입자물리학 표준모형

Kwang Sik JEONG

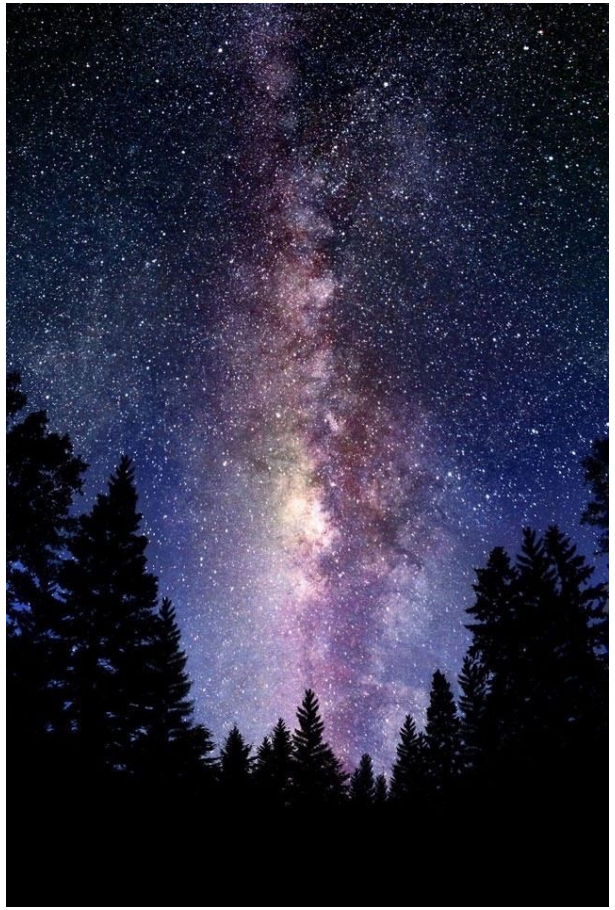
부산대학교
PUSAN NATIONAL UNIVERSITY

KIAS Particle Physics Summer Camp 2023

21 – 24 August 2023

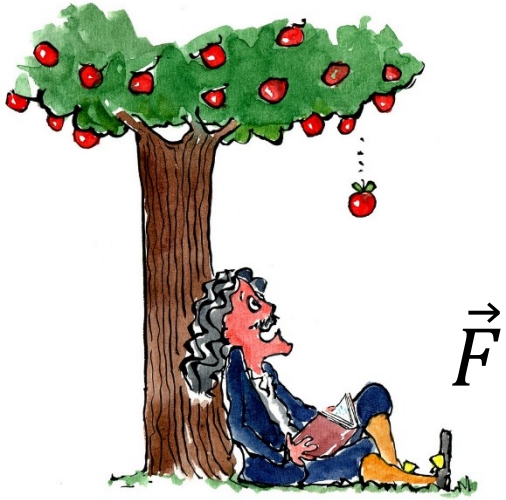
Introduction

물리학



자연 현상을 이론 및 법칙을 바탕으로 이해

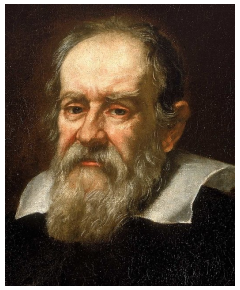
→ 원인과 결과 사이의 수학적 관계



뉴턴 역학

$$\vec{F} = \frac{d}{dt} m\vec{v}$$

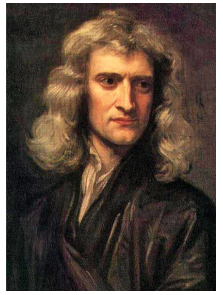




Galileo Galilei 1564-1642



Johannes Kepler 1571-1630



Isaac Newton 1643-1727
Principia, 1687 자연철학의 수학적원리

현대물리학

상대성이론

자연법칙은 모든 관찰자에게 동일하다

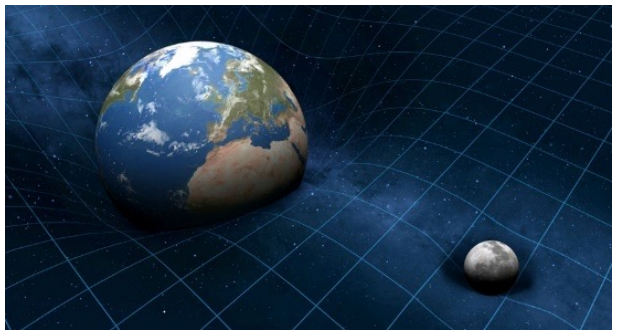
- **특수상대성이론**

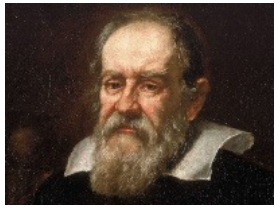
  시간 + 공간 → 시공간

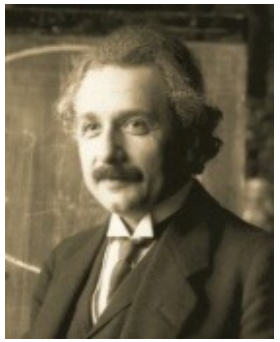
  장(field)에 의한 힘의 전달 e.g. 전기장, 자기장

- **일반상대성이론**

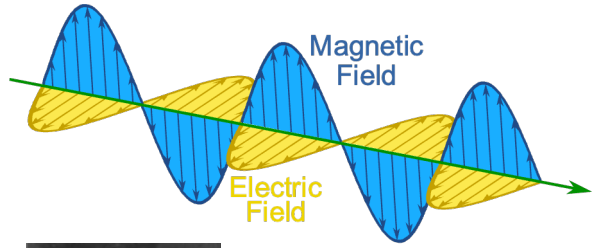
  에너지에 의해 시공간이 휘어짐 → 중력







Albert Einstein 1879-1955



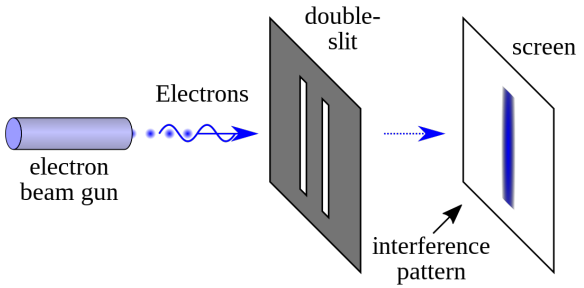


James Clerk Maxwell 1831-1879

현대물리학

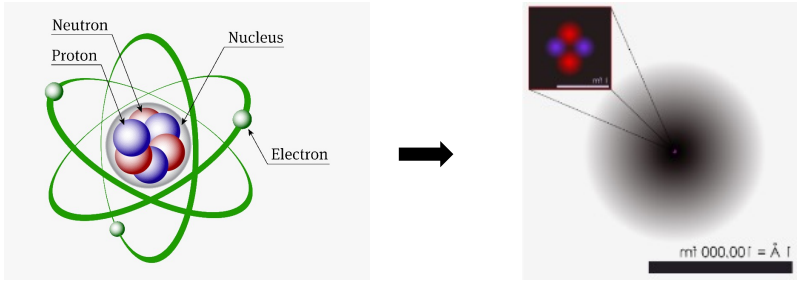
양자역학

일어날 수 있는 모든 상태들이 중첩(superposition)되어 있다



물질파

$$\lambda = \frac{h}{p}$$



- **불확정성 원리** → 다양하고 복잡한 자연 현상



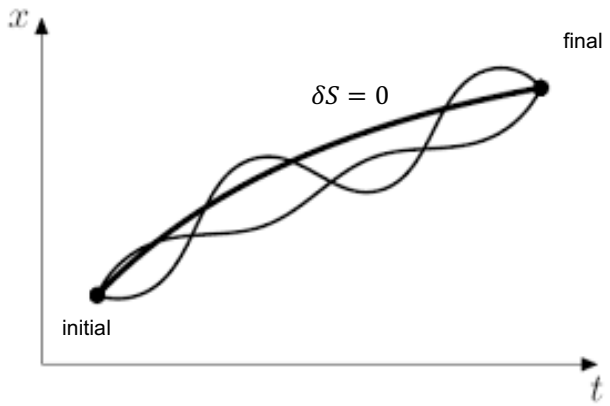
Planck, Einstein, Bohr, de Broglie, Born, Dirac, Heisenberg
Pauli, Schroedinger, Feynman, Compton, Neumann, Fermi

물리계의 시간 변화

Principle of the stationary action

Classical equation of motion (path) is determined by the extreme of the action

$\delta S=0 \Rightarrow$ classical path



Spacetime: $x^{\mu} = (ct, x, y, z)$

Action: $S = \int cdtdxdydz\, \mathcal{L}\,(x^{\mu})$

Lagrangian density

자연상수

Speed of light

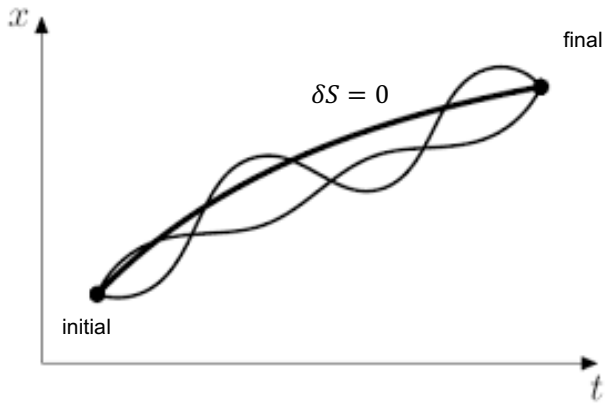
$C \simeq 3\times10^{8}$m/s

물리계의 시간 변화

Path integral

Quantum mechanical evolution is determined by sum over all possible paths

Contribution of amplitude of a path determined by the action



$$\langle \text{final} | \text{initial} \rangle = \sum_{\text{path}} e^{-\frac{i}{\hbar} S[\text{path}]}$$

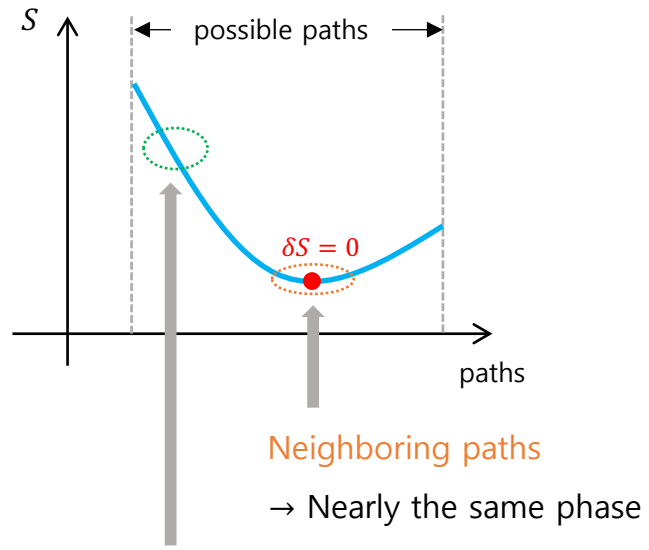
자연상수

Reduced Planck constant

$\hbar \simeq 1.05 \times 10^{-34}\,\text{J}\cdot\text{s}$

물리계의 시간 변화

Quantum effects



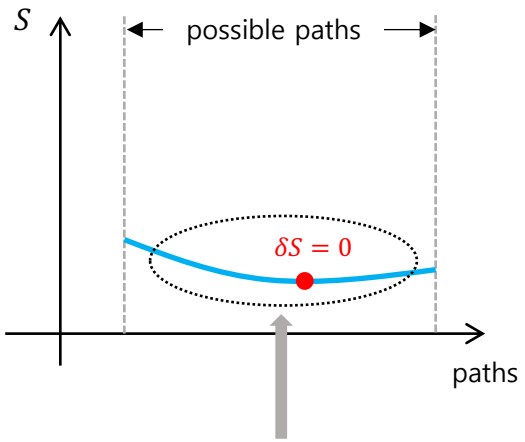
Neighboring paths

→ Nearly the same phase

Paths with $\Delta S \gg \hbar$

→ **Destructive interference** $\left|e^{iS/\hbar}\right|^2$

Classical system



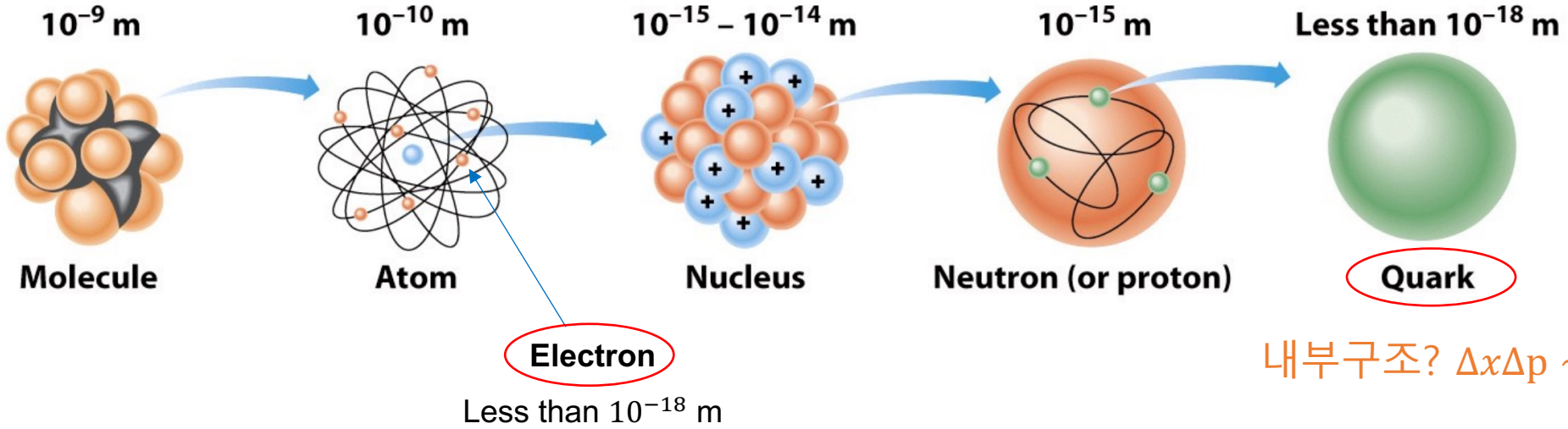
Paths with $\Delta S \sim \hbar$

→ **All paths are important**

입자물리학

기본입자(elementary particles)와 그들 사이의 상호작용을 연구

- 전자, 쿼크: 내부 구조가 없는 기본 입자



내부구조? $\Delta x \Delta \mathrm{p} \sim \hbar$

상대론 + 양자역학

→ 입자 생성 $(E > mc^2)$ 및 소멸

→ 입자와 반입자

입자물리학

- 전자의 성질 *1897년, 음극선*

  질량: $m_e \simeq 9.1\times10^{-31}$kg → 관성, 중력

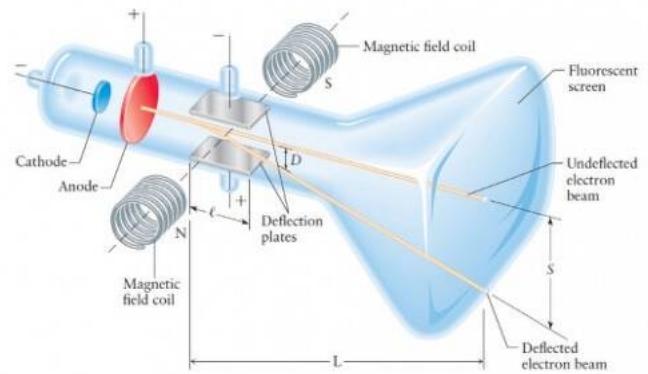
  전기전하: $-e \simeq -1.6\times10^{-19}$ C → 전자기력

  스핀: 1/2 → 통계학적 성질

  Fermion (Pauli's exclusion principle)



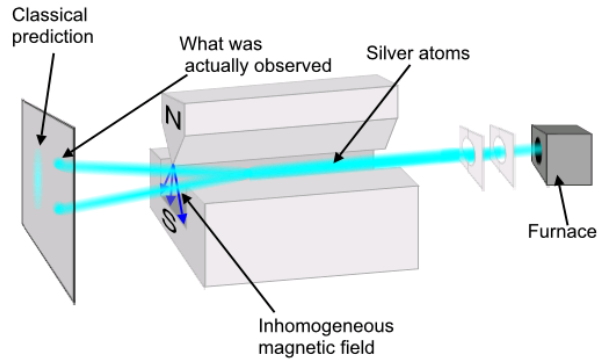
Joseph John Thomson 1856-1940



- 기본입자의 구분: 질량, 전하, 스핀

  대칭성과 관련된 물리량임

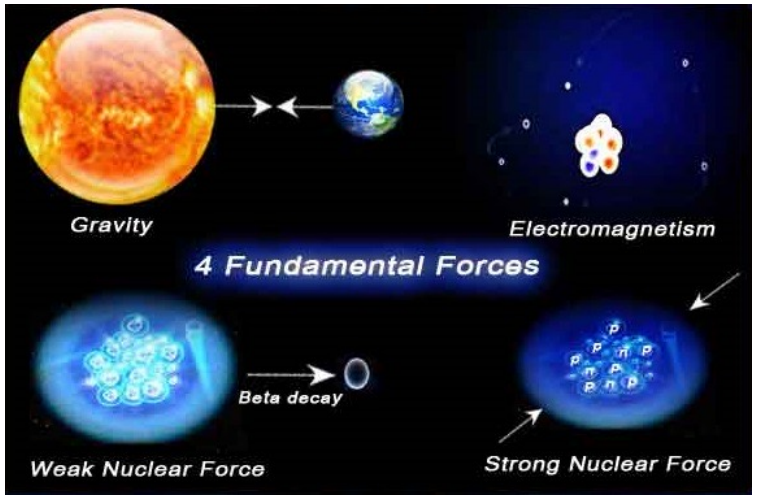
  모든 Lorentz 좌표계에서 같은 값을 갖음



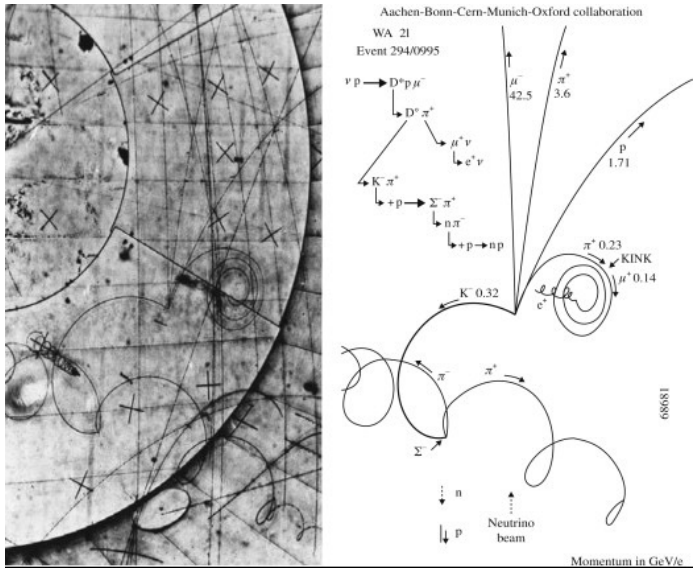
Stern–Gerlach experiment

표준모형

우리 우주의 기본입자와 그들 사이의 상호작용인 전자기력, 약력, 강력을 기술

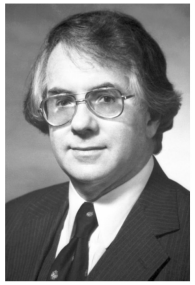


- 실험을 통해 10^{-19}m 까지의 자연 현상을 잘 설명함을 확인함
- 중력은 제외 됨 전자와 전자 사이의 전기력과 중력 크기 비교



particle tracks seen from a bubble chamber (left) and their interpretations (right).

