Equivariant Localization in Supergravity

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KIAS-P99103
QMW-PH-99-15
IASSNS-HEP-99/110
hep-th/9912082

Monopole Dynamics and BPS Dyons in $N = 2$ Super-Yang-Mills Theories

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ABSTRACT

We determine the low energy dynamics of monopoles in pure $N = 2$ Yang-Mills theories for points in the vacuum moduli space where the two Higgs fields are not aligned. The dynamics is governed by a supersymmetric quantum mechanics with potential terms and four real supercharges. The corresponding superalgebra contains a central charge but nevertheless supersymmetric states preserve all four supercharges. The central charge depends on the sign of the electric charges and consequently so does the BPS spectrum. We focus on the $SU(3)$ case where certain BPS states are realised as zero-modes of a Dirac operator on Taub-NUT space twisted by the tri-holomorphic Killing vector field. We show that the BPS spectrum includes hypermultiplets that are consistent with the strong- and weak-coupling behaviour of the Seiberg-Witten theory.

arXiv

QMW-PH-00-0
KIAS-P0004
hep-th/000803

General Low Energy Dynamics of Supersymmetric Monopoles

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ABSTRACT

We derive general low energy dynamics of monopoles and dyons in $N = 2$ and $N = 4$ supersymmetric Yang-Mills theories by utilising a collective coordinate expansion. The resulting new kind of supersymmetric quantum mechanics incorporates the effects of multiple Higgs fields, both in the $N = 2$ vector multiplet and hypermultiplets, having non-vanishing expectation values.



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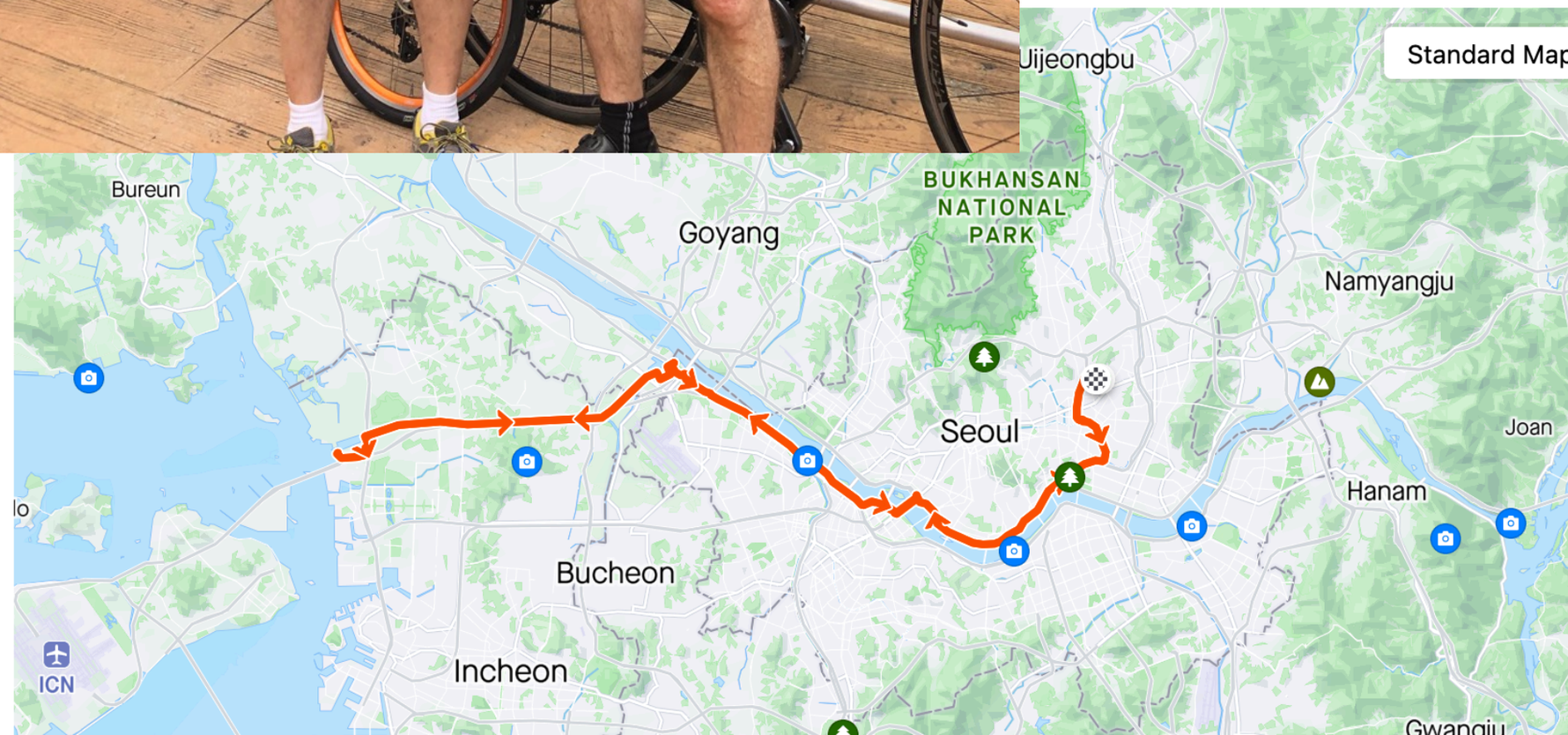
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Equivariant Localization in Supergravity

