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Study of three-body B decays in PQCD

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In coloration with ZT Zou, HN Li



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Outline

- Motivation
- Theoretical framework in PQCD Approach
- Some results and discussions
- Summary



Motivation

01 They can help us to test the factorization approach, which has been used in two-body decays successfully.

02 They also help us to test SM and probe the NP signal

03 They offer us opportunities to study the line-shape of intermediate states.

04 Combing with the two-body decays and the measurements, we can try to understand the sources of strong phases.

05 The study of three-body decays helps us to test the SU(3) asymmetry.

The status of 3-body decays of B meson

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Experiment

Large amounts of data from CDF, D0, Belle, BaBar, LHCb (Most with BFs ~ 10^{-5})

$$B^- \to \pi^+\pi^-\pi^-, K^-\pi^+\pi^-, K^+K^-\pi^-, K^+K^-K^-, K^-K_SK_S$$

$$\overline{B}^0 \to \overline{K}^0\pi^+\pi^-, K^-\pi^+\pi^0, K^0K^-\pi^+, K^+K^-\pi^0, K^+K^-\overline{K}^0, K_SK_SK_S$$

$B_{(s)} \to DPP$ $\quad$ $B \to \mathrm{DDP}$

$B^0_{(s)} \to \mathrm{X}_{c\bar{c}}(\pi\pi, KK, K\pi \ldots)$

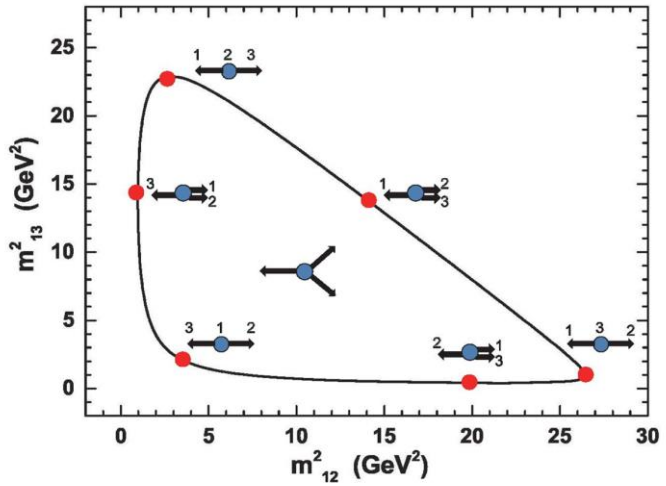
Theory

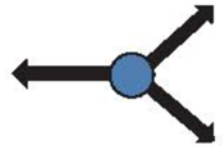
- Factorization Approach — H.Y. Cheng, C.K. Chua, Y. Li,…
- QCD Factorization — A. Furman, B.El Bennich, R. Kaminski, T. Mannel, X.H. Guo, Y.D. Yang, Z.H. Zhang,…
- PQCD — HN. Li, WF Wang, YLI
- Symmetry — X.G. He, G.N. Li, D. Xu, J.L. Rosner, M. Gronau,…
- Sum rules — Ulf-G. Meißner (Light-cone), A. Khodjamirian (QCD)

The data tells us that

- the nonresonant signal in charm decays is small, less than 10% ;
- the nonresonant fraction is about (70–100)% in $B \to KKK$ decays;
- the nonresonant contributions share about 17–40% in $B \to K\pi\pi$ decays;
- The fraction of nonresonant contribution is about 35% in the $B \to \pi\pi\pi$ decay.
- Hence, the nonresonant three-body decays play an essential role in penguin-dominated B decays. For example $B \to KKK$ decays.

Dalitz plot analysis





The central region corresponds to the case where all three final particles fly apart with large energy (E≈mB/3).

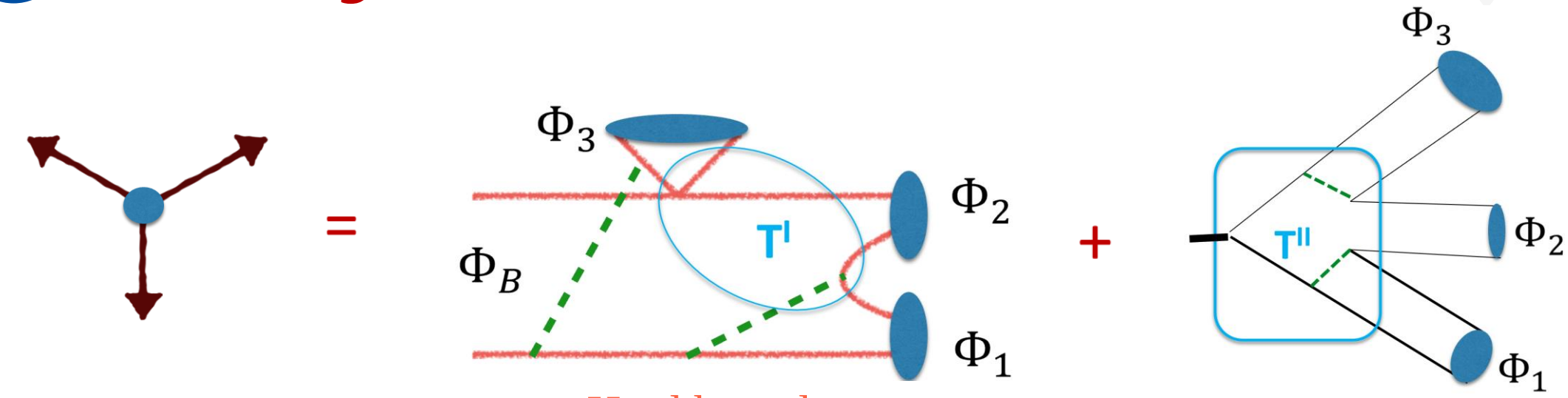
The corners correspond to the case in which one of final particles is approximately at rest (i.e. soft), and the other two fly back-to-back with large energy (E ≈mB/2).



