서울시립대학교
UNIVERSITY OF SEOUL

Pinning Without Disorder in Active Ising Model

Jae Dong Noh (University of Seoul)

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Motility-Induced Pinning in Flocking System with Discrete Symmetry

Chul-Ung Woo and Jae Dong Noh
Department of Physics, University of Seoul, Seoul 02504, Korea
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We report a motility-induced pinning transition in the active Ising model for an active self-propelled particle system with discrete symmetry. This model was known to exhibit a liquid-gas type flocking phase transition, but a recent study reveals that the polar order is metastable due to droplet excitation. Using extensive Monte Carlo simulations, we demonstrate that, for an intermediate alignment interaction strength, the steady state is characterized by traveling local domains, which renders the polar order short-ranged in both space and time. We further demonstrate that interfaces between colliding domains become pinned as the alignment interaction strength increases. A resonating back-and-forth motion of individual self-propelled particles across interfaces is identified as a mechanism for the pinning. We present an analytic argument for the motility-induced pinning transition by incorporating the resonance mechanism into the hydrodynamic theory. The resulting steady state consists of a network of pinned interfaces interconnected by particle currents. While the polar order is still short-ranged in space, the particle currents mediated by the pinned interfaces can exhibit long-range temporal correlations.

arXiv:2403.10106

see also P28 by Chul-Ung Woo

Overview

Ordering of Many-Particle Systems

- particle $i = 1, \cdots, N$: position $\mathbf{r}_i$ and "spin" $\mathbf{v}_i$

- Traditional equilibrium setup

  $\mathbf{v}_i$'s are fluctuating satisfying the detailed balance while $\mathbf{r}_i$'s are quenched.

  Order-disorder phase transitions are universal.

- Out of equilibrium

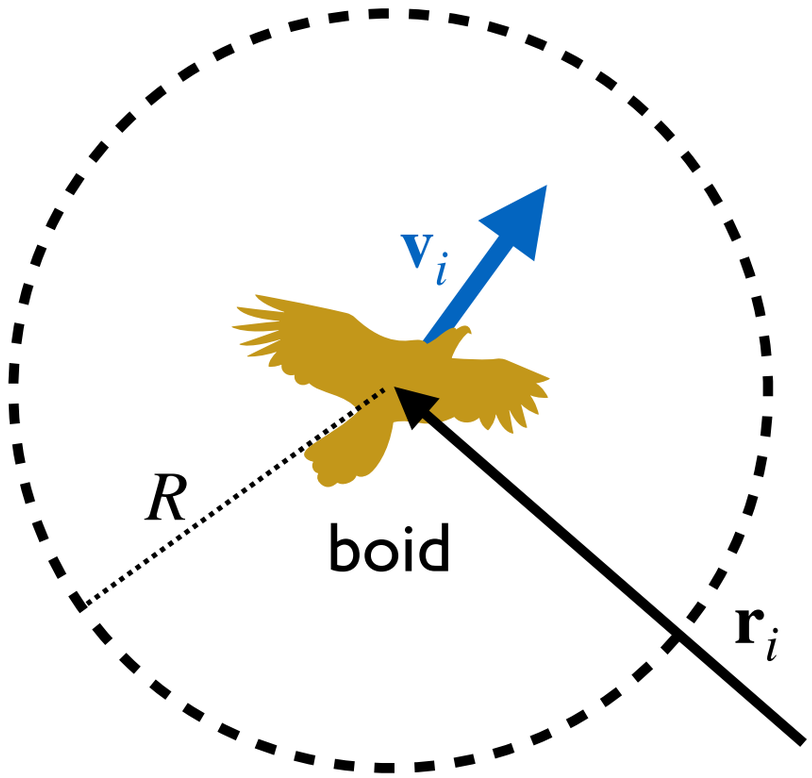
  - Dynamics of $\mathbf{v}_i$'s breaks the detailed balance while $\mathbf{r}_i$'s are still quenched.

  - active matters with motility (both $\mathbf{v}_i$'s and $\mathbf{r}_i$'s are fluctuating)

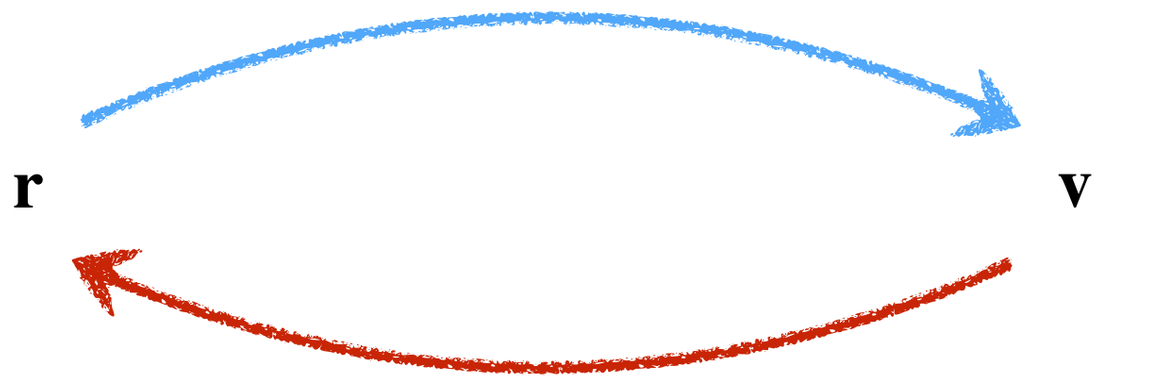
    self-propelled particles, active Brownian particles, swarmalators, …

Model for Flocking B(ird)oids by Vicsek et al.

Vicsek et al, PRL (1995)



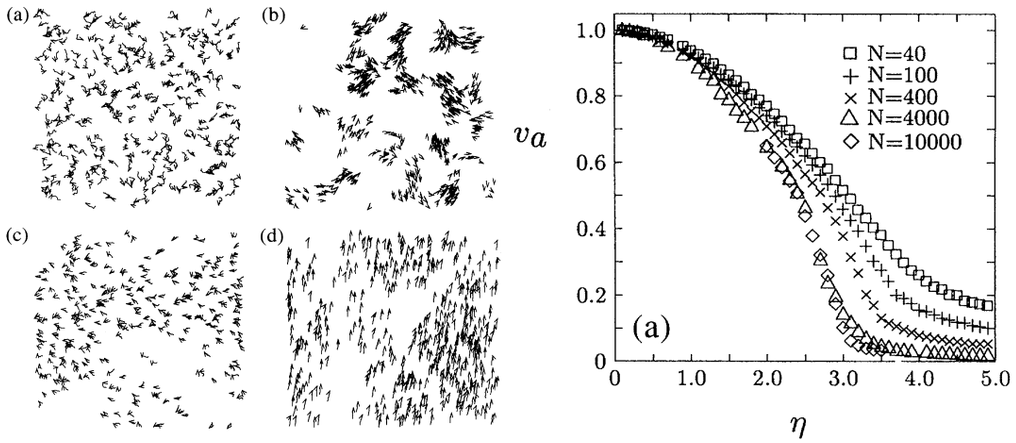
position sets spin-spin interaction network



spin steers position

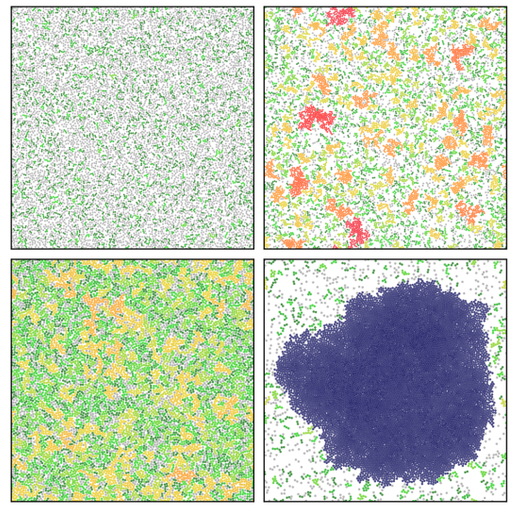
Striking Features

- Spontaneous breaking of continuous symmetry even in 2D
  - long range order in the Vicsek model
  - Mermin-Wagner theorem in equilibrium



