Introduction to Collider Physics Part I

Chan Beom Park

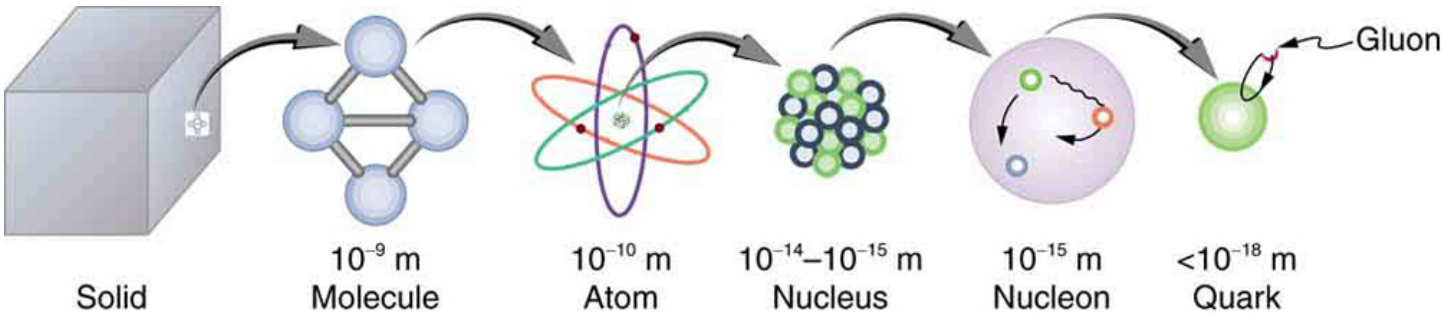
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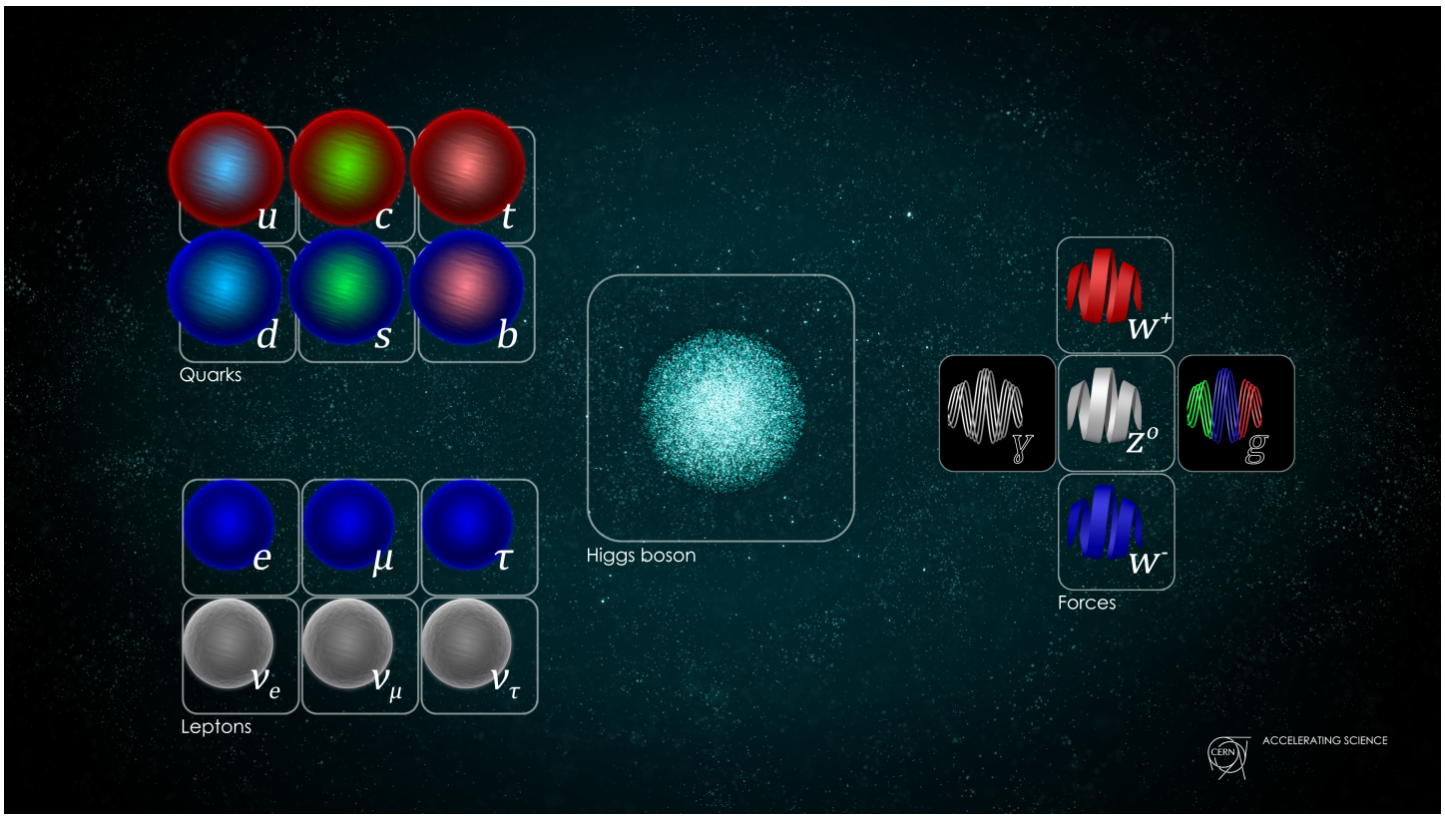
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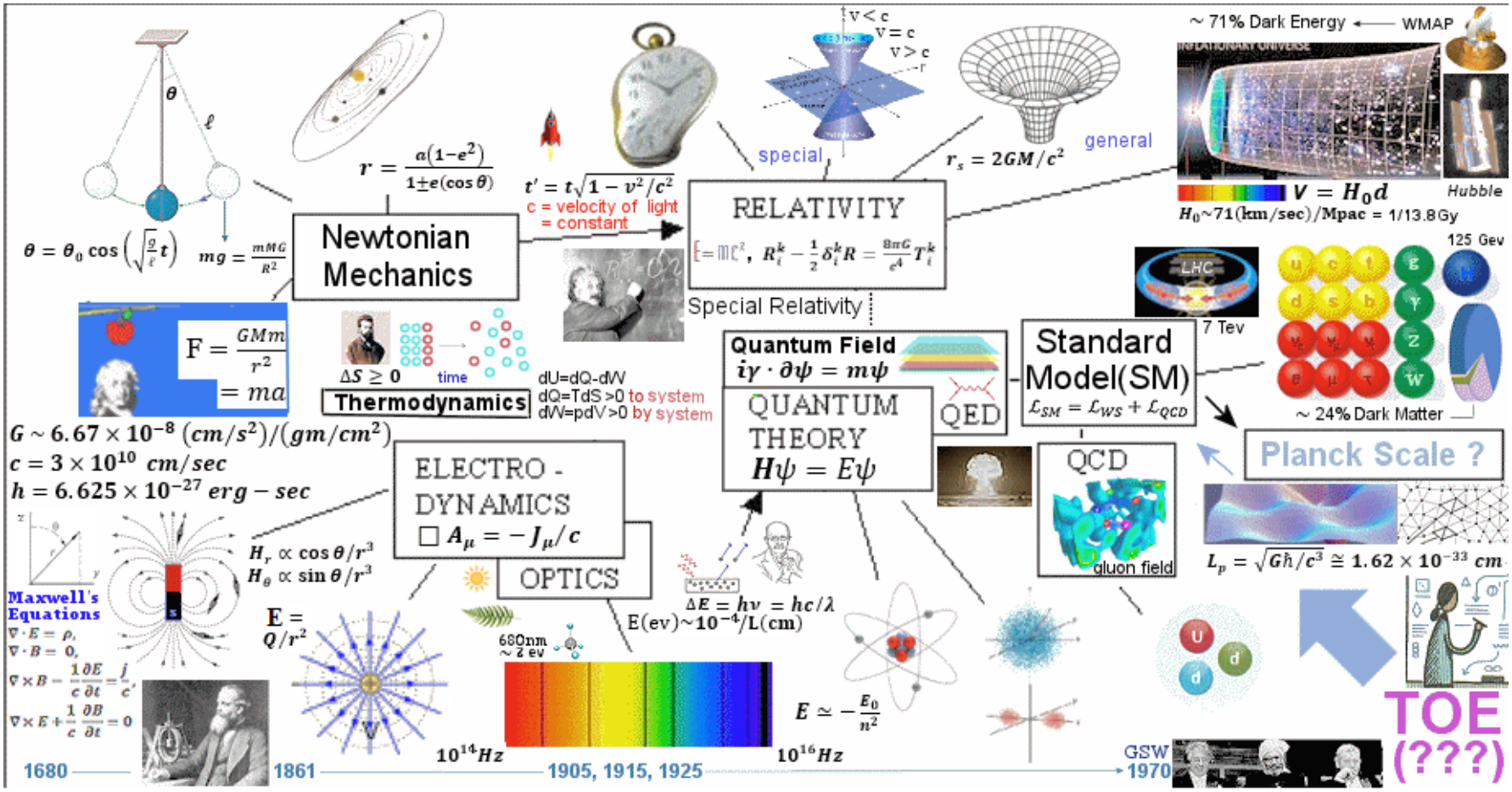
22 August 2023