표준모형

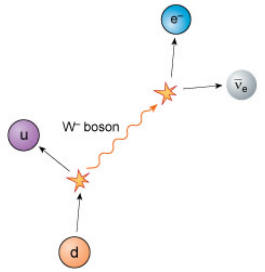


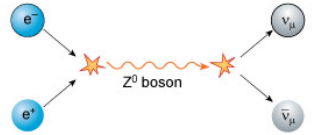


전자기력, 약력

- 이론: Sheldon Lee Glashow, Abdus Salam, Steven Weinberg

Nobel prize in 1979 for "their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current"



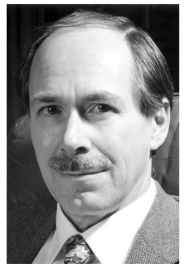
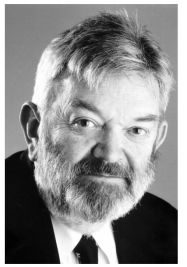


- 실험: Carlo Rubbia, Simon van der Meer (CERN)

Nobel prize in 1984 for "their decisive contributions to the large project which led to the discovery of the field particles W and Z, communicators of weak interaction."



표준모형

전자기력, 약력

- Gerardus ‘t Hooft, Martinus J.G. Veltman

Nobel prize in 1999 for "elucidating the quantum structure of electroweak interactions in physics"

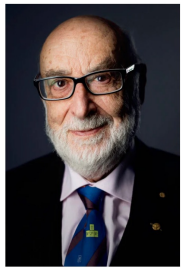
- Yoichiro Nambu



Nobel prize in 2008 for “the discovery of the mechanism of spontaneous broken symmetry in subatomic physics”

- Francois Englert, Peter W. Higgs

Nobel prize in 2013 for “the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, confirmed by the ATLAS and CMS experiments at CERN’s Large Hadron Collider”



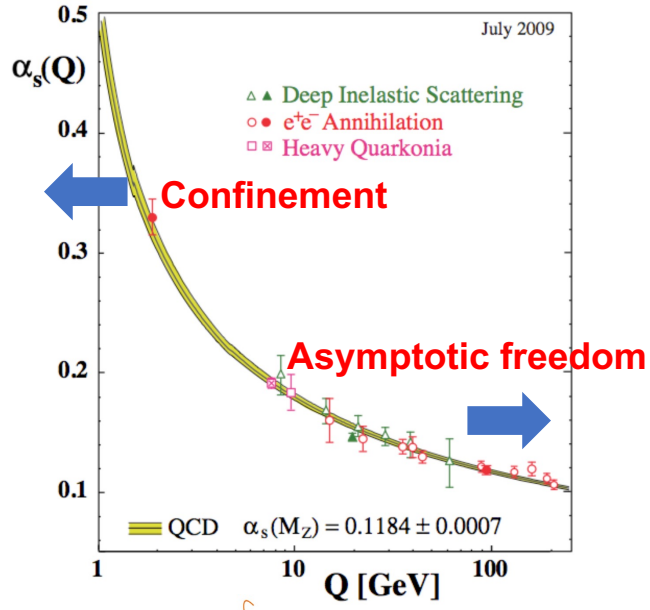

표준모형



강력

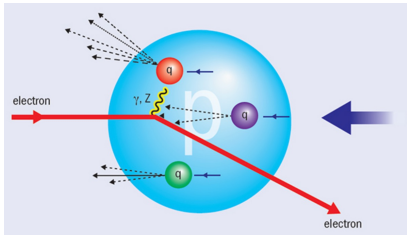
- 이론: David J. Gross, H. David Politzer, Frank Wilczek

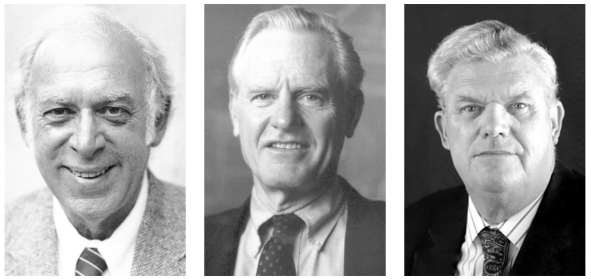
Nobel prize in 2004 for "the discovery of asymptotic freedom in the theory of the strong interaction"

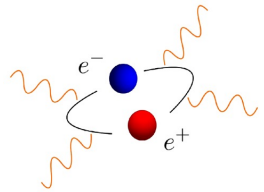


- 실험: Jerome I. Friedman, Henry W. Kendall, Richard E. Taylor

Nobel prize in 1990 for "their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"

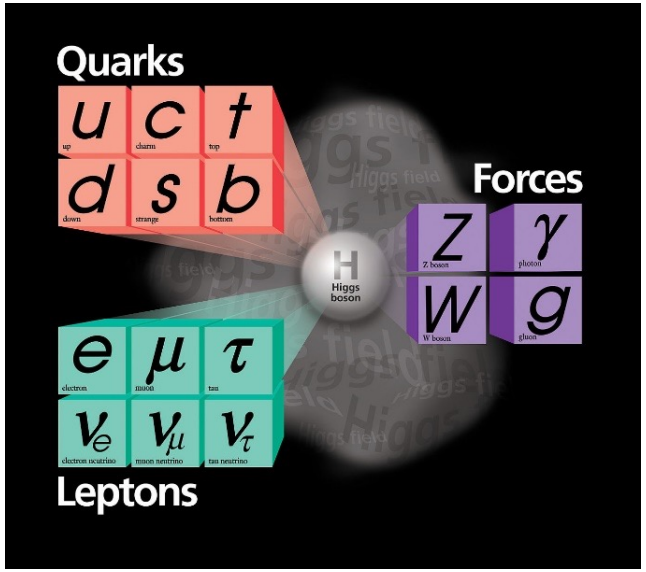


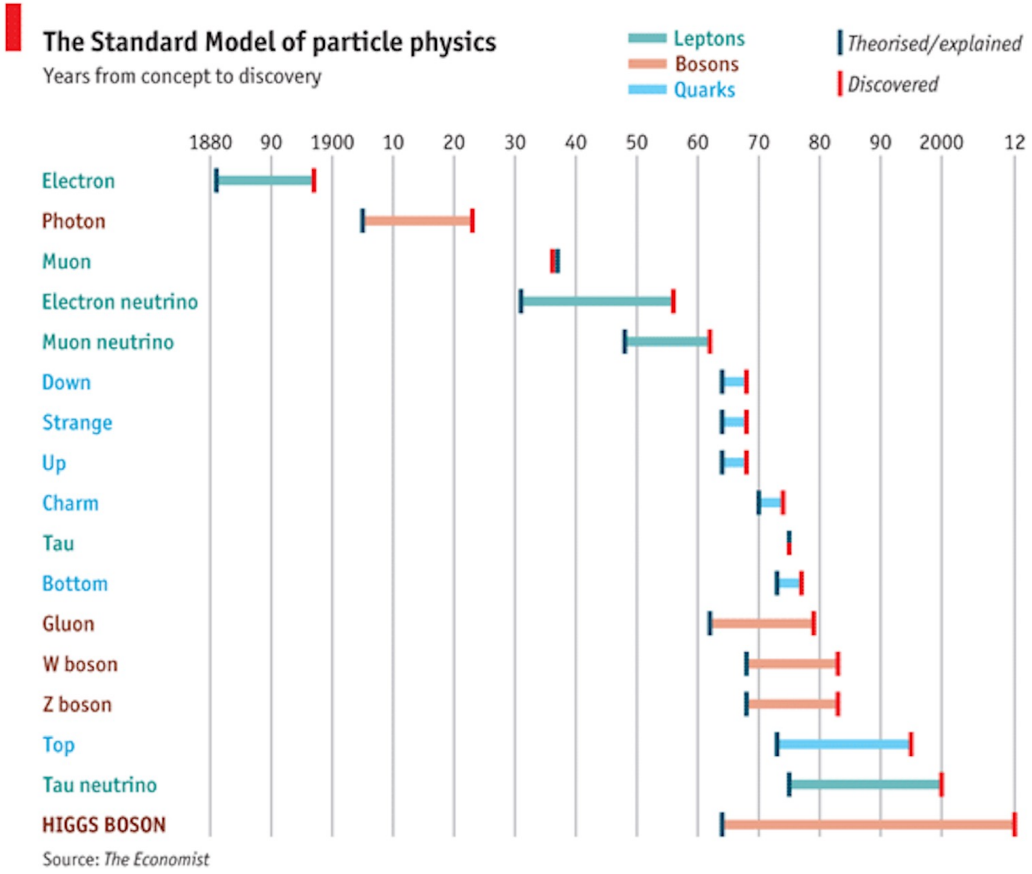




표준모형 기본입자

기본입자 17개로 구성





Dirac equation 1930

→ positron discovery 1932

muon discovery 1937

(theoretically unexpected)

Higgs mechanism 1964

EW theory 1967

→ Higgs discovery 2012

Theoretical Framework

Quantum Field Theory

Natural Units

Natural constants: Speed of light c, Planck constant $\hbar$, Newton constant G

- Natural unit: $c = \hbar = 1$

  $c = 1 \Rightarrow$ [length]=[time], [energy]=[mass]

  $\hbar = 1 \Rightarrow$ [energy]=1/[time]

  참고: $1\text{eV} \simeq 1.6\times10^{-19}\text{J}$, $\hbar c \simeq 197\text{MeV}\cdot\text{fm}$



시공간: $x^\mu = (t, x, y, z)$

양자효과: $\sum e^{iS[\text{path}]}$

- 단위 변환: $1\text{kg} \simeq 5.01\times10^{26}\text{GeV}$, $1\text{m} \simeq 5.07\times10^{15}\text{GeV}^{-1}$, $1\text{sec} \simeq 1.52\times10^{24}\text{GeV}^{-1}$

- Spacetime metric (Minkowski space): $\eta^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

Quantum Field Theory

Framework to describe elementary particles and their interactions

- Special relativity + Quantum mechanics

- Action: $S = \int d^4x\, \mathcal{L}$

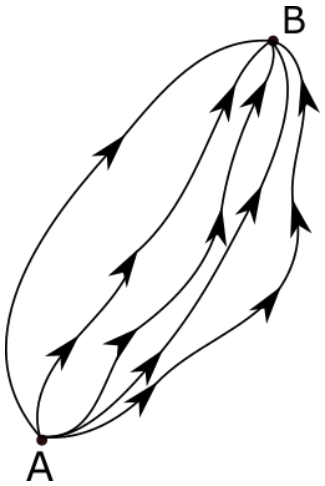
$\partial_\mu \phi \equiv \partial\phi/\partial x^\mu$

Lagrangian density: function of a field and its spacetime derivative

$$\mathcal{L} = \mathcal{L}(\phi, \partial_\mu \phi)$$

Classical field equation of motion: $\delta S = 0$

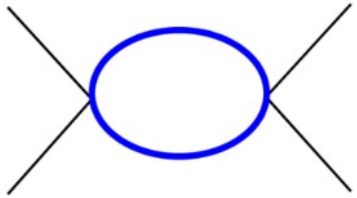
Quantum effects: path integral $\langle \text{final}|\text{initial}\rangle = \sum e^{iS} = \int D\phi\, e^{iS}$



Quantum Field Theory

Theory: Action + **Regularization** + **Renormalization**

divergence in loop diagrams (quantum effects, $\Delta x \Delta \mathrm{p} \sim 1, \Delta t \Delta E \sim 1$)



$$\int_{-\infty}^{\infty} d^4p \ (\dots)$$

four-momentum: $P^{\mu} = (E, \vec{p})$

- Probability amplitudes for elementary particle interactions

(1) **Regularization:** cutoff regularization, dimensional regularization

(2) **Counter terms:** added to absorb infinities

Quantum Field Theory

(3) **Renormalization:** to determine how to absorb the infinities into counter terms

→ Arbitrary: **renormalization scale μ** (unphysical)

- Perturbation theory: expansion with respect to *renormalized parameters*

depending on μ

energy scale of the process

generally involving logs, $\ln(E/\mu)$ and $\ln(M/\mu)$

mass of particle

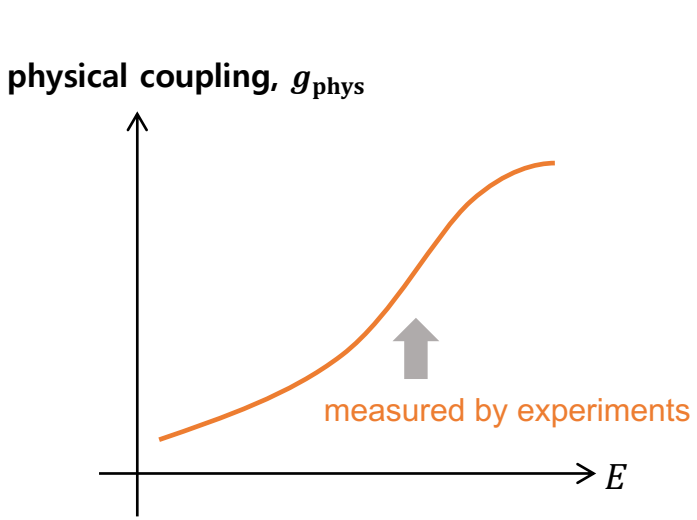
Physical quantities should be independent of the renormalization scale!

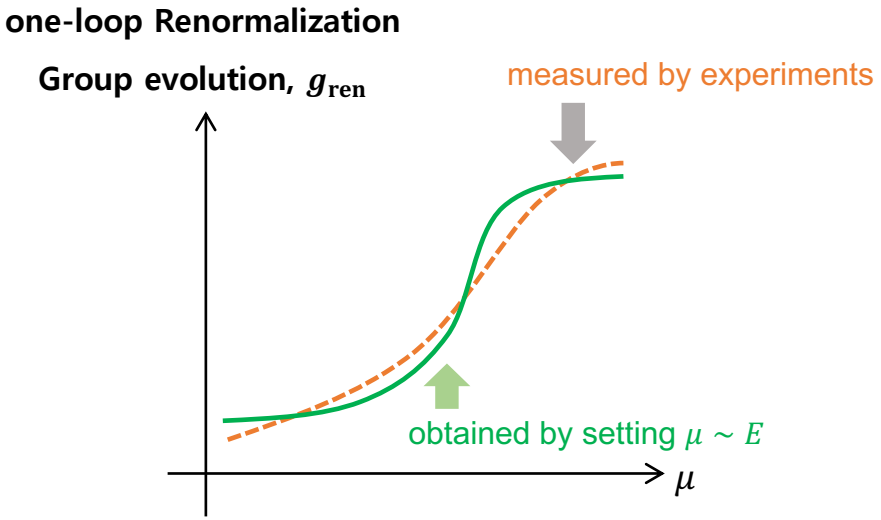
→ One can set $\boldsymbol{\mu \sim E}$ or $\boldsymbol{M}$ to reduce the sensitivity to μ when truncating the perturbation series

Quantum Field Theory

Energy dependence of a physical coupling

$$g_{\text{phys}}(E) = \sum_{n=0}^{n=\infty} c_n \, g_{ren}^{n}(E, \mu)$$ ← Renormalization scale independent





Quantum Field Theory

Theory: Symmetry

$\Delta S =$(path-independent) under transformations

- Define an elementary particle according to how the corresponding field transforms
- Determine interactions among elementary particles

- **Noether theorem**

  Continuous symmetry ↔ **Conserved quantity** e.g. energy, momentum



Amalie Emmy Noether 1882-1935

Symmetry

Maxwell equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{E} = -\nabla \Phi - \frac{\partial \mathbf{A}}{\partial t}$$

Gauge invariance

E and B remain the same under

$$\phi \to \phi - \frac{\partial \Lambda}{\partial t} \text{ and } \vec{A} \to \vec{A} + \nabla \Lambda$$

Charge conservation

- **Covariant formulation**

Four potential: $A^\mu = (\phi, \vec{A})$ → EM field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Lorentz vector

$$F^{\alpha\beta} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix}$$

Maxwell equations: $\partial_\mu F^{\mu\nu} = \mu_0 J^\nu \quad \text{and} \quad \partial_\mu \tilde{F}^{\mu\nu} = 0$

Charge conservation: $\partial_\mu J^\mu = 0$

Symmetry

Group theory and representation

Symmetries in quantum field theory

$\Delta S =$(path-independent)

where the action reads $S = \int d^4x\, \mathcal{L}(\phi, \partial_\mu \phi)$

Spacetime symmetry

Line element, $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$, remains the same under *Poincare transformations*

- **Lorentz transformations**: $dx^\mu \to dx'^\mu = \Lambda^\mu_\nu dx^\nu$ for constants Λ^μ_ν (3 spatial rotations + 3 boosts)

  $\Longrightarrow$ conserved quantity: angular momentum, position of the center of mass

- **Translations**: $x^\mu \to x'^\mu = x^\mu +$ constant

  $\Longrightarrow$ conserved quantity: four-momentum $P^\mu = (E, \vec{p})$

Symmetry

Spacetime symmetry

Transformation of fields under Lorentz symmetry

$$\Phi^a(x) \rightarrow \Phi'^a(x') = R^a_b(\Lambda)\Phi^b(x)$$

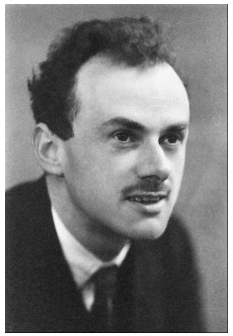
Determined by the spin of the field

$R^a_b(\Lambda)$ representation labelled by two indices (n,m) for n, m=0, 1/2, 1, 3/2 ,…

- **spin 0**: (0,0) → Scalar field, $\phi'(x') = \phi(x)$
- **spin 1/2**: (1/2,0) and (0,1/2) → Left and right fermions c.f. Dirac equation for electron

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

- **spin 1**: (1/2,1/2) → Vector field, $A'^\mu(x') = \Lambda^\mu_\nu A^\nu(x)$



Paul Adrien Maurice Dirac, 1902-1984

Spin-statistics theorem: integer spin (boson) , half-integer spin (fermion)