Vicsek et al, PRL (1995)

- Motility Induced Phase Separation
  - condensation via repulsive interaction
- Motility Induced Interface Pinning



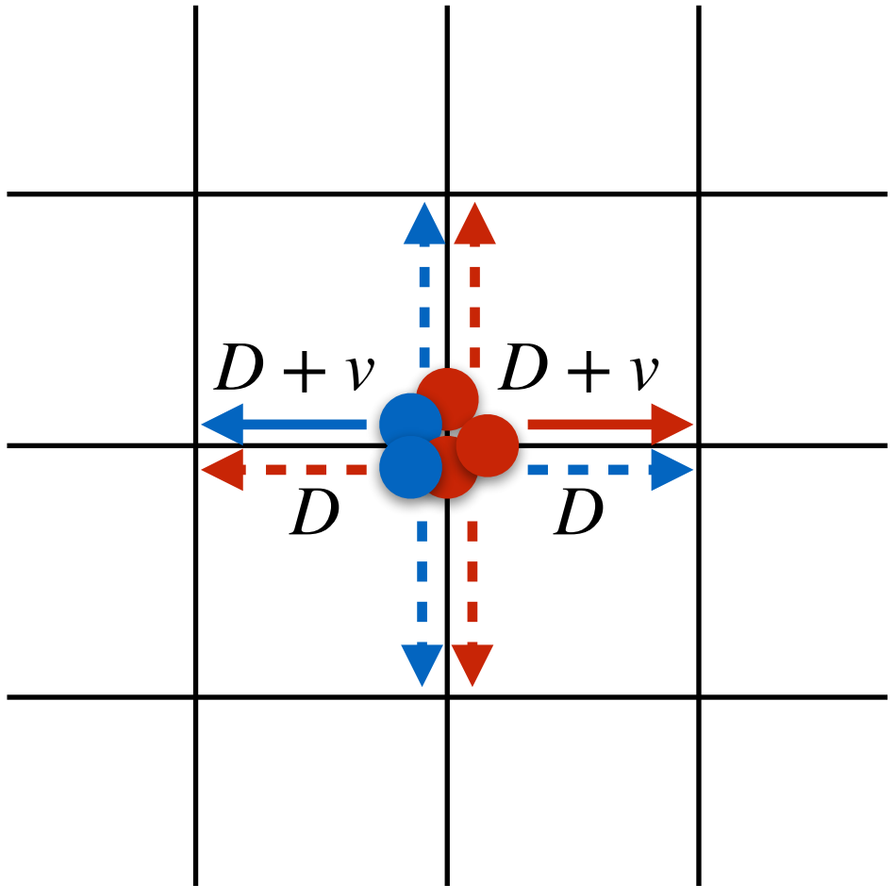
Fily and Marchetti, PRL (2012)

Introduction

Active Ising Model

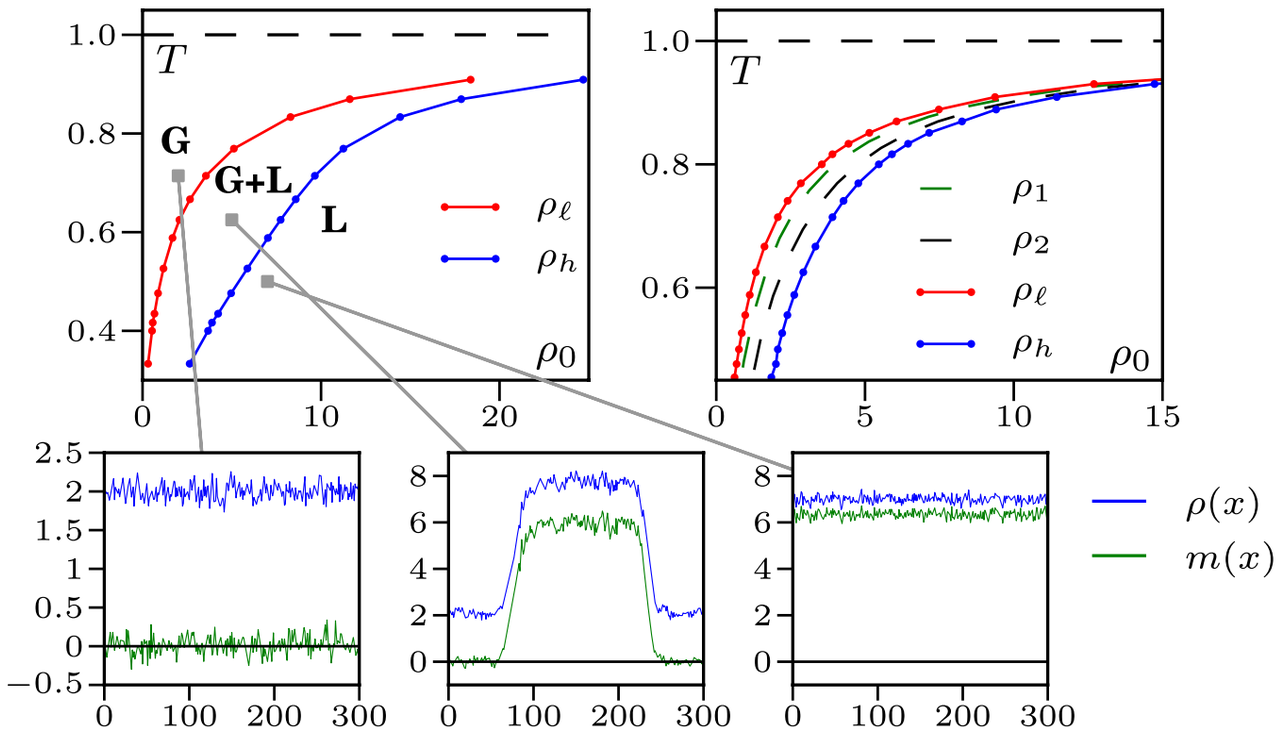
Solon and Tailleur, PRL (2013)

- flocking model with discrete Z_2 symmetry
  - particle density $\rho_0 = N/L^2$
  - $\mathbf{r}_i \in \mathbb{Z}^2$
  - Ising spin $s_i = +1, -1$
  - hopping (D) and self-propulsion (v)
  - spin flip ($s \to -s$) rate $e^{-\beta m_{\mathbf{r}} s/\rho_{\mathbf{r}}}$
    (ferromagnetic Ising model)
  - flocking order parameter $m = \langle m_{\mathbf{r}} \rangle$

