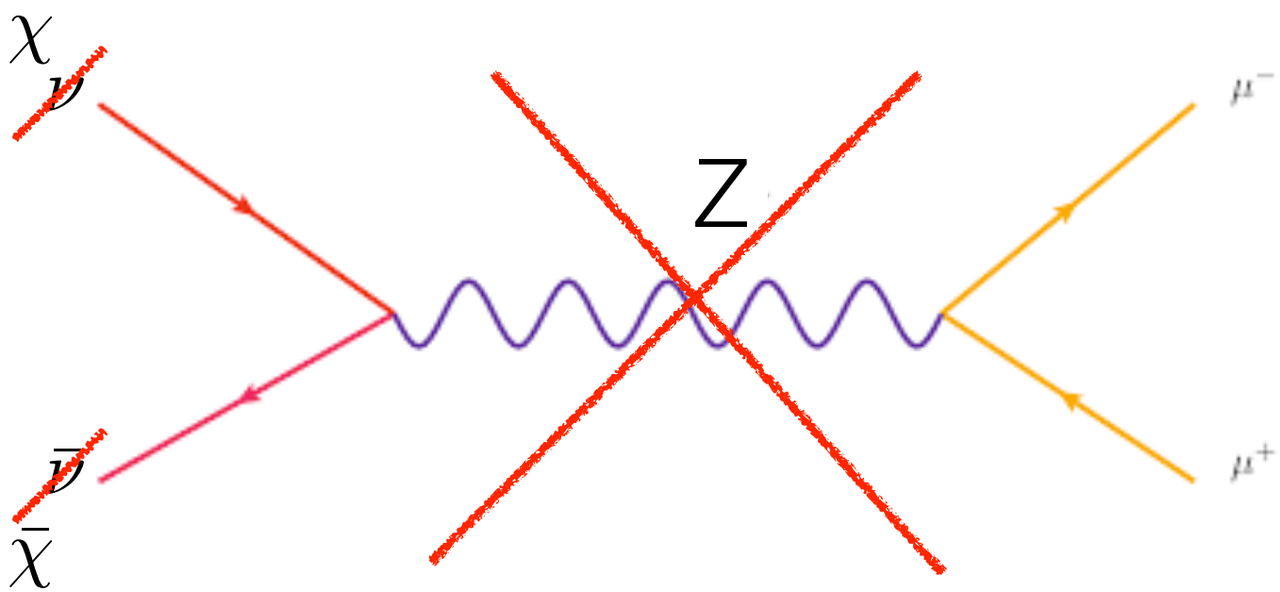
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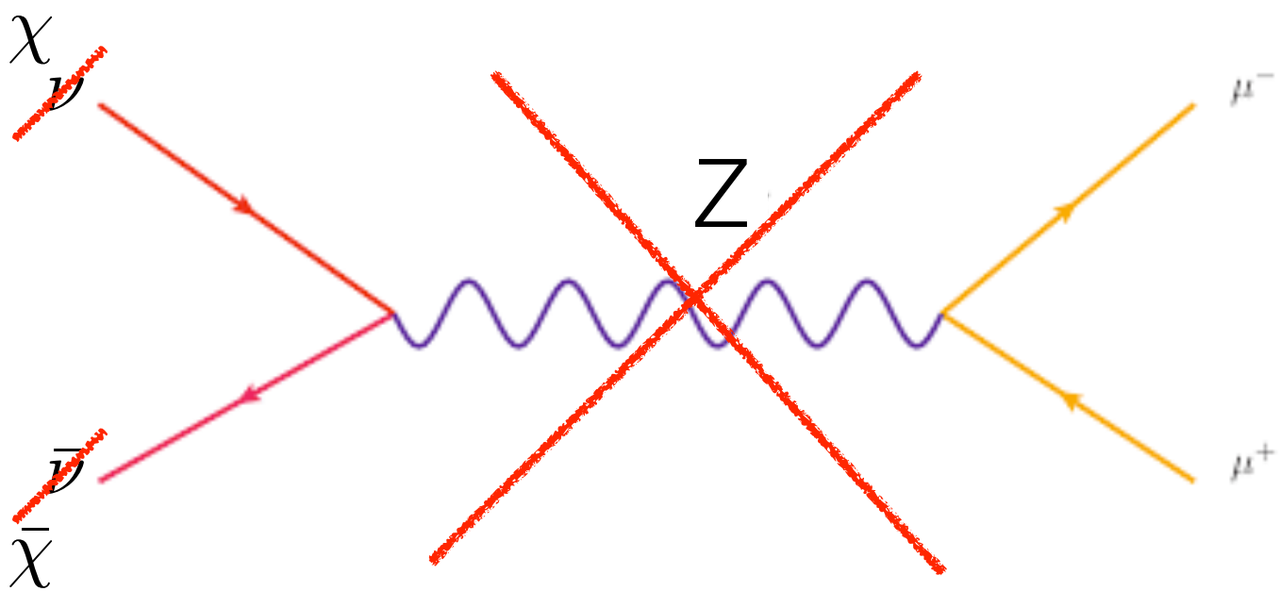
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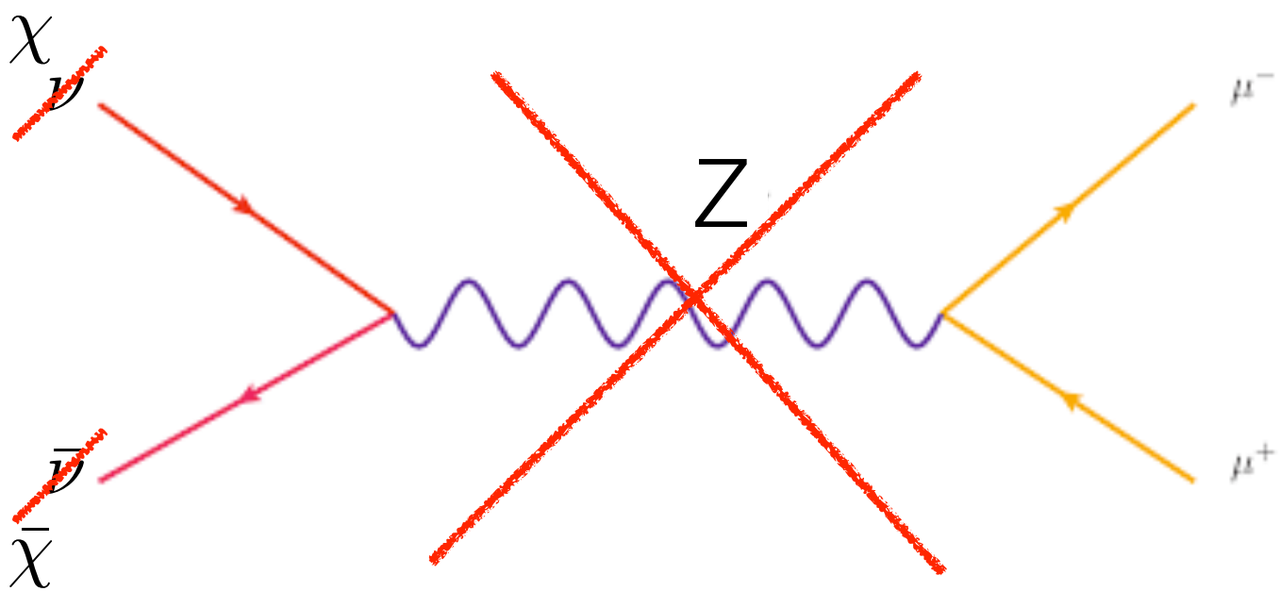
$\sigma \simeq G_F^2 m_\chi^2$

$$x_{\rm f.o.} \approx 20 \quad \text{for } m_\chi \simeq 1\,{\rm GeV}$$

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$$\Omega_\chi = \frac{\rho_{\chi,0}}{\rho_0} = \frac{m_\chi n_{\chi,0}}{\rho_0} = \frac{m_\chi T_0^3}{\rho_0} \frac{n_{\chi,0}}{T_0^3}$$

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$$= \frac{T_0^3}{\rho_0} x_{\mathrm{f.o.}} \frac{n_{\chi,\mathrm{f.o.}}}{T_{\mathrm{f.o.}}^2}$$

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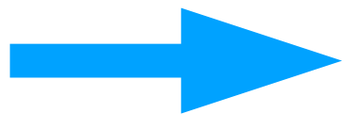
Relative relic density

$$\Omega_\chi = \frac{\rho_{\chi,0}}{\rho_0} = \frac{m_\chi n_{\chi,0}}{\rho_0} = \frac{m_\chi T_0^3}{\rho_0}\frac{n_{\chi,0}}{T_0^3} = \frac{m_\chi T_0^3}{\rho_0}\frac{n_{\chi,\mathrm{f.o.}}}{T_{\mathrm{f.o.}}^3}$$

$$n(T_{\mathrm{f.o.}}) \simeq \frac{T_{\mathrm{f.o.}}^2}{\sigma M_P} \qquad = \frac{T_0^3}{\rho_0} x_{\mathrm{f.o.}} \frac{n_{\chi,\mathrm{f.o.}}}{T_{\mathrm{f.o.}}^2} = \frac{T_0^3}{\rho_0 M_P}\frac{x_{\mathrm{f.o.}}}{\langle\sigma v\rangle}$$

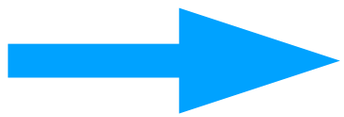
Thermal relic: cold relic

$$\Omega_\chi = \frac{T_0^3}{\rho_0 M_P} \frac{x_{\rm f.o.}}{\langle \sigma v \rangle}$$



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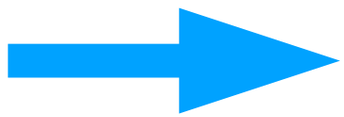


$$\left(\frac{\Omega_\chi}{0.2}\right) = \frac{x_{\rm f.o.}}{20} \frac{10^{-9}\,{\rm GeV}^{-2}}{\langle \sigma v \rangle}$$

$$\rho_0 = \frac{3H_0^2}{8\pi G_N} \simeq 10^{-29}\,{\rm g/cm}^3 \qquad T_0 \simeq 2.7\,{\rm K} \sim 10^{-4}\,{\rm eV}$$

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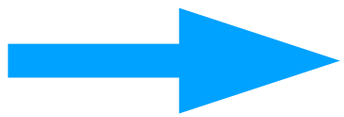
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Quiz

$$\rho_0 = \frac{3H_0^2}{8\pi G_N} \simeq 10^{-29}\,{\rm g/cm}^3 \qquad T_0 \simeq 2.7\,{\rm K} \sim 10^{-4}\,{\rm eV}$$

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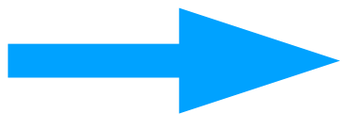
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For $m_\chi = \mathcal{O}({\rm GeV})$

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For $m_\chi = \mathcal{O}({\rm GeV}) \longrightarrow x_{\rm f.o.} \approx 20 \longrightarrow \Omega_\chi \sim 0.2$

if $\langle \sigma v \rangle \sim 10^{-9}\,{\rm GeV}^{-2} \sim 10\,{\rm pb}$

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For $m_\chi = \mathcal{O}({\rm GeV}) \longrightarrow x_{\rm f.o.} \approx 20 \longrightarrow \Omega_\chi \sim 0.2$

$10^{-26}\,{\rm cm^3/s}$

if $\langle \sigma v \rangle \sim 10^{-9}\,{\rm GeV}^{-2} \sim 10\,{\rm pb}$

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$10^{-26}\,{\rm cm^3/s}$

if $\langle \sigma v \rangle \sim 10^{-9}\,{\rm GeV}^{-2} \sim 10\,{\rm pb}$

$v \approx 0.3$

Weak interaction

Thermal relic: cold relic

$$\Omega_\chi = \frac{T_0^3}{\rho_0 M_P} \frac{x_{\rm f.o.}}{\langle \sigma v \rangle}$$

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$10^{-26}\,{\rm cm^3/s}$

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$v \approx 0.3$

Weak interaction

Weak scale mass

Thermal relic: cold relic

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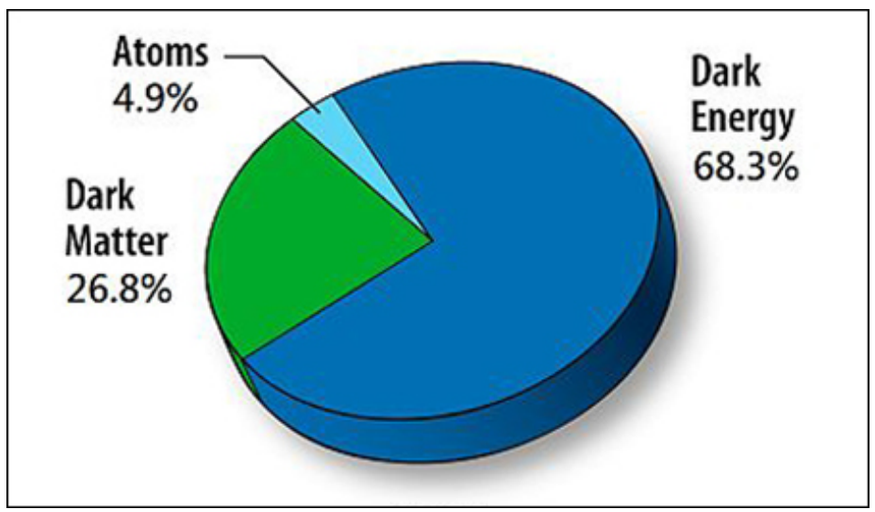
if $\langle \sigma v \rangle \sim 10^{-9}\,{\rm GeV}^{-2} \sim 10\,{\rm pb}$ ($10^{-26}\,{\rm cm}^3/{\rm s}$)

$v \approx 0.3$

Weak interaction

about 20%

Weak scale mass



Thermal relic: cold relic

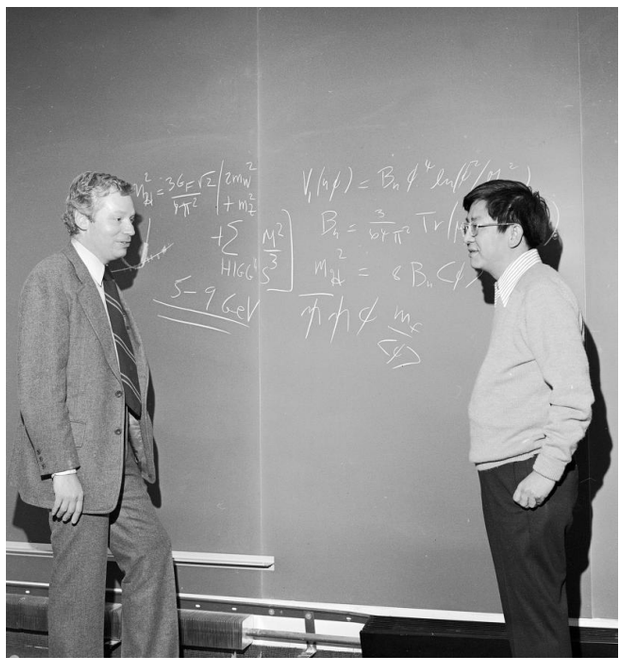
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Weakly Interacting Massive Particle (WIMP)

Weak scale mass & interaction



PHYSICAL REVIEW LETTERS

VOLUME 39 — 25 JULY 1977 — NUMBER 4

Cosmological Lower Bound on Heavy-Neutrino Masses

Benjamin W. Lee[a]
Fermi National Accelerator Laboratory,[b] *Batavia, Illinois 60510*

and

Steven Weinberg[c]
Stanford University, Physics Department, Stanford, California 94305

(Received 13 May 1977)

The present cosmic mass density of possible stable neutral heavy leptons is calculated in a standard cosmological model. In order for this density not to exceed the upper limit of 2×10^{-29} g/cm^3, the lepton mass would have to be *greater* than a lower bound of the order of 2 GeV.

Boltzmann equation

Evolution of the number density of DM out of equilibrium with SM



thermal bath

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle \left(n^2 - n_{\rm EQ}^2 \right)$$

$$n_\chi = n_{\bar{\chi}} = n$$

Boltzmann equation

Evolution of the number density of DM out of equilibrium with SM



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$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle \left(n^2 - n_{\rm EQ}^2 \right)$$

Liouville operator

$n_\chi = n_{\bar{\chi}} = n$

Change in time
of the phase space density

$$\frac{1}{a^3} \frac{d}{dt} \left(a^3 \cdot n \right) = \frac{dn}{dt} + 3Hn$$

Boltzmann equation

Evolution of the number density of DM out of equilibrium with SM



$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle \left(n^2 - n_{\rm EQ}^2\right)$$

Liouville operator

Collision operator
(kinetic equilibrium &
$T \ll E - \mu$, i.e., classical)

$n_\chi = n_{\bar\chi} = n$

Change in time
of the phase space density

Number of particles/phase-space volume lost or gained/unit time

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3 \cdot n\right) = \frac{dn}{dt} + 3Hn$$

$$n_{\rm EQ}^2 \langle\sigma v\rangle \left(\frac{n_{\rm SM}^2}{n_{\rm EQ,\,SM}^2} - \frac{n_\chi^2}{n_{\rm EQ,\,\chi}^2}\right)$$

Boltzmann equation

Evolution of the number density of DM out of equilibrium with SM



thermal bath

Cross over when

$n_1 H \sim n_1 n_2 \langle \sigma v \rangle$

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle \left(n^2 - n_{\rm EQ}^2 \right)$$

Liouville operator

$n_\chi = n_{\bar{\chi}} = n$

Collision operator
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Boltzmann equation

In terms of comoving number density $Y \equiv \frac{n}{s}$

Boltzmann equation

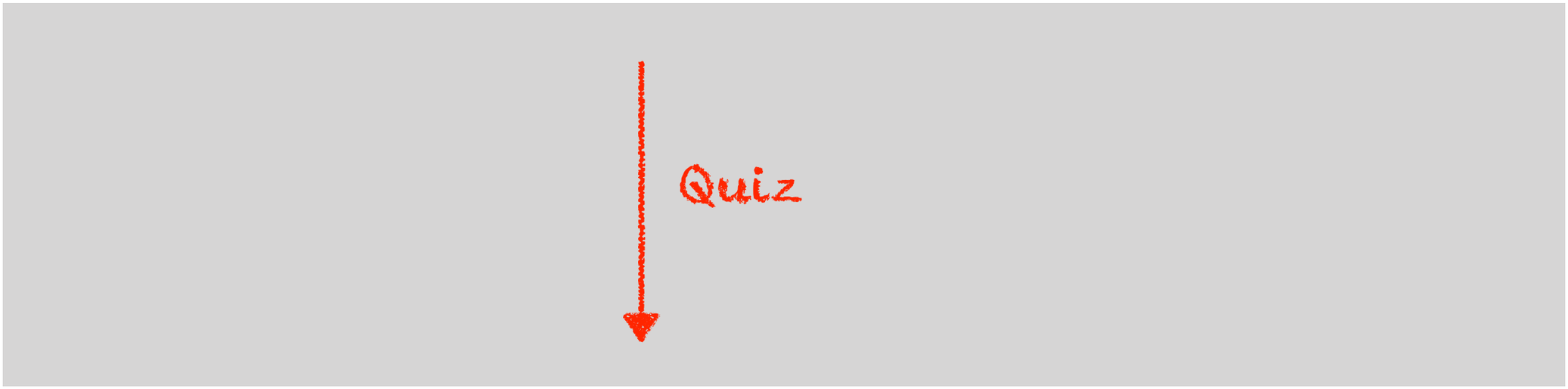
In terms of comoving number density $Y \equiv \frac{n}{s}$

Total entropy density of the Universe $sa^3 = \text{constant}$

Boltzmann equation

In terms of comoving number density $Y \equiv \dfrac{n}{s}$

Total entropy density of the Universe $sa^3 = \text{constant}$



$$\frac{dY}{dx} = \frac{1}{3H}\frac{ds}{dx}\langle\sigma v\rangle\left(Y^2 - Y_{\rm EQ}^2\right)$$

$Y_{\rm EQ} \equiv n_{\rm EQ}/s$

$$\frac{x}{Y_{\rm EQ}}\frac{dY}{dx} = -\frac{\Gamma_{\rm ann.}}{H}\left(\frac{Y^2}{Y_{\rm EQ}^2} - 1\right)$$

$\Gamma_{\rm ann.} = n_{\rm EQ}\langle\sigma v\rangle$

equilibrium annihilation rate

Boltzmann equation

$$\frac{x}{Y_{\mathrm{EQ}}}\frac{dY}{dx} = -\frac{\Gamma_{\mathrm{ann.}}}{H}\left(\frac{Y^2}{Y_{\mathrm{EQ}}^2} - 1\right)$$

Boltzmann equation

$$\frac{x}{Y_{\rm EQ}}\frac{dY}{dx} = -\frac{\Gamma_{\rm ann.}}{H}\left(\frac{Y^2}{Y_{\rm EQ}^2} - 1\right)$$

RD at freeze-out $H \simeq T^2/M_P = m_\chi^2/x^2 M_P$

Boltzmann equation

Solve this differential equation!

$$\frac{x}{Y_{\rm EQ}}\frac{dY}{dx} = -\frac{\Gamma_{\rm ann.}}{H}\left(\frac{Y^2}{Y_{\rm EQ}^2}-1\right)$$

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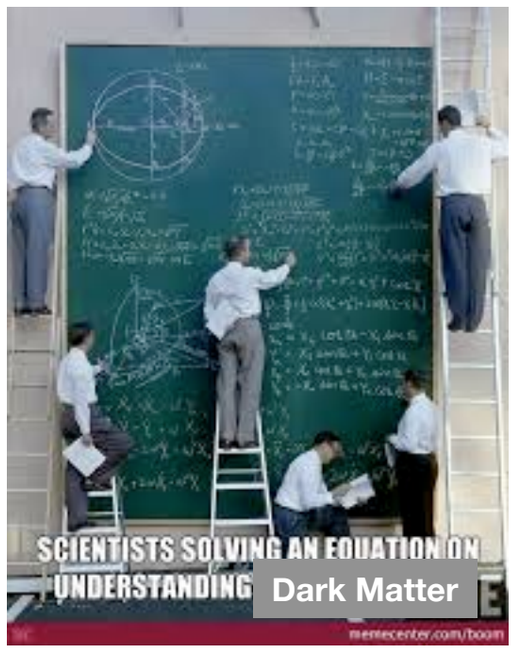
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FYI: No analytic solution
(Riccati equation)



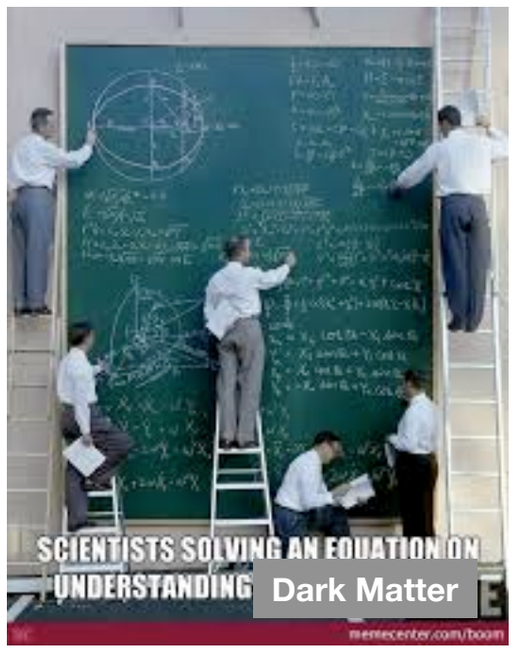
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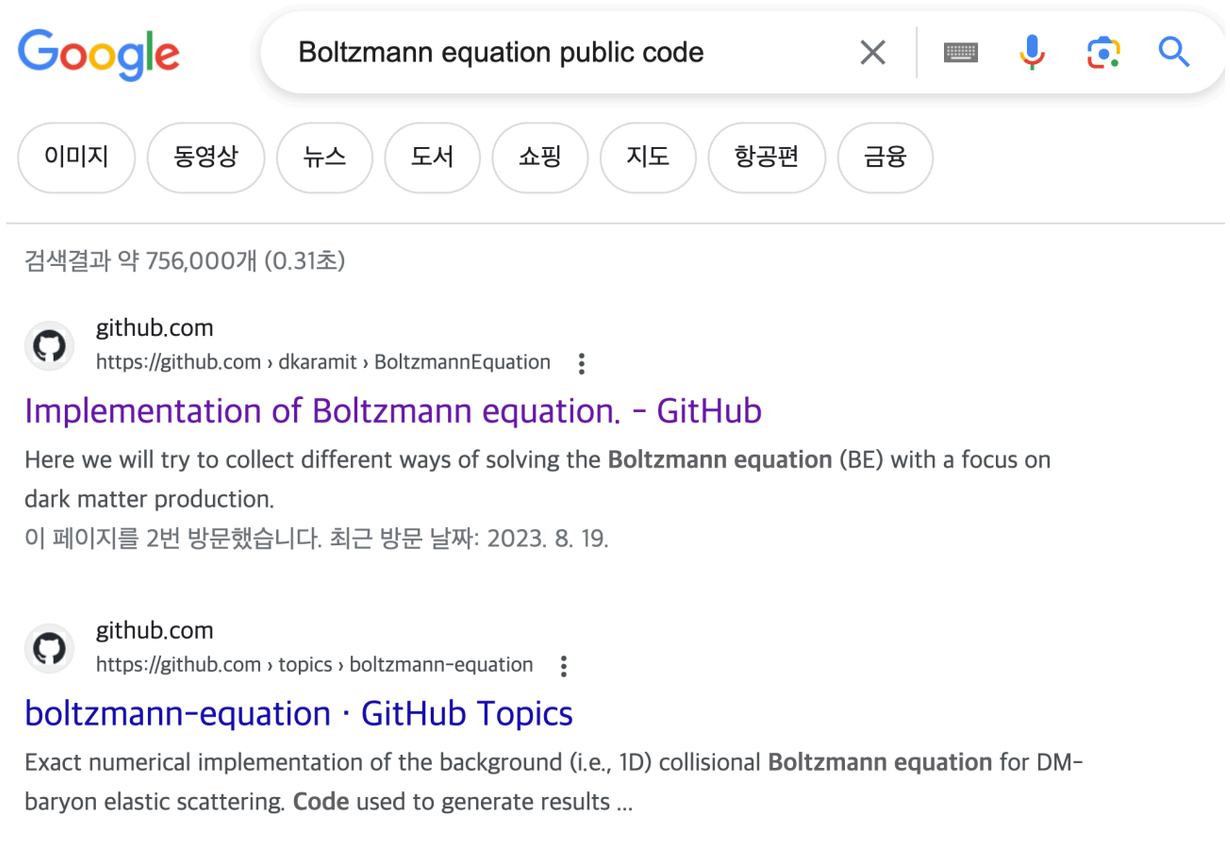
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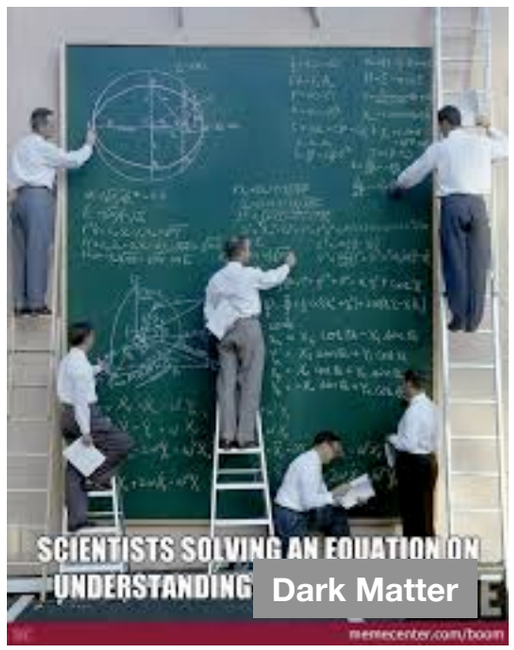
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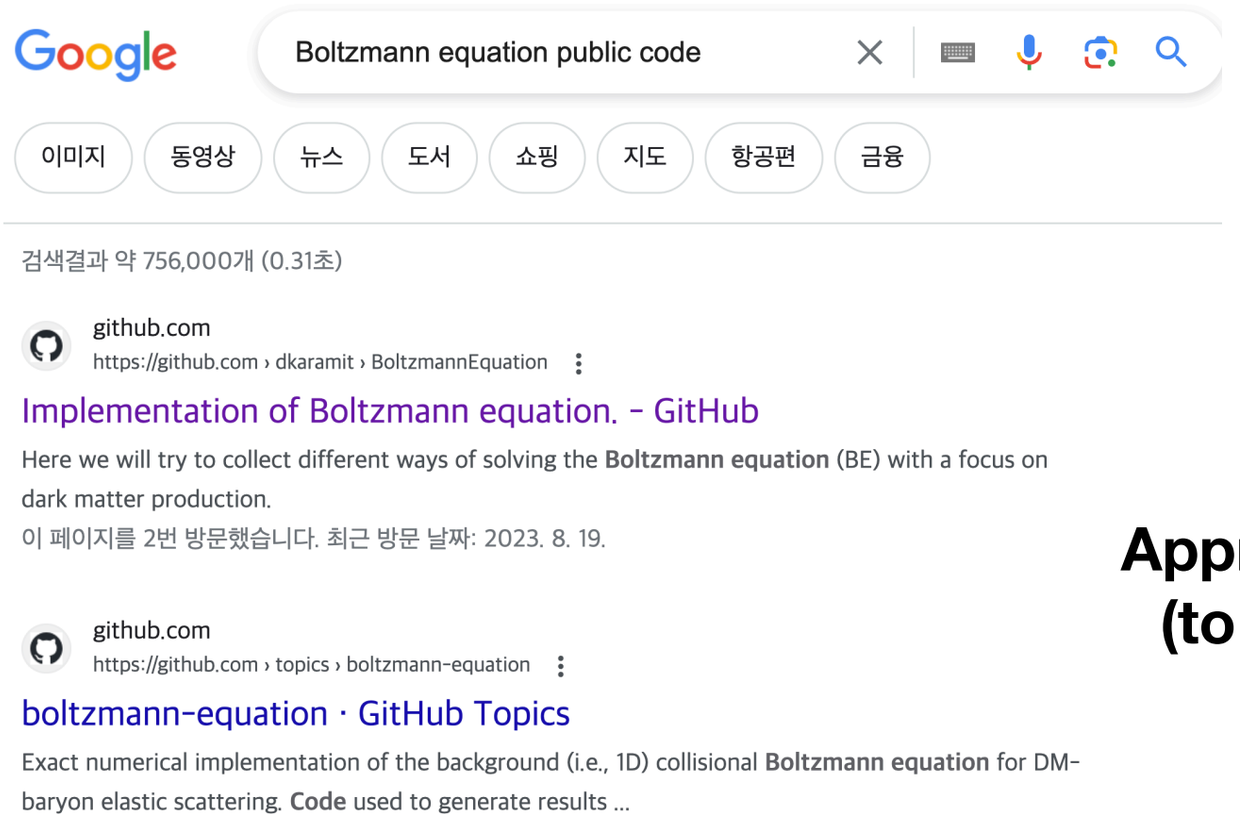
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Approximate analytic solution?
(to understand more clearly)

Boltzmann equation

$$\frac{x}{Y_{\rm EQ}}\frac{dY}{dx} = -\frac{\Gamma_{\rm ann.}}{H}\left(\frac{Y^2}{Y_{\rm EQ}^2} - 1\right)$$

RD at freeze-out $H \simeq T^2/M_P = m_\chi^2/x^2 M_P$

Long before freeze-out $\Gamma_{\rm ann.} \gg H$

Particles are in equilibrium $Y = Y_{\rm EQ}$

Approximate analytic solution?
(to understand more clearly)

Boltzmann equation

$$\frac{x}{Y_{\rm EQ}}\frac{dY}{dx} = -\frac{\Gamma_{\rm ann.}}{H}\left(\frac{Y^2}{Y_{\rm EQ}^2} - 1\right)$$

RD at freeze-out $H \simeq T^2/M_P = m_\chi^2/x^2 M_P$

$Y(\infty) \simeq Y_{EQ}(x_{f.o.}) \gg Y_{EQ}(\infty)$

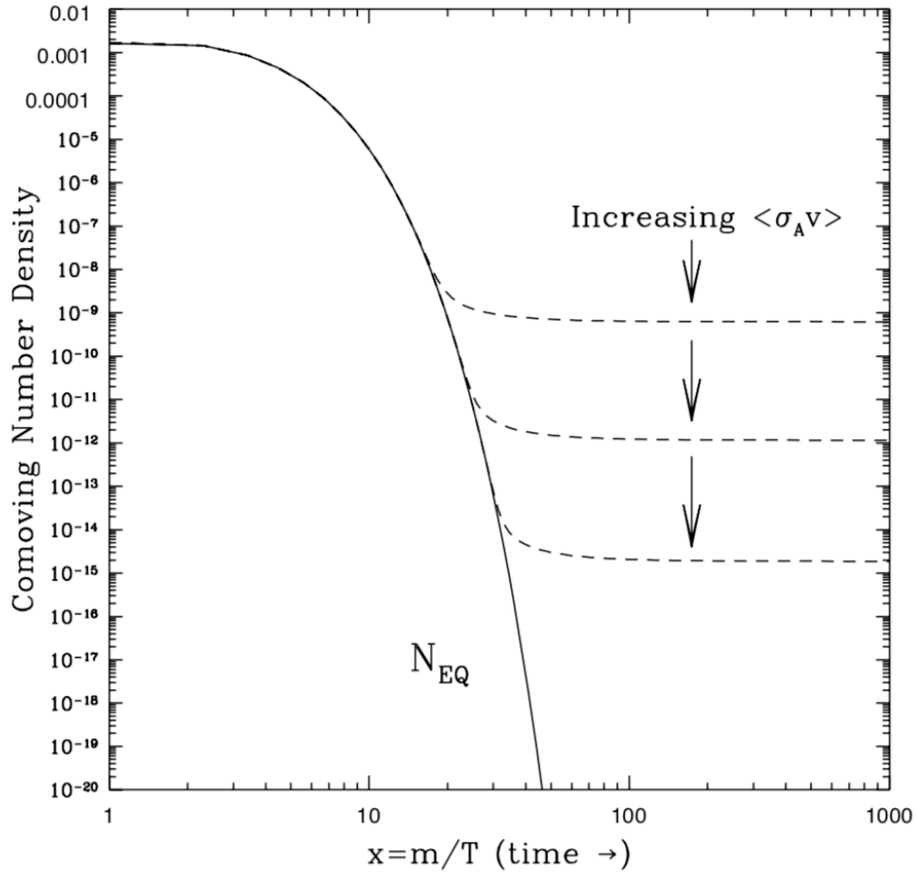
Long after freeze-out $\Gamma_{\rm ann.} \ll H$ → $\frac{dY}{dx} \to 0$ Y: almost constant

Long before freeze-out $\Gamma_{\rm ann.} \gg H$

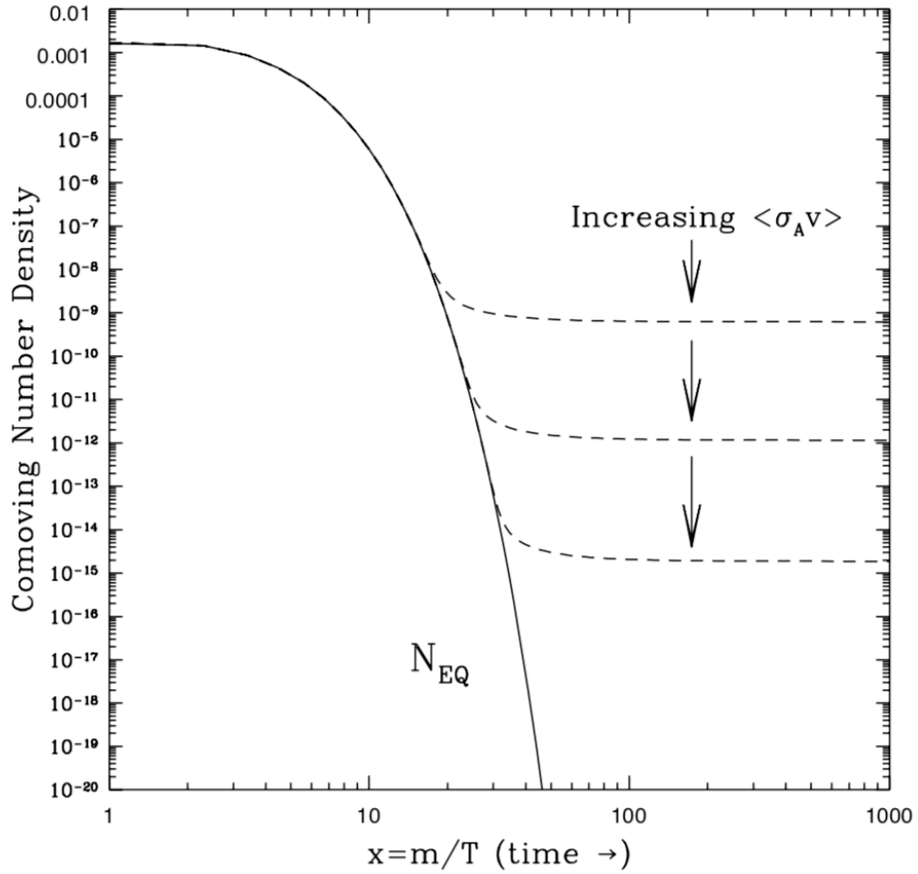
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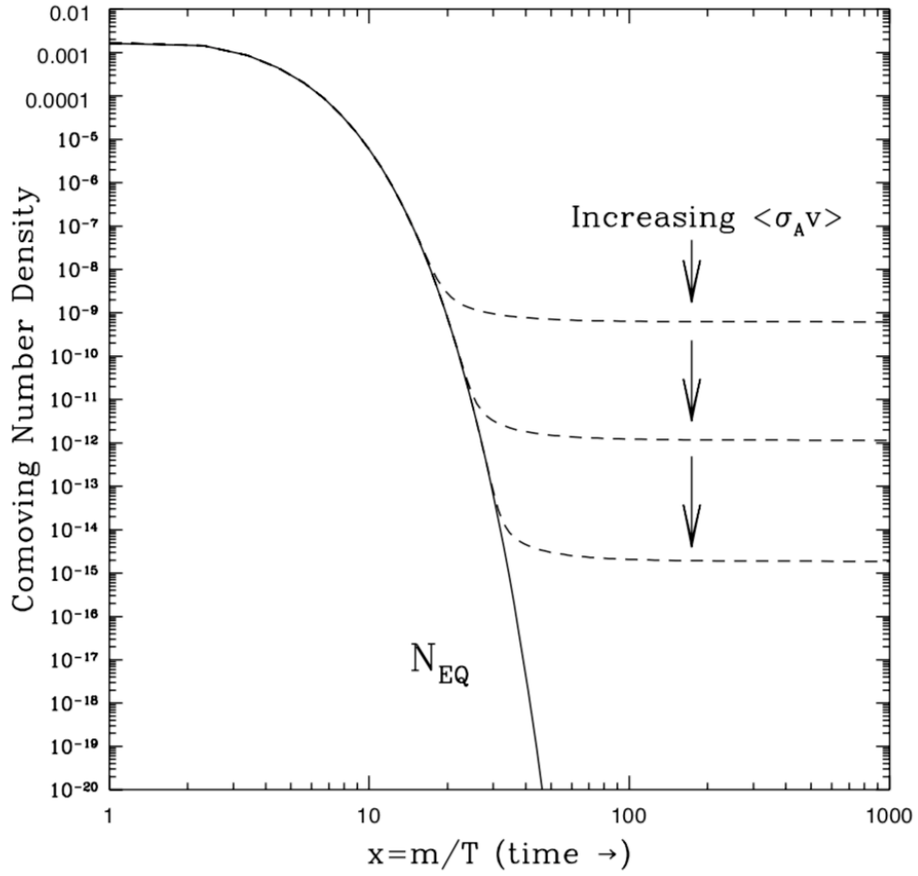
Boltzmann equation



$$\frac{dY}{dx} = \frac{1}{3H}\frac{ds}{dx}\langle\sigma v\rangle\left(Y^2 - Y_{\rm EQ}^2\right)$$