Jerome Gauntlett

with

Pietro Benetti-Genolini and James Sparks (x4)

Alice Lüscher and Yusheng Jiao (x1)

Introduction

- Equivariant localization:
  Powerful new method to compute supersymmetric observables in supergravity without solving the supergravity equations of motion!

- Several applications in supersymmetric holography:

  - Susy $AdS_D \times M$ solutions of D=10/11 SUGRA, dual to $d = D - 1$ SCFTs; compute central charge, Δ of some op's.

  - Susy black hole solutions in AdS ; compute S_{BH}

  - SCFTs on M ; compute $I = -\log Z$

Susy solutions of supergravity (general D)

- Equations of motion for bosonic fields

  Einstein equations: $R_{\mu\nu} = ...$

  Matter/flux equations of motion

- Supersymmetry $\Leftrightarrow$ Killing spinors ϵ

$$\nabla\epsilon + (\text{flux}\cdot\Gamma)\epsilon = 0$$

$$(\text{flux}\cdot\Gamma)\epsilon = 0$$

Susy solutions with an R-symmetry Killing vector:

$$\xi = \bar{\epsilon}\Gamma^{\mu}\epsilon\partial_{\mu} \qquad\qquad \mathcal{L}_{\xi}\epsilon \equiv \xi^{\mu}\nabla_{\mu}\epsilon + \frac{1}{8}d\xi_{\mu\nu}\Gamma^{\mu\nu}\epsilon = iq\epsilon$$

Very general

Define equivariant exterior derivative: $d_\xi = d - \xi \lrcorner$

Satisfies $(d_\xi)^2 = -\mathcal{L}_\xi$

There is a corresponding equivariant cohomology

More general spinor bi-linears:

$\Psi_{(r)} = \bar{\epsilon}\Gamma_{(r)}\epsilon$ where $\Gamma_{(r)} \equiv \frac{1}{r!}\Gamma_{\mu_1\ldots\mu_r} dx^{\mu_1} \wedge \ldots dx^{\mu_r}$

$\tilde{\Psi}_{(r)} = \bar{\epsilon}^c\Gamma_{(r)}\epsilon$

Results from around 20 years ago: [Gauntlett,Pakis 02] [Gauntlett,Martelli,Pakis,Waldram 02]

The Killing spinor ϵ defines a G-structure

$\Leftrightarrow$

Algebraic and Differential relations on $\Psi_{(r)}$, $\tilde{\Psi}_{(r)}$

New Observation:

Construct multiforms $\Phi = \Phi_{2n} + \Phi_{2n-2} + \ldots \Phi_0$

that are polynomials in the supergravity fields and the bilinears $\Psi_{(r)}$ which are equivariantly closed: $d_\xi \Phi = 0$

Comments:

- Only need to impose subset of the KSEs/equations of motion:
  Such structures exist partially off-shell
- Typically several Φ exist in a given theory
- Many physical susy quantities can be computed using the equivariant cohomology of Φ . The topology and R-symmetry is enough information - don't need to solve SUGRA pdes!
  - just need to assume the solution exists

Berline-Vergne-Atiyah-Bott theorem

Let M be a ξ invariant closed submanifold, 2n-dimensional

$$\int_M \Phi = \sum_{\substack{F \\ \mathrm{codim} F = 2k}} \frac{1}{d_F} \frac{(2\pi)^k}{\prod_{i=1}^k b_i} \int_F \frac{f^*\Phi}{\prod_{i=1}^k \left[1 + \frac{2\pi}{b_i} c_1(L_i)\right]} .$$