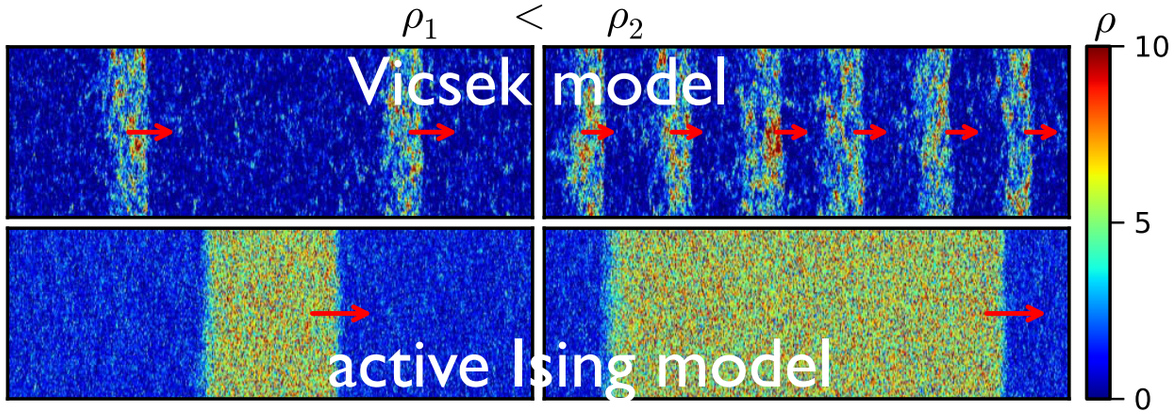


Liquid-Gas Transition

Solon and Tailleur, PRL (2013)



Solon, Chate, and Tailleur, PRL (2015)

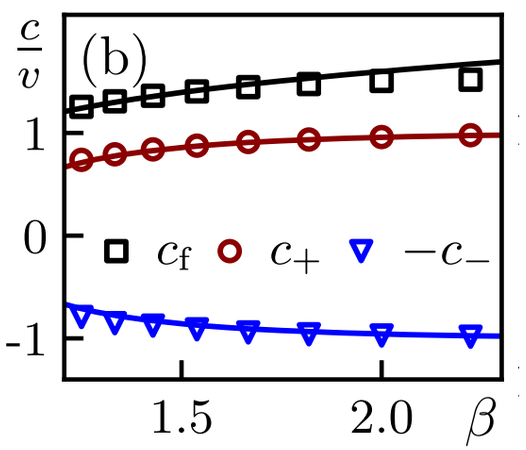


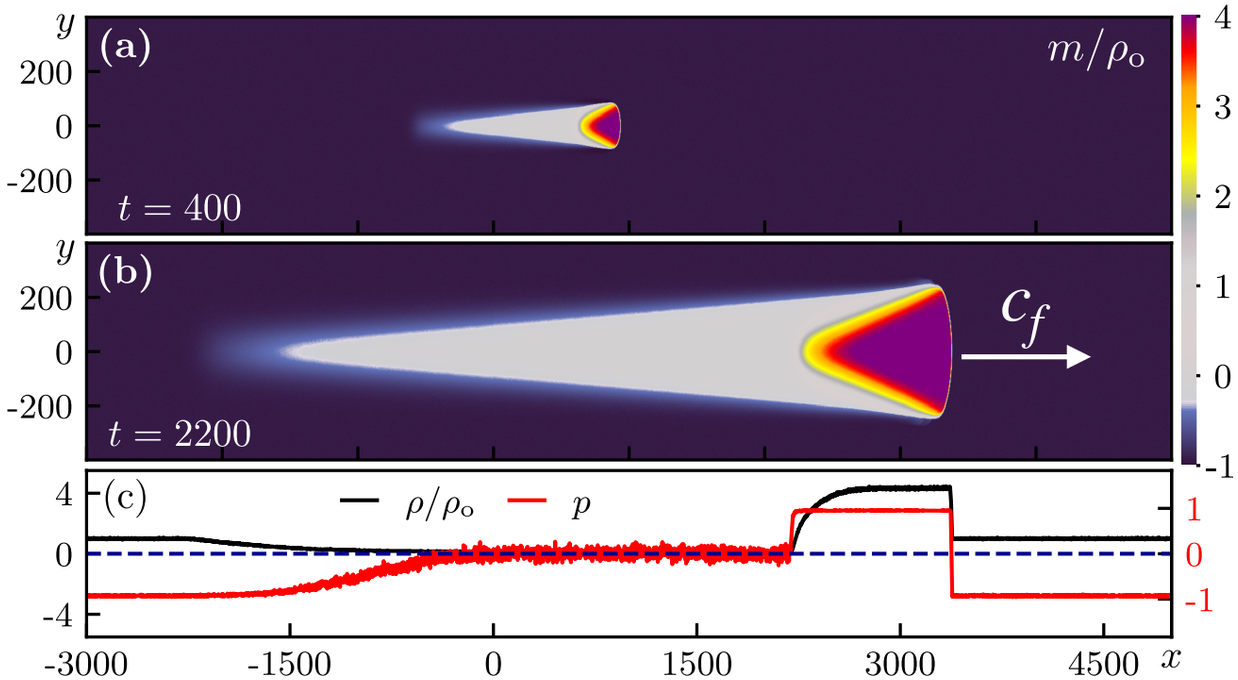
coexistence of a disordered gas state ($m = 0$)
and an ordered liquid state ($m \neq 0$)

AIM Ordered State is Metastable

Benvegnen et al., PRL (2023)

