Constraining Cosmology with Large Scale Structure

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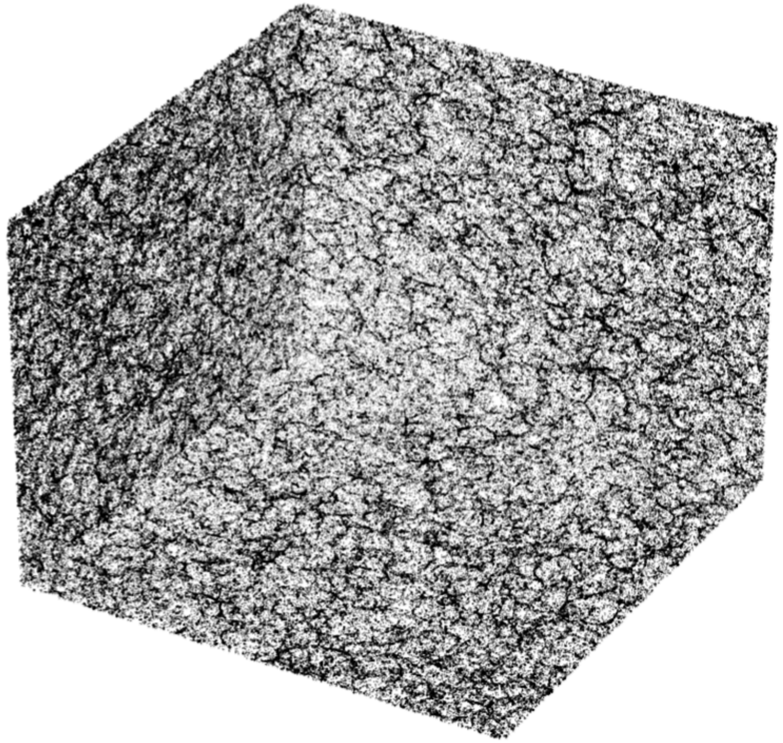
In collaboration with : Changbom Park, Pravabati Chingangbam, Sungwook E. Hong, Juhan Kim, Ho Seong Hwang, Priya Goyal, Motonari Tonegawa

GENUS (QUICKLY)

- Convert the angular/redshift information of a galaxy catalog into a three-dimensional point distribution (assume a cosmological model)
- Generate a continuous density field by smoothing
- Create a boundary of constant field value (in two dimensions, a perimeter)
- Count the number of connected regions and holes



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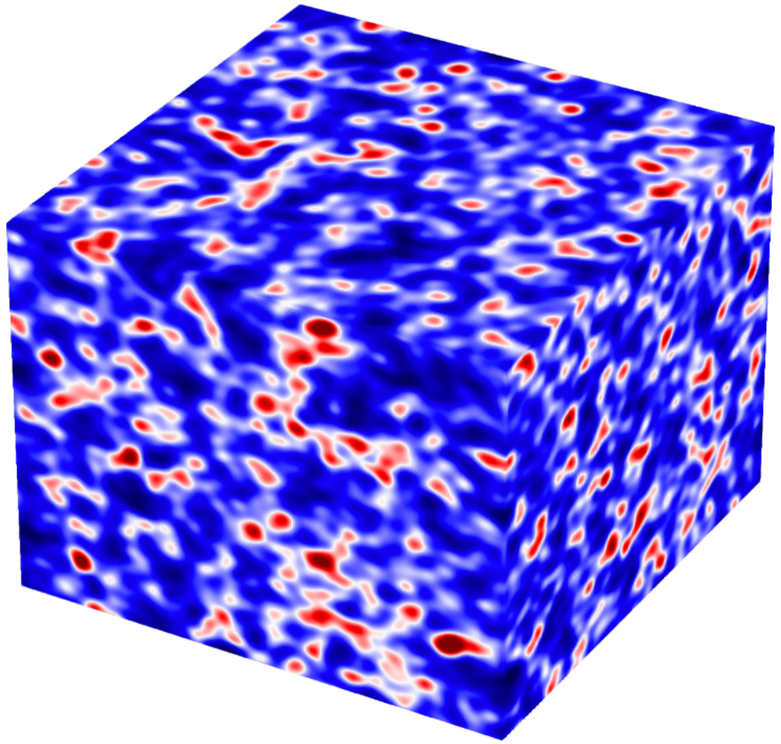

$\delta(x, y, z)$

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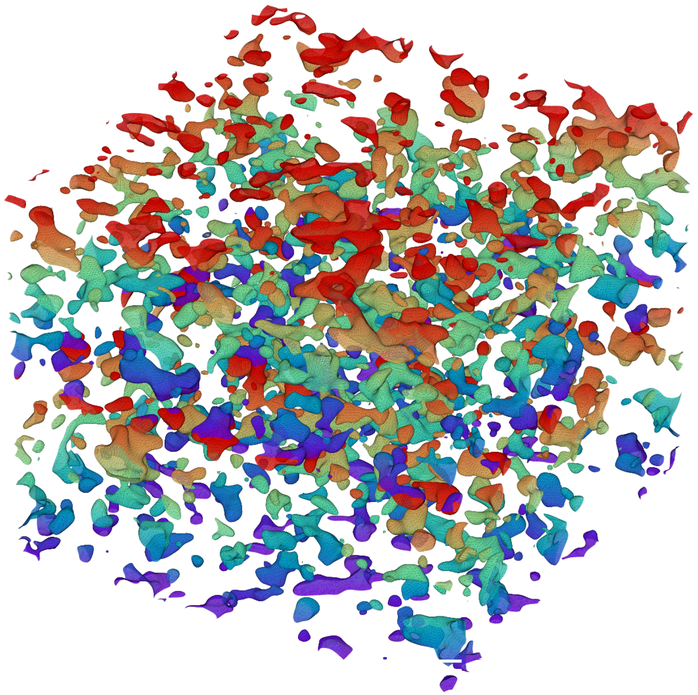
- Measure the morphology and topology of the excursion sets

