Introduction to Collider Physics
Part II

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KIAS Particle Physics Summer Camp 2023

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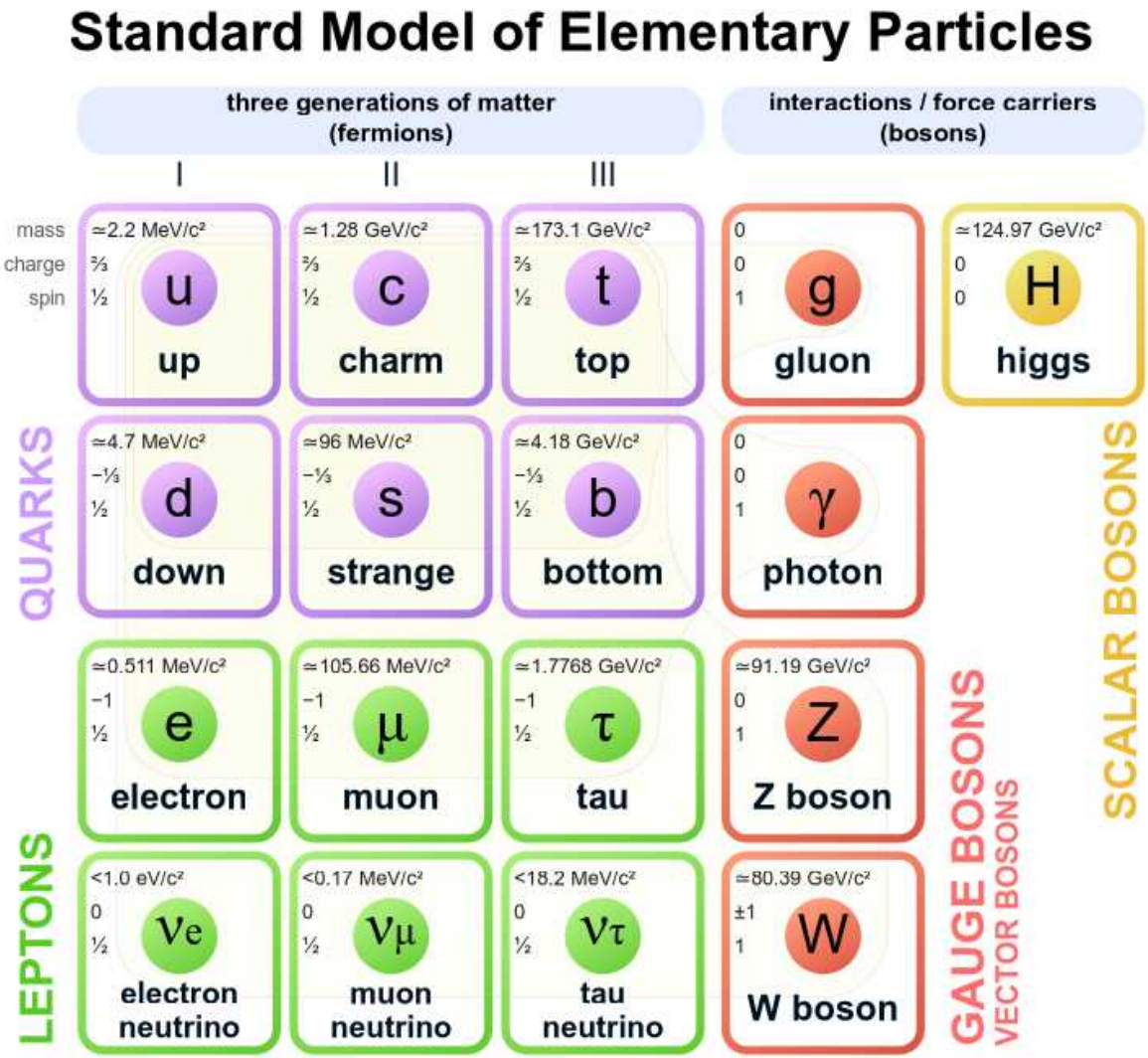
Shiva at CERN: the "Destroyer and Benefactor."

Collider objects

- Detectors *detect* particles. Which particles could be detected?

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Collider objects

- Strongly-interacting particles (quark and gluons) become electrically charged or neutral hadrons (protons, pions, $\cdots$).
  - Free quarks are never observed: $e^+e^- \to q\bar{q} \to$ hadrons.
  - Exception: top quarks decay before hadronization because $\Gamma_t = 1.42$ GeV $\gg \Lambda_{\text{QCD}} \simeq 200$ MeV (the lifetime is too short $\Rightarrow$ it *decays* before hadronizing: $t \to bW$).

Collider objects

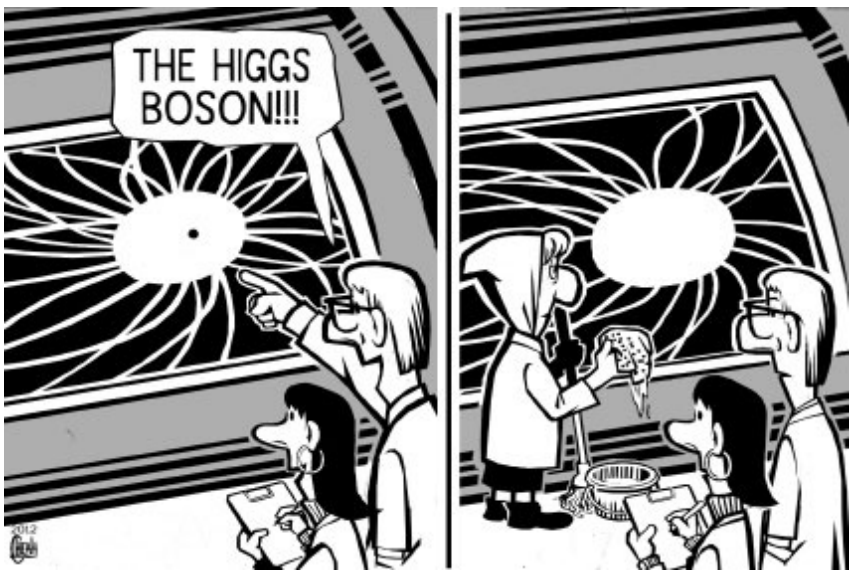
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- Heavy electroweak bosons (W, Z and Higgs bosons) decay promptly (within $\mathcal{O}(10^{-25}$ s)).
  - *e.g.*, $h \to W^+ + W^- \to u\bar{d} + e^-\bar{\nu}_e$.

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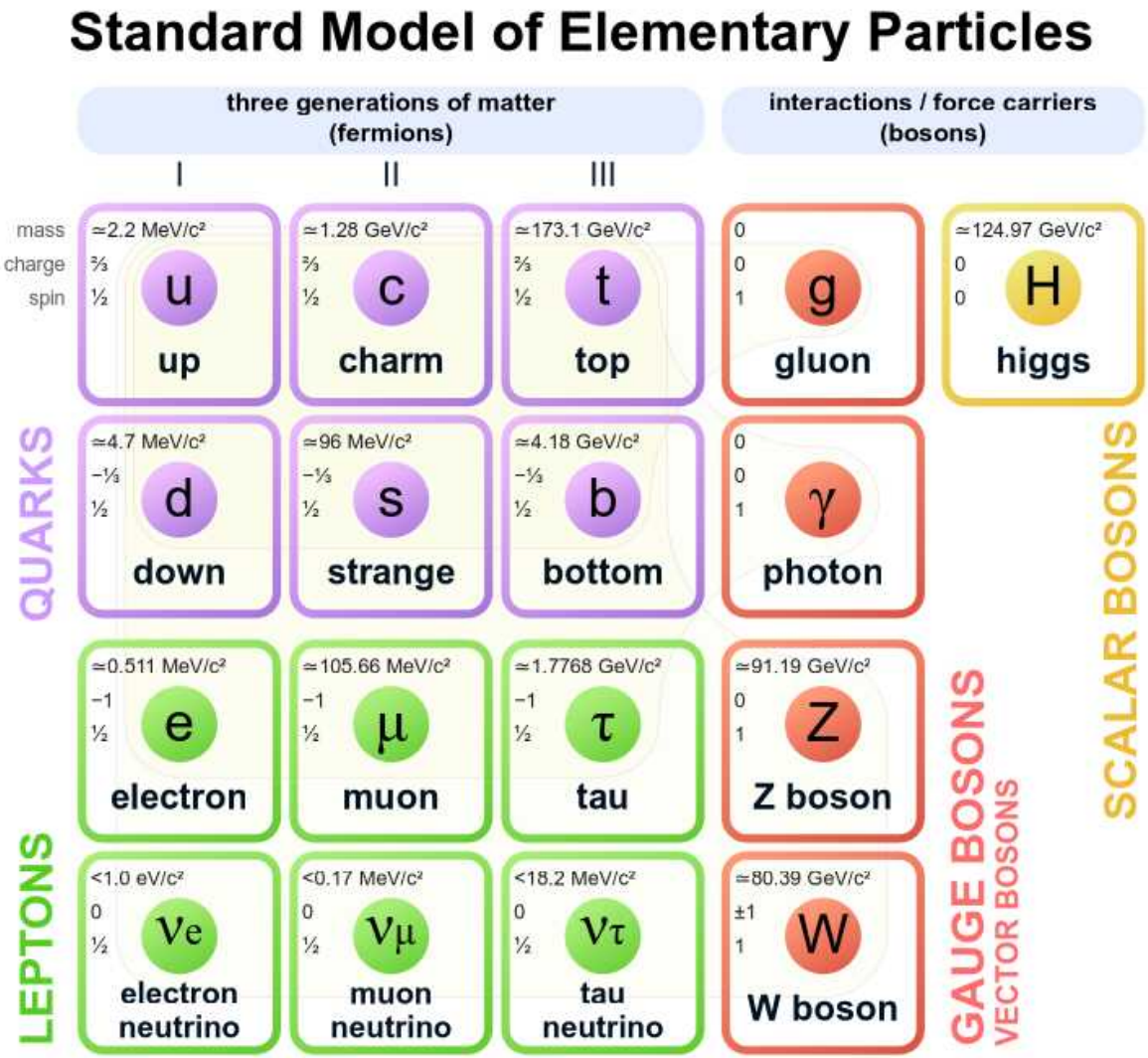
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From *Cheah's 'Event Horizon' page*

