Solitons, branes and QCD

- M-theory for nuclear physics -

Koji Hashimoto (Kyoto U)

ArXiv:1003.4988 (hep-th)
ArXiv:1005.4412 (hep-th)
ArXiv:1006.3612 (hep-th)
ArXiv:1103.5688 (hep-th)
ArXiv:1203.5386 (hep-th)
ArXiv:1902.07444 (hep-th)
ArXiv: 2103.03563 (hep-th)

w/ P. Yi (KIAS)
Y. Matsuo (Kindai U)
T. Morita (Shizuoka U)
N. Iizuka (Osaka U)
S. Aoki (Kyoto YITP)

$$S = \frac{1}{g_s l_s} \int dt \ \mathrm{tr} \left((D_0 X^M)^2 - l_s^{-4} [X^M, X^N]^2 + \text{fermions} \right)$$

M(atrix) theory

[Banks, Fischler, Shenker, Susskind (1996)]

$$\begin{aligned} S &= \frac{\lambda N_c}{54\pi} M_{\rm KK} \int dt \ \mathrm{tr} \ \Bigg[(D_0 X^M)^2 - \frac{2}{3} M_{\rm KK}^2 (X^4)^2 + D_0 \bar{w}_i^{\dot{\alpha}} D_0 w_{\dot{\alpha} i} - \frac{1}{6} M_{\rm KK}^2 \bar{w}_i^{\dot{\alpha}} w_{\dot{\alpha} i} \\ &\quad - \frac{3^6 \pi^2}{4 \lambda^2 M_{\rm KK}^4} \left(\vec{D} \right)^2 + i \vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{X}^{\dot{\beta}\alpha} X_{\alpha\dot{\alpha}} + i \vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{w}_i^{\dot{\beta}} w_{\dot{\alpha} i} \Bigg] \quad + 4 N_c \int dt \ \mathrm{tr} \ A_0 \end{aligned}$$

Nuclear matrix model
(M-theory for nuclear physics)

[Iizuka, Yi, KH (2010)]

Preprint typeset in JHEP style - HYPER VERSION

CERN-PH-TH-2010-062
KIAS-P10007
RIKEN-TH-186
RIKEN-MP-1

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A Matrix Model for Baryons and Nuclear Forces

Koji Hashimoto[*,a], **Norihiro Iizuka**[†,b] **and Piljin Yi**[§,c]

[*] *Nishina Center, RIKEN, Saitama 351-0198, Japan*
[†] *Theory Division, CERN, CH-1211 Geneva 23, Switzerland*
[‡] *School of Physics, Korea Institute for Advanced Study, Seoul 130-722, Korea*
[a] *E-mail:* koji(at)riken.jp
[b] *E-mail:* norihiro.iizuka(at)cern.ch
[c] *E-mail:* piljin(at)kias.re.kr

Abstract: We propose a new matrix model describing multi-baryon systems. We derive the action from open string theory on the wrapped baryon vertex D-branes embedded in the D4-D8 model of large N_c holographic QCD. The positions of k baryons are unified into $k \times k$ matrices, with spin/isospin of the baryons encoded in a set of k-vectors. Holographic baryons are known to be very small in the large 't Hooft coupling limit, and our model offers a better systematic approach to dynamics of such baryons at short distances. We compute energetics and spectra ($k = 1$),

September, 1998

Talk at APCTP

1/4 BPS states in Super Yang-Mills theory from Multi-pronged strings

K. HASHIMOTO

Department of Physics, Kyoto University

Collaborated work with H. HATA *and* N. SASAKURA

hep-th/9803127 (Phys.Lett. B431 (1998) 303)

hep-th/9804164 (to be published in Nucl.Phys.B)

Plan of this talk

1. **Introduction**
2. **BPS Solutions**
3. **Various aspects of the Solutions**
4. **Conclusion and Discussions**

First page of my slides at KIAS workshop 1998

Dec 1997, Bergman

hep-th/9712211
HUTP-97/A104

Three-Pronged Strings and 1/4 BPS States in N=4 Super-Yang-Mills Theory

Oren Bergman*

Lyman Laboratory of Physics
Harvard University
Cambridge, MA 02138

December 1997

arXiv:hep-th/9712211v1 22 Dec 1997

Abstract

We provide an explicit construction of 1/4 BPS states in four-dimensional $N = 4$ Super-Yang-Mills theory with a gauge group $SU(3)$. These states correspond to three-pronged strings connecting three D3-branes. We also find curves of marginal stability in the moduli space of the theory, at which the above states can decay into two 1/2 BPS states.

March 1998, Hata, Sasakura, KH

KUNS-1499
HE(TH) 98/03
hep-th/9803127

arXiv:hep-th/9803127v1 16 Mar 1998

3-String Junction and BPS Saturated Solutions in $SU(3)$ Supersymmetric Yang-Mills Theory

Koji HASHIMOTO*, Hiroyuki HATA† and Naoki SASAKURA‡

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

March, 1998

Abstract

We construct BPS saturated regular configurations of $\mathcal{N} = 4$ $SU(3)$ supersymmetric Yang-Mills theory carrying non-parallel electric and magnetic charges. These field theory BPS states correspond to the string theory BPS states of 3-string junctions connecting three different D3-branes by regarding the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory as an effective field theory on parallel D3-branes.

April 1998

Hata, Sasakura, KH

KUNS-1503
HE(TH)98/05
hep-th/9804164

arXiv:hep-th/9804164v1 25 Apr 1998

Multi-Pronged Strings and BPS Saturated Solutions in $SU(N)$ Supersymmetric Yang-Mills Theory

Koji HASHIMOTO*, Hiroyuki HATA† and Naoki SASAKURA‡

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

April, 1998

Abstract

Extending our previous work on $SU(3)$, we construct spherically symmetric BPS saturated regular configurations of $\mathcal{N} = 4$ $SU(N)$ supersymmetric Yang-Mills theory preserving 1/4 supersymmetry, and investigate their features. We also give exact solutions in the case some of the free parameters of the general solutions take certain values. These field theory BPS states correspond to the string theory BPS states of multi-pronged strings connecting N different D3-branes by regarding the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory as an effective field theory on parallel D3-branes. We compare our solutions with multi-pronged strings in string picture.