Scale hierarchy

The quest for elementary particles and fundamental interactions

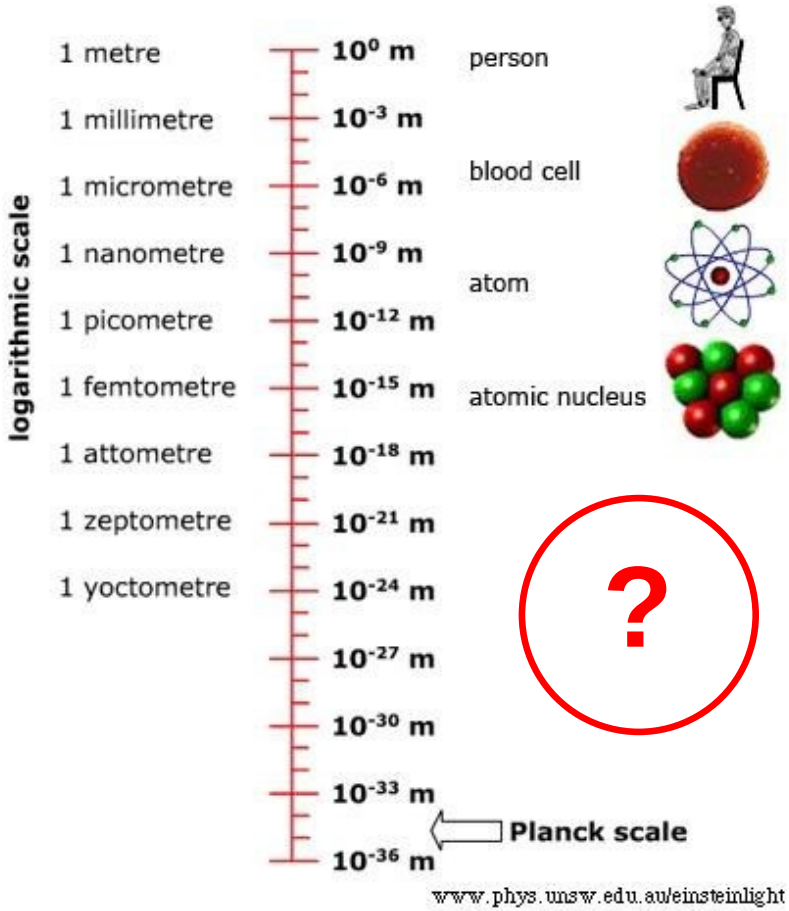





Newtonian Mechanics
RELATIVITY
Special Relativity
special
general
Thermodynamics
dU=dQ-dW
dQ=TdS >0 to system
dW=pdV >0 by system
time
Quantum Field
QUANTUM THEORY
QED
QCD
gluon field
Standard Model(SM)
ELECTRO - DYNAMICS
OPTICS
Maxwell's Equations
c = velocity of light = constant
~ 71% Dark Energy
WMAP
Hubble
~ 24% Dark Matter
125 Gev
7 Tev
LHC
Planck Scale ?
TOE (???)
1680
1861
1905, 1915, 1925
GSW 1970

Scale hierarchy

The quest for elementary particles and fundamental interactions



- What lies beyond the Standard Model?

Search for Physics beyond the Standard Model







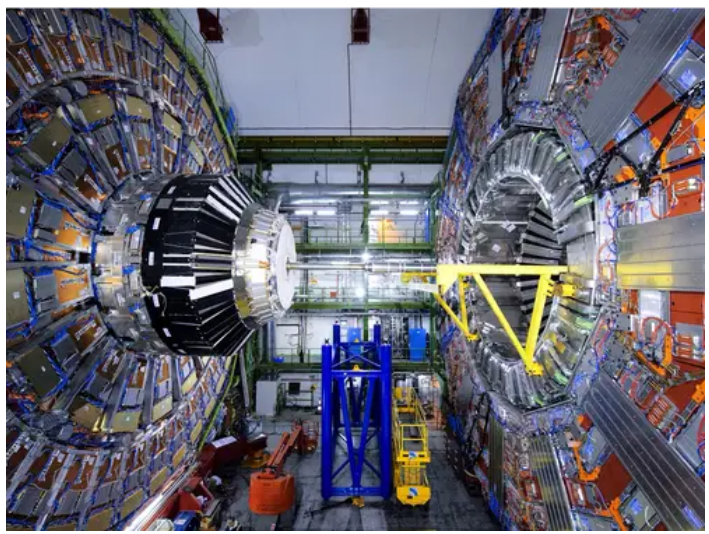
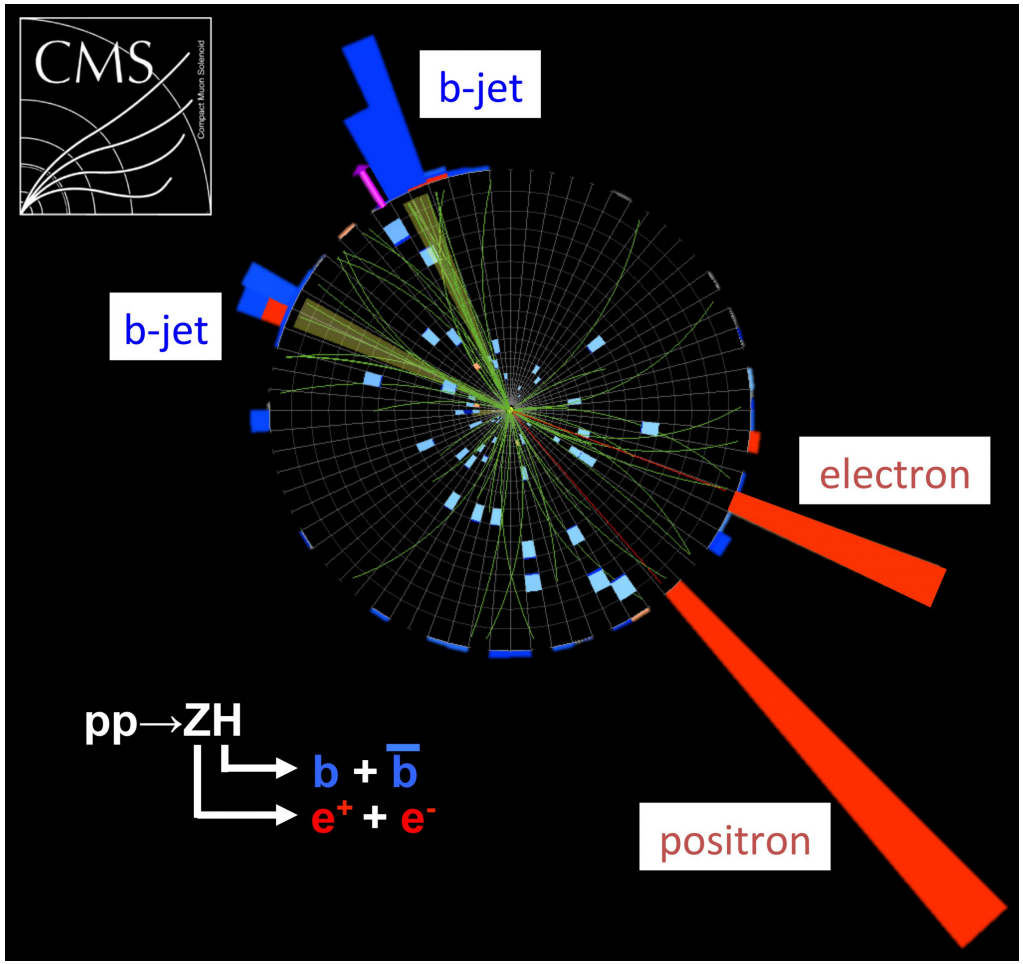




Image credit: science.org

"I think we've got it." — CERN Director General Rolf-Dieter Heuer

- The **Higgs boson** was discovered at the Large Hadron Collider (LHC), CERN, 2012.



- At colliders, we can
  - discover new particles,
  - measure the properties of elementary particles.