- $f : F \hookrightarrow M$, where $\xi = 0$
- b_i are the weights of action of ξ on $N_F = \oplus_{i=1}^k L_i$

$$\xi = \sum_{i=1}^k b_i \frac{\partial}{\partial \phi_i}$$

- Normal space can have orbifold singularities $N_F \cong \mathbb{R}^{2k}/\Gamma$ with Γ a finite group of order d_F

Berline-Vergne-Atiyah-Bott theorem

$$\int_M \Phi = \sum_{\dim\, 0} \frac{1}{d_{F_0}} \frac{(2\pi)^n}{b_1 \cdots b_n} \Phi_0$$

$$+ \sum_{\dim\, 2} \frac{1}{d_{F_2}} \frac{(2\pi)^{n-1}}{b_1 \cdots b_{n-1}} \int \left[\Phi_2 - \Phi_0 \sum_{1 \le i \le n-1} \frac{2\pi}{b_i} c_1(L_i) \right]$$

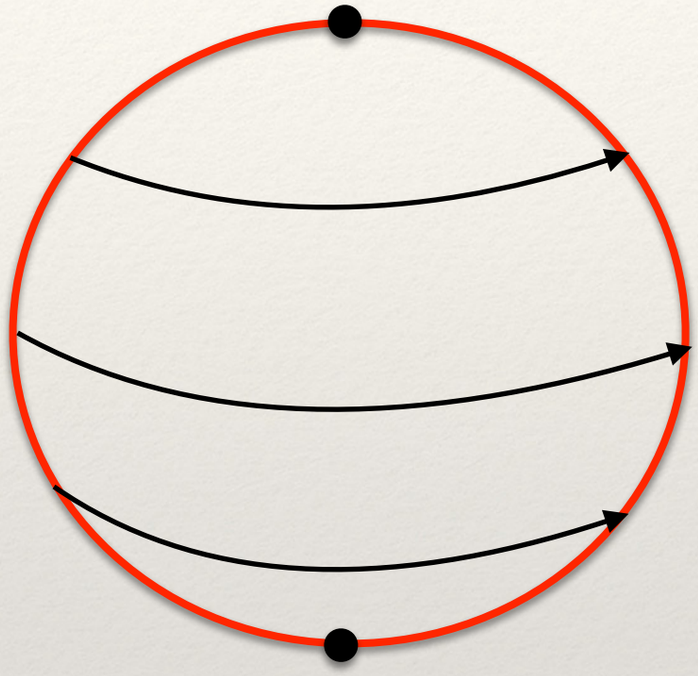
$$+ \ldots$$

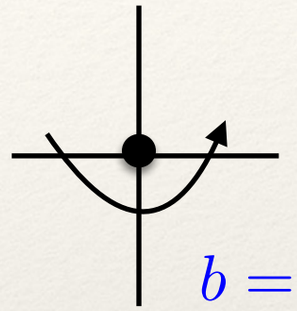
Simple example: S^2

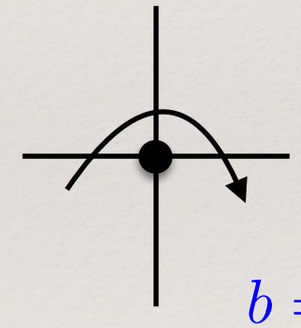
$$ds^2 = d\theta^2 + \sin^2\theta d\phi^2 \qquad \xi = \partial_\phi$$

$$\Phi = \sin\theta d\theta \wedge d\phi + \cos\theta$$

$$d_\xi \Phi = 0$$







$$\int_{S^2} \mathrm{vol} = \int_{S^2} \Phi$$

$$= 2\pi\left(\frac{\cos\theta}{b}\Big|_N + \frac{\cos\theta}{b}\Big|_S\right)$$

$$= 2\pi\left(\frac{1}{1} + \frac{-1}{-1}\right) = 4\pi$$

Example 1: $AdS_5 \times M_6$ [Gauntlett,Martelli,Sparks,Waldram 04]

- Consider most general solutions of D=11 supergravity that are dual to $\mathcal{N}=1$ SCFTs in d=4

$$ds^2 = e^{2\lambda}\Big[ds^2(AdS_5) + ds^2(M_6)\Big]$$

$G_{(4)}$ a four-form on M_6 ; flux quantisation $G_{(4)} \in H^4(M_6, \mathbb{Z})$