The central part of the edge corresponds to the case in which two particles move collinearly with large energy and the other energetic particle recoils back.



Center region



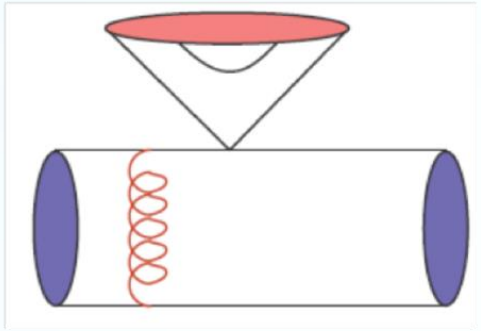
Hard kernels

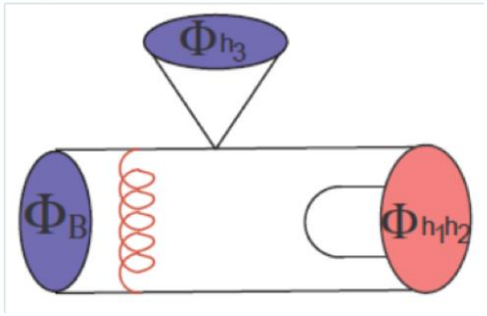
$$\langle P_1P_2P_3|\mathcal{O}_i|B\rangle_c = T_i^I \otimes F^{B\to P_1} \otimes \Phi_{P_2} \otimes \Phi_{P_3} + T_i^{II} \otimes \Phi_B \otimes \Phi_{P_1} \otimes \Phi_{P_2} \otimes \Phi_{P_3}$$

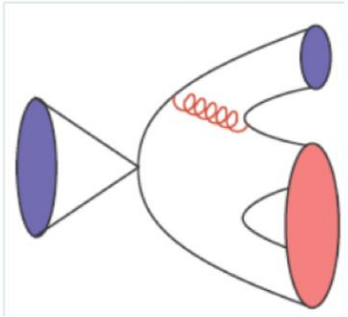
- In this region, two **hard gluons** are needed, so this contributions should be both αs- and power-suppressed.
- Totally, there are 256 Feynman diagrams with complicate kinematics, we are in progress for about 5 years.

Edge-Regions： Quasi-two-body B Decays









$$\langle P_1P_2P_3|\mathcal{O}_i|B\rangle_c = T_i^I \otimes F^{B\to P_1} \otimes \Phi_{P_2P_3} + T_i^{II} \otimes F^{B\to P_1P_2} \otimes \Phi_{P_3} + T_i^{III} \otimes \Phi_B \otimes \Phi_{P_1} \otimes \Phi_{P_2P_3}$$

Hard kernels

Transition form factor

Two-meson wave function

Quasi-two-body B Decays

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Three-body nonleptonic B decays in perturbative QCD

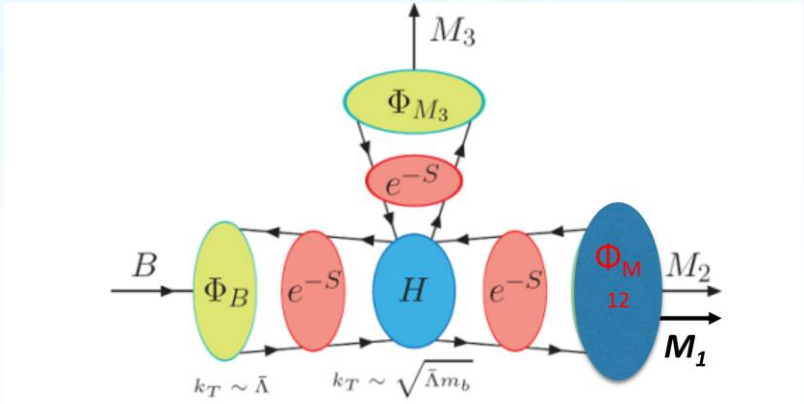
Chuan-Hung Chen, Hsiang-Nan Li

Abstract

We develop perturbative QCD formalism for three-body nonleptonic B meson decays. Leading contributions are identified by defining power counting rules for various topologies of amplitudes. The analysis is simplified into the one for two-body decays by introducing two-meson distribution amplitudes. This formalism predicts both nonresonant and resonant contributions, and can be generalized to baryonic decays.