- spontaneous nucleation of a droplet
- counter-propagating and growing

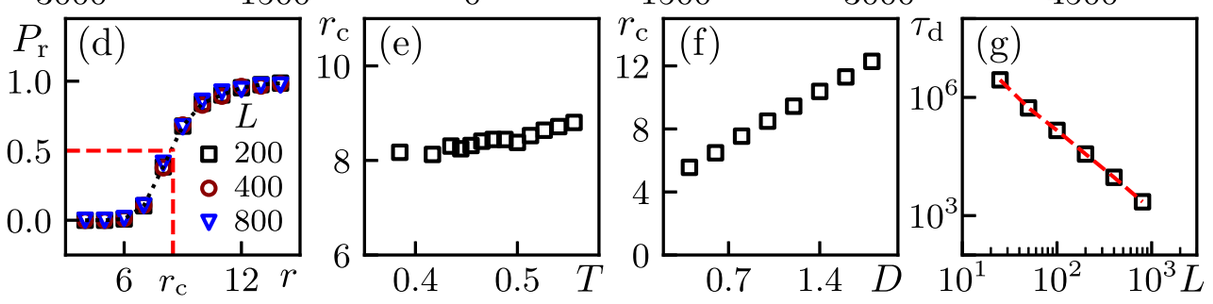




- characteristic nucleation size
- nucleation time

$$\propto L^{-2} \text{ (local event)}$$

$$\propto e^{D/D_0} \text{ with } D_0 \simeq 0.1$$



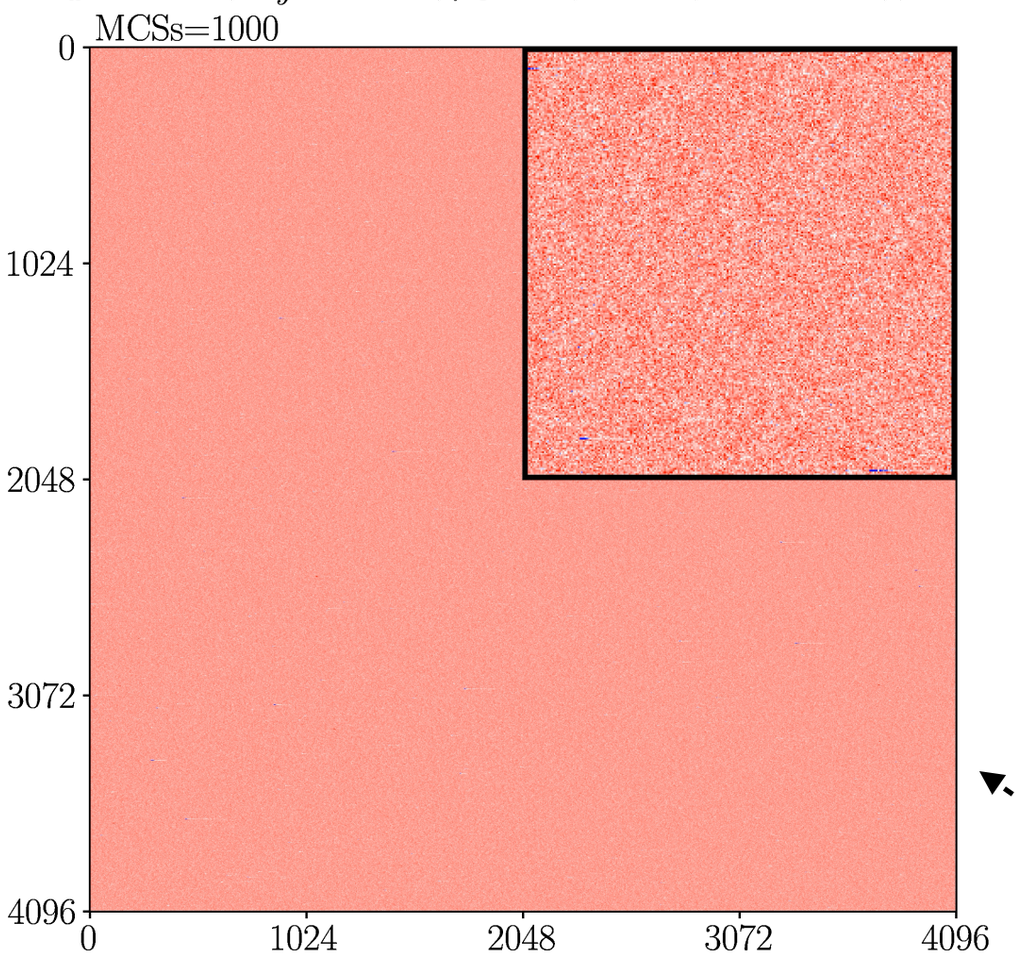
survival prob. characteristic droplet size nucleation time $\tau_d \sim L^{-2}$

- Droplet excitation disrupts the homogeneous long range order.

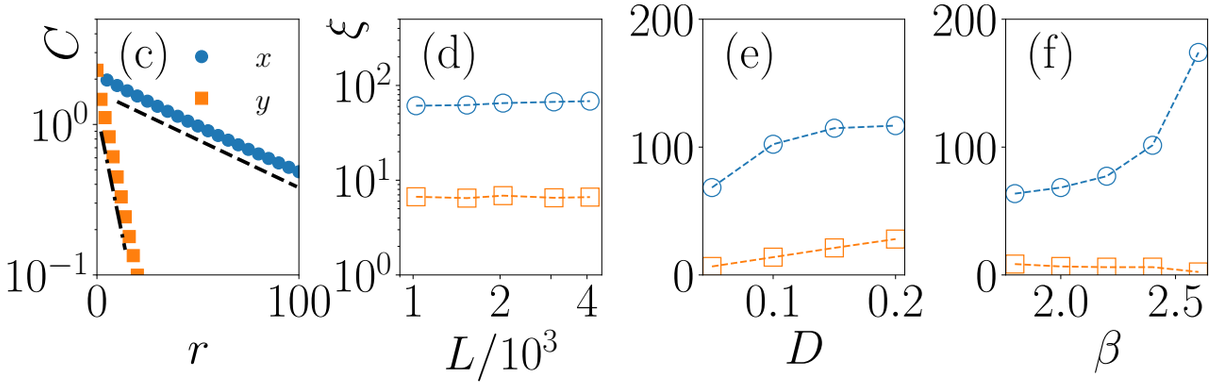
- questions: phase diagram, new phase(s), …

Disordered droplet phase

$L_x = 4096, L_y = 4096, \rho_0 = 8, v = 1, D = 0.05, \beta = 2$

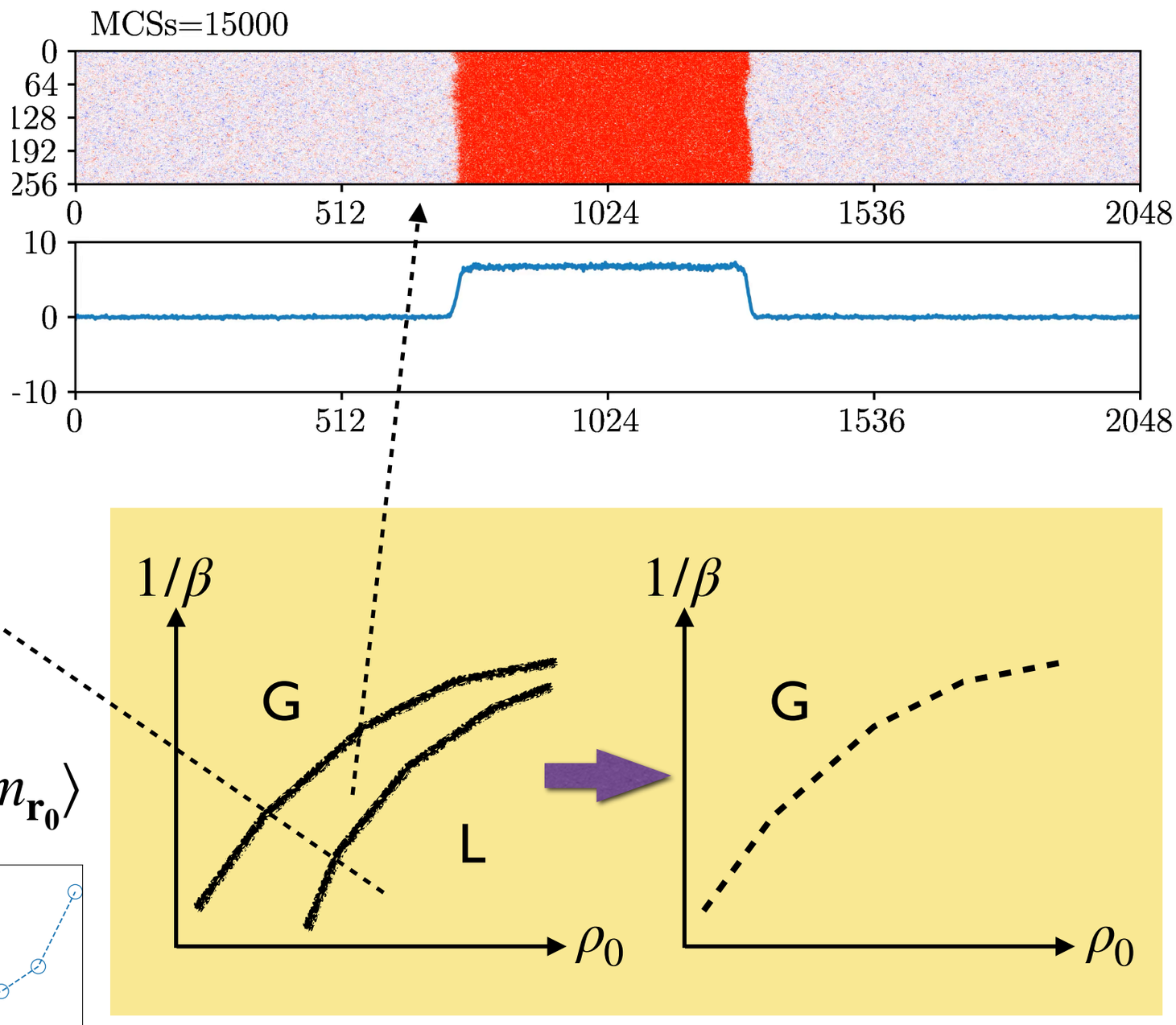


correlation function $C(\mathbf{r}) = \langle m_{\mathbf{r}+\mathbf{r_0}} m_{\mathbf{r_0}} \rangle$



finite correlation lengths

$L_x = 2048, L_y = 256, \rho_0 = 3.00, v = 1.00, D = 0.10, \beta = 2.00$

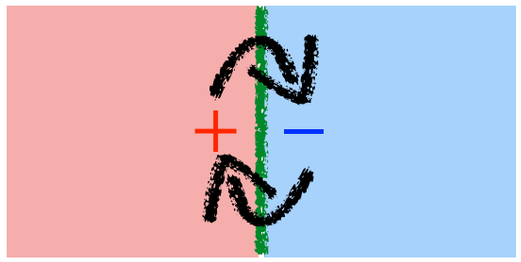


Motility-Induced Interface Pinning

Low Temperature Limit ($\beta \gg 1$)