- Susy: D=11 spinor $\varepsilon = \psi \otimes \epsilon$

  where ψ a Killing spinor on AdS_5 and ϵ on M_6 satisfies the KSEs:

$$\Big[\nabla_m + \frac{i}{2}\gamma_m\gamma_* - \frac{1}{4}(e^{-3\lambda}G\cdot\gamma)_m\Big]\epsilon = 0$$

$$\Big[\gamma\cdot d\lambda - i\gamma_* + \tfrac{1}{3!}(e^{-3\lambda}G\cdot\gamma)\Big]\epsilon = 0$$

- Construct bi-linears

$$1 = \bar{\epsilon}\epsilon \qquad \sin\zeta = -i\bar{\epsilon}\gamma_*\epsilon$$

$$K = \bar{\epsilon}\gamma_{(1)}\epsilon \qquad \xi^\flat = \tfrac{1}{3}\bar{\epsilon}\gamma_{(1)}\gamma_*\epsilon$$

$$Y = -i\bar{\epsilon}\gamma_{(2)}\epsilon \qquad Y' = \bar{\epsilon}\gamma_{(2)}\gamma_*\epsilon$$

Comments:

- $\xi^\flat$ is a one-form dual to an R-symmetry Killing vector - it is dual to the R-symmetry of the dual d=4 SCFT

- More bilinears exist using ϵ^c, which we won't need (all were used in the G-structure analysis of [Gauntlett,Martelli, Sparks,Waldram 04])

$\Rightarrow$ Using only a subset of information in KSEs/equations of motion

- Convenient definition: $y = \frac{1}{2} e^{3\lambda} \sin\zeta$ $\qquad dy = e^{3\lambda} K$

- Equivariantly closed forms:

$$\Phi = e^{9\lambda} \mathrm{vol}_6 + \frac{1}{12} e^{9\lambda} * Y - \frac{1}{36} y e^{6\lambda} Y - \frac{1}{162} y^3 \qquad d_\xi \Phi = 0$$

$$a \propto \int_{M_6} \Phi$$

i.e. the a central charge localizes!

$$\Phi^G = G_{(4)} - \frac{1}{3} e^{3\lambda} Y' + \frac{1}{9} y \qquad d_\xi \Phi^G = 0$$

Flux quantization can be implemented via localization!

$$c \int_{C_4} G_{(4)} \in \mathbb{Z} \qquad C_4 \in H_4(M_6, \mathbb{Z})$$

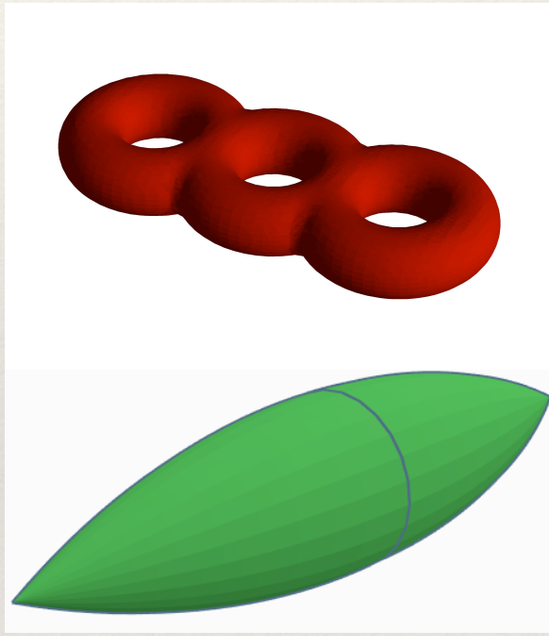
Putting these tools to work ("computing a without trying")

- Specify topology of M_6 and action of R-symmetry
- Compute integrals via BVAB

- Solutions arising from wrapping M5-branes $\Sigma \subset CY_3$ with

a) $\Sigma = \Sigma_g$ a genus g Riemann surface

[Maldacena,Nunez 00][Bah,Beem,Bobev,Wecht 12]



b) $\Sigma = \Sigma(n_+, n_-)$ a spindle

[Ferrero,Gauntlett,Martelli,Sparks 21]

The CY can be viewed as the total space of

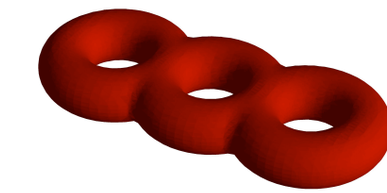
$\mathcal{O}(-p_1) \oplus \mathcal{O}(-p_2) \to \Sigma$ with $p_1 + p_2 = \chi(\Sigma)$