Genus = number of connected regions - number of holes



$\delta(x, y, z) = \nu$

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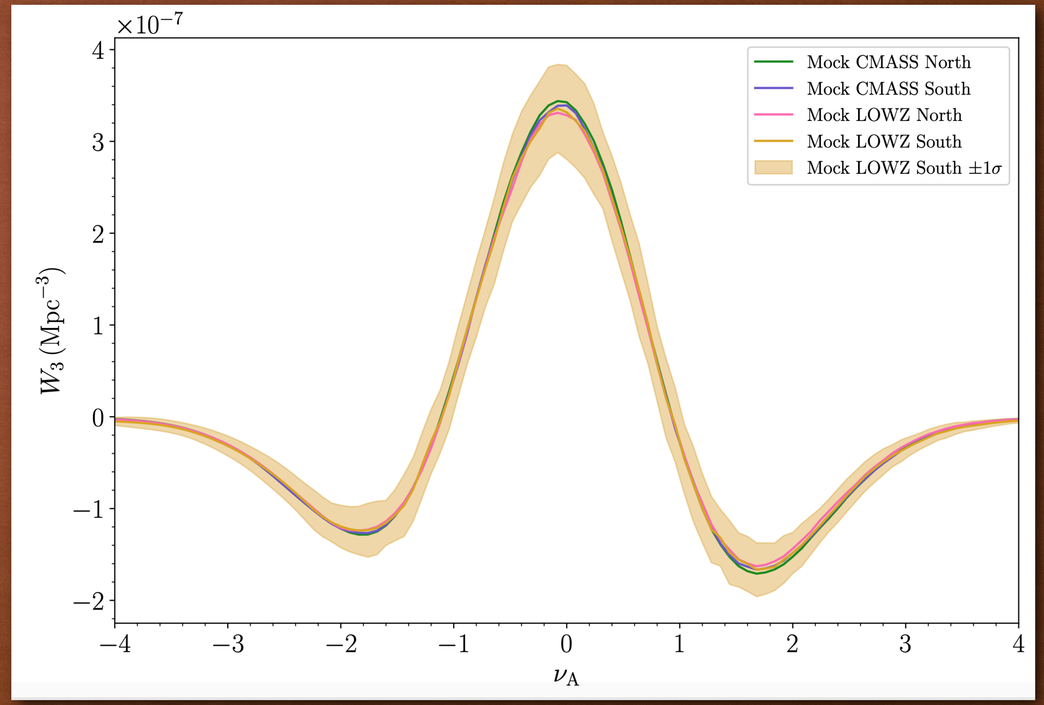
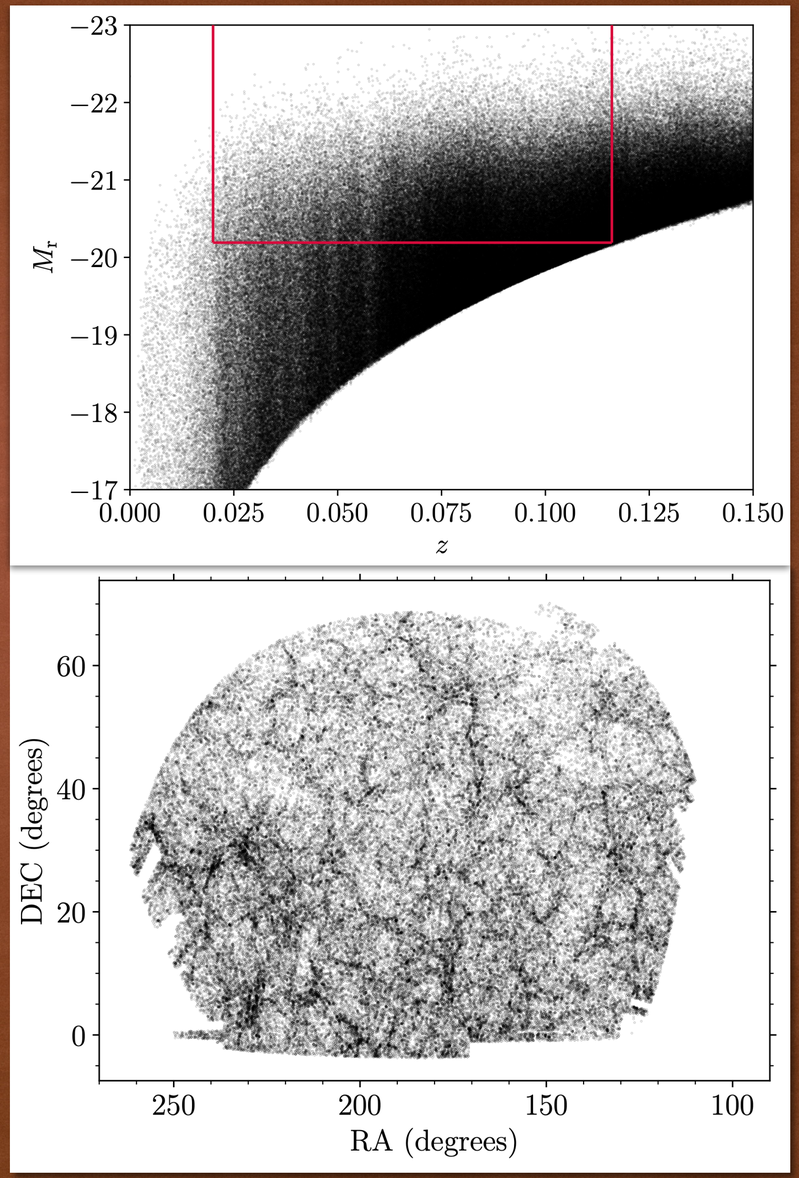


CONSTRAINTS FROM CURRENT SURVEYS

- SDSS MGS —
  - $\sim 600{,}000$ Galaxies
  - Magnitude limited
  - High Completeness, contiguous footprint
  - Dense Sample, $\bar{r} = \bar{n}^{-1/3} \sim 6\,h^{-1}\,\mathrm{Mpc}$
  - Modest volume —
  - $z \sim 0.1, f_{\mathrm{sky}} \sim 0.15, V \sim (250\,h^{-1}\,\mathrm{Mpc})^3$



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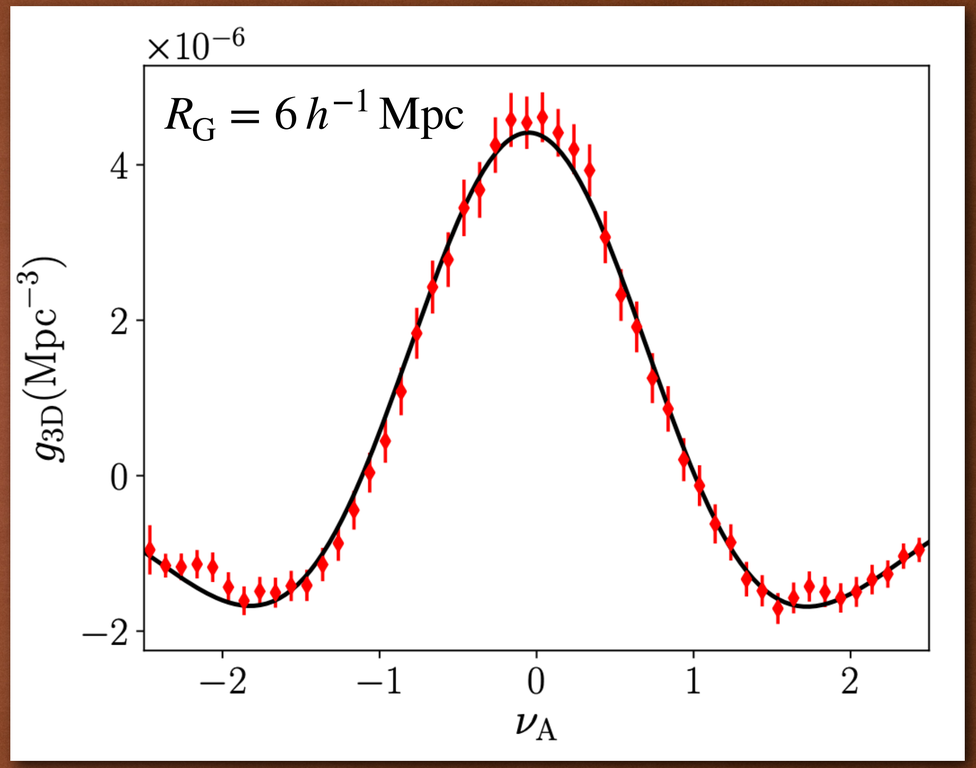
$$\sigma_i^2 = \frac{1}{2\pi^2}\int_0^\infty k^2 dk k^{2+2i} P(k) W^2(kR_G)$$