Table of contents

1. Basic knowledge for collider physics

2. Phase space and kinematics

References

- Michael E. Peskin, Daniel V. Schroeder, "An Introduction to Quantum Field Theory," Westview (1995).
- Vernon D. Barger, Roger J. N. Phillips, "Collider Physics," CRC Press (1996).
- R. K. Ellis, W. J. Stirling, B. R. Weber, "QCD and Collider Physics," Cambridge (1996).
- Tao Han, "Collider phenomenology: Basic knowledge and techniques," arXiv:hep-ph/0508097.
- Tilman Plehn, "Lectures on LHC Physics," arXiv:0910.4182.
- Maxim Perelstein, "Introduction to Collider Physics," arXiv:1002.0274.
- Matthew D. Schwartz, "TASI Lectures on Collider Physics," arXiv:1709.04533.


LET'S GET STARTED

Collider as a microscope



- A **microscope** can be used to observe small objects.

Collider as a microscope



- A **microscope** can be used to observe small objects.

- The basic components of microscopes:
  - incoming beam of particles (photon or electron),
  - target off of which the particles scatter,
  - detector that collects and processes the scattered particles and presents the data,
  - observer (brain or computer).

Collider as a microscope



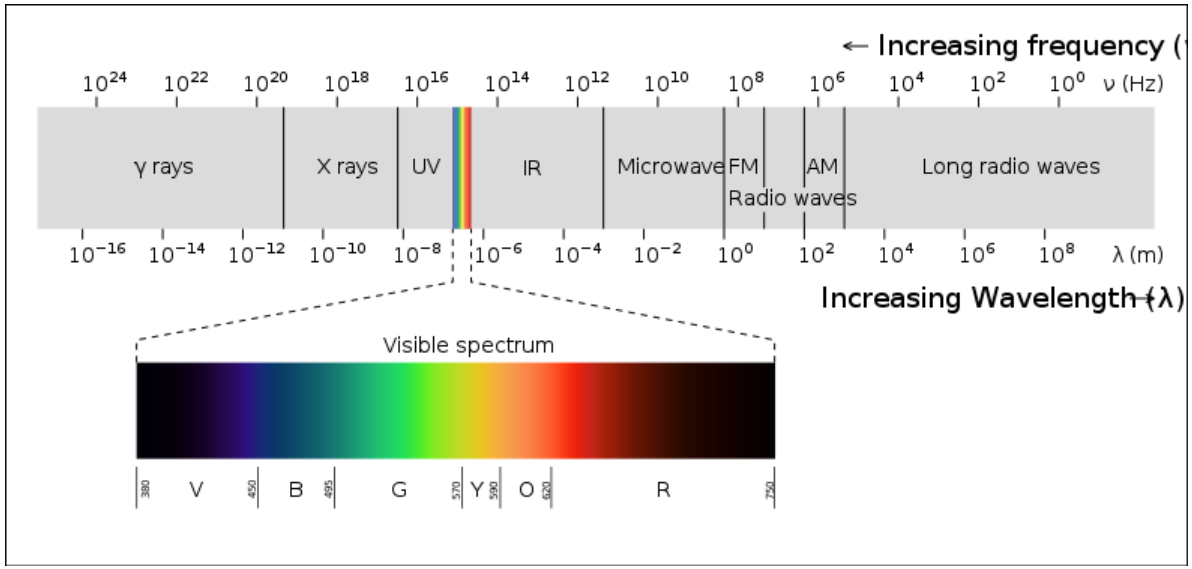
- A **microscope** can be used to observe small objects.
  - Particle physics studies the nature of *smallest* elementary particles.
- The basic components of microscopes:
  - incoming beam of particles (photon or electron),
  - target off of which the particles scatter,
  - detector that collects and processes the scattered particles and presents the data,
  - observer (brain or computer).
- Collider experiments consist of the same components.

Collider as a microscope

- Could we look at the inside of the proton or even *quarks and gluons* under a microscope? (*Why do we need high-energy colliders?*)

Collider as a microscope

- Could we look at the inside of the proton or even *quarks and gluons* under a microscope? (*Why do we need high-energy colliders?*)



- To look into the length scales much smaller than the proton ($\ll 10^{-15}$ m), we must increase the energies of incoming particles.

$$\lambda \propto \frac{1}{E}$$

Colliders are microscopes to explore the structure and the interaction of particles at the shortest possible length scale.

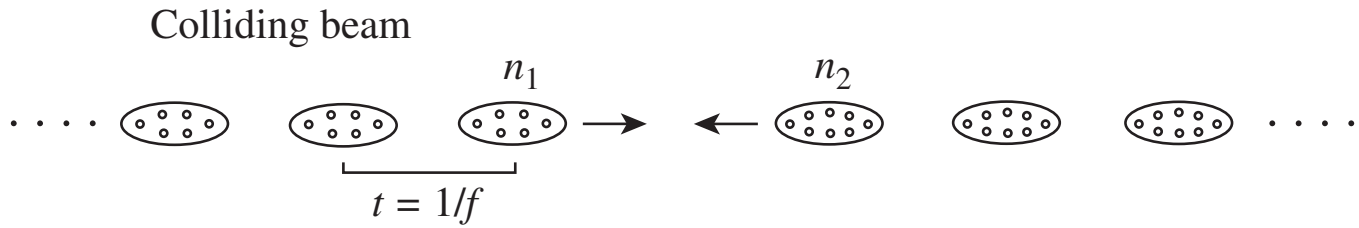
- What to collide?
  - *electrically charged* so that they can be **accelerated** by electric field.
  - *stable* so that they can be put in a **storage** ring for reuse.

Electrons and protons are good candidates for serving as the colliding beams.

- Lepton *vs* hadron colliders.

Collider

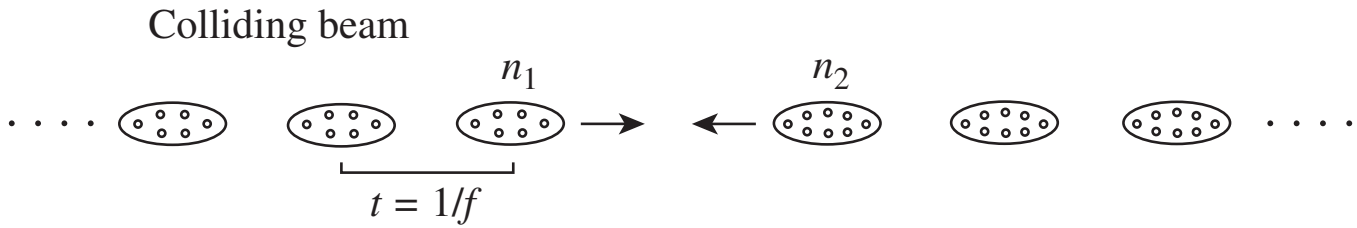
Luminosity



- colliding beam: "bunches" of particles (e or p).
- n_1, n_2: number of particles in each beam,
- f: beam crossing frequency,
- A: beams' cross-sectional area.

Collider

Luminosity



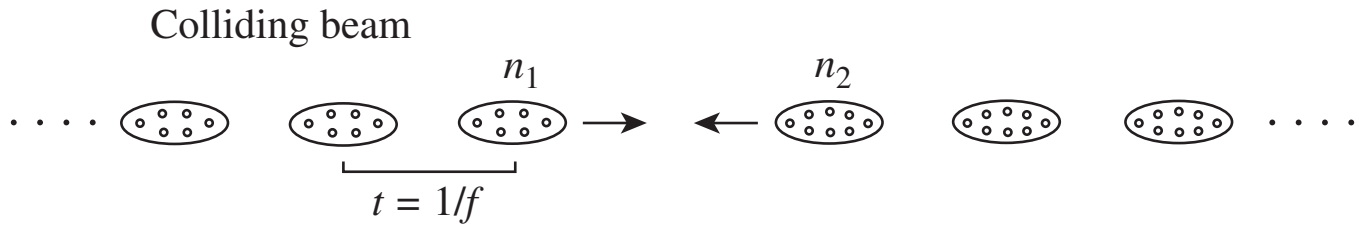
- colliding beam: "bunches" of particles (e or p).
- n_1, n_2: number of particles in each beam,
- f: beam crossing frequency,
- A: beams' cross-sectional area.
- the **number** of particles passing each other
  per **unit time**
  per **transverse area** = (instantaneous) luminosity,

$$\boxed{\mathcal{L} \propto \frac{f n_1 n_2}{A}}$$

(Do not confuse with Lagrangian density $\mathcal{L}$!)

Collider

Cross section



# of scattering "events" per unit time = reaction rate

$$= \boxed{\sigma \times \mathcal{L}}$$

- σ: (scattering) cross section.
- The number of scattering events:

$$\boxed{N_{\text{event}} = \sigma \times \int \mathcal{L}(t)dt}$$

- In a nutshell, collider = experiment counting N_{event}.

Collider

Cross section

$$\mathcal{L} \propto \frac{f n_1 n_2}{A}$$

$$N_{\text{event}} = \sigma \times \int \mathcal{L}(t) dt$$



Collider

Cross section: Natural unit

$$\mathcal{L} \propto \frac{f n_1 n_2}{A}$$

$$N_{\text{event}} = \sigma \times \int \mathcal{L}(t) dt$$

- The cross section σ has a unit of $(\text{area}) = (\text{length})^2$ and the integrated luminosity $\int \mathcal{L} dt$ is $(\text{length})^{-2}$.