Followed from causality, positive energy, and positive norm in the Hilbert space

Symmetry

Internal symmetry

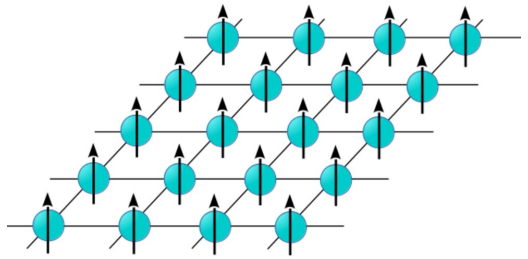
Transformation of fields

$$\Phi^a(x) \to \Phi'^a(x) = M^a_b \Phi^b(x)$$

- **Global symmetry**

  M^a_b: spacetime independent

  $\Longrightarrow$ Conserved quantity: global charge



e.g. theory of a complex scalar field

$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - m^2 \phi \phi^* - \lambda \phi^2 \phi^{*2}$

U(1) charge

determines how ϕ transforms under U(1)

$\to$ Lagrangian is invariant under U(1) transformation, $\phi(x) \to \phi'(x) = e^{iq\alpha}\phi(x)$, for constant α

Symmetry

- **Local (gauge) symmetry**

  M_b^a: spacetime dependent $M_b^a = M_b^a(x)$

  $\Longrightarrow$ Conserved quantity: gauge charge

  Spacetime derivative, $\partial_\mu \Phi^a$, is not covariant under local symmetry transformations

  $\rightarrow$ **Covariant derivative**

  $D_\mu \Phi^a$ such that $D_\mu \Phi'^a = M_b^a(x) D_\mu \Phi^b$

Gauge field (spin=1, real)

$$D_\mu = \partial_\mu + igqA_\mu$$

Gauge coupling

Gauge charge

determines how ϕ transforms

Symmetry

- **Local (gauge) symmetry**

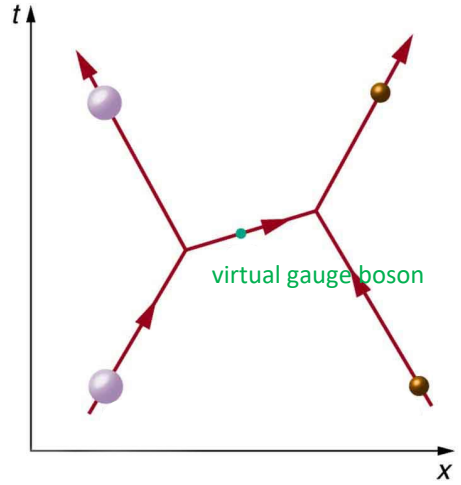
$$D_\mu = \partial_\mu + igqA_\mu$$

Gauge interaction: Interaction between Φ^a and A_μ

→ ***Fundamental force!***

Connection between Symmetry and Dynamics

Gauge boson: force mediator



virtual particle ($P_\mu P^\mu \neq m^2$): quantum effects

Gauge symmetry should **exact**, otherwise violation of **unitarity** and **renormalizability**

→ Massless gauge bosons ($A_\mu A^\mu$ is not allowed)

→ Constraint on the fermion contents of the theory

→ Gauge charge: absolutely conserved

Symmetry

- **Local (gauge) symmetry**

U(1) gauge symmetry

$\mathcal{L} = (D_\mu \phi)(D^\mu \phi^*) - m^2 \phi \phi^* - \lambda \phi^2 \phi^{*2}$ with $D_\mu = \partial_\mu + iqgA_\mu$

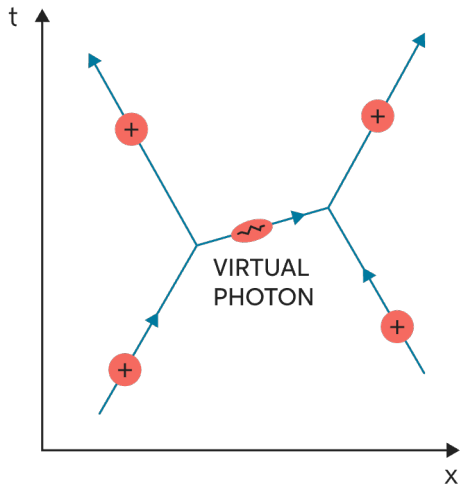
→ Lagrangian is invariant under U(1) transformation

$\phi(x) \rightarrow \phi'(x) = e^{iq\alpha(x)} \phi(x)$

$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) - \frac{1}{g} \partial_\mu \alpha$

e.g. Gauge symmetry of electromagnetism, $U(1)_{\mathrm{EM}}$

- gauge charge: electric charge
- gauge boson: photon



Feynman diagram

Symmetry Breaking

Symmetry of the action: ΔS =(path-independent)

- Spontaneous symmetry breaking

  The ground state does not respect the symmetry

  Lorentz symmetry allows a scalar field to have a nonzero vacuum expectation value (VEV)

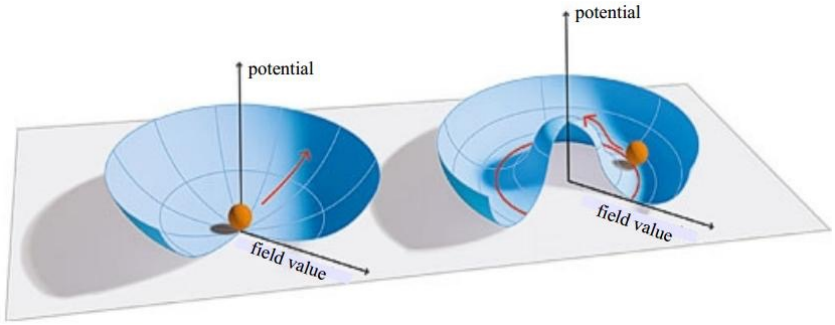
  $\langle\phi\rangle$ =(spacetime-independent) ← Determined by the minimum of the scalar potential $V(\phi)$

- Scalar bosons from a complex scalar ϕ: small fluctuations around the vacuum

Their masses are determined by the curvature of $V(\phi)$ at the vacuum

(1) $\langle\phi\rangle = 0 \rightarrow$ Real and imaginary components have the same mass

(2) $\langle\phi\rangle \neq 0 \rightarrow$ **Spontaneous symmetry breaking**



The phase component becomes massless ⬅ **Nambu-Goldstone boson**

Symmetry Breaking

Spontaneous breaking of global U(1) symmetry

$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - V(\phi, \phi^*)$ with $V = -\frac{1}{2}\mu^2 \phi\phi^* + \frac{1}{4}\lambda\phi^2\phi^{*2}$

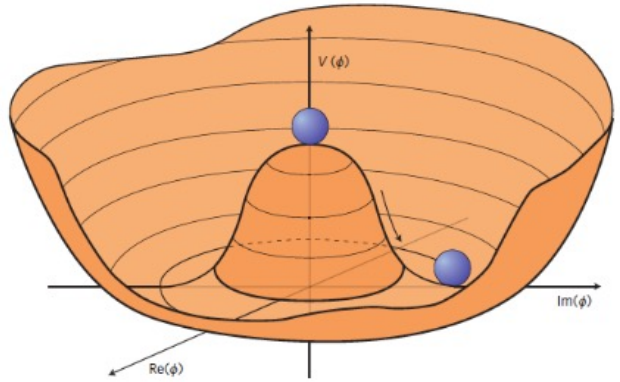
Lagrangian is invariant under U(1), but the ground state is not.

VEV of the scalar field: $v \equiv \sqrt{2}\,\langle|\phi|\rangle = \sqrt{2}\,\frac{\mu}{\sqrt{\lambda}} \neq 0$

Lagrangian in the canonical basis: $\phi = \frac{1}{\sqrt{2}}(\mathrm{v} + \rho(x))e^{\frac{ia(x)}{\sqrt{2}v}}$

$\mathcal{L} = \frac{1}{2}(\partial\rho)^2 + \frac{1}{2}(\partial a)^2 - V(\rho)$

Massless Nambu-Goldstone boson



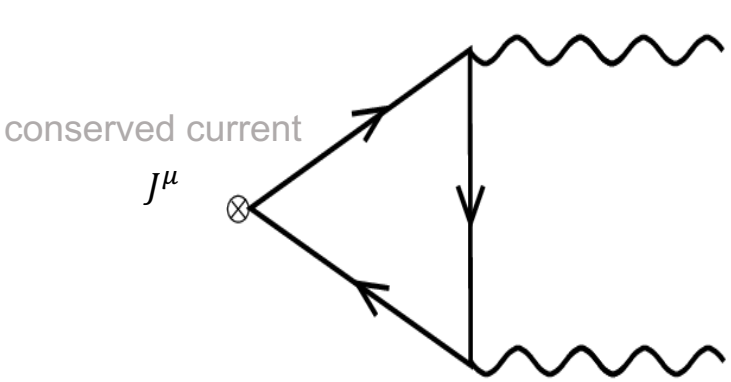
Spontaneous breaking of gauge U(1) symmetry → **Higgs mechanism** (later)

Symmetry Breaking

Symmetry of the action: $\Delta S =$(path-independent)

- Symmetry breaking by quantum effects

Coupled to gauge fields, fermions ψ with *chiral symmetry* can break the symmetry of the action



$$\Rightarrow \Delta\mathcal{L}_{\text{breaking}} = \epsilon \frac{c_\psi}{32\pi^2} F^{\mu\nu}\tilde{F}_{\mu\nu} \quad \text{under } \Phi \to \Phi + \epsilon\delta\Phi$$

Anomaly

$$\langle \text{final}|\text{initial}\rangle = \int DA_\mu D\phi D\bar{\psi}D\psi \; e^{iS[A_\mu,\phi,\bar{\psi},\psi]}$$

not invariant, $\Delta\mathcal{L}_{\text{breaking}}$

- Global anomaly: $\Delta\mathcal{L}_{\text{breaking}}$ is a total derivative $\Longrightarrow$ Relevant only nonperturbatively
- Gauge anomaly: violation of unitarity and renormalizability $\Longrightarrow$ Should be cancelled

Effective Theory

Low energy effective theory (approximate description)

- UV theory with renormalizable local operators

  $\mathcal{L}_{\text{UV}} = \mathcal{L}_H(\phi_H, \phi_L) + \mathcal{L}(\phi_L)$

  where ϕ_H (ϕ_L) are fields describing heavy (light) particles above (below) a scale Λ

- **Low energy effective theory of light fields**

Constructed by integrating out heavy fields: $\int D\phi_H D\phi_L e^{iS[\phi_H,\phi_L]}$

Below the cutoff scale Λ, $\mathcal{L}_{\text{eff}} = \mathcal{L}(\phi_L) + \sum_{n,i} \frac{c_i^{(n)}}{\Lambda^{n-4}} O_i^{(n)}(\phi_L)$

local operator of mass dim= n

Dimensionless Wilson coefficients $c_i^{(n)}$: Effects of heavy particles and high energy modes

Effective Theory

Low energy effective theory of light fields

- Contribution of the local operator $O_i^{(n)}$ to a process at energy scale $E \ll \Lambda$

$$c_i^{(n)} \left(\frac{E}{\Lambda}\right)^{n-4}$$

→ **Operators of three types**

(1) $n < 4$: Relevant

(2) $n = 4$: Marginal

(3) $n > 4$: Irrelevant (non-renormalizable)

infinite number of counter terms to regulate loop divergences

Effective Theory

Low energy effective theory of light fields

- **Matching: adjustment of coefficients at a scale around Λ**

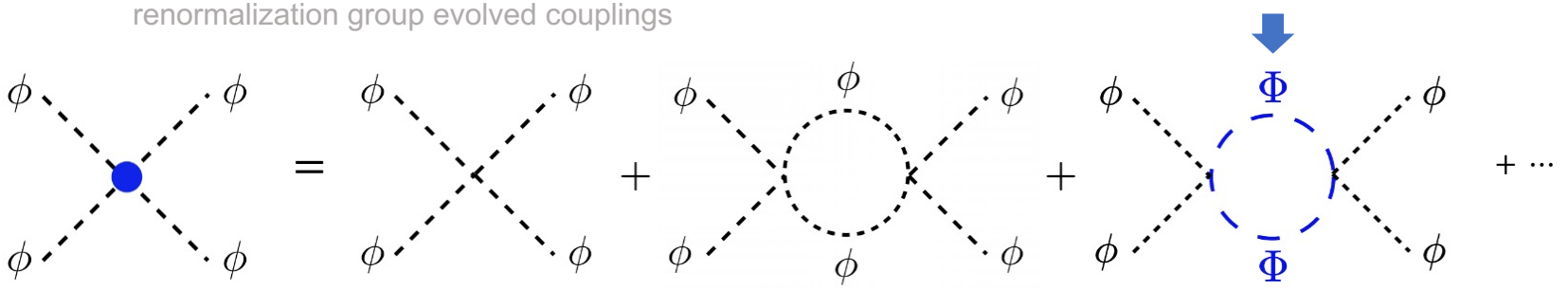
Same physical predictions at low energy scales

e.g. Quartic coupling of a scalar field

$$\lambda_{\text{eff}}(\mu) = \lambda_{\text{UV}}(\mu) + (\text{threshold corrections})$$

renormalization group evolved couplings

Heavy particle loops typically contain $\ln(\Lambda/\mu)$



e.g. hadrons in QCD below Λ_{QCD}

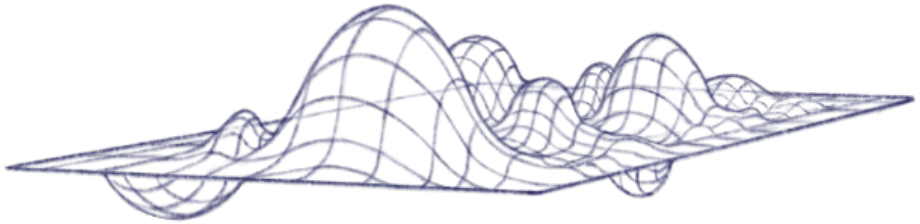
- For a case of strongly coupled theory, effective theory of relevant degrees of freedom

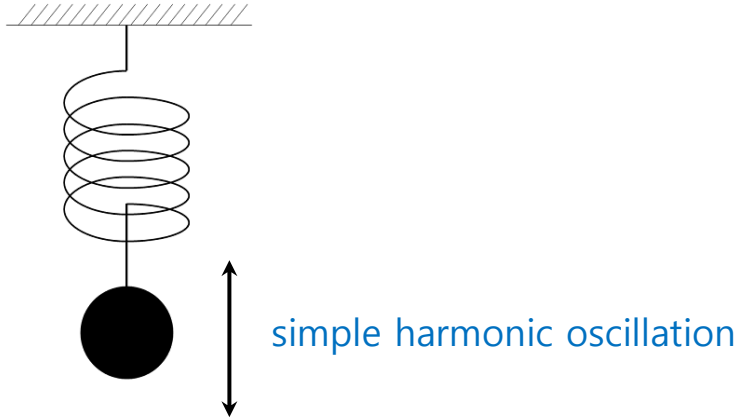
Physical Objects

Physical objects from FIELDS

- **Point-like particles elementary excitations**

Small linear perturbations of fields around the ground state





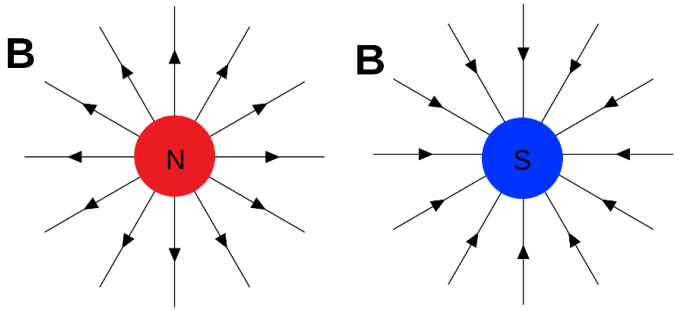
Physical Objects

- **Solitons**

Solution of the classical field equation, δS=0

- Lumps of fields of finite size
- Similar to particles, but without quantization
- Existence and stability due to non-linearity of the field equation

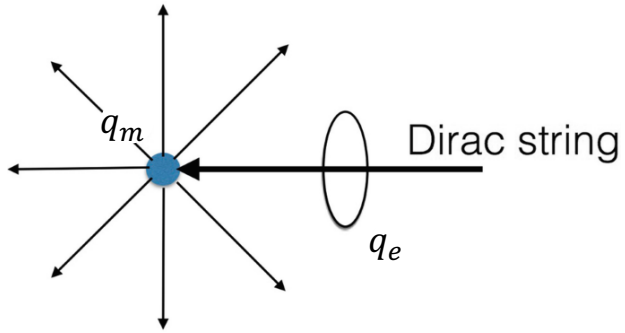
e.g. magnetic monopole, domain wall, cosmic string, …



(size) $\sim \frac{1}{m}$

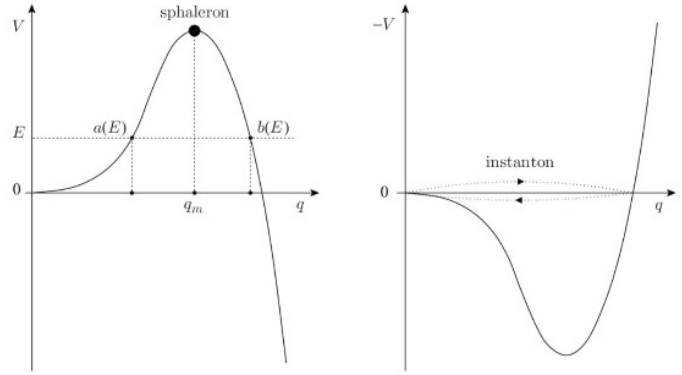
(energy) $\sim \frac{m}{g}$

m: mass of the elementary particle
g: gauge coupling



c.f. Dirac quantization: $\frac{q_e q_m}{2\pi\hbar} \in Z$

Physical Objects



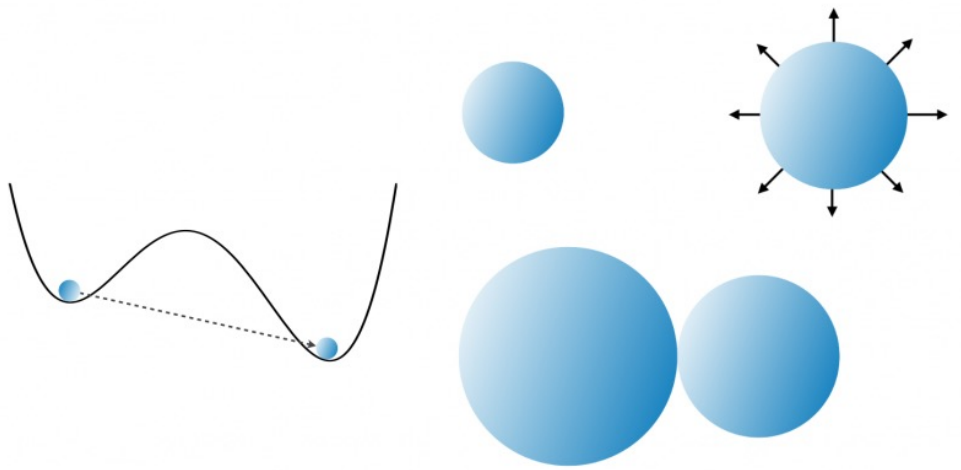
- **Instantons**

Localized, finite-action solution of the Euclidean field equations

Wick rotation $it \rightarrow \tau$

$$S = \int dt \left(\frac{1}{2} m \left(\frac{dq}{dt}\right)^2 - V(q)\right) \rightarrow iS_E = \int d\tau \left(\frac{1}{2} m \left(\frac{dq}{d\tau}\right)^2 + V(q)\right)$$