*hasshan@gauge.scphys.kyoto-u.ac.jp. Supported in part by Grant-in-Aid for Scientific Research from Ministry of Education, Science and Culture (#3160).
†hata@gauge.scphys.kyoto-u.ac.jp. Supported in part by Grant-in-Aid for Scientific Research from Ministry of Education, Science and Culture (#09640346).
†sasakura@gauge.scphys.kyoto-u.ac.jp

Kimyeong, Piljin

CU-TP-890
CLNS-98/1556
hep-th/9804174

arXiv:hep-th/9804174v1 27 Apr 1998

Dyons in $N = 4$ Supersymmetric Theories and Three-Pronged Strings

Kimyeong Lee [a][1] *and Piljin Yi* [b][2]

[a]Physics Department, Columbia University, New York, NY 10027

[b]F.R. Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853

ABSTRACT

We construct and explore BPS states that preserve 1/4 of supersymmetry in $N = 4$ Yang-Mills theories. Such states are also realized as three-pronged strings ending on D3-branes. We correct the electric part of the BPS equation and relate its solutions to the unbroken abelian gauge group generators. Generic 1/4-BPS solitons are not spherically symmetric, but consist of two or more dyonic components held apart by a delicate balance between static electromagnetic force and scalar Higgs force. The instability previously found in three-pronged string configurations is due to excessive repulsion by one of these static forces. We also present an alternate construction of these 1/4-BPS states from quantum excitations around a magnetic monopole, and build up the supermultiplet for arbitrary (quantized) electric charge. The degeneracy and the highest spin of the supermultiplet increase linearly with a relative electric charge. We conclude with comments.

[1]electronic mail: klee@phys.columbia.edu
[2]electronic mail: piljin@mail.lns.cornell.edu

Kimyeong, Piljin

from the usual BPS states that break only half of supersymmetry. For instance, suppose that we have $SU(n)$ broken down to $U(1)^{n-1}$. Pick a pair of roots $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ with $\boldsymbol{\alpha}^2 = \boldsymbol{\beta}^2 = 1$ and $\boldsymbol{\alpha} \cdot \boldsymbol{\beta} = -1/2$. A state of magnetic charge $m\boldsymbol{\alpha} + m\boldsymbol{\beta}$ and of electric charge $n\boldsymbol{\alpha}$ would then be 1/4-BPS.

Now the question is how these 1/4-BPS states are realized on the field theory side. One might be tempted to look for a spherically symmetric soliton. In fact, very recently, a special class of 1/4-BPS states in $SU(3)$ theory was found in a spherically symmetric ansatz [6]. In terms of roots, these BPS configurations carry magnetic charge of $2\boldsymbol{\alpha} + 2\boldsymbol{\beta}$. However, as will become clear in later sections, the existence of these solutions is quite accidental and fails to illuminate how the general 1/4-BPS dyons are constructed in the field theory language. One severe problem is that if their electric charge is, say, of the form $q\boldsymbol{\alpha}$, the real number[4] q is determined uniquely by the Higgs vev's. (In the spherically symmetric case of the total magnetic charge, $\boldsymbol{\alpha} + \boldsymbol{\beta}$, for instance, q has to vanish for all vev's.) Because of this, at generic point of vacuum moduli space, BPS configurations of properly quantized electric charge (q = integer) cannot be realized as a spherically symmetric classical soliton.

In general, we expect the BPS configurations to be of an elongated shape. Roughly speaking,

Appendix A: The ADHM Formalism

The ADHM formalism [9] for k instantons of the SU(n) gauge theory starts with a $(n+2k)\times 2k$ matrix

$$\Delta = \begin{pmatrix} \lambda_{n\times 2k} \\ \mu_{2k\times 2k} \end{pmatrix} + \begin{pmatrix} 0 \\ I_{2k\times 2k} \end{pmatrix} x, \tag{A1}$$

where $x = x_\alpha e_\alpha$ and $e_\alpha = (i\sigma_j, 1)$ [9]. Finding the $(n+2k)\times 2k$ matrix v such that

$$\Delta^\dagger v = 0, \quad v^\dagger v = I_{n\times n}, \tag{A2}$$

we can construct the anti-hermitian gauge field

$$A_\alpha = v^\dagger \partial_\alpha v. \tag{A3}$$

The condition for the field strength to be self-dual is that

$$(\Delta^\dagger \Delta)_{2k\times 2k} = f^{-1}_{k\times k} I_{2\times 2}. \tag{A4}$$

This implies that $\mu = \mu_\alpha e_\alpha$ with hermitian matrices $(\mu_\alpha)_{k\times k}$ and that

$$i\eta^i_{\alpha\beta}[\mu_\alpha, \mu_\beta] + tr_2(\sigma^i \lambda^\dagger \lambda) = 0, \tag{A5}$$

Another encounter… January 2009

Youngman, Sangmin, Piljin

arXiv:0902.4048v2 [hep-th] 9 Mar 2009

KIAS-P09008
SU-ITP-09/10

Holographic Deuteron and Nucleon-Nucleon Potential

Youngman Kim[♠♡], Sangmin Lee[◇] and Piljin Yi[♠]

[♠] *School of Physics, Korea Institute for Advanced Study, Seoul 130-722, Korea*
[♡] *Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 790-784, Korea*
[◇] *Department of Physics, Seoul National University, Seoul 151-747, Korea*

Abstract

We compute the potential between a pair of nucleons in the D4-D8 holographic QCD. In the large 't Hooft coupling limit, $\lambda \gg 1$, the hadronic size of the baryon is small $\sim 1/\sqrt{\lambda}M_{KK}$, and their interaction with mesons are well approximated by a set of dimension four and five operators. The nucleon-nucleon potential emerges from one-boson exchange picture involving massless pseudo-scalars and an infinite tower of spin one mesons. We find in particular that ρ meson exchanges are dominated by a dimension five derivative coupling of tensor type, whereas for ω mesons and axial mesons, such tensor couplings are completely absent. The potential is universally repulsive $\sim 1/r^2$ at short distance, and has the usual long-distance attractive behavior $\sim -1/r^3$ along a isosinglet and spin triplet channel. Both the large N_c form and the finite N_c form are given. In the former, a shallow classical minimum of depth $\sim 0.1 M_{KK}N_c/\lambda$ forms at around $rM_{KK} \simeq 5.5$.