Collider

Cross section: Natural unit

$$\boxed{\begin{aligned} \mathcal{L} &\propto \frac{f n_1 n_2}{A} \\ N_{\text{event}} &= \sigma \times \int \mathcal{L}(t) dt \end{aligned}}$$

- The cross section σ has a unit of (area) $=$ (length)^2 and the integrated luminosity $\int \mathcal{L} dt$ is (length)$^{-2}$.
- In particle physics, we use the natural unit:

$$\boxed{\hbar = c = 1}$$

i.e., dimensionless.

$$[\text{energy}] \times [\text{time}] = \frac{[\text{length}]}{[\text{time}]} = 1.$$

Because $E = mc^2 = m$, we have

$$[\text{length}] = [\text{time}] = [\text{energy}]^{-1} = [\text{mass}]^{-1} \Rightarrow \boxed{\sigma \text{ has } [\text{energy}]^{-2} = [\text{mass}]^{-2}}$$

- In the natural unit, the electron mass is 0.511 MeV $(= 9.109 \times 10^{-28}$ g) and proton mass is 1 GeV $(= 1.67 \times 10^{-24}$ g).

Collider

Cross section

$$\mathcal{L} \propto \frac{f n_1 n_2}{A}$$

$$N_{\text{event}} = \sigma \times \int \mathcal{L}(t) dt$$

- Typically, we use 1 b ("barn") = 10^{-24} cm^2 = 100 fm^2 for the cross section.

In the year 1942, physicists discussed a unit name for the cross sections of atomic nuclei.

- Candidates: "Oppenheimer" or "Bethe" after leading physicists.
- Discussions took place on the campus of Purdue University in Indiana. The "barn" is a dominant feature of Midwestern U.S. farmlands.
  - "Barn" is easier to say than "Oppenheimer."
  - "Bethe" sounds like "beta."
  - The "barn" began appearing in the internal reports at the lab in Los Alamos.



(K. Metropolis, Stanford report, July 21, 2004.)

$$\mathcal{L} \propto \frac{f n_1 n_2}{A}$$
$$N_{\text{event}} = \sigma \times \int \mathcal{L}(t) dt$$

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$$1 \text{ cm}^2 = 10^{24} \text{ b} = 10^{33} \text{ nb}$$

Collider

Cross section

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- 1 year $\simeq \pi \times 10^7$ s. If a collider only operates about $1/\pi$ of the time a year, we can take 1 year $\to 10^7$ s.

$$\mathcal{L} = 10 \text{ fb}^{-1}/\text{year} \simeq 1 \text{ nb}^{-1} \text{ s}^{-1} = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$$

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- **Exercise** 1: $\sigma(e^+e^- \to Z) \simeq 40$ nb on m_Z.
  How many events do we have for $\int \mathcal{L} dt = 200$ pb^{-1}?

$$\mathcal{L} \propto \frac{f n_1 n_2}{A}$$
$$N_{\text{event}} = \sigma \times \int \mathcal{L}(t) dt$$

- The integrated luminosity is *designed by* experimentalists.
- The physics of particles is encoded into the cross section (that theorists must calculate).

$$\sigma \propto \int (\text{phase space}) \times |\text{matrix element}|^2$$

- phase space: a space of all possible momenta of final-state particles, subject to conservation of energy and momentum.
- matrix element ($\mathcal{M}$): scattering amplitude of the given reaction process.
  - it contains *all* the physics that depends on the details of the particle interactions.

Table of contents

1. Basic knowledge for collider physics

2. **Phase space and kinematics**

Phase space

Consider a process:

$$a + b \to 1 + 2 + \cdots + n.$$

Phase space

Consider a process:

$$a + b \to 1 + 2 + \cdots + n.$$

- We have to impose the condition of four-momentum conservation:

$$\boxed{E_a + E_b = \sum_{i=1}^{n} E_i, \quad \boldsymbol{p}_a + \boldsymbol{p}_b = \sum_{i=1}^{n} \boldsymbol{p}_i}$$

with $E_i^2 = |\boldsymbol{p}_i|^2 + m_i^2$ $(i = a, b, 1, \cdots, n)$.

Phase space

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  with $E_i^2 = |\boldsymbol{p}_i|^2 + m_i^2$ $(i = a, b, 1, \cdots, n)$.

- The condition defines $3n - 4$ dimensional surface of the *final-state* momenta $\boldsymbol{p}_i$, called the phase space.
- The phase-space integral is

$$\boxed{\int d\Pi_n = \left(\prod_{i=1}^{n} \int \frac{d^3 p_i}{2E_i (2\pi)^3} \right) (2\pi)^4 \delta^{(4)} \left(p_a + p_b - \sum_{i=1}^{n} p_i \right)}$$

- For a $1 \to n$ process, there are $3n - 7$ variables by ignoring three-momentum of the decaying particle.

Phase space

One-body final state

$$\boxed{\int d\Pi_n = \left(\prod_{i=1}^{n} \int \frac{d^3 p_i}{2E_i (2\pi)^3}\right) (2\pi)^4 \delta^{(4)} \left(p_a + p_b - \sum_{i=1}^{n} p_i\right)}$$

For $a + b \to 1$,

$$d\Pi_1 = (2\pi) \frac{d^3 p_1}{2E_1} \delta^{(4)} (p_a + p_b - p_1)$$

Note that (**Exercise 2**)

$$\frac{d^3 p_1}{2E_1} = d^4 p_1 \delta(p_1^2 - m_1^2) \theta(E_1),$$

where $d^3 p_1 = dp_{1x} dp_{1y} dp_{1z}$ and $d^4 p_1 = dE_1 d^3 p_1$. Therefore,

$$\begin{aligned} d\Pi_1 &= 2\pi \int d^4 p_1 \delta(p_1^2 - m_1^2) \delta^{(4)} (p_a + p_b - p_1) \\ &= \boxed{2\pi \delta(s - m_1^2)} \end{aligned}$$

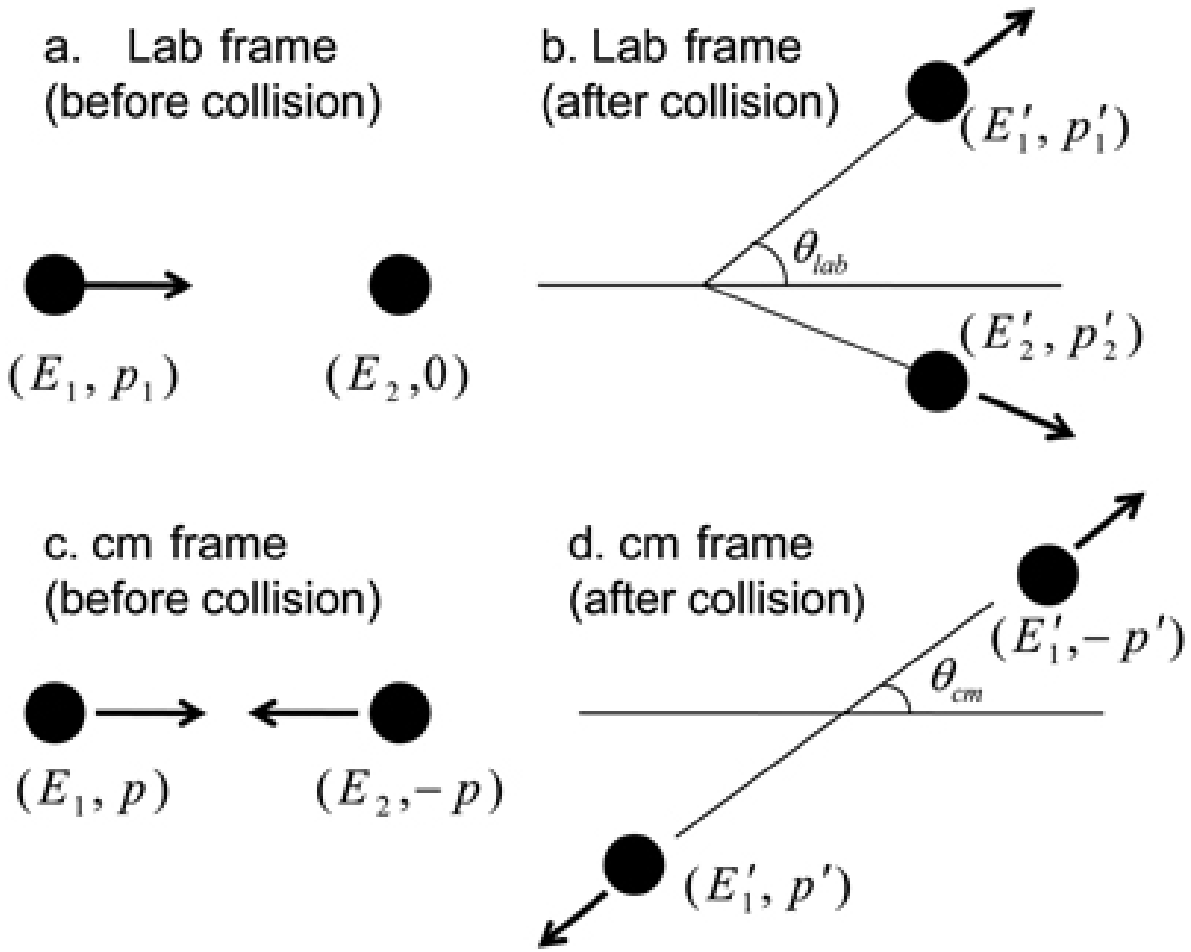
where $s = (p_a + p_b)^2$ is the collision energy or center-of-mass (CM) energy squared.