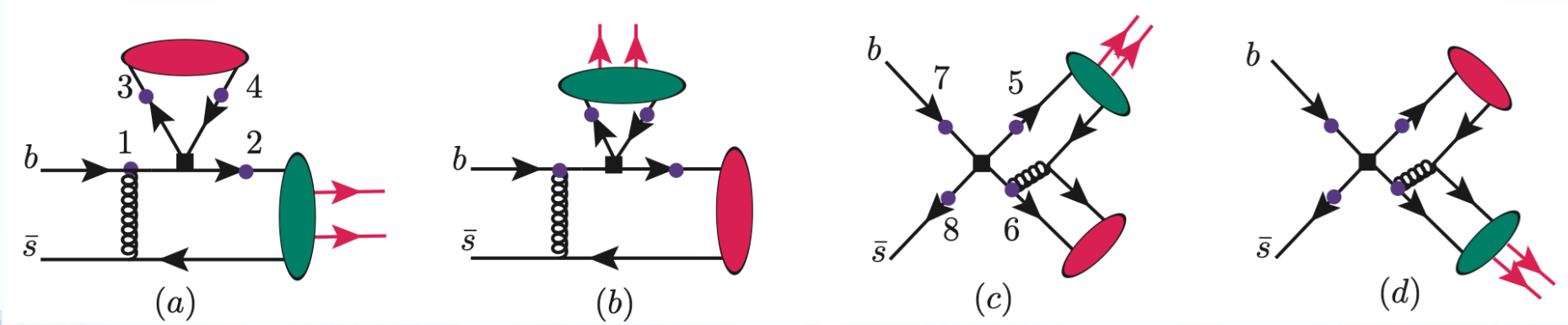
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PQCD framework for quasi 2-body B decays



$$\Phi_B \otimes \mathcal{H} \otimes \Phi_{M_1 M_2} \otimes \Phi_{M_3} \exp\left\{-s(s,p) - 2\int_{1/b}^{t} \frac{d\mu}{\mu}\gamma_q(\alpha_s(\mu))\right\}$$

- Nonperturbative parts are factorized into universal wave functions
- The hard part can be calculated perturbatively.
- Parton k_T smears end - point singularity

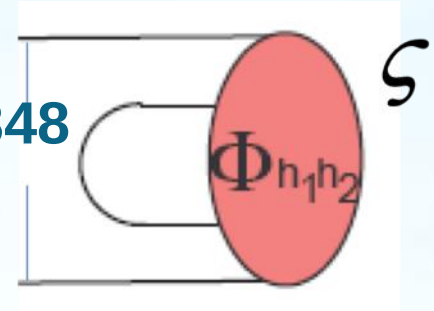


LCDA of the two-meson pair

- **For the P-wave $K\pi$ pair**

$$\Phi_P = \frac{1}{\sqrt{2N_c}}\left[\not{P}\phi_P(z,\zeta,\omega) + \omega\phi_P^s(z,\zeta,\omega) + \frac{\not{P}_1\not{P}_2 - \not{P}_2\not{P}_1}{\omega(2\zeta-1)}\phi_P^t(z,\zeta,\omega)\right]$$

$$\begin{aligned}
\phi_P(z,\zeta,\omega) &= \frac{3F_P^{\parallel}(\omega)}{\sqrt{2N_C}}z(1-z)\left[1+a_1C_1^{3/2}(t)+a_2C_2^{3/2}(t)\right](2\zeta-1-\xi),\\
\phi_P^s(z,\zeta,\omega) &= \frac{3F_P^{\perp}(\omega)}{2\sqrt{2N_C}}\left[t(1+a_st)-a_s2z(1-z)\right](2\zeta-1),\\
\phi_P^t(z,\zeta,\omega) &= \frac{3F_P^{\perp}(\omega)}{2\sqrt{2N_C}}t\left[t+a_t(3t^2-1)\right](2\zeta-1),
\end{aligned}$$



$$\frac{F_P^{\perp}}{F_P^{\parallel}} \approx \frac{f_V^T}{f_V},$$

- **For the P -wave form factor $F_P^{\parallel}$,we adopt the RBW model**

$$F_P^{\parallel}(\omega) = \frac{cm_0^2}{m_0^2 - \omega^2 - im_0\Gamma(\omega)}$$

$K^*(980), K^*(1410), K^*(1680)$ **should be considered.**

- **For S-wave $K\pi$-pair,**

$$\Phi_S = \frac{1}{2\sqrt{N_c}}\left[\not{P}\phi_S(z,\zeta,\omega) + \omega\phi_S^s(z,\zeta,\omega) + \omega(\not{n}\not{v} - 1)\phi_S^t(z,\zeta,\omega)\right]$$

Scalar form factor

$$\phi_S(z,\zeta,\omega) = \frac{6}{2\sqrt{2N_c}} F_S(\omega) z(1-z)\left[\frac{1}{\mu_S} + B_1 C_1^{3/2}(1-2z) + B_3 C_3^{3/2}(1-2z)\right],$$

$$\phi_S^s(z,\zeta,\omega) = \frac{1}{2\sqrt{2N_c}} F_S(\omega),$$

$$\phi_S^t(z,\zeta,\omega) = \frac{1}{2\sqrt{2N_c}} F_S(\omega)(1-2z),$$

SU(3) breaking, tiny,
suppressed by the difference of quark masses

S-wave form factor

- The form factor is often parameterized in the relativistic Breit-Wigner (RBW) model.
- However, the RBW function is a good model for narrow resonances that are well separated from other resonant or non-resonant contributions of the same spin.
- RBW model is broken down in the Kπ S-wave because the $K_0^*(1430)$ resonance interferes strongly with a slowly varying nonresonant contribution.