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Boltzmann equation

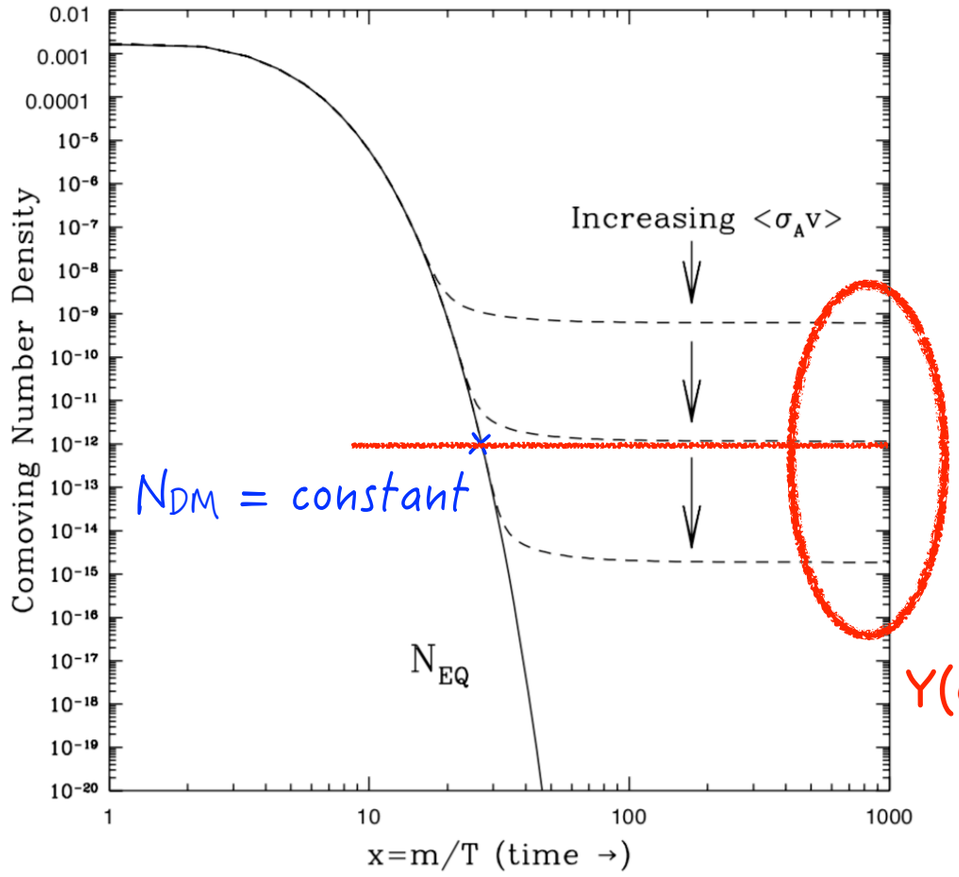


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Boltzmann equation



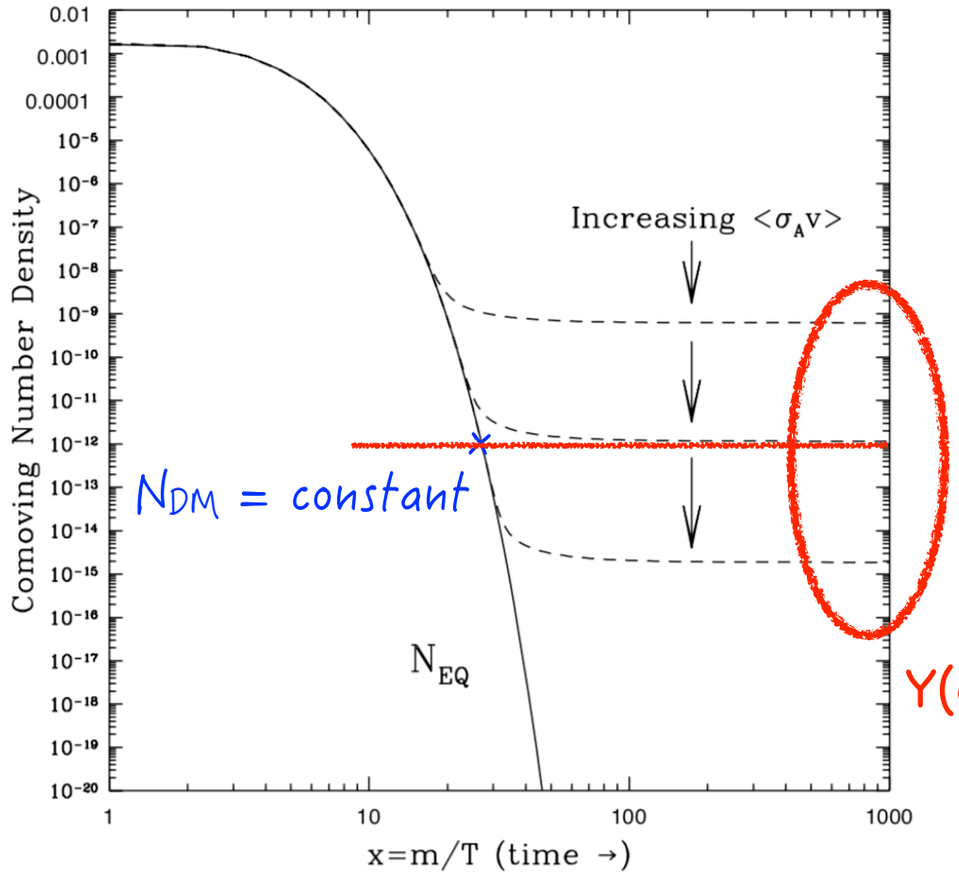
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$Y(\infty) \simeq Y_{EQ}(x_{f.o.})$

Boltzmann equation



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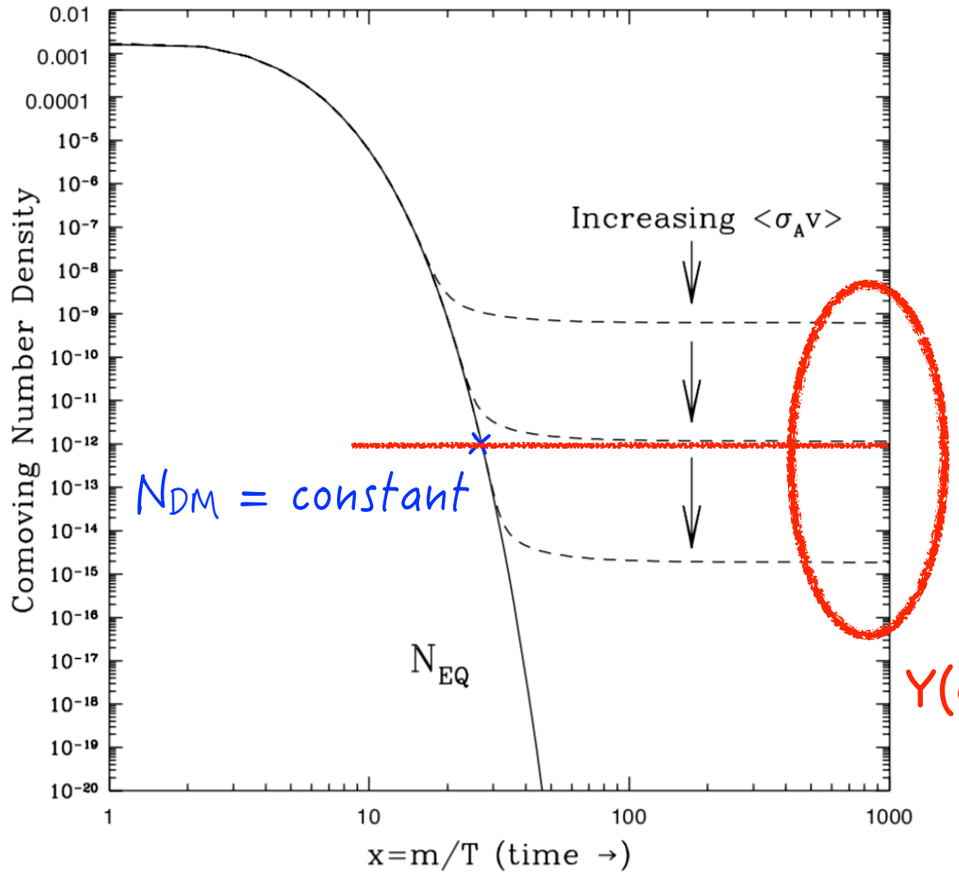
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Separate the T dependence from H and s: $H \equiv H(m)x^{-2}$, $s \equiv s(m)x^{-3}$

for constant g_* g_{*S}

Boltzmann equation



$$\frac{dY}{dx} = \frac{1}{3H}\frac{ds}{dx}\langle\sigma v\rangle\left(Y^2 - Y_{\rm EQ}^2\right)$$

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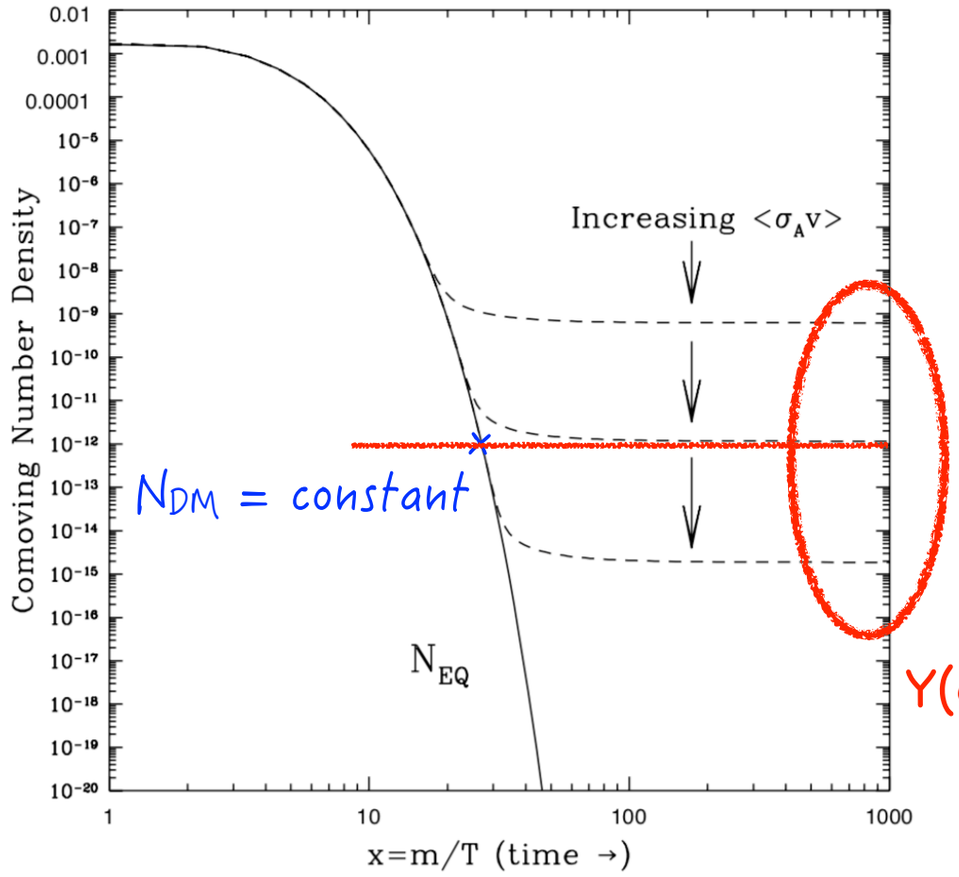
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for constant g_* g_{*S}

$$\frac{dY}{dx} = -\frac{\langle\sigma v\rangle s}{Hx}Y^2 \longrightarrow \frac{dY}{dx} = -\frac{\langle\sigma v\rangle s(m)}{H(m)x^2}Y^2 \longrightarrow \frac{dY}{Y^2} = -\frac{\langle\sigma v\rangle s(m)}{H(m)}\frac{dx}{x^2}$$

Boltzmann equation



$$\frac{dY}{dx} = \frac{1}{3H}\frac{ds}{dx}\langle\sigma v\rangle\left(Y^2 - Y_{\rm EQ}^2\right)$$

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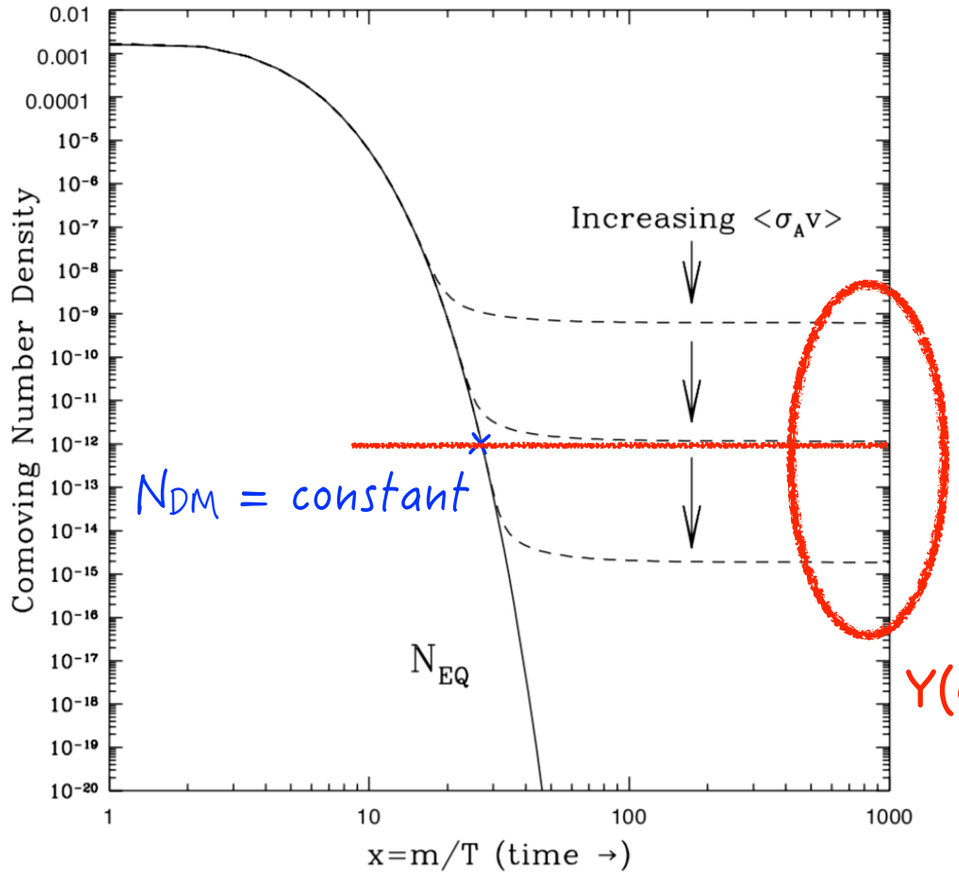
$$\longrightarrow \quad -\frac{\langle\sigma v\rangle s}{Hx}Y^2 \qquad \text{for} \quad x \gg x_{\rm f.o.}$$

Separate the T dependence from H and s: $H \equiv H(m)x^{-2}$, $s \equiv s(m)x^{-3}$

for constant g_* g_{*S}

$$\int_{x_{\rm f.o.}}^{\infty}\frac{dY}{Y^2} = -\frac{s(m)}{H(m)}\int_{x_{\rm f.o.}}^{\infty}dx\,\frac{\langle\sigma v\rangle}{x^2} \equiv -\frac{s(m)}{H(m)}J(x_{\rm f.o.})$$

Boltzmann equation



$$\frac{dY}{dx} = \frac{1}{3H}\frac{ds}{dx}\langle\sigma v\rangle\left(Y^2 - Y_{\rm EQ}^2\right)$$

$$= -\frac{\langle\sigma v\rangle s}{Hx}\left(Y^2 - Y_{\rm EQ}^2\right)$$

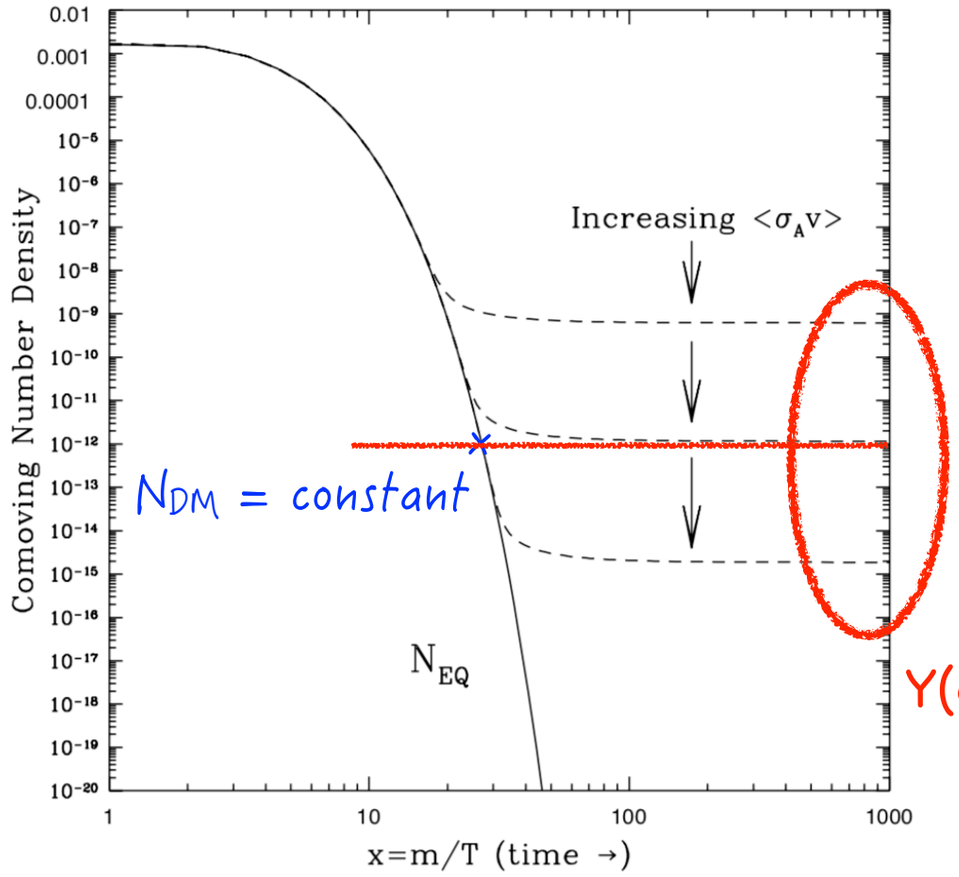
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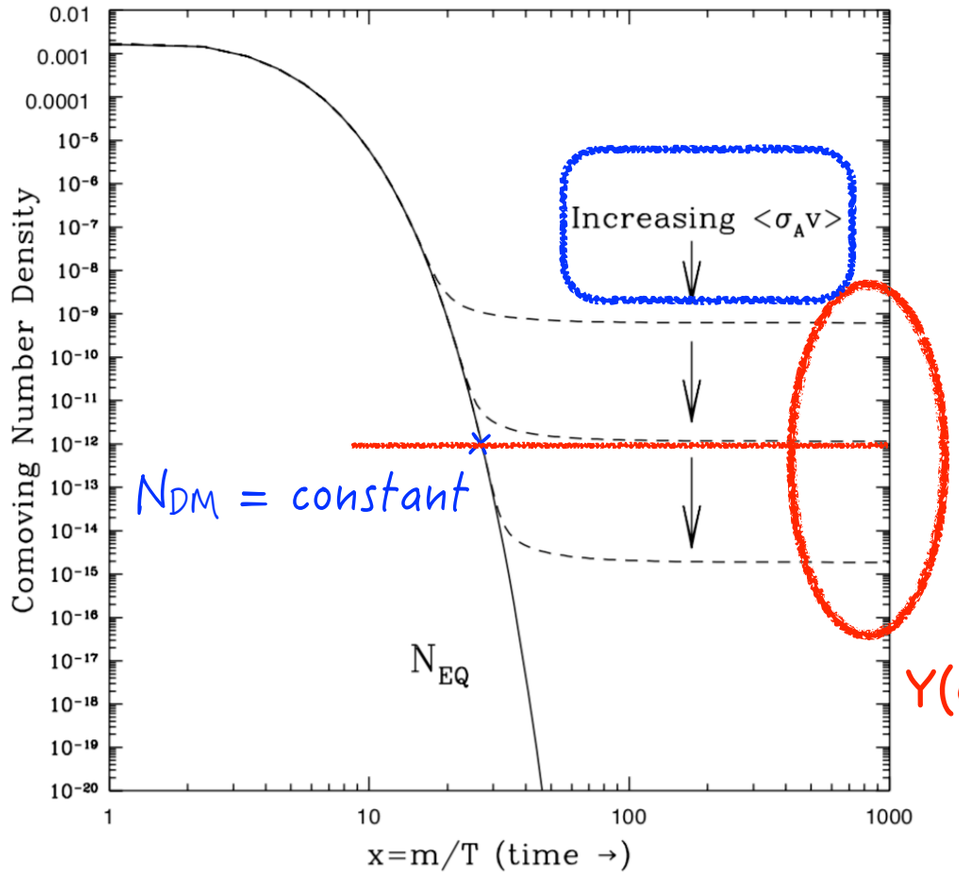
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$$\longrightarrow \quad Y(x_{\rm f.o.}) \gg Y_{\rm EQ}(x_{\rm f.o.}) \simeq Y_\infty$$

$$\boxed{Y_\infty \simeq -\frac{H(m)}{s(m)J(x_{\rm f.o.})}}$$

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$$\boxed{Y_\infty \simeq -\frac{H(m)}{s(m)J(x_{\rm f.o.})}}$$

Boltzmann equation

Relic density of DM at the present day $H(m) \propto m^2\,,\ s(m) \propto m^3$ $J(x_{\rm f.o.}) = \int_{x_{\rm f.o.}}^{\infty} dx\, \frac{\langle \sigma v \rangle}{x^2}$

$$\Omega = \frac{\rho_{\chi,0}}{\rho_0} = \frac{Y_\infty m s_0}{\rho_0} \simeq -\frac{H(m) m s_0}{\rho_0 s(m) J(x_{\rm f.o.})} = \frac{3\sqrt{10}}{4\pi} \frac{\sqrt{g_*}}{g_{*S}} \frac{s_0}{\rho_0 J(x_{\rm f.o.})}$$

Radiation Dominated $H = \frac{\pi}{3}\sqrt{\frac{g_*}{10}}\frac{T^2}{M_P}$ $s = \frac{2\pi^2}{45} g_{*S} T^3$

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for $T_{\rm f.o.} \gtrsim 1\,{\rm MeV}$

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K_i: Bessel functions

$s = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2$

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$$\longrightarrow \quad a + 6b/x + \mathcal{O}\left(1/x^2\right) \quad \text{for} \quad v \ll 1$$

s-wave p-wave

non-relativistic

σ: slowly changing with s

Boltzmann equation

With the expansions up to $1/x$, $\quad J(x_{\mathrm{f.o.}}) \simeq a/x_{\mathrm{f.o.}} + 3b/x_{\mathrm{f.o.}}^2$

Boltzmann equation

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$$\Omega h^2 \simeq \frac{0.38}{\sqrt{g_*}} \frac{s_0 x_{\rm f.o.}}{\rho_0 \left(a + 3b/x_{\rm f.o.}\right)} \simeq 0.1 \left(\frac{x_{\rm f.o.}}{20}\right) \left(\frac{60}{g_*}\right)^{1/2} \frac{3 \times 10^{-26}\,{\rm cm}^3/{\rm s}}{\langle \sigma v \rangle}$$

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$$\langle \sigma v \rangle \simeq a + b \langle v^2 \rangle$$

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$10^{-9}\,{\rm GeV}^{-2}$

$$\langle \sigma v \rangle \simeq a + b\langle v^2 \rangle$$

Griest, Seckel, PRD 1991

$$\chi\bar{\chi} \longrightarrow X\bar{Y}$$

Exceptions

- Resonance

  $\chi\bar{\chi} \to Z \to X\bar{Y}$ where $m_Z \sim 2m_\chi$

- Forbidden channel
  (annihilation below threshold)

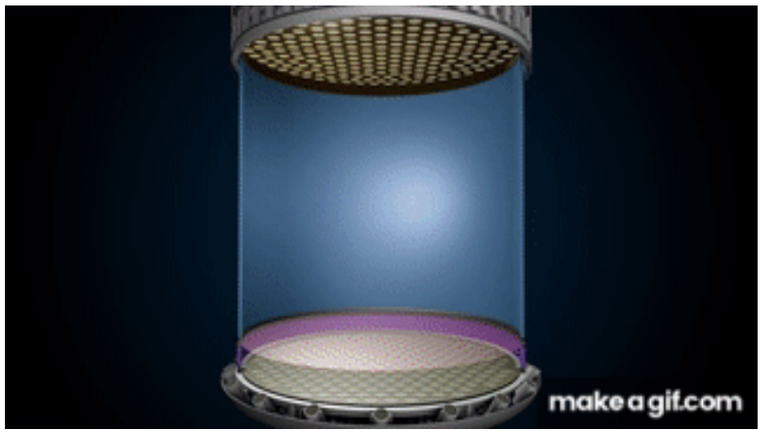
  $2m_\chi \lesssim m_X + m_Y$ Forbidden at T=0 but allowed with enough T>0

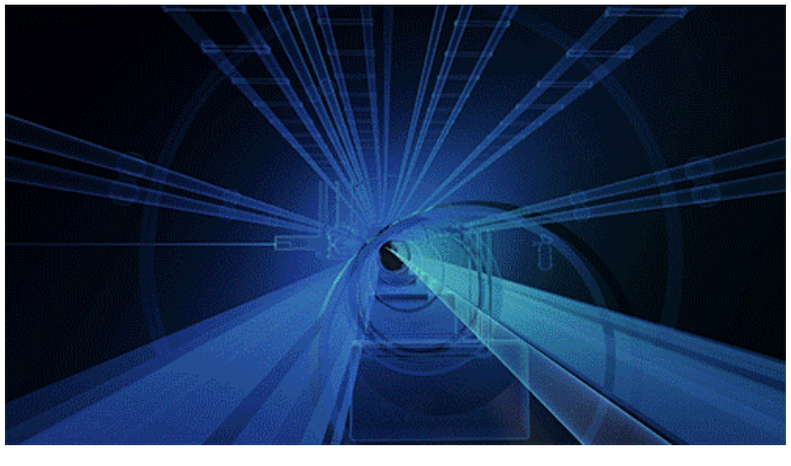
- Co-annihilation

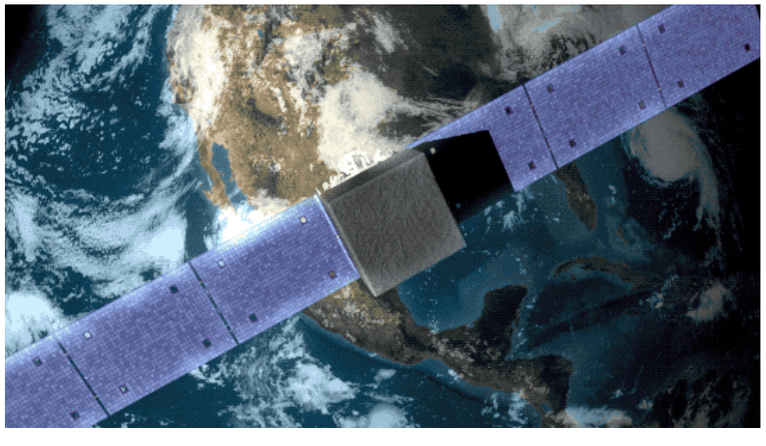
  $\chi_1\bar{\chi}_2 \to X\bar{Y}$ where $m_{\chi_1} \sim m_{\chi_2}$

Experimental Probes of Dark Matter

- Direct detection
- Indirect detection
- Collider search

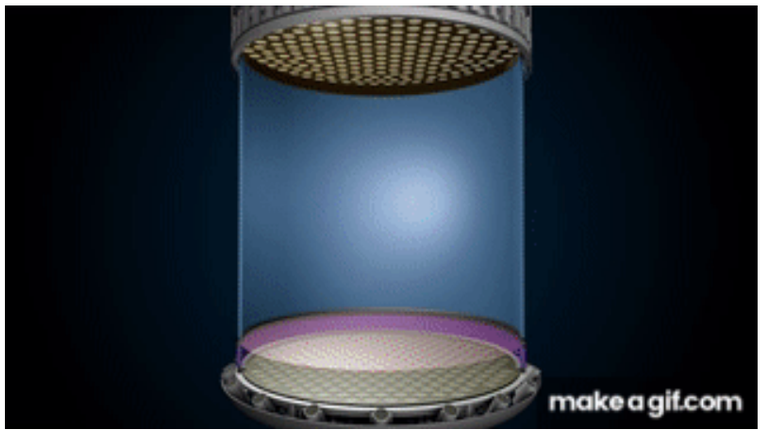


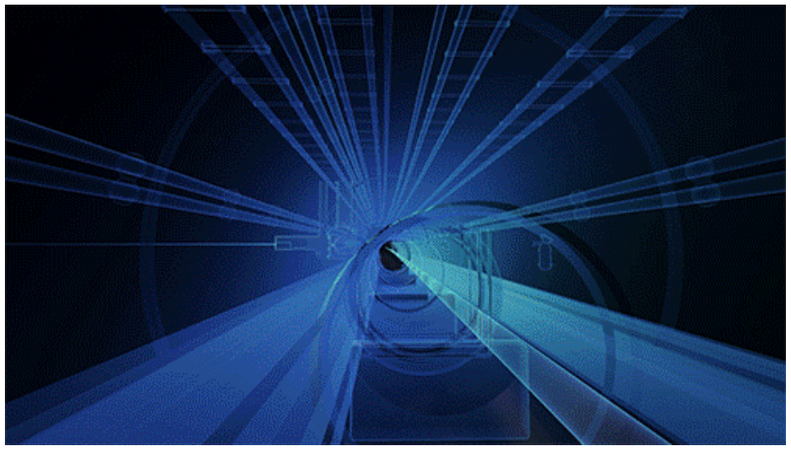


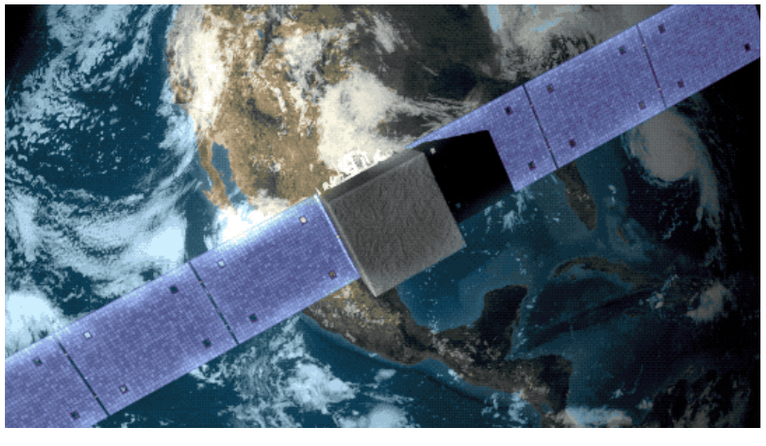


Experimental Probes of Dark Matter

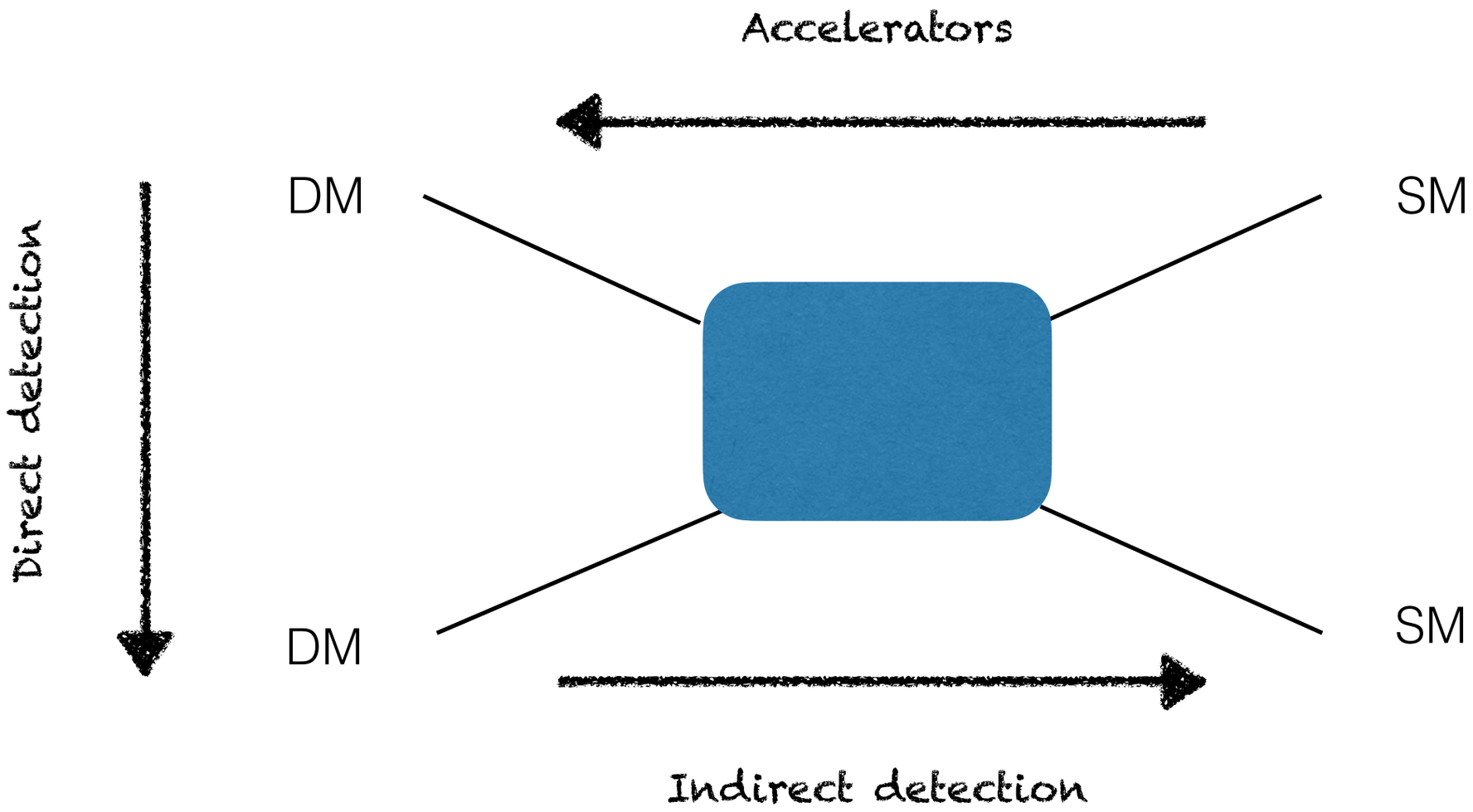
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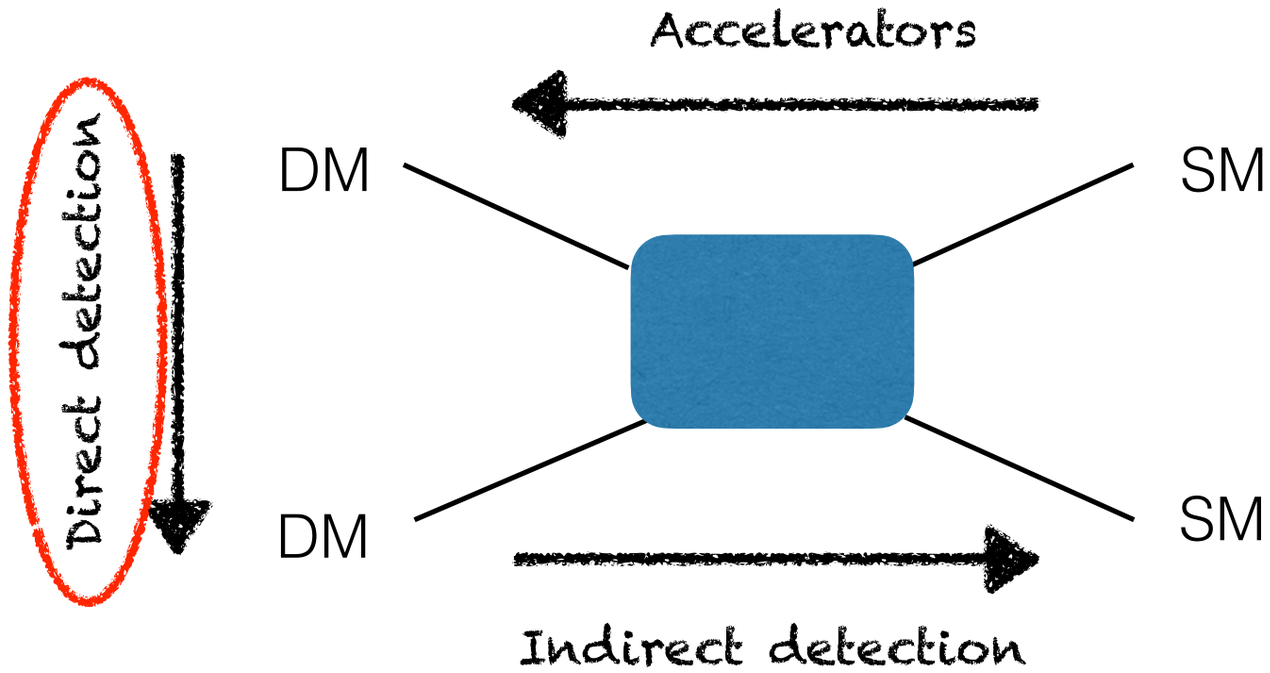