Phase space

Two-body final state

For $a + b \to 1 + 2$, consider the CM frame:

$$\boldsymbol{p}_1 + \boldsymbol{p}_2 = \mathbf{0} \quad \text{or} \quad \boldsymbol{p}_1 = -\boldsymbol{p}_2 \quad (\text{'back-to-back'})$$



Phase space

Two-body final state

For $\underline{a + b \to 1 + 2}$,

$$
\begin{aligned}
d\Pi_2 &= \frac{1}{(2\pi)^2} \frac{d^3p_1 d^3p_2}{2E_1 2E_2} \delta^{(4)}(p_a + p_b - p_1 - p_2) \\
&= \frac{1}{(2\pi)^2} \frac{dp_1 p_1^2 d\Omega}{2E_1 2E_2} \delta(E_{\mathrm{CM}} - E_1 - E_2) \quad (\because \boldsymbol{p}_2 = -\boldsymbol{p}_1) \\
&= \frac{p_1^2 d\Omega}{16\pi^2 E_1 E_2} \left(\frac{p_1}{E_1} + \frac{p_1}{E_2} \right)^{-1} \\
&= \boxed{\frac{d\cos\theta d\phi}{16\pi^2} \frac{|\boldsymbol{p}_1|}{E_{\mathrm{CM}}}}.
\end{aligned}
$$

- Recall that the two-body phase space has $3n - 4 = 2$ d.o.f., *i.e.*, the *polar* and *azimuthal* angles θ and ϕ.
- For reactions symmetric about the collision axis (no ϕ-dependence),

$$
d\Pi_2 = \frac{d\cos\theta}{8\pi} \frac{|\boldsymbol{p}_1|}{E_{\mathrm{CM}}}
$$

Phase space

Two-body final state: CM frame

In the CM frame

$$p_1^\mu = (E_1, \boldsymbol{p}_1) = (E_1, p_{1x}, p_{1y}, p_{1z}), \quad p_2^\mu = (E_2, -\boldsymbol{p}_1) = (E_2, -p_{1x}, -p_{1y}, -p_{1z}),$$

where

$$E_1^2 = |\boldsymbol{p}_1|^2 + m_1^2, \quad E_2^2 = |\boldsymbol{p}_1|^2 + m_2^2,$$

and $E_1 + E_2 = E_{\mathrm{CM}} = \sqrt{s}$.

Phase space

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- Then, we find that (**Exercise** 3)

$$E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \quad E_2 = \frac{s - m_1^2 + m_2^2}{2\sqrt{s}}, \quad |\boldsymbol{p}_1| = \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{2\sqrt{s}},$$

where $\lambda(x, y, z) \equiv (x - y - z)^2 - 4yz = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ is the Källén function.

Phase space

Two-body final state: CM frame

In the CM frame

$$p_1^\mu = (E_1, \boldsymbol{p}_1) = (E_1, p_{1x}, p_{1y}, p_{1z}), \quad p_2^\mu = (E_2, -\boldsymbol{p}_1) = (E_2, -p_{1x}, -p_{1y}, -p_{1z}),$$

where

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where $\lambda(x, y, z) \equiv (x - y - z)^2 - 4yz = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ is the Källén function.

- Thus,

$$d\Pi_2 = \frac{d\cos\theta d\phi}{16\pi^2} \frac{|\boldsymbol{p}_1|}{E_{\rm CM}}$$

$$= \frac{d\cos\theta d\phi}{32\pi^2} \frac{\lambda^{1/2}\left(s, m_1^2, m_2^2\right)}{s} = \boxed{\frac{1}{8\pi}\lambda^{1/2}\left(1, \frac{m_1^2}{s}, \frac{m_2^2}{s}\right) dx_1 dx_2}$$

where $\cos\theta = 2x_1 - 1$ and $\phi = 2\pi x_2$ with $0 \le x_1, x_2 \le 1$.

Phase space

Two-body final state

$$\boxed{\begin{aligned} d\Pi_1 &= 2\pi\delta(s - m_1^2), \\ d\Pi_2 &= \frac{1}{8\pi}\lambda^{1/2}\left(1, \frac{m_1^2}{s}, \frac{m_2^2}{s}\right) dx_1 dx_2 \end{aligned}}$$

The phase space volume is scaled down by

$$\boxed{\frac{d\Pi_2}{s\, d\Pi_1} \approx \frac{1}{16\pi^2}}$$

so-called the *phase space suppression* factor.

- The phase space volume with each additional final state particle scales down by the similar factor (∼ loop factor).

Phase space

Two-body final state: Mandelstam variables

The **Mandelstam variables** are a useful set of Lorentz-invariant variables for the two-body kinematics.

For $a + b \to 1 + 2$ (so $p_a + p_b = p_1 + p_2$),

$$\boxed{\begin{aligned} s &= (p_a + p_b)^2 = (p_1 + p_2)^2, \\ t &= (p_a - p_1)^2 = (p_b - p_2)^2 \\ &= m_a^2 + m_1^2 - 2(E_a E_1 - \boldsymbol{p}_a \cdot \boldsymbol{p}_1), \\ u &= (p_a - p_2)^2 = (p_b - p_1)^2 \\ &= m_a^2 + m_2^2 - 2(E_a E_2 - \boldsymbol{p}_a \cdot \boldsymbol{p}_2). \end{aligned}}$$

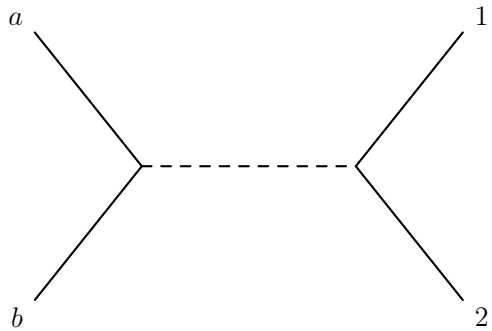
- They are not independent (**Exercise** 4):

$$s + t + u = m_a^2 + m_b^2 + m_1^2 + m_2^2$$

- $E_{\text{CM}} = \sqrt{s}$ is the CM energy:
  e.g., proton-proton collisions of $\sqrt{s} = 13.6$ TeV at the LHC.

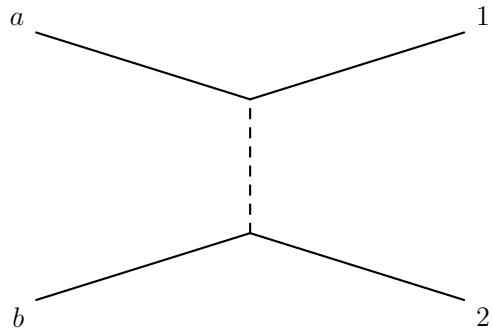
Phase space

Two-body final state: Mandelstam variables



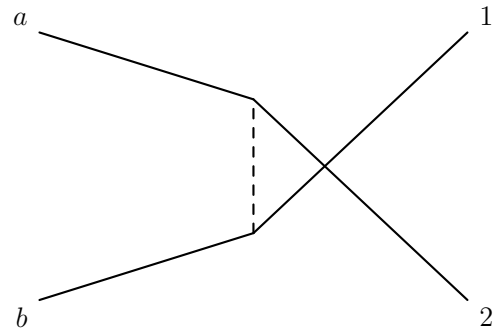
s-channel

$$\mathcal{M} \propto \frac{1}{s - m_\phi^2}$$



t-channel

$$\mathcal{M} \propto \frac{1}{t - m_\phi^2}$$



u-channel

$$\mathcal{M} \propto \frac{1}{u - m_\phi^2}$$

- Each channel leads to a characteristic angular dependence of the cross section.

Phase space

Three-body final state

For

$$a + b \to 1 + 2 + 3$$

i.e., 2-to-3 process, there are $\underline{3n - 4 = 5}$ d.o.f. which is pretty large.