$$S^{(0)} = \frac{\langle \delta^3 \rangle}{\sigma_0^4}, \qquad S^{(1)} = \frac{3}{2}\frac{\langle \delta |\nabla\delta|^2 \rangle}{\sigma_0^2\sigma_1^2}, \qquad S^{(2)} = -\frac{9}{4}\frac{\langle |\nabla\delta|^2 \nabla^2\delta \rangle}{\sigma_1^4}$$



$$g_{3D}(\nu_v) = \frac{1}{(2\pi)^2}\left(\frac{\sigma_1}{\sqrt{3}\sigma_0}\right)^3 e^{-\nu_v^2/2}\left(H_2(\nu_v) + \left[(S^{(1)} - S^{(0)})H_3(\nu_v) + (S^{(2)} - S^{(0)})H_1(\nu_v)\right]\sigma_0 + \mathcal{O}(\sigma_0^2)\right)$$

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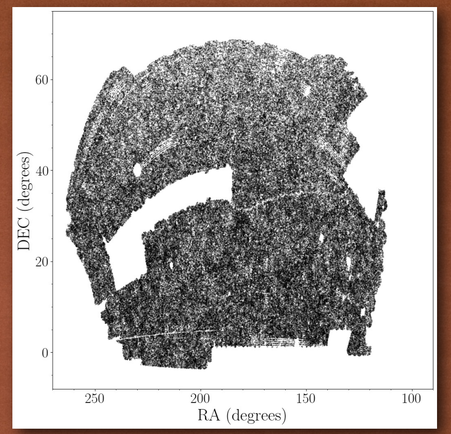
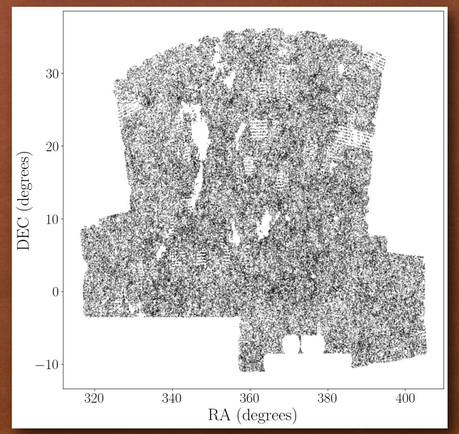
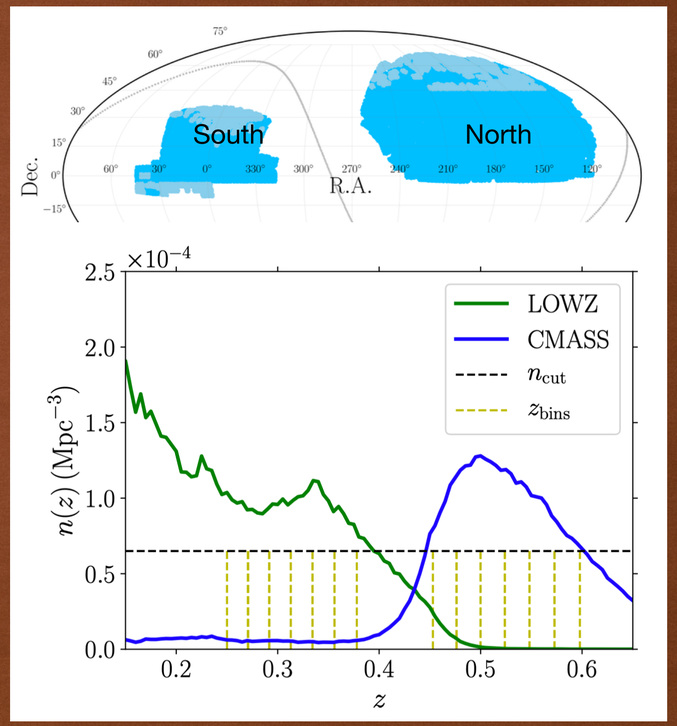
CONSTRAINTS FROM CURRENT SURVEYS

- BOSS —
  - $\sim 1{,}300{,}000$ Galaxies
  - Larger volume $0.2 \lesssim z \lesssim 0.7$, $V \sim 1.5\,h^{-3}\,\mathrm{Gpc}^3$
  - Two footprints, north and south sky
  - Colour selection (not magnitude limited)
  - Sparse sample — $\bar{r} \sim 16\,h^{-1}\,\mathrm{Mpc}$
  - Complicated angular selection function
  - Highly biased sample $b \sim 2$

LOWZ







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CONSTRAINTS FROM CURRENT SURVEYS