Sakai, Sugimoto, KH

arXiv:0901.4449v3 [hep-th] 9 Mar 2010

IU-MSTP-77
IPMU 09-0010
RIKEN-TH-147

Nuclear Force from String Theory

Koji Hashimoto[1], Tadakatsu Sakai[2], and Shigeki Sugimoto[3]

[1] *Theoretical Physics Laboratory, RIKEN, Wako 351-0198, Japan*
[2] *Department of Physics, Faculty of Sciences, Ibaraki University, Mito 310-8512, Japan*
[3] *Institute for the Physics and Mathematics of the Universe, The University of Tokyo, Chiba 277-8568, Japan*

Abstract

We compute the nuclear force in a holographic model of QCD on the basis of a D4-D8 brane configuration in type IIA string theory. The repulsive core of nucleons is important in nuclear physics, but its origin has not been well understood in strongly coupled QCD. We find that the string theory via gauge/string duality deduces this repulsive core at a short distance between nucleons. Since baryons in the model are realized as solitons given by Yang-Mills instanton configuration on flavor D8-branes, ADHM construction of two instantons probes well the nucleon interaction at short scale, which provides the nuclear force quantitatively. We obtain a central force, as well as a tensor force, which is strongly repulsive as suggested in experiments and lattice results. In particular, the nucleon-nucleon potential $V(r)$ (as a function of the distance) scales as r^{-2}, which is peculiar to the holographic model. We compare our

Taiwan String Theory Workshop 2010

December 20 to 23, 2009 @NTNU, Taiwan

	20 (Sun)	21 (Mon)	22 (Tue)	23 (Wed)
09:00 | 10:30	Feng	Minwalla	Hashimoto	Cheng
11:00 | 12:00	(local)	(local)	(local)	(local)
13:30 | 15:00	Gustavsson	Yoshida	Yi	Song
15:30 | 16:30	(local)	(local)	(local)	(local)

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A Matrix Model for Baryons and Nuclear Forces

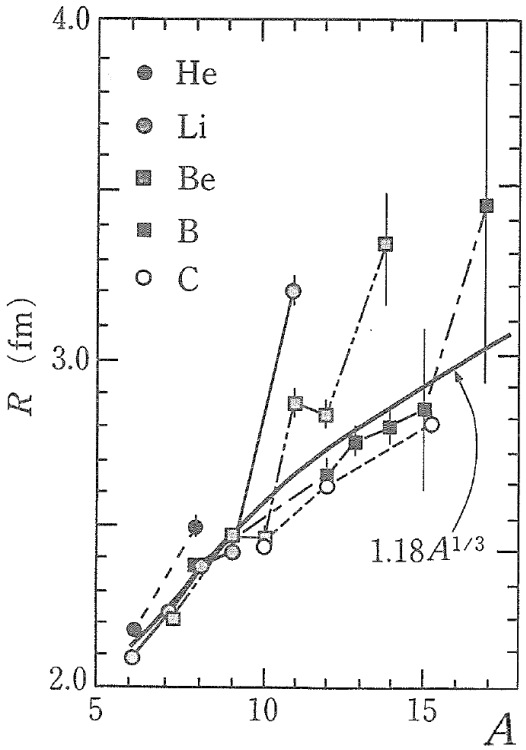
Koji Hashimoto[*,a], **Norihiro Iizuka**[†,b] **and Piljin Yi**[§,c]

[*] *Nishina Center, RIKEN, Saitama 351-0198, Japan*
[†] *Theory Division, CERN, CH-1211 Geneva 23, Switzerland*
[‡] *School of Physics, Korea Institute for Advanced Study, Seoul 130-722, Korea*
[a] *E-mail:* koji(at)riken.jp
[b] *E-mail:* norihiro.iizuka(at)cern.ch
[c] *E-mail:* piljin(at)kias.re.kr

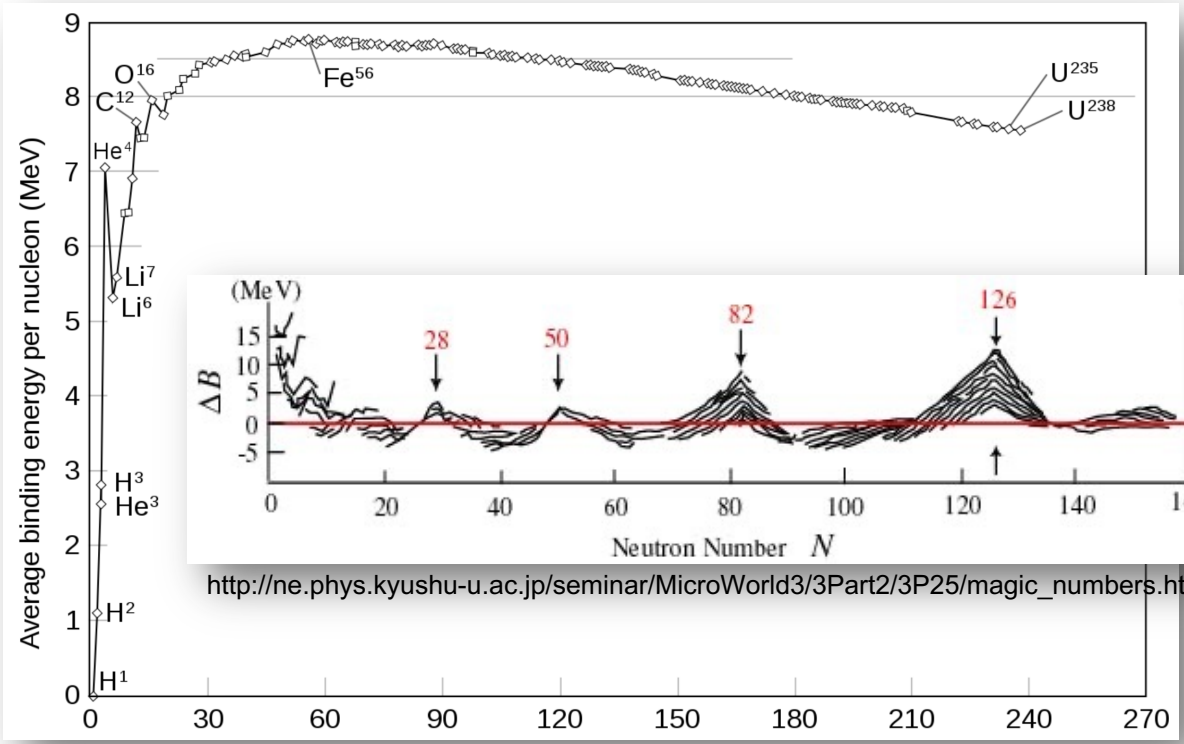
Abstract: We propose a new matrix model describing multi-baryon systems. We derive the action from open string theory on the wrapped baryon vertex D-branes embedded in the D4-D8 model of large N_c holographic QCD. The positions of k baryons are unified into $k \times k$ matrices, with spin/isospin of the baryons encoded in a set of k-vectors. Holographic baryons are known to be very small in the large 't Hooft coupling limit, and our model offers a better systematic approach to dynamics of such baryons at short distances. We compute energetics and spectra ($k = 1$),