- In practice, three-body decays (1-to-3 process) are often important: $\mu \to e\bar{\nu}_e\nu_\mu$, $h \to WW^* \to W + e\bar{\nu}_e$.
- $\underline{3n - 7 = 2}$ d.o.f. for the three-body decays of a heavy particle at rest $p = (M, \mathbf{0})$.

$$\begin{aligned} d\Pi_3 &= \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}(2\pi)^4\delta^{(4)}(p_1 + p_2 + p_3 - p) \\ &= \frac{1}{8(2\pi)^5}\frac{d^3p_1 d^3p_2}{E_1E_2E_3}\delta(E_1 + E_2 + E_3 - M) \end{aligned}$$

Phase space

Three-body final state

For decays of unpolarized particles (no ϕ dependence),

$$d\Pi_3 = \frac{1}{32\pi^3}\frac{p_1 dp_1}{E_1 E_2 E_3}(p_2 dp_2)(p_1 p_2 d\cos\theta)\delta(E_1 + E_2 + E_3 - M).$$

- $E_{1,2}^2 = p_{1,2}^2 + m_{1,2}^2$ leads to $E_{1,2}dE_{1,2} = p_{1,2}dp_{1,2}$.
- $E_3^2 = p_3^2 + m_3^2 = (\boldsymbol{p}_1 + \boldsymbol{p}_2)^2 + m_3^2 = \boldsymbol{p}_1^2 + \boldsymbol{p}_2^2 + 2p_1 p_2 \cos\theta + m_3^2$

  $\Rightarrow E_3 dE_3 = p_1 p_2 d\cos\theta$ for given p_1 and p_2.

$$\begin{aligned} d\Pi_3 &= \frac{1}{32\pi^3} dE_1 dE_2 dE_3 \delta(E_1 + E_2 + E_3 - M) \quad (\leftarrow E_3 = M - E_1 - E_2) \\ &= \frac{1}{32\pi^3} dE_1 dE_2 \end{aligned}$$

Phase space

Three-body final state

We introduce Lorentz-invariant variables as follows:

(*c.f.*, the Mandelstam variables).

$$x_1 = (p - p_1)^2 = (p_2 + p_3)^2 = M^2 + m_1^2 - 2ME_1,$$
$$x_2 = (p - p_2)^2 = (p_1 + p_3)^2 = M^2 + m_2^2 - 2ME_2,$$
$$x_3 = (p - p_3)^2 = (p_1 + p_2)^2 = M^2 + m_3^2 - 2ME_3.$$

- In literature, $x_1 = m_{23}^2 = s_{23}$, $x_2 = m_{13}^2 = s_{13}$, $x_3 = m_{12}^2 = s_{12}$.

$$x_1 + x_2 + x_3 = M^2 + m_1^2 + m_2^2 + m_3^2.$$

- $dx_1 = -2MdE_1$ and $dx_2 = -2MdE_2$. Therefore,

$$\boxed{d\Pi_3 = \frac{dx_1 dx_2}{128\pi^3 M^2}}$$

Consider a process

$$a + b \to \phi \to 1 + 2$$

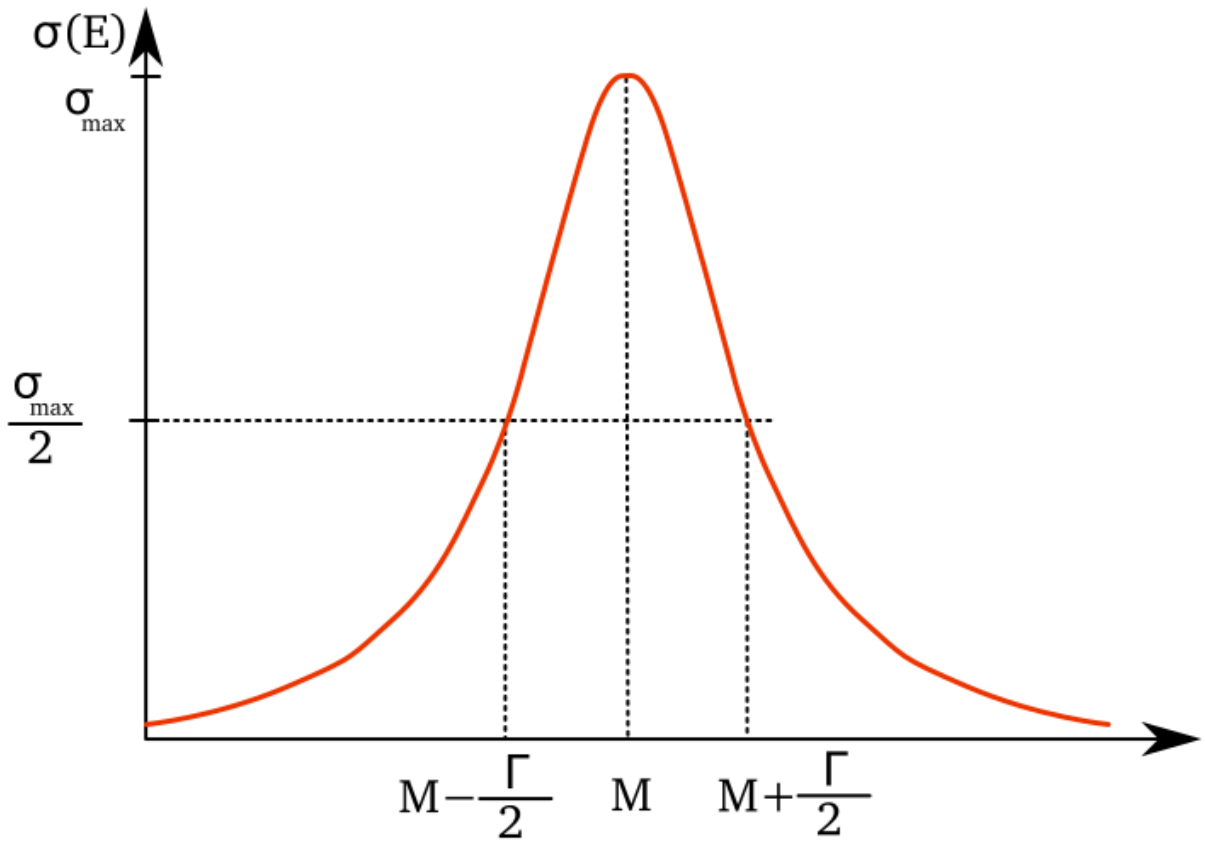
i.e., there is a virtual particle in the intermediate state (s-channel). The cross section is

$$\sigma \propto \left| \text{the propagator of } \phi \right|^2$$
$$= \left| \frac{i}{s - m_\phi^2 + i m_\phi \Gamma_\phi} \right|^2 = \boxed{\frac{1}{(s - m_\phi^2)^2 + m_\phi^2 \Gamma_\phi^2}}$$

- This is known as the Breit-Wigner distribution (Cauchy distribution), the characteristic shape of resonance.
- Γ_ϕ is the *decay rate* of the unstable particle ϕ ($1/\Gamma_\phi$ is the lifetime).
  - It has [mass] dimension.
- The full-width at half-maximum of the distribution is $2 m_\phi \Gamma_\phi$. Therefore, we sometimes use the word decay width.

Phase space

Resonance



$$\sigma \propto \frac{1}{(s - m_\phi^2)^2 + m_\phi^2 \Gamma_\phi^2}$$

Phase space

Resonance and Narrow Width Approximation

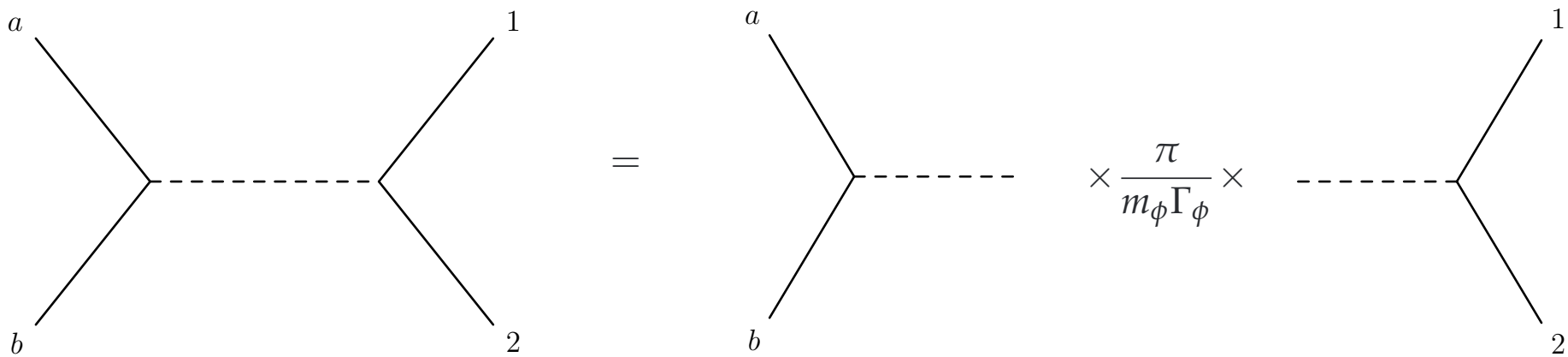
$$\sigma \propto \frac{1}{(s-m_\phi^2)^2+m_\phi^2\Gamma_\phi^2}$$
$$= \frac{1}{\Gamma_\phi m_\phi^3}\frac{\Gamma_\phi/m_\phi}{(s/m_\phi^2-1)^2+\Gamma_\phi^2/m_\phi^2}$$
$$\xrightarrow{\Gamma_\phi/m_\phi\to 0} \frac{\pi}{\Gamma_\phi m_\phi^3}\delta(s/m_\phi^2-1) = \boxed{\frac{\pi}{\Gamma_\phi m_\phi}\delta(s-m_\phi^2)}$$
$$\left(\because \lim_{\epsilon\to 0}\frac{\epsilon}{\epsilon^2+x^2} = \pi\delta(x) : \text{ Cauchy distribution}\right)$$