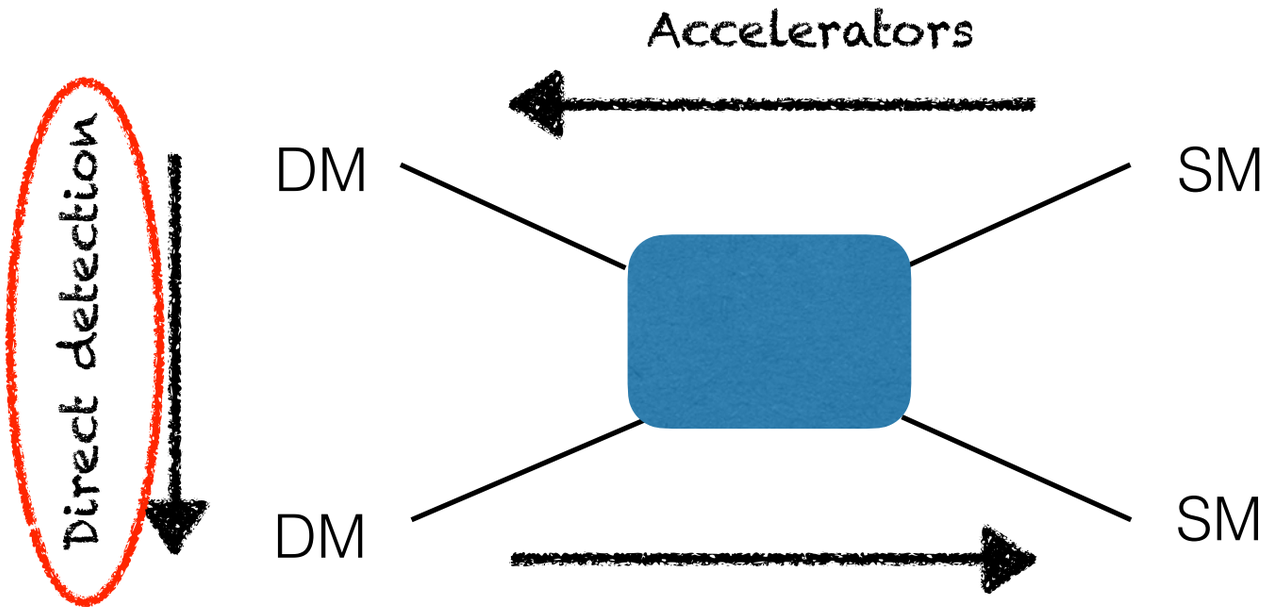
Probe of Dark Matter

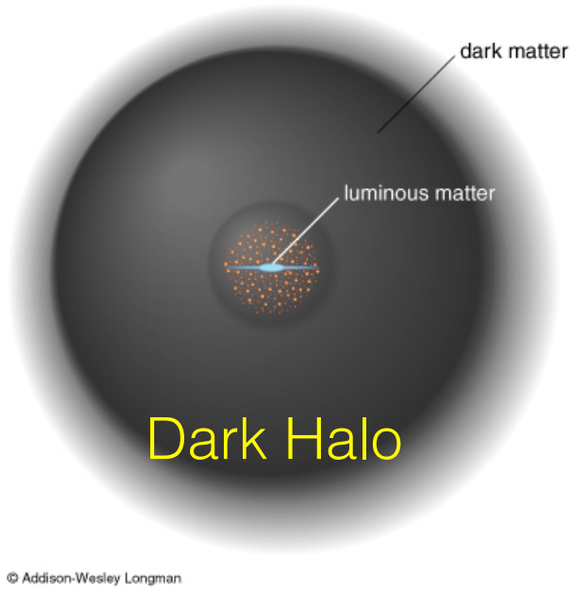


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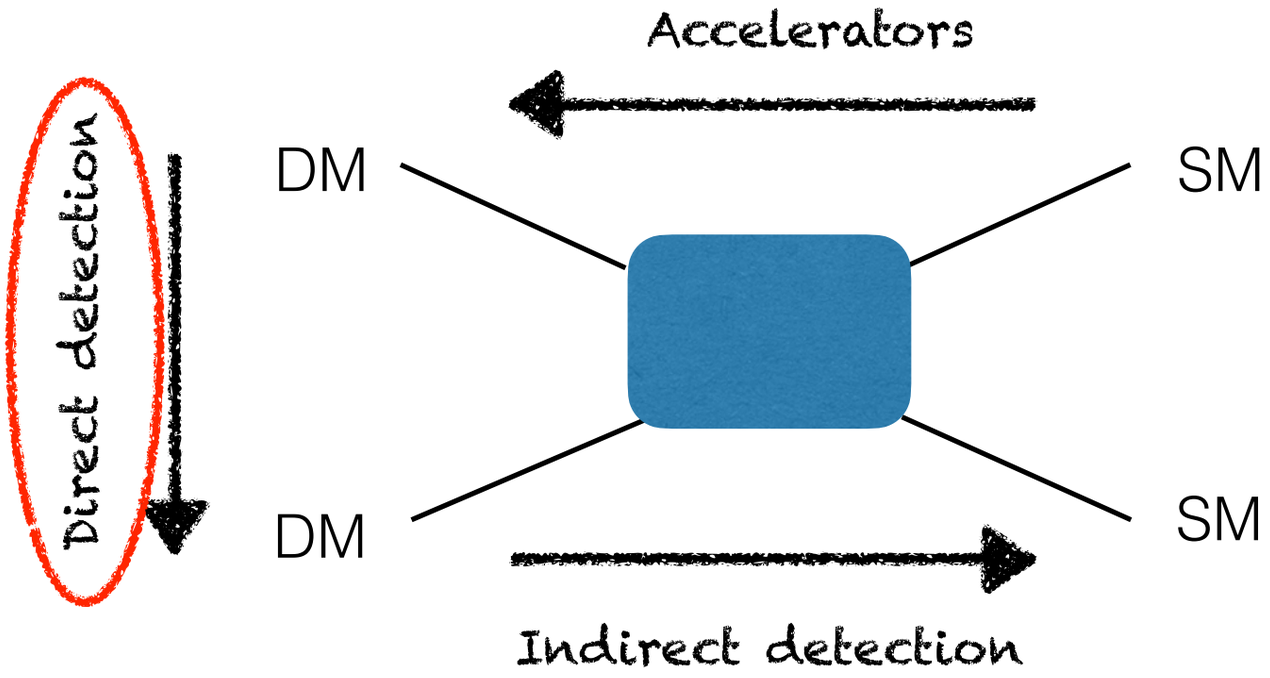


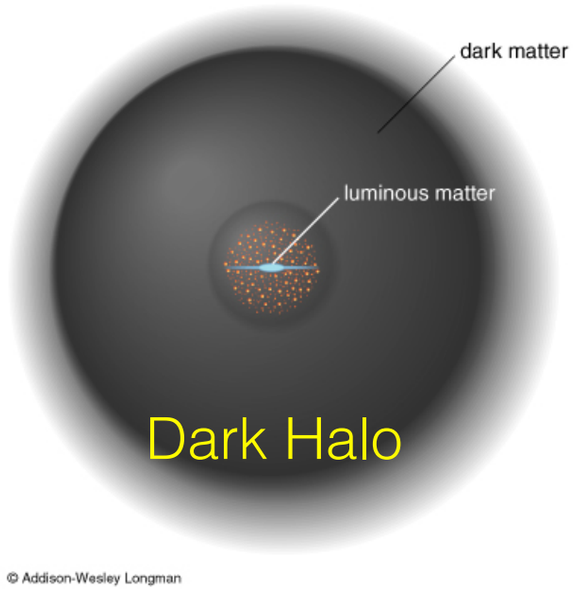
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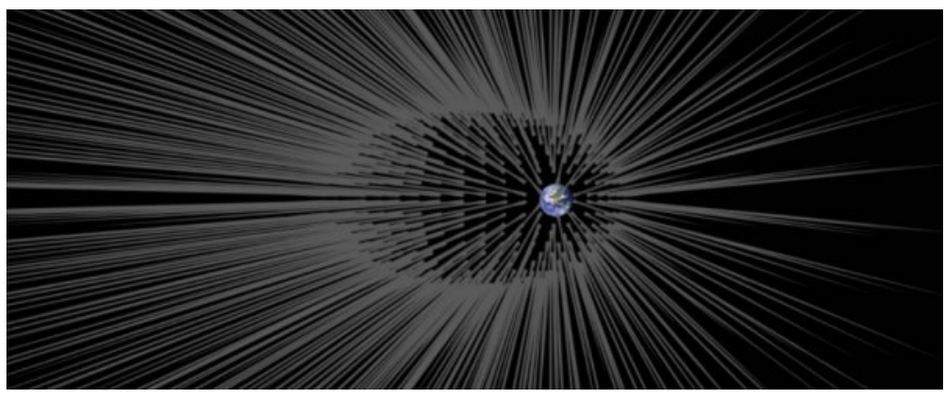




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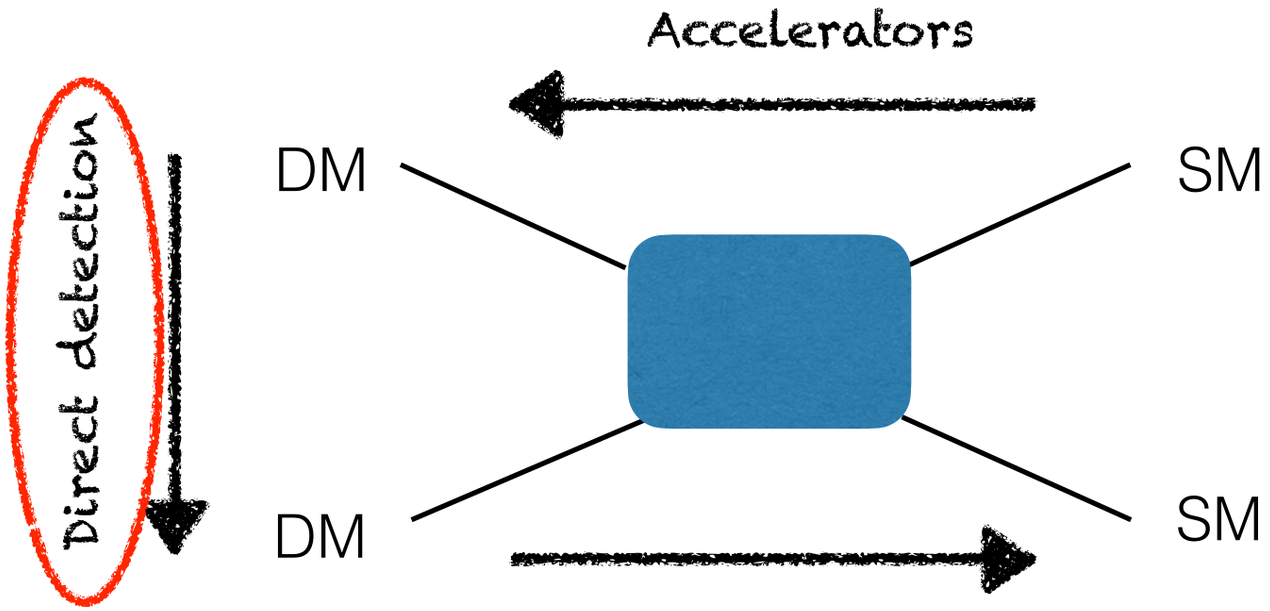


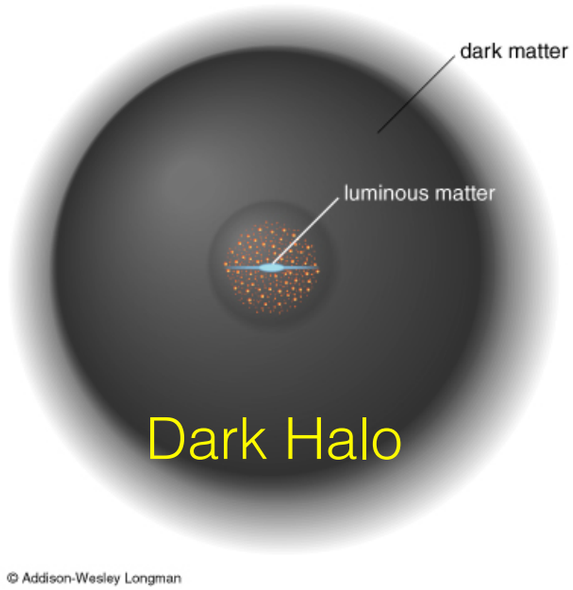


태양계가 우리 은하를 공전하는
속도 = 평균 230km/s

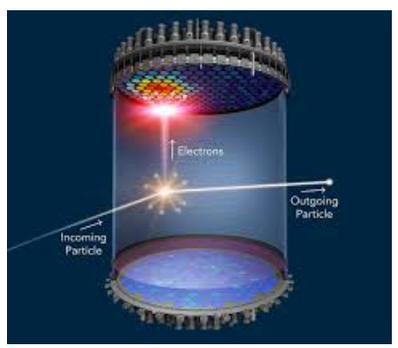
수백개의 암흑물질이 평균 230km/s의
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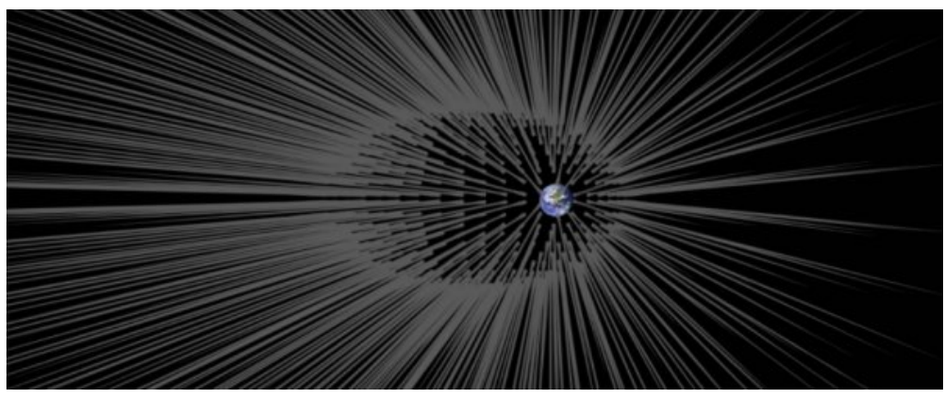
Probe of Dark Matter





underground

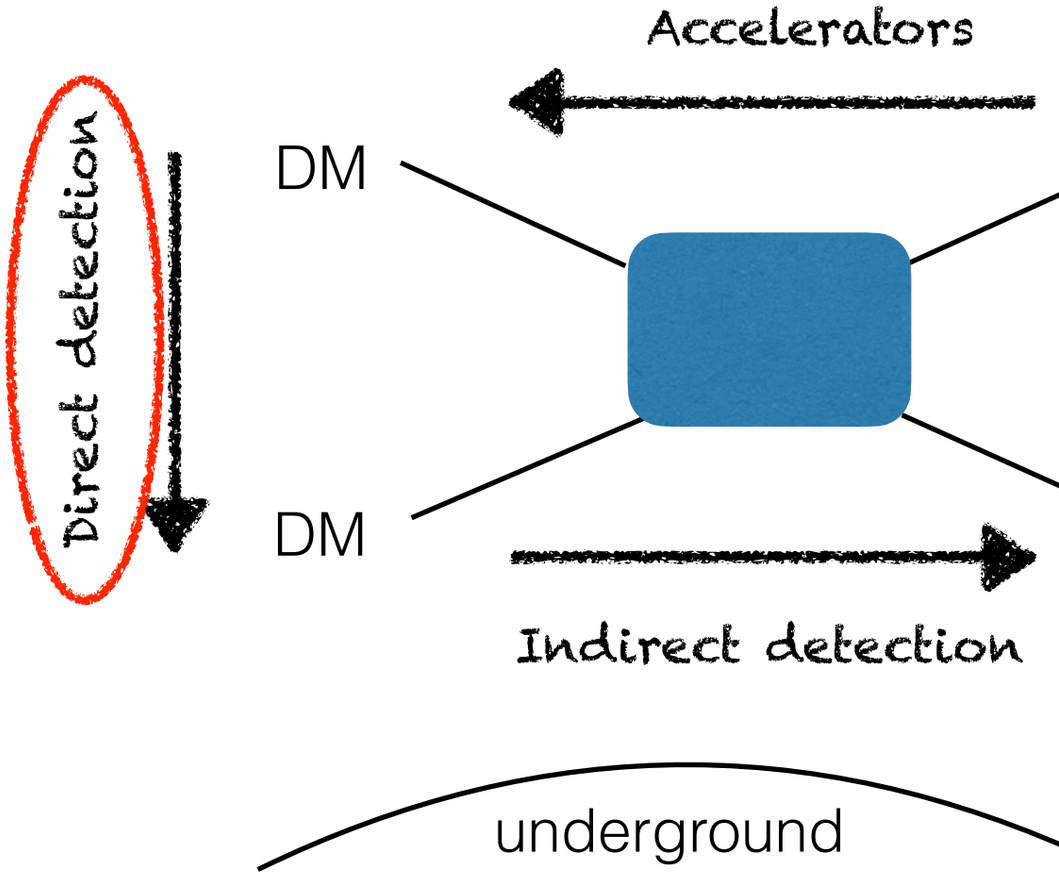


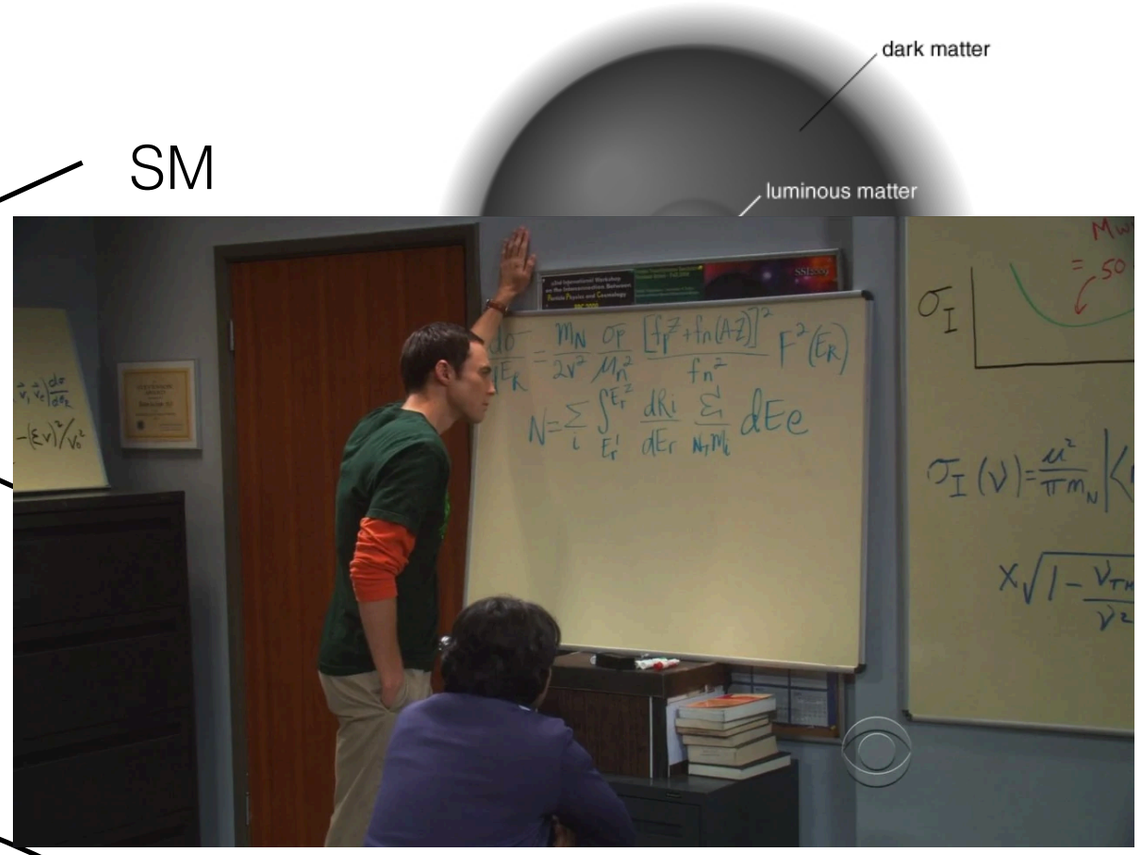


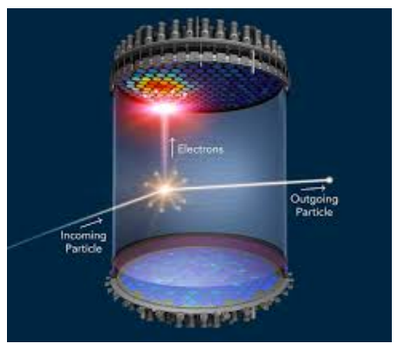
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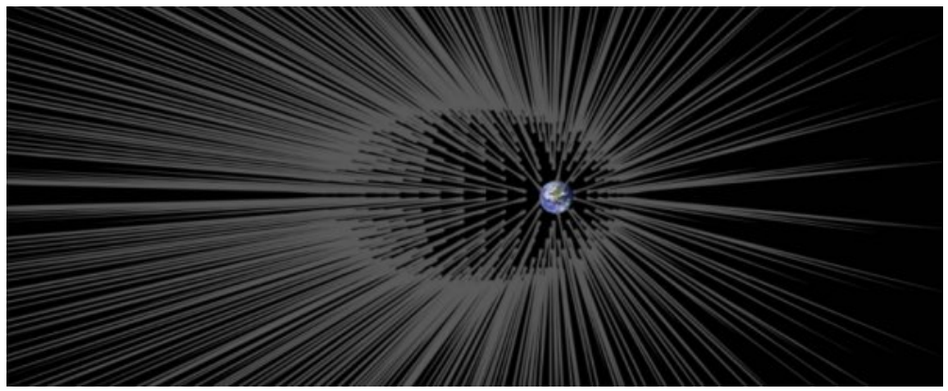
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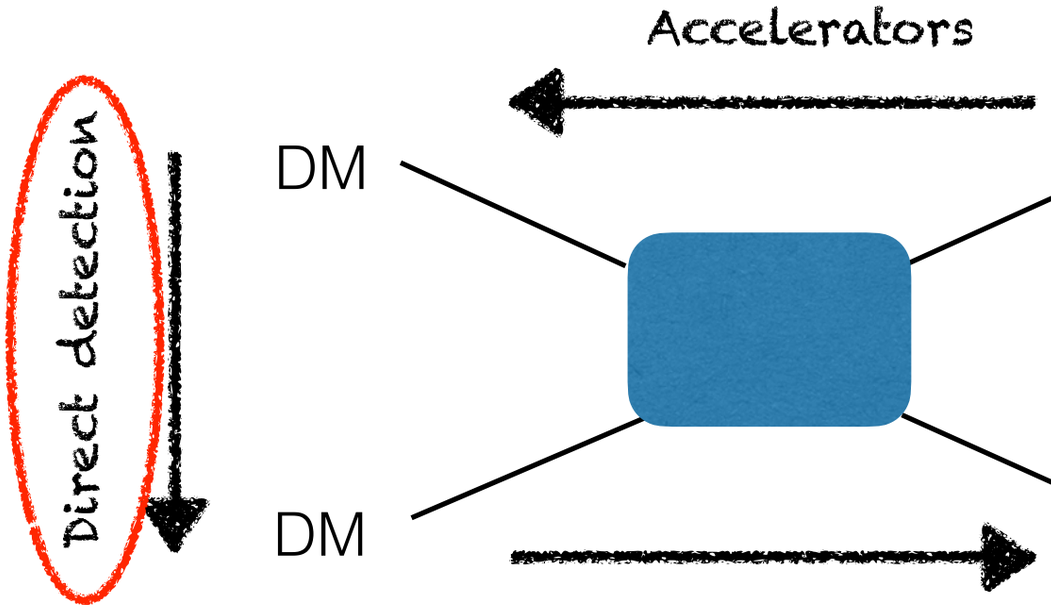


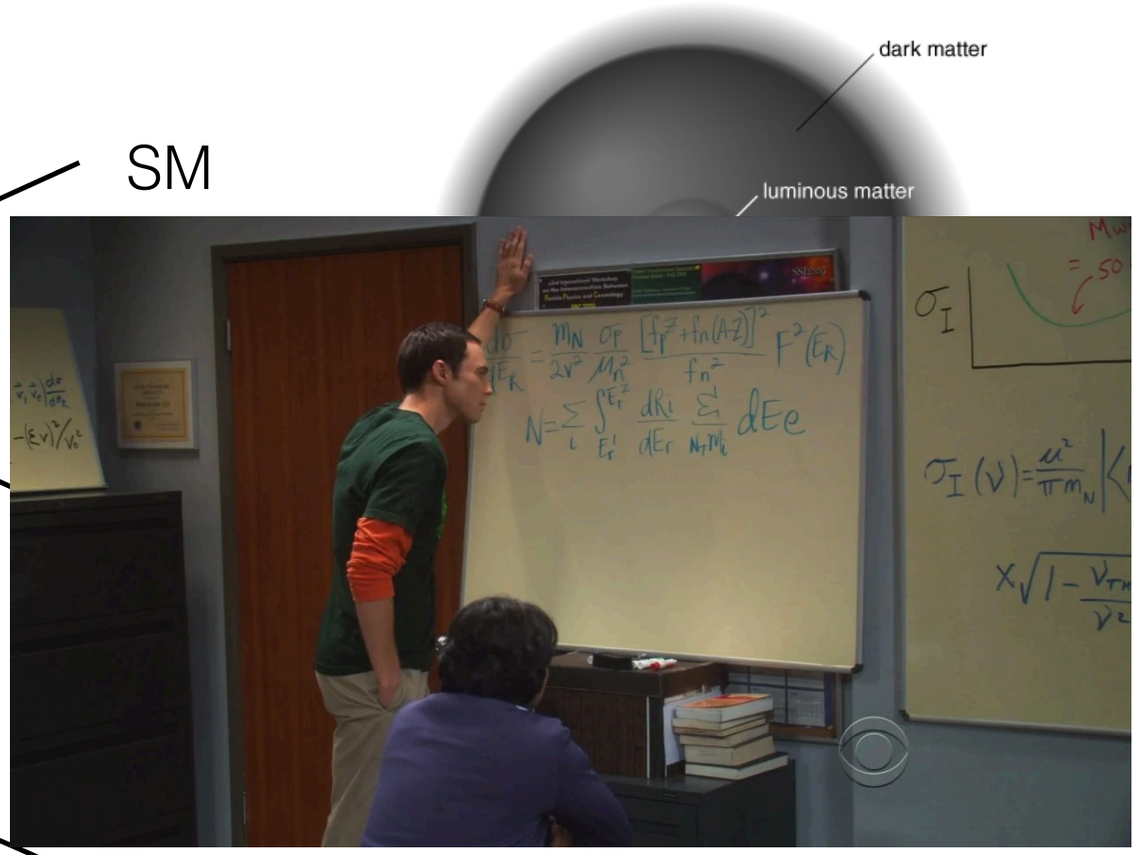


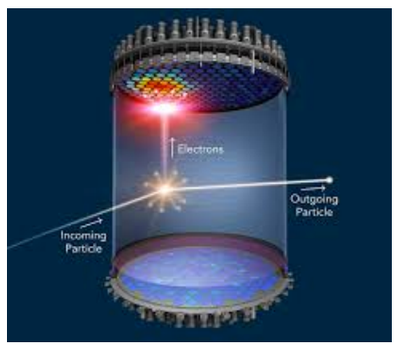
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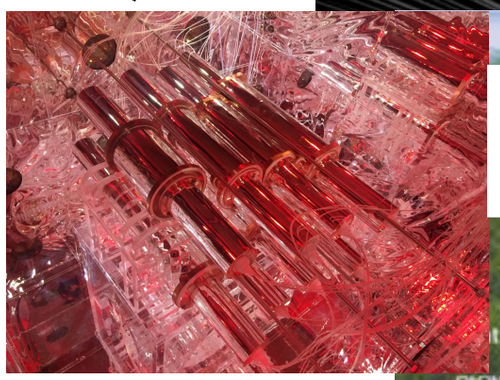
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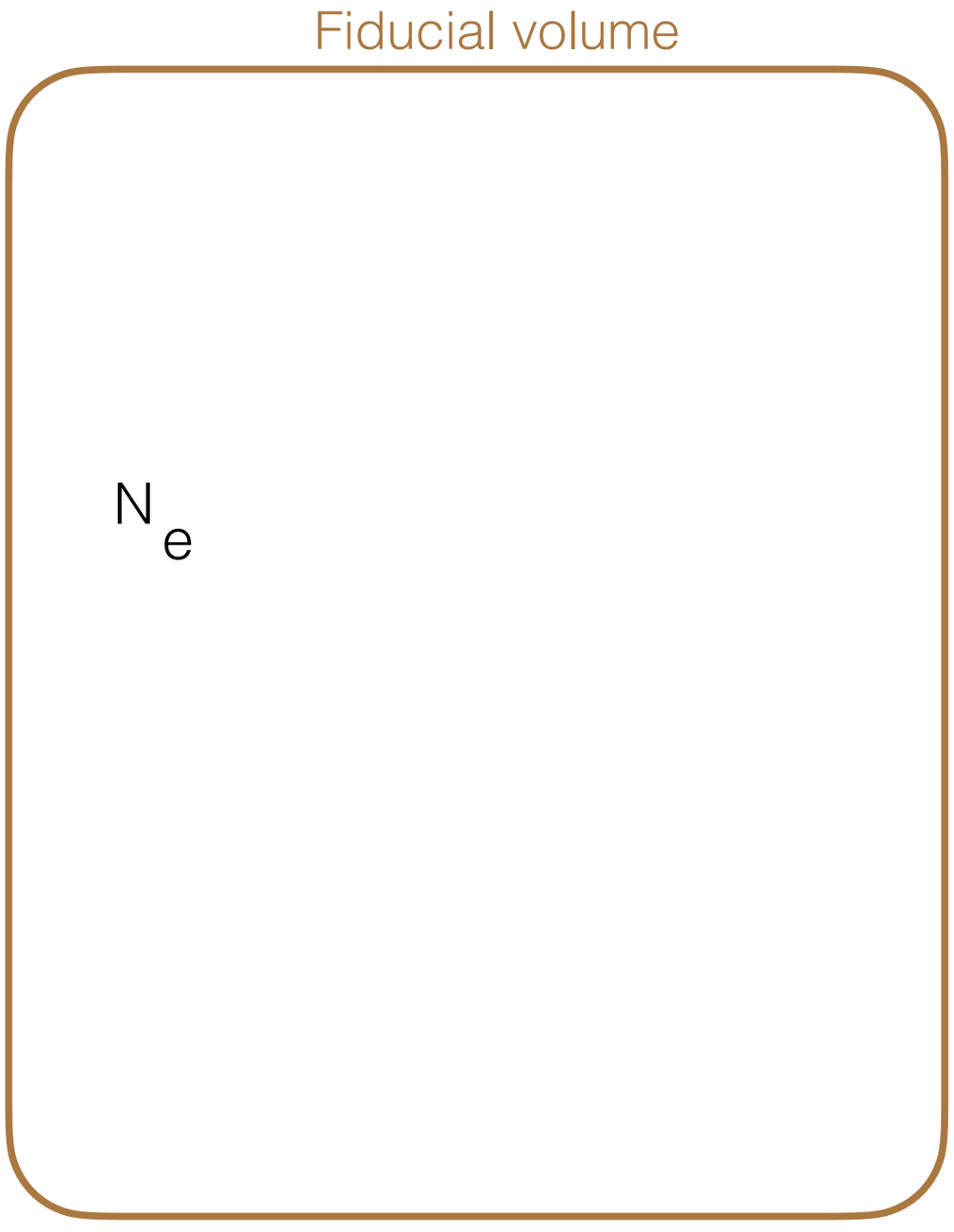




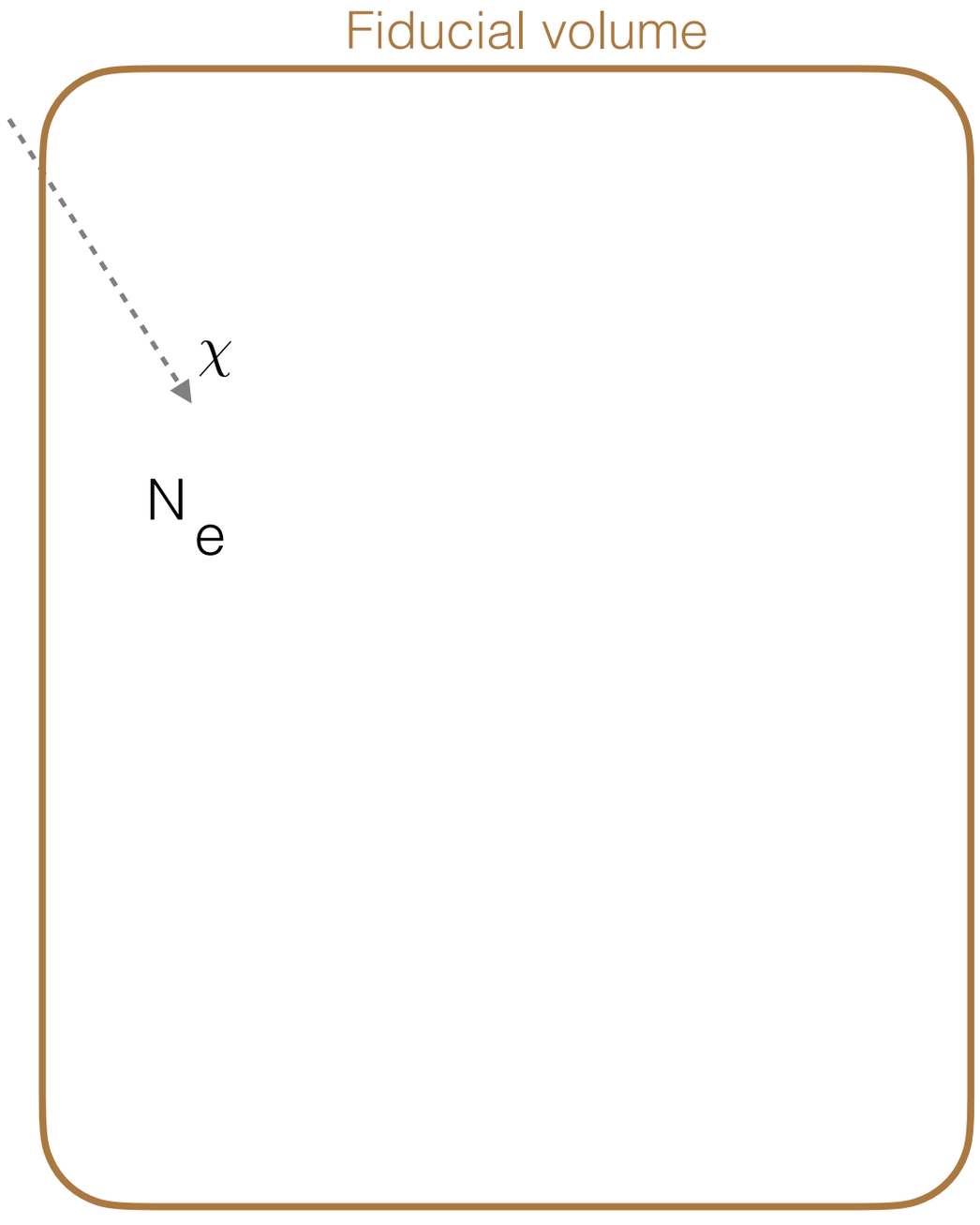
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수백개의 암흑물질이 평균 230km/s의
속도로 나를 지나감