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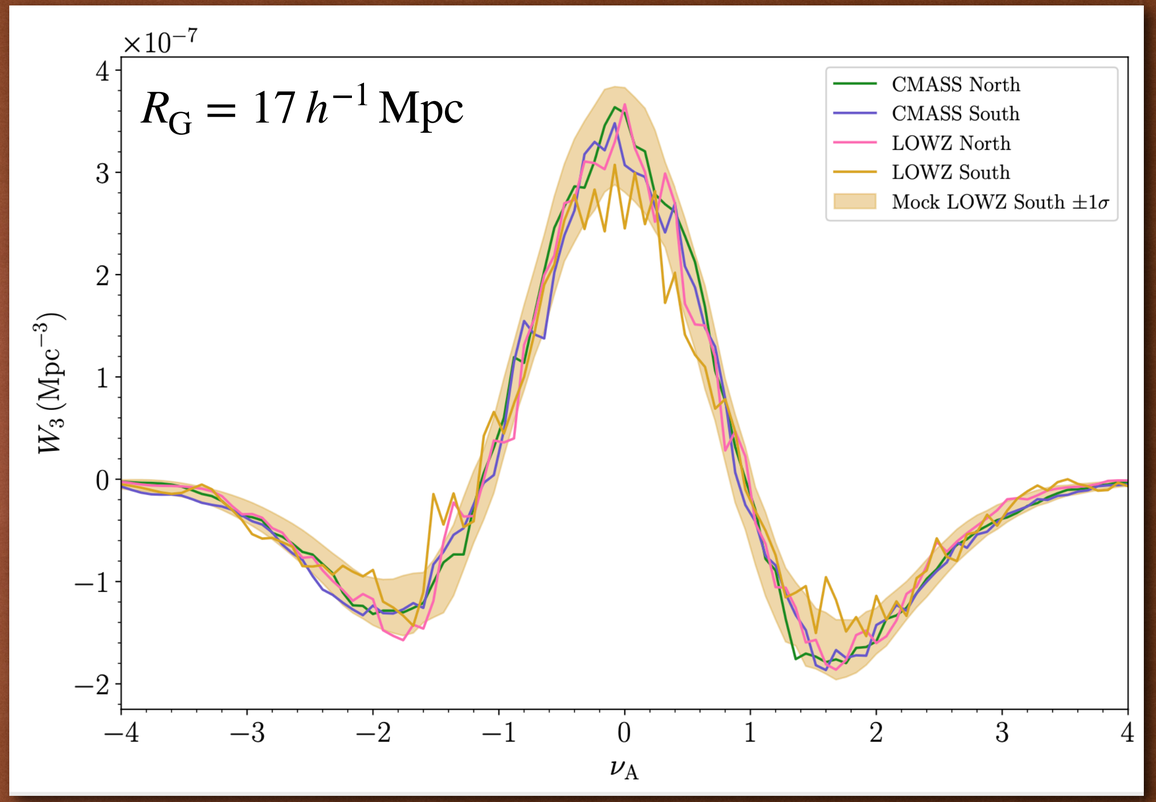
  - $\sim$ 1,300,000 Galaxies
  - Larger volume $0.2 \lesssim z \lesssim 0.7$, $V \sim 1.5\,h^{-3}\,\mathrm{Gpc}^3$
  - Two footprints, north and south sky

$$\sigma_i^2 = \frac{1}{2\pi^2}\int_0^\infty k^2 dk k^{2+2i} P(k) W^2(kR_G)$$

$$S^{(0)} = \frac{\langle \delta^3 \rangle}{\sigma_0^4}, \qquad S^{(1)} = \frac{3}{2}\frac{\langle \delta |\nabla\delta|^2 \rangle}{\sigma_0^2\sigma_1^2}, \qquad S^{(2)} = -\frac{9}{4}\frac{\langle |\nabla\delta|^2 \nabla^2\delta \rangle}{\sigma_1^4}$$



$$g_{3\mathrm{D}}(\nu_v) = \frac{1}{(2\pi)^2}\left(\frac{\sigma_1}{\sqrt{3}\sigma_0}\right)^3 e^{-\nu_v^2/2}\left(H_2(\nu_v) + \left[(S^{(1)} - S^{(0)})H_3(\nu_v) + (S^{(2)} - S^{(0)})H_1(\nu_v)\right]\sigma_0 + \mathcal{O}(\sigma_0^2)\right)$$

CONSTRAINTS FROM CURRENT SURVEYS

- From these curves, what can we say about cosmology?

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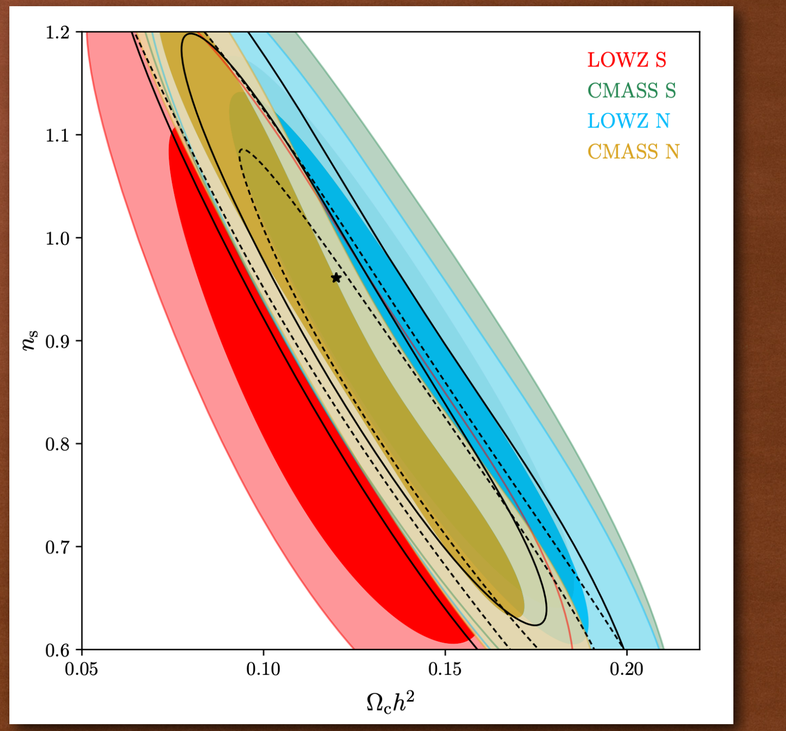
Data/MF	$\Omega_c h^2\, n_s$	a_0	a_2	χ^2_r
CMASS	0.110 ± 0.006	0.006 ± 0.012	0.183 ± 0.021	1.32
LOWZ	0.111 ± 0.008	0.007 ± 0.020	0.082 ± 0.036	1.07

Planck $\Omega_c h^2 n_s = 0.116 \pm 0.001$



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CONSTRAINTS FROM CURRENT SURVEYS