- This is called the narrow width approximation (**NWA**).
- Near a resonance we can treat the resonant particle as being on-shell ($\because \delta(s-m_\phi^2)$).
- When $\Gamma_\phi \gtrsim m_\phi$, ϕ decays very fast so it does not form sharp resonance.
  - Identifying a resonance with a particle only makes sense when $\Gamma_\phi < m_\phi$.

Phase space

Resonance and Narrow Width Approximation

For $\Gamma_\phi \ll m_\phi$,



$$\boxed{\sigma(a+b\to\phi\to 1+2) = \sigma(a+b\to\phi)\times \mathrm{BR}(\phi\to 1+2)}$$

where

$$\begin{aligned}\mathrm{BR}(\phi\to 1+2) &= \frac{1}{2m_\phi\Gamma_\phi}\int d\Pi_2\,|\mathcal{M}(\phi\to 1+2)|^2 \\ &= \frac{\Gamma(\phi\to 1+2)}{\Gamma_\phi}\end{aligned}$$

is the "branching ratio" (BR) of ϕ decaying to $1+2$.

E.g., the Higgs boson has mass $m_h = 125$ GeV and $\Gamma_h = 4 \times 10^{-3}$ GeV, thus $\Gamma_h / m_h = \mathcal{O}(10^{-5}) \ll 1$. The NWA can be applied to the Higgs processes!

- The decay modes of the Higgs: $h \to f\bar{f}$, where $f = b, \tau, W, Z, \gamma, \ldots$
- We compute all the $\Gamma(h \to f\bar{f})$.
- Total decay width: $\Gamma_h = \sum_f \Gamma(h \to f\bar{f})$.
- Branching ratios: $\mathrm{BR}(h \to f\bar{f}) = \Gamma(h \to f\bar{f}) / \Gamma_h$.
- We compute the cross section of $pp \to h$ (at the LHC).
- Now we have $\sigma(pp \to h \to f\bar{f}) = \sigma(pp \to h)\mathrm{BR}(h \to f\bar{f})$.

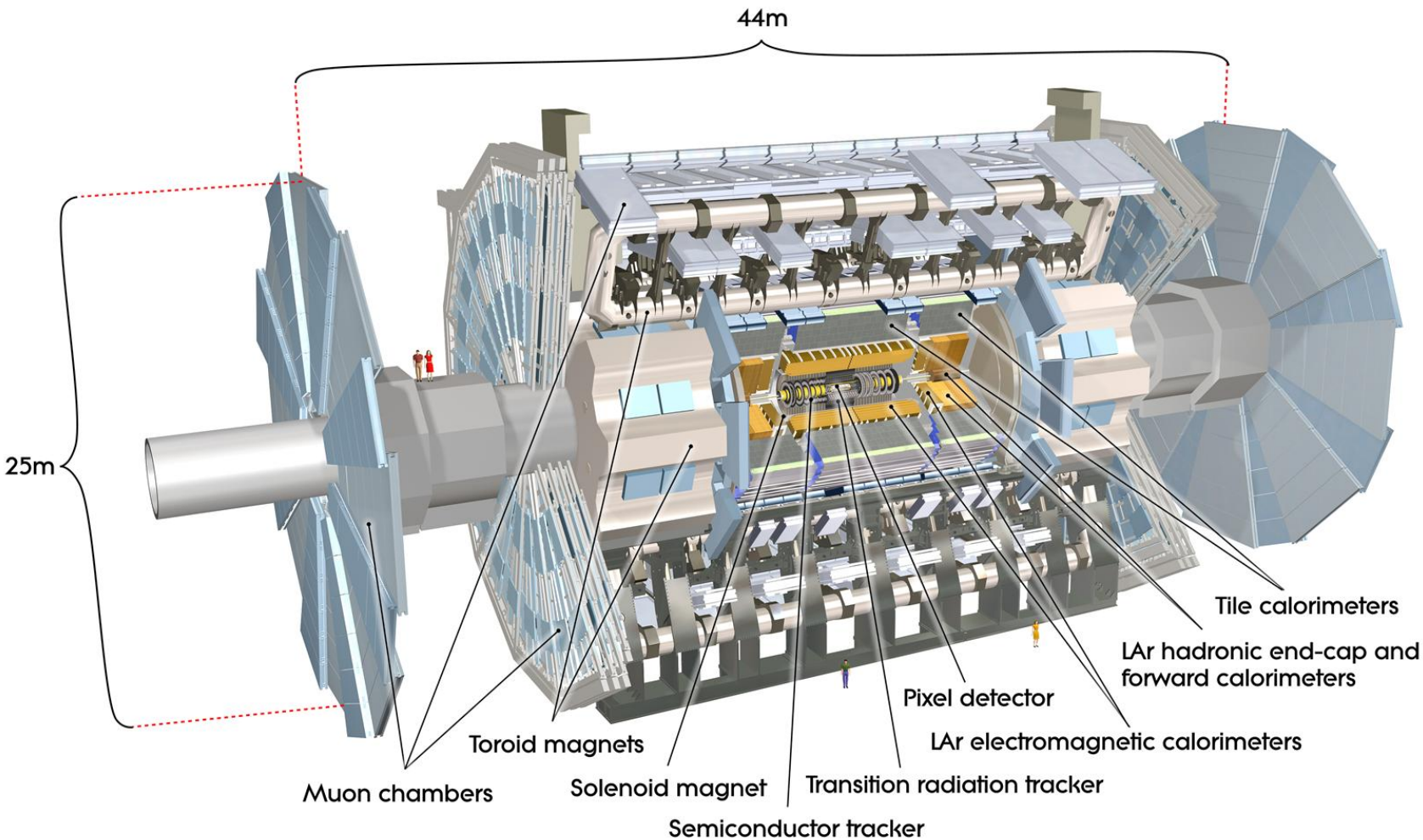
The NWA can be applied not only to the production, but also the decay processes.

- $\Gamma(t \to bW \to be\nu) = \Gamma(t \to bW) \times \mathrm{BR}(W \to e\nu)$
  because $\Gamma_t / m_t = 1.42/173 = \mathcal{O}(10^{-2}) \ll 1$.

Question?