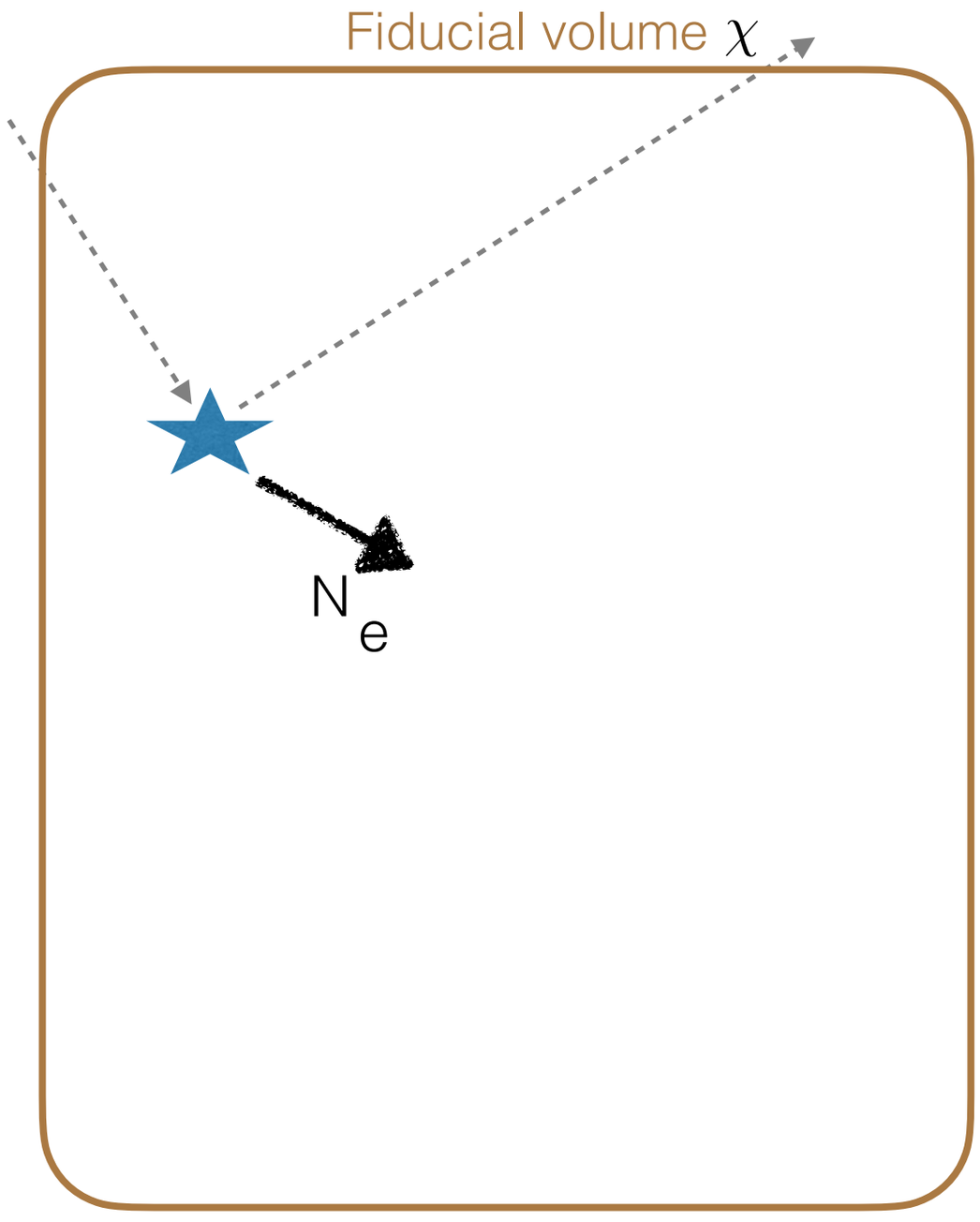
Direct Detection of Dark Matter



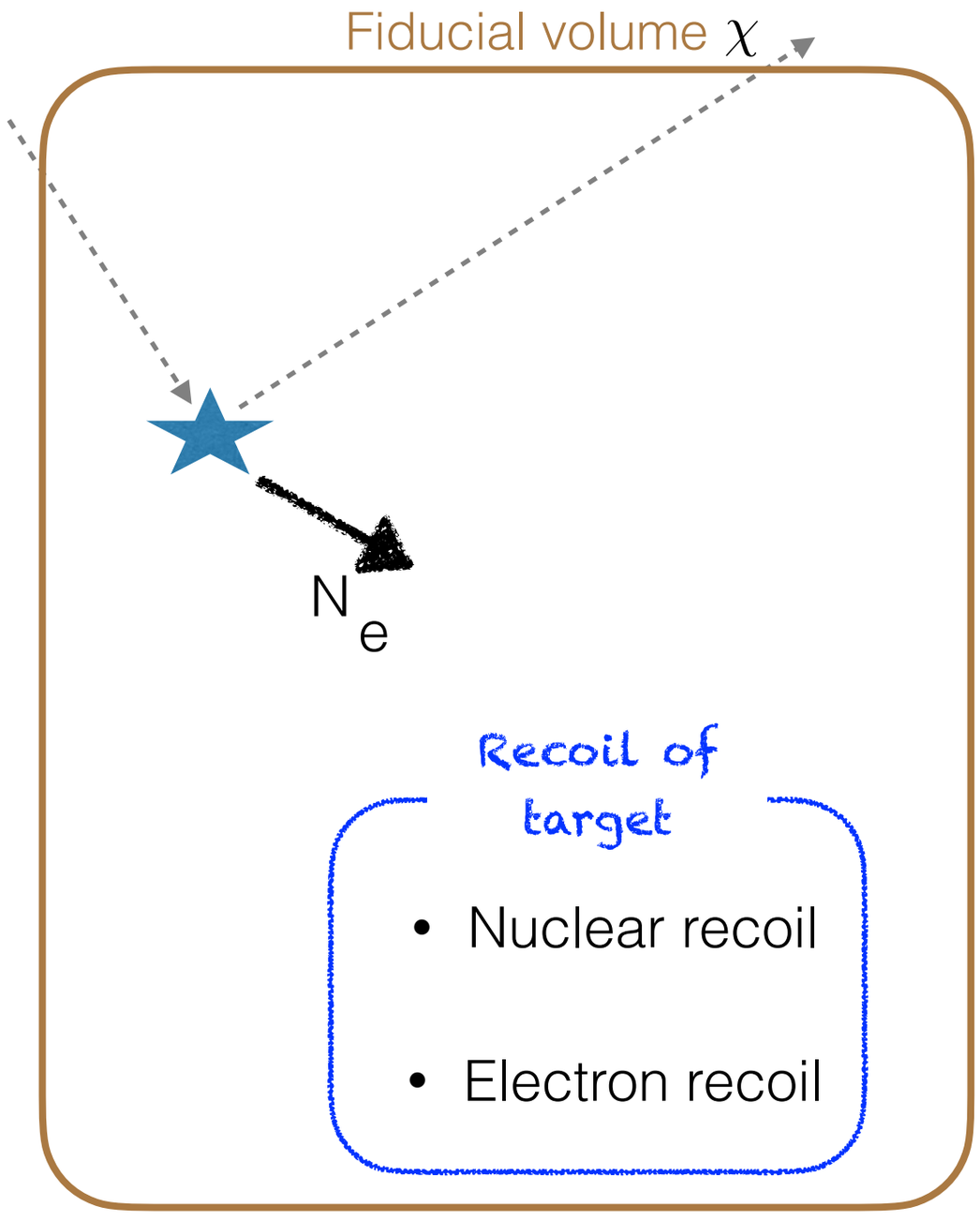
Direct Detection of Dark Matter



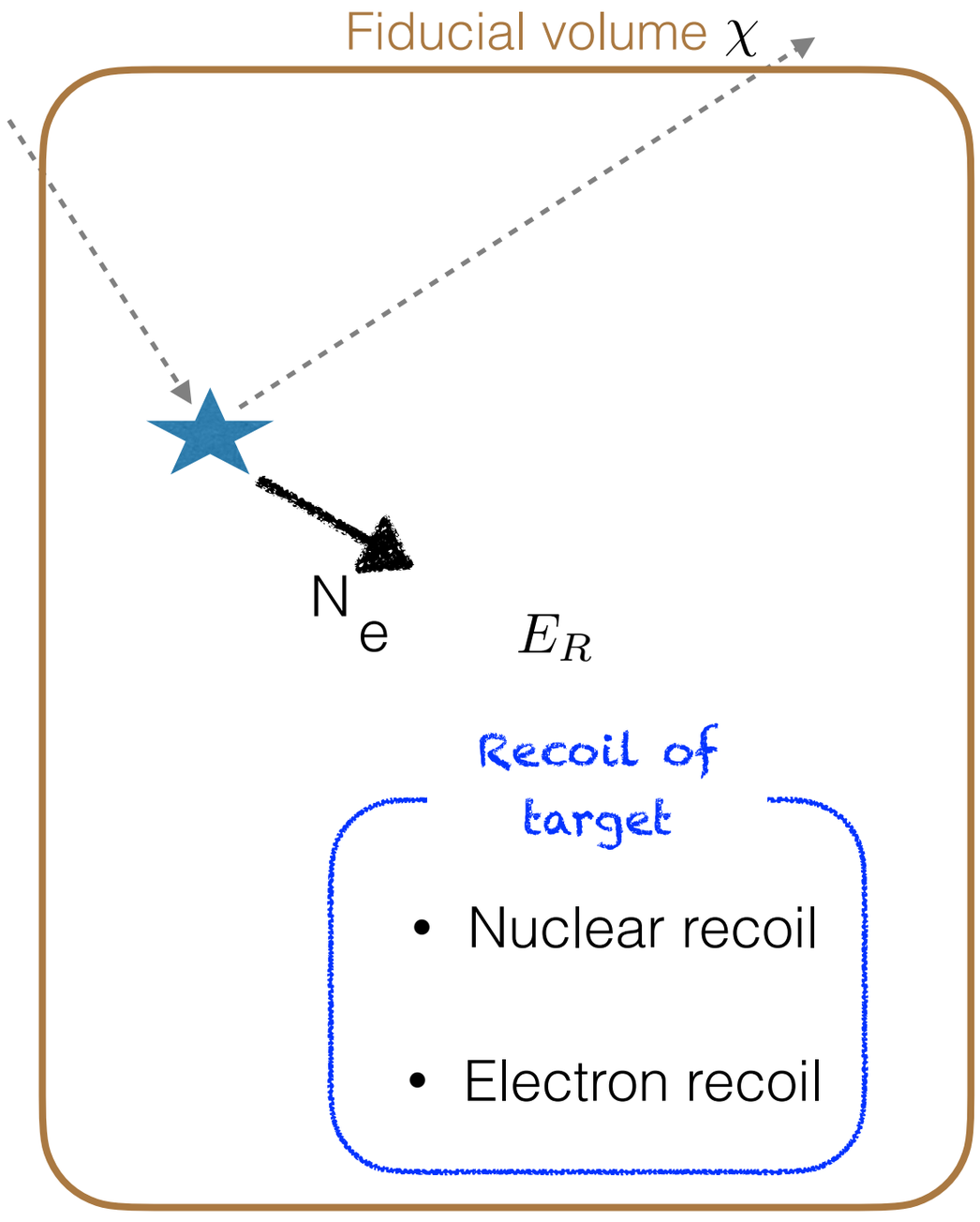
Direct Detection of Dark Matter



Direct Detection of Dark Matter

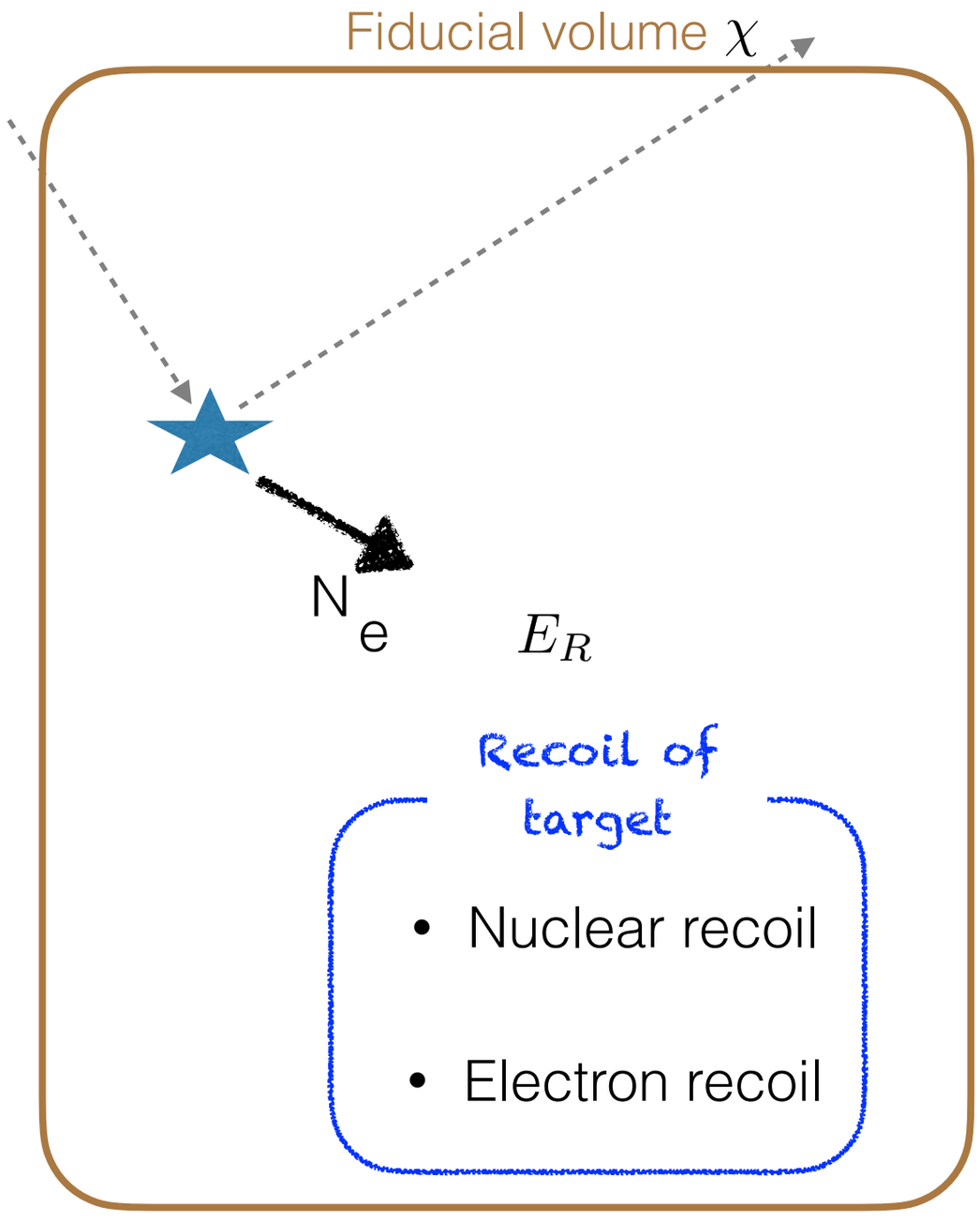


Direct Detection of Dark Matter



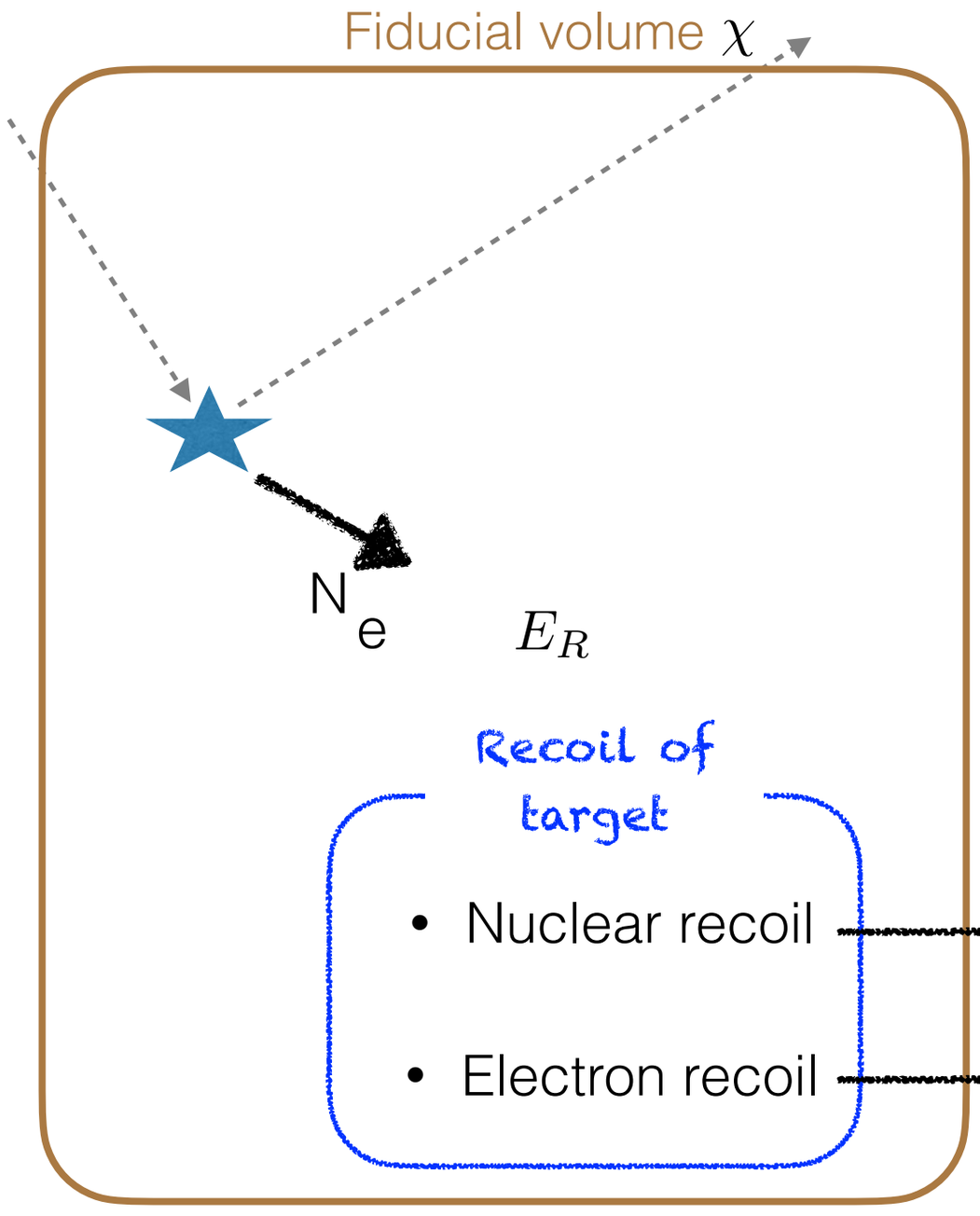
Recoil energy

Direct Detection of Dark Matter



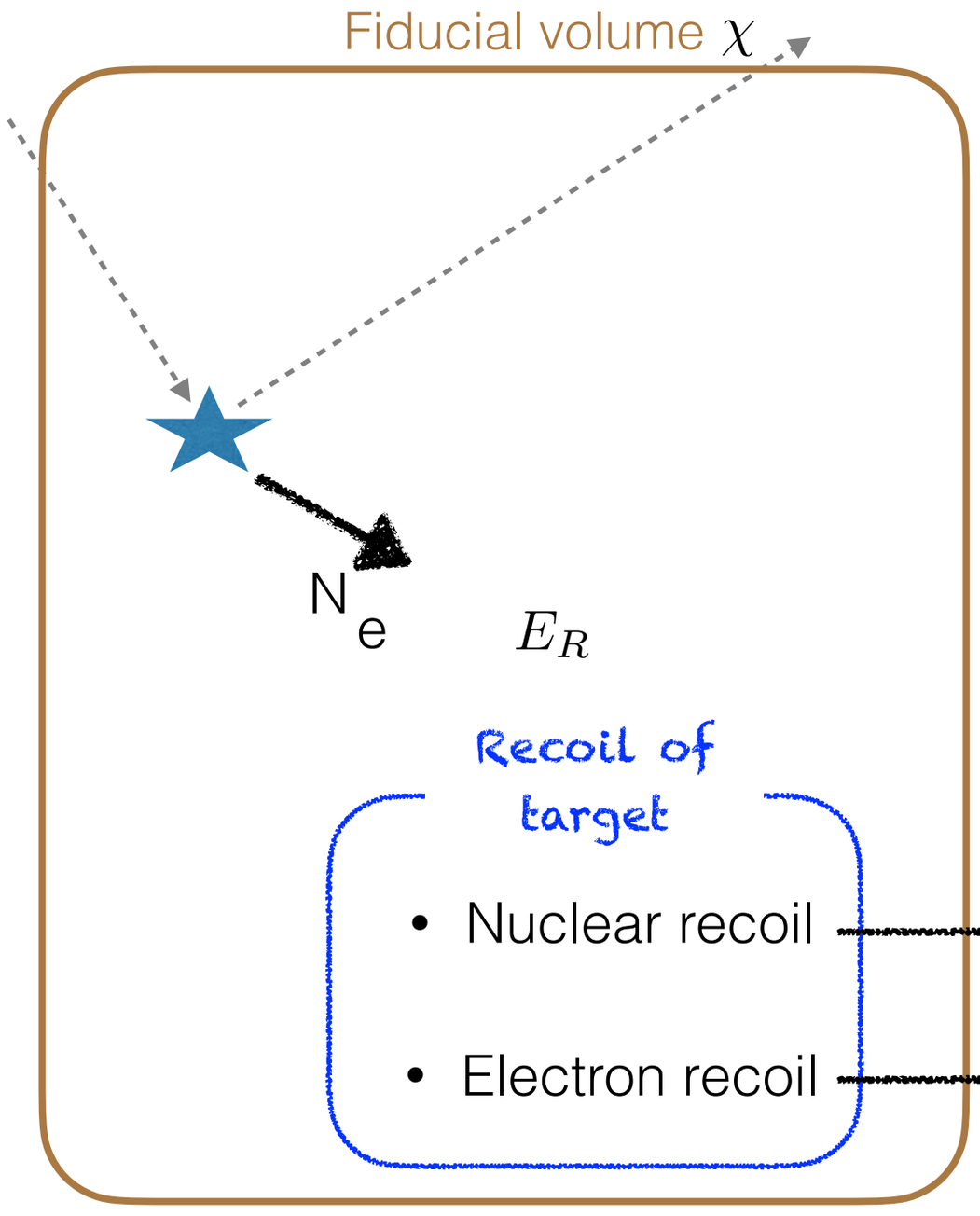
Recoil energy — kinetic energy of the recoiled target

Direct Detection of Dark Matter



Recoil energy — kinetic energy of the recoiled target

Direct Detection of Dark Matter



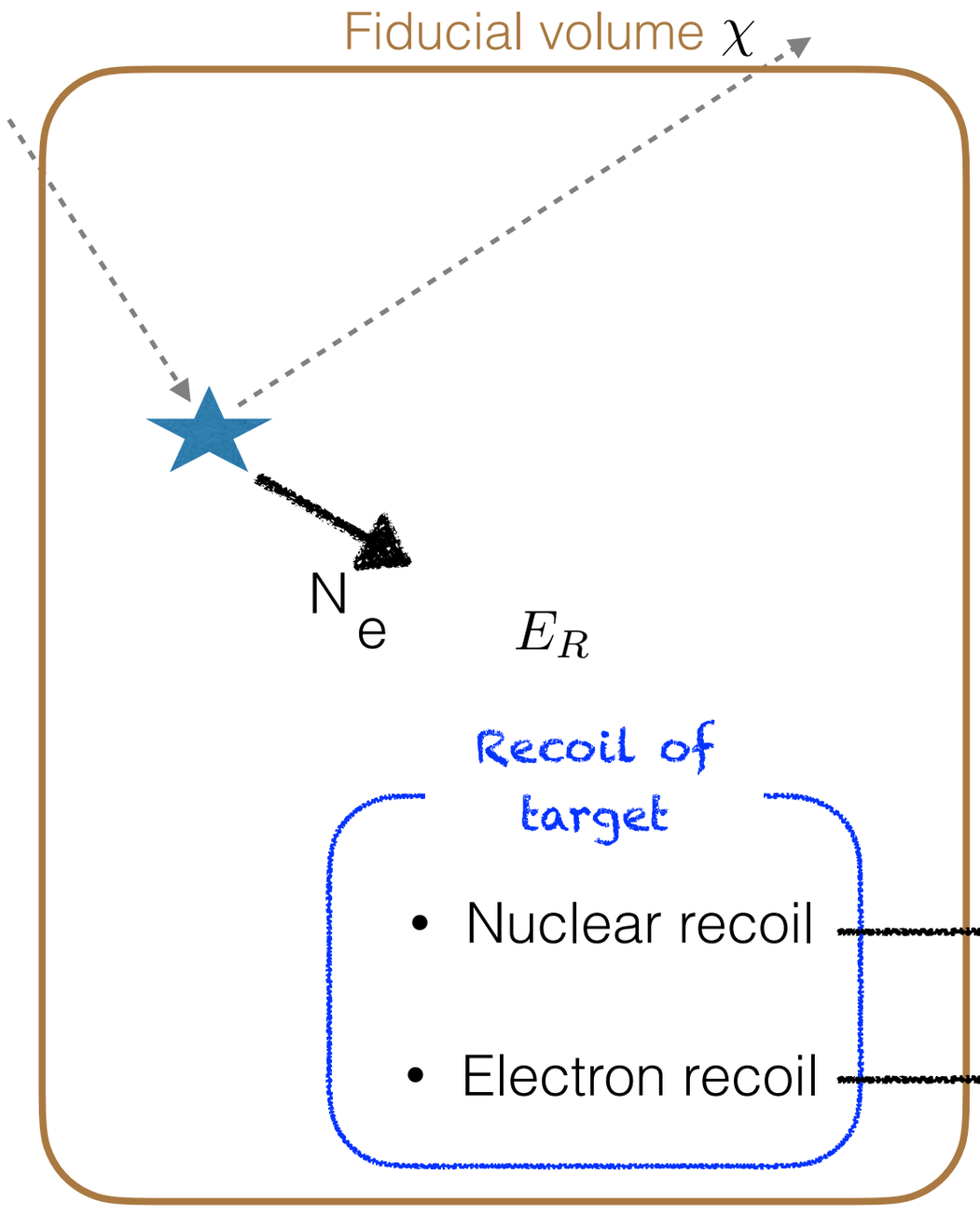
Recoil energy — kinetic energy of the recoiled target

$m_\chi \gtrsim \mathcal{O}(1\,\mathrm{GeV})$

$m_\chi \ll \mathcal{O}(1\,\mathrm{GeV})$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

Direct Detection of Dark Matter



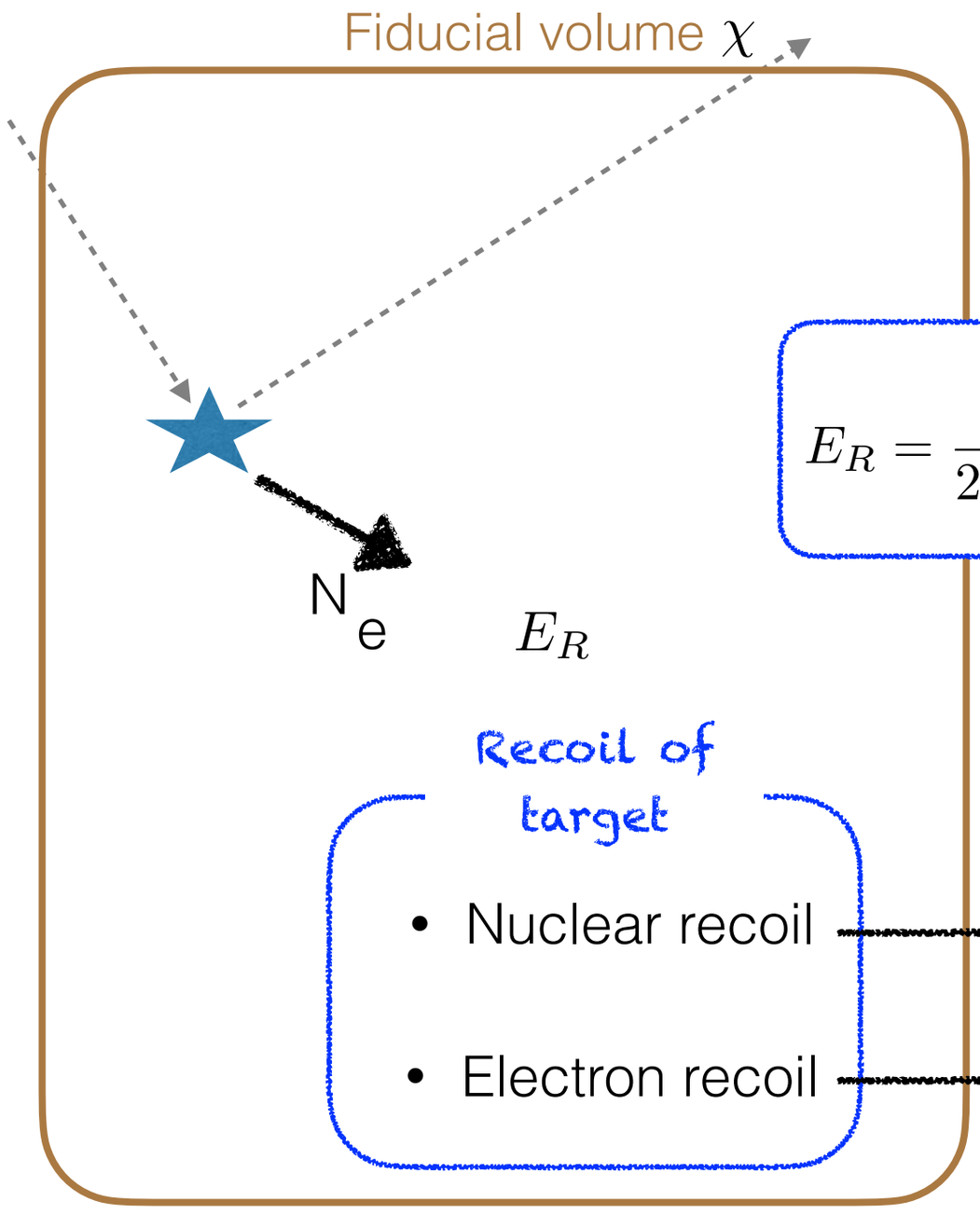
Recoil energy — kinetic energy of the recoiled target

$m_\chi \gtrsim \mathcal{O}(1\,\mathrm{GeV})$ WIMP

$m_\chi \ll \mathcal{O}(1\,\mathrm{GeV})$

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Direct Detection of Dark Matter



Recoil energy — kinetic energy of the recoiled target

$$E_R = \frac{q^2}{2m_N} \simeq 50\,\mathrm{keV}\left(\frac{m_\chi}{100\,\mathrm{GeV}}\right)^2\left(\frac{100\,\mathrm{GeV}}{m_N}\right)$$

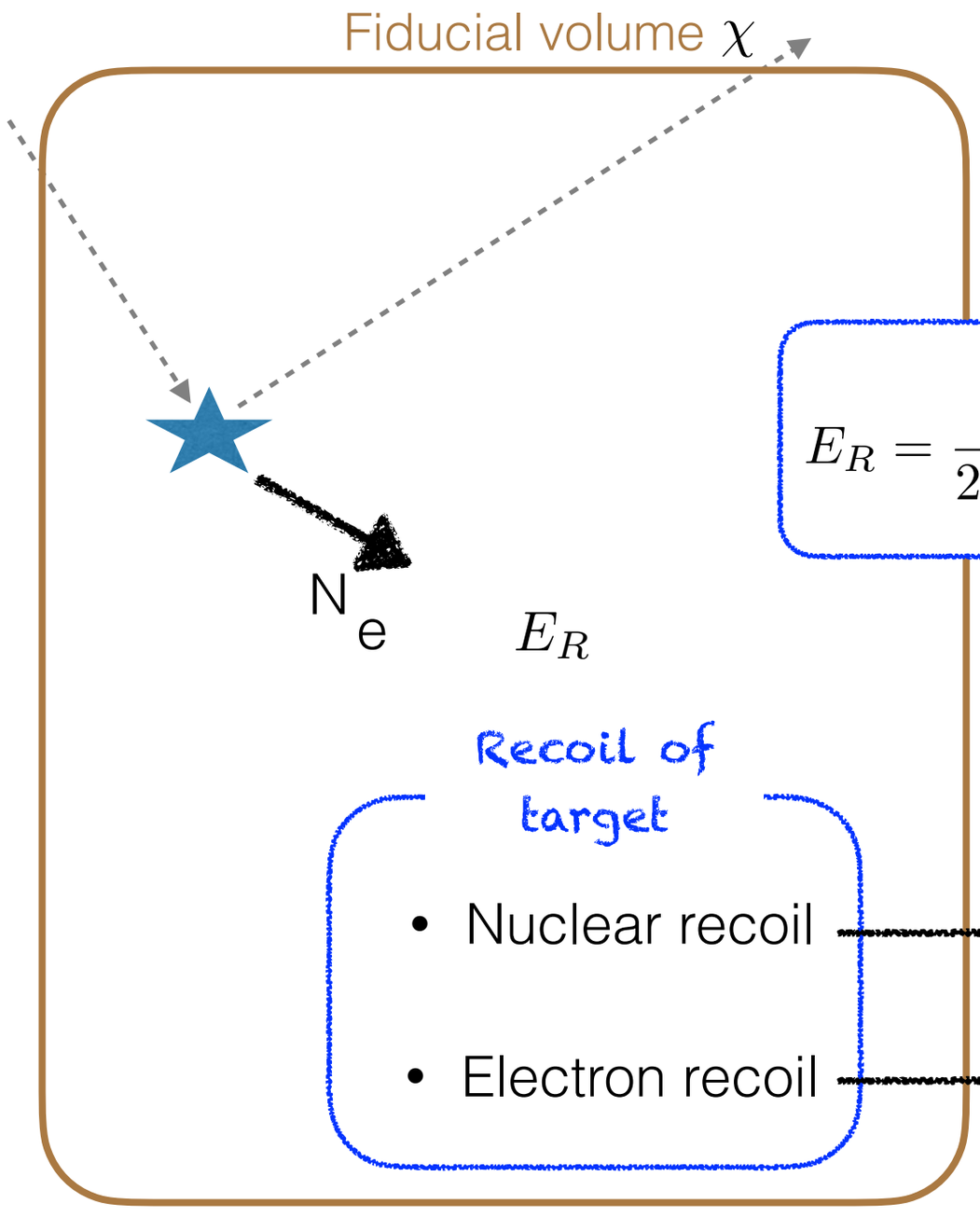
$q \sim m_\chi v \sim m_\chi \times 10^{-3}$ momentum transfer

$m_\chi \gtrsim \mathcal{O}(1\,\mathrm{GeV})$ WIMP

$m_\chi \ll \mathcal{O}(1\,\mathrm{GeV})$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

Direct Detection of Dark Matter



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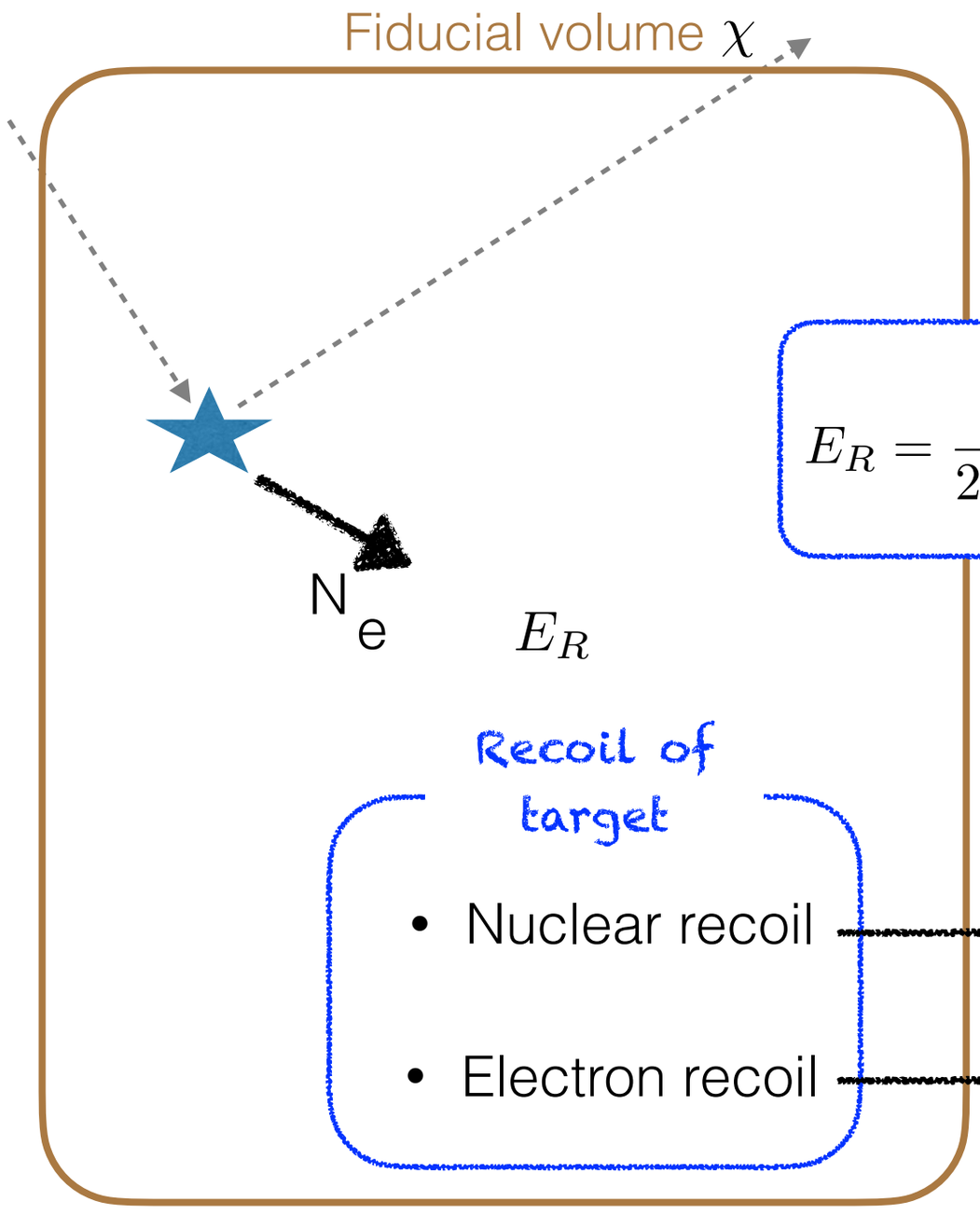
Most of the experiments (Xe, Ar, NaI) are sensitive for $E_R \lesssim 50$ keV

Recoil of target

- Nuclear recoil → $m_\chi \gtrsim \mathcal{O}(1\,\mathrm{GeV})$ WIMP
- Electron recoil → $m_\chi \ll \mathcal{O}(1\,\mathrm{GeV})$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

Direct Detection of Dark Matter



Recoil energy — kinetic energy of the recoiled target

$$E_R = \frac{q^2}{2m_N} \simeq 50\,\mathrm{keV}\left(\frac{m_\chi}{100\,\mathrm{GeV}}\right)^2\left(\frac{100\,\mathrm{GeV}}{m_N}\right)$$

$q \sim m_\chi v \sim m_\chi \times 10^{-3}$ momentum transfer

Most of the experiments (Xe, Ar, NaI) are sensitive for $E_R \lesssim 50$ keV « nuclear binding energy (1-10 MeV)

- Nuclear recoil → $m_\chi \gtrsim \mathcal{O}(1\,\mathrm{GeV})$ WIMP
- Electron recoil → $m_\chi \ll \mathcal{O}(1\,\mathrm{GeV})$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

Direct Detection of Dark Matter

The differential rate per unit detector mass

$$\frac{dR}{dE_R} = \frac{n_\chi}{m_N}\langle v \frac{d\sigma}{dE_R}\rangle$$

Nucleus mass Volume swept/time/E_R

Direct Detection of Dark Matter

The differential rate per unit detector mass

$$\frac{dR}{dE_R} = \frac{n_\chi}{m_N}\langle v \frac{d\sigma}{dE_R}\rangle$$

Nucleus mass Volume swept/time/E_R

$$\rho_\odot = m_\chi n_\chi$$

Direct Detection of Dark Matter

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Nucleus mass Volume swept/time/E_R

$$\rho_\odot = m_\chi n_\chi$$

Determining the local density is very important!