0712.1605

- LASS line-shape is developed to evaluate the combined amplitude, the expression of which is given by

1711.09854

$$F_S(\omega) = \frac{\omega}{|p_1|(\cot\delta - i)} + e^{2i\delta}\frac{m_0\Gamma_0\frac{m_0}{|p_0|}}{m_0^2 - \omega^2 - im_0\Gamma_0\frac{|p_1|}{\omega}\frac{m_0}{|p_0|}},$$

Nonresonance

resonance

$$\cot\delta = \frac{1}{a|p_1|} + \frac{r|p_1|}{2}$$

Numerical Results and Discussions

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Decay Modes	PQCD	EXP Unit:10^{-6}
$B_s \to K^{\pm}(K^{*\pm}(892) \to) \overline{(K)}^0\pi^{\pm}$	$12.2^{+3.8+4.7+0.5}_{-2.3-2.8-0.0}$	$12.4 \pm 0.8 \pm 0.5 \pm 2.7 \pm 1.3$
$B_s \to K^{\pm}(\overline{(K)}^0\pi^{\pm})^{*\ 1}_0$	$26.0^{+11.7+7.9+1.0}_{-9.2-6.1-0.8}$	$24.9 \pm 1.8 \pm 0.5 \pm 20.0 \pm 2.6$
$B_s \to K^{\pm}(K^{*\pm}_0(1430) \to) \overline{(K)}^0\pi^{\pm}$	$20.9^{+8.8+6.9+0.7}_{-7.2-4.7-0.2}$	$19.4 \pm 1.4 \pm 0.4 \pm 15.6 \pm 2.0 \pm 0.3$
$B_s \to K^{\pm}(\overline{(K)}^0\pi^{\pm})_{NR}$	$13.1^{+6.0+3.7+0.4}_{-5.0-2.9-0.5}$	$11.4 \pm 0.8 \pm 0.2 \pm 9.2 \pm 1.2 \pm 0.5$
$B_s \to K^{\pm}(K^{*\pm}_2(1430) \to) \overline{(K)}^0\pi^{\pm}$	$4.4^{+2.2+1.7+0.5}_{-1.5-1.1-0.0}$	$3.4 \pm 0.8 \pm 0.8 \pm 5.4 \pm 0.4$
$B_s \to \overline{(K)}^0(\overline{(K)}^{*0}(892) \to)K^{\mp}\pi^{\pm}$	$11.4^{+4.1+5.1+1.3}_{-1.6-3.0-0.0}$	$13.2 \pm 1.9 \pm 0.8 \pm 2.9 \pm 1.4$
$B_s \to \overline{(K)}^0(K^{\mp}\pi^{\pm})^*_0$	$25.8^{+11.8+8.1+1.1}_{-9.1-5.8-0.1}$	$26.2 \pm 2.0 \pm 0.7 \pm 7.3 \pm 2.8$
$B_s \to \overline{(K)}^0(\overline{(K)}^*_0(1430) \to)K^{\mp}\pi^{\pm}$	$20.1^{+8.9+7.0+1.0}_{-6.9-4.4-0.1}$	$20.5 \pm 1.6 \pm 0.6 \pm 5.7 \pm 2.2 \pm 0.3$
$B_s \to \overline{(K)}^0(K^{\mp}\pi^{\pm})_{NR}$	$12.9^{+5.9+3.7+0.4}_{-4.9-2.9-0.5}$	$12.1 \pm 0.9 \pm 0.3 \pm 3.3 \pm 1.3 \pm 0.5$
$B_s \to \overline{(K)}^0(\overline{(K)}^*_2(1430) \to)K^{\mp}\pi^{\pm}$	$3.4^{+1.6+1.4+0.3}_{-1.1-0.9-0.0}$	$5.6 \pm 1.5 \pm 0.6 \pm 7.0 \pm 0.6$

Eur.Phys.J.C 80 (2020) 6, 517
JHEP 06 (2019) 114

Numerical Results and Discussions

- Compared with the mass, the width of K^* is narrow enough, the narrow-width approximation (NWA) works well here. Under this approximation, the quasi-two-body decay with the resonance R can be factorized as

$$\mathcal{B}(B \to RP \to P_1P_2P) = \mathcal{B}(B \to RP) \times \mathcal{B}(R \to P_1P_2).$$

- Using the averaged $B(K^*(892) \to K\pi) \approx 100\%$, we then estimate the branching ratios of the corresponding two-body decays as

Approach	$B_s \to K^{\pm}K^{*\mp}$	$B_s \to \overline{K}^0 \overline{K}^{*0}$
QCDF [122]	$21.6^{+12.1}_{-7.8}$	$20.6^{+12.4}_{-8.0}$
SCET [123]	$19.7^{+5.3}_{-4.5}$	$18.7^{+5.2}_{-4.4}$
Flavor Symmetry [124]	16.01 ± 0.91	15.65 ± 0.87
PQCD [125]	$10.7^{+3.7}_{-2.5}$	$11.6^{+4.0}_{-2.6}$
This Work	$18.3^{+9.0}_{-5.4}$	$17.1^{+9.9}_{-5.2}$
Exp [29]	18.6 ± 4.7	19.8 ± 5.7