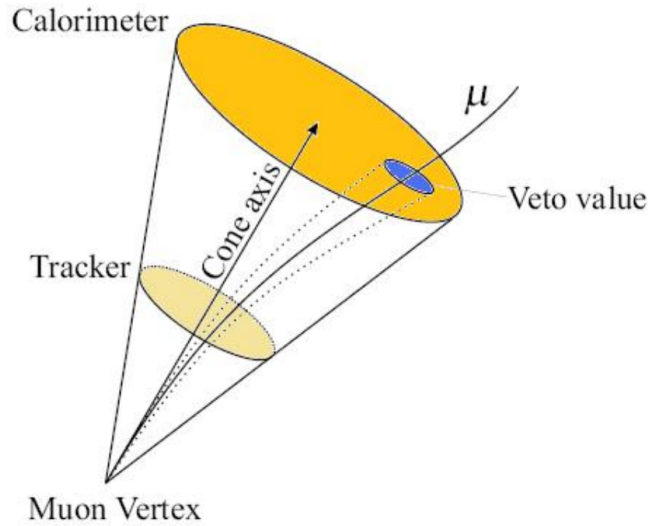
Collider objects

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Collider objects

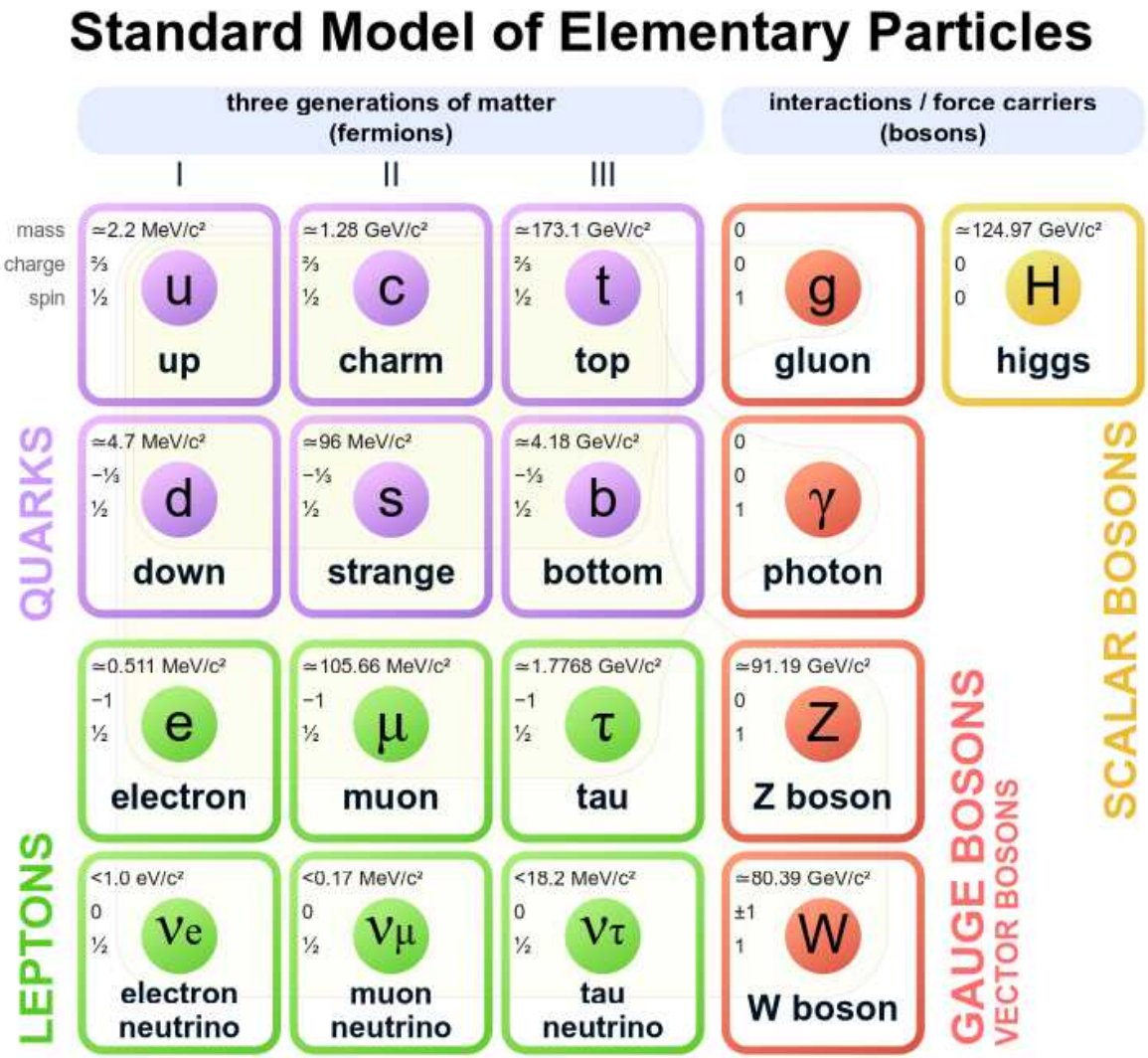
- The electron is stable and the muon is practically stable.
  - $c\tau \simeq 658.6$ m $\gg$ the diameter of the detector is $\mathcal{O}(10\text{ m})$
  - Typically, we are interested in *isolated* electrons and muons: no other hard objects around them.
    - Charged leptons at colliders = isolated electrons or muons



- Isolated *hard* photon is also of great interest.
- The tau leptons decay before detected: $c\tau \simeq 87\ \mu$m. What we detect is tau jets.

Collider objects

- Detectors *detect* particles. Which particles could be detected?



Collider objects

- Neutrinos escape all the detections: missing energy.

- Neutrino momenta can be estimated by momentum conservation:

$$\boldsymbol{p}_\nu + \sum \boldsymbol{p}_{\text{observed}} = 0 \quad \text{(CM frame)}$$

$$\Rightarrow \boxed{\boldsymbol{p}_\nu = -\sum \boldsymbol{p}_{\text{observed}}}$$

(observed: hadrons, isolated leptons, τ jets, photons)

- Because neutrino is nearly massless $(m_\nu = 0)$,

$$E_\nu = |\sum \boldsymbol{p}_{\text{observed}}| = E_{\text{miss}} = \not{E}.$$

Collider objects

All the collider signals may described by

- hadrons (jets),
- isolated electrons and muons (isolated leptons),
- τ jets,
- photons,
- missing energy.

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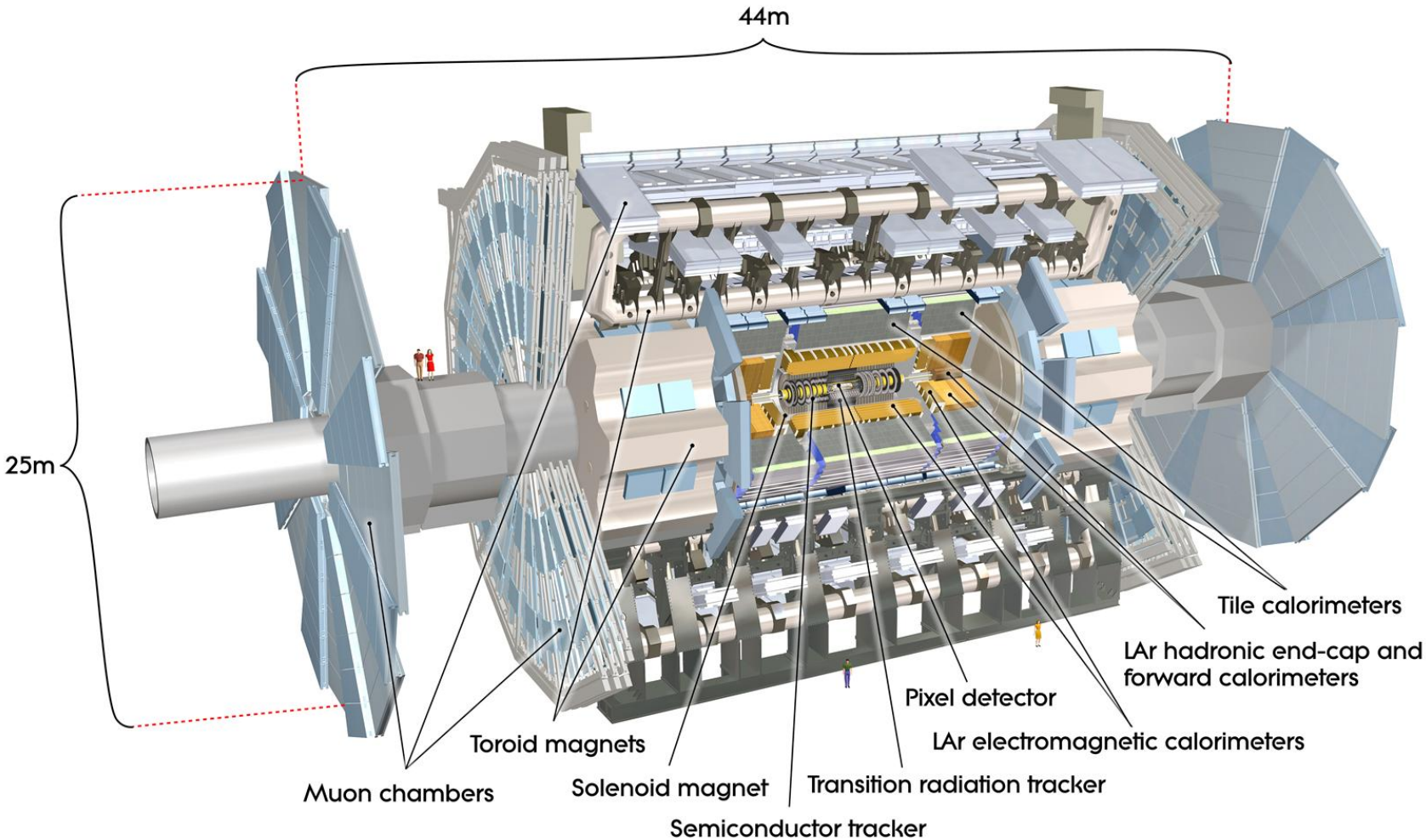
e.g., $h \to W^+ + W^- \to u\bar{d} + e^- \bar{\nu}_e$ has the final state (signal) of

two jets (u and $\bar{d}$),

one isolated electron (e^-),

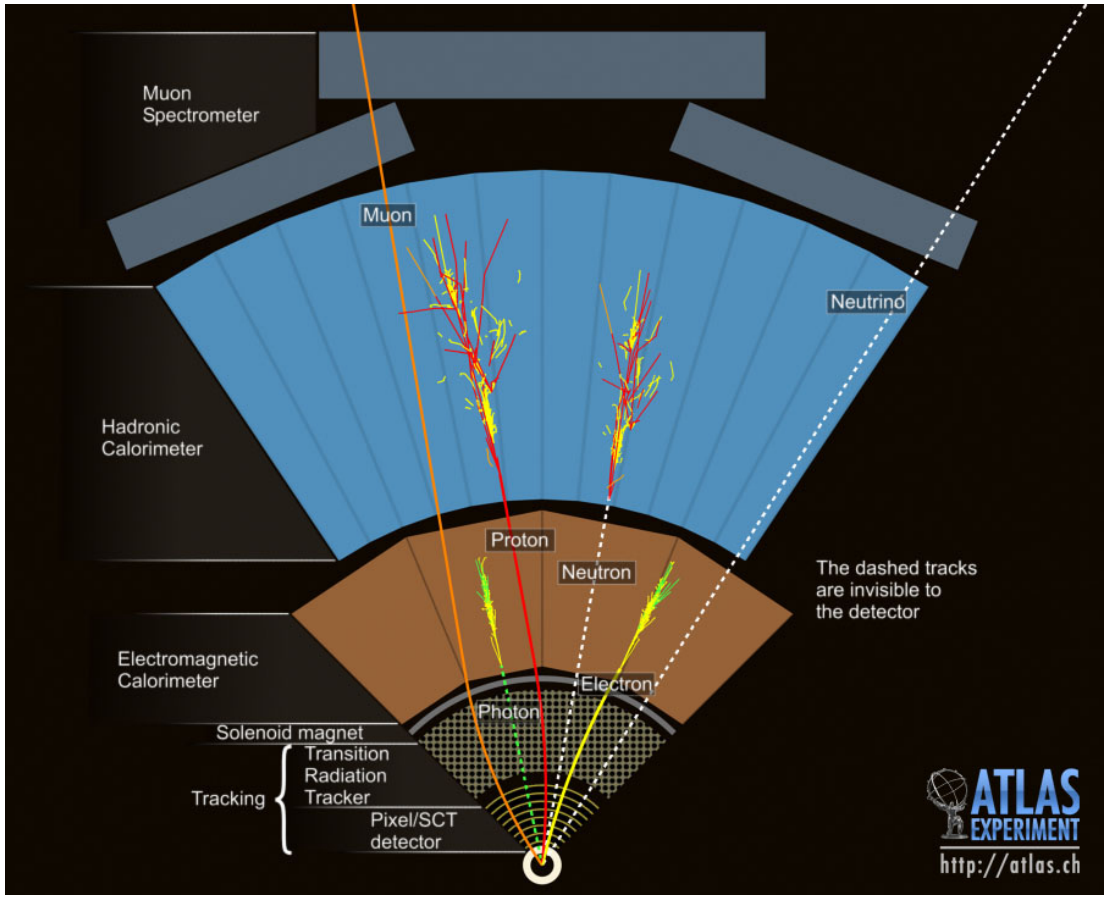
and missing energy (ν_e).