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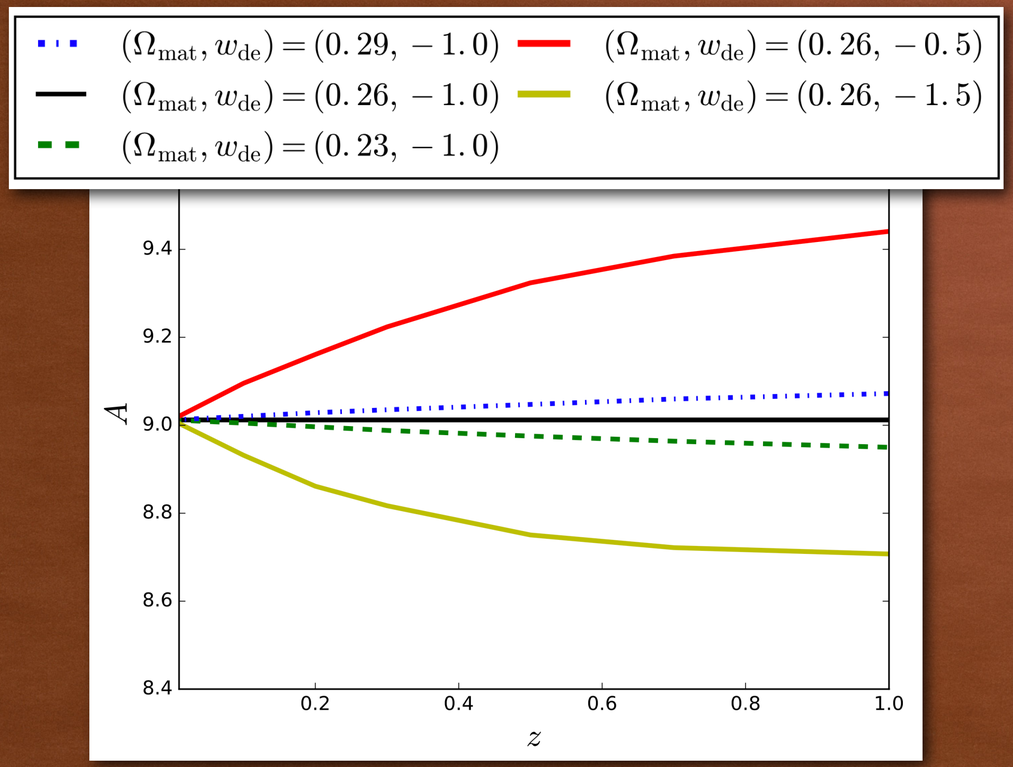
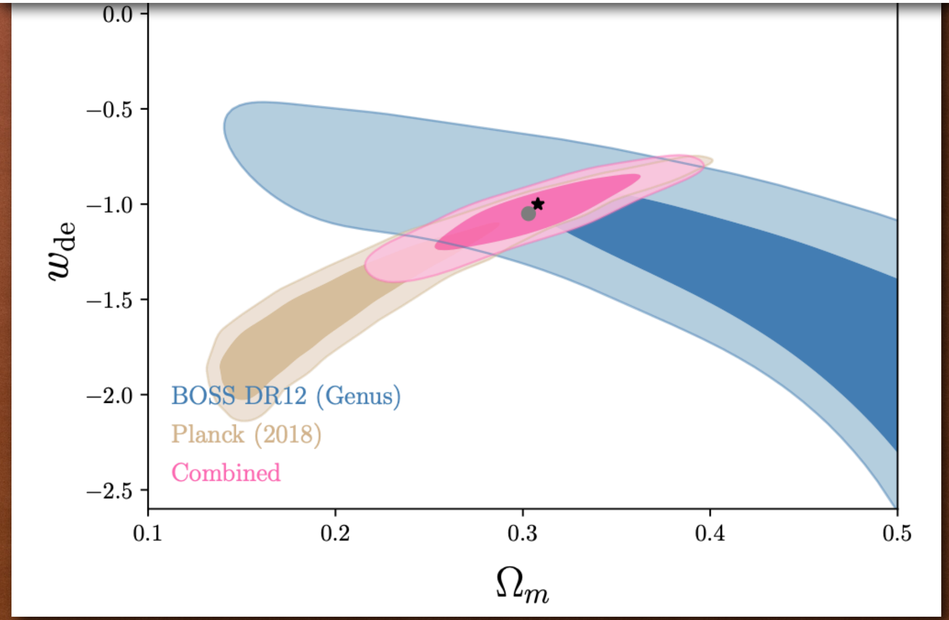


Data	$\Omega_{\rm m}$	$w_{\rm de}$
Genus (BOSS+MGS)	$0.507^{+0.104}_{-0.126}$	$-2.24^{+1.07}_{-1.14}$
Genus (BOSS+MGS) + Planck (2018)	0.303 ± 0.036	$-1.05^{+0.13}_{-0.12}$



(Park and Kim, 2009)

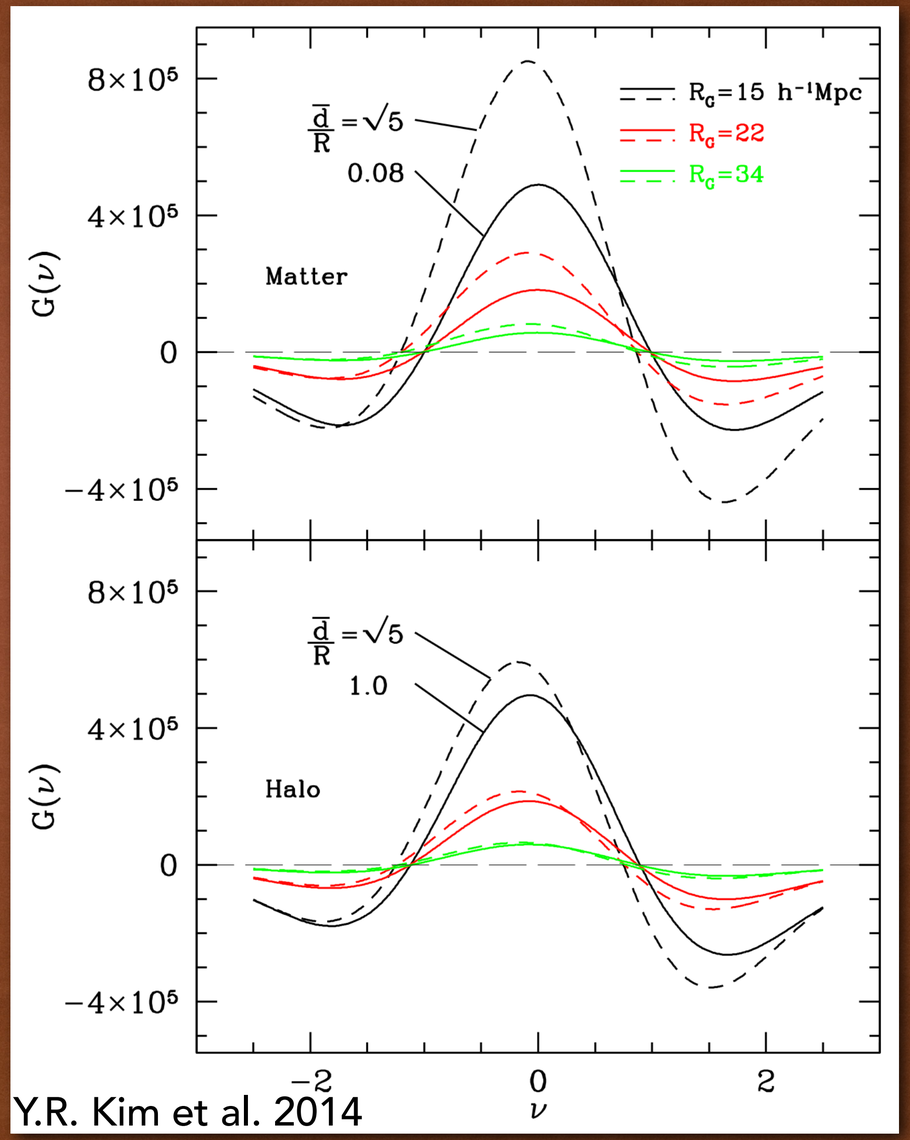
THEORETICAL ISSUES

- Most significant contaminant — shot noise
- Increases amplitude, introduces non-Gaussianity
- Shot noise is not just Gaussian white noise

