NSPCS 2024

JULY 22-25
5F Conference Hall, KIAS

Power-efficiency trade-off for finite-time quantum harmonic Otto heat engines

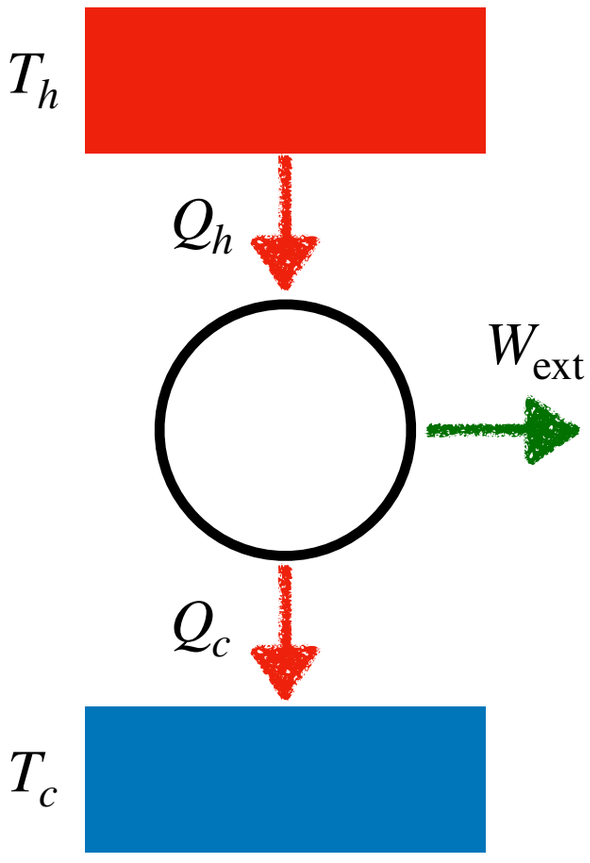
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KIAS KOREA INSTITUTE FOR ADVANCED STUDY

apctp

Power-efficiency trade-offs

$$P = \frac{W_{\text{ext}}}{\tau_{\text{cyc}}} \text{ : power} \qquad \eta = \frac{W_{\text{ext}}}{Q_h} \text{ : efficiency}$$