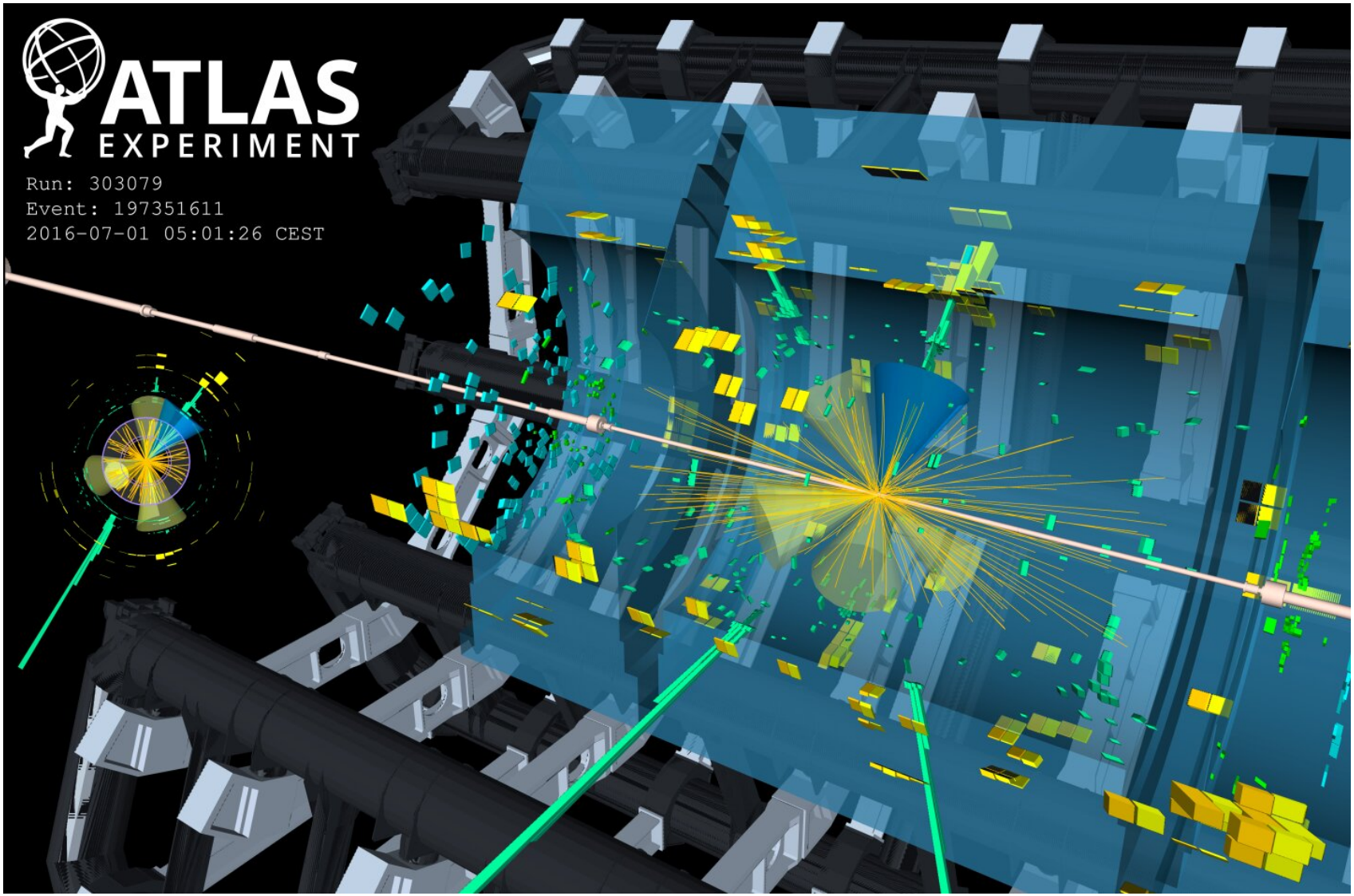


Collider



The ATLAS detector at the LHC

Collider



A collision event at the ATLAS detector at the LHC

Lab frame kinematics

Consider the four-momentum of a particle,

$$p^\mu = (E, p_x, p_y, p_z)$$

(in a Cartesian coordinate system), where $E = \sqrt{|\boldsymbol{p}|^2 + m^2} = \sqrt{p_x^2 + p_y^2 + p_z^2 + m^2}$.

- The beam axis (parallel to the incoming beams) is conventionally set to be the z-axis.
- The polar angle θ is the angle of the particle w.r.t. the z-axis (longitudinal direction).
- The azimuthal angle ϕ is the angle around the beam axis.

$$p^\mu = (E, |\boldsymbol{p}| \sin\theta \cos\phi, |\boldsymbol{p}| \sin\theta \sin\phi, |\boldsymbol{p}| \cos\theta)$$

(*i.e.*, spherical coordinate system)

Lab frame kinematics

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(*i.e.*, spherical coordinate system)

- However, the polar angle is not a good observable because it is not invariant under longitudinal boost.
- Transverse momentum is invariant (frame-independent) under longitudinal boost:

$$\boxed{p_T = \sqrt{p_x^2 + p_y^2}} = |\boldsymbol{p}| \sin\theta$$

Thus, we could write

$$p^\mu = (E, p_T \cos\phi, p_T \sin\phi, p_z).$$

instead of using the polar angle θ (cylindrical coordinate system).

Lab frame kinematics

Rapidity

Under Lorentz boost along the z-axis,

$$\begin{pmatrix} E' \\ p_z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta_z \\ -\gamma\beta_z & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix}$$
$$= \begin{pmatrix} \cosh y & -\sinh y \\ -\sinh y & \cosh y \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix} = \mathbf{\Lambda}(y) \begin{pmatrix} E \\ p_z \end{pmatrix}$$

($\gamma^2 - \gamma^2\beta_z^2 = 1 = \cosh^2 y - \sinh^2 y$, *i.e.*, unit hyperbola)

- y is called (longitudinal) rapidity (hyperbolic angle).

$$\cosh y = \gamma, \quad \sinh y = \gamma\beta_z$$
$$\Rightarrow y = \operatorname{artanh}\beta_z = \operatorname{artanh}\frac{p_z}{E} = \boxed{\frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)}$$
$$\left(\because \operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)\right)$$

- **Exercise** 5:

  The difference of rapidity ($\Delta y = y_1 - y_2$) is *invariant* under Lorentz boost.

Lab frame kinematics

Transverse energy

From the definition of the rapidity,

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) = \frac{1}{2}\ln\frac{(E+p_z)^2}{(E-p_z)(E+p_z)} = \frac{1}{2}\ln\frac{(E+p_z)^2}{p_T^2+m^2}$$
$$= \ln\frac{E+p_z}{E_T}.$$

- $E_T = \sqrt{p_T^2+m^2} = \sqrt{p_x^2+p_y^2+m^2}$ is transverse energy.
  - It is invariant under Lorentz boost along the z-axis.
- Because $p_z/E = \tanh y = \sinh y/\cosh y$, we find $E = E_T\cosh y$ and $p_z = E_T\sinh y$. The particle four-momentum can be written as

$$\boxed{p^\mu = (E_T\cosh y,\ p_T\cos\phi,\ p_T\sin\phi,\ E_T\sinh y)}$$

Lab frame kinematics

$$y = \ln \frac{E + p_z}{E_T} = \ln \frac{\sqrt{E_T^2 + p_z^2} + p_z}{E_T}$$

$$\Rightarrow \frac{dy}{dp_z} = \frac{\sqrt{E_T^2 + p_z^2} + p_z}{E_T^2 + p_z^2 + p_z\sqrt{E_T^2 + p_z^2}} = \frac{E + p_z}{E^2 + p_z E} = \frac{1}{E}$$

- dy is invariant under Lorentz boost, so is dp_z/E.
- The element of phase-space integral can be written as

$$\frac{d^3p}{E} = p_T dp_T d\phi \frac{dp_z}{E} = \boxed{p_T dp_T\, d\phi\, dy}$$
$$= E_T dE_T\, d\phi\, dy \quad \left(\because\ E_T^2 = p_T^2 + m^2\right)$$

- All the variables, p_T, E_T, ϕ, y, are *boost-invariant* along the longitudinal direction.

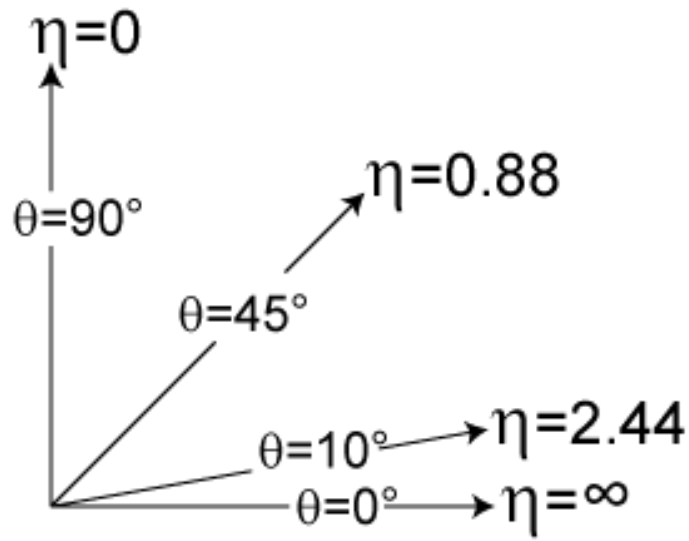
Lab frame kinematics

Pseudorapidity

In the massless limit $(m \to 0)$, $E = |\boldsymbol{p}|$, thus

$$
\begin{aligned}
y &= \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right) = \frac{1}{2} \ln \left(\frac{|\boldsymbol{p}| + p_z}{|\boldsymbol{p}| - p_z} \right) \\
&= \frac{1}{2} \ln \left(\frac{|\boldsymbol{p}| + |\boldsymbol{p}| \cos\theta}{|\boldsymbol{p}| - |\boldsymbol{p}| \cos\theta} \right) = \frac{1}{2} \ln \left(\frac{1 + \cos\theta}{1 - \cos\theta} \right) \\
&= \boxed{- \ln \tan \frac{\theta}{2} = \eta}
\end{aligned}
$$

- η is the pseudorapidity.
- In collider physics, the pseudorapidity is preferred over the polar angle ($\theta = 2 \operatorname{artanh}(e^{-\eta})$).



Lab frame kinematics

In collider physics, the angular separation between particles is defined by

$$\boxed{\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}}$$

which is also boost-invariant.