- **Tunneling** between quantum systems
- **Decay of a false vacuum** by bubble nucleation



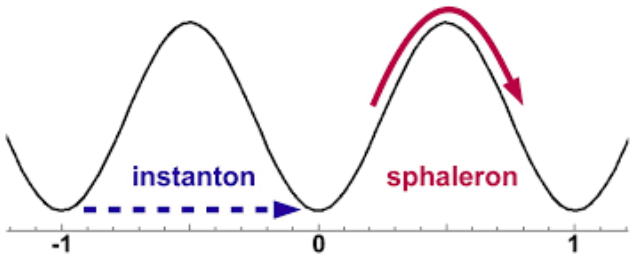
bubbles of the true vacuum

Physical Objects

- **Sphalerons**

Static, unstable, and finite-energy solution of the classical field equation

- Classical transition between vacua
- Possible at high energy scales



- Non-Abelian gauge theory: topologically non-trivial vacuum structure with an infinite number of ground states Chern-Simons number

  → Anomaly has physical effects due to instantons

Standard Model

Fermions

Essence of the SM

Gauge symmetry

SM is based on **three gauge symmetries**, $\mathbf{SU(3)_C \times SU(2)_L \times U(1)_Y}$

Describe strong (QCD), weak, and electromagnetic interactions

Left-handed fermions (1/2,0) are charged under $\mathrm{SU}(2)_L$, but right-handed ones (0,1/2) are not

All elementary particles are **massless** before the electroweak phase transition

Higgs mechanism

Spontaneous symmetry breaking of $\mathbf{SU(2)_L \times U(1)_Y}$ down to $\mathbf{U(1)_{EM}}$

Elementary particles get **massive**

Symmetries

Continuous symmetries of the SM

- **Poincare symmetry:** translation + Lorentz invariance
- **Gauge symmetries:** $\mathbf{SU(3)_C \times SU(2)_L \times U(1)_Y}$ → **Fundamental forces**
- **Global symmetries**

  (1) Two **accidental** global symmetries $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_L$

  → Conservation of baryon and lepton number (quark $B = 1/3$ and $L = 0$, lepton $B = 0$ and $L = 1$)

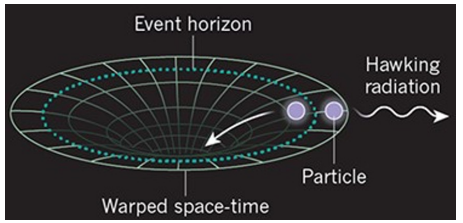
  → $B + L$ is anomalous under electroweak gauge group ($B - L$ is conserved)

  (2) Approximate global symmetry: **flavor symmetry**

  (only broken by the weak interactions)

c.f. Global symmetry: explicitly broken by quantum gravity effects

→ higher dimensional operators with dim> 4 suppressed by the Planck mass



$$M_{Pl} = \frac{1}{\sqrt{8\pi G}} \simeq 2.4 \times 10^{18}\ \text{GeV}$$

Symmetries

Discrete symmetries

- **Charge conjugation C**: $Q \to -Q$ (opposite charges)
- **Parity transformation P**: $x \to -x,\ y \to -y,\ z \to -z$
- **Time reversal T**: $t \to -t$

	C	P	T
Q (charge)	$-Q$	Q	Q
$\vec{p}$ (momentum)	$\vec{p}$	$-\vec{p}$	$-\vec{p}$
s (spin angular momentum)	s	s	$-s$

- **CPT theorem: the action is invariant under CPT** if nature is described by a local, Lorentz invariant field theory with the usual connection between spin and statistics

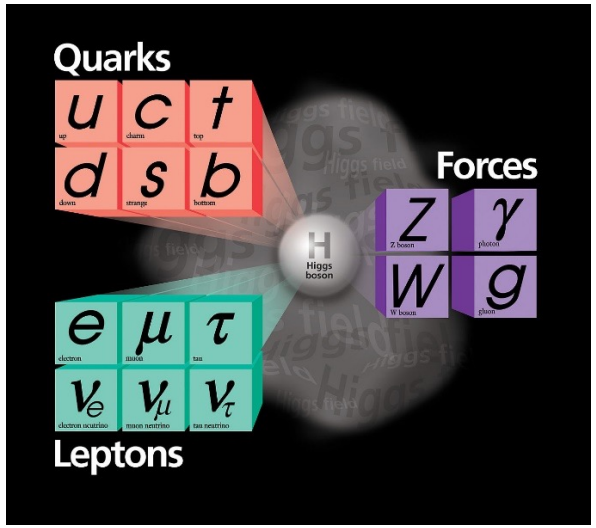
  Locality: $\mathcal{L} = \mathcal{L}(x)$

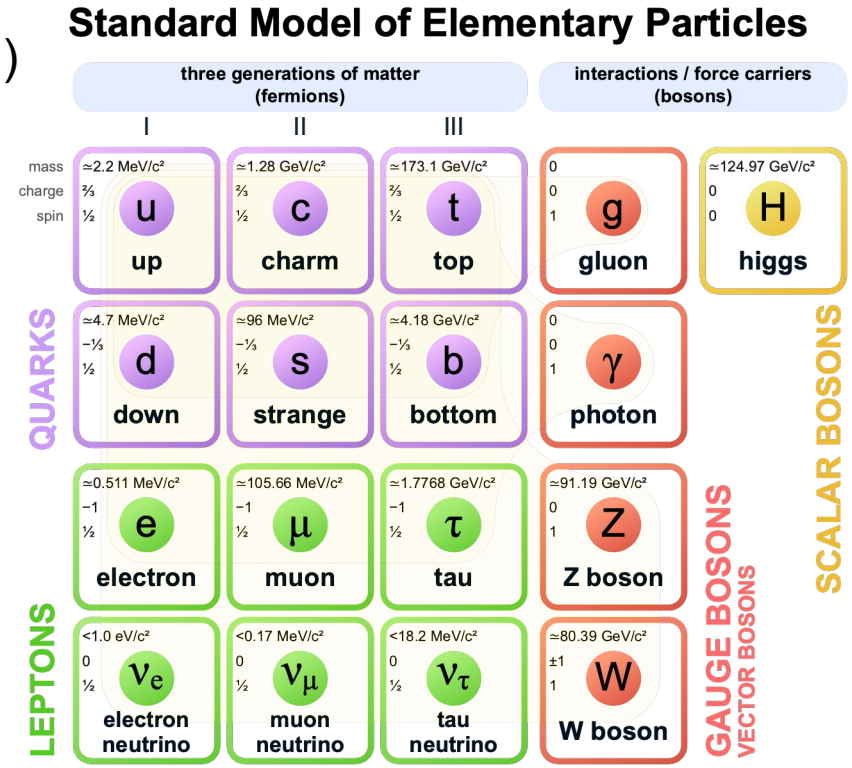
  Lorentz invariance: $\mathcal{L}$ =(Lorentz scalar) ← Required by relativity

  Unitarity: $\mathcal{L} = \mathcal{L}^{\dagger}$

Field Contents

- Scalar (spin=0): **Higgs field**
- Fermion (spin=1/2): **Quarks** (6 flavors), **Leptons** (6 flavors)
- Vector (spin=1): **Photon, W and Z bosons, Gluons**





Fermions

Fermion (spin=1/2)

Pauli's exclusion principle → **Matter**

- **Leptons**

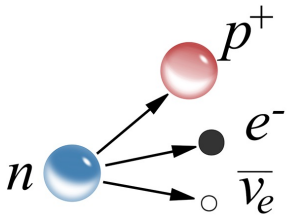
3 charge leptons: electric charge $= -1$

electron: stable

muon (lifetime $\simeq 2\times10^{-6}$s), tau (lifetime $\simeq 3\times10^{-13}$s)

3 neutrinos: electric charge $= 0$

massless in the SM (but tiny masses: neutrino oscillations)





Wolfgang Ernst Pauli, 1900-1958

c.f. muons produced when cosmic rays hit the atmosphere 15km above the earth surface

Fermions

- **Quarks**

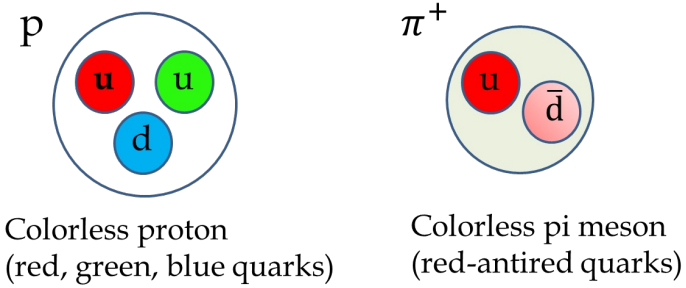
3 up-type quarks: electric charge $= 2/3$

3 down-type quarks: electric charge $= -1/3$

Three colors: **R**, **G**, **B**

Confinement at energy scales below $\Lambda_{QCD} \simeq 1GeV$

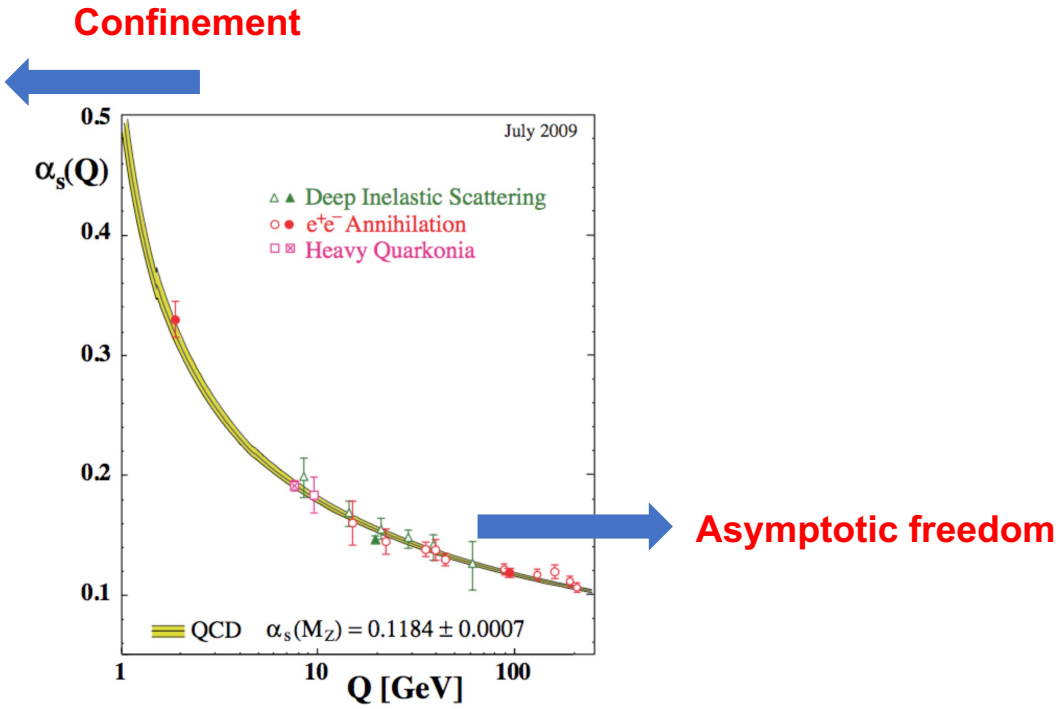
→ Hadrons (colorless): meson ($q\bar{q}$), baryon (qqq)



Colorless proton
(red, green, blue quarks)

Colorless pi meson
(red-antired quarks)

Mass of hadron is mostly QCD binding energy



Standard Model

Interactions

How SM Particles Interact?

Fundamental forces: Gauge interactions

- **Covariant derivatives for $\mathbf{SU(3)_C \times SU(2)_L \times U(1)_Y}$**

$$D^\mu = \partial^\mu - i g_1 \frac{Y}{2} B^\mu - i g_2 \frac{\tau_i}{2} W_i^\mu - i g_3 \frac{\lambda_a}{2} G_a^\mu$$

Gauge fields: B^μ, $W^\mu_{i=1,2,3}$, $G^\mu_{a=1,2,\ldots,8}$ → Gauge bosons (force mediator)

Gauge couplings: g_1, g_2, g_3 → Universal for all representations

Gauge charges: Y, τ_i, λ_a (hyper charge, weak charge, color charge)

charge for non-Abelian gauge group: fixed by representation

Field Contents

Fields of the SM

Representations of $\mathbf{SU(3)_C \times SU(2)_L \times U(1)_Y}$

Particle(s)	Field(s)	Charge	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Quarks	Q_i	$(2/3, -1/3)$	1/2	**3**	**2**	1/3
3 generations	u_{Ri}	2/3	1/2	$\bar{\mathbf{3}}$	**1**	4/3
	d_{Ri}	$-1/3$	1/2	$\bar{\mathbf{3}}$	**1**	$-2/3$
Leptons	L_i	$(0, -1)$	1/2	**1**	**2**	-1
3 generations	l_{Ri}	-1	1/2	**1**	**1**	-2
Gluons	G^a_μ	0	1	**8**	**1**	0
W bosons	$W^{1,2}_\mu$	± 1	1	**1**	**3**	0
Photon, Z boson	W^3_μ, B_μ	0	1	**1**	**3, 1**	0
Higgs boson	H	0	0	**1**	**2**	1

Lagrangian

Lagrangian invariant under $\mathbf{SU(3)_C \times SU(2)_L \times U(1)_Y}$

$$\mathcal{L}_{SM} = -\frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2g^2} \mathrm{Tr}(W_{\mu\nu} W^{\mu\nu}) - \frac{1}{2g_s^2} \mathrm{Tr}(G_{\mu\nu} G^{\mu\nu})$$

← kinetic terms of gauge fields

$$+ \bar{Q}_i i \not{D} Q_i + \bar{L}_i i \not{D} L_i + \bar{u}_i i \not{D} u_i + \bar{d}_i i \not{D} d_i + \bar{e}_i i \not{D} e_i$$

← kinetic terms of fermion fields

$$+ \left(Y_u^{ij} \bar{Q}_i u_j \tilde{H} + Y_d^{ij} \bar{Q}_i d_j H + Y_l^{ij} \bar{L}_i e_j H + \mathrm{h.c.} \right)$$

← Higgs interactions to fermions

(Yukawa interaction)

$$+ (D_\mu H)^\dagger (D^\mu H) - \lambda (H^\dagger H)^2 + \mu^2 H^\dagger H + \frac{\theta}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}(G_{\mu\nu} G_{\rho\sigma}).$$

kinetic term of the Higgs field

Higgs potential

No mass terms allowed for fermions and gauge fields

- **How many independent (physical) parameters are there in $\mathcal{L}_{SM}$?**

Lagrangian

- **Field strength tensors**

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_1 \epsilon_{ijk} W^j_\mu W^k_\nu$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu$$

} Non Abelian gauge groups

- **Higgs mechanism:** $\langle |H| \rangle \neq 0$

  Spontaneous symmetry breaking of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ down to $\mathrm{U}(1)_{\mathrm{EM}}$

  → Masses to quarks, charged leptons, electroweak gauge bosons ($W^\pm$ and Z)

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu)$$

$$Z^0_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}}(g_1 W^3_\mu - g_2 B_\mu)$$

$$A_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}}(g_2 W^3_\mu + g_1 B_\mu)$$ ← for U(1)_EM (massless)

weak mixing angle

$$\sin\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}$$

Gauge Interactions: EM

Electroweak interactions, $\mathbf{SU(2)_L \times U(1)_Y}$

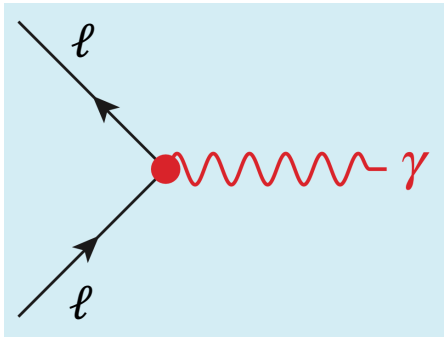
- **Electromagnetic interaction**

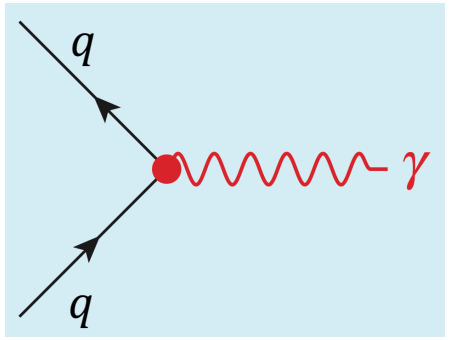
  Gauge charge: **electric charge**

  Gauge boson: **photon** (massless, no self interaction) → **Long range force**

Interaction coupling: fine-structure constant $\alpha_{\mathrm{EM}} \equiv \frac{e^2}{4\pi} \simeq \frac{1}{137}$ at low energy scales

Basic vertices:





Figures: Julia Woithe et al

charged lepton: $\ell = e, \mu, \tau$ quark: $q = u, c, t$ and d, s, b

Universality of gauge couplings (no difference between generations): experimentally confirmed

Gauge Interactions: Weak

Electroweak interactions, $\mathbf{SU(2)_L \times U(1)_Y}$

- **Weak interaction**

  Gauge charge: **weak charge**

  Gauge boson: $\boldsymbol{W^{\pm}}$ (80.4 GeV, electric charge $= \pm 1$) and $\boldsymbol{Z}$ (91.2 GeV, electric charge $= 0$)

  → Potential mediated by a mediator with mass m: $V(r) \sim \frac{e^{-mr}}{r}$

  Very short range force: $\sim \frac{1}{m_W} \sim 0.001$ fm

Interaction coupling: $\alpha_W \simeq 10^{-6}$ at low energy scales

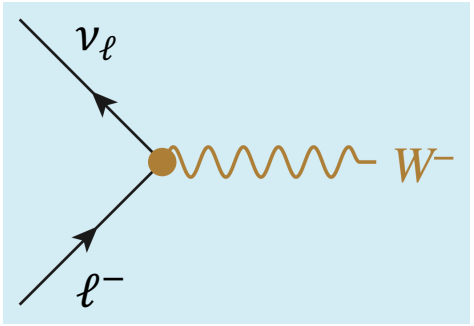
Flavor symmetry: conserved in strong and EM interactions, but can be broken by weak interaction