Challenges in Nuclear Physics

1. Magic number = 2,8,20,28,50,…
2. Nuclear density saturation
3. Universal nuclear binding energy ~ 8 [MeV]



[『大学院原子核物理』(講談社)]



http://ne.phys.kyushu-u.ac.jp/seminar/MicroWorld3/3Part2/3P25/magic_numbers.htm

$$S = \frac{\lambda N_c}{54\pi} M_{\rm KK} \int dt\ {\rm tr}\ \Big[(D_0 X^M)^2 - \frac{2}{3} M_{\rm KK}^2 (X^4)^2 + D_0 \bar{w}_i^{\dot{\alpha}} D_0 w_{\dot{\alpha} i} - \frac{1}{6} M_{\rm KK}^2 \bar{w}_i^{\dot{\alpha}} w_{\dot{\alpha} i}$$
$$- \frac{3^6 \pi^2}{4\lambda^2 M_{\rm KK}^4} \left(\vec{D}\right)^2 + i\vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{X}^{\dot{\beta}\alpha} X_{\alpha\dot{\alpha}} + i\vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{w}_i^{\dot{\beta}} w_{\dot{\alpha} i} \Big] + 4N_c \int dt\ {\rm tr}\ A_0$$

ArXiv/1003.4988 Iizuka, Yi, KH

0. Nuclear Matrix Model (2 slides)

1. Magic Numbers (7 slides)

2. Nuclear density saturation (6 slides)

3. Nuclear binding energy (2 slides)

0. Nuclear Matrix Model

ArXiv/1003.4988 Iizuka, Yi, KH

0-1. Effective description of multi-baryon is matrix

0-2. Nuclear matrix model to nuclear physics

0-1

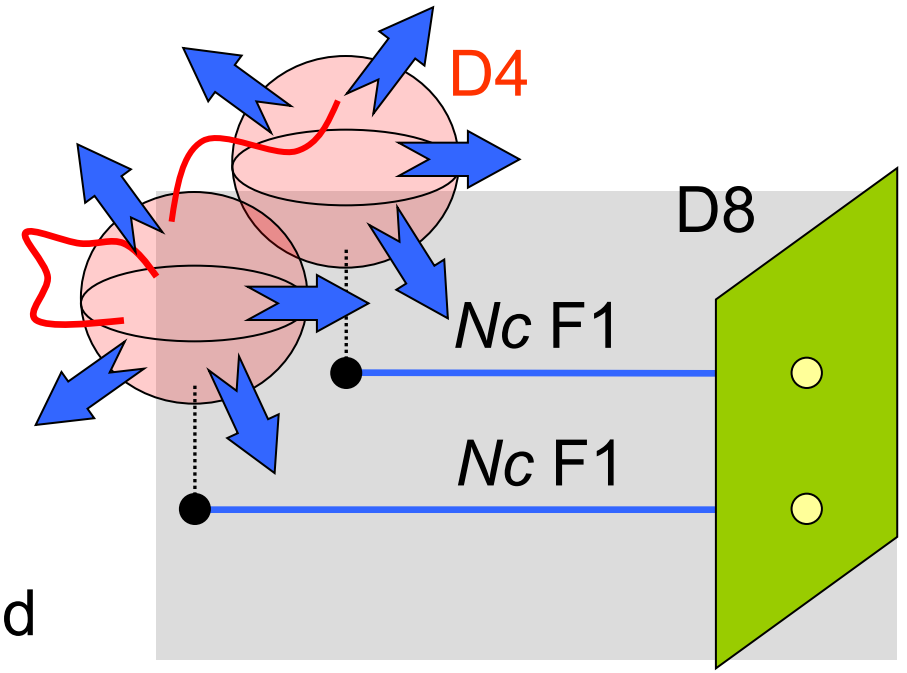
Multi-baryons = matrix

Effective theory for *A* baryons

= Baryonic D4-brane action in Sakai-Sugimoto

= Matrix quantum mechanics with CS term

- D4-D4 strings
  → *A* x *A* matrix
- D4-D8 strings
  → *A* x N_f matrix
- RR flux penetrating S^4
  → CS term
- S^4 direction unnecessary
  → Dim. reduced to 1d



0-2 Nuclear Matrix Model to Nuclear physics

[Iizuka,Yi, KH 1003.4988]

$$S = \frac{\lambda N_c}{54\pi} M_{\rm KK} \int dt\ {\rm tr}\ \left[(D_0 X^M)^2 - \frac{2}{3} M_{\rm KK}^2 (X^4)^2 + D_0 \bar{w}_i^{\dot{\alpha}} D_0 w_{\dot{\alpha} i} - \frac{1}{6} M_{\rm KK}^2 \bar{w}_i^{\dot{\alpha}} w_{\dot{\alpha} i} \right.$$
$$\left. - \frac{3^6 \pi^2}{4\lambda^2 M_{\rm KK}^4} \left(\vec{D}\right)^2 + i\vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{X}^{\dot{\beta}\alpha} X_{\alpha\dot{\alpha}} + i\vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{w}_i^{\dot{\beta}} w_{\dot{\alpha} i} \right] + 4N_c \int dt\ {\rm tr}\ A_0$$

$X^M\ (M = 1,2,3,4)$: A x A Hermitian matrix

Eigenvalues of $X^{M=1,2,3}$ = location of A baryons

Only 2 parameters : $M_{\rm KK},\ \lambda = N_c g_{\rm QCD}^2$

Nuclear physics action

$$S \sim \int dt \left[\sum_{i=1}^{A} \frac{1}{2} m_N |\partial_t \vec{x}^{(i)}|^2 + \sum_{i \neq j} V(x^{(i)} - x^{(j)}, \cdots) + \cdots \right]$$

$$S = \frac{\lambda N_c}{54\pi} M_{\rm KK} \int dt\ {\rm tr}\ \Big[(D_0 X^M)^2 - \frac{2}{3} M_{\rm KK}^2 (X^4)^2 + D_0 \bar{w}_i^{\dot{\alpha}} D_0 w_{\dot{\alpha} i} - \frac{1}{6} M_{\rm KK}^2 \bar{w}_i^{\dot{\alpha}} w_{\dot{\alpha} i}$$
$$- \frac{3^6 \pi^2}{4\lambda^2 M_{\rm KK}^4} \left(\vec{D}\right)^2 + i\vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{X}^{\dot{\beta}\alpha} X_{\alpha\dot{\alpha}} + i\vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{w}_i^{\dot{\beta}} w_{\dot{\alpha} i} \Big] + 4N_c \int dt\ {\rm tr}\ A_0$$