- Each particle
  jumps to a random neighbor site at rate $4D$ (slow),
  self-propels to a preferred direction at rate v (slow), and
  aligns its spin state at rate $\simeq e^{\beta}$ (fast)

- a domain wall or an interface between two magnetic domains



domain wall
interface

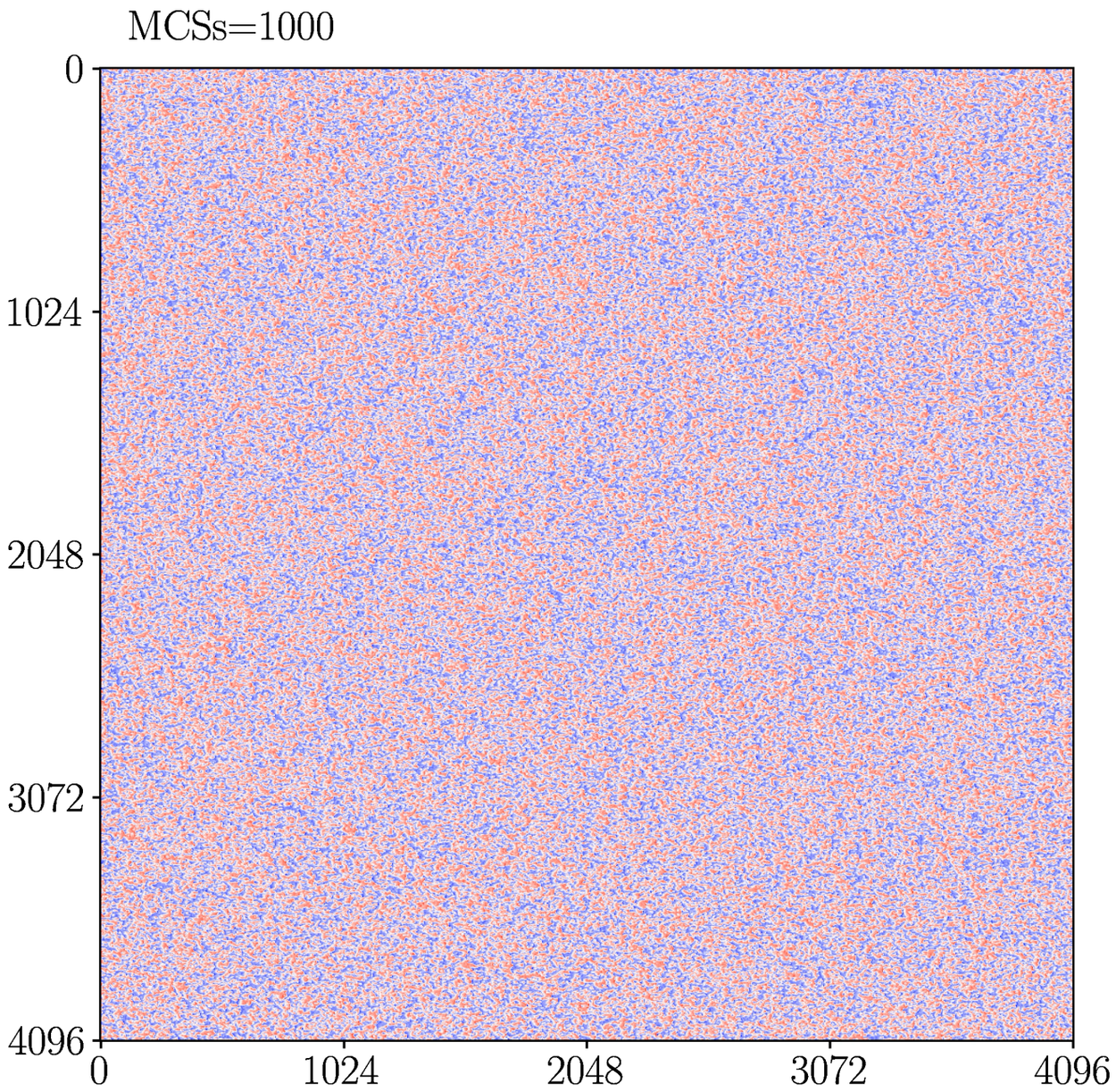
The resonating back-and-forth oscillatory motion of individual particles pins the interface
$\Rightarrow$ motility-induced interface pinning

MIIP prevents the system from evolving into an absorbing ordered state even when $\beta = \infty$

- Without activity ($v = 0$), the AIM at $\beta = \infty$ reduces to a voter model.