Gauge Interactions: Weak

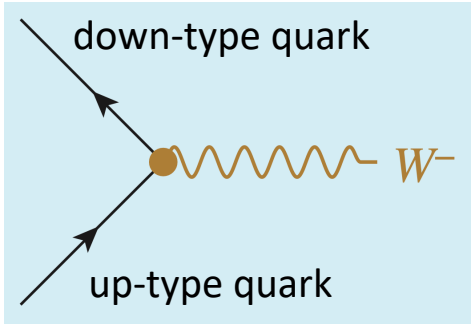
Electroweak interactions, $\mathbf{SU(2)_L \times U(1)_Y}$

- **Weak interaction**

Basic vertices:

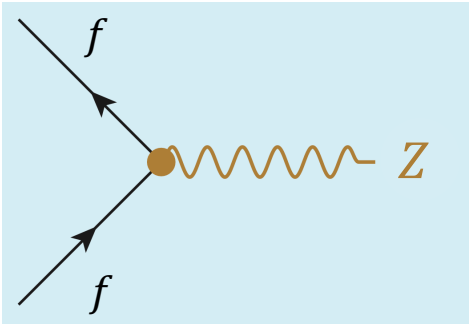


charged lepton: $\ell^- = e, \mu, \tau$



interfamily mixing

because quarks are massive



f = quarks and leptons

GIM mechanism: No flavor-changing neutral current at tree-level in the SM

Neutrino: Detection via weak interactions

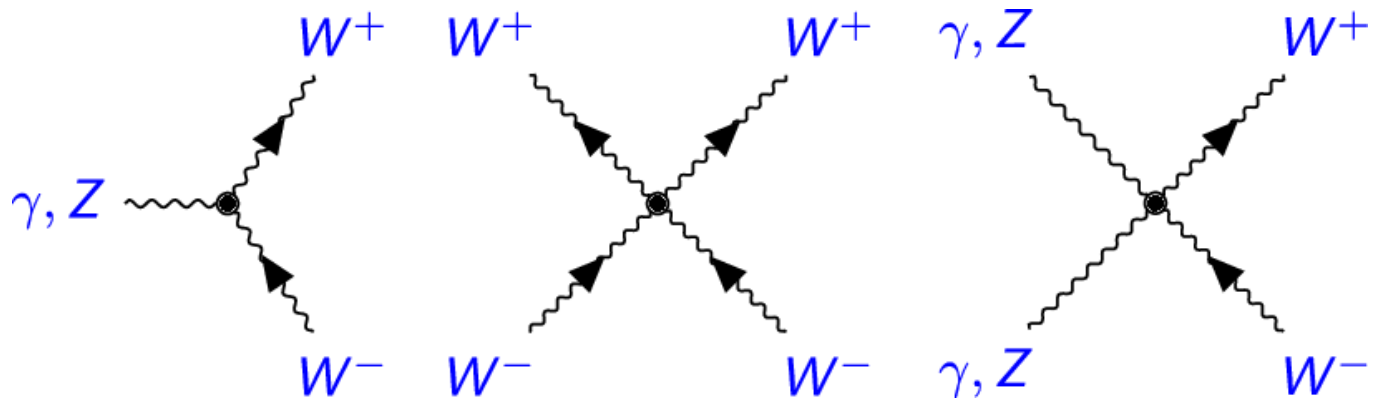
Gauge Interactions: Weak

Electroweak interactions, $\mathbf{SU(2)_L \times U(1)_Y}$

- **Weak interaction**

Interaction among electroweak gauge bosons

Basic vertices:



Gauge Interactions: Strong (QCD)

Strong interaction, $\mathbf{SU(3)_C}$

- **Strong interaction**

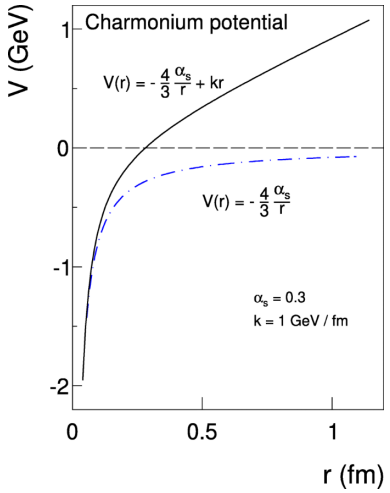
  Gauge charge: **color charges R, G, B**

  Gauge boson: **gluons** (massless, 8 types)

Potential between quarks: $V(r) \approx kr - \frac{4}{3}\frac{\alpha_s}{r}$

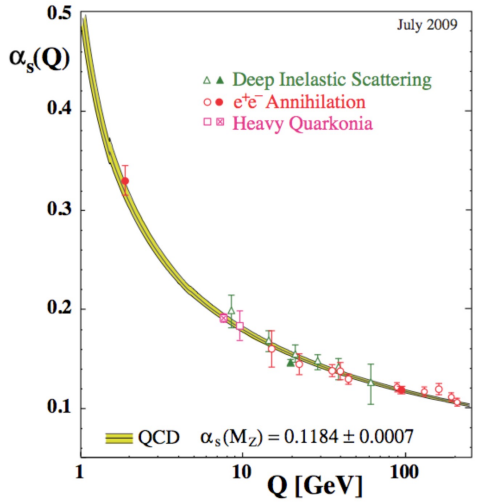
linear term (long range)

Coulomb-like term



⇒ **Color confinement**

No isolated quarks



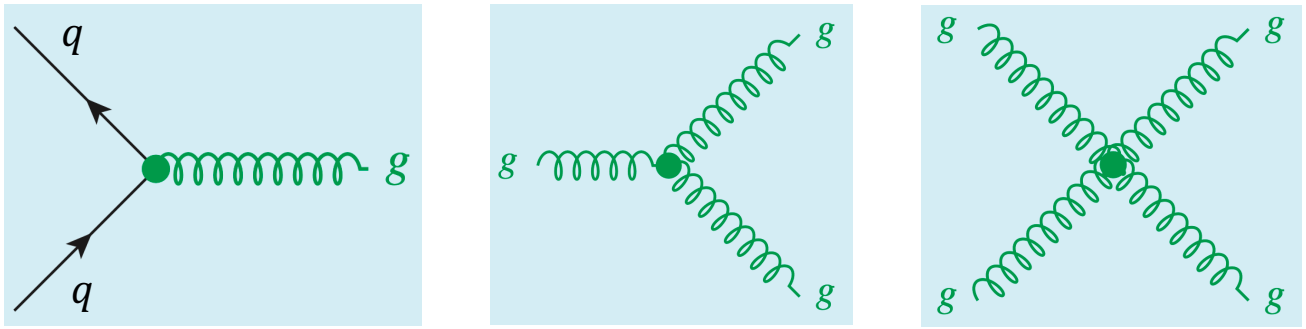
Very short range force ~ 1 fm

Interaction coupling: $\alpha_s \simeq 1$ at low energy scales → **Nonperturbative!**

Gauge Interactions: Strong (QCD)

Strong interaction, $\mathbf{SU(3)_C}$

Basic vertices:

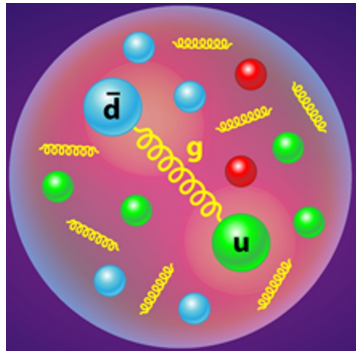


q = quarks

gluon self-interactions

Hadron (colorless bound state of quarks): meson ($q\bar{q}$), baryon (qqq)

mediator of nuclear force



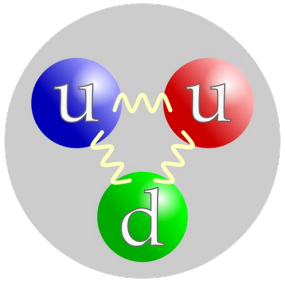
π^+ meson

Gauge Interactions: Strong (QCD)

Strong interaction, $\mathbf{SU(3)_C}$

u (2.3 MeV), d (4.8 MeV)

- **Nuclei: nuclear force mediated by mesons**



Proton

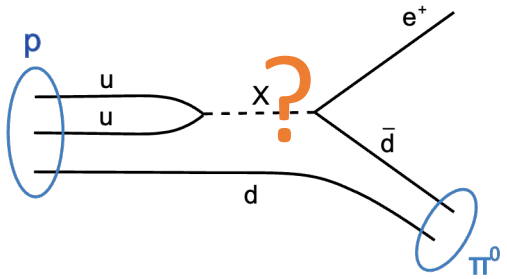
mass: 0.938 GeV

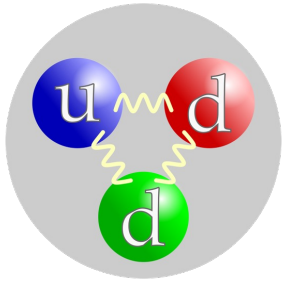
spin: 1/2

electric charge: $+1$

size: 1.68 fm $\simeq 5.07\ \mathrm{GeV}^{-1}$

lifetime $> 1.6\times10^{34}$ years





Neutron

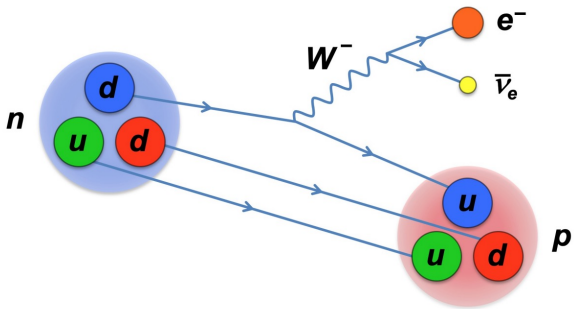
mass: 0.940 GeV

spin: 1/2

electric charge: 0

lifetime $\simeq 8.8\times10^{2}$ s

(stable inside the nucleus)



Yukawa Interactions

Interaction between fermions mediated by a scalar boson

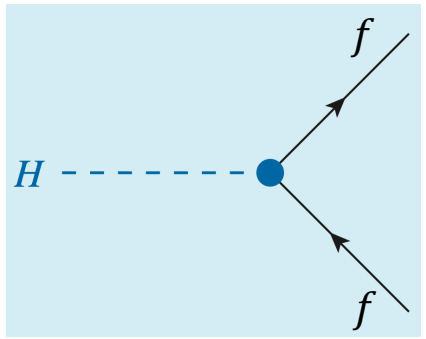
Force mediator: **Higgs boson** (spin $= 0$, mass $m_h \simeq 125$ GeV, electric charge $= 0$)

- Potential: $V(r) = -y^2 \frac{e^{-m_h r}}{r}$ → **Universally attractive**

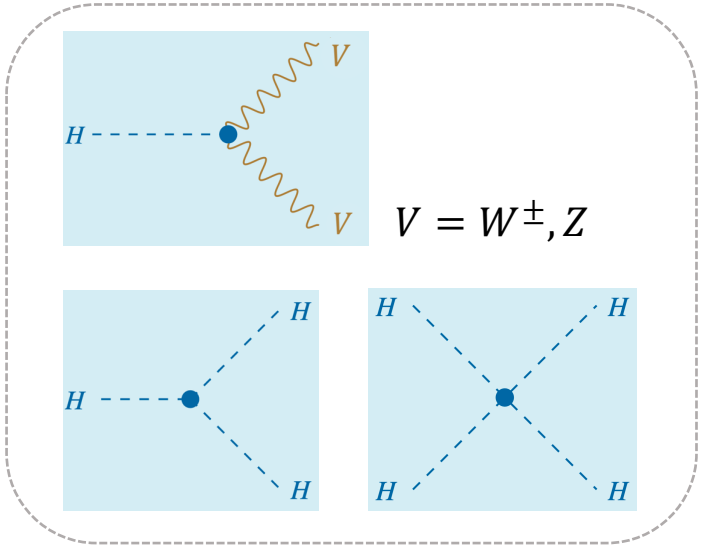
  Short range force due to massive mediator

  Higgs mechanism: Yukawa coupling ∝ (fermion mass)

- **Vertices:**



f = quarks, charged leptons



Gauge Couplings

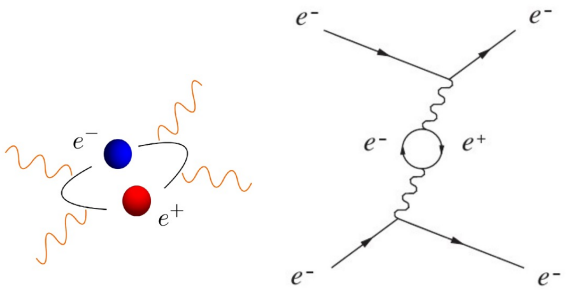
One-loop renormalization group evolution (RGE)

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i \quad \text{where } \alpha_i \equiv \frac{g_i^2}{4\pi} \text{ for } i = 1,2,3 \text{ and } t = \ln \mu$$

Beta function coefficients in the SM with n_f families

$$b_i = \left(\frac{1}{10} + \frac{4}{3} n_f, \; -\frac{43}{6} + \frac{4}{3} n_f, \; -11 + \frac{4}{3} n_f \right)$$

vacuum polarization

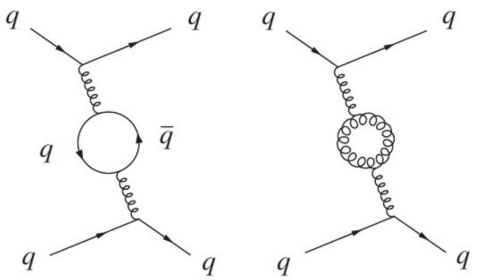


dipole (virtual electron and positron)

- Contribution to the beta function coefficient

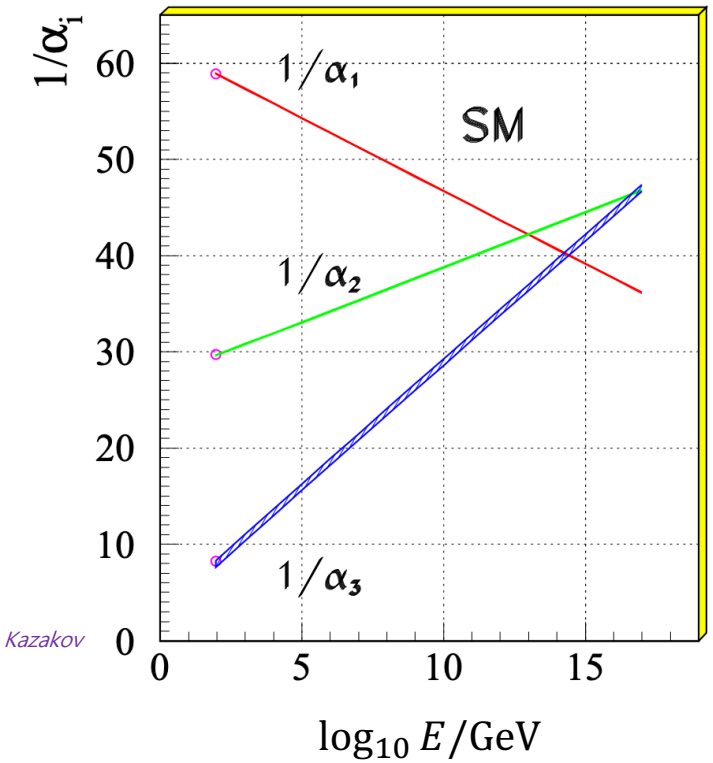
  Gauge interactions with fermions and bosons give a **positive contribution** (**screening**)

  Gauge self-interactions give a **negative contribution** (**anti-screening**)



Gauge Couplings

RGE of gauge couplings



D. I. Kazakov

Grand Unified Theory (GUT) around 10^{15}GeV?

quantization of electric charge, magnetic monopole, …

Standard Model

Higgs Sector

Spontaneously Broken Gauge Symmetry

Complex scalar with gauge U(1) symmetry

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \left(D_\mu\phi\right)\left(D^\mu\phi^*\right) - V(\phi,\phi^*) \text{ with } V = -\frac{1}{2}\mu^2\phi\phi^* + \frac{1}{4}\lambda\phi^2\phi^{*2}$$

Spontaneous breaking: $\langle|\phi|\rangle = \frac{v}{\sqrt{2}}$ with $v = \frac{\mu}{\sqrt{\lambda}}$

- **Canonical field basis**

(1) Lagrangian in terms of $\rho(x), a(x)$ and $A_\mu(x)$ where $\phi = \frac{1}{\sqrt{2}}\left(\mathrm{v} + \rho(x)\right)e^{\frac{ia(x)}{\sqrt{2}v}}$

(2) Gauge transformation $\phi \to e^{iq\alpha}\phi$, $A_\mu \to A_\mu - \frac{1}{g}\partial_\mu\alpha$ by taking $\alpha(x) = -\frac{1}{q}\frac{a(x)}{\sqrt{2}}$ *unitary gauge*

$$\Longrightarrow \mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\partial\rho)^2 + \frac{1}{2}q^2g^2v^2A^\mu A_\mu + \frac{1}{2}q^2g^2\rho^2A^\mu A_\mu - V(\rho)$$

Mass term for the gauge boson

Longitudinal model = NG boson $a(x)$

Higgs mechanism!