$$P(k,z) \rightarrow P(k,z) + \frac{1}{\bar{n}}$$

- The matter density is a random field PDF $\Phi(\delta_m)$
- We actually measure the galaxy point process

$$P(\delta_g, \delta_m) = \Psi(\delta_g \,|\, \delta_m)\Phi(\delta_m)$$



Y.R. Kim et al. 2014

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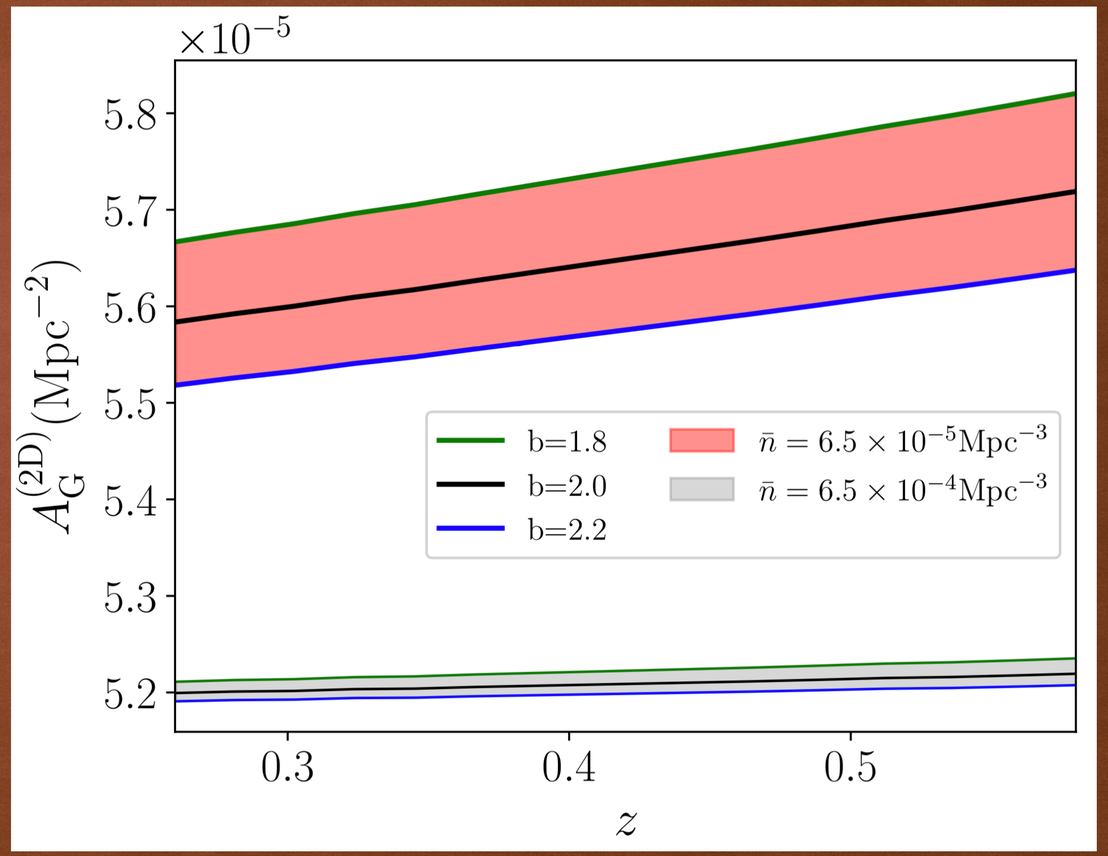
$$P(\delta_g, \delta_m) = \Psi(\delta_g \,|\, \delta_m)\Phi(\delta_m)$$