Unit:10^{-6}

NWA is invalid in $B \to M_1(f_0(980) \to)KK$ decays

If NWA works well in $B^+ \to K^+ f_0(980) \to K^+K^+K^-$ and $B^+ \to K^+ f_0(980) \to K^+\pi^+\pi^-$ decays, the ratio between the $f_0(980) \to K^+K^-$ and $f_0(980) \to \pi^+\pi^-$ can be defined as

$$\mathcal{R}_1 \equiv \frac{\mathcal{B}(f_0(980) \to K^+K^-)}{\mathcal{B}(f_0(980) \to \pi^+\pi^-)} = \frac{\mathcal{B}(B^+ \to K^+ f_0(980) \to K^+K^+K^-)}{\mathcal{B}(B^+ \to K^+ f_0(980) \to K^+\pi^+\pi^-)} \sim 1.0^{+0.5}_{-0.4}$$

But:

- $\mathcal{R} = 0.69 \pm 0.32$ between $B \to KK^+K^-$ and $B \to K\pi^+\pi^-$ decays by BABAR. Phys. Rev. D 74, 032003 (2006)
- While $\mathcal{R} = 0.92 \pm 0.07$ if the parameters of $f_0(980)$ are adopted from BES
- Meanwile $\mathcal{R} \sim 0.625 \pm 0.21$ by BES using $J/\psi \to \phi f_0(980) \to \phi\pi^+\pi^-$ and $J/\psi \to \phi f_0(980) \to K^+K^-$ decays

Phys. Lett. B 607, 243–253 (2005)

- BES also obtained $\mathcal{R} = 0.25^{+0.22}_{-0.20}$ by $J/\psi \to \gamma\chi_{c0} \to \gamma f_0(980) f_0(980) \to \gamma\pi^+\pi^-K^+K^-$ and $J/\psi \to \gamma\chi_{c0} \to \gamma f_0(980) f_0(980) \to \gamma\pi^+\pi^-\pi^+\pi^-$

Phys. Rev. D 70, 092002 (2004)

Phys. Rev. D 72, 092002 (2005)

- $\mathcal{R} = 0.37^{+0.23}_{-0.13}$ in $B_s \to J/\psi\, f_0(980) \to J/\psi\, \pi^+\pi^-$ and $B_s \to J/\psi\, f_0(980) \to J/\psi\, \pi^+\pi^-$

Eur. Phys. J. C 79(9), 792 (2019)

NWA is invalid in $B \to M_1(f_0(980) \to)KK$ decays

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- **It seems that we hardly can reach a reliable and universal $\mathcal{R}$, and even the PDG have not performed the averaged value using the current experimental data.**

- **The above-mentioned discrepancy can be resolved by noting that NWA for the resonant three-body decay is applicable only when the two-body decays $B \to RM$ and $R \to M_1M_2$ are kinematically allowed and the resonance is narrow. However, as the decay $f_0(980) \to KK$ is kinematically barely or even not allowed, the off resonance peak effect of the resonant state will become important.**

$$\mathcal{B}(B \to RP \to P_1P_2P) = \mathcal{B}(B \to RP) \times \mathcal{B}(R \to P_1P_2).$$

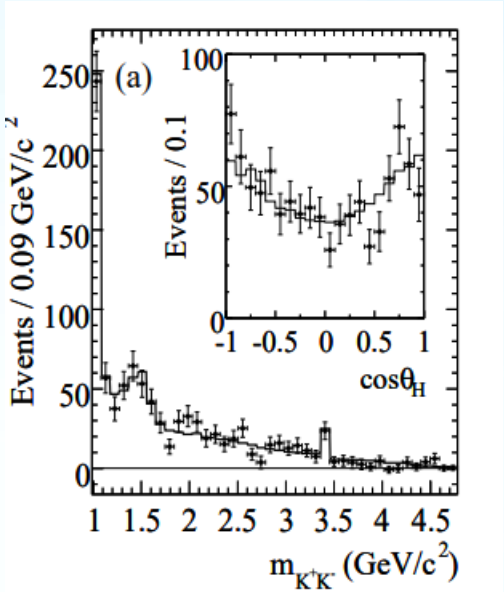
Eur. Phys. J. C (2020) 80:394

The $f_X(1500)$ puzzle

- Measurements of $B^0 \to K^+K^-K_s$ and $B^\pm \to K^+K^-K^\pm$ from BaBar and Belle have shown evidences of a wide structure in the m_{KK} spectrum around 1.5 GeV. It is assumed as a single scalar resonance; however a vector hypothesis could not been ruled out.

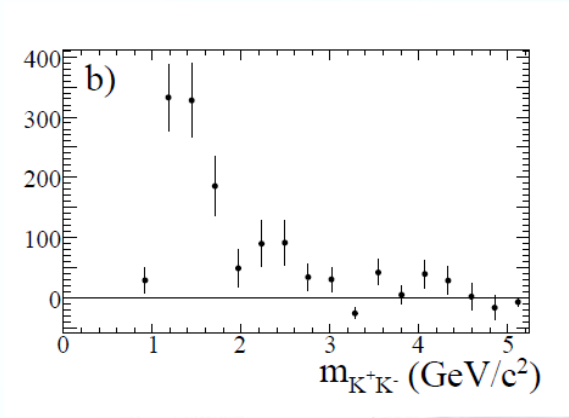
Phys. Rev. D 71, 092003(2005); Phys. Rev. D 82, 073011(2010).

Phys. Rev. D 74, 032003(2006); Phys. Rev. Lett 99, 161802(2007);



- The BaBar measurement of $B^+ \to K^+K^-\pi^+$ appears to show an enhancement around 1.5 GeV. The structure is either a vector meson or something exotic.