Higgs Sector

Higgs field in the Standard Model

Spin=0 (scalar), $\mathbf{SU(3)_C}$ singlet, $\mathbf{SU(2)_L}$ doublet, hypercharge $= 1$

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- **Lagrangian for the Higgs**

$$\mathcal{L}_H = \left(D_\mu H\right)^\dagger (D^\mu H) - V(H, H^\dagger) \quad \text{with} \quad V\left(H, H^\dagger\right) = -\mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2$$

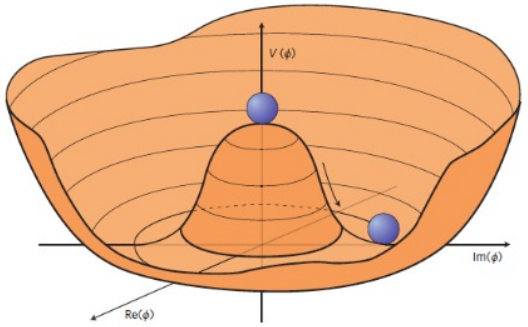
the only mass parameter in the SM

LHC results

$\boldsymbol{\mu \simeq 88}$ **GeV,** $\boldsymbol{\lambda \simeq 0.13}$

at the weak scale

- **Spontaneous symmetry breaking**



$$\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$\Longrightarrow \mathbf{SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}}$$

Higgs Mechanism

Higgs mechanism

- **Massive gauge bosons**

From $(D_\mu H)^\dagger (D^\mu H)$, one obtains $\frac{v^2}{8}\left[g^2\left((W_\mu^1)^2+(W_\mu^2)^2\right)+(gW_\mu^3-g'B_\mu)^2\right]$

mass for W boson

$$\frac{1}{2}\left(\frac{g\,v}{2}\right)^2 W_\mu^\dagger W^\mu \quad \text{where} \quad W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right)$$

Gauge boson masses: proportional to v

$$m_W = \frac{g\,v}{2}$$

$$m_Z = \frac{v}{2}\sqrt{g^2+g'^2}$$

mass for Z boson

$$Z_\mu \equiv \frac{1}{\sqrt{g^2+g'^2}}\left(gW_\mu^3 - g'B_\mu\right)$$

Higgs Mechanism

Higgs mechanism

- **Massive fermions**

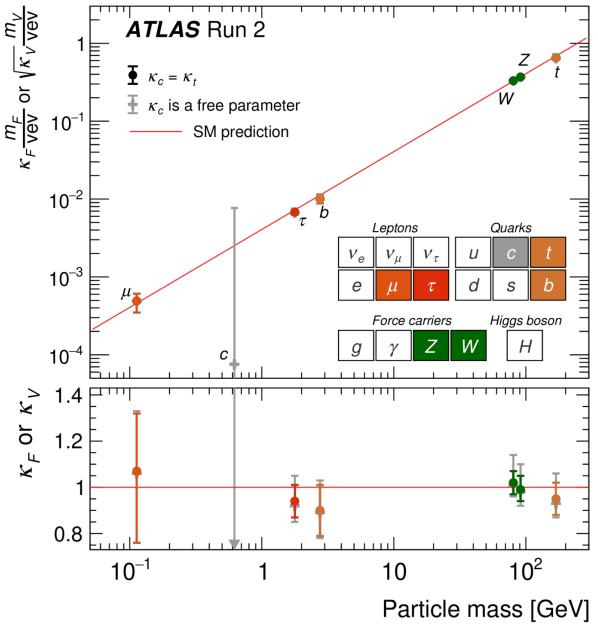
  From the Yukawa interactions, one obtains

$$\frac{y_e\ v}{\sqrt{2}}(\bar{e}_L e_R + \bar{e}_R e_L) + \frac{y_u\ v}{\sqrt{2}}(\bar{u}_L u_R + \bar{u}_R u_L) + \frac{y_d\ v}{\sqrt{2}}(\bar{d}_L d_R + \bar{d}_R d_L)$$

  → **Masses to charged fermions**

Fermion masses: proportional to v and Yukawa coupling

$$m_f = \frac{y_f v}{\sqrt{2}} \text{ for } f = e_i, u_i, d_i \ (i = 1,2,3)$$



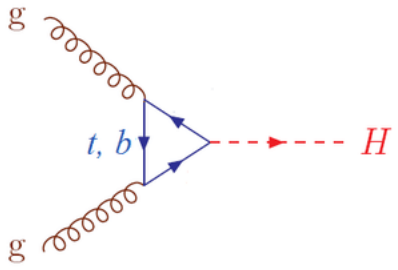
Higgs VEV

$v \simeq 246$ **GeV**

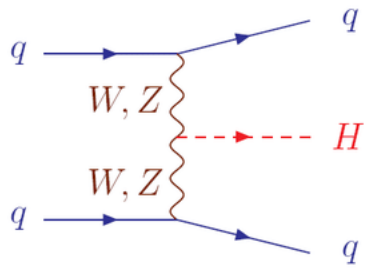
Higgs Boson

Higgs boson discovery at Large Hadron Collider at CERN

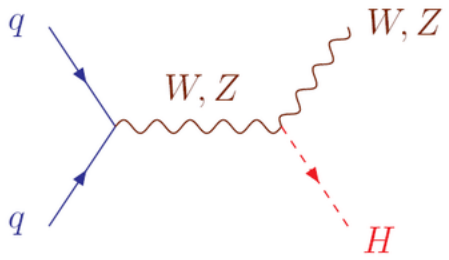
- **Higgs production**



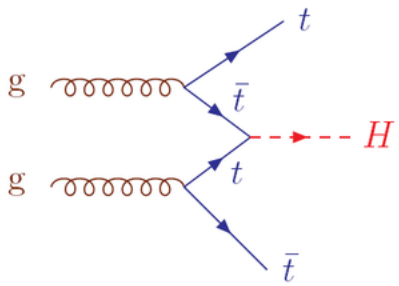
gluon fusion (88%)



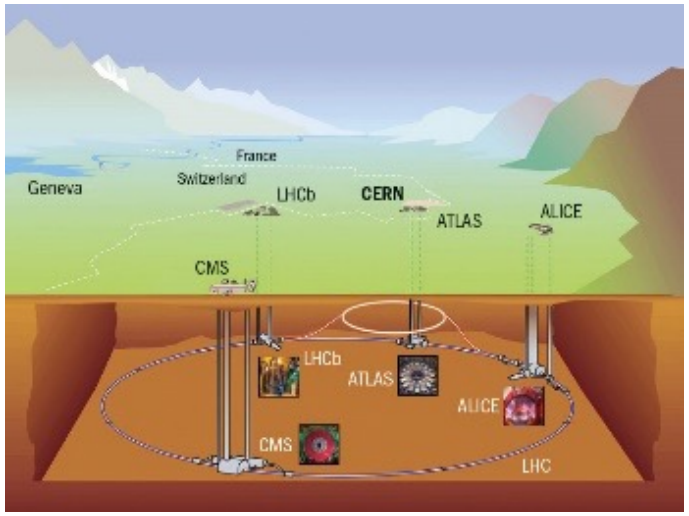
vector boson fusion (7%)

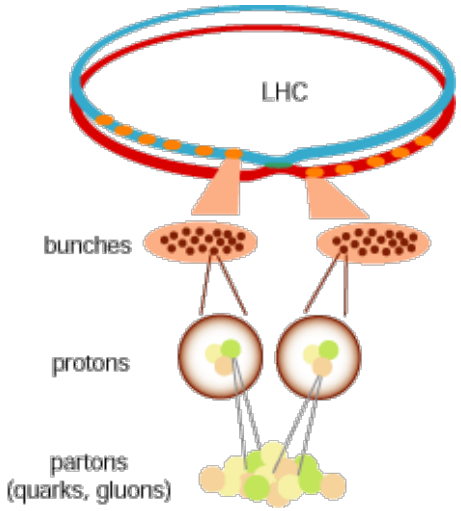


associated production with W/Z (3%)



associated production with top (1%)

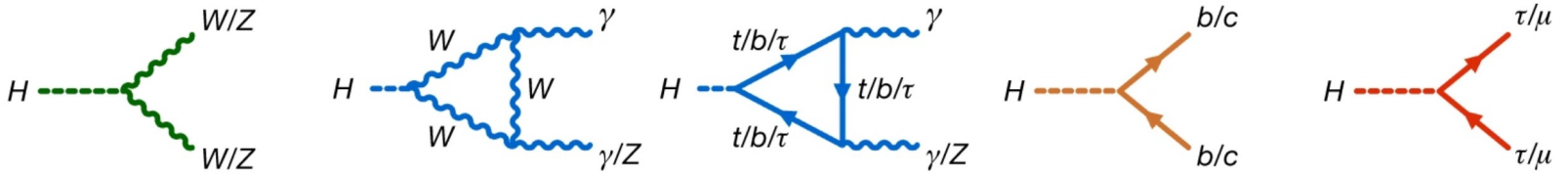


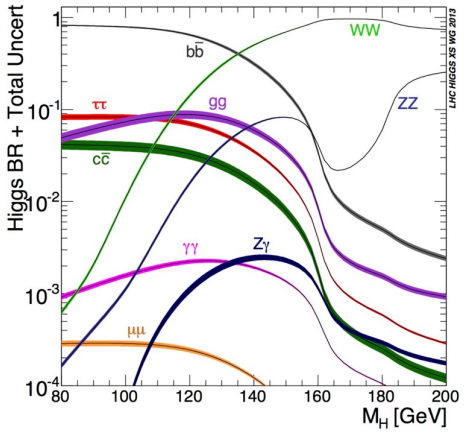


Higgs Boson

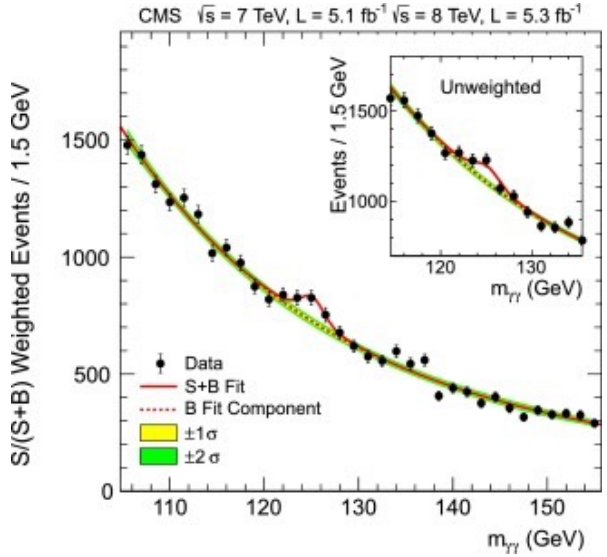
Higgs boson discovery at Large Hadron Collider at CERN

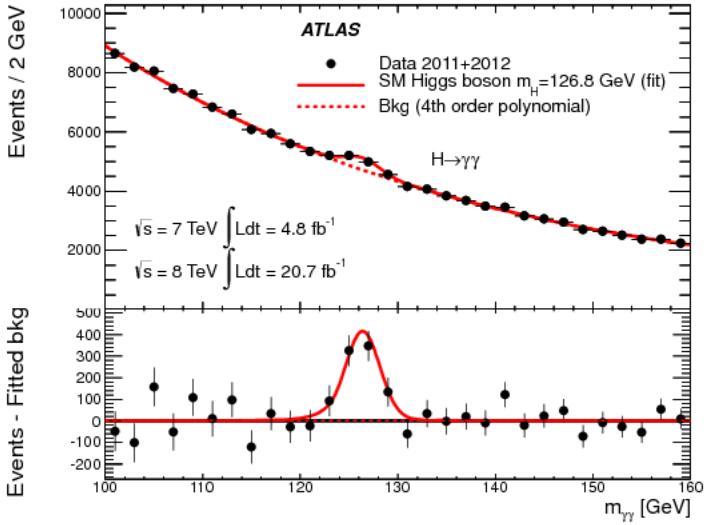
- **Higgs decay** (lifetime 1.6×10^{-22} sec)





- **Higgs boson signals**





Higgs boson mass

$$m_h \simeq 125\text{GeV}$$

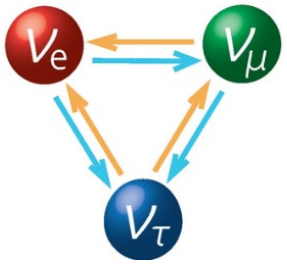
Beyond the Standard Model

Observational Hints

Beyond the SM

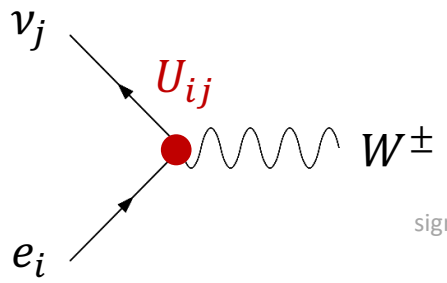
Experimental evidences

Neutrino Masses and Mixing

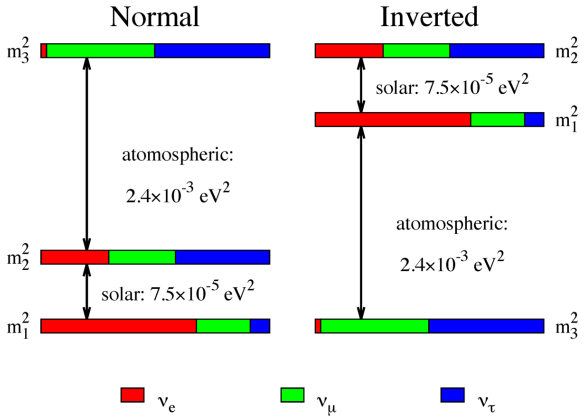


Neutrino oscillation experiments

Weak eigenstate = mixture of mass eigenstates



sign from matter effects →



parameter	best fit $\pm1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	$7.59^{+0.20}_{-0.18}$	7.24–7.99	7.09–8.19
Δm^2_{31} $[10^{-3}\text{eV}^2]$	2.45 ± 0.09 $-(2.34^{+0.10}_{-0.09})$	$2.28 - 2.64$ $-(2.17 - 2.54)$	$2.18 - 2.73$ $-(2.08 - 2.64)$
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	0.28–0.35	0.27–0.36
$\sin^2\theta_{23}$	0.51 ± 0.06 0.52 ± 0.06	0.41–0.61 0.42–0.61	0.39–0.64
$\sin^2\theta_{13}$	$0.010^{+0.009}_{-0.006}$ $0.013^{+0.009}_{-0.007}$	≤ 0.027 ≤ 0.031	≤ 0.035 ≤ 0.039

Cosmological observations

$\sum m_\nu < 0.1$ eV

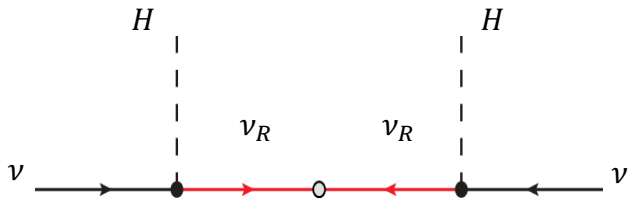
- **Neutrinos are massless in the SM**

Neutrino Mass

How to generate tiny neutrino masses?

- **Majorana** $\nu = \nu^{-}$ $(\Delta L = 2)$

  Seesaw mechanism: $L = y\,\bar{\ell}_L \tilde{H} \nu_R + \frac{1}{2} M_R \overline{\nu_R} \nu_R^c \;\rightarrow\; L_{eff} = \frac{y^2}{M_R}\left(\bar{\ell}_L \tilde{H}\right)\left(\tilde{H}^T \ell^c\right)$



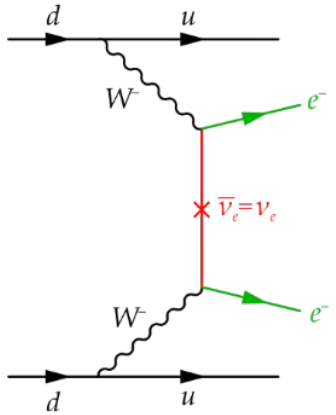
$$m_\nu \simeq \frac{y^2 \langle H \rangle^2}{M_R} \sim 0.1\text{eV}$$

seesaw scale: $M_R \sim y^2 \times 10^{14}$GeV

- Dirac $\nu \neq \nu^{-}$ $(\Delta L = 0)$

  Tiny Yukawa coupling: $L = y\,\bar{\ell}_L \tilde{H} \nu_R \;\rightarrow\; m_\nu = y\langle H \rangle \sim 0.1\text{eV}$

  $\beta\beta 0\nu$ to test Majorana nature of neutrino:



Beyond the SM

Dark Matter

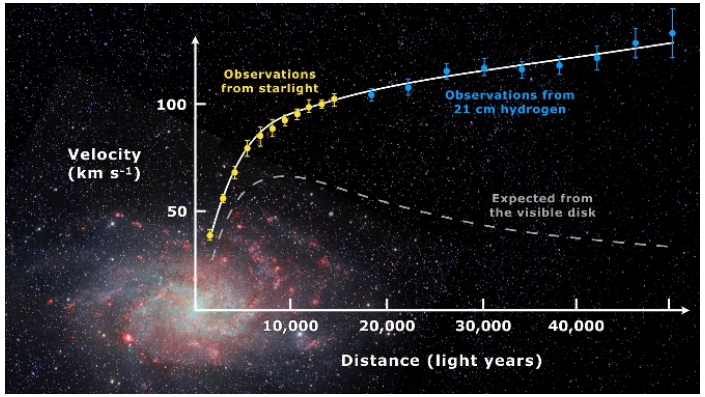
Non-baryonic cold dark matter

Observed motions of stars and galaxies

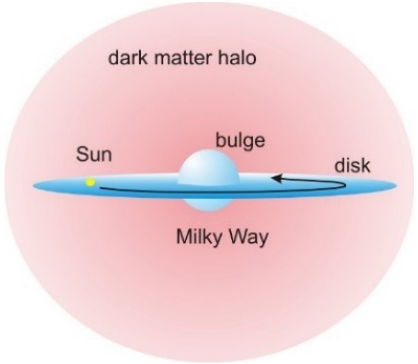
Mass distribution measured with gravitational lensing



Bullet cluster: gravitational lensing + X-ray

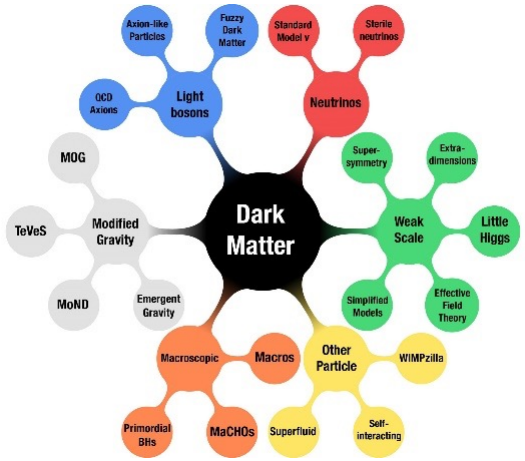


Rotation curve of galaxy



- **No candidate for cold dark matter in the SM**

Dark Matter



Various models in a broad range of masses

Stability + Production mechanism

Active researches on dark matter detection strategy

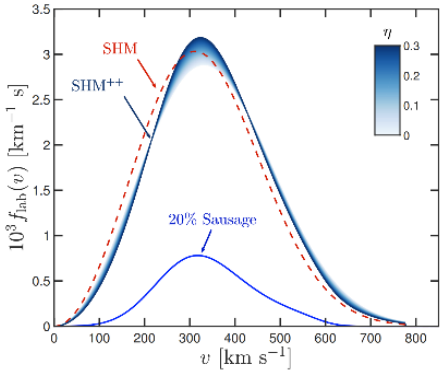
Dwarf galaxies

- Bosonic: de Broglie wavelength $<$ kpc $\Rightarrow$ heavier than 10^{-22}eV
- Fermionic: phase space density limit by Pauli exclusion principle $\Rightarrow$ heavier than about keV