- Next generation of galaxy surveys will mitigate this effect



NEXT GENERATION

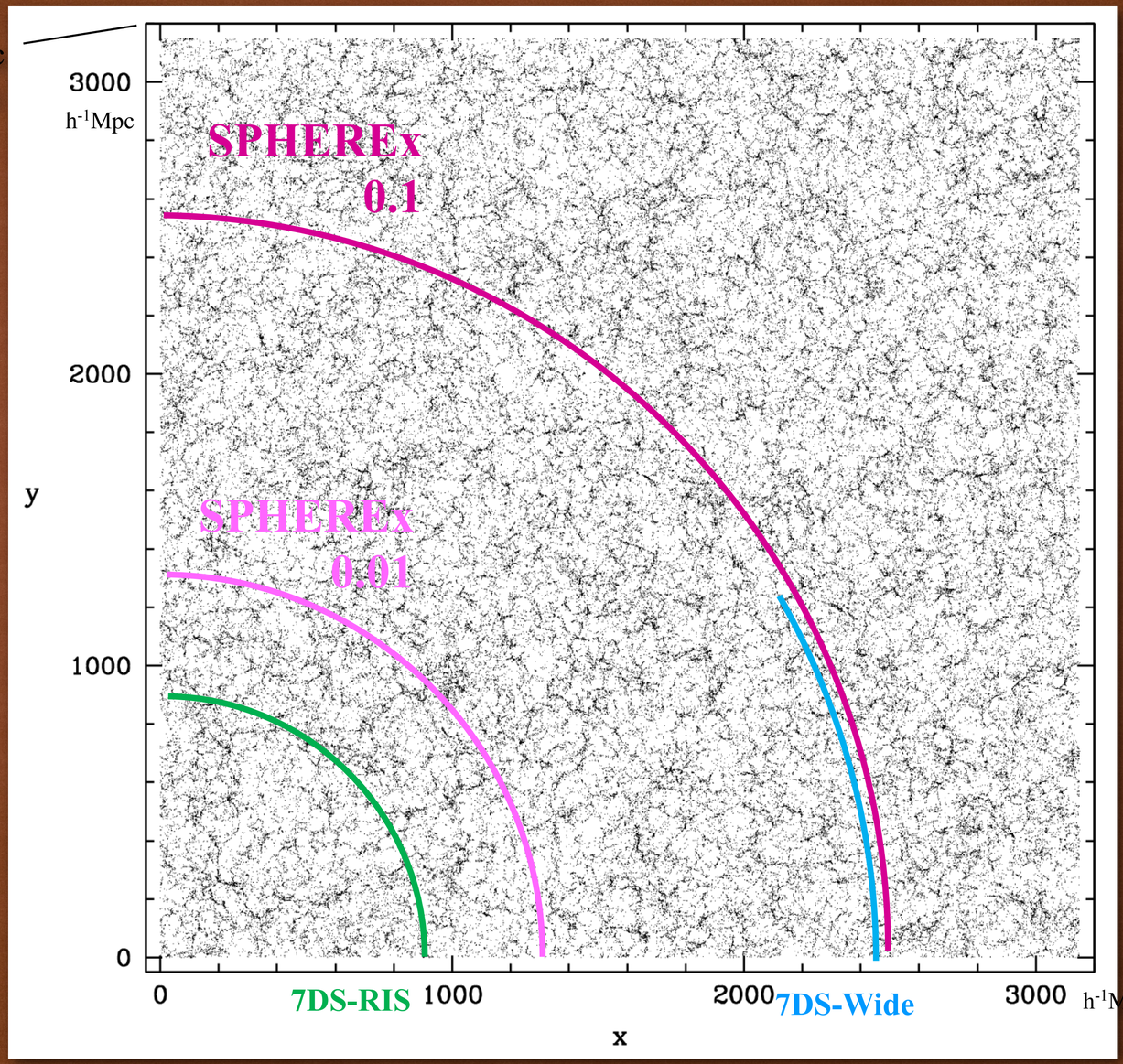
3150h^{-1}Mpc
= 4375Mpc

Mock galaxies (HR4)
with $M>M_{lim}$ =-21.6 and d_{bar}=9 h^{-1} Mpc

7DS-RIS: d_{max}=1200 Mpc (z=0.3)
for m_{lim}=19mag and M_{abs}=-21.6
20,000 deg^2

SPHEREx $|\delta z|/(1+z) < 0.01$ sample
d_{max}=1800 Mpc **(z~0.48)**
All sky

SPHEREx $|\delta z|/(1+z) < 0.1$ sample
d_{max}=3500 Mpc **(z~1.1)**
All sky



Slide courtesy of Changbom Park

NEXT GENERATION

- 7DS-RIS + 7DS-Wide Combined sample — effective survey volume $\sim (2260\,\mathrm{Mpc})^3$+SPHEREx all-sky sample
- Topology statistical uncertainty $< 0.7\,\%$ (currently ~ few%)
- Can constrain primordial non-Gaussianity

$$B_{\rm gr}(k_1,k_2,k_3,z) = 2\left[F_2(\mathbf{k}_1,\mathbf{k}_2)P(k_1,z)P(k_2,z) + \mathrm{cyc.}\right]$$

$$B_{\rm pr}(k_1,k_2,k_3,z) = \frac{2f_{\rm NL}}{D(z)}\left[\frac{P(k_1,z)P(k_2,z)M(k_3)}{M(k_1)M(k_2)} + \mathrm{cyc.}\right]$$

$$S^{(0)} = \frac{1}{8\pi^4\sigma_0^4}\int_0^\infty dk_1\int_0^\infty dk_2\int_{-1}^{1} d\mu k_1^2k_2^2B(k_1,k_2,k_{12},z)W(k_1R_G)W(k_2R_G)W(k_{12}R_G)$$

$$g_{\rm 3D}(\nu_v) = \frac{1}{(2\pi)^2}\left(\frac{\sigma_1}{\sqrt{3}\sigma_0}\right)^3 e^{-\nu_v^2/2}\left(H_2(\nu_v) + \left[(S^{(1)}-S^{(0)})H_3(\nu_v) + (S^{(2)}-S^{(0)})H_1(\nu_v)\right]\sigma_0 + \mathcal{O}(\sigma_0^2)\right)$$

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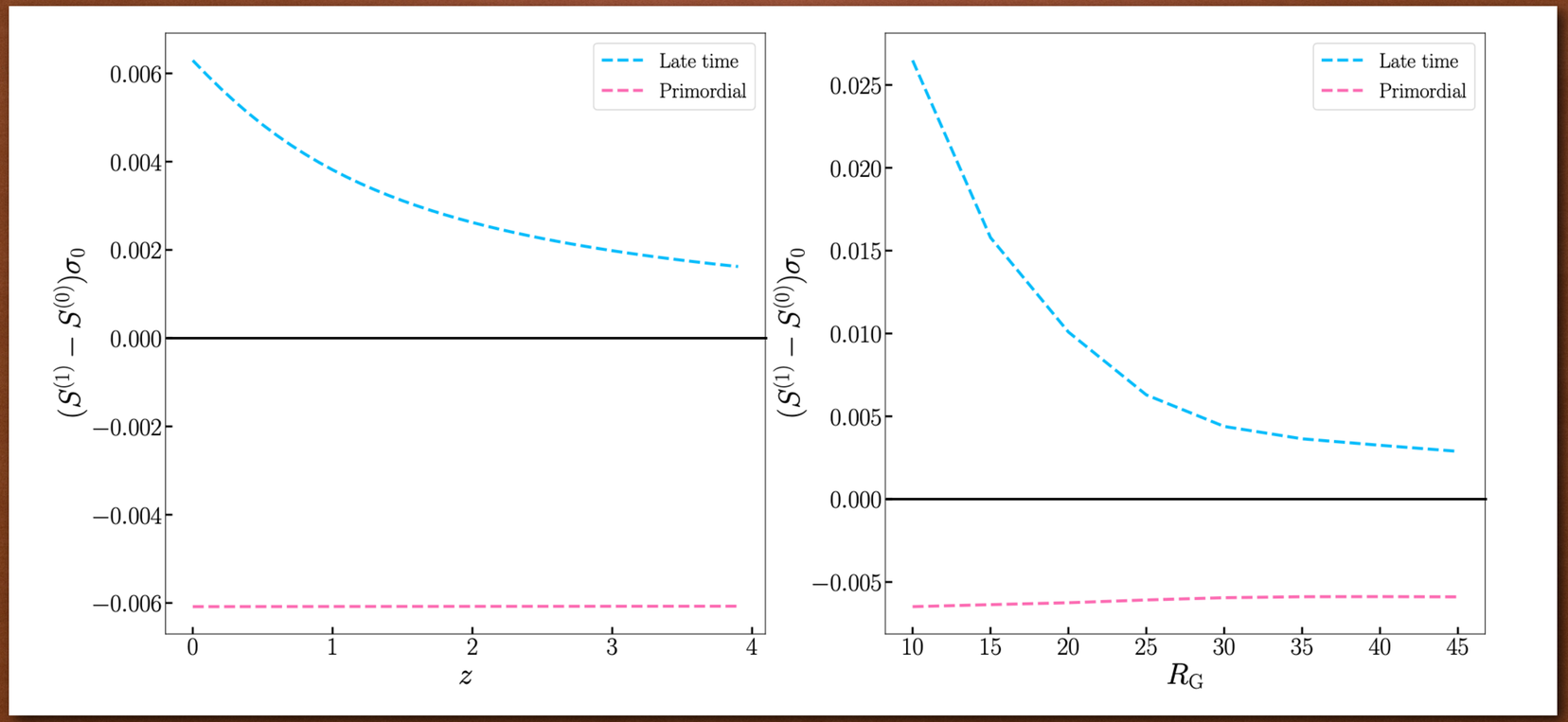