IF this gives rise to a d=4 CFT there will be a $AdS_5 \times M_6$ solution with $S^4 \to M_6 \to \Sigma$ and the fibration determined by

$S^4 \subset \mathbb{C}_1 \oplus \mathbb{C}_2 \oplus \mathbb{R}$ with $\mathbb{C}_i$ twisted using $\mathcal{O}(-p_i)$

And the solution will have an R-symmetry Killing vector

a) $\Sigma = \Sigma_g$: M5-branes wrapping a Riemann surface



Let ∂_{φ_i} rotate the $\mathbb{C}_i$ and hence give KVs acting S^4

Write $\xi = b_1 \partial_{\varphi_1} + b_2 \partial_{\varphi_2}$

For $b_i \neq 0$ the KV has fixed points at N and S poles of S^4 and the fixed point set is Σ_g

Spinor at the poles is chiral $\Rightarrow b_1 + b_2 = 1$

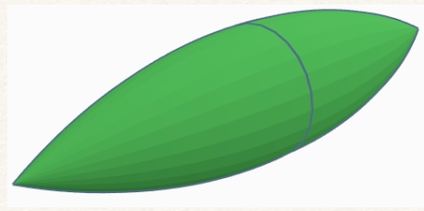
Now use BVAB/localisation:

- Flux quantisation $$c\int_{S^4} G_{(4)} = N$$
- Central charge $$a = -\frac{9}{8} b_1 b_2 (b_1 p_2 + b_2 p_1) N^3$$

Comments:

- These are off-shell results. On-shell result obtained by extremizing $a(b_i)$ subject to $b_1 + b_2 = 1$ and agrees with sugra solution
- Result agrees with off-shell computation in field theory
- SUGRA computation using BVAB is much simpler than constructing sugra solution

b) $\Sigma = \Sigma(n_+, n_-)$: M5-branes wrapping a spindle



Now write $\xi = b_1 \partial_{\varphi_1} + b_2 \partial_{\varphi_2} + b_0 \partial_{\varphi_0}$

where ∂_{φ_0} is the KV which rotates the spindle

For $b_i \neq 0$ the KV now has 4 isolated fixed points located at the N and S poles of the S^4 and the + and - poles of the spindle. Now find:

$$a = \frac{9}{16 b_0}[(b_1^+ b_2^+)^2 - (b_1^- b_2^-)^2] N^3$$

with constraints: $b_1^\pm + b_2^\pm = 1 \mp \frac{b_0}{n_\pm}$ $\qquad b_i^+ - b_i^- = -\frac{p_i}{n_+ n_-}$

- After extremization one obtains the result from the explicit SUGRA solution
- Result agrees with off-shell computation in field theory [Ferrero,Gauntlett,Martelli,Sparks 21]
  (the form is a "gravitational block" [Hosseini,Hristov,Zaffaroni 19])

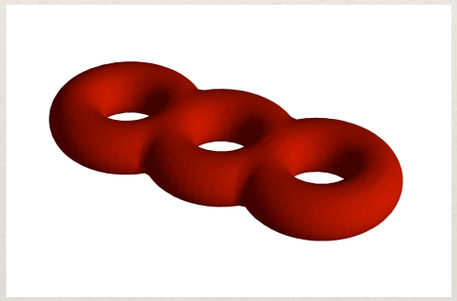
Example 2: D=2 Gauged Supergravity and black hole entropy

- Consider D=4 $\mathcal{N}=2$ gauged SUGRA coupled to matter

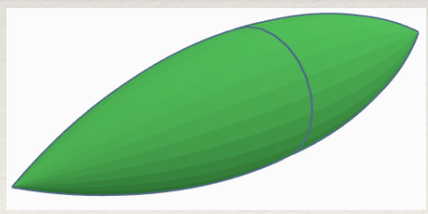
- Focus on $AdS_2 \times M_2$ solutions $\qquad F^I = p^I \mathrm{vol}(M_2)$

- These arise as the near horizon limit of susy magnetically charged black holes with event horizons:

$M_2 = \Sigma_g$



$M_2 = \Sigma(n_+, n_-)$



black holes are accelerating

- Find that the black entropy can be evaluated using localisation. New version of attractor mechanism.