ArXiv/1003.4988 Iizuka, Yi, KH

0. Nuclear Matrix Model (2 slides)

1. Magic Numbers (7 slides)

2. Nuclear density saturation (6 slides)

3. Nuclear binding energy (2 slides)

1. Magic Numbers

ArXiv/1902.07444 Matsuo, Morita, KH

1-1. Strategy to solve the quantum mechanics
1-2. Ground state solving gauge constraint
1-3. A=1 : Nucleon and Delta
1-4. A=3 : H_3 and He_3
1-5. A=4 : He_4
1-6. Magic numbers
1-7. Comparison to quark model / shell model

1-1 Strategy to solve the quantum mechanics

1. Approximate $X(t)$ and $w(t)$ as harmonic oscillators.
2. Build states satisfying gauge constraints.
3. Include interactions as perturbation.

	field	index	gauge $U(A)$	global $SU(N_f)$	global $SU(2) \times SU(2)$
Matter field	$X^M(t)$	$M = 1, 2, 3, 4$	adj.	$\mathbf{1}$	$(\mathbf{2}, \mathbf{2})$
Matter field	$w_{\dot{\alpha} i}(t)$	$\dot{\alpha} = 1, 2;\ i = 1, \cdots, N_f$	$\mathbf{A}$	$\mathbf{N_f}$	$(\mathbf{1}, \mathbf{2})$
Gauge field	$A_0(t)$		adj.	$\mathbf{1}$	$(\mathbf{1}, \mathbf{1})$
Auxiliary field	$D_s(t)$	$s = 1, 2, 3$	adj.	$\mathbf{1}$	$(\mathbf{1}, \mathbf{3})$

$$S = \frac{\lambda N_c}{54\pi} M_{\rm KK} \int dt\ {\rm tr}\ \left[(D_0 X^M)^2 - \frac{2}{3} M_{\rm KK}^2 (X^4)^2 + D_0 \bar{w}_i^{\dot{\alpha}} D_0 w_{\dot{\alpha} i} - \frac{1}{6} M_{\rm KK}^2 \bar{w}_i^{\dot{\alpha}} w_{\dot{\alpha} i} \right.$$
$$\left. - \frac{3^6 \pi^2}{4\lambda^2 M_{\rm KK}^4} \left(\vec{D} \right)^2 + i \vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{X}^{\dot{\beta}\alpha} X_{\alpha\dot{\alpha}} + i \vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{w}_i^{\dot{\beta}} w_{\dot{\alpha} i} \right] + 4 N_c \int dt\ {\rm tr}\ A_0$$

1-2 Ground state solving gauge constraint

$$|\psi_0\rangle = \left(\epsilon_{a_1 \cdots a_A} w_{a_1} \cdots w_{a_A}\right)^{N_c} |0\rangle$$

Creation operator $w_a^{\dot{\alpha} i}$

$$\begin{cases} \dot{\alpha} = 1, 2 : \text{Spacetime spinor} \\ i = 1, 2 \cdots, N_f : \text{Flavor} \\ a = 1, 2, \cdots, A : U(A) \text{ baryon} \end{cases}$$

This state satisfies the gauge constraint :

$$0 = \frac{\delta S}{\delta A_0} = Q_{U(A)} - N_c \mathbf{1}$$

$$\Rightarrow \begin{cases} Q_{SU(A)}|\psi\rangle = 0 & \text{State is } SU(A)\text{-inv.} \\ Q_{U(1)_B}|\psi\rangle = N_c A|\psi\rangle & \text{State has } N_c A \ w\text{'s.} \end{cases}$$

1-3

$A = 1$: Nucleon and Delta

$N_c = 3, N_f = 2$

$$|\psi_0\rangle = \left(\epsilon_{a_1 \cdots a_A} w_{a_1} \cdots w_{a_A}\right)^{N_c} |0\rangle$$

$$|\psi_0\rangle = \mathcal{S}^{(J,I)=(\frac{1}{2},\frac{1}{2})} \mathcal{S}^{(J,I)=(\frac{1}{2},\frac{1}{2})} \mathcal{S}^{(J,I)=(\frac{1}{2},\frac{1}{2})} |0\rangle$$

$$\mathcal{S} = \underbrace{w^{\dot{\alpha}_1 i_1}}$$

one w

$$\Rightarrow \quad (J, I) = \left(\tfrac{1}{2}, \tfrac{1}{2}\right), \quad \left(\tfrac{3}{2}, \tfrac{3}{2}\right)$$

Nucleon Delta

Mass difference
from QM perturbation : $\langle H_{\rm int} \rangle = \dfrac{4\lambda}{9\pi N_c} M_{\rm KK} I(I+1)$

$A = 3$: H_3 and He_3

$N_c = 3, N_f = 2$

$$|\psi_0\rangle = \left(\epsilon_{a_1 \cdots a_A} w_{a_1} \cdots w_{a_A}\right)^{N_c} |0\rangle$$

$$|\psi_0\rangle = \mathcal{S}^{(J,I)=(\frac{1}{2},\frac{1}{2})} \mathcal{S}^{(J,I)=(\frac{1}{2},\frac{1}{2})} \mathcal{S}^{(J,I)=(\frac{1}{2},\frac{1}{2})} |0\rangle$$

$$\mathcal{S} = \epsilon_{a_1 a_2 a_3} \underbrace{w^{\dot{\alpha}_1 i_1}_{a_1} w^{\dot{\alpha}_2 i_2}_{a_2} w^{\dot{\alpha}_3 i_3}_{a_3}}_{\text{Three } w\text{s}} : \ (J, I) = \left(\tfrac{1}{2}, \tfrac{1}{2}\right)$$

$$\Rightarrow \quad (J, I) = \left(\tfrac{1}{2}, \tfrac{1}{2}\right), \quad \left(\tfrac{3}{2}, \tfrac{3}{2}\right)$$