Detector



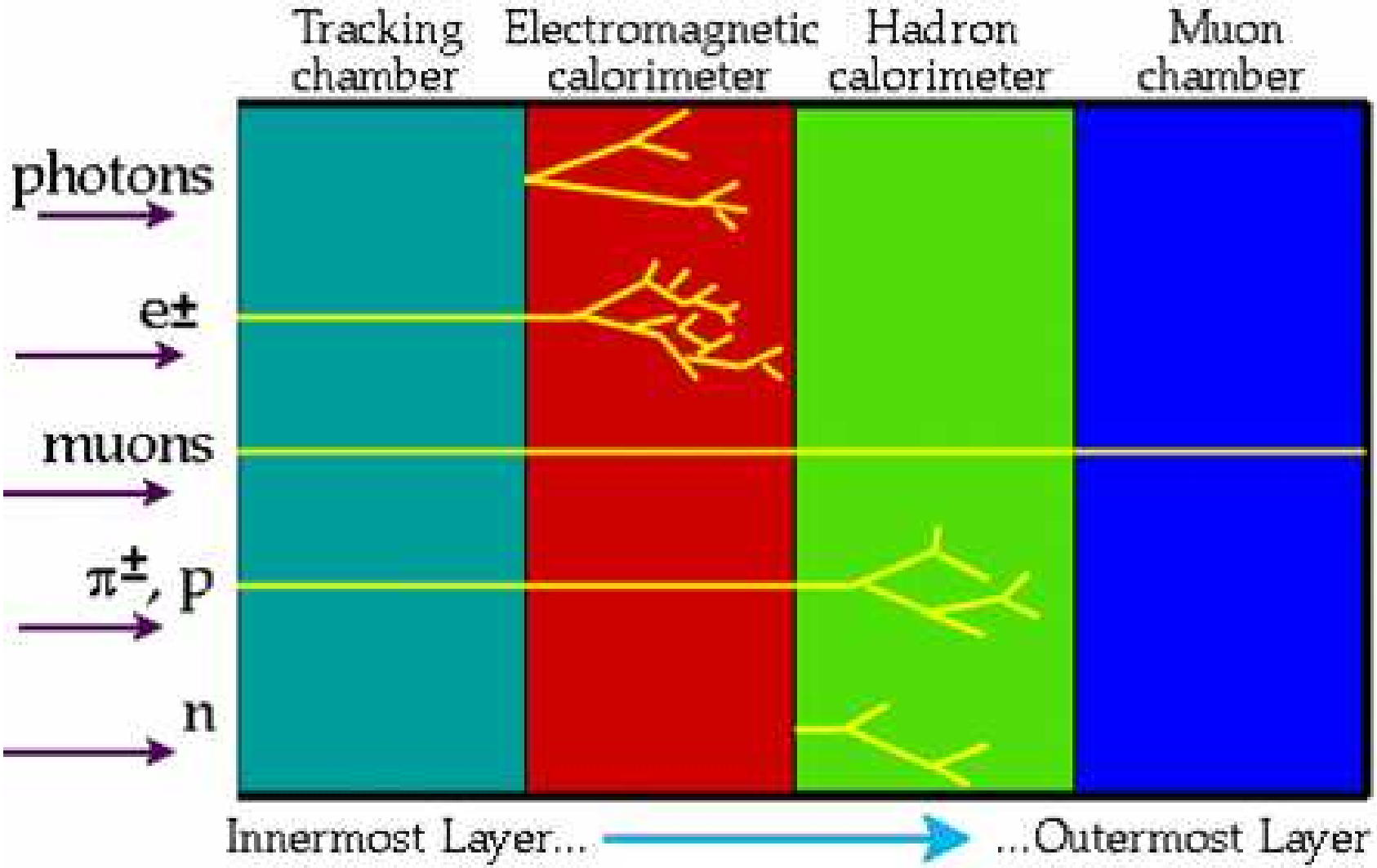
The ATLAS detector at the LHC

Detector



- Tracker: trajectories of charged particles
- Electromagnetic calorimeter (ECAL): Electromagnetic shower by electrons and photons
- Hadron calorimeter (HCAL): Showers by hadrons
- Muon chamber: detects muons which pass the calorimeters

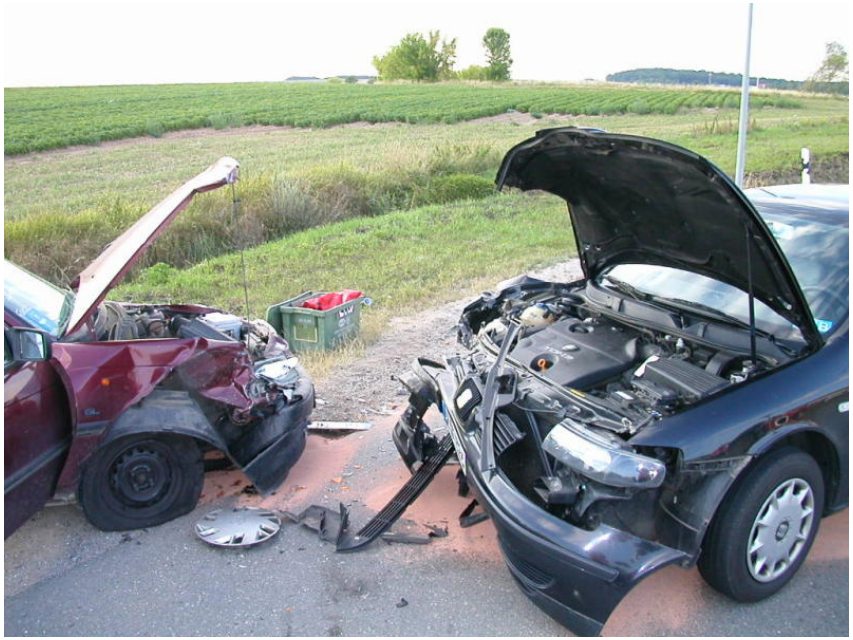
Detector


Tracking chamber
Electromagnetic calorimeter
Hadron calorimeter
Muon chamber
photons
e±
muons
π±, p
n
Innermost Layer...
...Outermost Layer

- Questions . . . ?



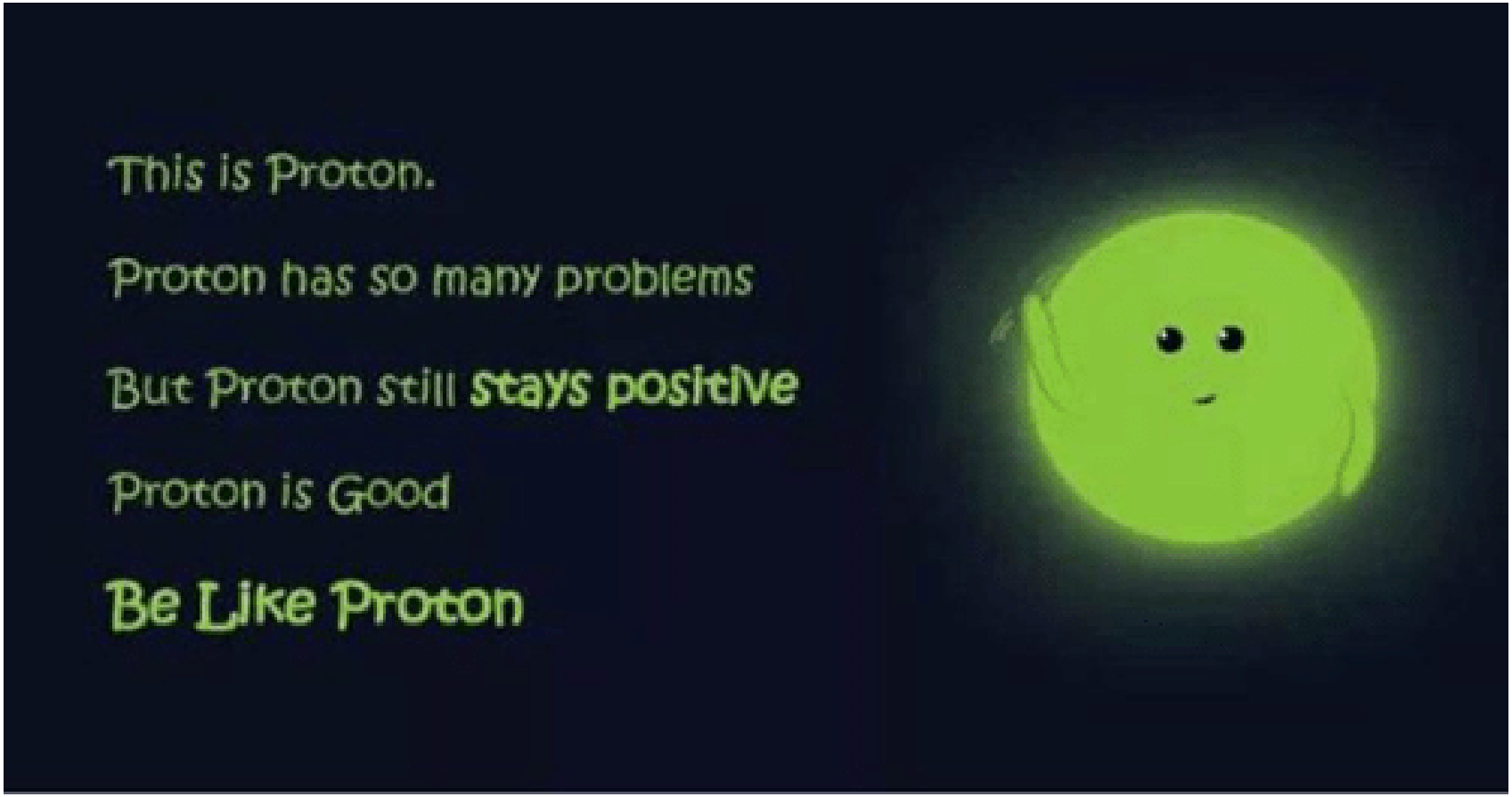
Collider



- What to collide?
  - *electrically charged* so that they can be accelerated by electric field.
  - *stable* so that they can be put in a storage ring for reuse.