Direct Detection of Dark Matter

The differential rate per unit detector mass

$$\frac{dR}{dE_R} = \frac{n_\chi}{m_N}\langle v \frac{d\sigma}{dE_R}\rangle = \frac{\rho_\chi}{m_\chi m_N}\int_{v_{\rm min}}^{v_{\rm max}} d^3v\, v\tilde{f}(\mathbf{v},t)\frac{d\sigma}{dE_R}$$

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Direct Detection of Dark Matter

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Nucleus mass

Volume swept/time/E_R

DM velocity distribution in the lab frame

Direct Detection of Dark Matter

The differential rate per unit detector mass

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Nucleus mass

Volume swept/time/E_R

DM velocity distribution in the lab frame

Astrophysical properties

Direct Detection of Dark Matter

The differential rate per unit detector mass

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Nucleus mass Volume swept/time/E_R DM velocity distribution in the lab frame

Astrophysical properties

For a given recoil energy $v_{\rm min} = \sqrt{\frac{E_R m_N}{2\mu_{\chi N}^2}}$

Direct Detection of Dark Matter

The differential rate per unit detector mass

$$\frac{dR}{dE_R} = \frac{n_\chi}{m_N}\langle v \frac{d\sigma}{dE_R}\rangle = \frac{\rho_\chi}{m_\chi m_N}\int_{v_{\rm min}}^{v_{\rm max}} d^3v\, v\tilde{f}(\mathbf{v},t)\frac{d\sigma}{dE_R}$$

Nucleus mass Volume swept/time/E_R DM velocity distribution in the lab frame

Astrophysical properties

For a given recoil energy $\quad v_{\rm min} = \sqrt{\frac{E_R m_N}{2\mu_{\chi N}^2}}$

In term of the galactic frame distribution $f(\mathbf{v})$ (Galilean transformation)

$$\tilde{f}(\mathbf{v}) = f(\mathbf{v} + \mathbf{v}_{\rm obs}(t))$$

Direct Detection of Dark Matter

The differential rate per unit detector mass

$$\frac{dR}{dE_R} = \frac{n_\chi}{m_N}\langle v \frac{d\sigma}{dE_R}\rangle = \frac{\rho_\chi}{m_\chi m_N}\int_{v_{\rm min}}^{v_{\rm max}} d^3v\, v\tilde{f}(\mathbf{v},t)\frac{d\sigma}{dE_R}$$

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$$\tilde{f}(\mathbf{v}) = f(\mathbf{v}+\mathbf{v}_{\rm obs}(t)) \qquad \mathbf{v}_{\rm obs}(t) = \mathbf{v}_\odot + \mathbf{v}_{\rm ES}$$

Direct Detection of Dark Matter

The differential rate per unit detector mass

$$\frac{dR}{dE_R} = \frac{n_\chi}{m_N}\langle v \frac{d\sigma}{dE_R}\rangle = \frac{\rho_\chi}{m_\chi m_N}\int_{v_{\rm min}}^{v_{\rm max}} d^3v\, v\tilde{f}(\mathbf{v},t)\frac{d\sigma}{dE_R}$$

Nucleus mass Volume swept/time/E_R DM velocity distribution in the lab frame

Astrophysical properties

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reference 220km/s 30km/s

Schoenrich et al., 2009 Lee, Lisanti, Shafdi, 2013

Direct Detection of Dark Matter

The differential rate per unit detector mass

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Nucleus mass — Volume swept/time/E_R — DM velocity distribution in the lab frame

Astrophysical properties

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reference 220km/s 30km/s

$$\mathbf{v}_{\rm obs}(t) \approx \mathbf{v}_\odot\,(1+\epsilon\cos[\omega(t-t_0)])$$

Schoenrich et al., 2009 — Lee, Lisanti, Shafdi, 2013

Direct Detection of Dark Matter

The differential rate per unit detector mass

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Nucleus mass Volume swept/time/E_R DM velocity distribution in the lab frame

Astrophysical properties

For a given recoil energy $\quad v_{\rm min} = \sqrt{\frac{E_R m_N}{2\mu_{\chi N}^2}}$

In term of the galactic frame distribution $f(\mathbf{v})$ (Galilean transformation)

$$f(\mathbf{v}+\mathbf{v}_{\rm obs}(t)) \simeq f(\mathbf{v}+\mathbf{v}_\odot) + \epsilon\cos[\omega(t-t_0)]f'(\mathbf{v}+\mathbf{v}_\odot)+\cdots$$

$$\frac{dR}{dE_R} = A_0 + A_1\cos[\omega(t-t_0)]f'(\mathbf{v}+\mathbf{v}_\odot)+\cdots$$

Direct Detection of Dark Matter

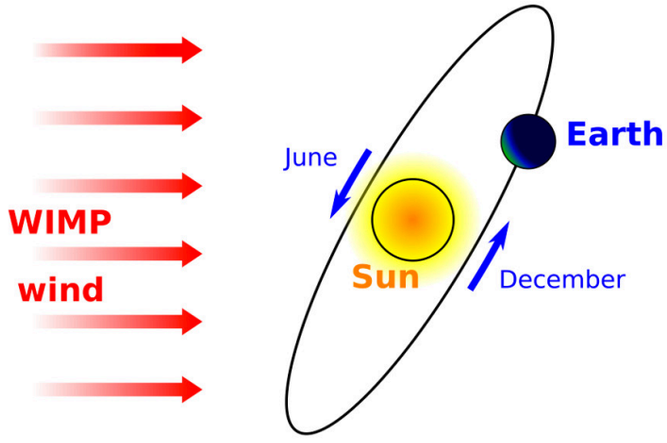
The differential rate per unit detector mass

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Nucleus mass Volume swept/time/E_R DM velocity distribution in the lab frame

Astrophysical properties

For a given recoil energy $v_{\rm min} = \sqrt{\frac{E_R m_N}{2\mu_{\chi N}^2}}$



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Annual modulation

Direct Detection of Dark Matter

The differential rate per unit detector mass

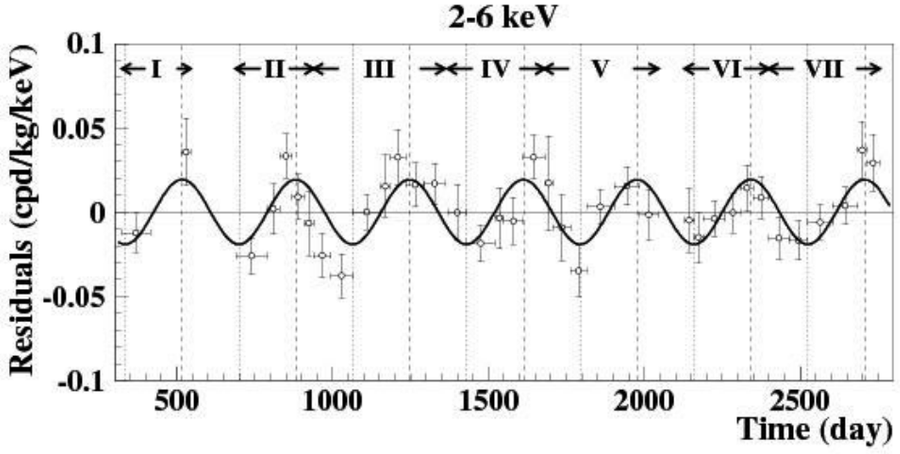
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Nucleus mc

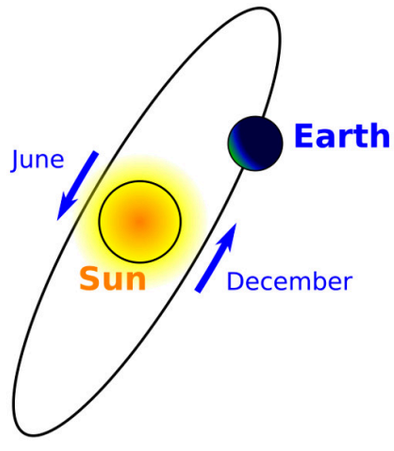
DM velocity distribution in the lab frame

Astrophysical

For a given







In term of the galactic frame distribution $f(\mathbf{v})$ (Galilean transformation)

$$f(\mathbf{v}+\mathbf{v}_{\rm obs}(t)) \simeq f(\mathbf{v}+\mathbf{v}_\odot) + \epsilon\cos[\omega(t-t_0)]f'(\mathbf{v}+\mathbf{v}_\odot)+\cdots$$

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Annual modulation

Direct Detection of Dark Matter

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Nucleus mass Volume swept/time/E_R DM velocity distribution in the lab frame

Particle (+ nuclear) properties

Direct Detection of Dark Matter

The differential rate per unit detector mass

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Nucleus mass Volume swept/time/E_R DM velocity distribution in the lab frame

Particle (+ nuclear) properties

Microscopic interaction: DM - quark (gluon) operator, e.g., $\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q$

Direct Detection of Dark Matter

The differential rate per unit detector mass

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Match to nucleon operators $\langle n|\bar{q}\gamma^\mu q|n\rangle$

Direct Detection of Dark Matter

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Nucleus mass Volume swept/time/E_R DM velocity distribution in the lab frame

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↓

Match to nucleon operators $\langle n|\bar{q}\gamma^\mu q|n\rangle$

↓

Consider coherent scattering with nucleus using nuclear wave functions

Direct Detection of Dark Matter

The differential rate per unit detector mass

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Nucleus mass — Volume swept/time/E_R — DM velocity distribution in the lab frame

Particle (+ nuclear) properties

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↓

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Take the non-relativistic limit

Direct Detection of Dark Matter

The differential rate per unit detector mass

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Nucleus mass Volume swept/time/E_R DM velocity distribution in the lab frame

Particle (+ nuclear) properties

Direct Detection of Dark Matter

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Nucleus mass Volume swept/time/E_R DM velocity distribution in the lab frame

Particle (+ nuclear) properties

For some operators (type of interactions),

$$\frac{d\sigma}{dE_R} \propto \frac{m_N}{v^2}\left[Z f_p + (A-Z) f_n\right]^2 F^2(q)$$

Direct Detection of Dark Matter

The differential rate per unit detector mass

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$$f_p \bar{\chi}\chi\bar{p}p + f_n \bar{\chi}\chi n n$$

Direct Detection of Dark Matter

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Nucleus mass — Volume swept/time/E_R — DM velocity distribution in the lab frame

Particle (+ nuclear) properties

For some operators (type of interactions),

$$f_p \propto \sum_{q=u,d,s}\frac{m_p}{m_q}f^p_{T_q} + \frac{2}{27}f^p_{TG}\sum_{q=c,b,t}\frac{m_p}{m_q}$$

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$$f_p\bar{\chi}\chi\bar{p}p + f_n\bar{\chi}\chi nn$$

Direct Detection of Dark Matter

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$$m_p f^p_{T_q} = \langle p|m_q\bar{q}q|p\rangle$$

$f_p\bar{\chi}\chi\bar{p}p + f_n\bar{\chi}\chi nn$

Nuclear form factor
(distribution of nucleons in a nucleus)

Direct Detection of Dark Matter

The differential rate per unit detector mass

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Nuclear form factor
(distribution of nucleons in a nucleus)

Proportional to the mass of the target nucleus!

Direct Detection of Dark Matter

The differential rate per unit detector mass

$$\frac{dR}{dE_R} = \frac{n_\chi}{m_N}\langle v \frac{d\sigma}{dE_R}\rangle = \frac{\rho_\chi}{m_\chi m_N}\int_{v_{\rm min}}^{v_{\rm max}} d^3v\, v\tilde{f}(\mathbf{v},t)\frac{d\sigma}{dE_R}$$

Nucleus mass Volume swept/time/E_R DM velocity distribution in the lab frame

Particle (+ nuclear) properties

For other type of interactions,

$$\frac{d\sigma}{dE_R} \propto \frac{m_N}{v^2}\frac{J+1}{J}\left[a_p\langle S_p\rangle + a_n\langle S_n\rangle\right]^2 F^2_{\rm SD}(q)$$

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Average spin contribution of p/n in the target nucleus

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Average spin contribution of p/n in the target nucleus

Proportional to the nuclear spin — Spin-dependent scattering

Not added coherently inside a nucleus
(typically only a few unpaired nucleons)

Only some isotopes can be used and bounds are weak.

Direct Detection of Dark Matter

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What we observe in the experiments: $N_{\rm event}$, E_R

Direct Detection of Dark Matter

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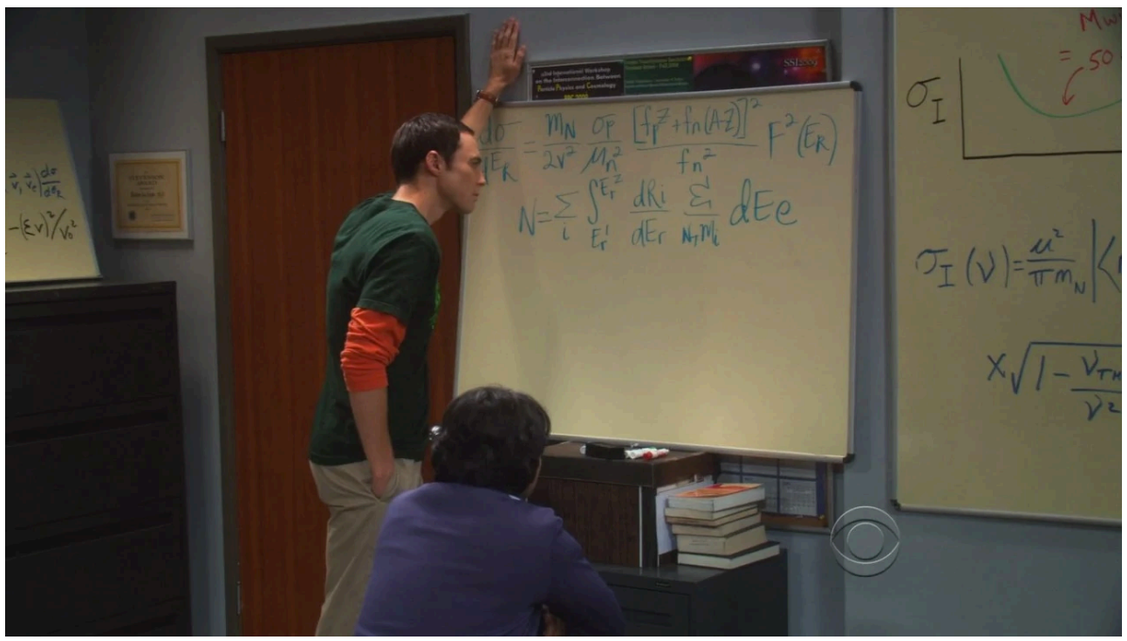
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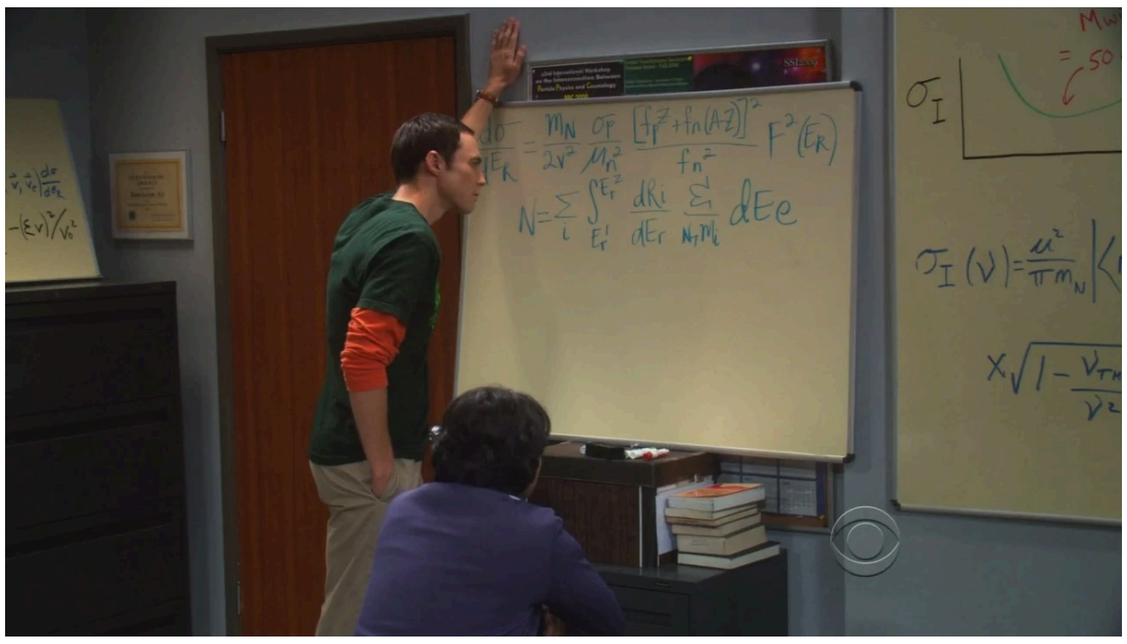


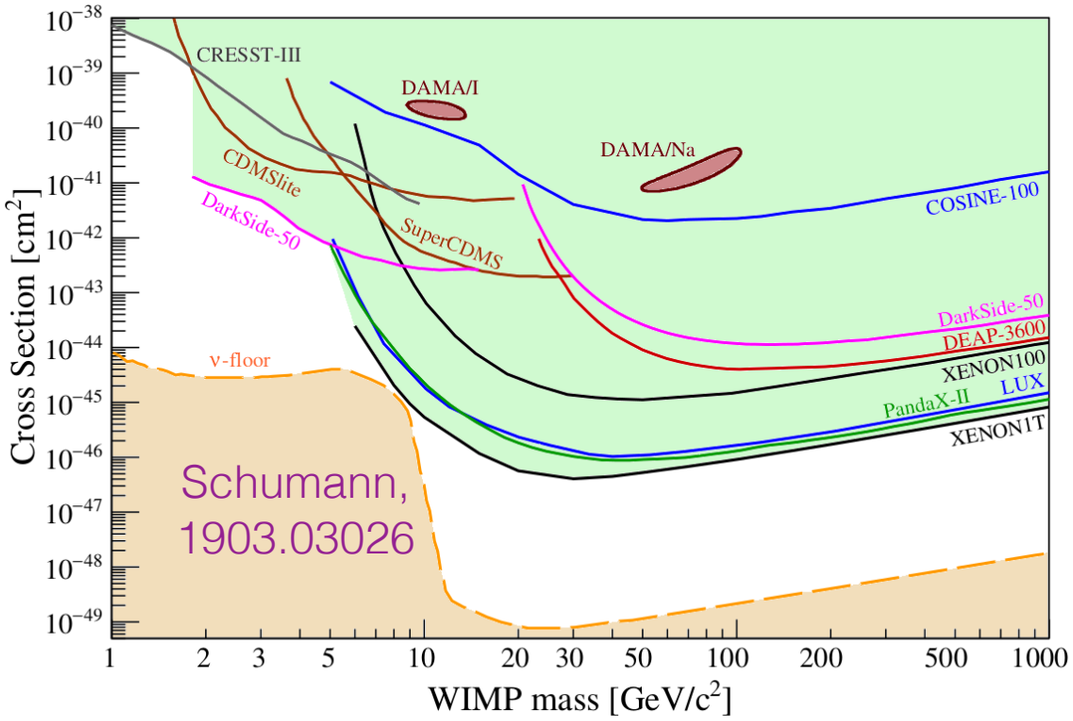
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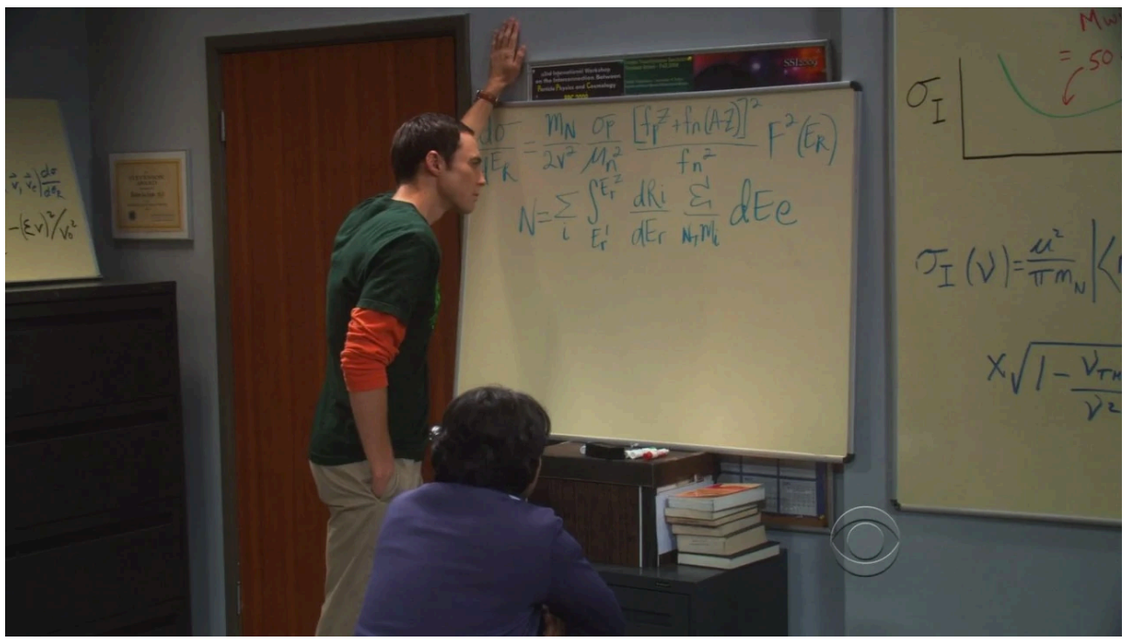


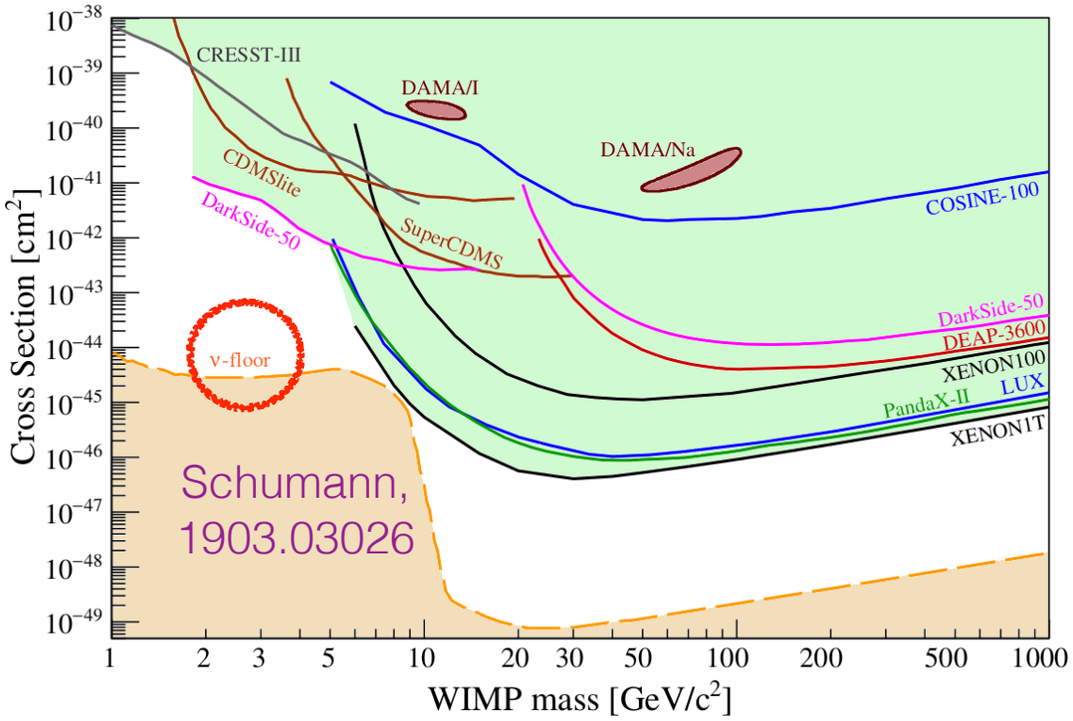
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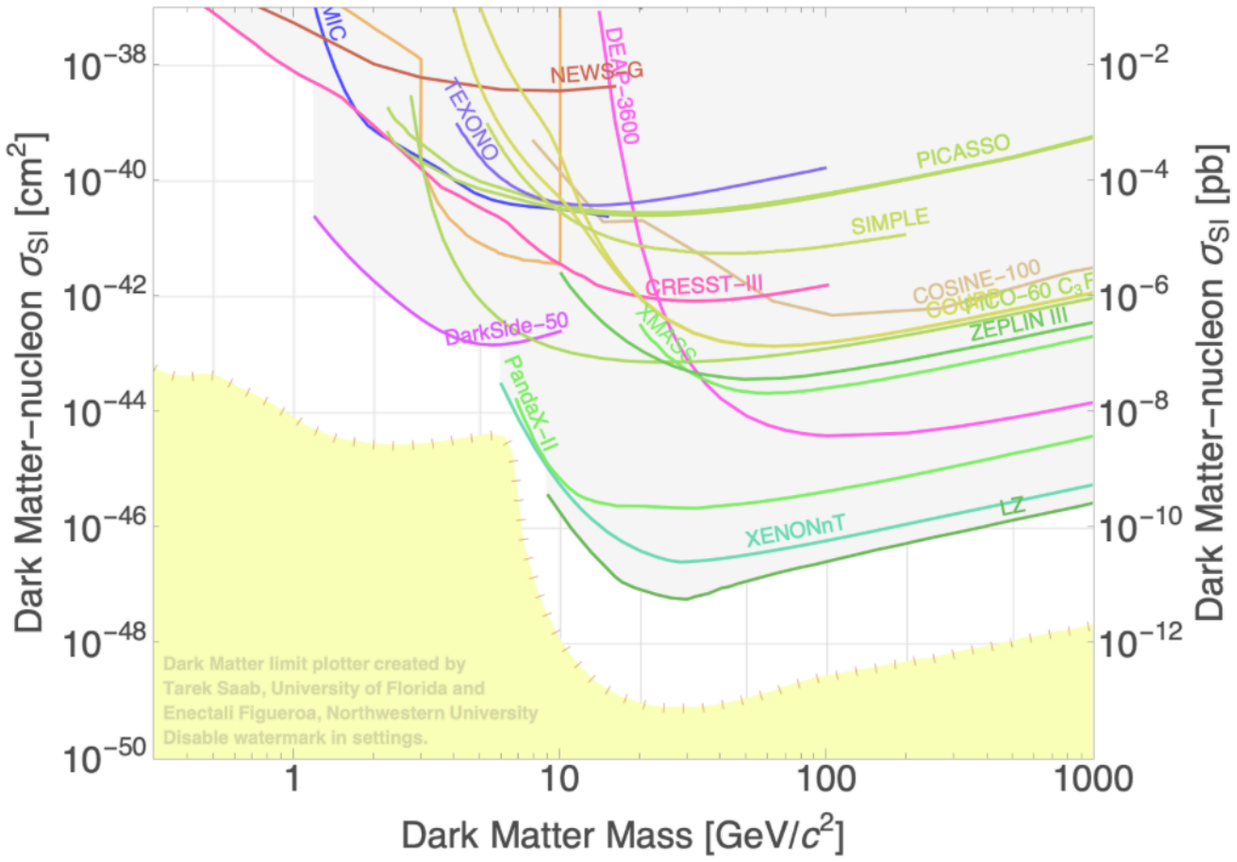
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Direct Detection of Dark Matter

DM limit plotter



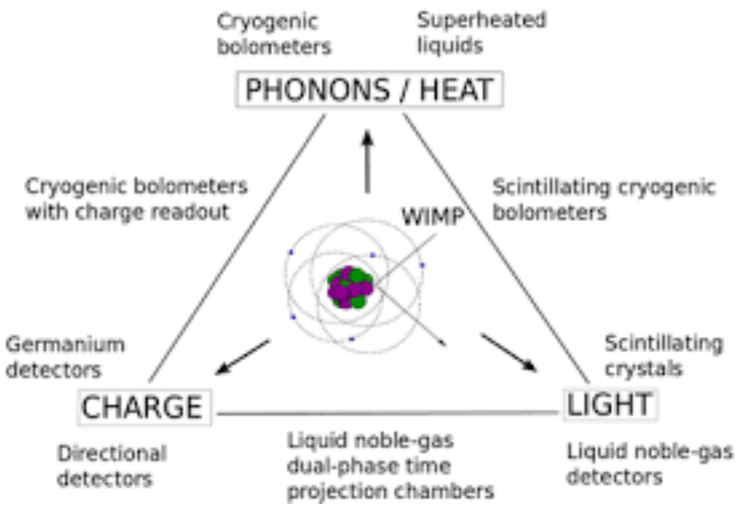


All Nuclear Recoil limits are scaled to a local dark matter density of 0.3 GeV/c^2

https://supercdms.slac.stanford.edu/dark-matter-limit-plotter

Direct Detection of Dark Matter

Target? observation method?

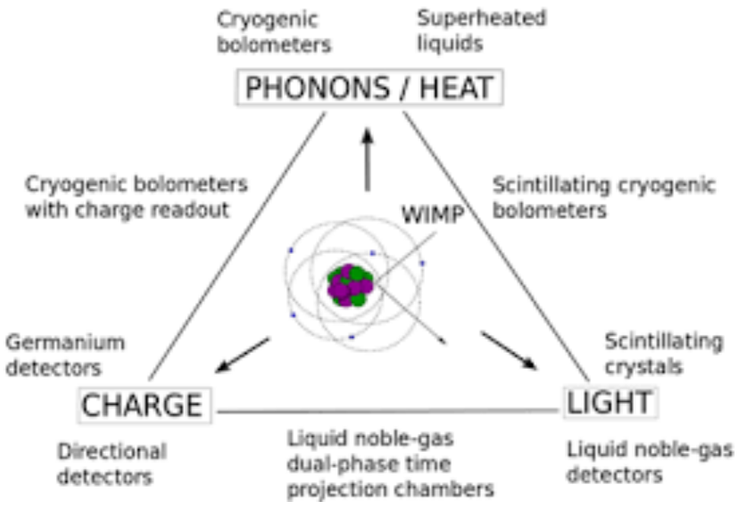


충돌한 핵자가 만드는 시그널

- 전기 시그널
- 빛 (scintillation)
- 극저온 상태에서 발생하는 열

Direct Detection of Dark Matter

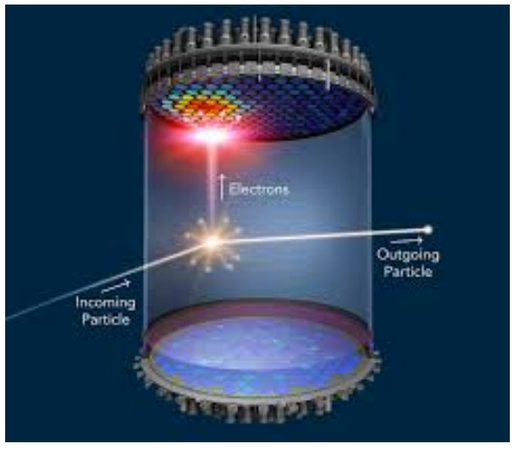
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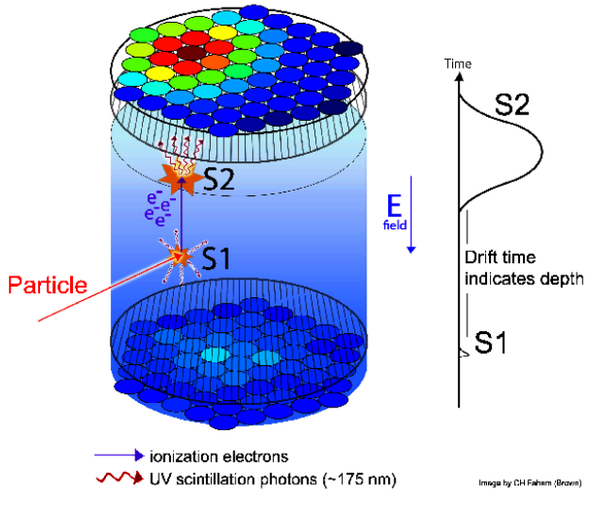


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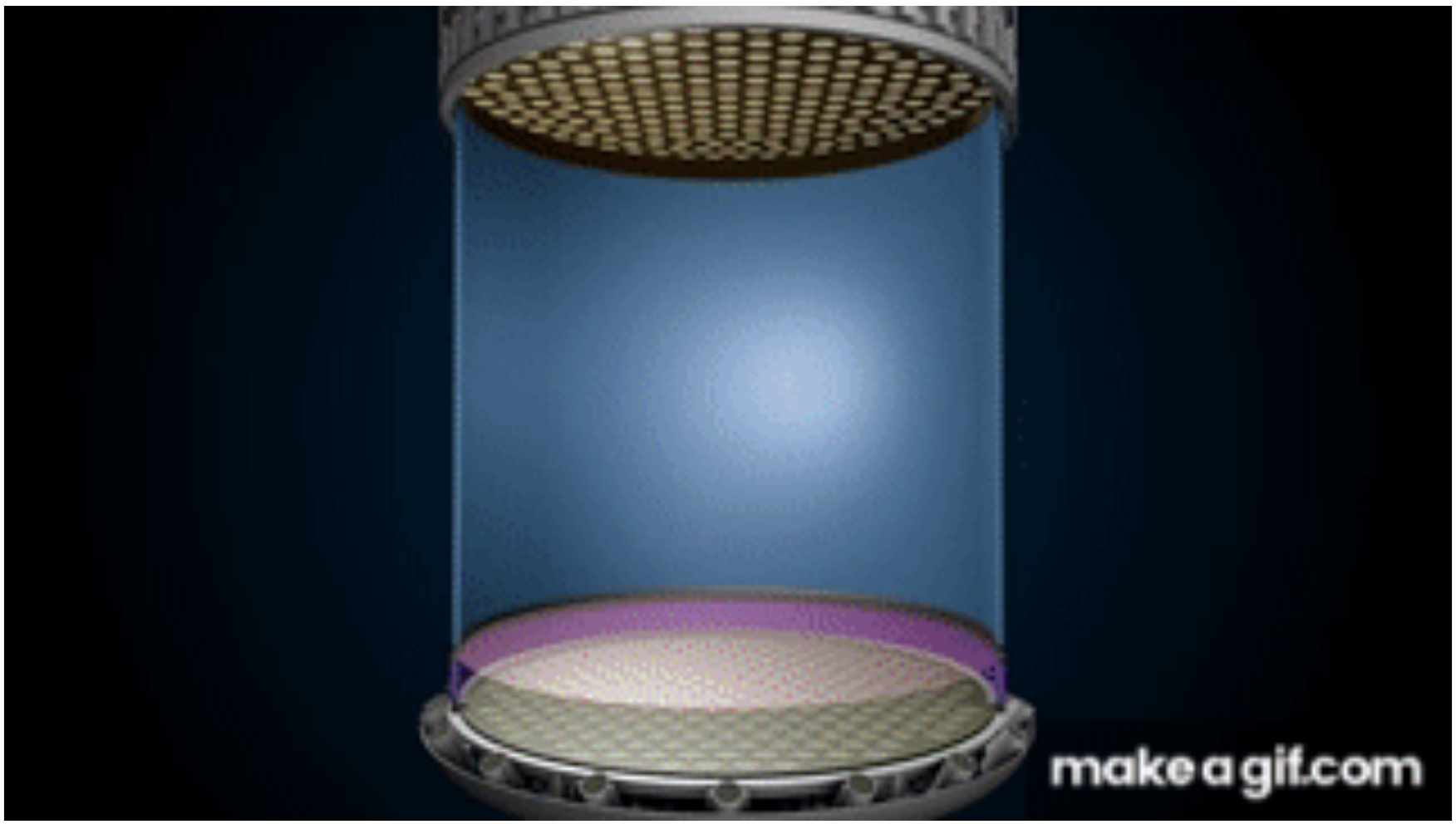
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Example: Xenon detectors (XENON1T, LUX-ZEPLIN, PANDA, ..)