H_3, He_3

1-5

$A = 4$: $\mathbf{He_4}$

$N_c = 3, N_f = 2$

$$|\psi_0\rangle = \left(\epsilon_{a_1 \cdots a_A} w_{a_1} \cdots w_{a_A}\right)^{N_c} |0\rangle$$

$$|\psi_0\rangle = (\mathcal{S})^3 |0\rangle$$

$$\mathcal{S} = \epsilon_{a_1 a_2 a_3 a_4} \epsilon_{\dot{\alpha}_1 \dot{\alpha}_2} \epsilon_{\dot{\alpha}_3 \dot{\alpha}_4} \epsilon_{i_1 i_3} \epsilon_{i_2 i_4} \underbrace{w_{a_1}^{\dot{\alpha}_1 i_1} w_{a_2}^{\dot{\alpha}_2 i_2} w_{a_3}^{\dot{\alpha}_3 i_3} w_{a_4}^{\dot{\alpha}_4 i_4}}_{\text{Four } w\text{ s}}$$

$$\Rightarrow \quad (J, I) = (0, 0)$$

$\mathbf{He_4}$

1-6

Magic numbers

Q. What if $A > 2N_f$? A. Use $(Xw)_a$ instead of w_a

Baryon #	Built state (for $N_c = 1, N_f = 1$)
$A = 1$	$\|\psi_0\rangle = \epsilon u_\uparrow \|0\rangle$
Magic number $A = 2$	$\|\psi_0\rangle = \epsilon u_\uparrow u_\downarrow \|0\rangle$
$A = 3$	$\|\psi_0\rangle = \epsilon uu(Xu)\|0\rangle$ $(\because \|\psi_0\rangle = \epsilon u_\uparrow u_\downarrow u_\uparrow \|0\rangle = 0)$
$A = 4$	$\|\psi_0\rangle = \epsilon uu(Xu)(Xu)\|0\rangle$
⋮	
Magic number $A = 8$	$\|\psi_0\rangle = \epsilon uu\underbrace{(Xu)\cdots(Xu)}_{\text{6 of } Xu}\|0\rangle$
$A = 9$	$\|\psi_0\rangle = \epsilon uu\underbrace{(Xu)\cdots(Xu)}_{\text{6 of } Xu}(XXu)\|0\rangle$
⋮	

∴ We have only 2 of u (spin ↑ and ↓) for w

∴ We have only $2\times3 = 6$ of Xu using X^1, X^2, X^3

Comparison to quark model / shell model

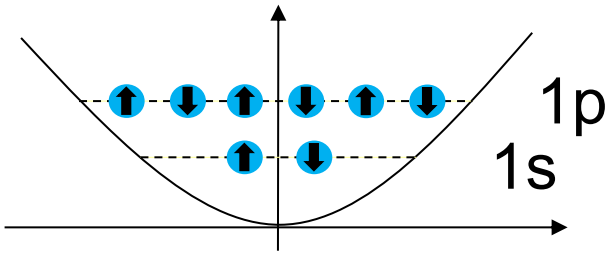
- No quark … but spectra are reproduced!

The boson field $w_a^{\dot{\alpha} i}$ is BaryonD4 - flavorD8 string, which carries the spin/flavor charge of a quark.
But: no color, charged under Baryon symmetry U(A).

- Quark motion? :

Xw means a "quark" w with angular momentum.

- Shell model



+ Woods-Saxon potential
+ Spin-Orbit int.

We have "Pauli principle" due to the U(A) constraint

$$S = \frac{\lambda N_c}{54\pi} M_{\rm KK} \int dt\ {\rm tr}\ \Big[(D_0 X^M)^2 - \frac{2}{3} M_{\rm KK}^2 (X^4)^2 + D_0 \bar{w}_i^{\dot{\alpha}} D_0 w_{\dot{\alpha} i} - \frac{1}{6} M_{\rm KK}^2 \bar{w}_i^{\dot{\alpha}} w_{\dot{\alpha} i}$$
$$- \frac{3^6 \pi^2}{4\lambda^2 M_{\rm KK}^4} \left(\vec{D}\right)^2 + i\vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{X}^{\dot{\beta}\alpha} X_{\alpha\dot{\alpha}} + i\vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{w}_i^{\dot{\beta}} w_{\dot{\alpha} i} \Big] + 4N_c \int dt\ {\rm tr}\ A_0$$

ArXiv/1003.4988 Iizuka, Yi, KH

0. Nuclear Matrix Model (2 slides)

1. Magic Numbers (7 slides)

2. Nuclear density saturation (6 slides)

3. Nuclear binding energy (2 slides)

2. Nuclear density saturation

ArXiv/1103.5668 Morita, KH
ArXiv/2103.03563 Matsuo, KH

2-1. Heavy nucleus is simpler (!)
2-2. Inevitable formation of atomic nucleus
2-3. Nuclear density saturation is seen
2-4. Analytic formula of nuclear radius
2-5. Finiteness of nucleus
2-6. A-dependence of charge radius, reproduced

2-1

Heavy nucleus is simpler (!)

At large *A*, we can drop *w(t)*

→ Nuclear matrix model

= Dimensionally reduced Yang-Mills theory

$$S = c \int dt \ \mathrm{tr} \left[\frac{1}{2} (D_t X^I)^2 - \frac{g^2}{4} [X^I, X^J]^2 \right]$$

Q. Solve this quantum mechanics at large *A*

2-2 Inevitable formation of atomic nucleus

$$S = c \int dt \ \mathrm{tr} \left[\frac{1}{2} (D_t X^I)^2 - \frac{g^2}{4} [X^I, X^J]^2 \right]$$

Classical config. : Diagonal $X^I = \begin{pmatrix} x^I_{(1)} & & \\ & x^I_{(2)} & \\ & & \cdots \end{pmatrix}$

However this is not realized quantum-mechanically.

For large *x* with $X = \begin{pmatrix} x & \delta x \\ \delta x & -x \end{pmatrix}$,

potential is too narrow as $V \sim g^2 x^2 (\delta x)^2$ [Lüscher '83]

Ground state :

Eigenvalues accumulated, baryons bound

2-3 Nuclear density saturation is seen

$$S = c \int dt \ \mathrm{tr} \left[\frac{1}{2} (D_t X^I)^2 - \frac{g^2}{4} [X^I, X^J]^2 \right]$$

'tHooft expansion for large *A* : $\lambda_A \equiv A g^2$ fixed

→ Dimensional analysis:

$$\frac{1}{A^2} \mathrm{tr} \langle X^I X^I \rangle = c \lambda_A^{-1/3} + \cdots$$

→ Nuclear density saturation is derived

$$\sqrt{r^2_{\mathrm{mean}}} \equiv \sqrt{\frac{1}{A} \mathrm{tr} \langle X^I X^I \rangle} \propto A^{1/3}$$

2-4 Analytic formula of nuclear radius

Large D, large A expansion with fixed $g^2\,A\,D$

Developped in [9811220 Hotta, Nishimura,Tsuchiya]
[0910.4526 Mandal, Mahato, Morita]

Result : $$\sqrt{r^2_{\rm mean}}\Big|_{T=0} = \frac{3^{5/2}\pi^{2/3}}{2^{5/6}5^{1/6}}\frac{A^{1/3}}{M_{\rm KK}(N_c\lambda^2)^{1/3}}$$