Electrons and protons are good candidates for serving as the colliding beams.

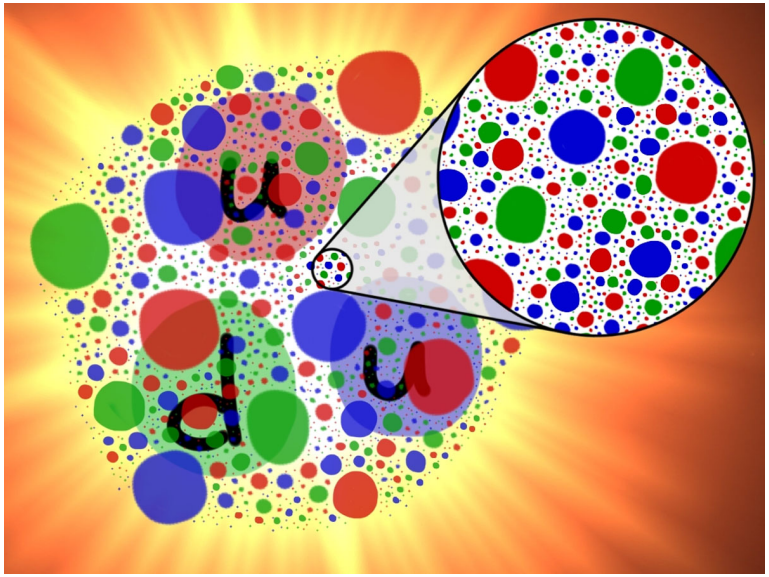
- Lepton *vs* hadron colliders.



Be like proton

Hadron collider

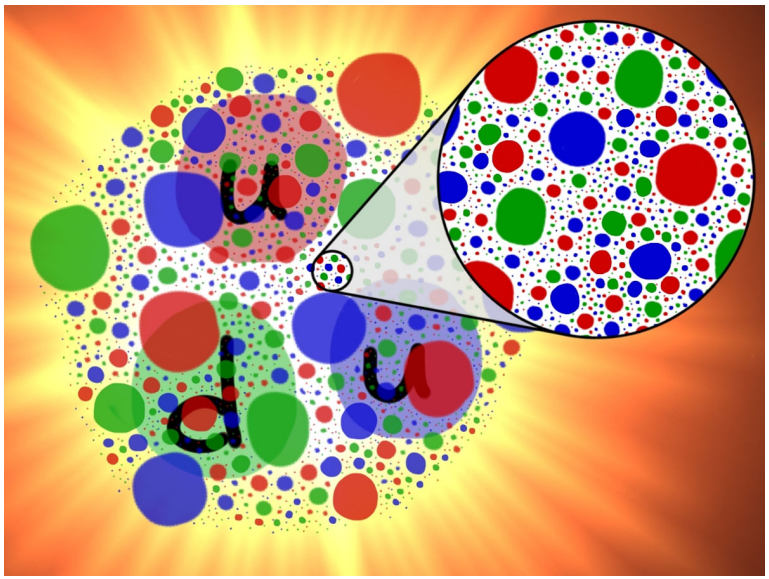
- Protons are composite particles (↔ elementary particles), made of "partons" of quarks and gluons.
  - The quarks and gluons are strongly interacting according to QCD.



picture taken from Fermilab at work

Hadron collider

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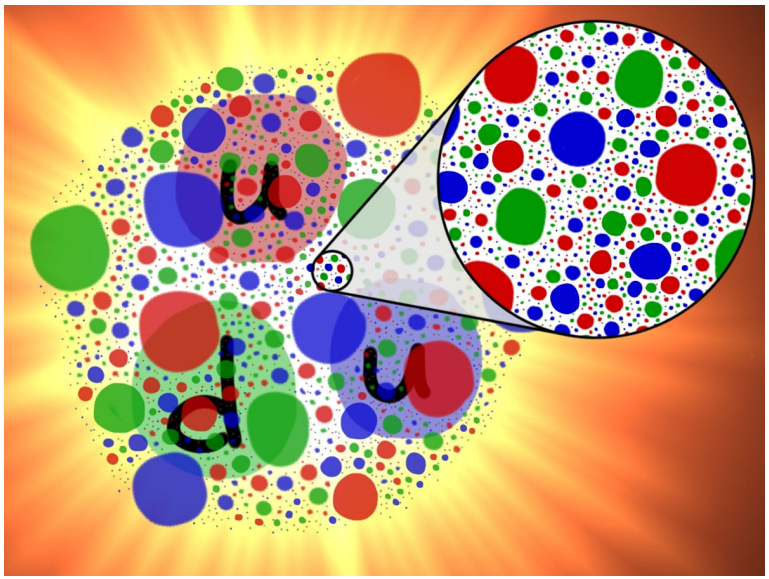


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- The proton is much heavier than the electron: $m_p / m_e \simeq 2000$.
- Due to the heavier mass, hadron colliders can provide much higher collision energies.

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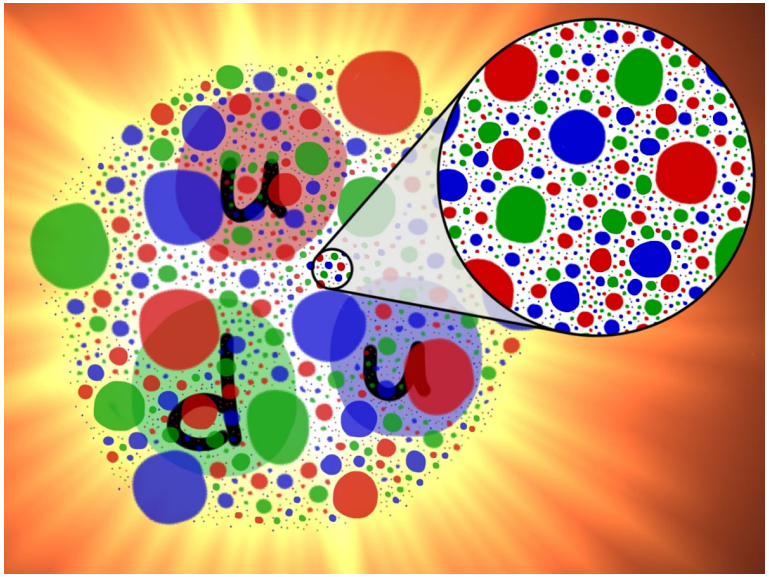


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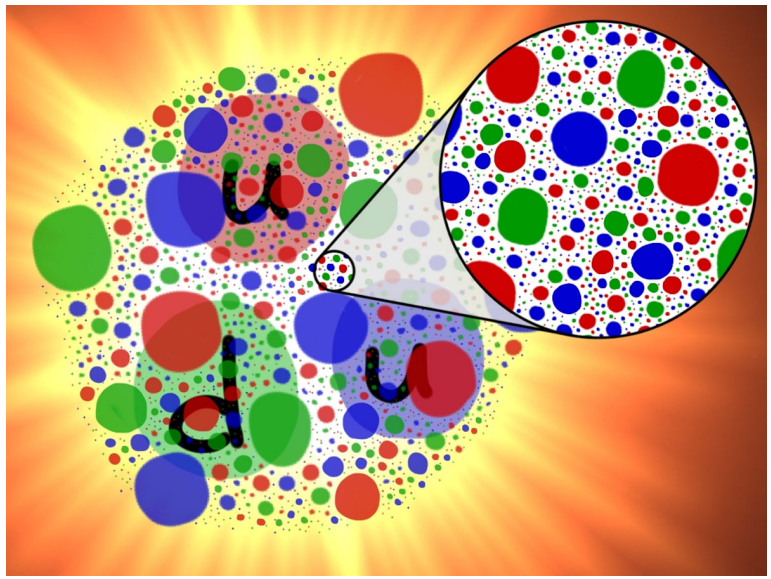


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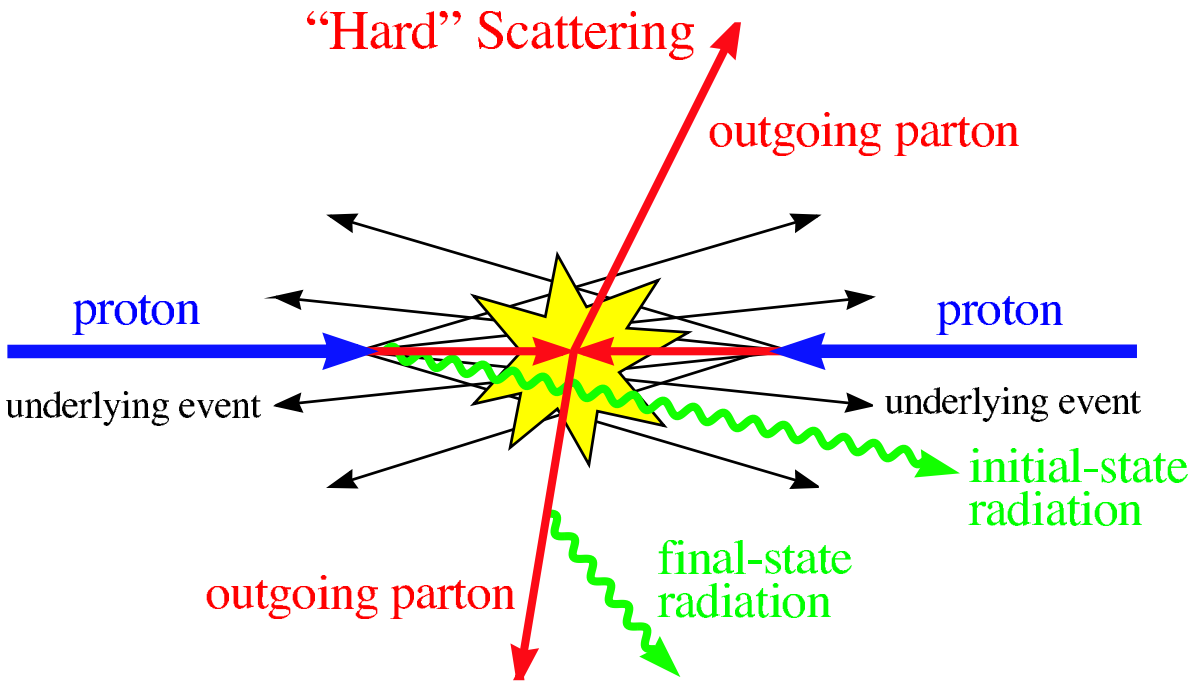


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- Because of strong interactions, the hadronic reactions yield large cross sections.
- Induced by initial parton combinations ($q\bar{q}$, qg, gg) *cf.*, e^+e^-, there are many possible reactions for different charge and spin states.
- However, the compositeness and the strong interactions can be *disadvantageous* in certain aspect.