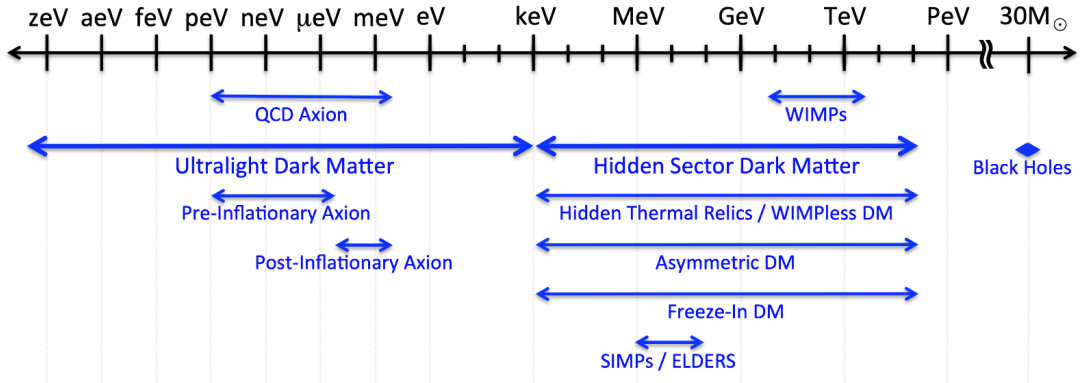
If once thermalized: structure formation $\Rightarrow$ heavier than keV

DM velocity distribution

Standard halo model

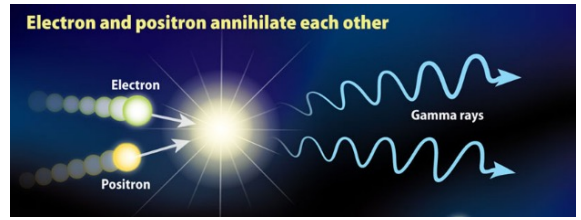


Evans, O'Hare, McCabe



Beyond the SM

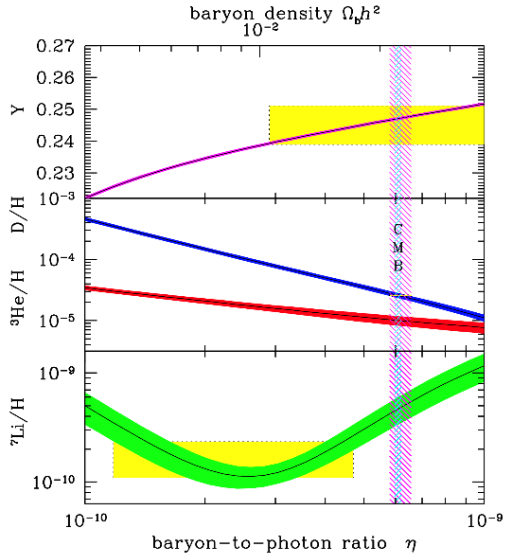
Baryon Asymmetry

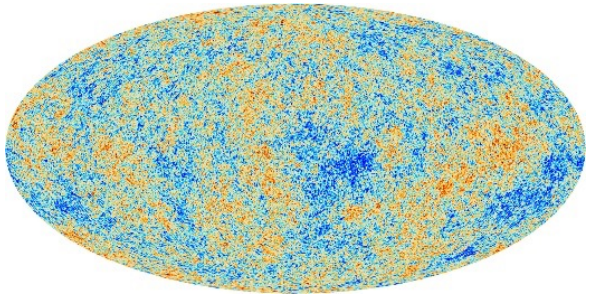


Non-observation of gamma ray burst from matter-antimatter annihilation

Big Bang Nucleosynthesis: abundance of light elements

Cosmic Microwave Background: sound speed of baryon-photon fluid



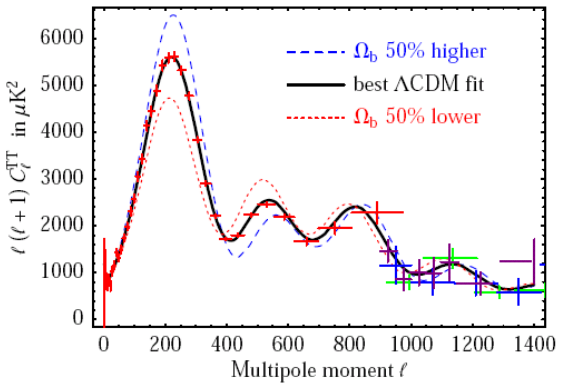


deuterium

helium-3,4,

lithium-7

temperature anisotropy (angular scale)



Baryogenesis

Baryogenesis

Sakharov conditions in CPT conserving background:

***B* violation**, **C and CP violation**, and B violating interactions **out of thermal equilibrium**

→ **Not enough in the SM**

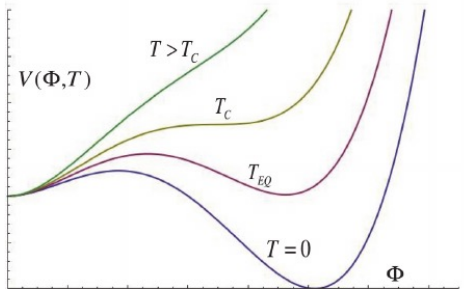
- **Electroweak phase transition:**

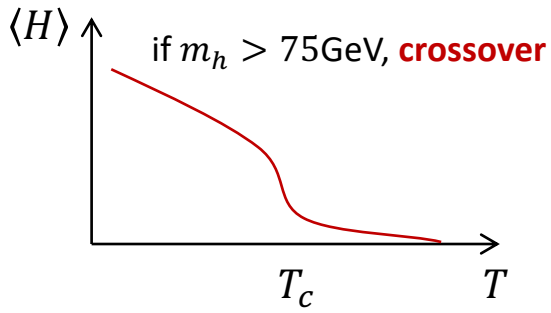
  (free potential) $= V +$ (thermal effects)

$$\Delta V_{\mathrm{TH}} \simeq \frac{1}{2} C_h T^2 h^2 - \frac{2}{3} E_h T h^3$$

$$C_h \simeq \frac{2m_W^2 + m_Z^2 + 2m_t^2}{4v^2} \simeq 0.16$$

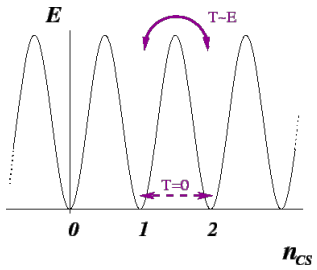
$$E_h \simeq \frac{3(2m_W^3 + m_Z^3)}{8\pi v^3} \simeq 1.4 \times 10^{-2}$$





Last period affecting baryon asymmetry

because B+L violation by rapid EW sphaleron transition in symmetric phase



Beyond the SM

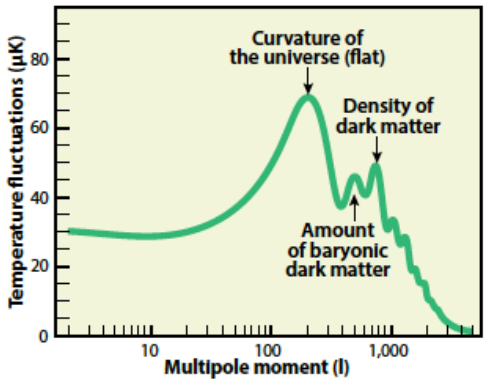
Cosmological Inflation

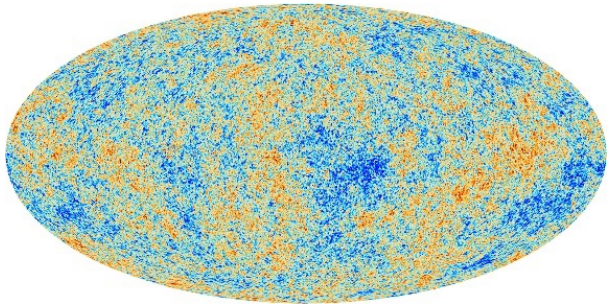
Exponential expansion & reheating for the standard hot Big Bang model

- Homogeneous and isotropic universe on large scales
- Nearly scale-invariant, Gaussian density perturbations

power spectrum of density perturbation

$\log_k P_\rho(k) \sim 1$



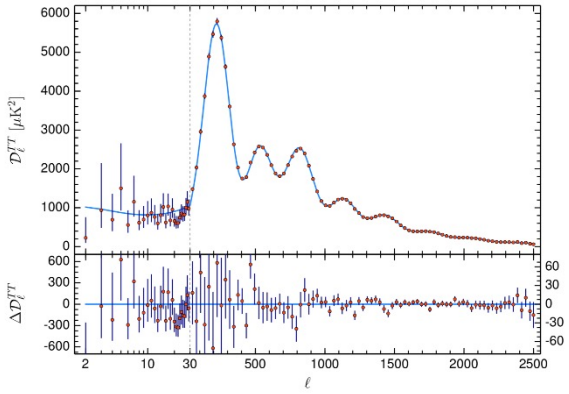


$\frac{\delta T}{T} \sim 10^{-5}$

sourced by density fluctuations

Temperature power spectrum

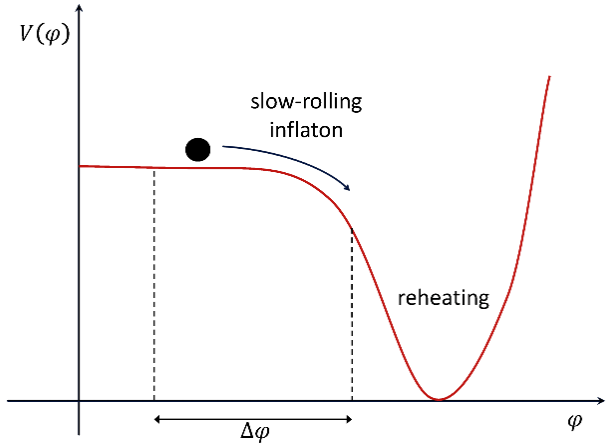
Planck 2018



- **Difficult to implement inflation in the SM**

Cosmic Inflation

Slow-roll inflation



Inflation during slow-roll era with $\epsilon \ll 1$ and $|\eta| \ll 1$

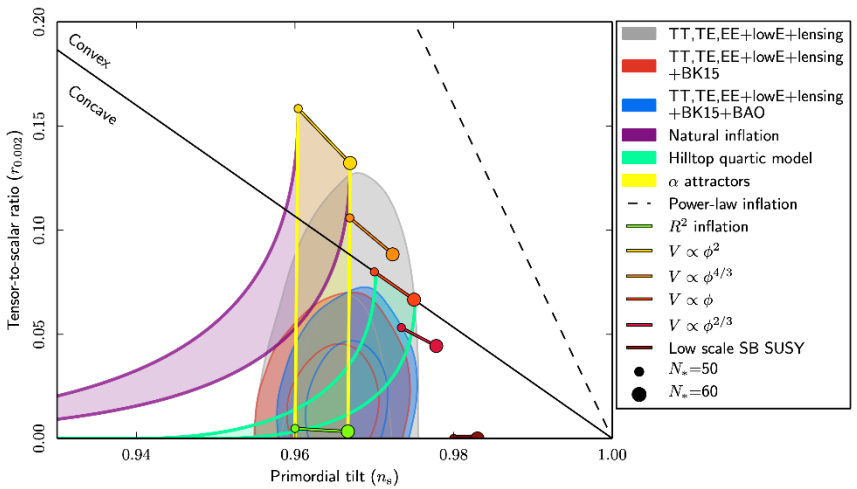
$$\epsilon \equiv \frac{1}{2} M_{Pl}^2 \left(\frac{V'}{V}\right)^2, \qquad \eta \equiv M_{Pl}^2 \frac{V''}{V}$$

Reheating: coherent oscillations of inflaton to convert energy to SM particles

- **Planck constraints**

$$n_s \approx 1 + 2\frac{V''}{V} - 3\left(\frac{V'}{V}\right)^2$$

$$r = \frac{A_t}{A_s} \approx 8\left(\frac{V'}{V}\right)^2$$



Beyond the Standard Model

Theoretical Puzzles

Beyond the SM

Theoretical puzzles

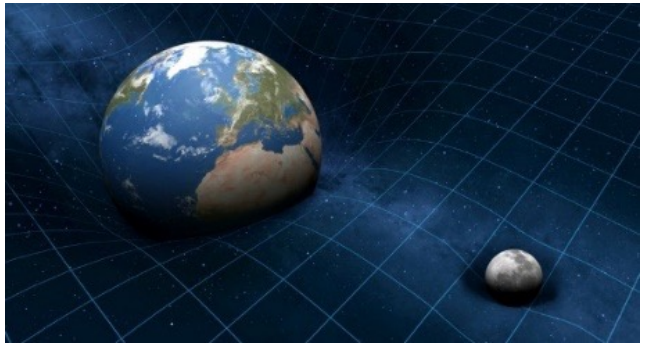
Quantum Gravity

- General relativity

  gravity = spacetime curvature

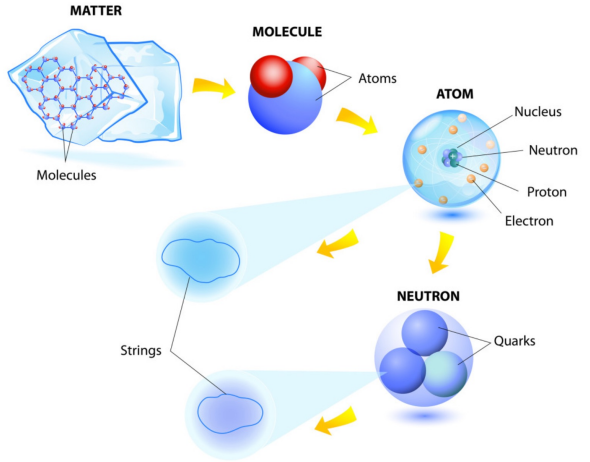
quantized

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi \boldsymbol{T_{\mu\nu}}$$



Gravitational force becomes comparable to other forces at Planck scale: $M_{Pl} \simeq 2.4 \times 10^{18}$GeV

Not renormalizable in QFT → String theory (?)



Beyond the SM

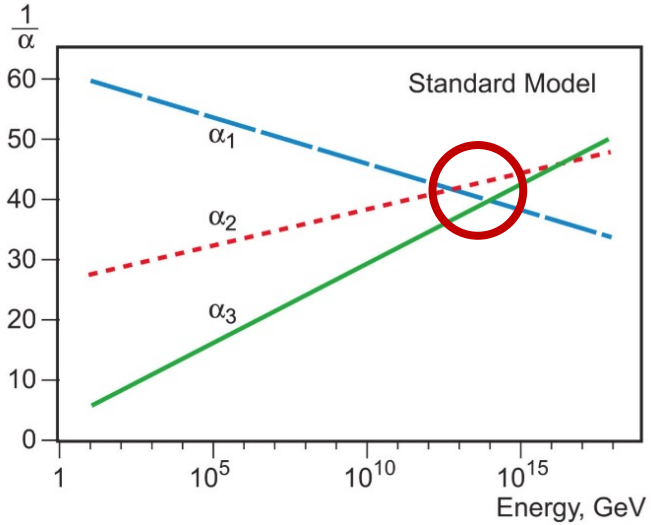
Gauge sector

Grand Unification

Hint from renormalization group flow

Charge quantization (magnetic monopole)

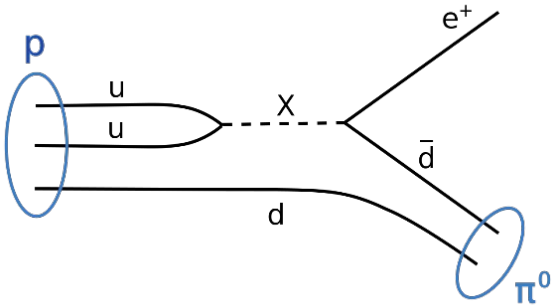
c.f. from anomaly cancellation? but scalars, Dirac fermions



Models: SU(5), SO(10), E_6, …

Proton decay by B and L violations mediated by heavy gauge bosons

Unification scale: $M_{\mathrm{GUT}} \sim 10^{16}\mathrm{GeV}$



proton lifetime > 10^{33} years

$$M_{\mathrm{GUT}} > 10^{16}\mathrm{GeV}\left(\frac{g_{\mathrm{GUT}}^2}{0.5}\right)^{1/2}$$

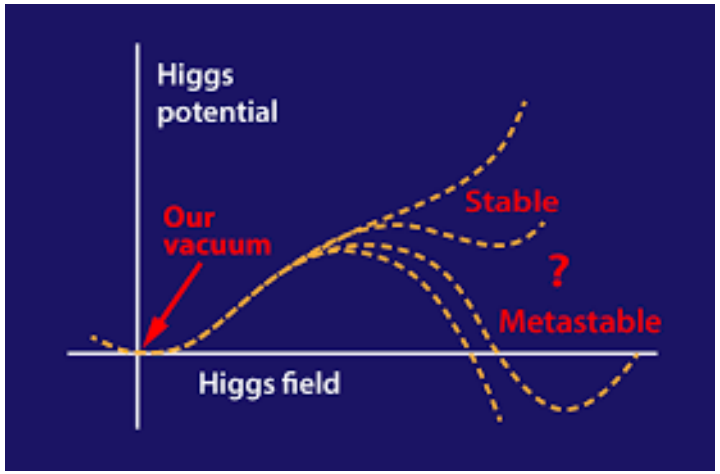
Beyond the SM

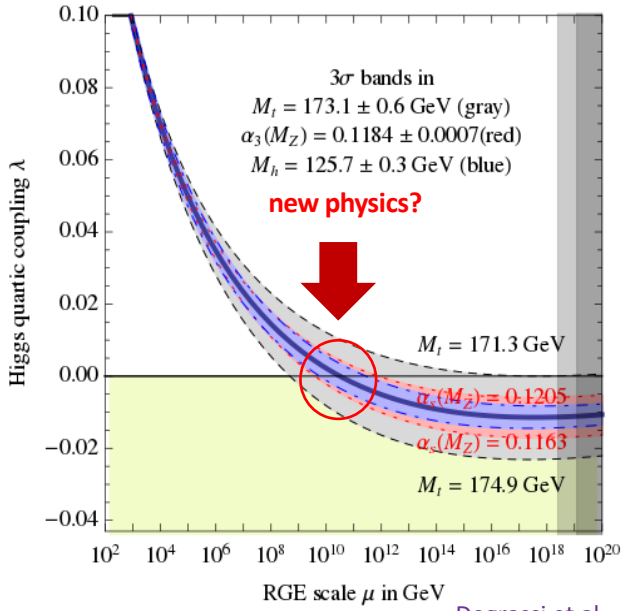
Higgs sector

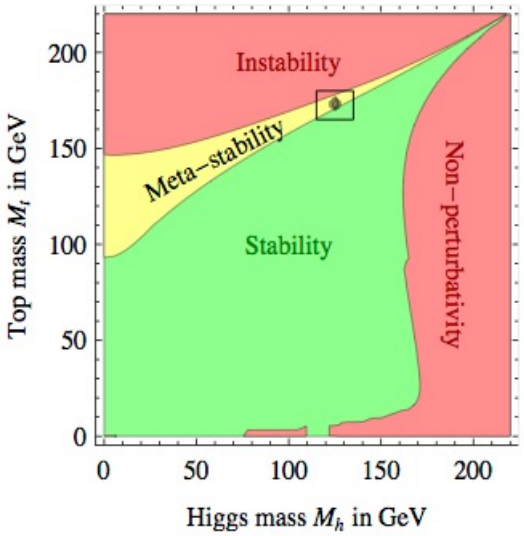
Vacuum stability

Higgs potential: $V = -\mu^2|H|^2 + \lambda|H|^4$ with $\mu \simeq 88\text{GeV}$ and $\lambda \simeq 0.13$ at the weak scale