AIM at large $\beta < \infty$

$L_x = 4096, L_y = 4096, \rho_0 = 4, v = 1, D = 1.00, \beta = 5$



random initial configuration

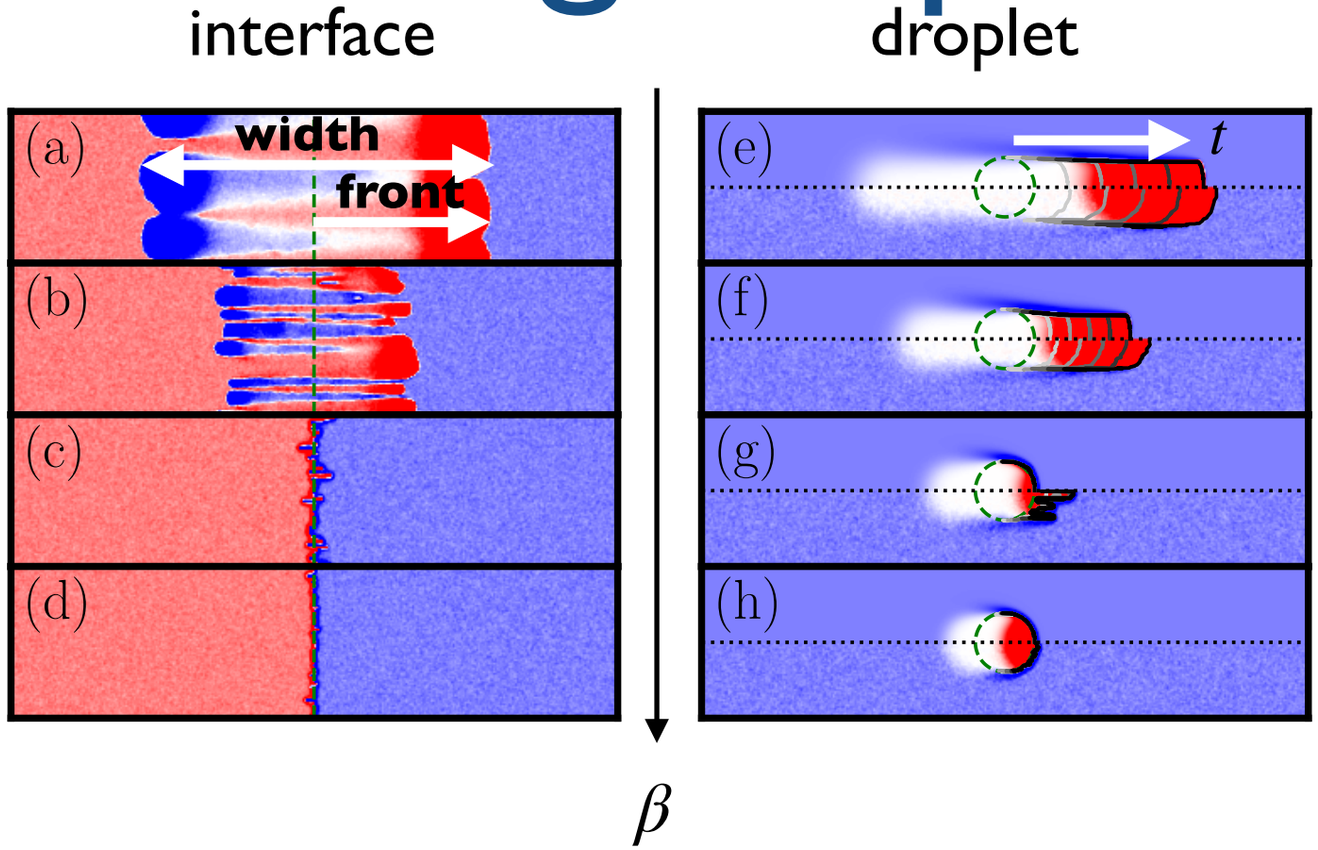
small droplets

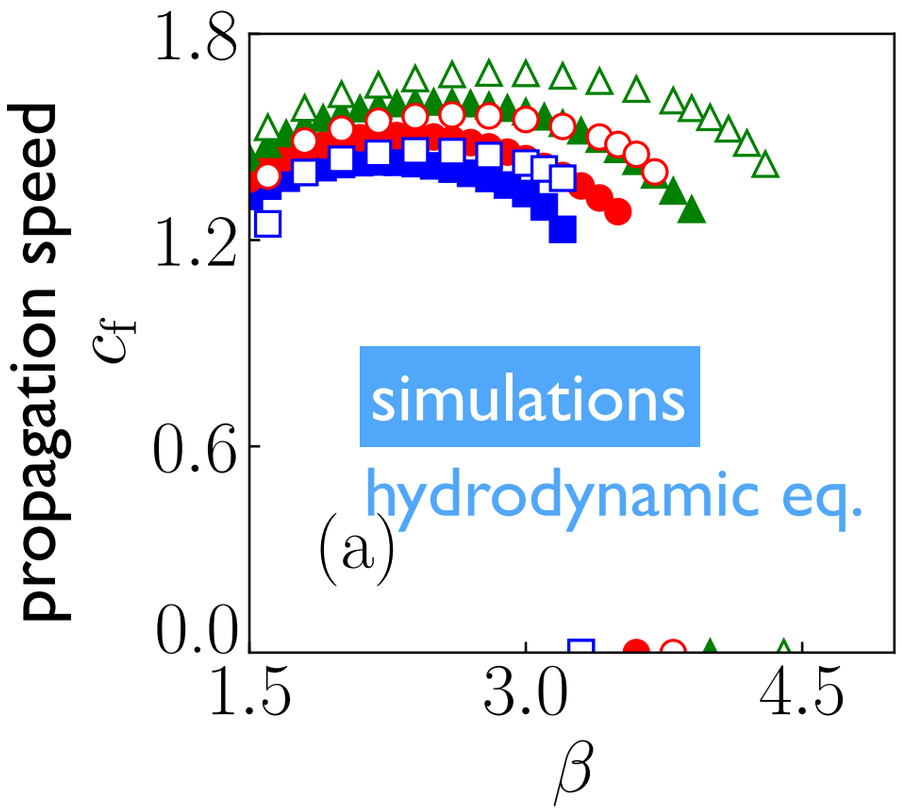
pinned interfaces

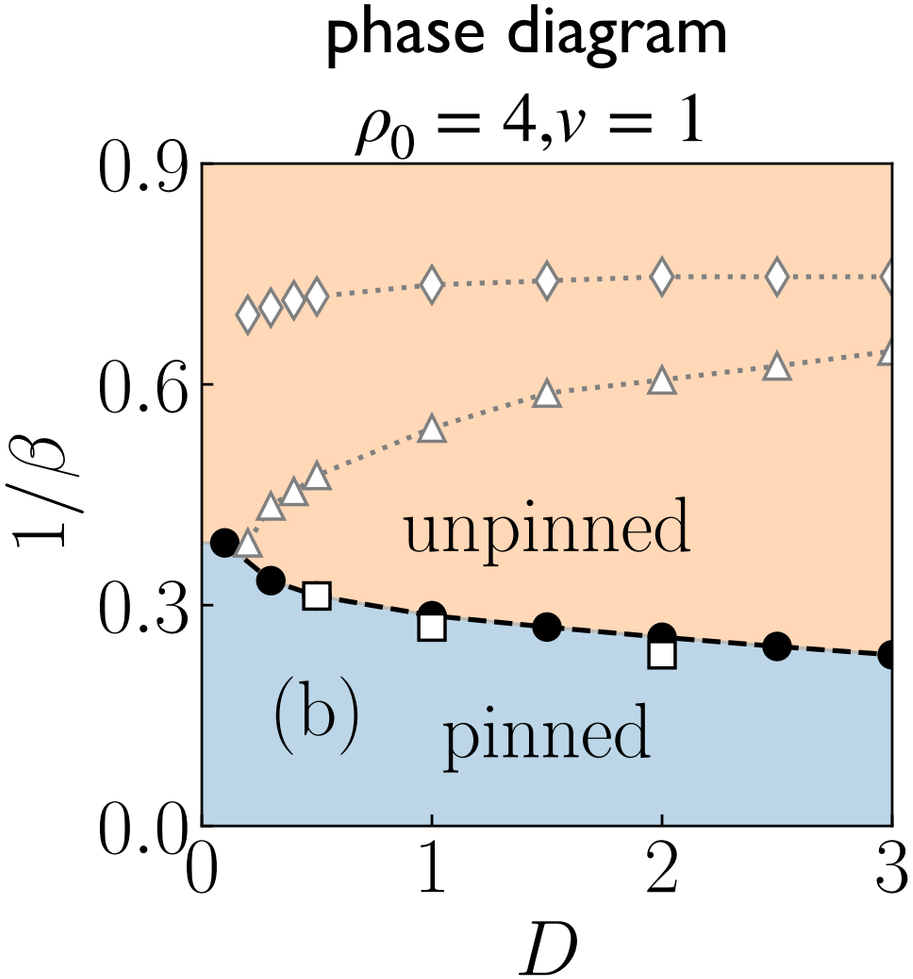
coarsening dynamics
(growth and shrink)