Hadron collider

Parton scattering



- Parton: constituents of hadrons (In QCD, they are quarks and gluons). It carries fractional energy of hadron (proton).

$$p = xP \quad (0 < x < 1).$$

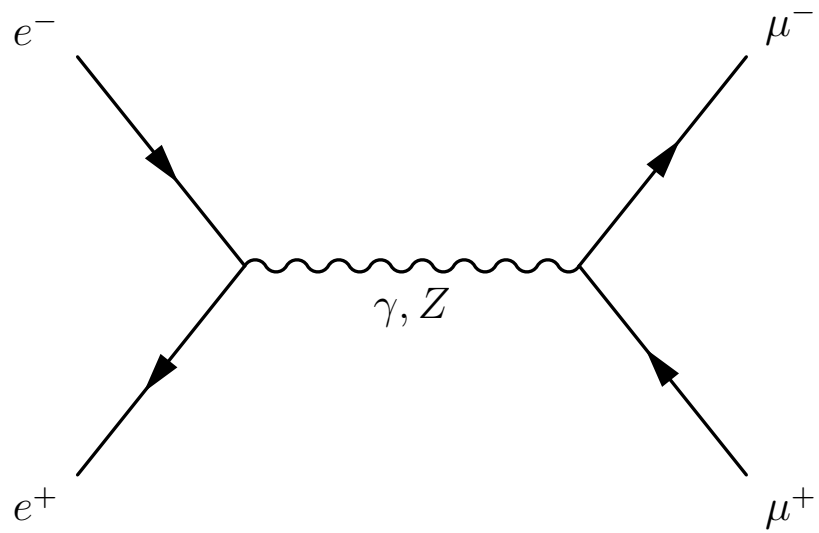
(p: energy-momentum of parton, P: that of proton)

- The probability of finding a parton q with energy fraction x is $\boxed{f_q(x)dx}$ where $f_q(x)$ is the parton distribution function (PDF).

Hadron collider

Parton scattering

Consider

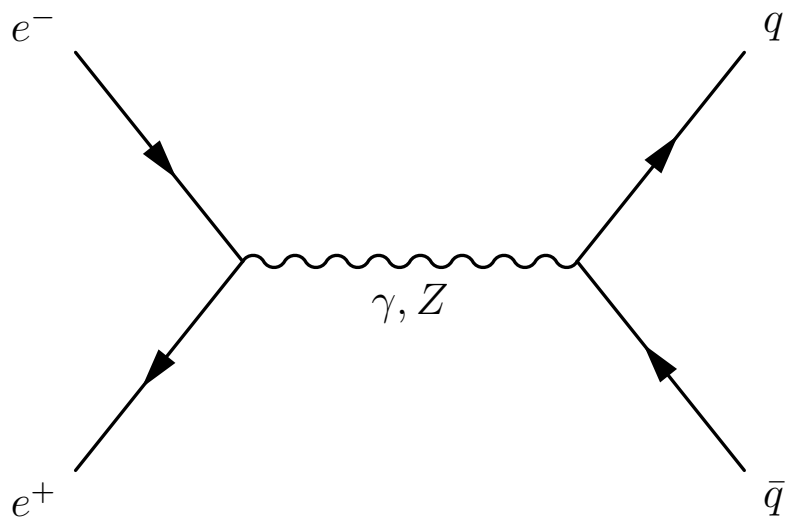


For $s = (p_{e^+} + p_{e^-})^2 \ll m_Z^2$,

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

Hadron collider

Parton scattering



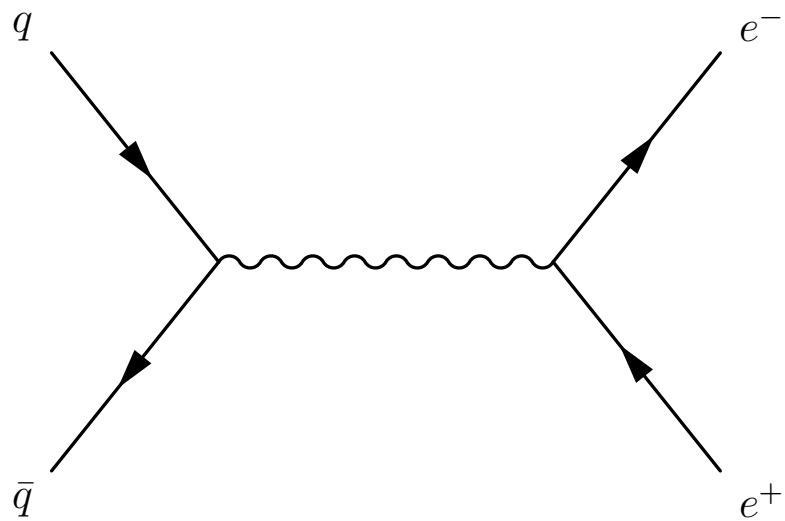
For electric charge Q_q of final-state quarks and $s \ll m_Z^2$,

$$\begin{aligned}\sigma(e^+e^- \to q\bar{q}) &= 3Q_q^2 \times \sigma(e^+e^- \to \mu^+\mu^-) \\ &= 3Q_q^2 \, \frac{4\pi\alpha^2}{3s}.\end{aligned}$$

The factor "3" is from the number of quark colors.

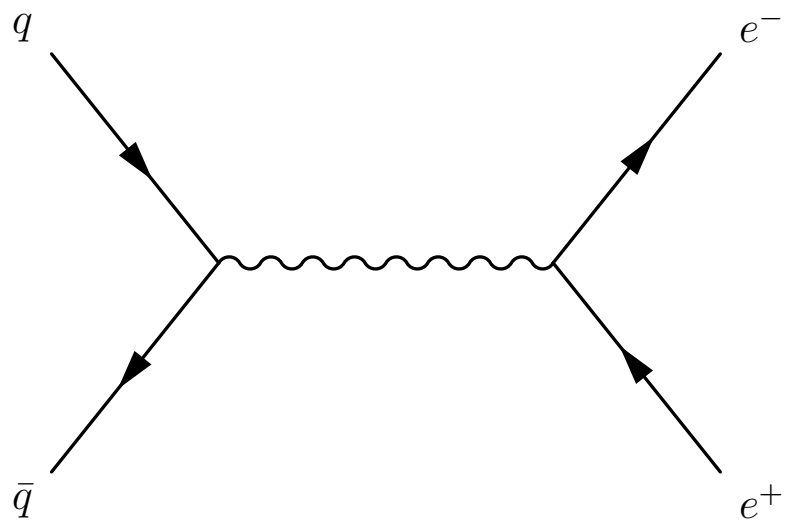
Hadron collider

Parton scattering: Drell-Yan process



Hadron collider

Parton scattering: Drell-Yan process

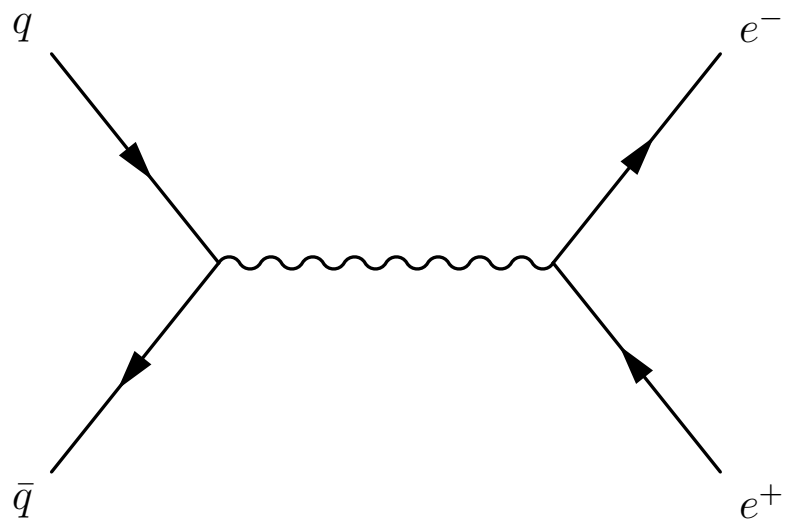


For $\hat{s} = (p_q + p_{\bar{q}})^2$: partonic collision energy,

$$\sigma(q\bar{q} \to e^+e^-) = \frac{1}{3} Q_q^2 \frac{4\pi\alpha^2}{3\hat{s}}$$ (average over initial spins, sum over final spins)

Hadron collider

Parton scattering: Drell-Yan process



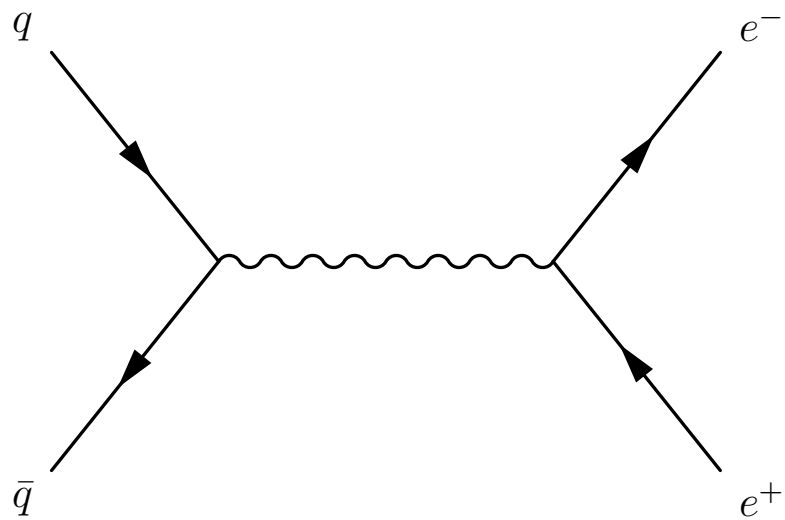
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- What would it be $\sigma(pp \to e^+e^-)$?
  - What we measure is the cross section of the proton-proton collision process, NOT that of parton-parton collision.
  - The momentum of parton is unknown. We have only *probabilities*, through parton distribution function.

Hadron collider

Parton scattering: Drell-Yan process



By taking into account the initial-state quark PDFs, the cross section is

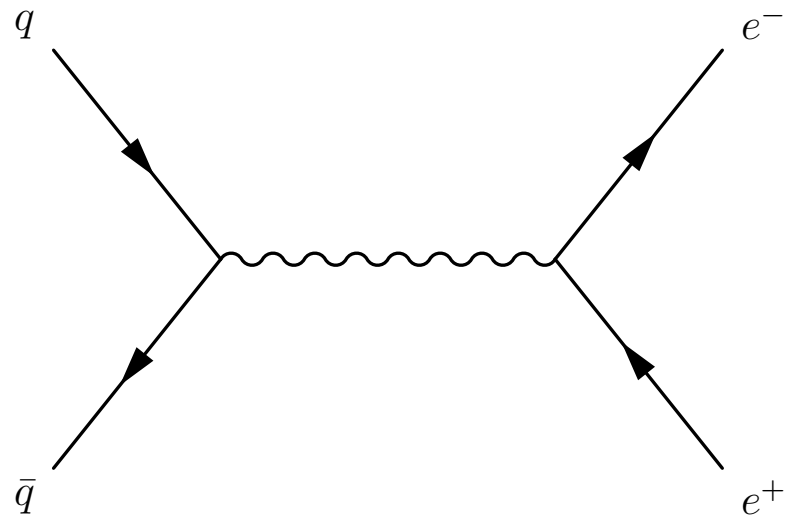
$$\sigma(pp \to Y + X) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_q f_q(x_1) f_{\bar{q}}(x_2)\, \sigma(q(p_1) + \bar{q}(p_2) \to Y)$$

where

- Y is the final state (Here, it is e^+e^-),
- X denotes anything else including remnants of the colliding protons.