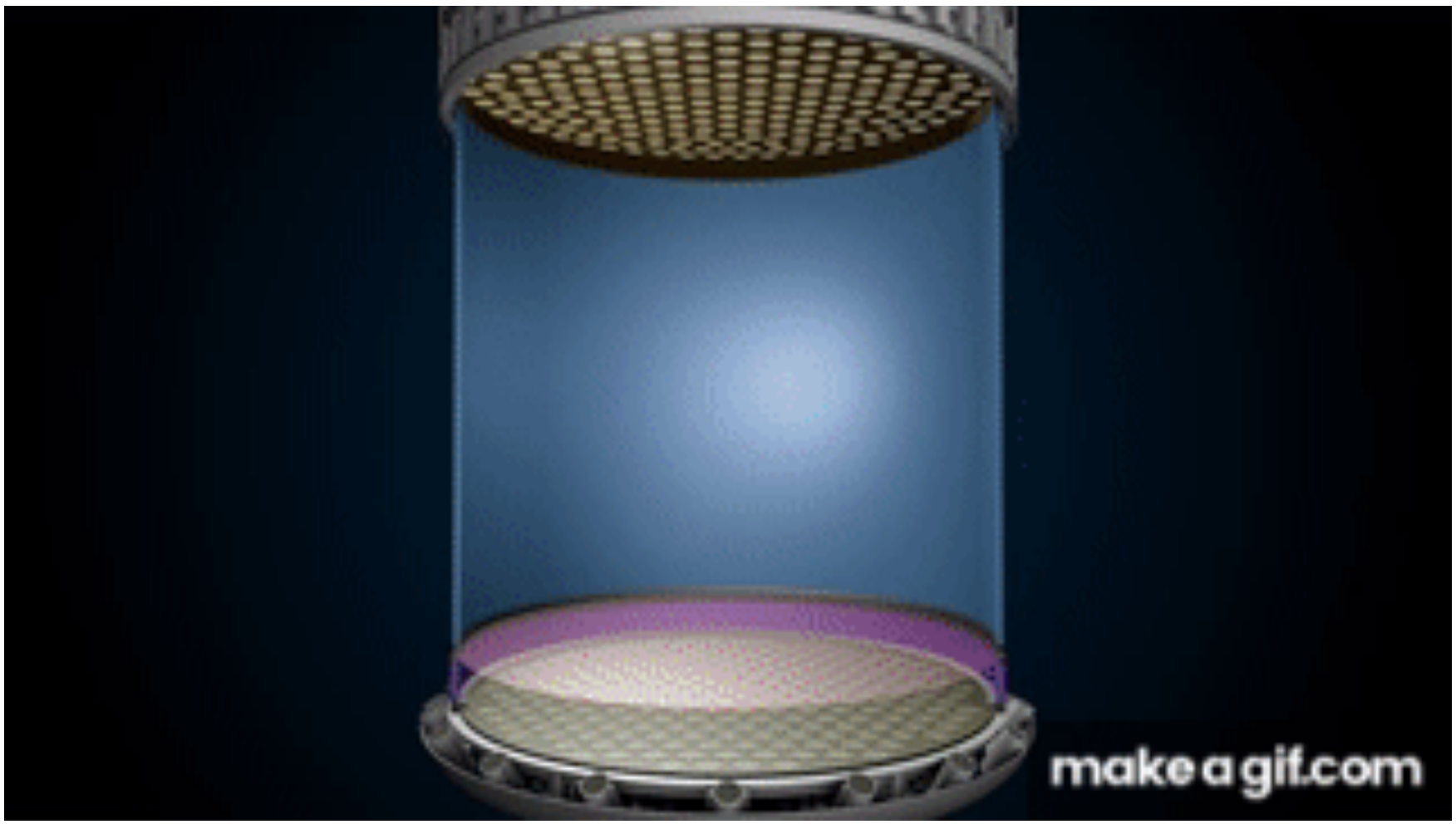


Direct Detection of Dark Matter



LUX-ZEPLIN

Direct Detection of Dark Matter

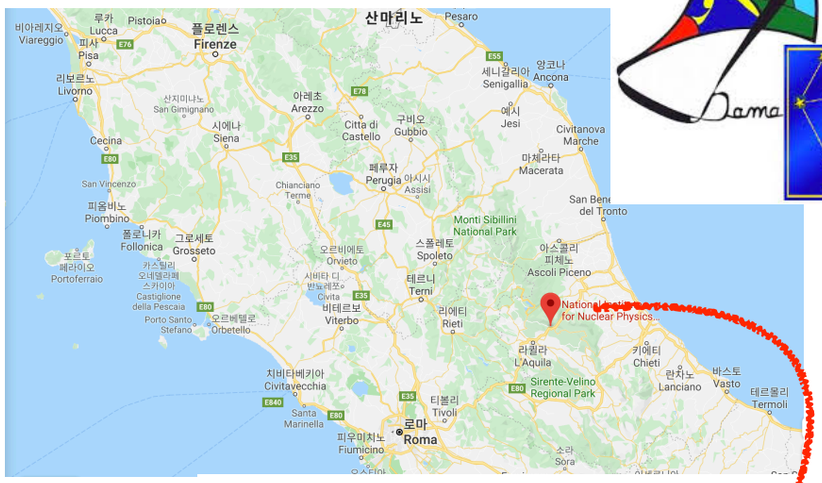


LUX-ZEPLIN

Direct Detection of Dark Matter

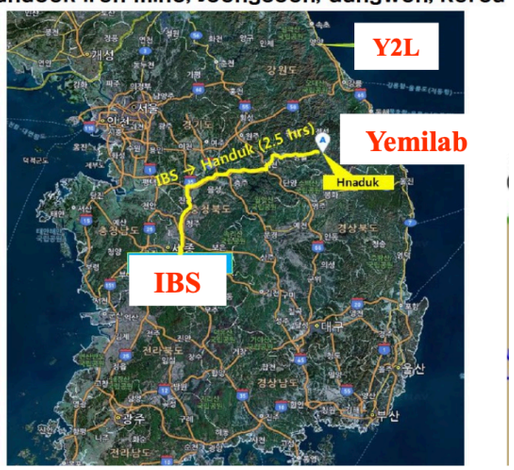
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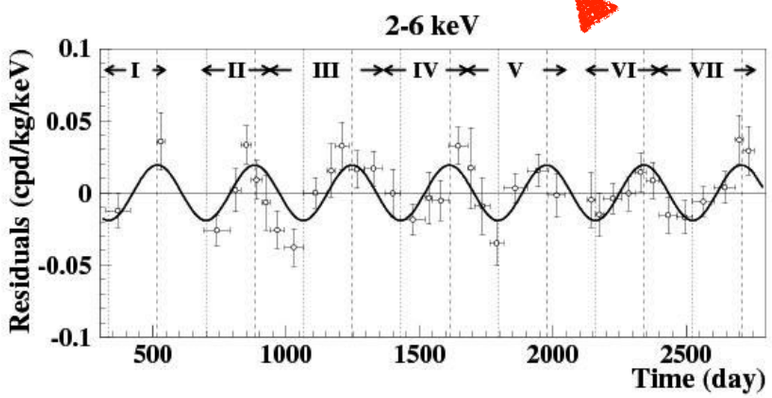






Handeok iron mine, Jeongseon, Gangwon, Korea

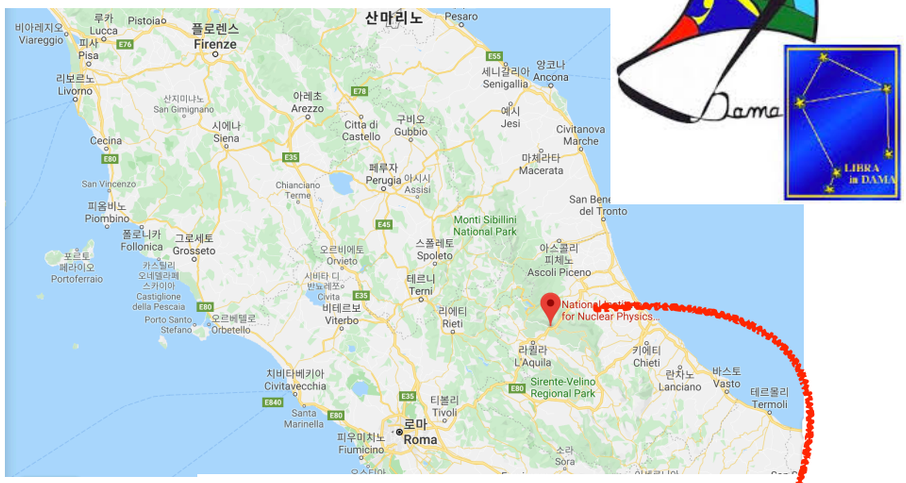




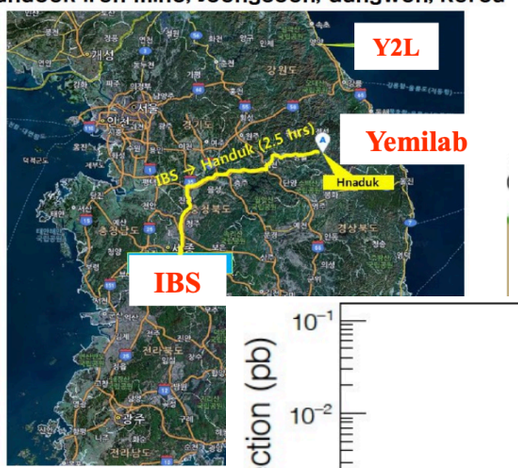
Direct Detection of Dark Matter

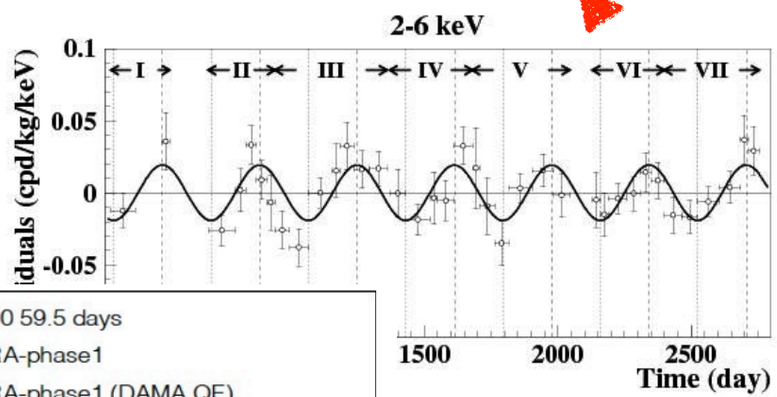
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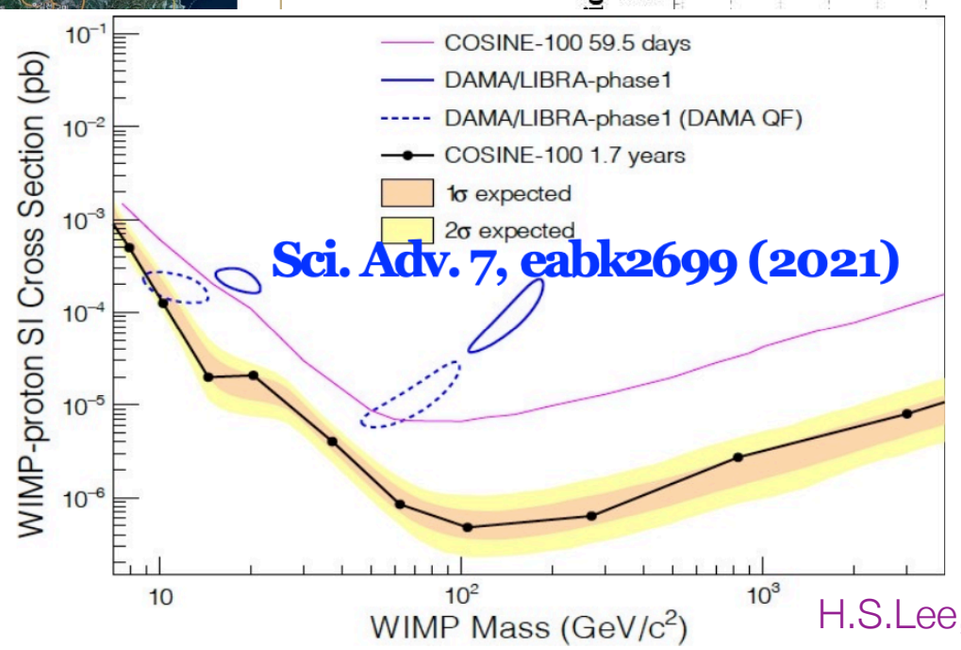




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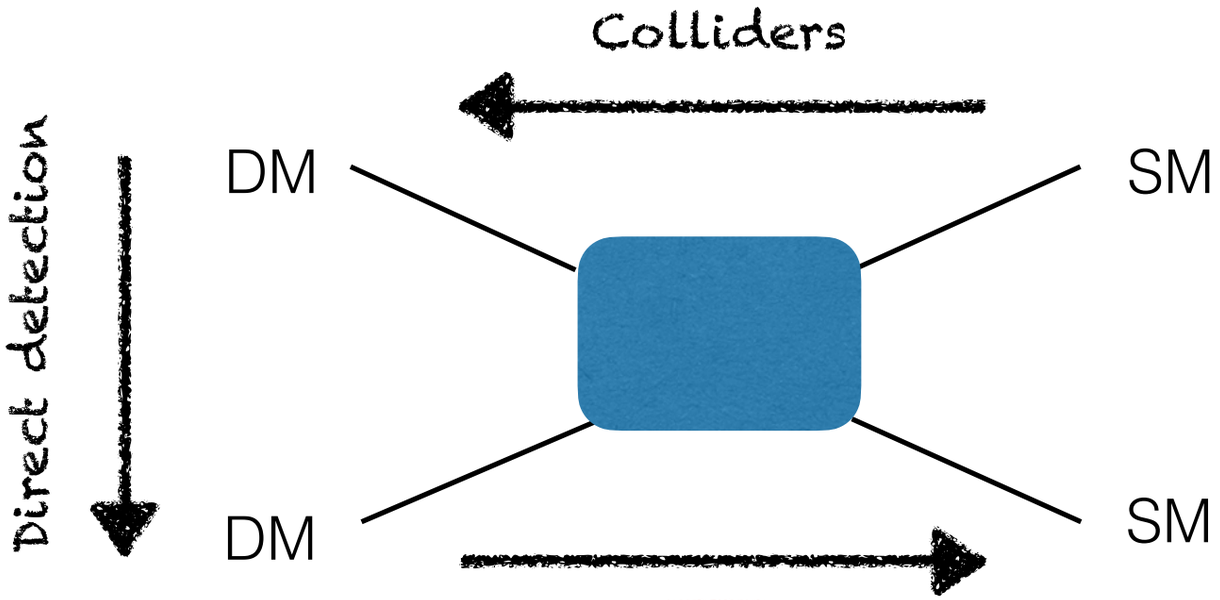


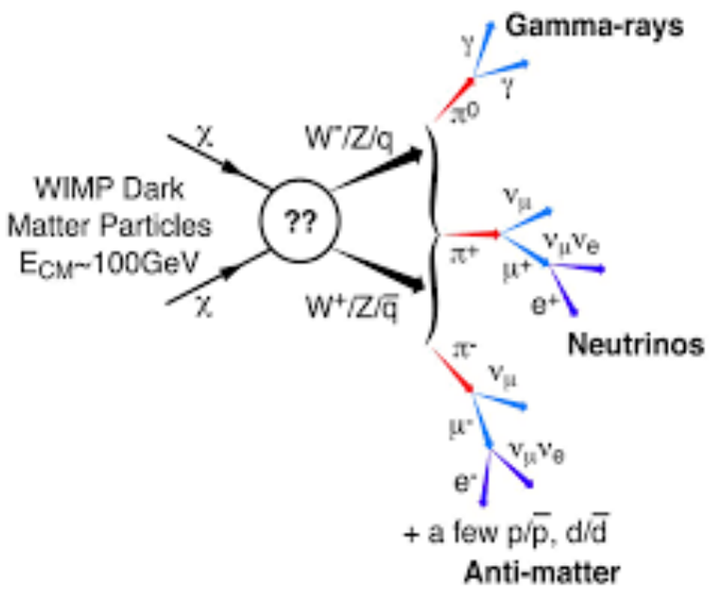


Sci. Adv. 7, eabk2699 (2021)

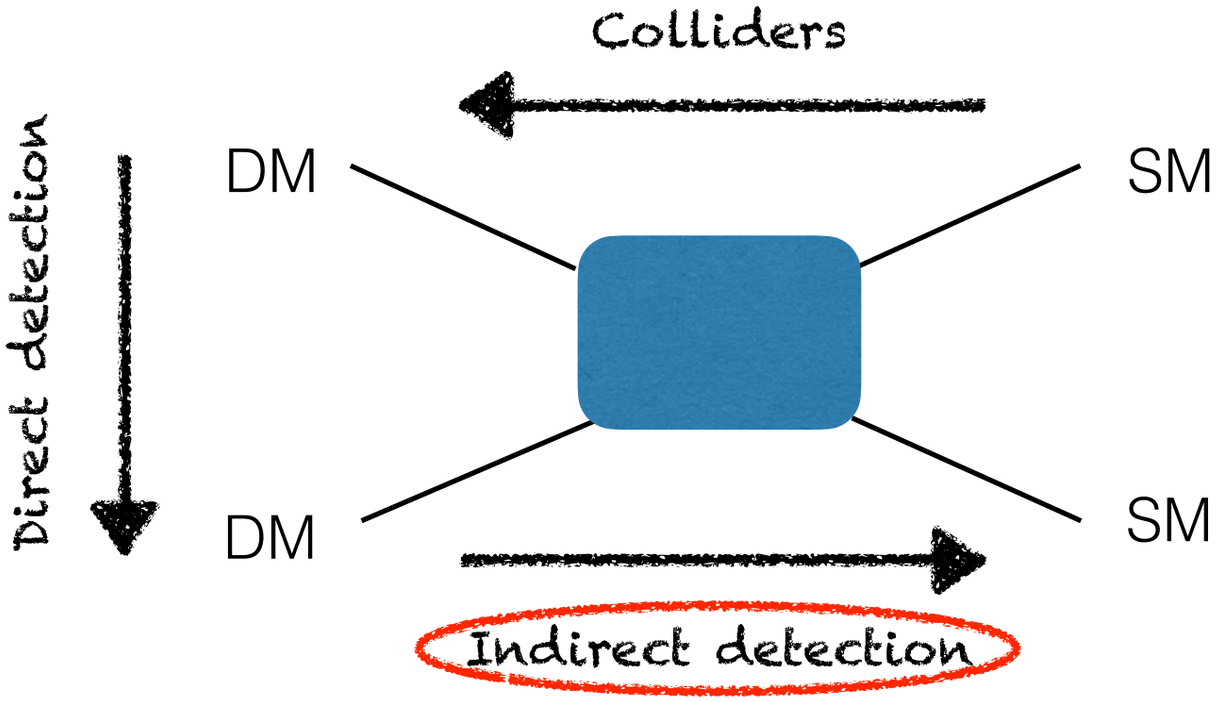
H.S.Lee, talk at PPC 2023

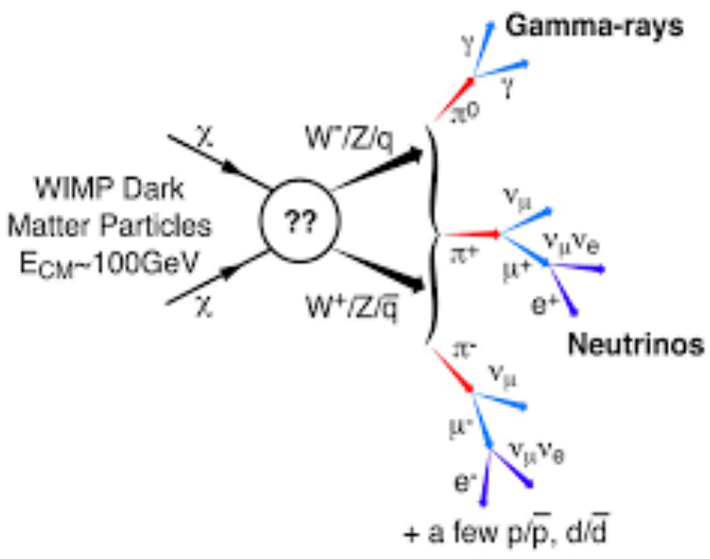
Indirect Detection of Dark Matter





Indirect Detection of Dark Matter





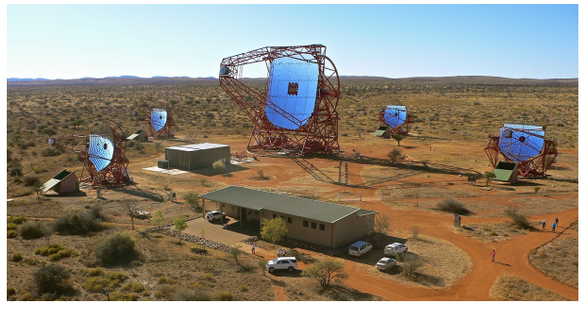
Fermi-LAT



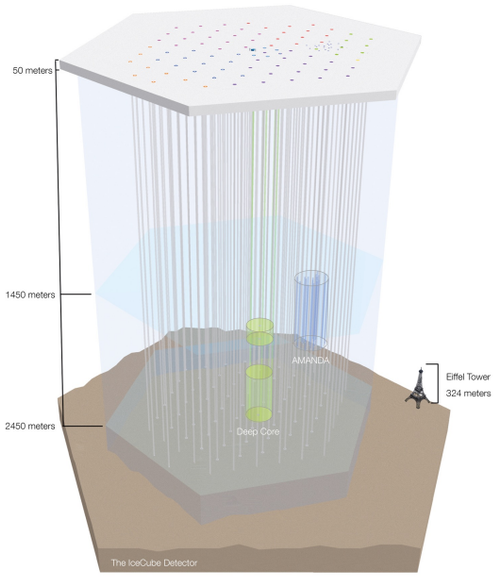
AMS-02



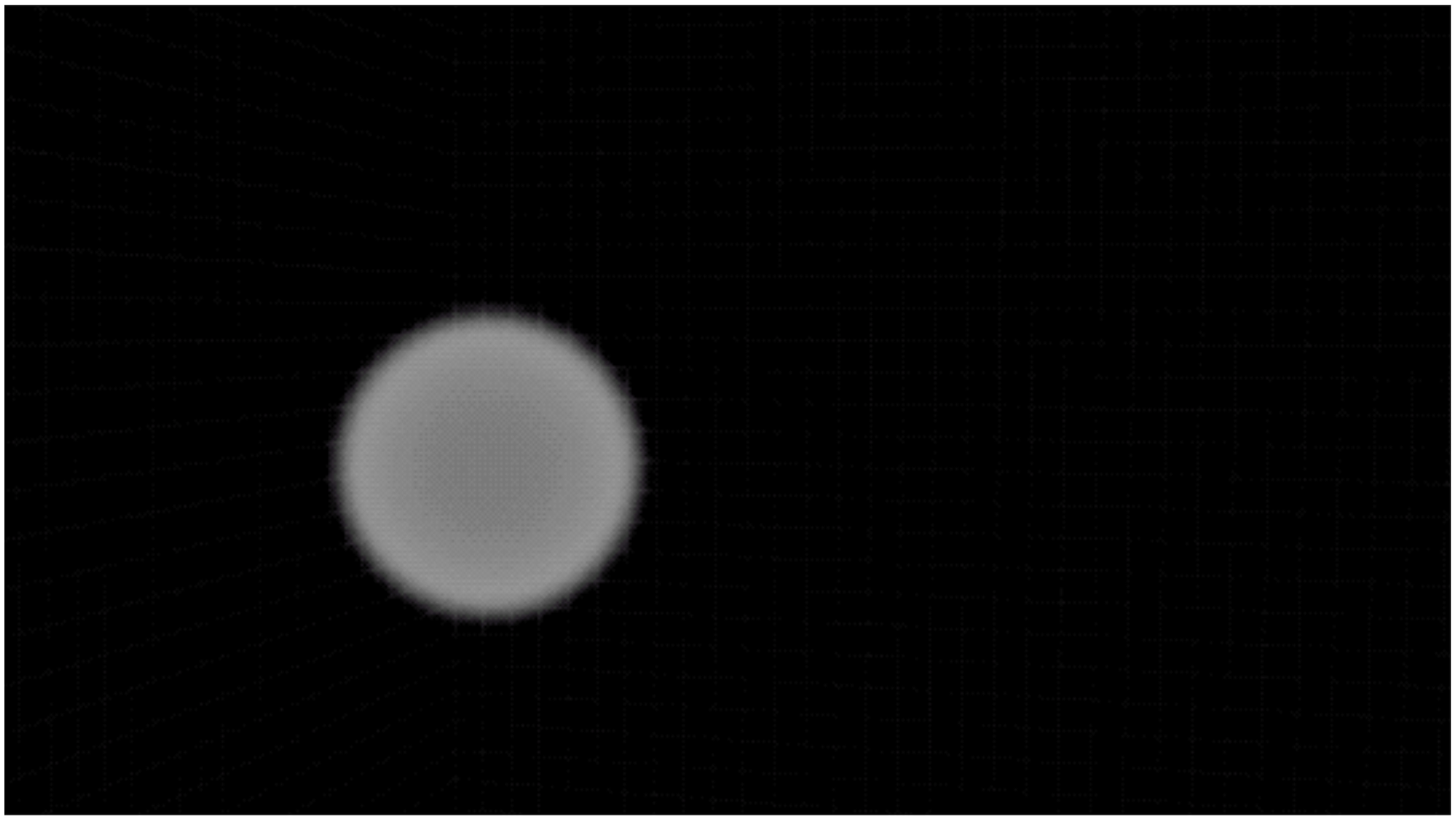
HESS



IceCube

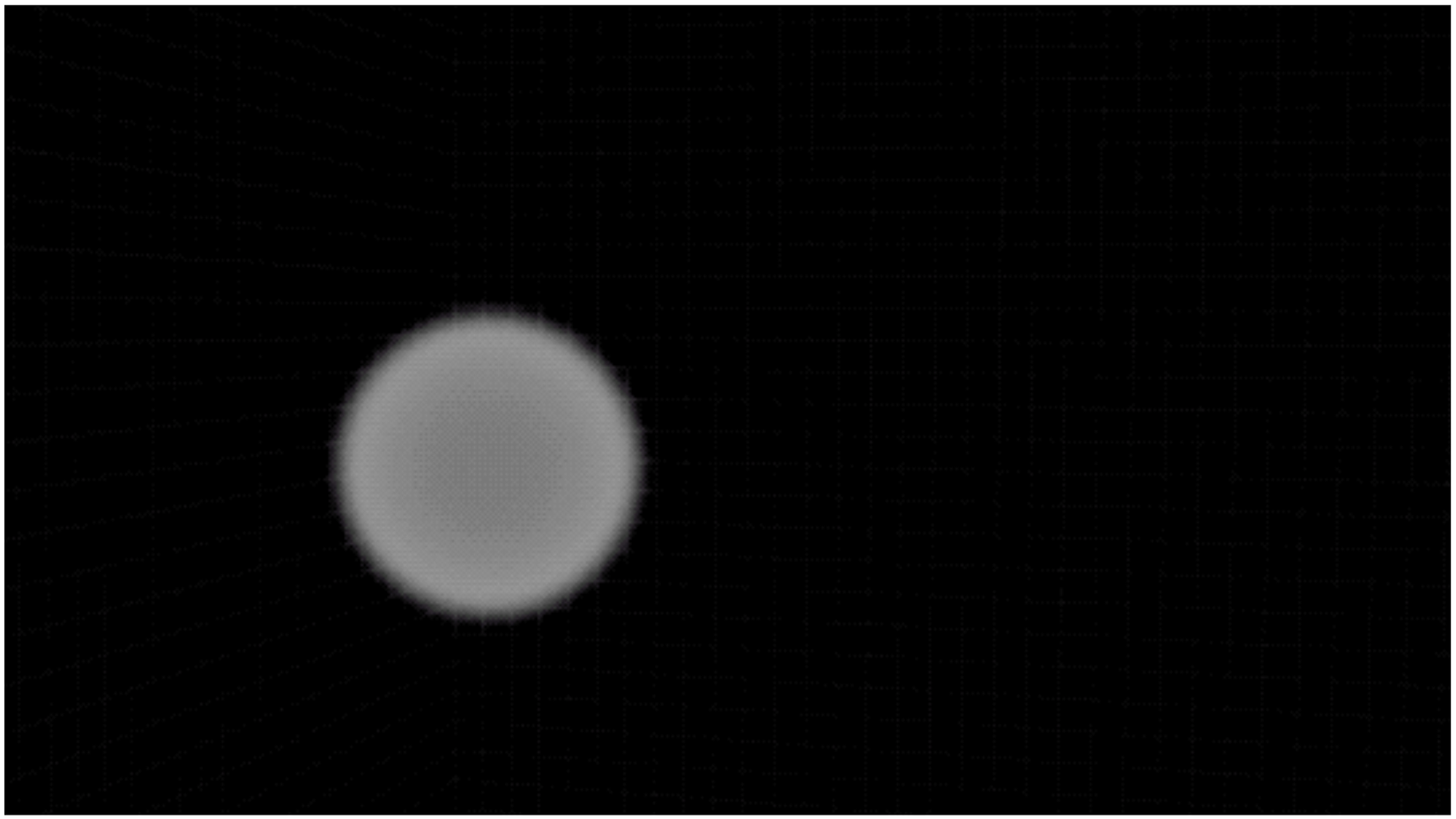


Indirect Detection of Dark Matter



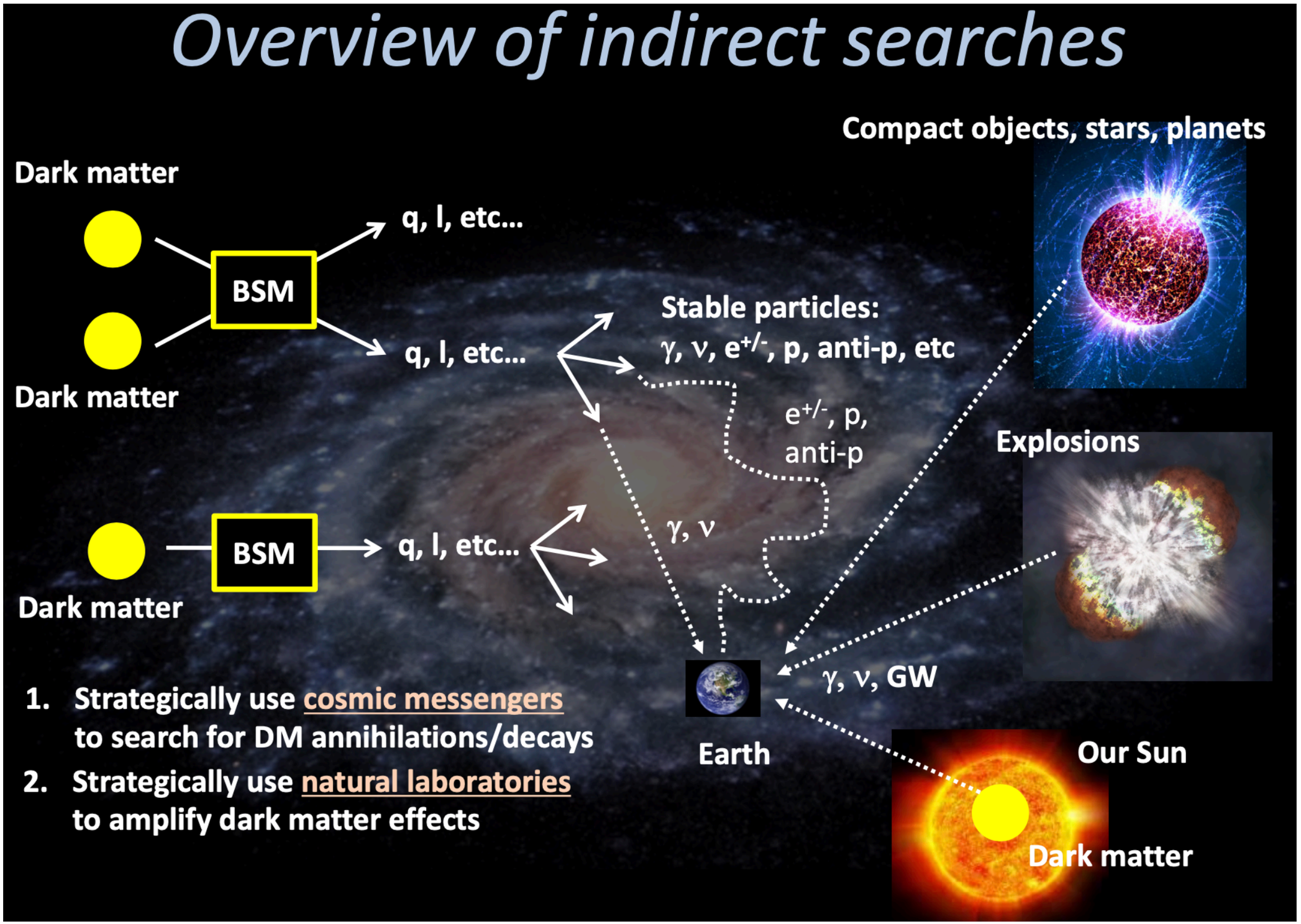
NASA, https://nasa.tumblr.com/post/180859237809/dark-matter-101-looking-for-the-missing-mass

Indirect Detection of Dark Matter



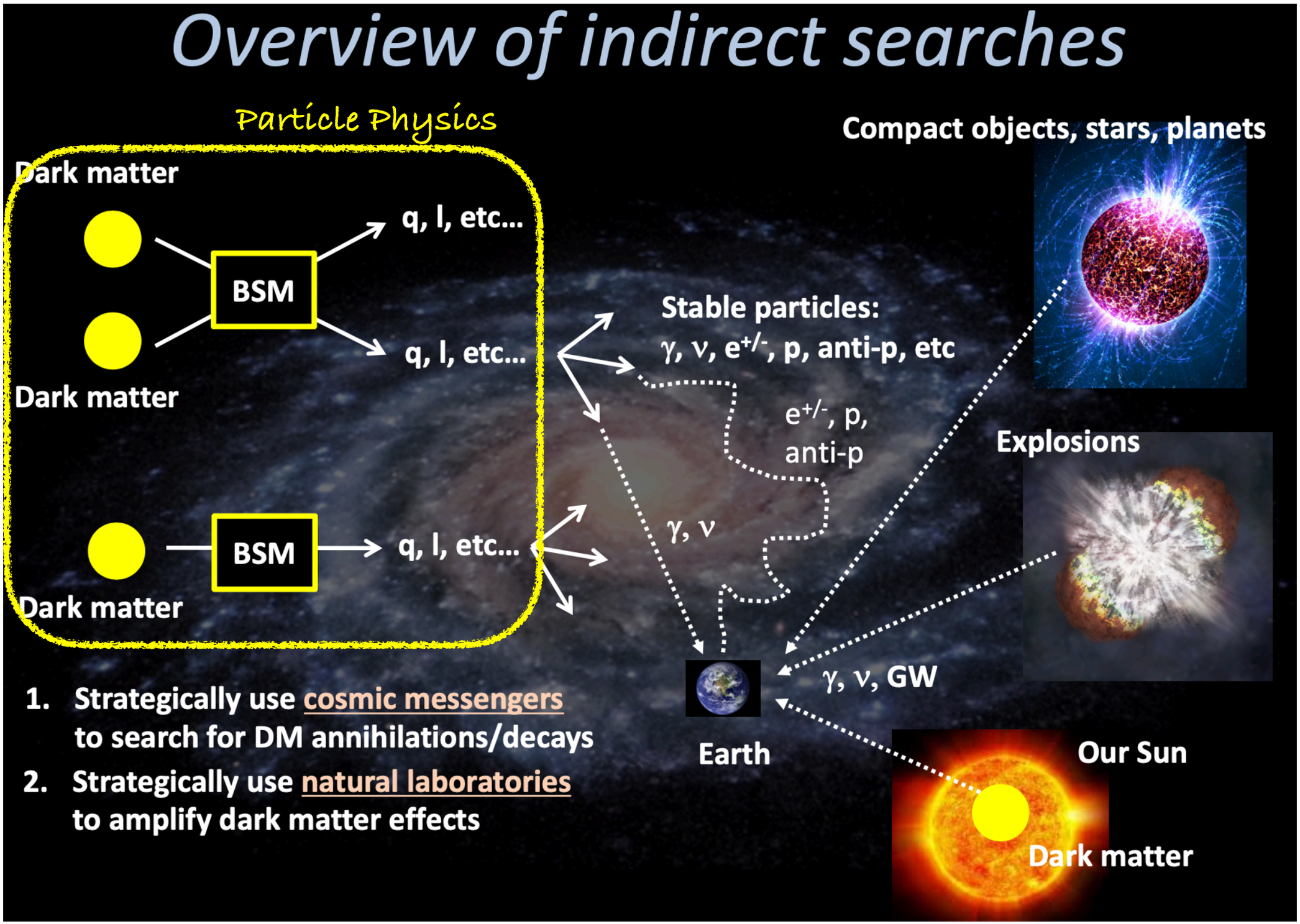
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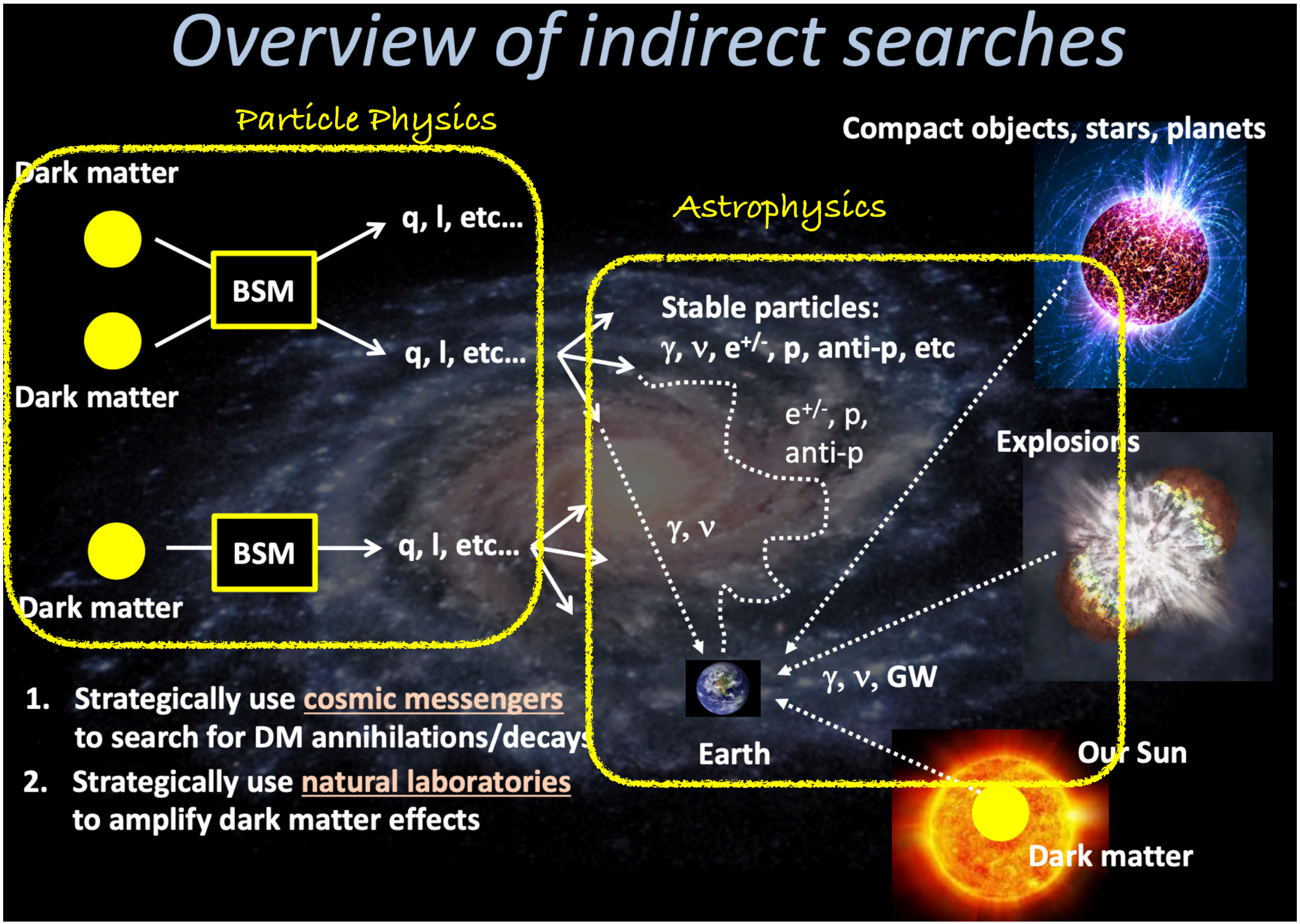
S. Horiuchi, talk at PPC 2023

Indirect Detection of Dark Matter



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Indirect Detection of Dark Matter

e.g., annihilation of Majorana DM to photon at the GC

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Spectrum of photons received at Earth/Volume/Time
where our telescope/detector has area A

Slatyer, 1710.05137

$$\frac{dN_\gamma}{dEdtdV} = \left(\frac{dN_\gamma}{dE}\right)_0 \frac{A}{4\pi r^2}\frac{n^2}{2}\langle\sigma v\rangle$$

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Prompt spectrum at the annihilation point (r from the Earth)

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Integrating over the line of sight & solid angle subtended by the object

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N-body & observations

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e.g., annihilation of Majorana DM to photon at the GC + far away?