leak currents

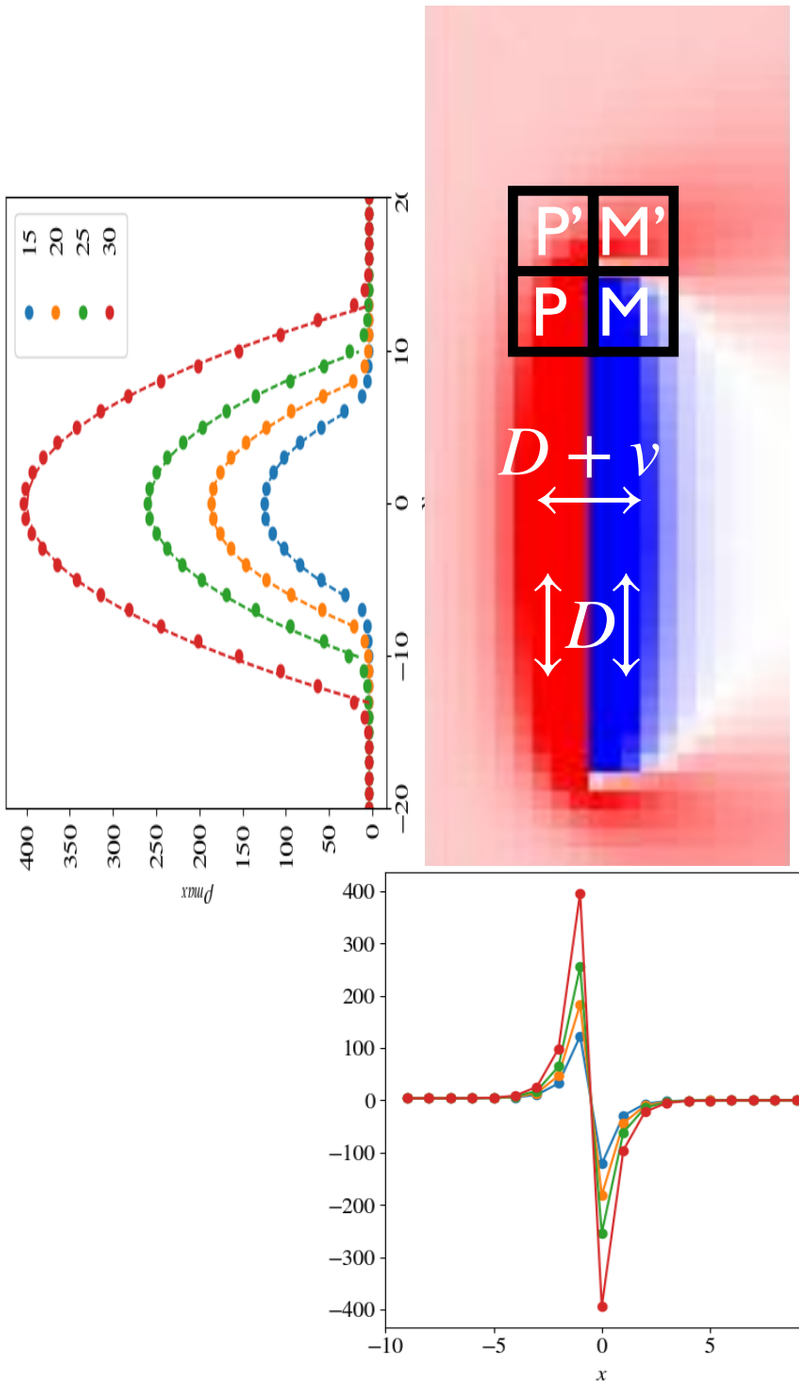
Pinning-Depinning Transition







Dynamics of a PI of Length l



single site model ($D \ll v$)

mean occupation numbers (from flux balance)

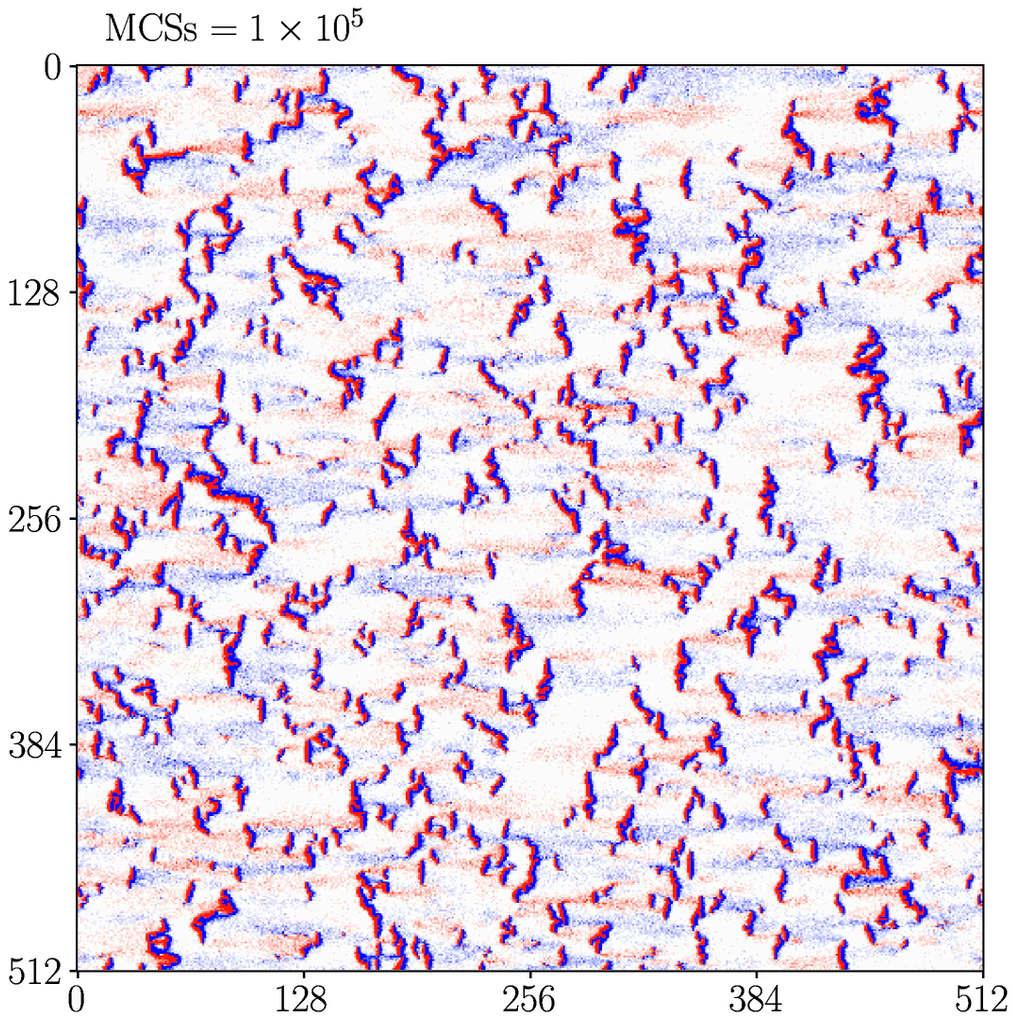
$$\langle N_P \rangle \simeq \langle N_M \rangle \simeq \frac{\rho_0 v l}{4D}, \ \langle N_{P'} \rangle \simeq \frac{\rho_0 l}{4}, \ \ \langle N_{M'} \rangle \simeq \frac{\rho_0 l}{2}$$

The PI grows if $N_{M\to M'} > N_{M'}$ or shrink if $N_{P\to M} > N_M$.