⇓

- rho meson mass $M_{\rm KK} \sim 0.95[\mathrm{GeV}]$
- Yukawa coupling $\lambda \sim 5.3$

$$\sqrt{r^2_{\rm mean}} \sim 0.7A^{1/3}\ [\mathrm{fm}]$$

Cf. Experiment $\sqrt{r^2_{\rm mean}} \simeq 1.0A^{1/3}\ [\mathrm{fm}]$

2-5

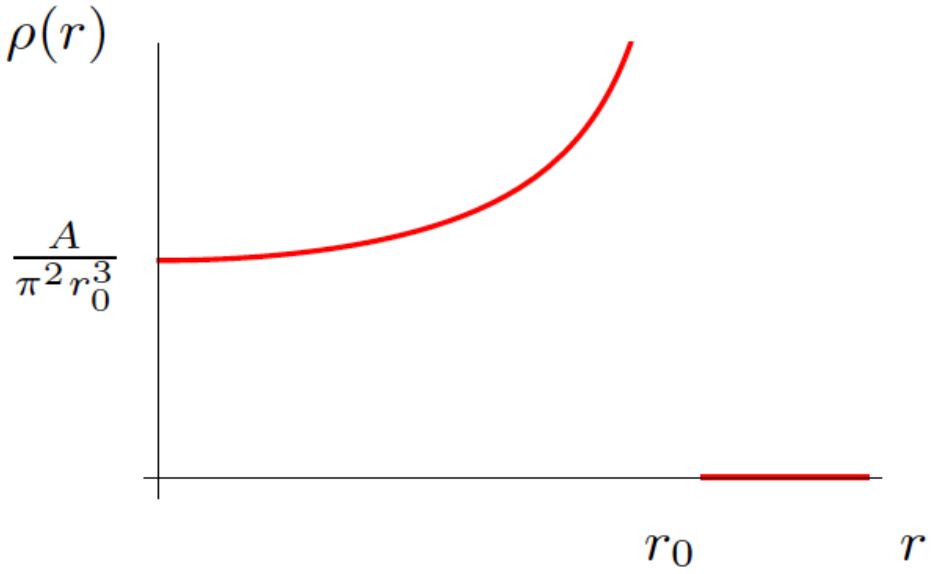
Finiteness of nucleus

Nuclear density via D-brane distribution formula :

[Taylor, vanRaamsdonk, 9904095]

$$\rho(r) = \int \prod_{\hat{i}=4}^{D} dx^{\hat{i}} \frac{1}{(2\pi)^D} \int d^D k \, e^{-ik\cdot x} \langle \mathrm{tr}_A \exp(ik \cdot Y) \rangle$$

$$= \begin{cases} \dfrac{A}{\pi^2 r_0^2 \sqrt{r_0^2 - r^2}} & (r < r_0) \\ 0 & (r_0 < r) \end{cases}$$



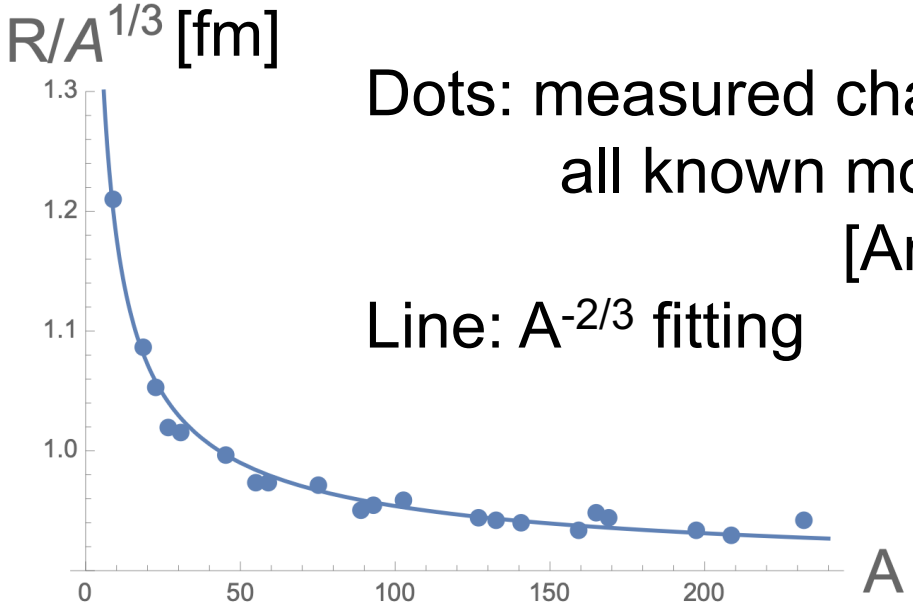
Finiteness of nucleus!

$$r_0 \sim 0.8 \times A^{1/3} \ [\mathrm{fm}]$$

2-6 A-dependence of charge radius, reproduced

Model prediction : $\dfrac{R}{A^{1/3}} = c_1\left(1 + c_2 A^{-2/3}\right)$

Experiments:



Dots: measured charge radius of all known mononuclidic nuclei [Angeli, Marinova '13]

Line: $A^{-2/3}$ fitting

Numerics comparison?

Fit: $c_1 = 0.89$ [fm] and $c_2 = 1.5$

Ours: $c_1 \simeq 2.4$ [fm] and $c_2 \simeq 0.57$

$$S = \frac{\lambda N_c}{54\pi} M_{\rm KK} \int dt\ {\rm tr}\ \Big[(D_0 X^M)^2 - \frac{2}{3} M_{\rm KK}^2 (X^4)^2 + D_0 \bar{w}_i^{\dot{\alpha}} D_0 w_{\dot{\alpha} i} - \frac{1}{6} M_{\rm KK}^2 \bar{w}_i^{\dot{\alpha}} w_{\dot{\alpha} i}$$
$$- \frac{3^6 \pi^2}{4\lambda^2 M_{\rm KK}^4} \left(\vec{D}\right)^2 + i\vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{X}^{\dot{\beta}\alpha} X_{\alpha\dot{\alpha}} + i\vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{w}_i^{\dot{\beta}} w_{\dot{\alpha} i} \Big] + 4N_c \int dt\ {\rm tr}\ A_0$$