- E.g. $\mathcal{N}=2$ gauged SUGRA coupled to n vector multiplets

Metric: $g_{\mu\nu}$

$n+1$ gauge fields: A^I $\qquad I=0,1,\ldots,n$

n complex scalars: $X^I(z^i)$ $\qquad i=1,\ldots,n$

parametrize special Kähler manifold $\mathcal{K}$

Holomorphic prepotential $\mathcal{F}(X^I)$

FI gauge parameters ξ^I

- Construct equivariantly closed forms using bilinears and then use BVAB…

- Result:

"Dressed scalars": $x^I \equiv e^{\lambda} e^{\mathcal{K}/2} X^I P$

$$S_{BH} = -\frac{\pi}{2G}\frac{1}{2b_0}[\mathcal{F}(x^I_+) - \sigma\mathcal{F}(x^I_-)]$$

$$\xi_I x^I_+ = 2 + \frac{2b_0}{n_+} \qquad \xi_I x^I_- = 2 - \sigma\frac{2b_0}{n_-}$$

$$p^I = \frac{1}{2b_0}(x^I_+ - x^I_-)$$

- Off-shell result; need to extremize over $b_0, x^I_\pm$

- Agrees with known sugra results

- For S^2 : set $n_\pm = 1$ and $b_0 \to 0$

$$S_{BH} = -\frac{\pi}{2G} p^I \partial_I \mathcal{F}(x^I)$$

Attractor result

$$\xi_I x^I = 2 \qquad \xi_I p^I = 2$$

- For STU model, uplift solution on S^7

$$AdS_2 \times Y_9 \qquad S^7 \to Y_9 \to \Sigma(n_+, n_-)$$

has a GK geometry

[Kim,Park 06][Gauntlett,Kim 07]

[Couzens,Gauntlett,Martelli,Sparks 18]
[Boido,Gauntlett,Martelli,Sparks 22][…]

- Can add hypermultiplets too….

Example3: D=4 Gauged Supergravity

- Minimal D=4 N=2 gauged supergravity Euclidean

$$I = -\frac{1}{16\pi G}\int_M (R_g + 6 - F^2)\mathrm{vol}_g + I_{\partial M}$$

$$I_{\partial M} = -\frac{1}{16\pi G}\int_{\partial M} \left(2K - 4 - R_h\right)\mathrm{vol}_h$$

$$(\nabla_\mu - iA_\mu + \tfrac{1}{2}\Gamma_\mu + \frac{i}{4}F_{\nu\rho}\Gamma^{\nu\rho}\Gamma_\mu)\epsilon = 0$$

- Solutions can be uplifted on SE_7 to get solutions of D=11 SUGRA dual to $d = 3, \mathcal{N} = 2$ SCFTs on ∂M

[Gauntlett, Varela 07]

On-shell action:

$$I = \left[\frac{1}{(2\pi)^2} \int_M \Phi \right] \frac{\pi}{2G_4} + I_{\partial M}$$

- Can't use BVAB directly:

  Assume ξ has fixed points only in the bulk

  Then bulk integral will give a fixed point contribution plus a divergent boundary piece. Remarkably, this boundary piece exactly cancels the terms in $I_{\partial M}$

- Result:

  The fixed points can either be "nuts" (fixed points) or "bolts" (fixed surfaces) [Gibbons,Hawking 77]

  The spinor has fixed chirality on the fixed point set

- On-shell action:

$$I = \left\{ \sum_{\substack{\text{fixed} \\ \text{points}_\pm}} \mp \frac{(b_1 \mp b_2)^2}{4 b_1 b_2} + \sum_{\substack{\text{fixed} \\ \Sigma_\pm}} \int_{\Sigma_\pm} \left[\frac{1}{2} c_1(T\Sigma_\pm) \mp \frac{1}{4} c_1(L) \right] \right\} \frac{\pi}{2 G_4}$$

[Benetti-Genolini 19]

- Comments:
  - Recovers results for known SUGRA solutions
  - Much more general
  - Prediction for dual SCFT $I = -\log Z^{SCFT}(\partial M)$
- Agrees with field theory computation using localization
- Generalizes- add matter

Final Comments

• New technique for computing BPS observables for supersymmetry solutions in SUGRA.

• Illustrated in diverse examples and easily recover many results for known solutions. Also gives results for solutions that can't be constructed in closed form.

• Other cases: $AdS_3 \times M_8$ of D=11 SUGRA

$AdS_3 \times M_2$ of D=5 gauged SUGRA +hypers

D=6 Euclidean SUGRA

• Many other cases to consider e.g odd dimensions

•Why does it work?

• Technique is predicated on solutions existing. Physics gives some constraints e.g. negative central charge. Is it possible that in some cases there are sufficient constraints?

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