Hadron collider

Parton scattering: Drell-Yan process



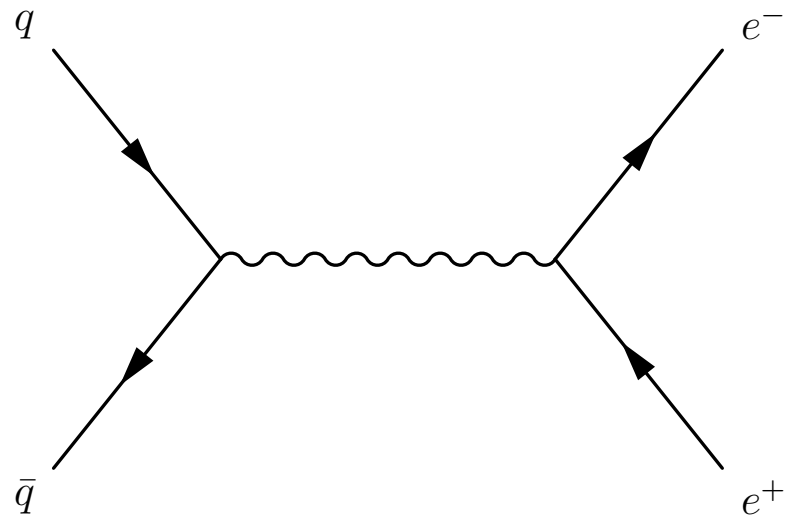
- In the proton-proton center-of-mass frame, the proton momenta are $(m_p = 0)$

$$P_1 = (E, 0, 0, E) \quad P_2 = (E, 0, 0, -E),$$

thus $s = (P_1 + P_2)^2 = 4E^2$ (= 13.6 TeV at the LHC).

Hadron collider

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- The initial partons carry fractional momenta:

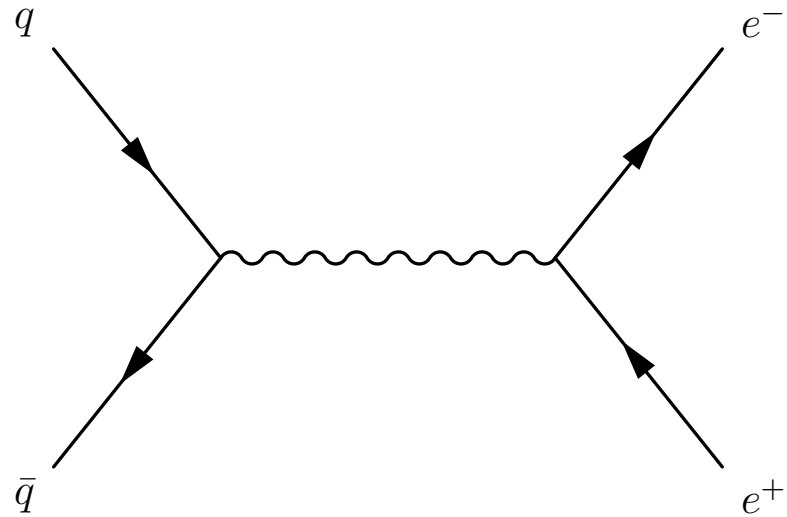
$$p_1 = x_1 P_1 = (x_1 E, 0, 0, x_1 E), \quad p_2 = x_2 P_2 = (x_2 E, 0, 0, -x_2 E).$$

The partonic collision energy is given as

$$\hat{s} = (p_1 + p_2)^2 = (x_1 + x_2)^2 E^2 - (x_1 - x_2)^2 E^2 = 4 x_1 x_2 E^2$$
$$= \boxed{x_1 x_2 s}$$

Hadron collider

Parton scattering: Drell-Yan process



- The partonic collision energy $\rightarrow$ energy of the mediating virtual photon

$$k = x_1 P_1 + x_2 P_2 = \frac{\sqrt{s}}{2}((x_1 + x_2),\, 0,\, 0,\, (x_1 - x_2))$$

  with mass $M = \sqrt{\hat{s}} = \sqrt{x_1 x_2 s}$.

- The boost factor is

$$\beta = \frac{k_z}{E_\gamma} = \frac{x_1 - x_2}{x_1 + x_2}.$$

  - Unless $x_1 = x_2$, the partonic system is *boosted*:

Lab frame = CM frame of protons $(\boldsymbol{P}_1 + \boldsymbol{P}_2 = \boldsymbol{0})$
$\neq$ CM frame of colliding partons $(\boldsymbol{p}_1 + \boldsymbol{p}_2 \neq \boldsymbol{0})$

- The Lorentz factor is

$$\gamma = \frac{E_\gamma}{M} = \frac{(x_1 + x_2)\sqrt{s}}{2\sqrt{\hat{s}}} = \frac{x_1 + x_2}{2\sqrt{x_1 x_2}} = \frac{1}{2}\left(\sqrt{\frac{x_1}{x_2}} + \sqrt{\frac{x_2}{x_1}}\right)$$
$$= \cosh y,$$

  where y is the rapidity.

- From the previous relations, we find

$$\hat{s} = x_1 x_2 s, \quad e^y = \sqrt{\frac{x_1}{x_2}}.$$

  We can now determine the parton energy fractions x_1 and x_2:

$$\boxed{x_1 = \sqrt{\frac{\hat{s}}{s}} e^y, \quad x_2 = \sqrt{\frac{\hat{s}}{s}} e^{-y}}$$

Hadron collider

Parton scattering: Drell-Yan process

- We can change the variables $(x_1, x_2) \to (\hat{s}, y)$ for $d^2\sigma/(dx_1 dx_2)$ using

$$\frac{\partial(\hat{s}, y)}{\partial(x_1, x_2)} = s = \frac{\hat{s}}{x_1 x_2}.$$

- Therefore, the cross section for the Drell-Yan process is

$$\frac{d^2\sigma}{d\hat{s}\, dy}(pp \to e^+e^- + X) = \frac{x_1 x_2}{\hat{s}} \frac{d^2\sigma}{dx_1 dx_2}$$

$$= \boxed{\sum_q x_1 f_q(x_1) x_2 f_{\bar{q}}(x_2)\, \frac{1}{3} Q_q^2\, \frac{4\pi\alpha^2}{3\hat{s}^2}}$$

with x_1 and x_2 determined by $\hat{s}$ and y,

$$x_1 = \sqrt{\frac{\hat{s}}{s}} e^y, \quad x_2 = \sqrt{\frac{\hat{s}}{s}} e^{-y}.$$

  - $\hat{s}$ and y can be measured by using the four-momenta of final-state electrons: $\hat{s} = (p_{e^+} + p_{e^-})^2$, $y = \text{arcosh}((E_{e^+} + E_{e^-})/\sqrt{\hat{s}})$.

For a proton having energy E,

$$\boxed{\begin{pmatrix}\text{probability of finding constituent } i \\ \text{carrying energy between } xE \text{ and } (x+dx)E\end{pmatrix} = f_i(x)dx}$$

- $f_i(x)$'s are the parton distribution functions (PDFs), corresponding to the *probability density function* in probability theory.

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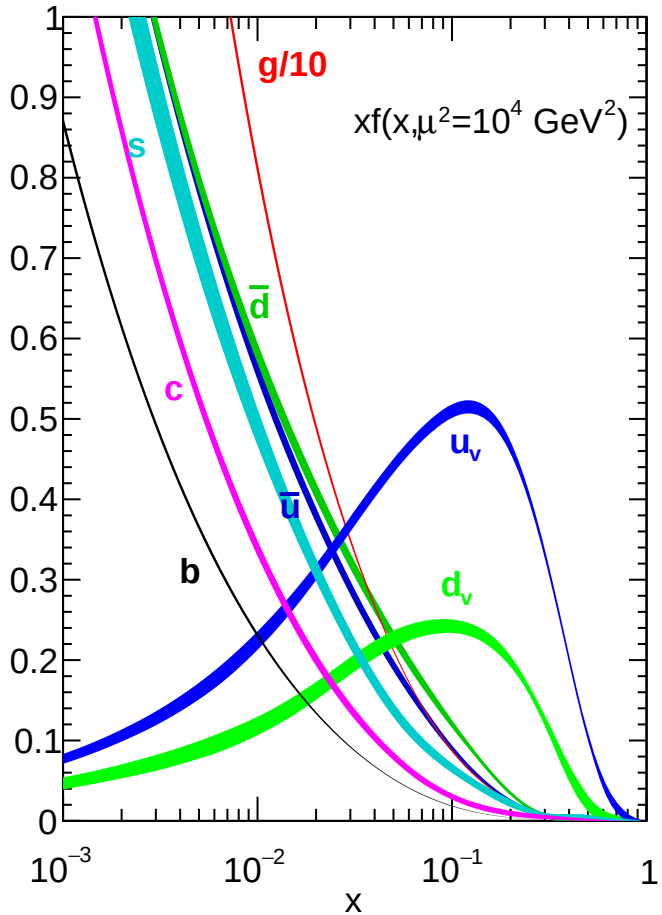
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- These cannot be computed within perturbation theory due to the non-perturbative nature of bound states like proton
  (quarks and gluons cannot be observed as free particles).
  - In other words, the structure of proton is not known in perturbative QCD.
  - Still, we can measure them by a fit to a number of experimental observables.

Hadron collider

Parton Distribution Functions

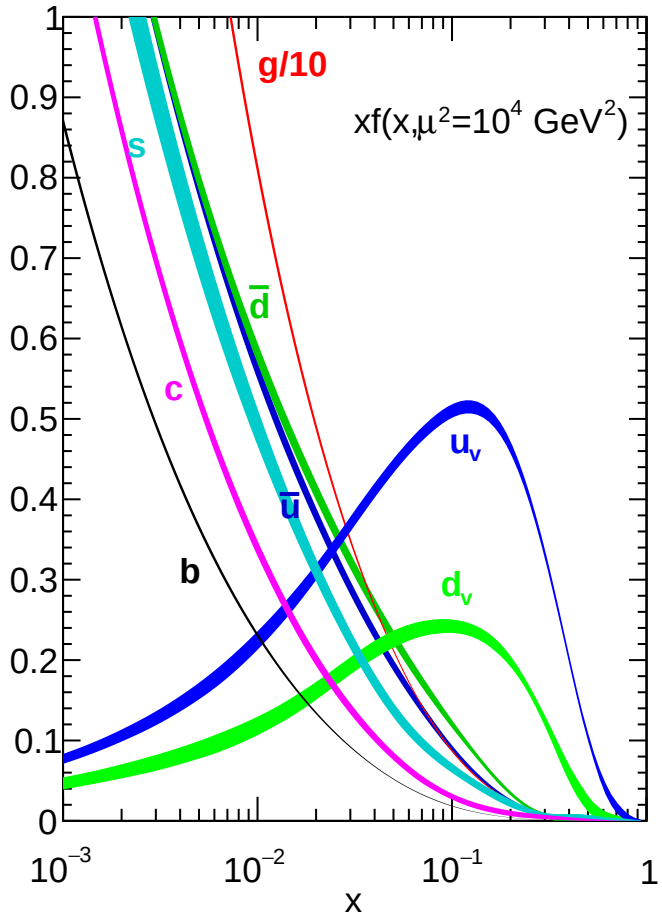
- u and d quarks dominate the PDFs at $x \gtrsim 0.1$ (valence quarks).