Phys. Rev. Lett. 99, 221801(2007).



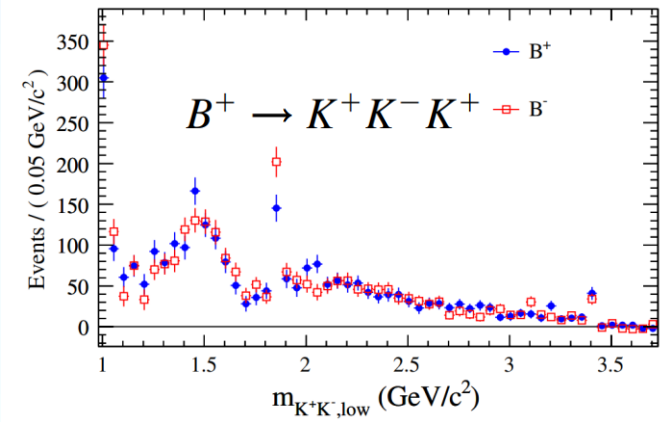
- The BaBar analysis of $B^\pm \to K_sK_s\pi^\pm$ finds no evidence of a possible $f_X(1500)$

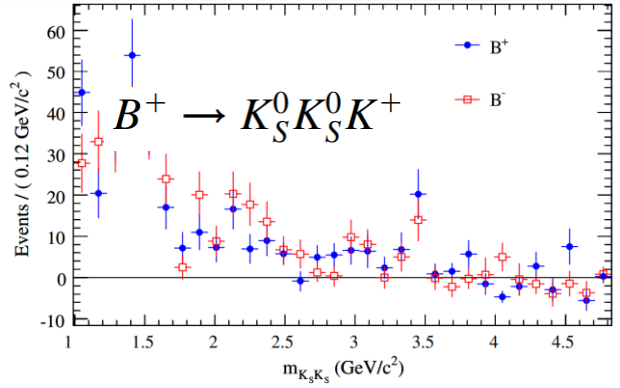
Phys. Rev. D 79, 051101(R) (2009).

The $f_X(1500)$ puzzle

- **Dalitz-plot analyses of $B^0 \to K^+K^-K_S$, $B^+ \to K^+K^-K^+$ and $B^+ \to K_SK_SK^+$ by BaBar**

Phys. Rev. D 85 , 112010 (2012)





Conclusion : The $f_X(1500)$ is not a single scalar resonance, but instead can be described by the sum of the well-established resonances $f_0(1500)$, $f_2(1525)$, and $f_0(1710)$.

- **But in the amplitude analysis of $B_s \to K_SK_SK_S$ by BaBar**

Phys. Rev. D 85, 054023 (2012)

"We do not observe any significant contribution from the so-called $f_X(1500)$ resonance seen in, for example, $B^+ \to K^+K^-K^+$. The peak in the invariant mass between 1.5 and 1.6 GeV can be described by the interference between the $f_0(1710)$ resonance and the nonresonant component."

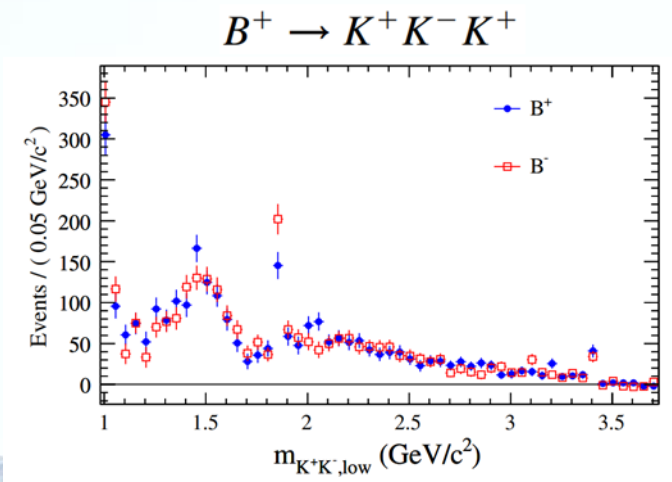
Is the $f_X(1500)$ a vector meson?

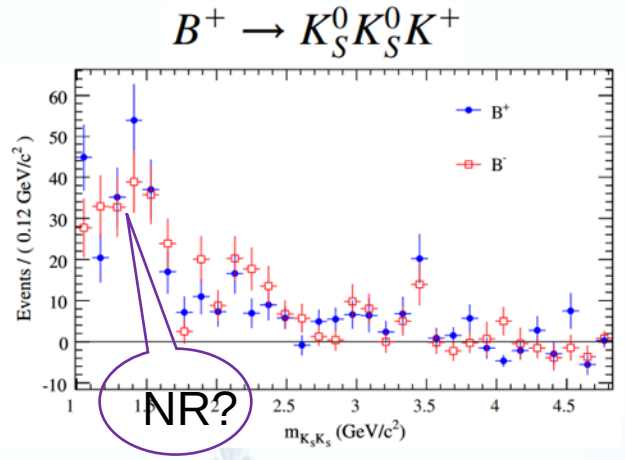
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- **2019, LHCb reported the amplitude analysis of $B^{\pm} \to \pi^{\pm}K^{+}K^{-}$ decays, where a resonance $\rho^0(1450)$ with wide width dominates the $K^{+}K^{-}$ system.**

$f_X(1500) = \rho^0(1450)$???