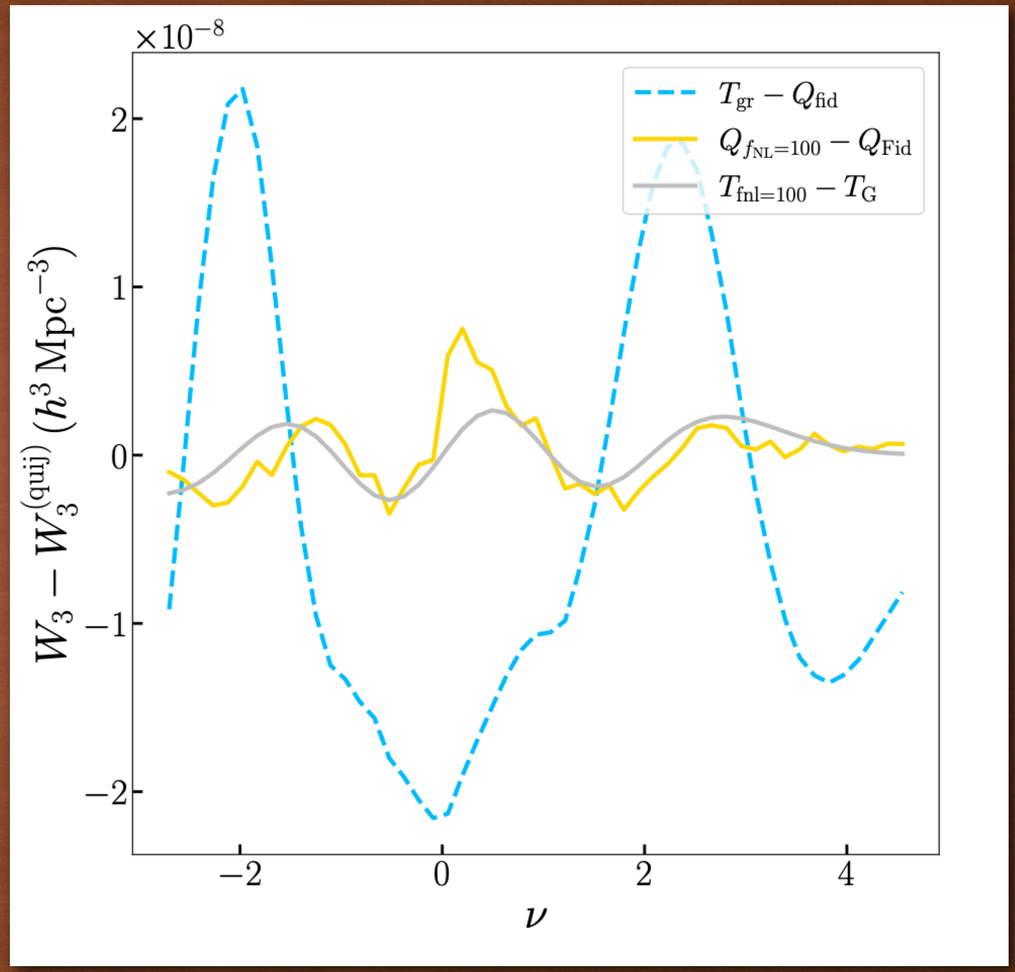
NEXT GENERATION

T_{gr} - Q_{fid} = difference between analytically and numerically calculated non-Gaussian MFs due to NL gravitational evolution

T_{fnl100} - T_G = Numerically measured non-Gaussian MFs in non-Gaussian simulations (f_{nl}=100): impact of f_{nl} on MFs

Q_{fnl100} - Q_{fid} = Numerically measured non-Gaussian MFs in non-Gaussian simulations (f_{nl}=100)

==> Effects of NL gravitational evolution have been effectively subtracted out and the remaining departure of MFs from Gaussian can be inferred



$V = 1\,h^{-3}\,\mathrm{Gpc}^3$

$R_{\rm G} = 25\,h^{-1}\,\mathrm{Mpc}$

NEXT GENERATION

- We can go beyond the Bispectrum, and search for contributions from the four and five point functions.

$$V_k(\nu) = \frac{1}{(2\pi)^{(k+1)/2}} \frac{\omega_3}{\omega_{3-k}\omega_k} \left(\frac{\sigma_1}{\sqrt{3}\sigma_0}\right)^k e^{-\nu^2/2} \Bigg[H_{k-1}(\nu) + \left[\frac{1}{6} S^{(0)} H_{k+2}(\nu) + \frac{k}{3} S^{(1)} H_k(\nu) + \frac{k(k-1)}{6} S^{(2)} H_{k-2}(\nu)\right] \sigma_0$$
$$+ \left\{ \frac{1}{72} (S^{(0)})^2 H_{k+5}(\nu) + \left(\frac{1}{24} K^{(0)} + \frac{k}{18} S^{(0)} S^{(1)}\right) H_{k+3}(\nu) + k \left[\frac{1}{8} K^{(1)} + \frac{k-1}{36} S^{(0)} S^{(2)} + \frac{k-2}{18} (S^{(1)})^2\right] H_{k+1}(\nu) \right.$$
$$+ k \left[\frac{k-2}{16} K_1^{(2)} + \frac{k}{16} K_2^{(2)} + \frac{(k-1)(k-4)}{18} S^{(1)} S^{(2)}\right] H_{k-1}(\nu)$$
$$\left. + k(k-1)(k-2) \left[\frac{1}{24} K^{(3)} + \frac{k-7}{72} (S^{(2)})^2\right] H_{k-3}(\nu) \right\} {\sigma_0}^2 + O\left({\sigma_0}^3\right) \Bigg],$$

- The next generation of surveys will probe more modes than the CMB, which is a two-dimensional surface.
- Non-Gaussianity is scale dependent, and the CMB and LSS can probe different scales.
- We must carefully eliminate observational sources of non-Gaussianity — shot noise, redshift space distortion, lensing, galaxy bias…

SUMMARY

- We can measure the topology of the galaxy distribution
- From the genus, we can infer cosmological parameters — primordial power spectrum index, fraction of dark matter, the equation of state of dark energy
- Current generation surveys are comparable in power to Planck
- The next generation will be significantly more constraining! Primordial non-Gaussianity, neutrino mass, warm dark matter etc.
- Much work to be done, to model the non-Gaussian components of the genus.

Thank you!