1st law

$$\Delta E = Q_h - Q_c + W_{\text{ext}} = 0$$

+

2nd law

$$\Delta S_{\text{tot}} = -\frac{Q_h}{T_h} + \frac{Q_c}{T_c} \geq 0$$

↓

$$\eta \leq 1 - \frac{T_c}{T_h} = \eta_C$$

reversible operation

$$P = \frac{W_{\text{ext}}}{\tau_{\text{cyc}}} \to 0$$

+

tighter lower bounds

$$\Delta S_{\text{tot}} \geq \chi J^2$$

with $J \in \{Q_h, Q_c, W_{\text{ext}}\}$

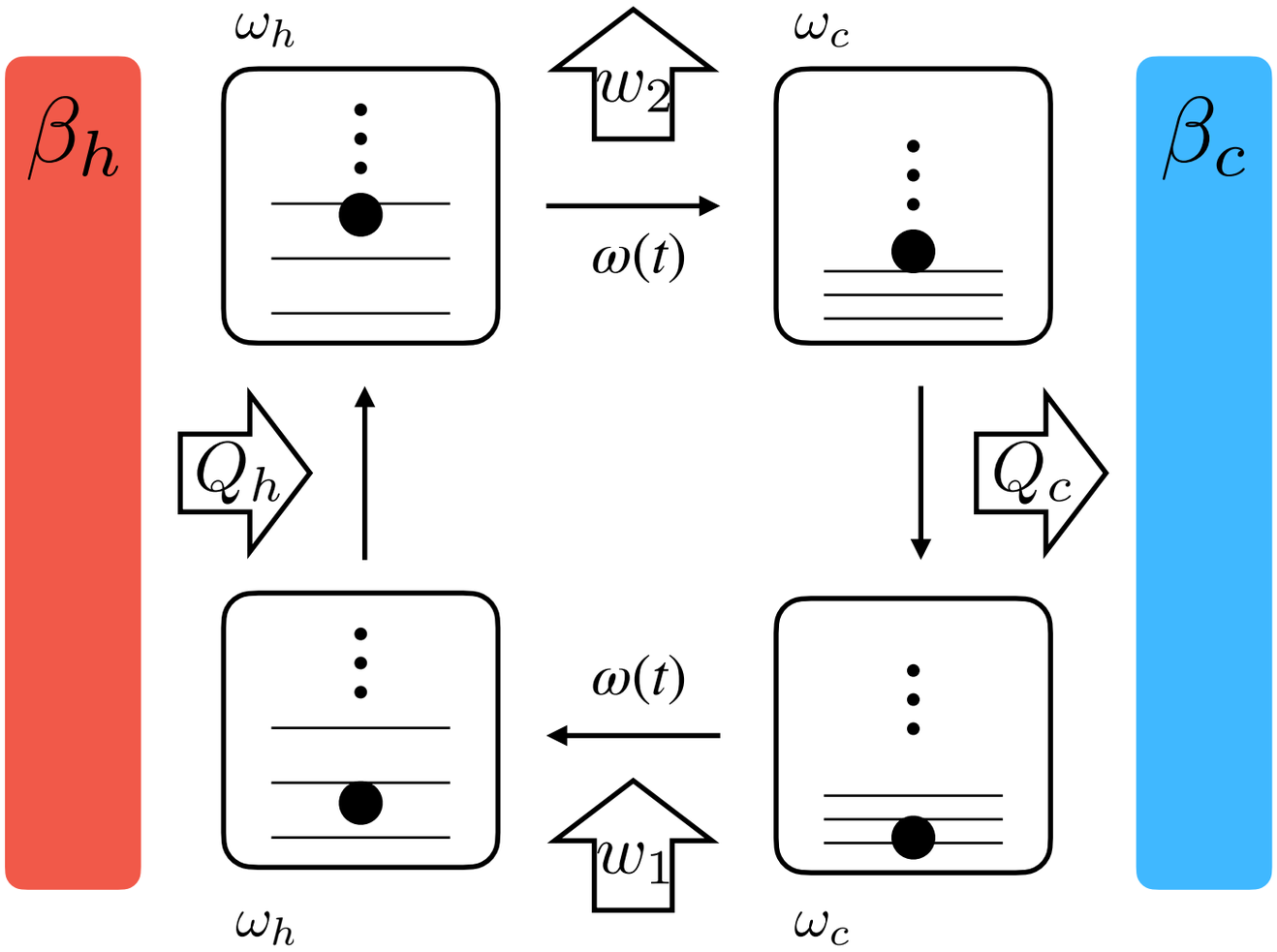
PRL'15 Barato/Seifert
PRE'18 Dechant/Sasa