$\Gamma(\rho(1450)) = 400 \pm 60$ MeV

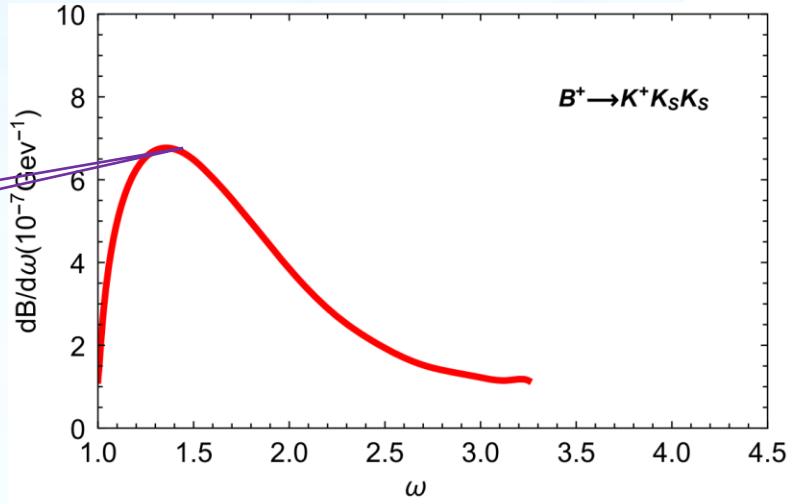


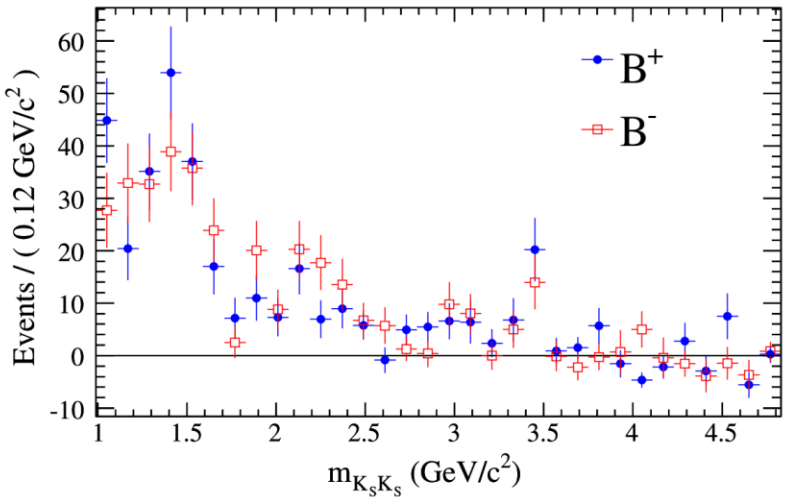


If $f_X(1500) = \rho^0(1450)$

$B^+ \to K_S^0 K_S^0 K^+$

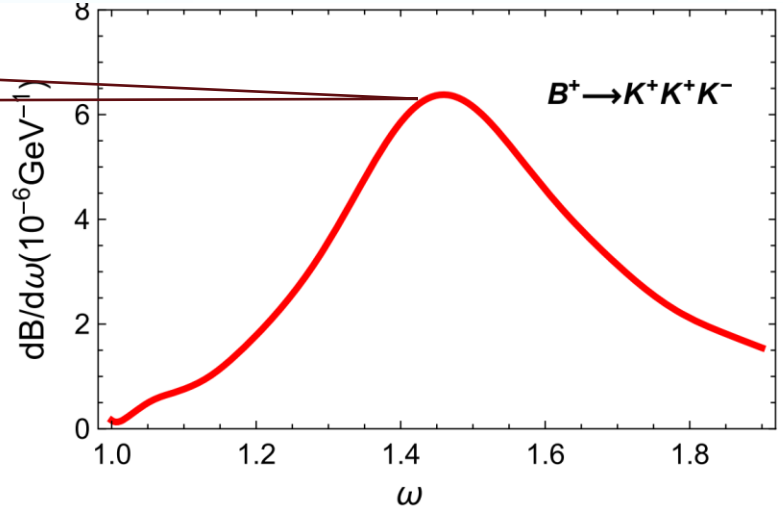
NR

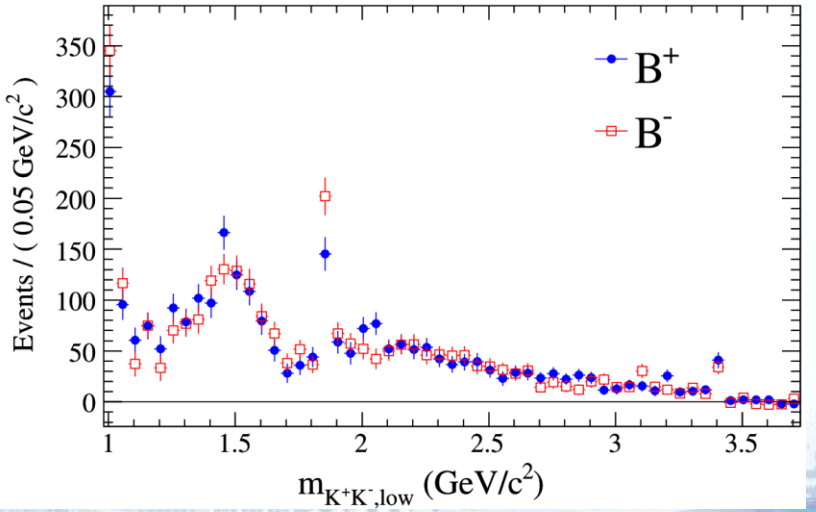


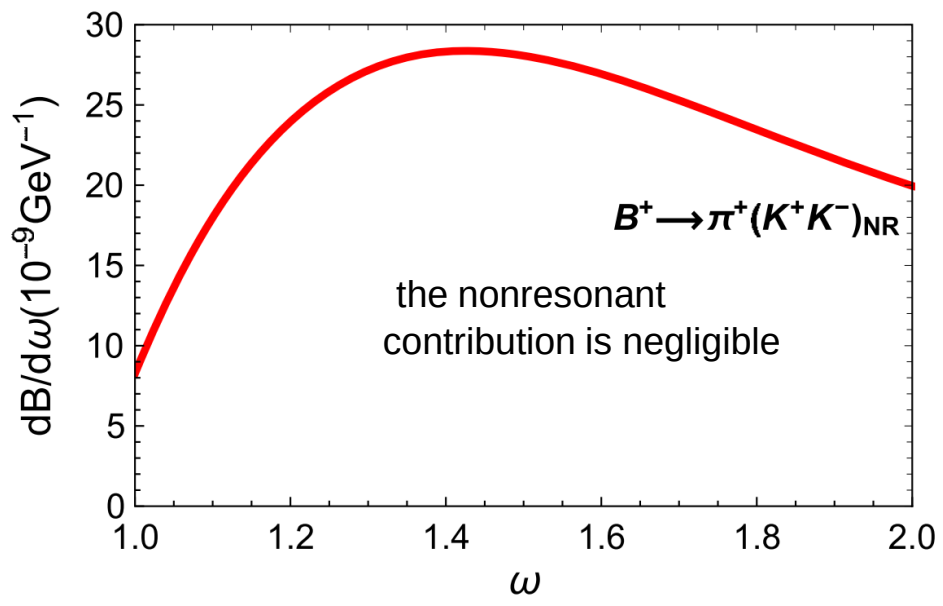


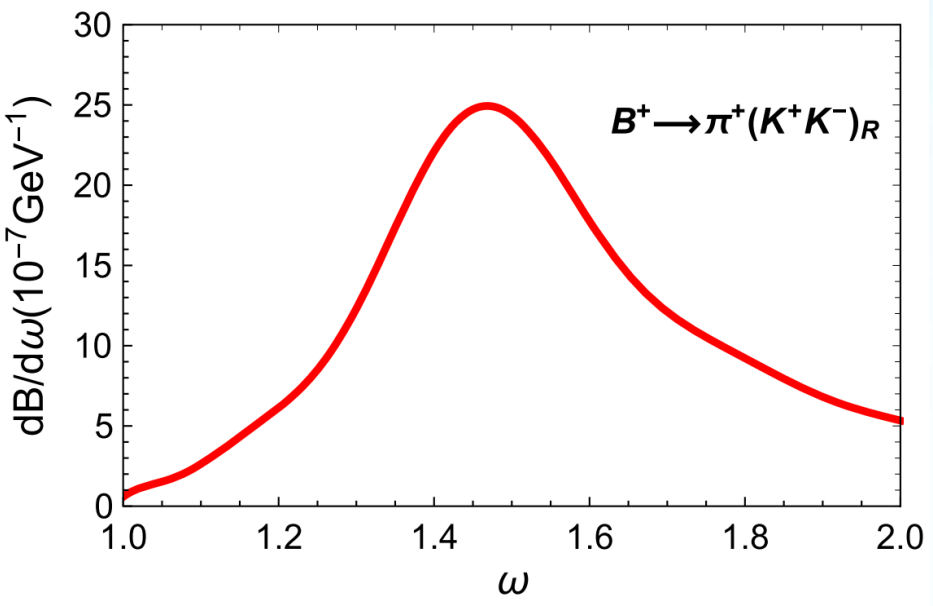
$B^+ \to K^+ K^- K^+$

R









Suggestion : $f_X(1500) = \rho^0(1450)$

$$\mathcal{B}(B^+ \to K^+\rho^0(1450) \to K^+K^+K^-) = (3.62^{+1.33+0.87+0.18}_{-1.42-0.81-0.24}) \times 10^{-6},$$

$$\mathcal{B}(B^0 \to K^0\rho^0(1450) \to K^0K^+K^-) = (7.49^{+4.36+3.76+0.35}_{-3.96-3.05-0.12}) \times 10^{-7},$$

$$\mathcal{B}(B^\pm \to \pi^\pm\rho^0(1450) \to \pi^\pm K^+K^-) = (1.60^{+0.21}_{-0.20}) \times 10^{-6}.$$

$$\mathcal{B}(B^\pm \to \pi^\pm\rho^0(1450) \to \pi^\pm K^+K^-) = (1.61^{+1.21}_{-0.87}) \times 10^{-6}.$$

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Summary

- 3-body decays of B meson are very important in heavy flavor physics.
- We studied quasi-two-body decays of B meson by introducing wave functions of meson pair with large uncertainties.
- However, we only calculated few parts of whole dalitz plot.
- We concluded that fx(1500) should be $\rho^0(1450)$.

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Thanks for your attention