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redshift

Prof. K.-Y. Choi's lecture

Prompt spectrum at the annihilation point (r from the Earth)

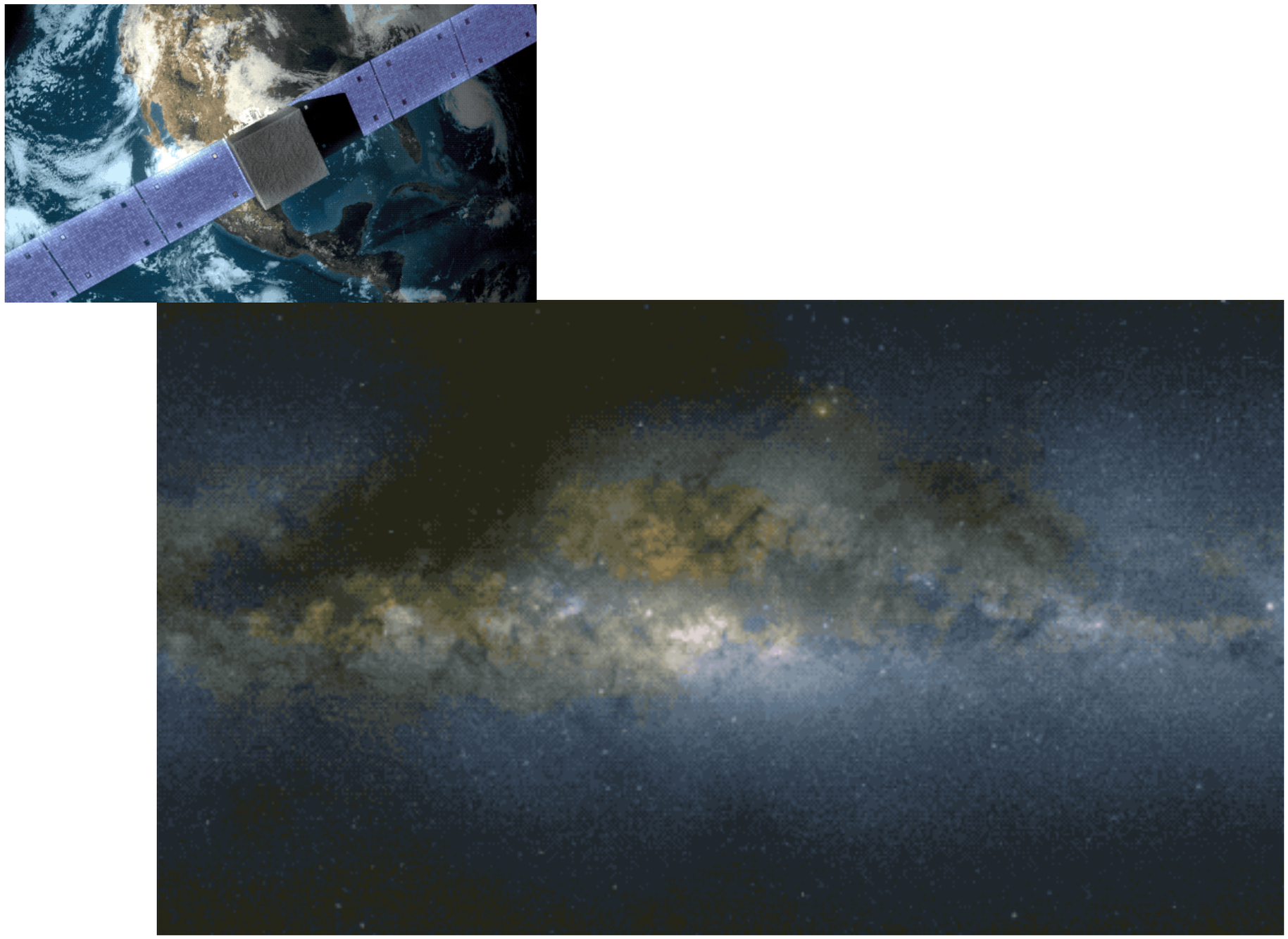
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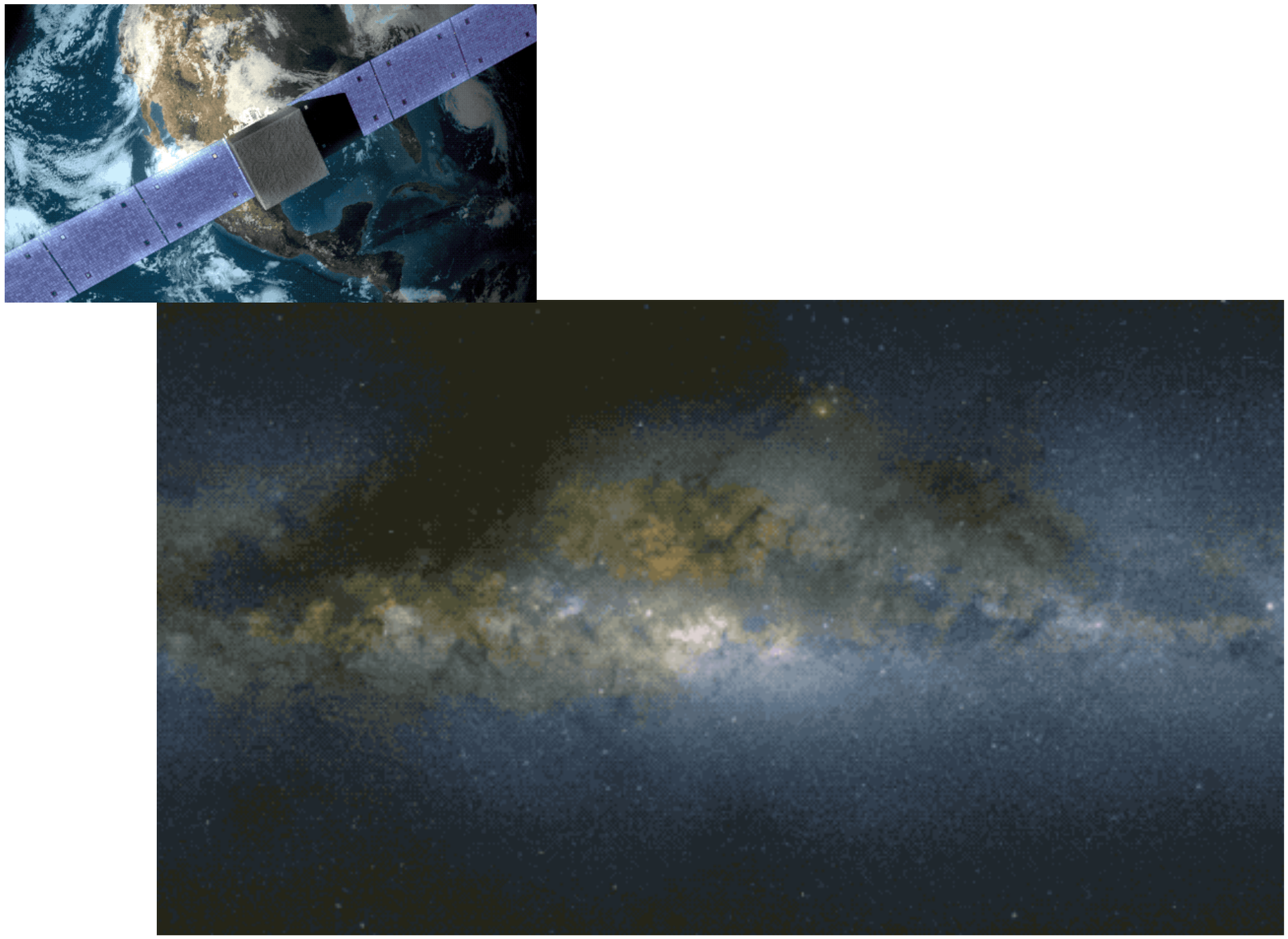
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Indirect Detection of Dark Matter



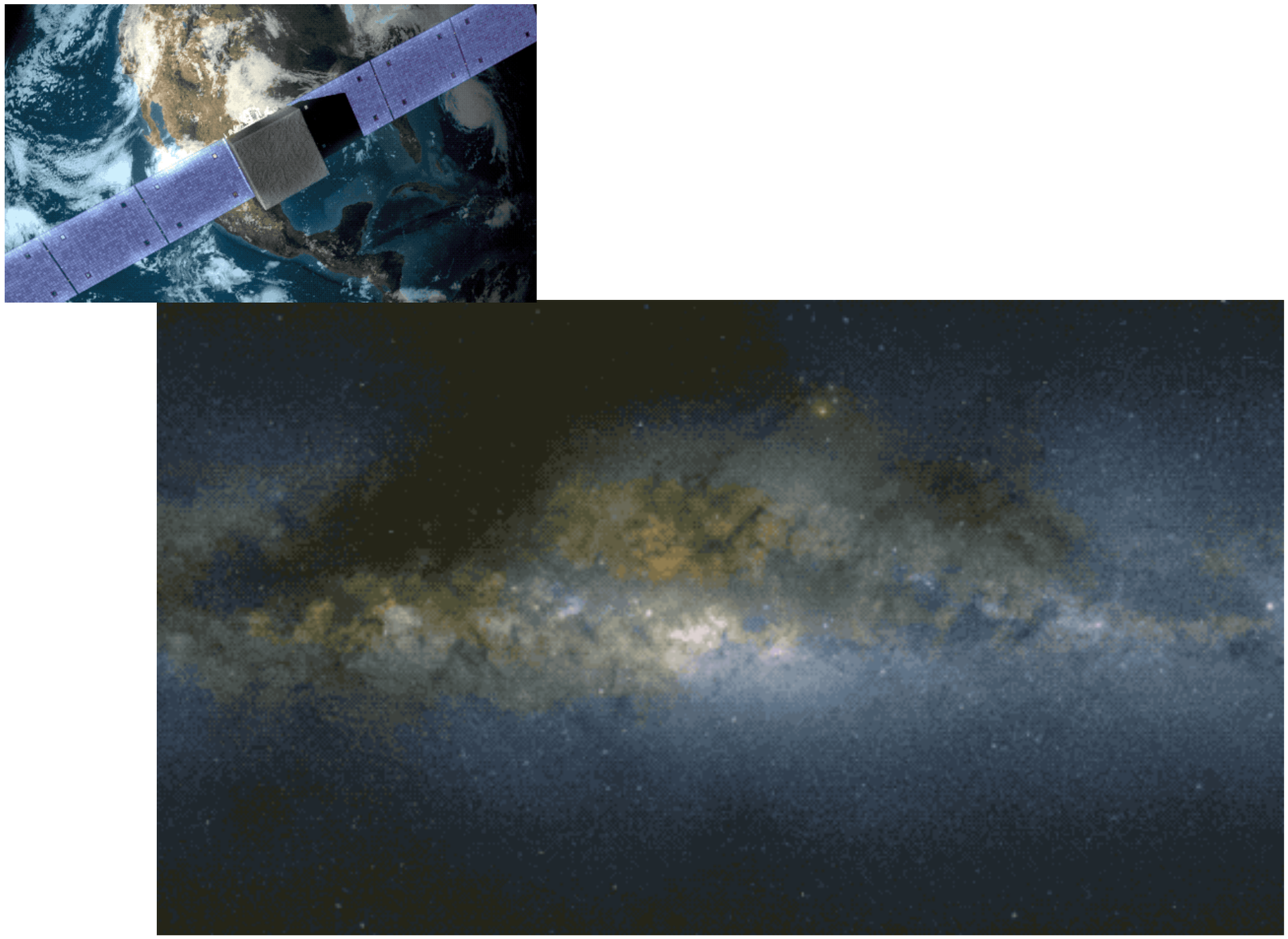
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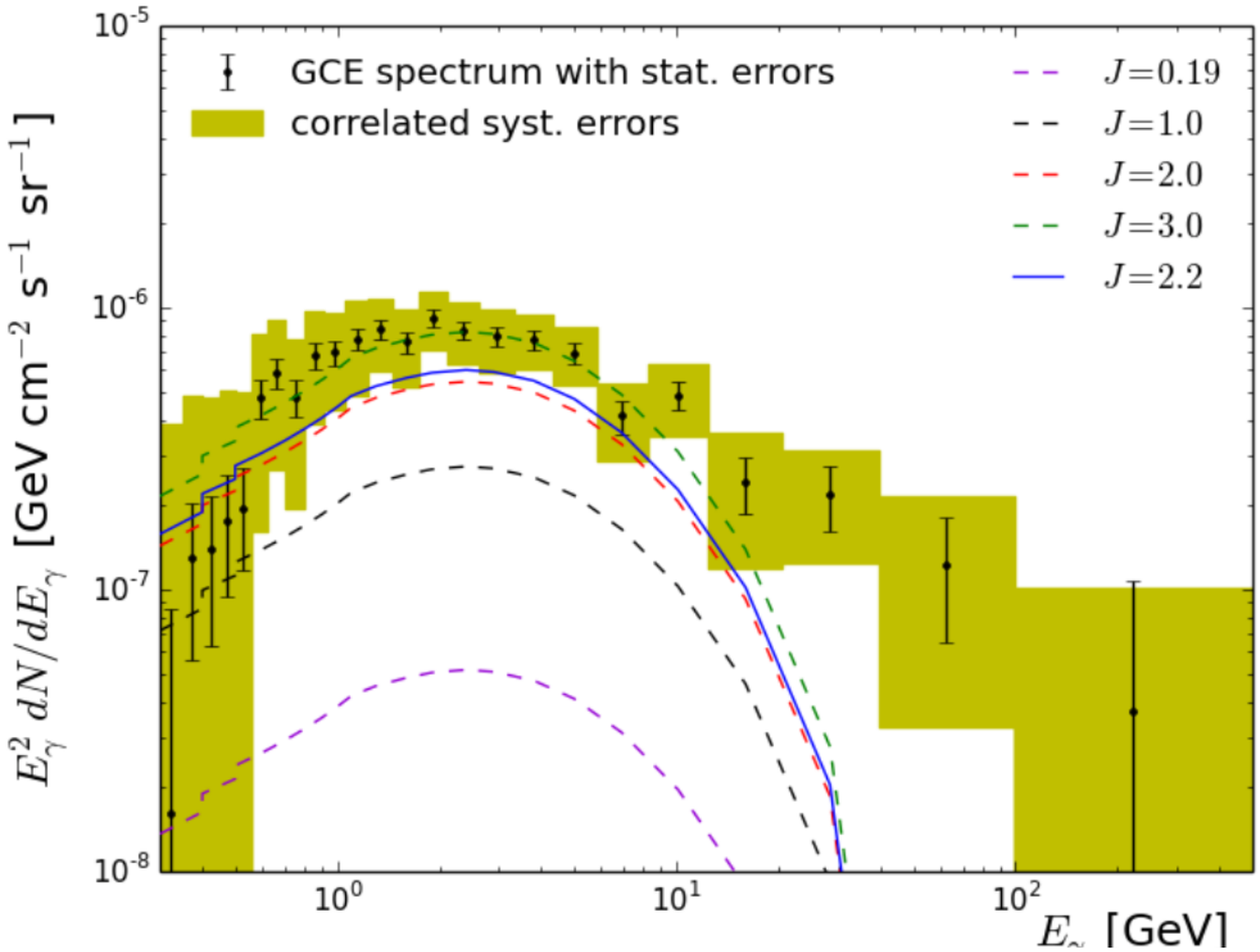
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Indirect Detection of Dark Matter



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Indirect Detection of Dark Matter



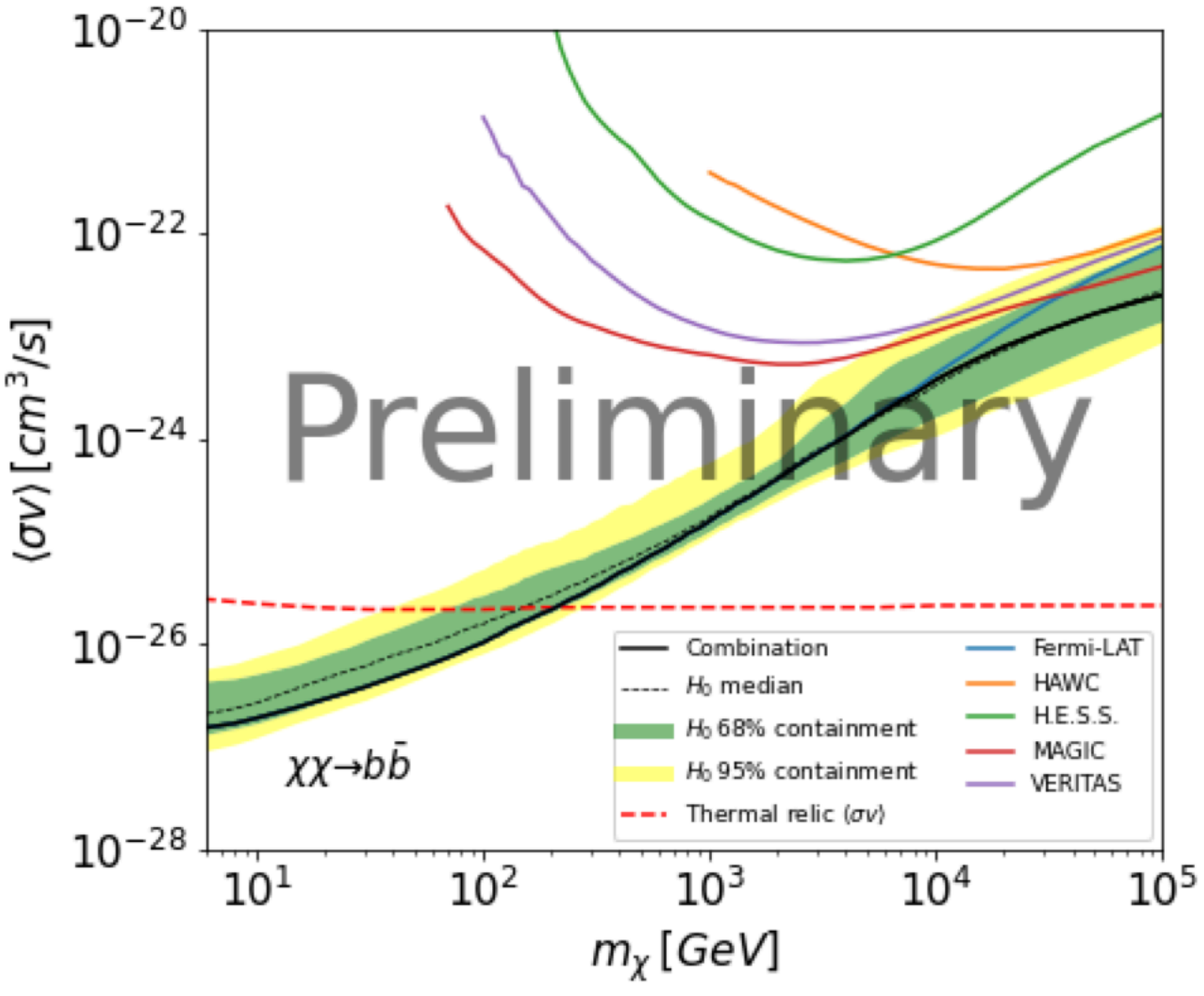
PHYSICAL REVIEW D **93,** 075023 (2016)

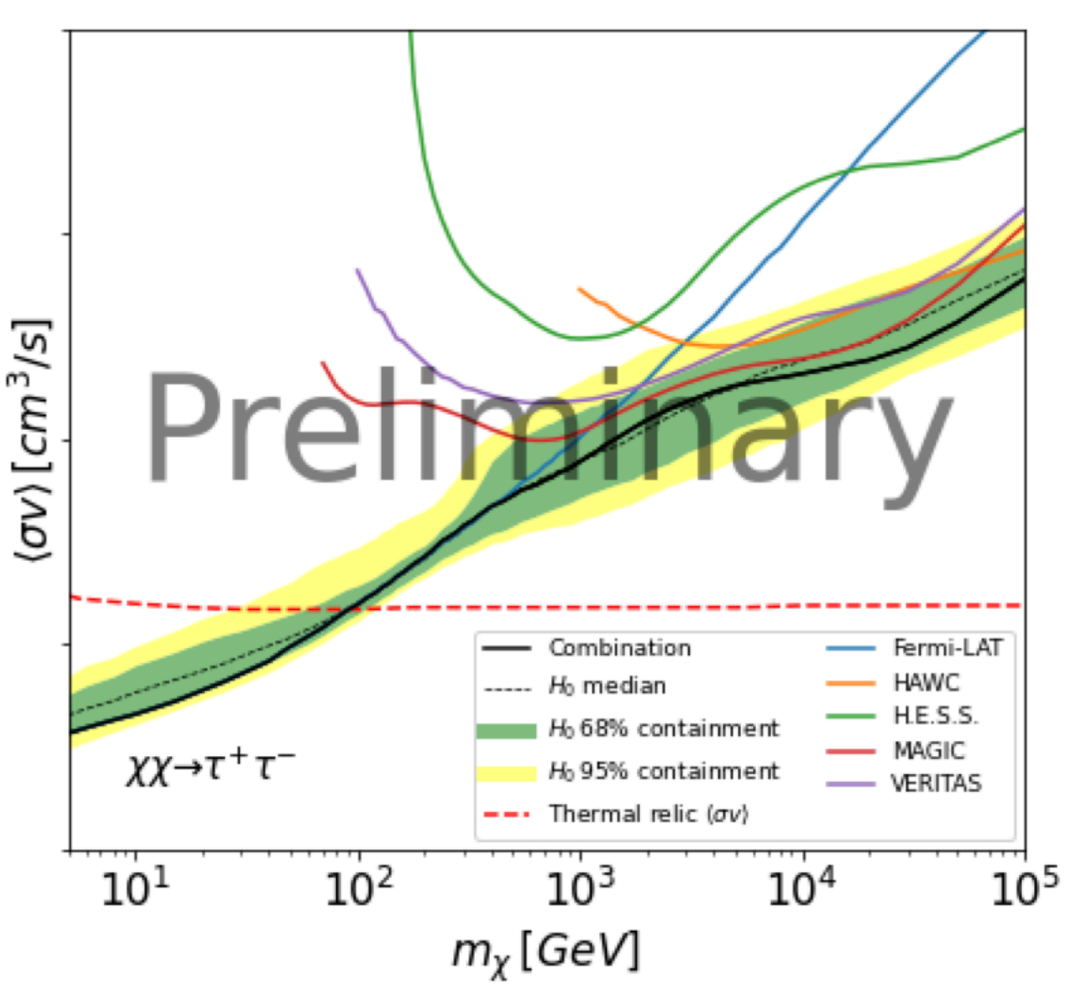
Secluded singlet fermionic dark matter driven by the Fermi gamma-ray excess

Yeong Gyun Kim,[1,2,*] Kang Young Lee,[3,†] Chan Beom Park,[2,‡] and Seodong Shin[4,§]

Indirect Detection of Dark Matter

e.g., photon from dSphs





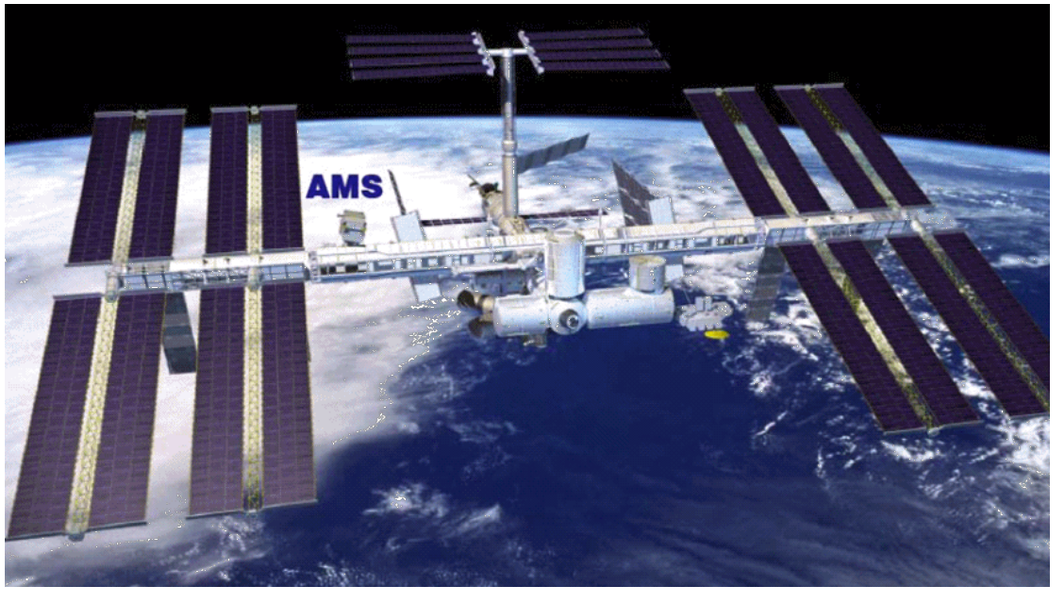
2108.13646

Hess, HAWK, VERITAS, H.E.S.S., Fermi-LAT

Indirect Detection of Dark Matter

Anti-proton excess

ETH Zurich



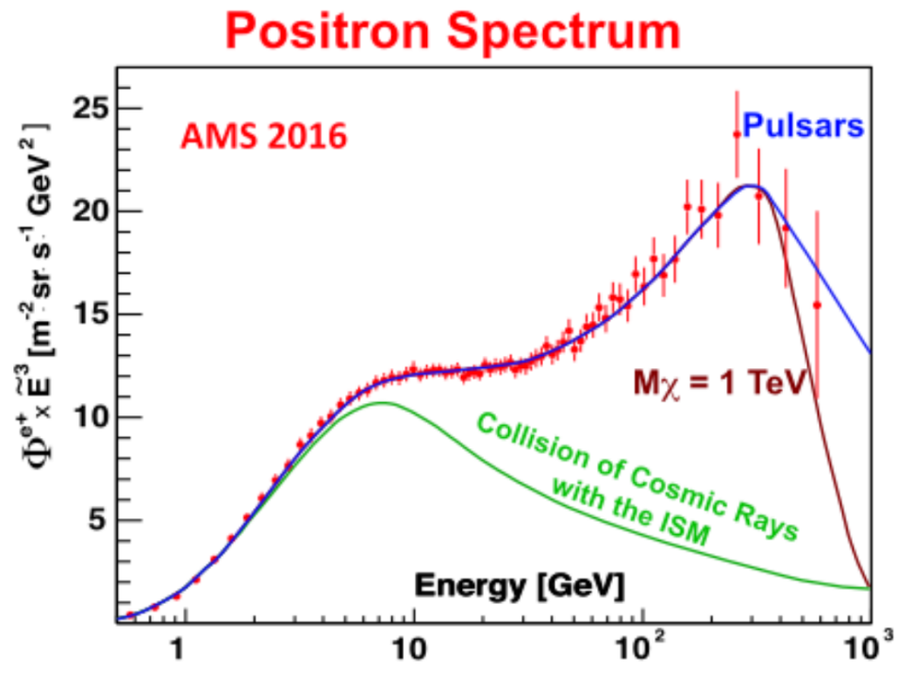
JHEP

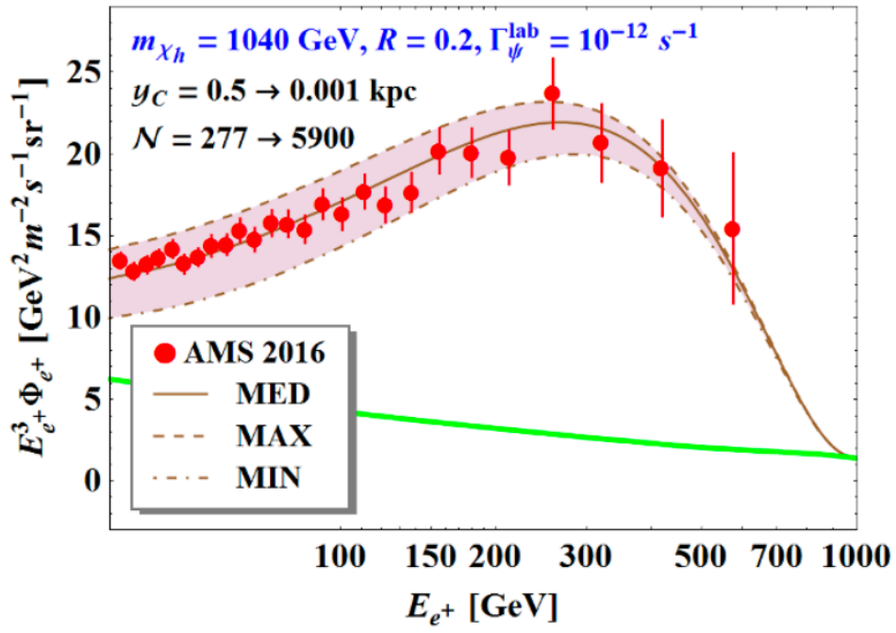
Published for SISSA by Springer

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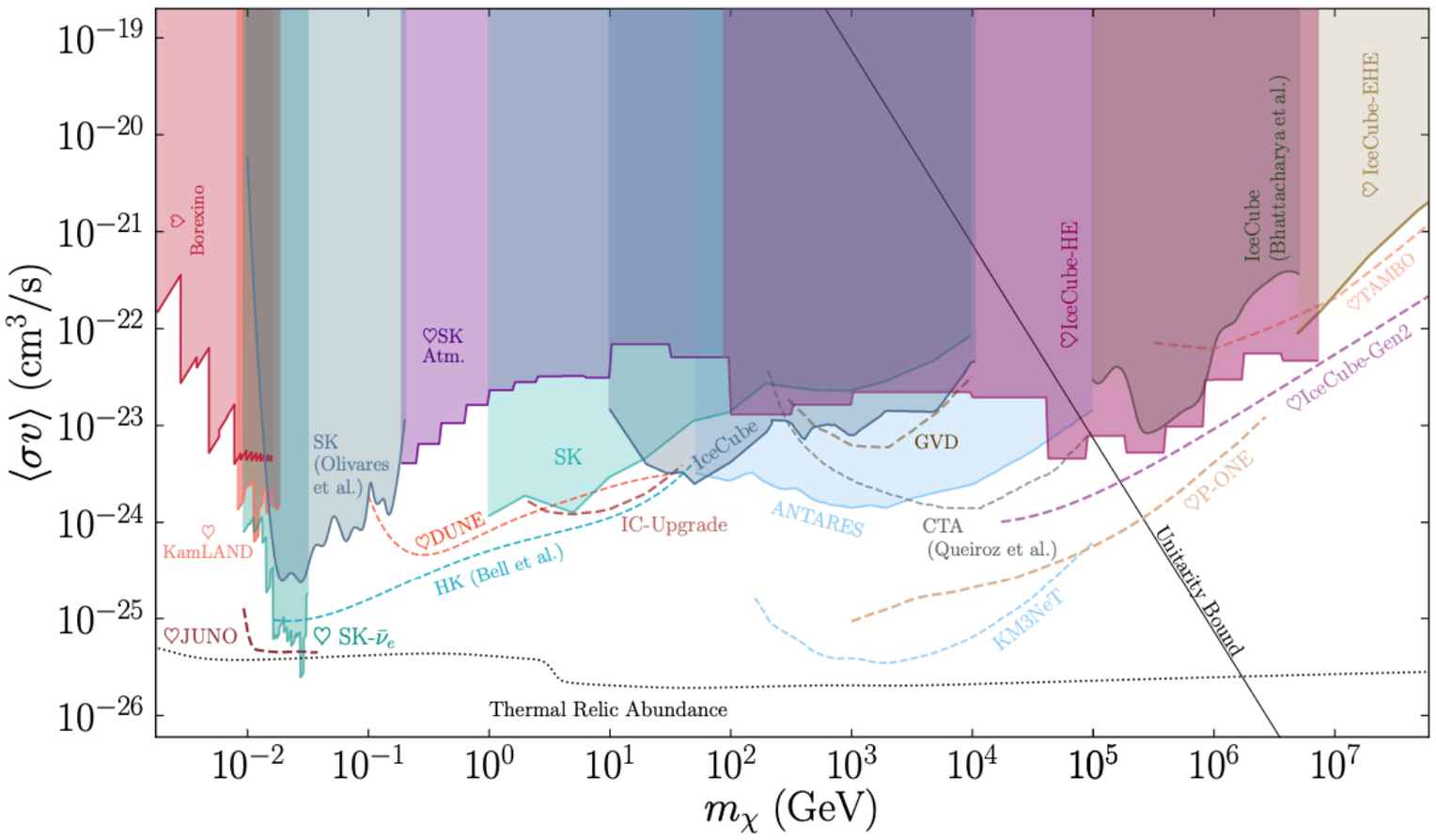
Dark matter "transporting" mechanism explaining positron excesses

Doojin Kim,[a] Jong-Chul Park[b,1] and Seodong Shin[c,d,1]





Indirect Detection of Dark Matter

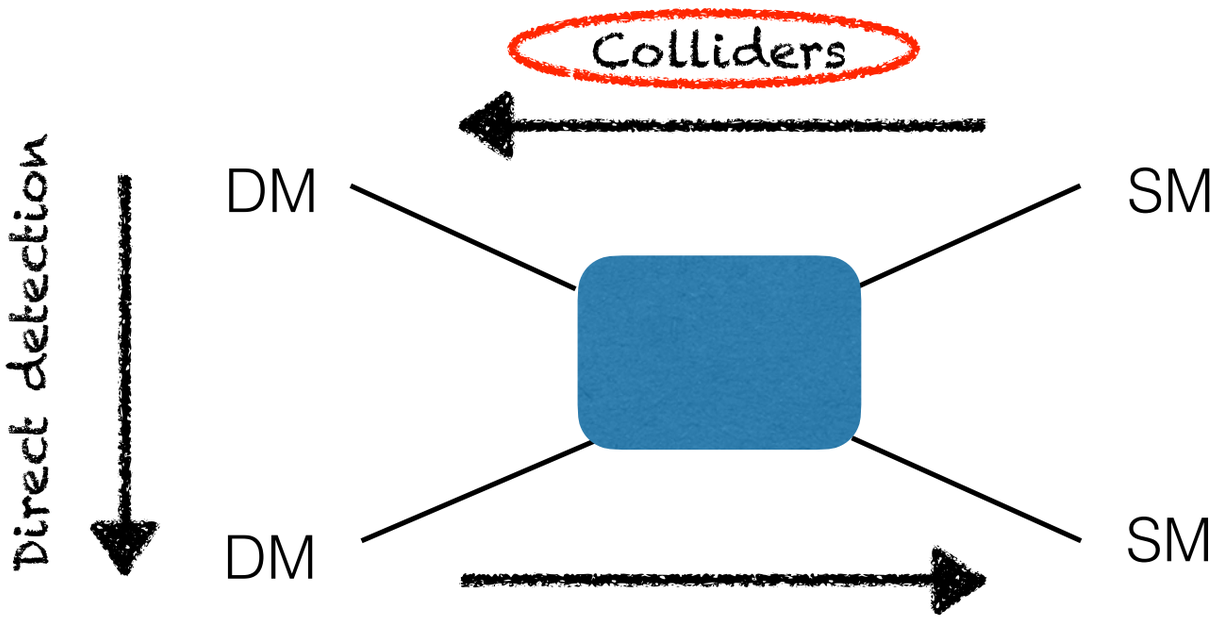


Arguelles et al., 1912.09486

- 90%CL limits and future sensitivities (5 yrs for neutrino experiments and 100 hours for CTA) for s-wave $\chi\chi \to \nu\nu$ where χ is Majorana

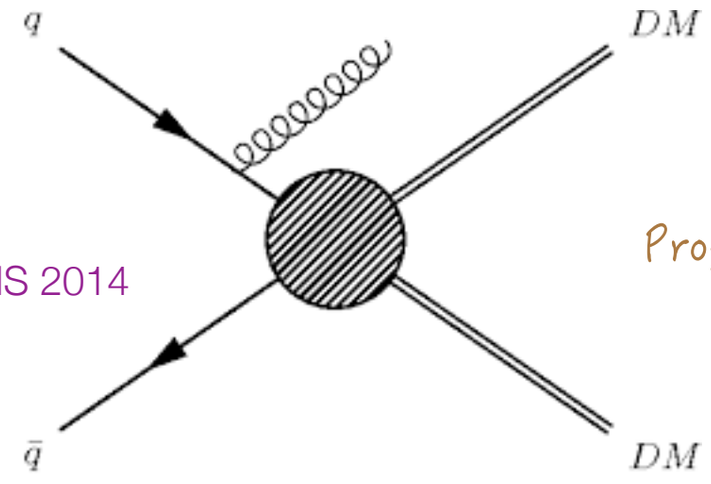
- Contributions both from Galactic Center and Extragalactic sources

Collider search of Dark Matter



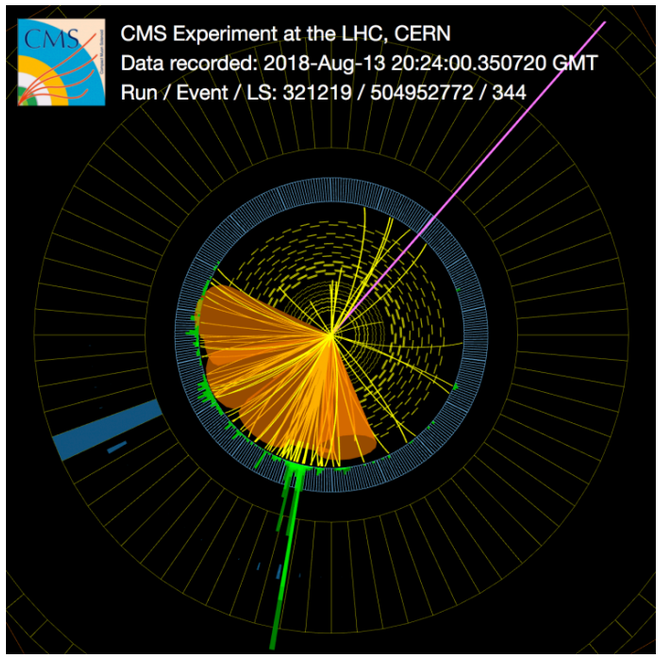


Indirect detection



CMS 2014

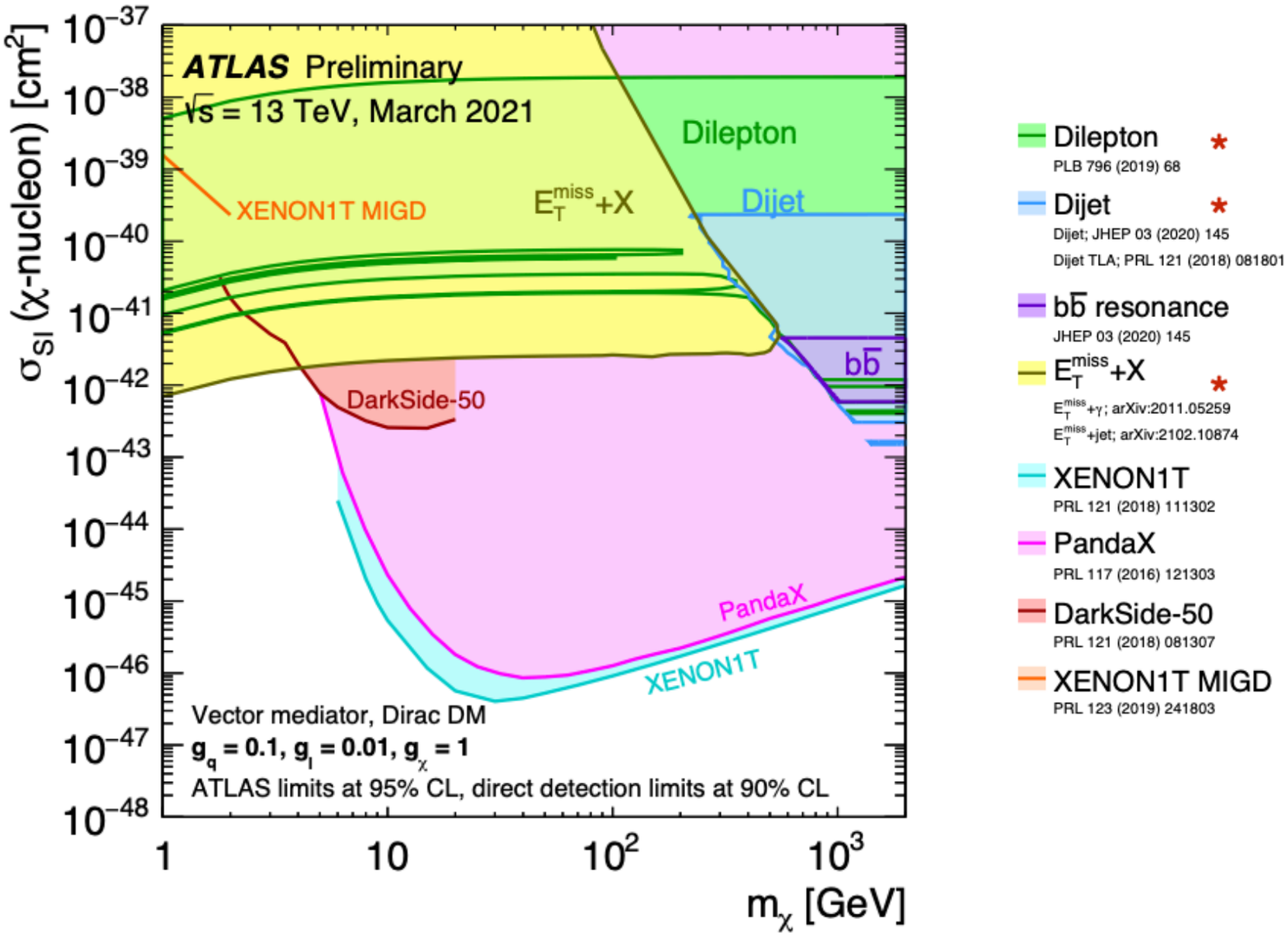
Prof. C.B. Park's lecture



Missing energy

Mono-jet, Mono-Higgs, …

Collider search of Dark Matter



ATL-PHYS-SLIDE-2021-528

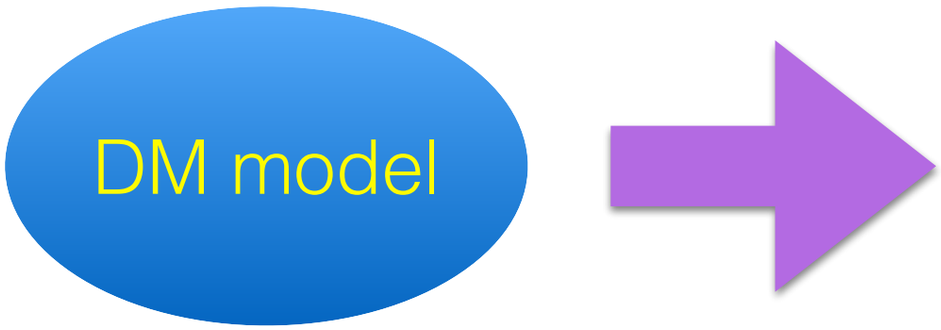
How to analyze the excesses or constraints?