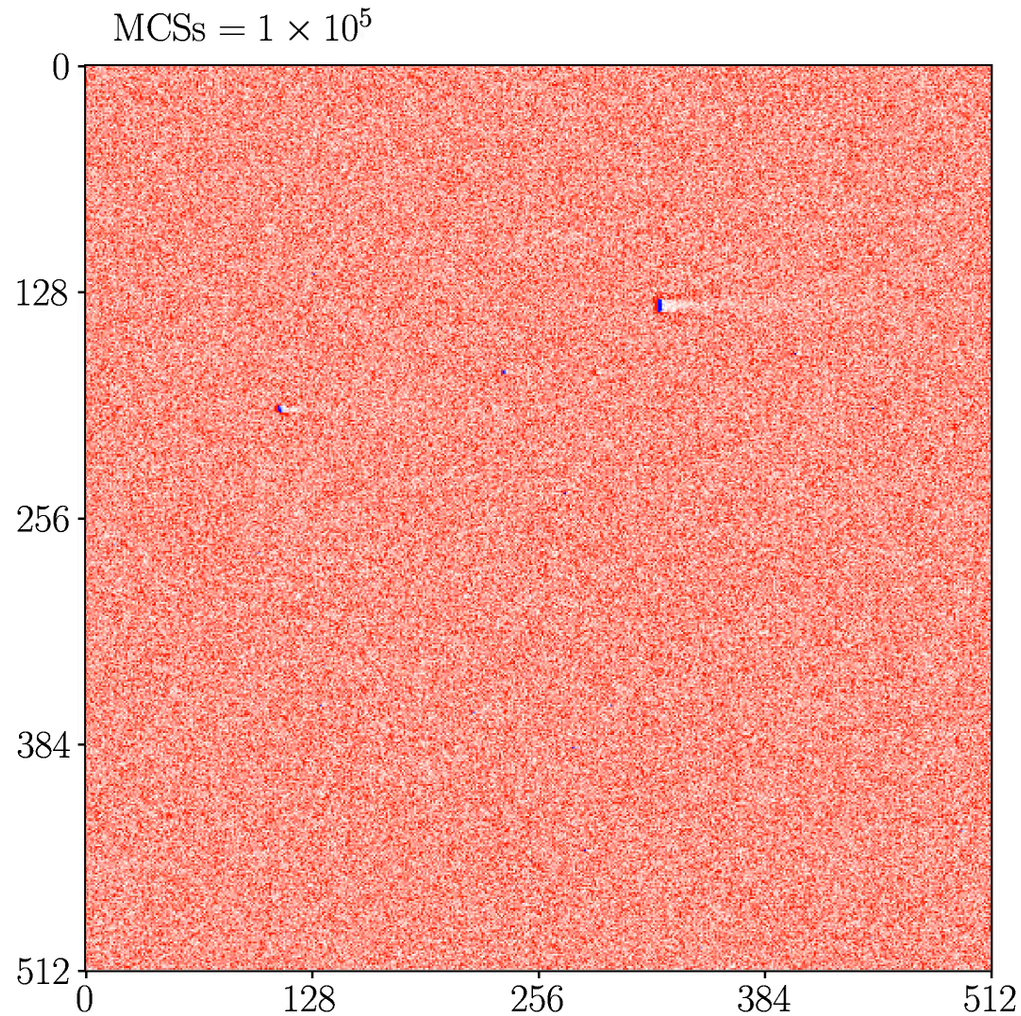
exponential scaling of characteristic time scales

$\tau_g \sim e^{l/l_g}$ and $\tau_s \sim e^{l/l_s}$ with $l_g = \frac{2vt_0}{\rho_0}$ and $l_s = \frac{8Dt_0}{\rho_0}$

Growth is dominant ($l_s \ll l_g$) for $D \ll v$.

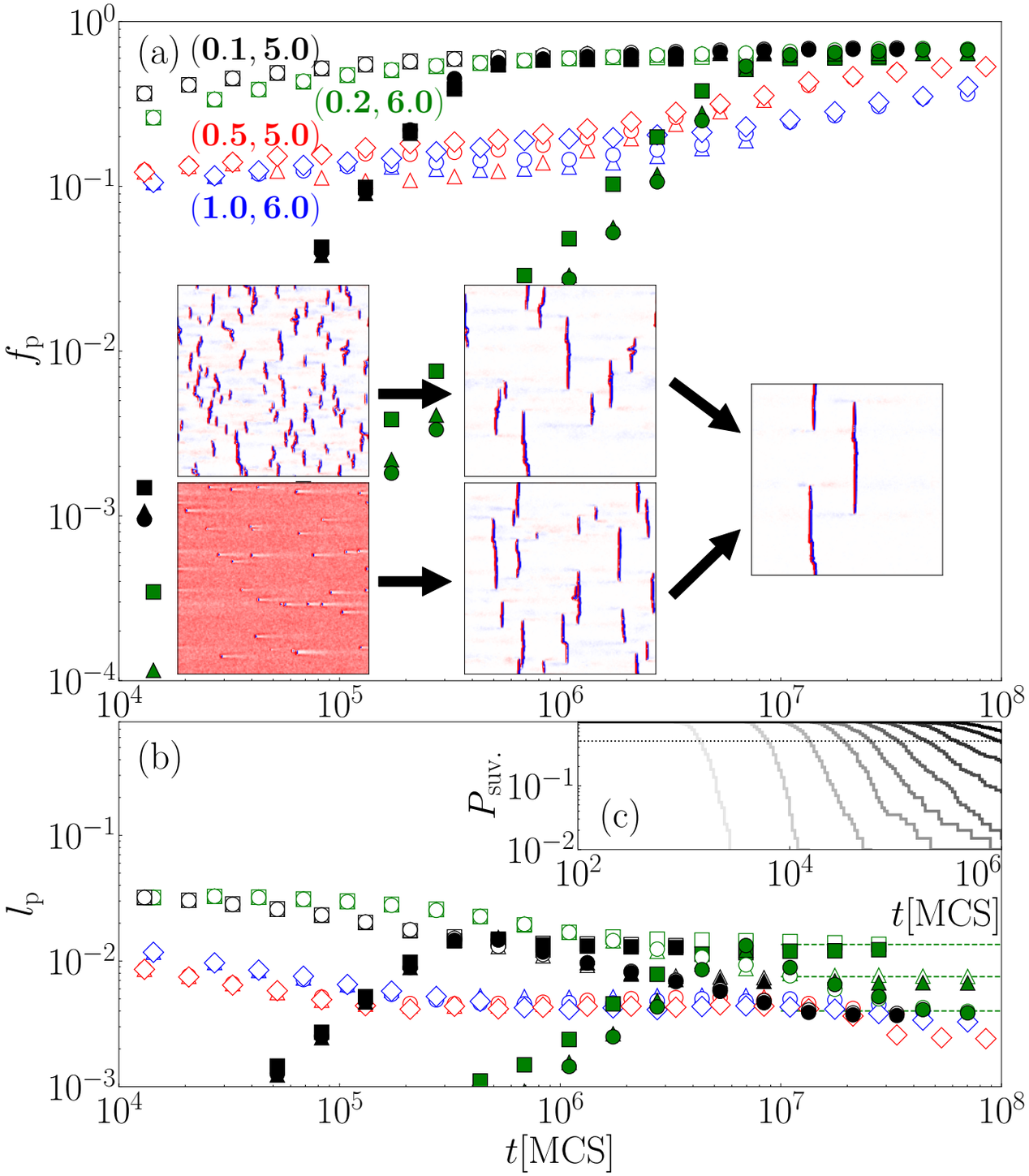


from a random initial state



from an ordered initial state

Nature of Pinned Phase



$f_{\rm p}$: fraction of particles trapped in PIs

$l_{\rm p}$: total length of PIs per site

for $D \lesssim 0.3$

random state →

coarsening → globally pinned state

ordered state → nucleation →

coarsening → globally pinned state

for $0.5 \lesssim D \lesssim 1$

random state

→coarsening → globally pinned state

ordered state → … (slow nucleation)

Discussions

- Metastability of the liquid state has to be reconsidered.

- Dynamics of PIs
  - for small D, we confirm numerically and analytically that PIs grow
    $\Rightarrow$ globally pinned state
  - for large D, more works are in order
    If PIs tend to shrink rather than to grow, the MIIP would protect the long range order from droplet excitations!

- Higher symmetry
  - active p-state clock model Chatterjee et al., EPL 2022, Solon et al., PRL 2022
  - numerical observation of MIIP for $p \leq 4$

Summary

- Pinning without disorder
  MIIP due to the resonating motion of individual particles

- PIs are characterized with slow dynamics.

- Phase diagram

- Are there another transition line within the pinned phase?