Higgs quartic coupling: $\frac{d\lambda(t)}{dt} = \frac{1}{16\pi^2}\left[24\lambda^2 - 6y_t^4 + 12\lambda y_t^2 + \cdots\right]$





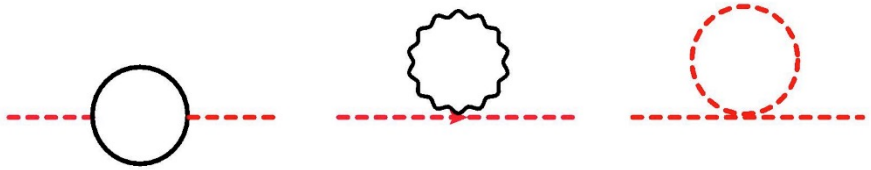


Degrassi et al

Beyond the SM

Higgs sector

Radiative correction to the Higgs mass squared parameter μ^2



$$\delta\mu^2 = \underbrace{-\frac{y^2}{8\pi^2}\Lambda^2}_{\text{fermion}} \quad \underbrace{+\frac{g^2}{8\pi^2}\Lambda^2}_{\text{gauge boson}} \quad \underbrace{+\frac{\lambda}{8\pi^2}\Lambda^2}_{\text{scalar boson}}$$

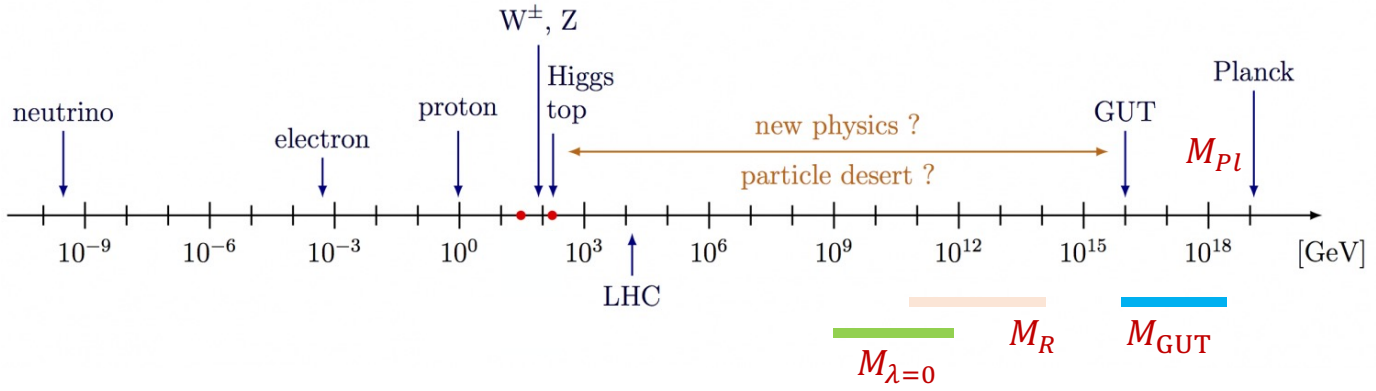
c.f. mass protection

Gauge boson: Gauge symmetry

Fermion: Chiral symmetry

EW hierarchy problem: Sensitive to unknown UV physics

How to stabilize the weak scale?

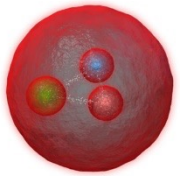


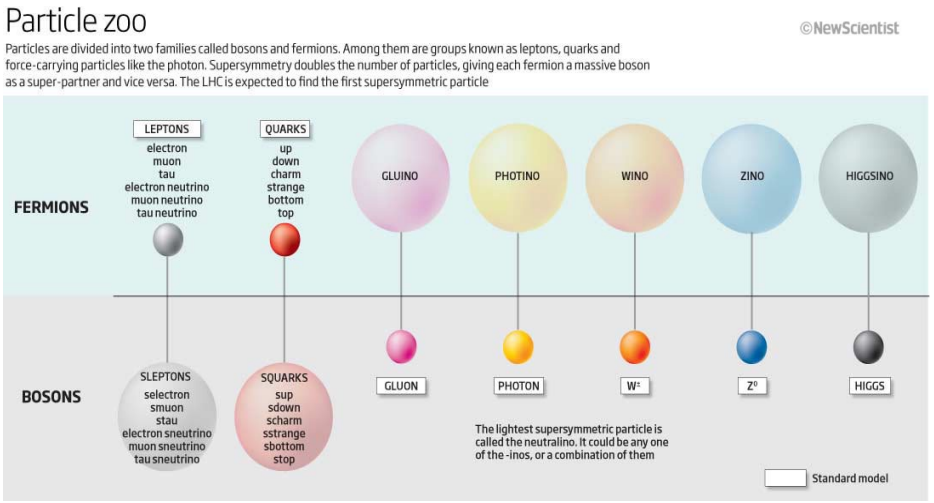
Gauge Hierarchy Problem

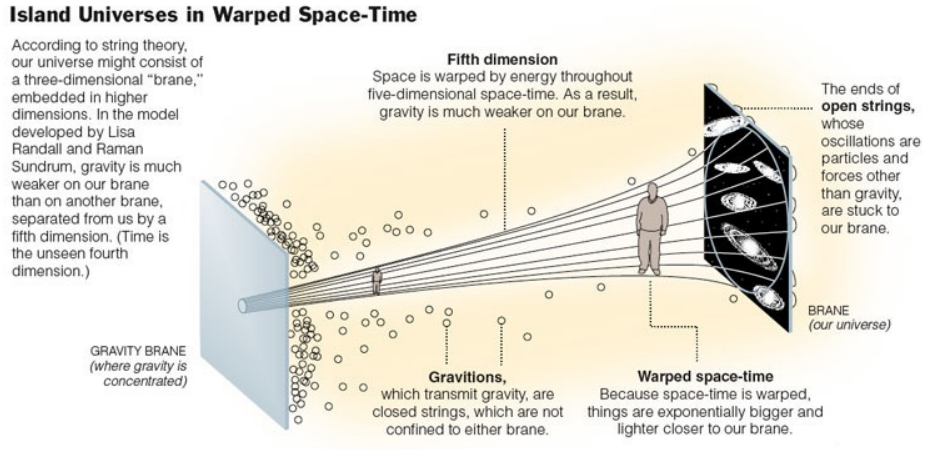
Higgs sector: Unnatural without new physics around TeV

EW naturalness has been considered as a guiding principle to BSM

Interesting ideas: Supersymmetry, Extra dimensions, Strong EWSB, …





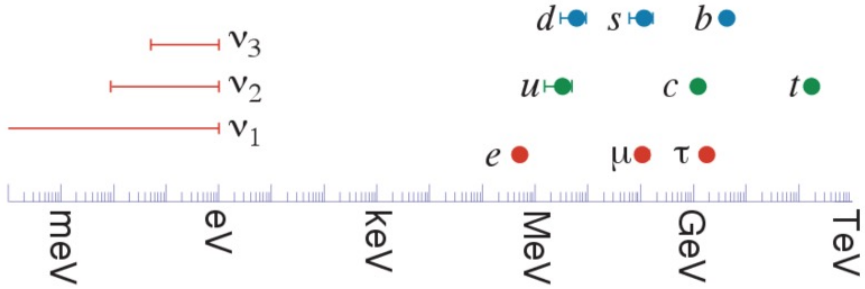


(often) WIMP as cold dark matter if realized around TeV

Beyond the SM

Fermion Sector

Quarks and leptons with Mass Hierarchy



Weak eigenstate = mixture of mass eigenstates

→ **CKM matrix** for quarks, **PMNS matrix** for leptons

→ **Flavor mixing and CP violation in charged weak current interactions**

Quark	Lepton
$\lvert U_{CKM}\rvert \simeq \begin{pmatrix} 1 & 0.2 & 0.005 \\ 0.2 & 1 & 0.04 \\ 0.005 & 0.04 & 1 \end{pmatrix}$	$\lvert U_{PMNS}\rvert \simeq \begin{pmatrix} 0.8 & 0.5 & 0.15 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$
3 small angles	2 large angles and 1 small angle
CP phase: $\delta_{CKM} \simeq 1$	CP phase: $\delta \simeq -\frac{\pi}{2}$ (?)

Beyond the SM

- **Quark sector**

  Why small mixing and hierarchical mass structure?

  Continuous family symmetry spontaneously broken by flavon ϕ

  e.g. Froggatt-Nielson mechanism

  → Dynamical Yukawa couplings: $y_{ij} = \lambda_{ij}\left(\frac{\langle\phi\rangle}{\Lambda}\right)^{n_{ij}}$ with $\lambda_{ij} = O(1)$

- **Lepton sector**

  Why one small and two large mixing angles?

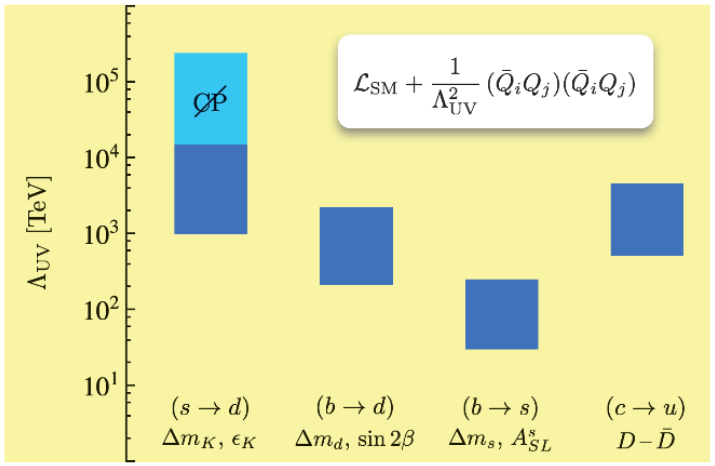
  Different from the quark sector

  - Fine-tuning? Non-Abelian discrete symmetries?
  - Seesaw + Froggatt-Nielson?

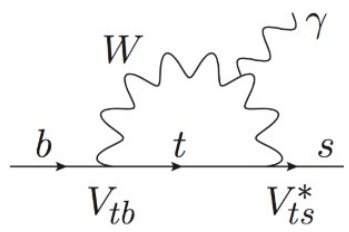
GIM Mechanism

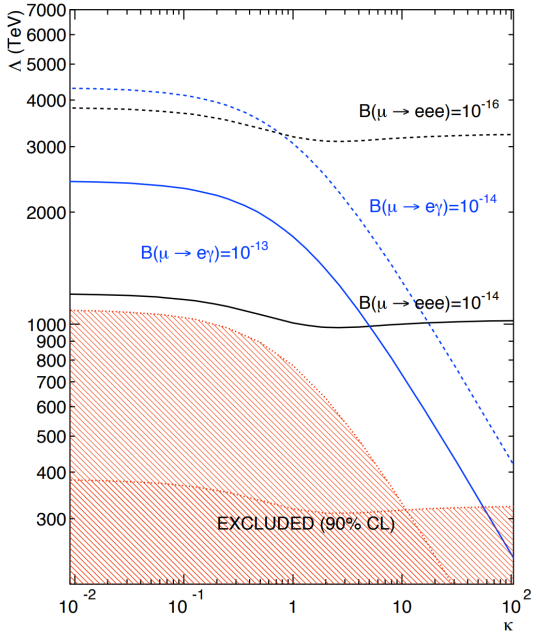
No flavor changing neutral current at tree-level in the SM (GIM)

→ Strong constraints on flavor and CP violations in BSM models



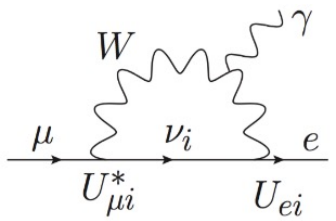
B. Grinstein





$$\frac{m_\mu}{(\kappa+1)\Lambda^2}\bar{\mu}_R \sigma_{\mu\nu} e_L F^{\mu\nu}$$

$$\frac{\kappa}{(1+\kappa)\Lambda^2}\bar{\mu}_L \gamma_\mu e_L \left(\bar{e}\gamma^\mu e\right)$$



Gouvea , Vogel

Beyond the SM

CP violation in the QCD sector

Two CP violating phases

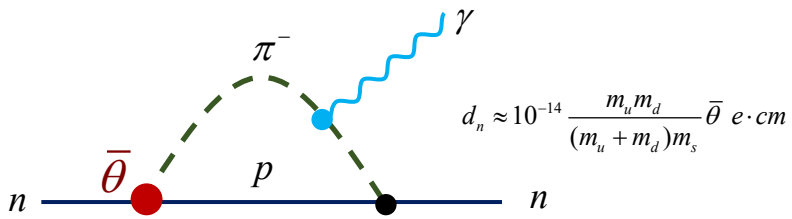
(1) $\boldsymbol{\delta_{\mathrm{CKM}}} \sim \arg(\det[y_u y_u^+, y_d y_d^+]) \simeq 1.2 \pm 0.3$

(2) $\boldsymbol{\bar{\theta}} = \theta + \arg(\det[y_u y_d]) \leftarrow$ **Physical if quarks are massive**

where topological QCD θ-term: $\mathcal{L}_{SM} \ni \frac{\theta}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$ (energy of θ-vacuum due to instantons)

c.f. theta-term of $\mathrm{SU}(2)_L$ → Rotated away by $\mathrm{U}(1)_{B+L}$ transformation

- **Electric dipole moment of neutron**



Experimental bound

$|\bar{\theta}| < 10^{-10}$

Why so tiny?

differet from δ_{CKM}

QCD Axion

Strong CP problem: How to make QCD CP conserving?

Peccei-Quinn solution

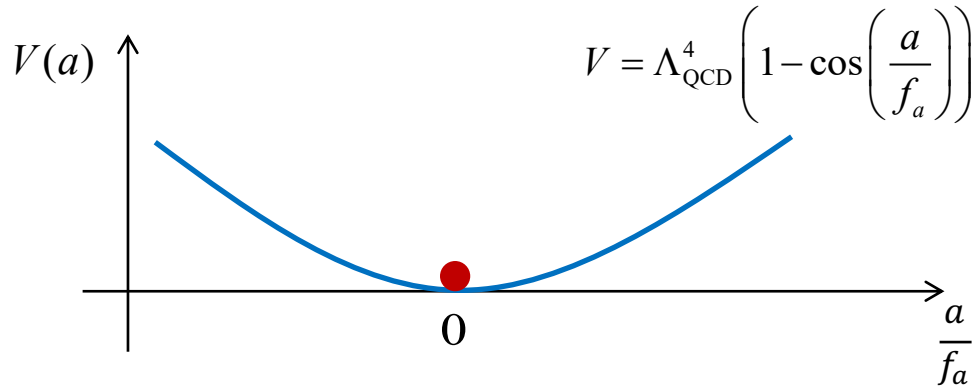
Promote to $\bar{\theta}$ a field, **axion**, anomalous coupled to gluons $\frac{1}{32\pi^2}\frac{a}{f_a}G_{\mu\nu}\tilde{G}^{\mu\nu}$

axion = **NG boson** associated with **spontaneously broken** $\mathbf{U(1)_{PQ}}$ at a energy scale ~ f_a

Properties of the axion: determined by the axion decay constant f_a

- **Axion potential**

Nonperturbatively generated by QCD instants after the QCD phase transition



Dynamical cancellation

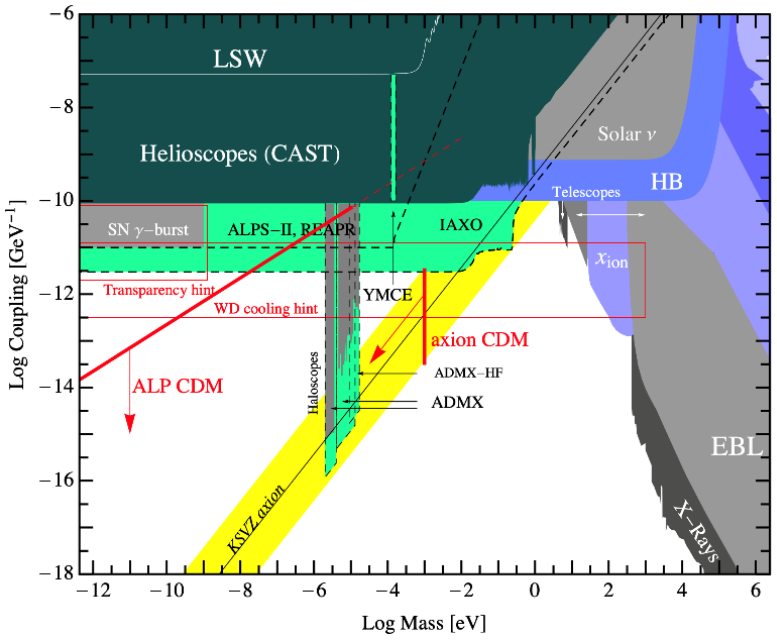
$$\bar{\theta} = \frac{\langle a \rangle}{f_a} = 0$$

QCD Axion

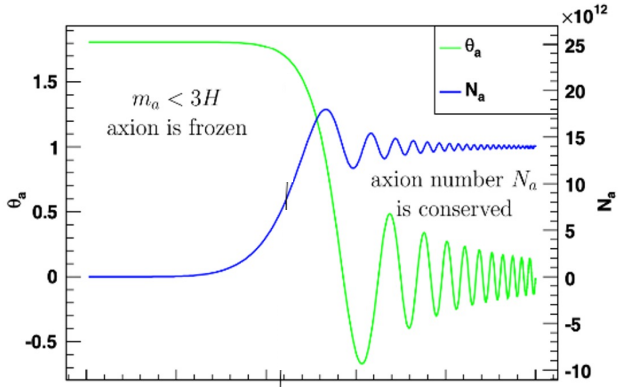
Axion properties

mass	axion-nucleon	axion-photon (model-dependent)
$m_a \sim 5\times10^{-6}\left(\frac{10^{12}\text{GeV}}{f_a}\right)\text{eV}$	$g_{aNN} \sim 10^{-12}\left(\frac{10^{12}\text{GeV}}{f_a}\right)$	$g_{a\gamma\gamma} \sim 10^{-15}\left(\frac{10^{12}\text{GeV}}{f_a}\right)\text{GeV}^{-1}$

- **Axion searches**



axion: dark matter candidate for f_a



Beyond the Standard Model

Extensions

Extension of the SM

Standard Model

Not a complete description of nature

Experimental results: Only **small deviations from the SM** prediction at scale **below TeV**

How to extend the SM? renormalizable interactions