Hadron collider

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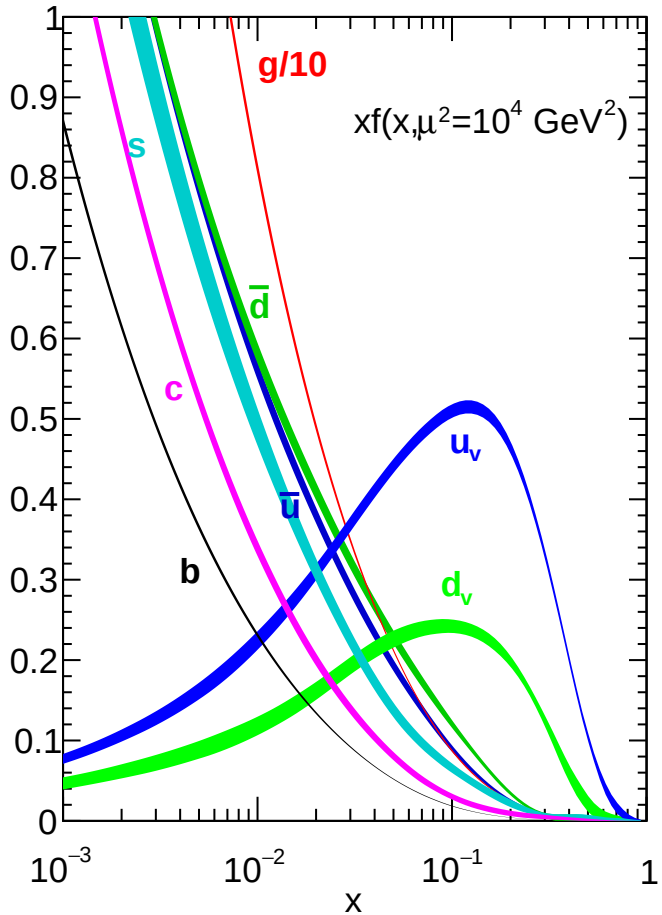
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- For lower x, the dominant component of the proton is the gluon.
  - At the LHC, most collisions with $\hat{s}$ up to 1 TeV are between gluons. We can call the LHC as a gluon collider.



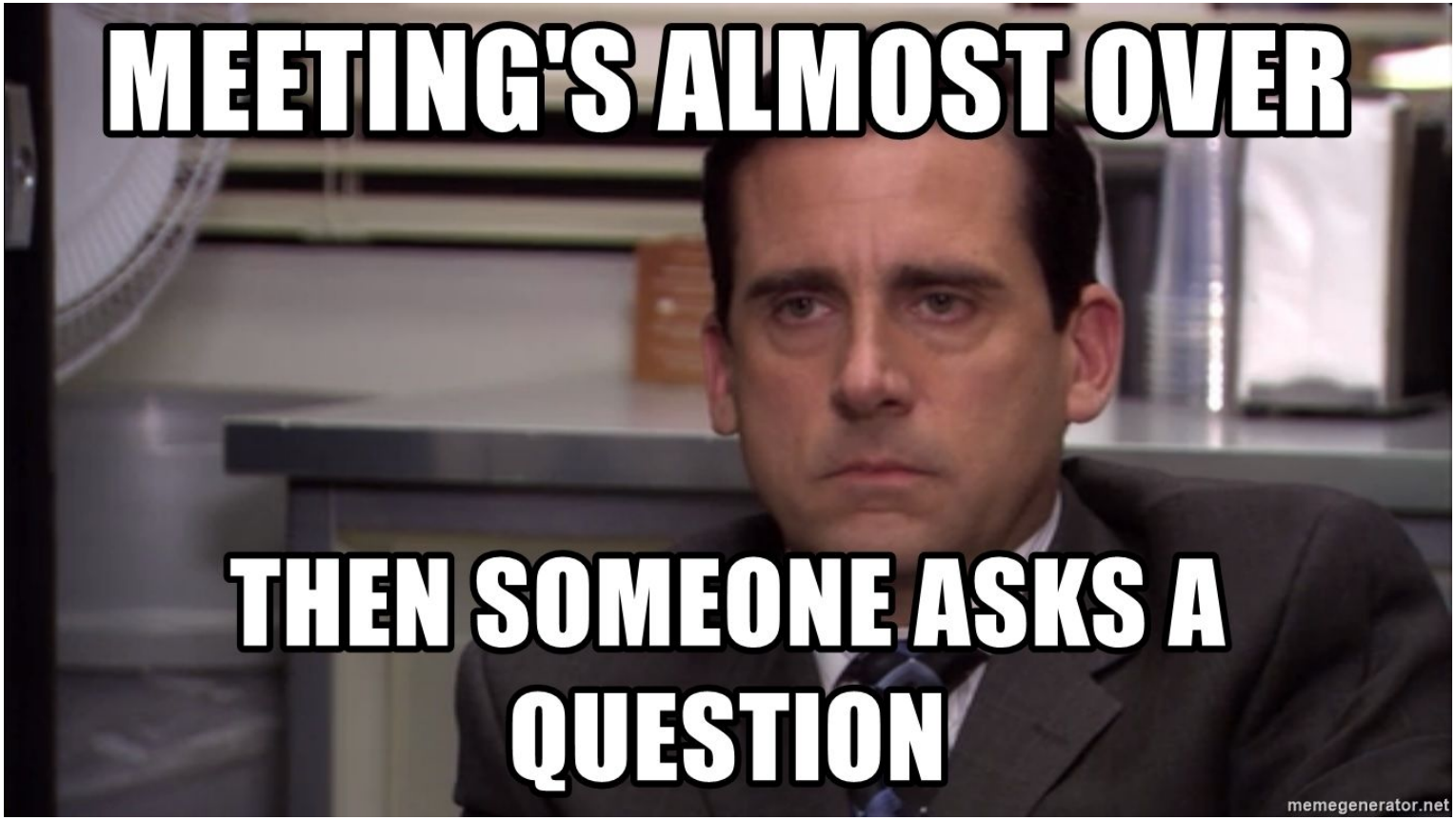
Hadron collider

Parton Distribution Functions

- u and d quarks dominate the PDFs at $x \gtrsim 0.1$ (valence quarks).
- For lower x, the dominant component of the proton is the gluon.
  - At the LHC, most collisions with $\hat{s}$ up to 1 TeV are between gluons. We can call the LHC as a gluon collider.
- Below $x \lesssim 0.01$, the PDFs of all light quarks are roughly the same (sea quarks).
  - Most of them come from gluon splittings, $g \to q\bar{q}$, which are flavor-blind and produce equal number of q and $\bar{q}$.



- Questions . . . ?


MEETING'S ALMOST OVER
THEN SOMEONE ASKS A QUESTION
memegenerator.net

Collider

- Parton-level
- Hadron-level
- Jet-level
- Detector-level

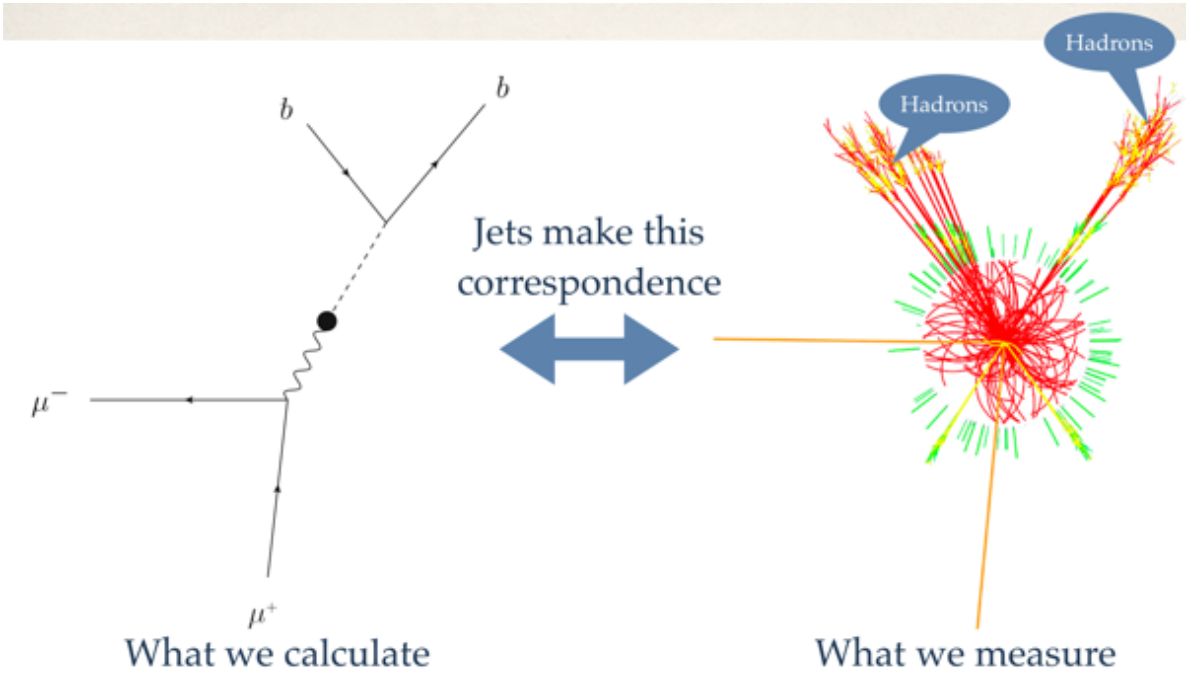


Collider

Jet-level

What we use in the collider analyses is jets, isolated leptons, τ-jets, photons, and missing energy.

- Jet: collimiated collections of hadrons, leptons, and photons.
- Isolated leptons and photons are objects that were not clustered into the jets.
- To construct jets, one must adopt jet algorithm: how to cluster the momenta of the final state particles into a certain number of jets.

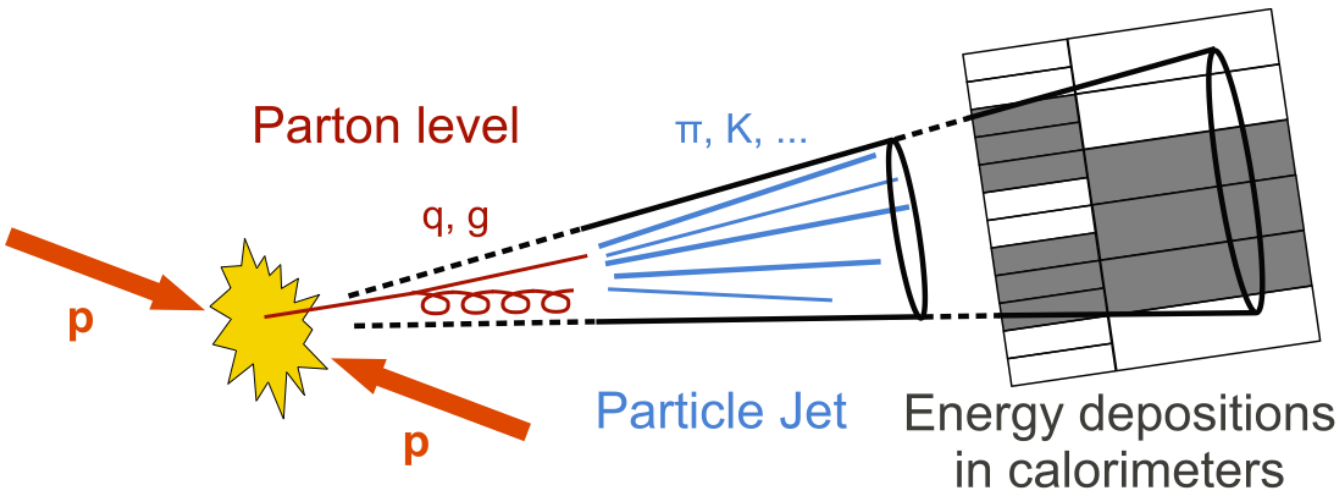


Collider

Detector-level

Detectors are not perfect, but has inherent limitations.