Submitted to the Proceedings of the US Community Study
on the Future of Particle Physics (Snowmass 2021)

Snowmass2021 Cosmic Frontier White Paper: Puzzling Excesses in Dark Matter Searches and How to Resolve Them

Rebecca K. Leane[*1,2], Seodong Shin[†3], Liang Yang[‡4], Govinda Adhikari[4], Haider Alhazmi[5], Tsuguo Aramaki[6], Daniel Baxter[7], Francesca Calore[8], Regina Caputo[9], Ilias Cholis[10], Tansu Daylan[11,12], Mattia Di Mauro[13], Philip von Doetinchem[14], Ke Han[15], Dan Hooper[16,17,18], Shunsaku Horiuchi[19,20], Doojin Kim[21], Kyoungchul Kong[22], Rafael F. Lang[23], Qing Lin[24,25], Tim Linden[26], Jianglai Liu[15,27,28], Oscar Macias[29], Siddharth Mishra-Sharma[30,31,32], Alexander Murphy[33], Meshkat Rajaee[3], Nicholas L. Rodd[34], Aditya Parikh[31], Jong-Chul Park[35], Maria Luisa Sarsa[36], Evan Shockley[18], Tracy R. Slatyer[32], Volodymyr Takhistov[20], Felix Wagner[37], Jingqiang Ye[38], Gabrijela Zaharijas[39], Yi-Ming Zhong[18], Ning Zhou[15], and Xiaopeng Zhou[40]

DM phenomenology



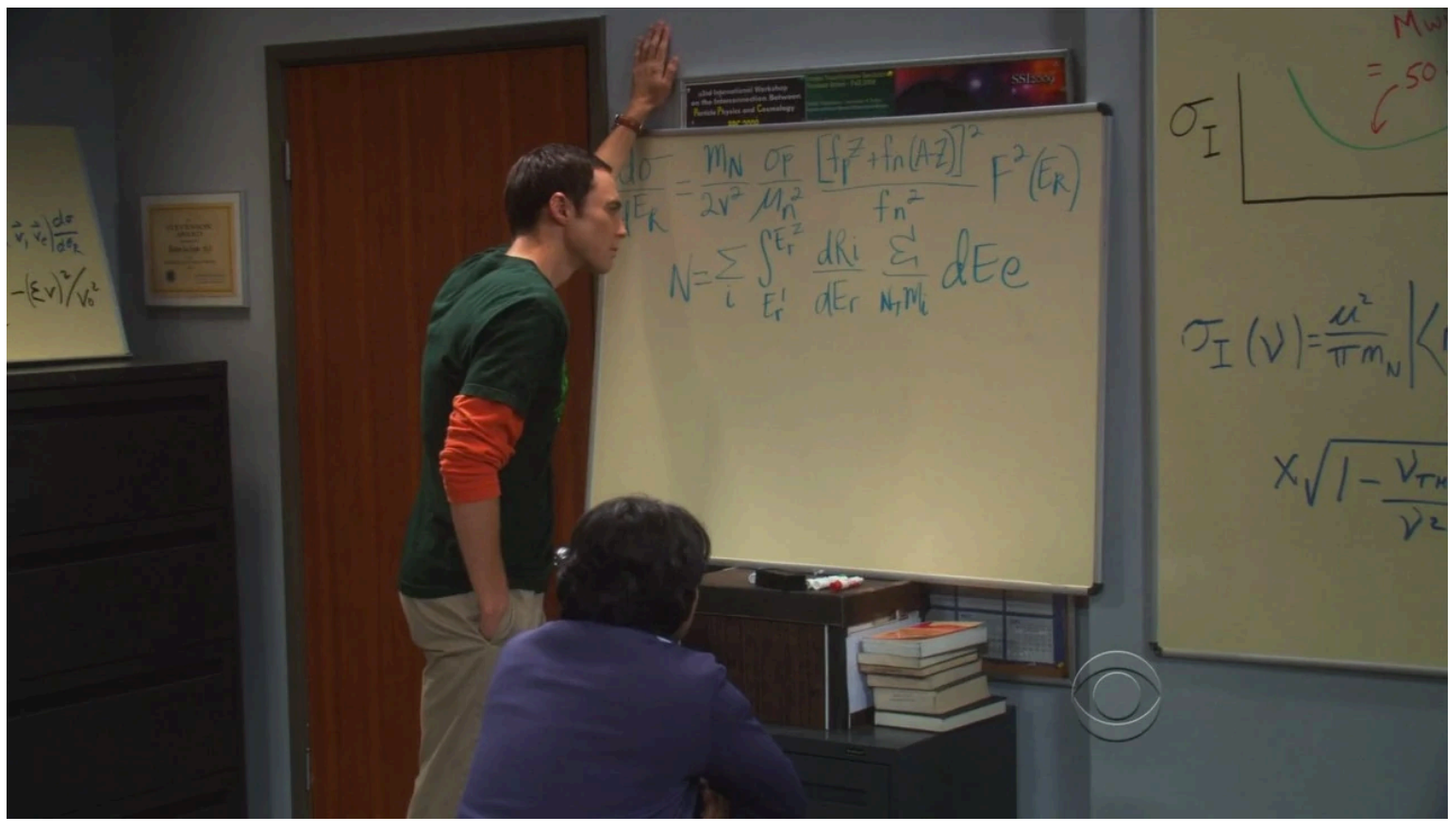
- Relic density? (interaction with SM)
- Direct detection?
- Indirect detection?
- Collider searches?

DM phenomenology

DM model

- Relic density? (interaction with SM)
- Direct detection?
- Indirect detection?
- Collider searches?

Calculate by yourself!!

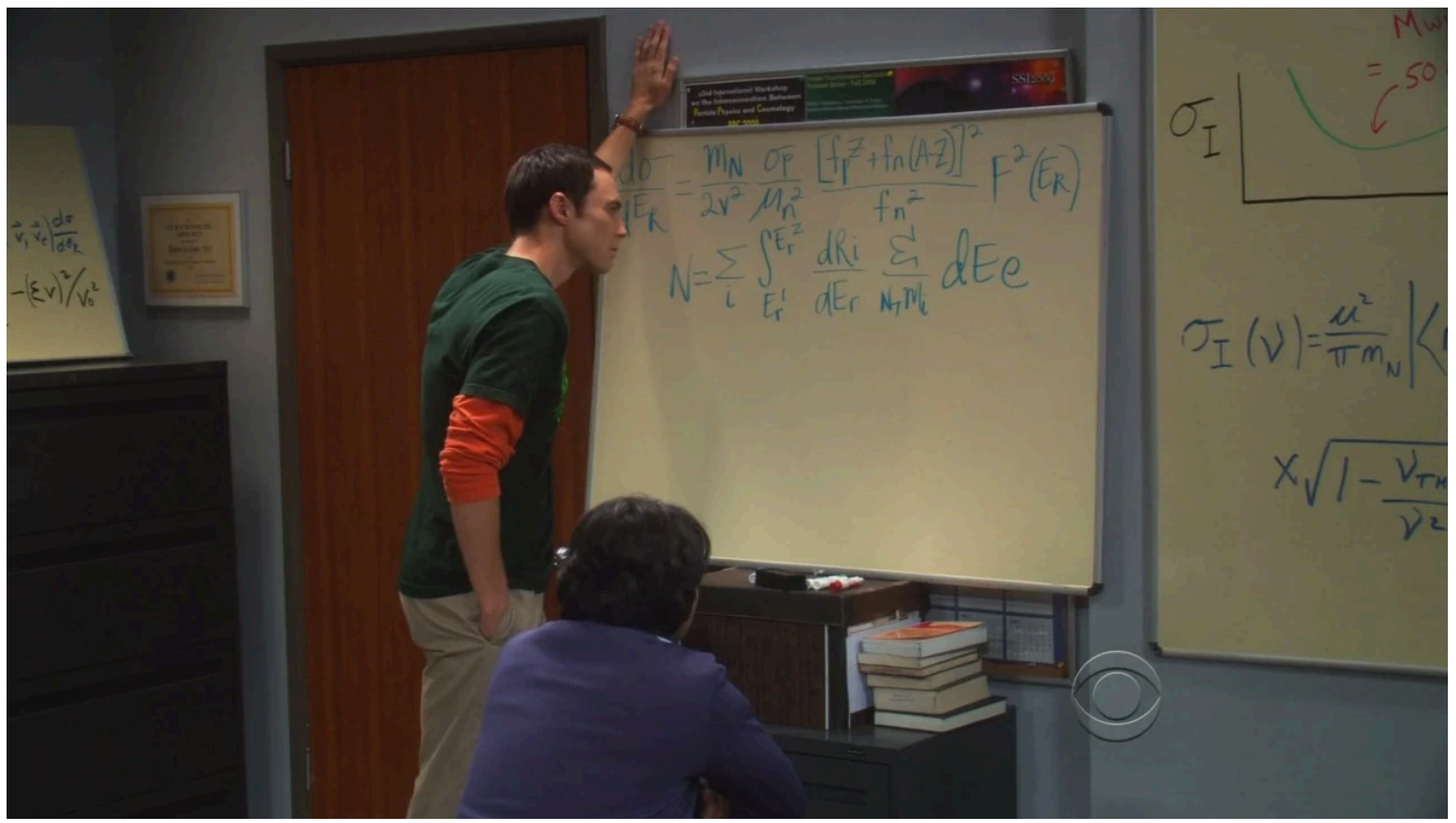


DM phenomenology

DM model

- Relic density? (interaction with SM)
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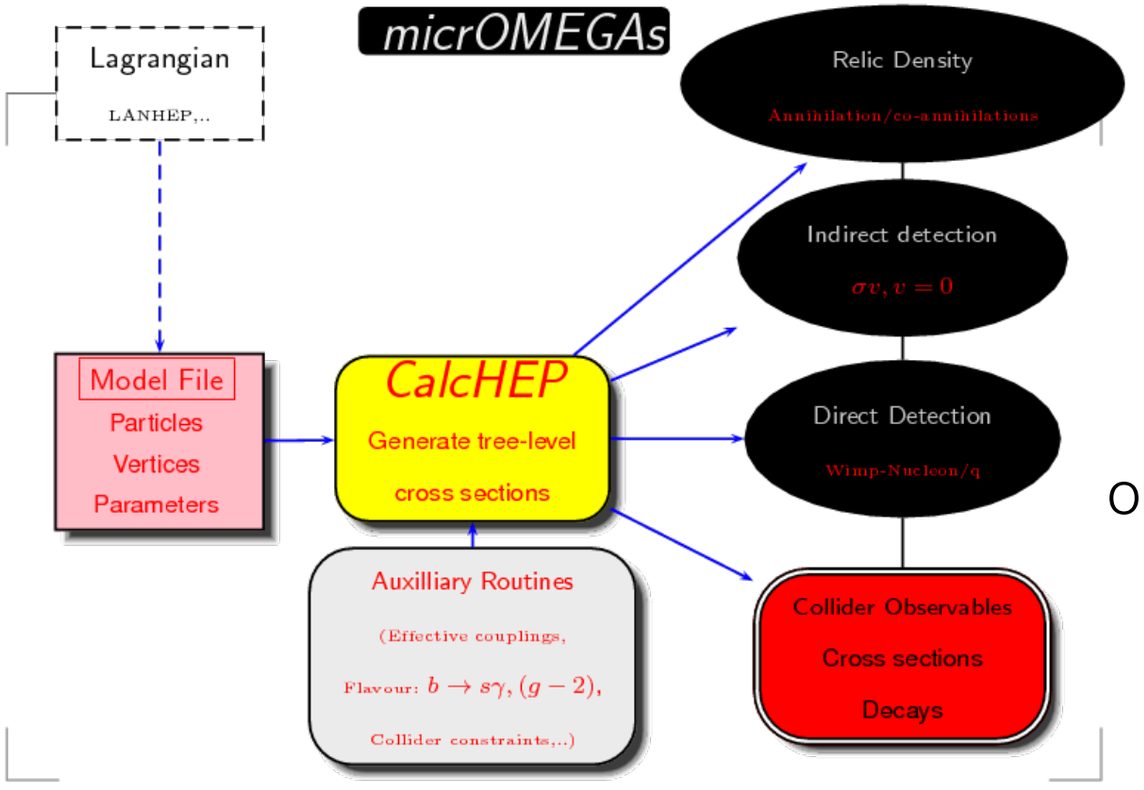
Calculate by yourself!!





DM phenomenology

Example: micrOmegas



https://lapth.cnrs.fr/micromegas/

or google “micrOmegas dark matter”

• `darkOmega(&Xf,fast,Beps,&err)`
calculates the dark matter relic density Ωh^2. This routine solves the differential evolution equation using the Runge-Kutta method. $X_f = Mcdm/T_f$ characterizes the freeze-out temperature which is defined by the condition $Y(T_f) = 2.5Y_{eq}(T_f)$. For asymmetric

`darkOmega` solves the differential equation for the abundance $Y(T)$ in the temperature interval `[Tend,Tstart]` defined by the conditions $Y(T_{start}) \approx 1.1Y_{eq}(T_{start})$, $Y(T_{end}) \approx 10Y_{eq}(T_{end})$. For temperatures below `Tend`, the contribution for Y_{eq} is neglected and the differential equation is integrated explicitely. The solution in the interval `[Tend,Tstart]`

WIMP candidates

- Neutralino in SUSY models
- Lightest odd-parity new particle in other models as well
- Simplified models: SM + DM + Mediator



thermal bath

WIMP candidates

- Neutralino in SUSY models
- Lightest odd-parity new particle in other models as well
- Simplified models: SM + DM + Mediator



thermal bath

- Higgs boson (new scalar): Higgs (scalar) portal
- Weak or new gauge boson: Vector portal
- Fermions (like neutrino): Fermionic portal

Example

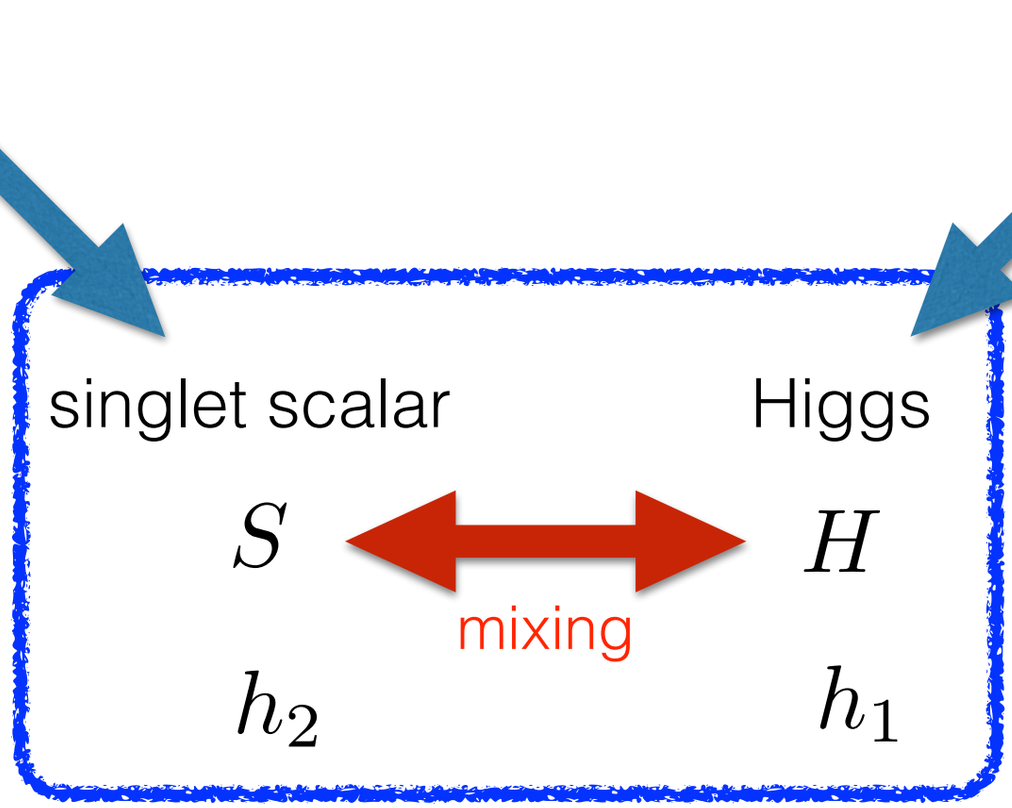
Singlet Fermionic Dark Matter

DM: singlet Dirac fermion

ψ

SM particles

$f, W, Z, \cdots$



Y.G. Kim, K.Y. Lee, **SS**, JHEP 0805, 100 [arXiv:0803.2932]

A renormalizable Higgs portal WIMP model

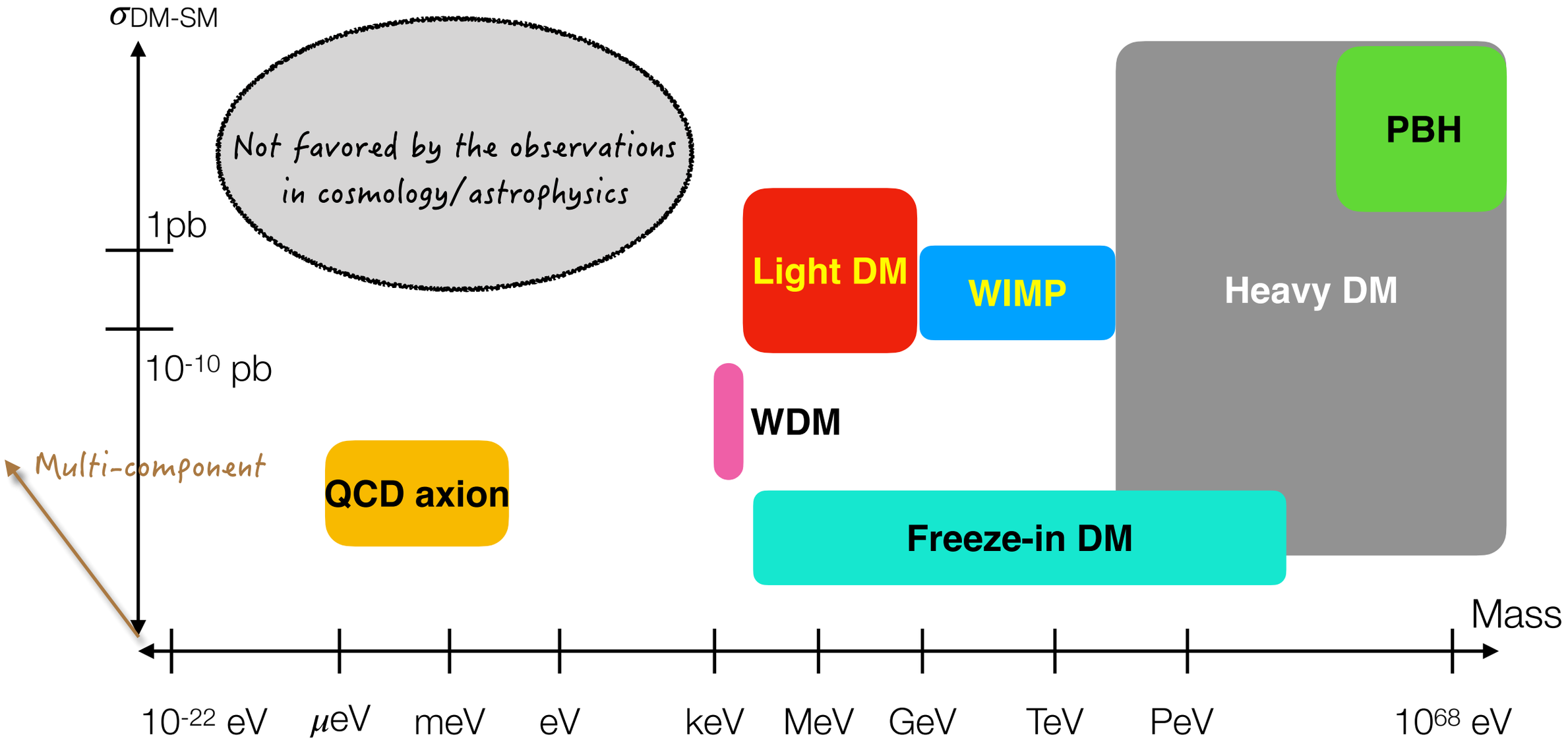
Future Direction of Dark Matter Research

- New theoretical ideas
- New experimental ideas

SM is very precise….

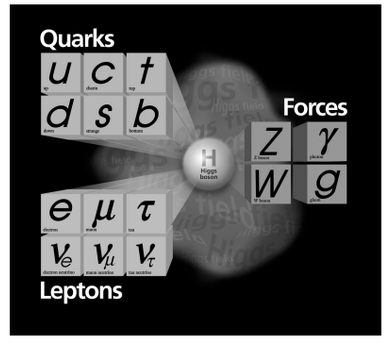


New Dark World beyond WIMP



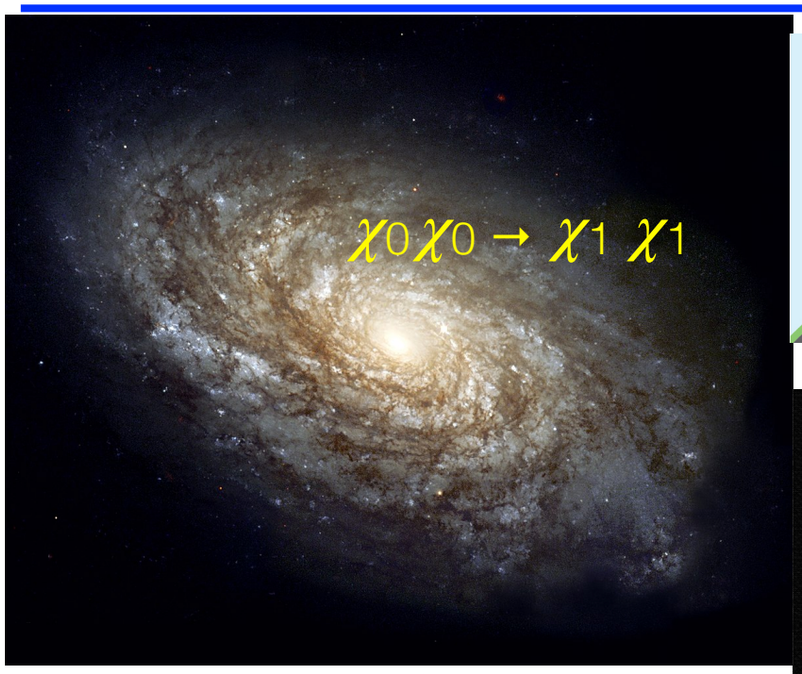
- Dark sector: multiple species of particles? Symmetries?

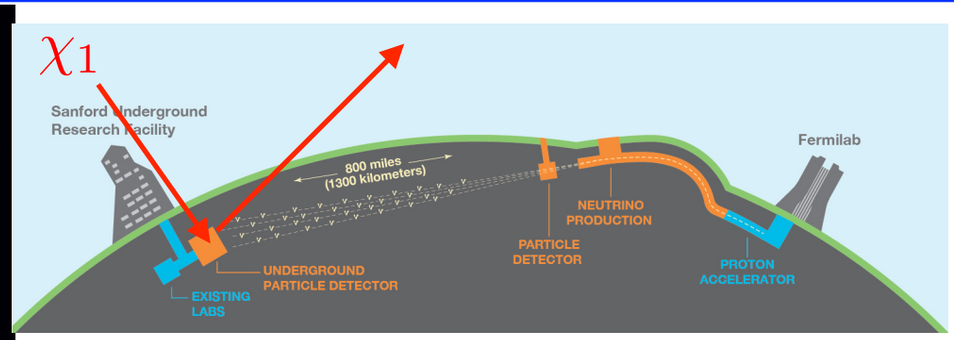
- Non-trivial structures give unique signals: e.g., iDM

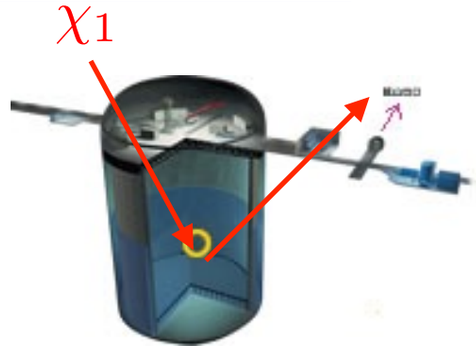


Smith, Weiner, PRD 2001

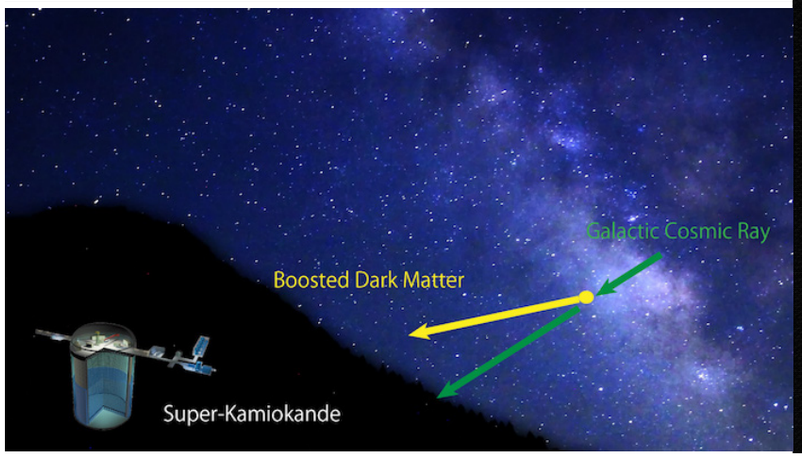
New Dark World beyond WIMP



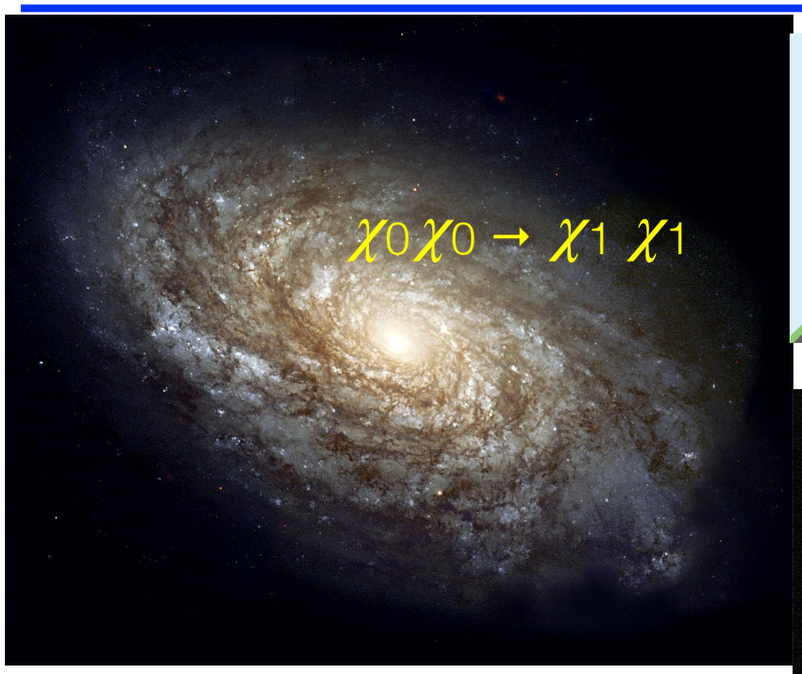


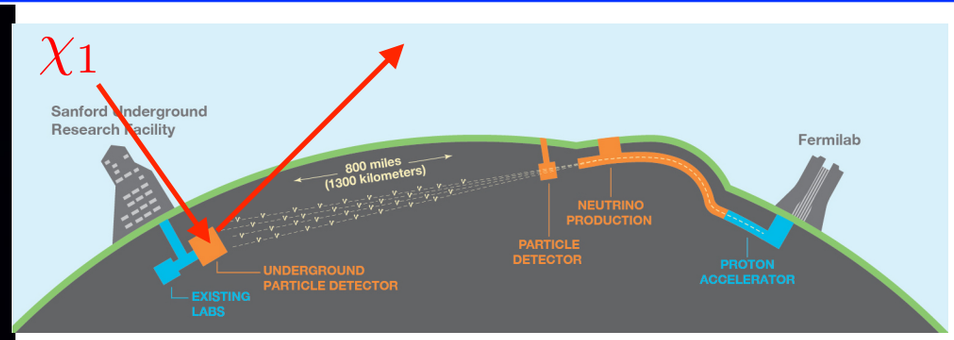


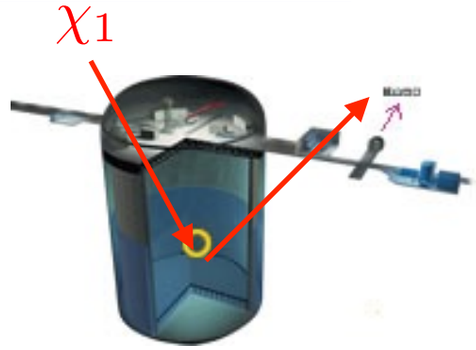




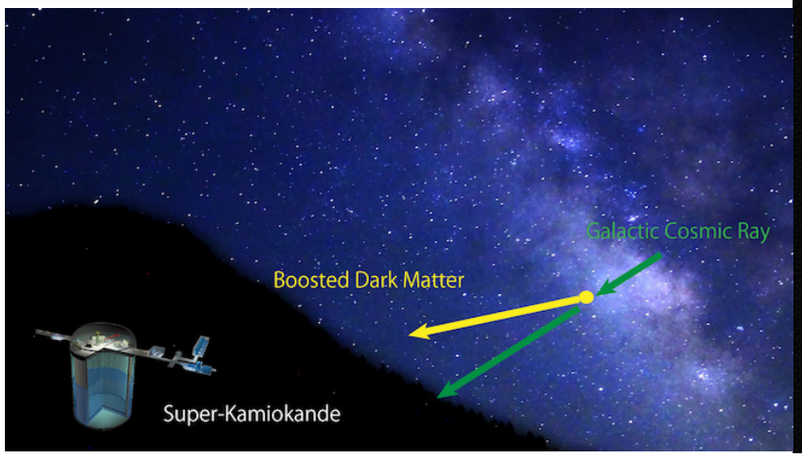
New Dark World beyond WIMP





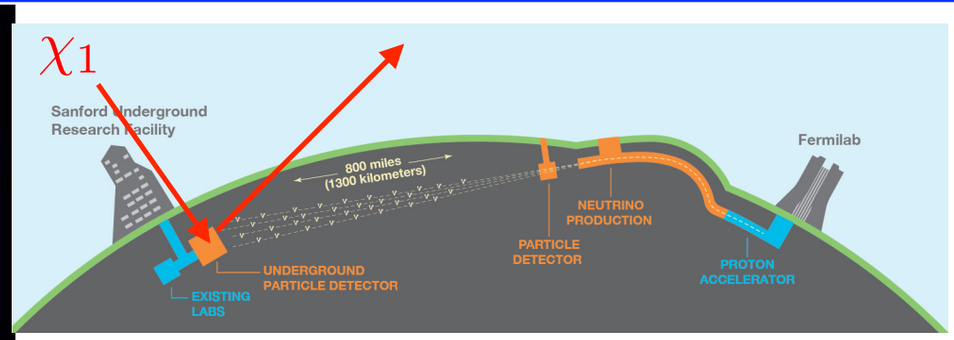


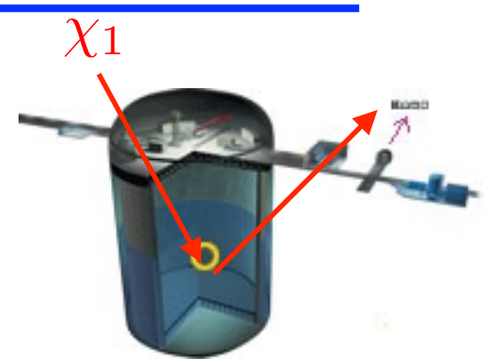




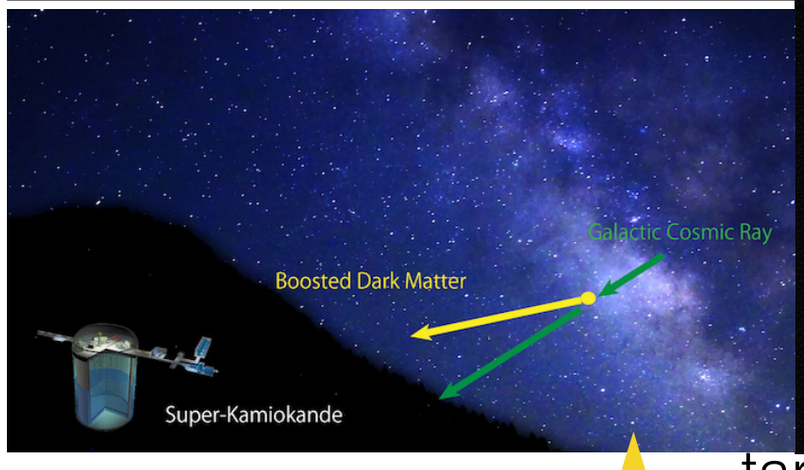
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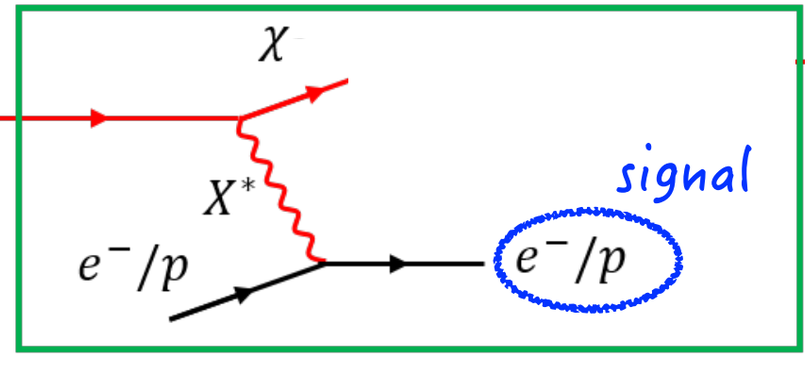


p/e

target

Low energy &
High intensity
beam

BOOSTED!
(beam E)



Detector

New Dark World beyond WIMP



Open questions

- Smallest scale physics and largest scale physics are closely correlated!

- **Dark Matter**: particle physics, astrophysics, cosmology (sometimes + nuclear physics)

- Exact DM profile in MW (or other galaxies)? local density $\rho_\odot$?

- New theoretical proposals (beyond WIMP): BSM theory, cosmology, astrophysical effect, collider phenomenology, intensity frontier experiments, …

- New experimental proposals

Search Prospects for Axion-like Particles at Rare Nuclear Isotope Accelerator Facilities

Wooyoung Jang,[1] Doojin Kim,[2] Kyoungchul Kong,[3] Youngjoon Kwon,[4] Jong-Chul Park,[5] Min Sang Ryu,[6,7] Seodong Shin,[8] Richard G. Van de Water,[9] Un-Ki Yang,[10] and Jaehoon Yu[1]

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Jump into **Dark Matter!**

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