ArXiv/1003.4988 Iizuka, Yi, KH

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3. Nuclear binding energy

ArXiv/2103.03563 Matsuo, KH

3-1. Build an effective nucleon potential
3-2. Binding energy, reproduced

4-1 Build an effective nucleon potential

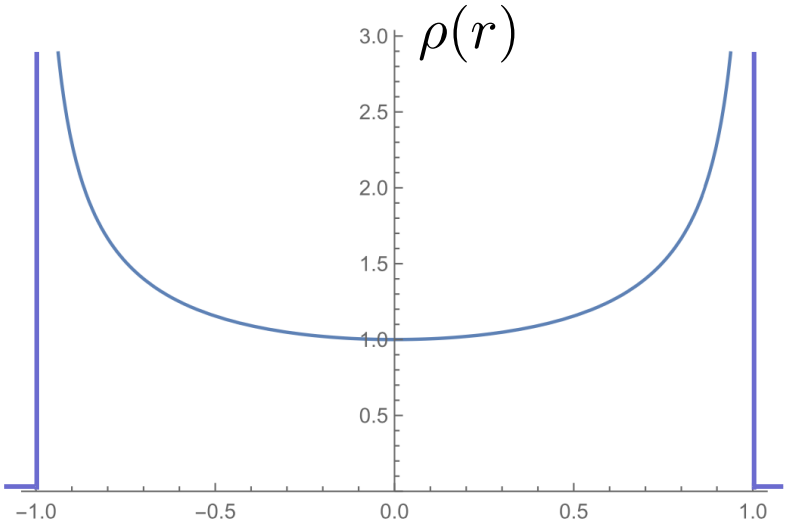
Nuclear density

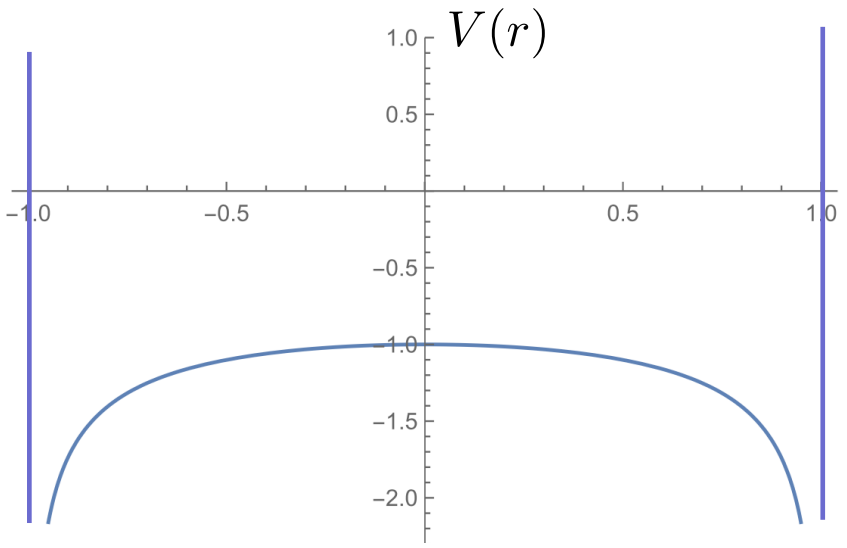
$$\rho(r) = \frac{A}{\pi^2 r_0^2 \sqrt{r_0^2 - r^2}}$$

WKB

Nucleon potential

$$V(r) = E_f - \frac{3^{2/3} A^{2/3}}{2^{2/3} r_0^{4/3} (r_0^2 - r^2)^{1/3}}$$





4-2 Binding energy, reproduced

Nuclear density

$$\rho(r) = \frac{A}{\pi^2 r_0^2 \sqrt{r_0^2 - r^2}}$$

WKB

Nucleon potential

$$V(r) = E_f - \frac{3^{2/3} A^{2/3}}{2^{2/3} r_0^{4/3} (r_0^2 - r^2)^{1/3}}$$

Nuclear binding energy (with $E_f = 0$)

$$B = 4\pi \int r^2 \rho(r) V(r) dr + 2\pi \int r^3 \rho(r)\, V'(r) dr$$

Potential energy

Kinetic term from virial theorem

Binding energy per nucleon matches the observed value

$$\frac{B}{A} = \left[\frac{3^{8/3}\Gamma\left(\frac{7}{6}\right)}{2^{2/3}\sqrt{\pi}\Gamma\left(\frac{2}{3}\right)} - \frac{3^{8/3}\sqrt{\pi}}{2^{2/3} 5 \Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{5}{6}\right)} \right] \lambda_r^{1/3} = 9.7 \text{ MeV}$$

$M_N = 939$ MeV

$M_\Delta = 1232$ MeV

$$S = \frac{\lambda N_c}{54\pi} M_{\rm KK} \int dt\ {\rm tr}\ \Big[(D_0 X^M)^2 - \frac{2}{3} M_{\rm KK}^2 (X^4)^2 + D_0 \bar{w}_i^{\dot{\alpha}} D_0 w_{\dot{\alpha} i} - \frac{1}{6} M_{\rm KK}^2 \bar{w}_i^{\dot{\alpha}} w_{\dot{\alpha} i}$$
$$- \frac{3^6 \pi^2}{4\lambda^2 M_{\rm KK}^4} \left(\vec{D}\right)^2 + i\vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{X}^{\dot{\beta}\alpha} X_{\alpha\dot{\alpha}} + i\vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{w}_i^{\dot{\beta}} w_{\dot{\alpha} i} \Big] + 4N_c \int dt\ {\rm tr}\ A_0$$

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$$S = \frac{1}{g_s l_s} \int dt \ \mathrm{tr} \left((D_0 X^M)^2 - l_s^{-4} [X^M, X^N]^2 + \text{fermions} \right)$$

M(atrix) theory

[Banks, Fischler, Shenker, Susskind (1996)]

$$\begin{aligned} S &= \frac{\lambda N_c}{54\pi} M_{\rm KK} \int dt \ \mathrm{tr} \ \Bigg[(D_0 X^M)^2 - \frac{2}{3} M_{\rm KK}^2 (X^4)^2 + D_0 \bar{w}_i^{\dot{\alpha}} D_0 w_{\dot{\alpha} i} - \frac{1}{6} M_{\rm KK}^2 \bar{w}_i^{\dot{\alpha}} w_{\dot{\alpha} i} \\ &\quad - \frac{3^6 \pi^2}{4 \lambda^2 M_{\rm KK}^4} \left(\vec{D} \right)^2 + i \vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{X}^{\dot{\beta}\alpha} X_{\alpha\dot{\alpha}} + i \vec{D} \cdot \vec{\tau}^{\ \dot{\alpha}}_{\dot{\beta}} \bar{w}_i^{\dot{\beta}} w_{\dot{\alpha} i} \Bigg] \quad + 4 N_c \int dt \ \mathrm{tr} \ A_0 \end{aligned}$$

Nuclear matrix model
(M-theory for nuclear physics)

[Iizuka, Yi, KH (2010)]