- imperfect calorimeters needs corrections on the jet energy, often called *jet energy scale*,
- finite energy resolution, often called *momentum smearing*,
- tagging/faking rates: mis-identification of particles,
- ...



They can be estimated by comparisons among simulations and data.

Collider variables

Invariant mass

To investigate the particle properties, we study various collider variables.

- The simplest collider variable is the invariant mass.
- We saw in the previous lecture a Breit-Wigner resonance in the invariant mass spectrum. For $pp \to Z \to e^+e^-$,

$$\begin{aligned} m_{e^+e^-}^2 &= (p_{e^+} + p_{e^-})^2 = (E_{e^+} + E_{e^-})^2 - |\boldsymbol{p}_{e^+} + \boldsymbol{p}_{e^-}|^2 \\ &= m_{e^+}^2 + m_{e^-}^2 + 2\left(E_{e^+}E_{e^-} - \boldsymbol{p}_{e^+} \cdot \boldsymbol{p}_{e^-}\right), \end{aligned}$$

where $m_e^2 = E_e^2 - |\boldsymbol{p}_e|^2$. If we neglect m_e,

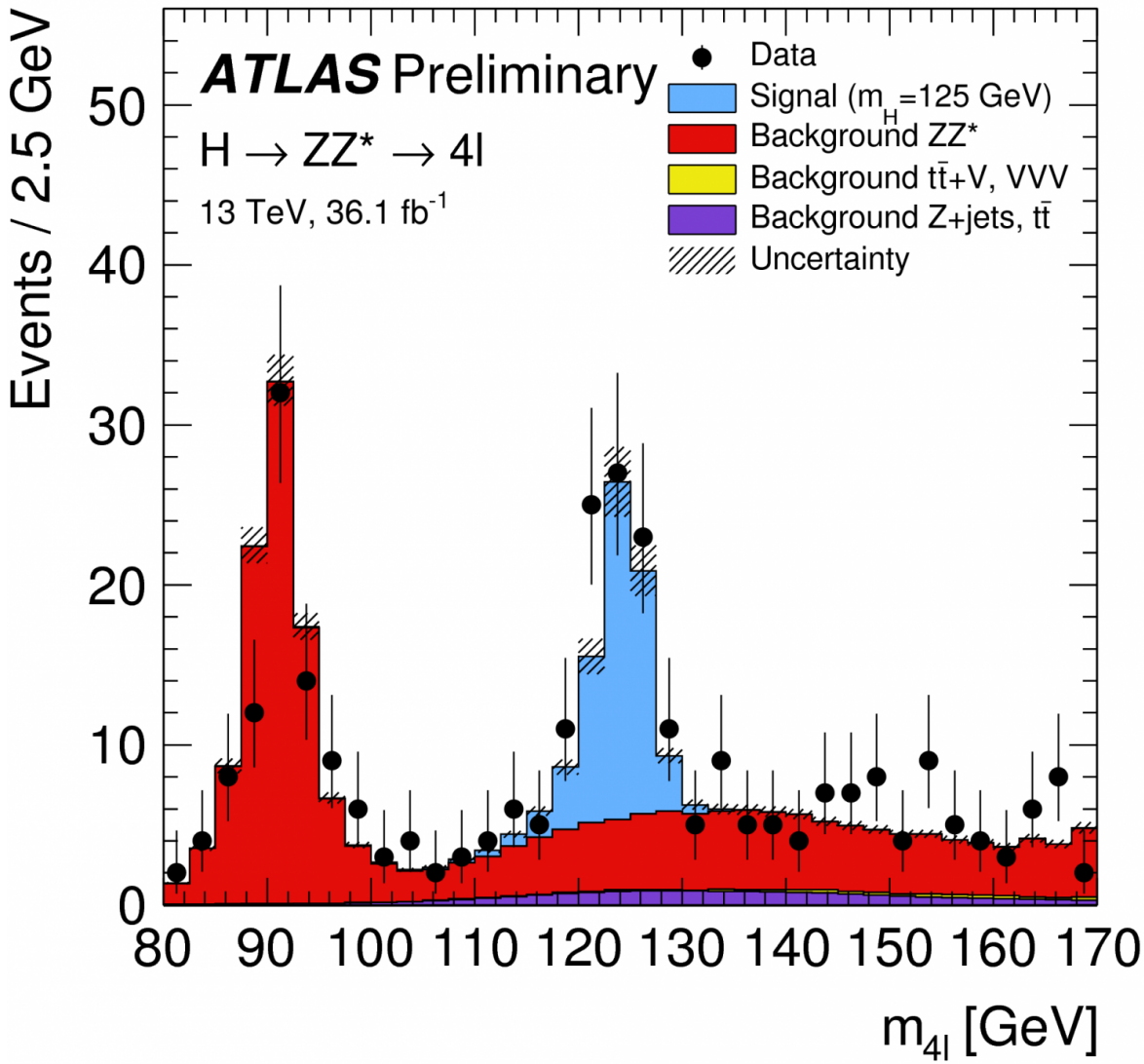
$$\boxed{m_{e^+e^-}^2 = 2E_{e^+}E_{e^-}(1 - \cos\theta_{e^+e^-})}$$

- It is straightforward to generalize the invariant mass variable to multi-body system. For $h \to ZZ \to e^+e^-\mu^+\mu^-$,

$$\begin{aligned} m_{e^+e^-\mu^+\mu^-}^2 &= \left(p_{e^+} + p_{e^-} + p_{\mu^+} + p_{\mu^-}\right)^2 \\ &= (E_{e^+} + E_{e^-} + E_{\mu^+} + E_{\mu^-})^2 - |\boldsymbol{p}_{e^+} + \boldsymbol{p}_{e^-} + \boldsymbol{p}_{\mu^+} + \boldsymbol{p}_{\mu^-}|^2 \\ &= m_h^2 \quad (\text{up to finite decay width}) \end{aligned}$$

Collider variables

Invariant mass



Collider variables

Invariant mass

For the coordinate system using E_T (transverse energy) and y (rapidity),

$$p^\mu = (E_T \cosh y,\ p_T \cos\phi,\ p_T \sin\phi,\ E_T \sinh y)$$

the invariant mass is written as

$$\begin{aligned} m_{ab}^2 &= (p_a + p_b)^2 = m_a^2 + m_b^2 + 2(E_a E_b - \boldsymbol{p}_a \cdot \boldsymbol{p}_b) \\ &= m_a^2 + m_b^2 + 2(E_a E_b - \boldsymbol{p}_{aT} \cdot \boldsymbol{p}_{bT} - p_{az} p_{bz}) \\ &= m_a^2 + m_b^2 + 2(E_{aT} E_{bT} \cosh y_a \cosh y_b - \boldsymbol{p}_{aT} \cdot \boldsymbol{p}_{bT} - E_{aT} E_{bT} \sinh y_a \sinh y_b) \\ &= m_a^2 + m_b^2 + 2(E_{aT} E_{bT} \cosh \Delta y - \boldsymbol{p}_{aT} \cdot \boldsymbol{p}_{bT}), \end{aligned}$$

where $\Delta y = y_a - y_b$.

Collider variables

We can always compute the invariant mass variable for the observed particle momenta.

- But what if there is an invisible particle like neutrino?



Collider variables

Transverse mass

Consider the $pp \to q\bar{q}' + X \to W^+ + X \to e^+\nu + X$ process (X are jets not associated with the W production).

- The longitudinal momentum of W has event-by-event different value: $p_z^W = p_z^q + p_z^{\bar{q}'} = (x_1 - x_2)P_z$.
  - W is boosted along the beam axis.
  - Because $p_z^W = p_z^e + p_z^\nu$, the longitudinal momentum of neutrino cannot be determined.
- Because of momentum conservation in the transverse plane: $\boldsymbol{p}_T^W + \boldsymbol{p}_T^X = 0$,

$$\boldsymbol{p}_T^W = \boldsymbol{p}_T^e + \boldsymbol{p}_T^\nu.$$

From momentum conservation, we find $\boxed{\boldsymbol{p}_T^\nu = -\boldsymbol{p}_T^e - \boldsymbol{p}_T^X}$, missing transverse momentum.

- A viable approach is that we ignore all the longitudinal components (ignoring m_e):

$$\begin{aligned} m_{e\nu}^2 &= (E^e + E^\nu)^2 - |\boldsymbol{p}^e + \boldsymbol{p}^\nu|^2 = (|\boldsymbol{p}^e| + |\boldsymbol{p}^\nu|)^2 - |\boldsymbol{p}^e + \boldsymbol{p}^\nu|^2 \\ &\to (|\boldsymbol{p}_T^e| + |\boldsymbol{p}_T^\nu|)^2 - |\boldsymbol{p}_T^e + \boldsymbol{p}_T^\nu|^2 \\ &= 2|\boldsymbol{p}_T^e||\boldsymbol{p}_T^\nu|(1 - \cos\phi) = 2E_T^e E_T^\nu (1 - \cos\phi) = m_{e\nu T}^2 \end{aligned}$$

Collider variables

Transverse mass

In general, for $pp \to Y \to v + \chi$, where v is a set of observed particles and χ is an invisible particle, the transverse mass can be obtained from the invariant mass by setting $y_\chi = y_v$.

$$m_{v\chi}^2 = m_v^2 + m_\chi^2 + 2(E_T^v E_T^\chi \cosh \Delta y - \boldsymbol{p}_T^v \cdot \boldsymbol{p}_T^\chi)$$
$$\xrightarrow{\Delta y = 0} \boxed{m_v^2 + m_\chi^2 + 2(E_T^v E_T^\chi - \boldsymbol{p}_T^v \cdot \boldsymbol{p}_T^\chi) = m_T^2}$$

Because $\cosh \Delta y \geq 1$,

$$\boxed{0 \leq m_T \leq m_{v\chi}}$$

- If we have a Breit-Wigner resonance $m_{v\chi} = m_Y$,

$$\frac{d^2\sigma}{dm_{v\chi}^2 dm_T^2} \propto \frac{\Gamma_Y m_Y}{(m_{v\chi}^2 - m_Y^2)^2 + \Gamma_Y^2 m_Y^2} \frac{1}{m_{v\chi}\sqrt{m_{v\chi}^2 - m_T^2}}.$$

  This leads to a "peak" around m_Y.

Collider variables

Transverse mass