↓

$$P \leq \chi' \eta (\eta_C - \eta)$$

steady-state heat engines
classical stochastic process

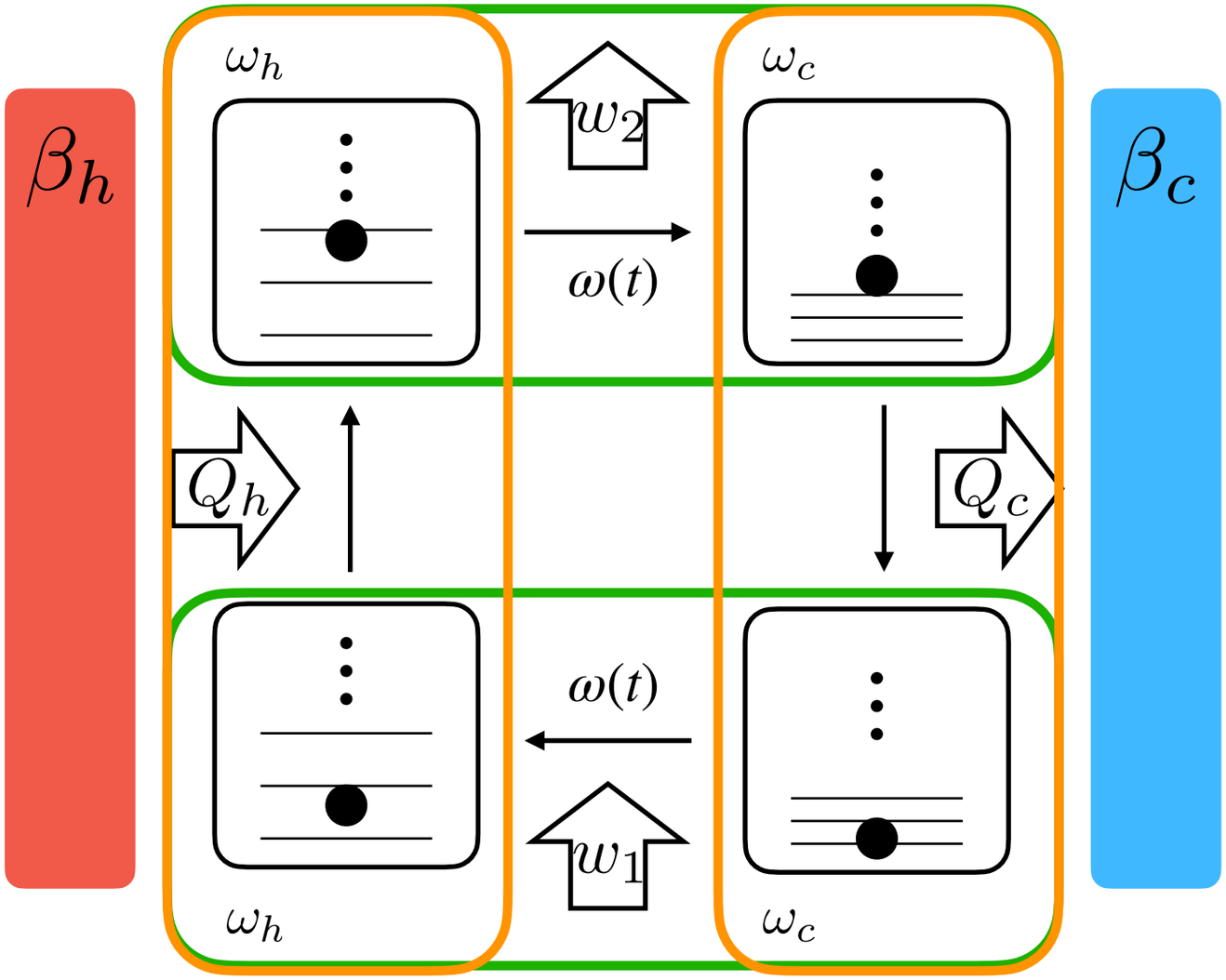
Quantum harmonic Otto heat engine



$$\hat{H}(t) = \hbar\omega(t)\left(\hat{a}^\dagger(t)\hat{a}(t) + \frac{1}{2}\right), \quad E(t) = \mathrm{tr}\{\hat{\rho}(t)\hat{H}(t)\}$$

$$S_{\mathrm{vN}}(t) = -\,\mathrm{tr}\{\hat{\rho}(t)\ln\hat{\rho}(t)\}$$

Time-evolution of harmonic oscillator



$$\hat{H}(t) = \hbar\omega(t)\left(\hat{a}^\dagger(t)\hat{a}(t) + \frac{1}{2}\right), \quad E(t) = \mathrm{tr}\{\hat{\rho}(t)\hat{H}(t)\}$$

$$S_{\mathrm{vN}}(t) = -\,\mathrm{tr}\{\hat{\rho}(t)\ln\hat{\rho}(t)\}$$

adiabatic processes

$$\frac{d}{dt}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H}(t), \hat{\rho}(t)]$$

$$\frac{dE(t)}{dt} = \mathrm{tr}\left(\hat{\rho}(t)\frac{d\hat{H}(t)}{dt}\right) = \dot{W}(t)$$

$$\frac{dS_{\mathrm{vN}}(t)}{dt} = 0$$

isochoric & isothermal processes

$$\frac{d}{dt}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}(t)] + \mathscr{D}\left(\hat{\rho}(t)\right)$$

$$\frac{dE(t)}{dt} = \mathrm{tr}\left(\hat{H}\frac{d\hat{\rho}(t)}{dt}\right) = \dot{Q}(t)$$

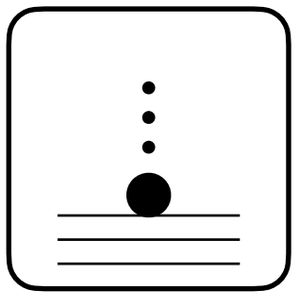
$$\frac{dS_{\mathrm{vN}}(t)}{dt} = \underbrace{\sigma}_{\geq 0} - \beta\dot{Q}$$

Phase-space approach

	moment generating function		probability distribution function
classical	$g(\xi,\xi^*) = \langle e^{-i(\xi\alpha+\xi^*\alpha^*)}\rangle$ $= \int f_{\rm cl}(\alpha,\alpha^*) e^{-i(\xi\alpha+\xi^*\alpha^*)} d^2\alpha$	Fourier transform $\longleftrightarrow$	$f_{\rm cl}(\alpha,\alpha^*) = \frac{1}{\pi^2}\int g(\xi,\xi^*) e^{i(\xi\alpha+\xi^*\alpha^*)} d^2\xi$
quantum	$g_1(\xi,\xi^*) = \mathrm{tr}\left\{\hat{\rho} e^{-i(\xi\hat{a}+\xi^*\hat{a}^\dagger)}\right\}$ $g_2(\xi,\xi^*) = \mathrm{tr}\left\{\hat{\rho} e^{-i\xi\hat{a}} e^{-i\xi^*\hat{a}^\dagger}\right\}$ $g_3(\xi,\xi^*) = \mathrm{tr}\left\{\hat{\rho} e^{-i\xi^*\hat{a}^\dagger} e^{-i\xi\hat{a}}\right\}$ $\vdots$	Fourier transform $\longleftrightarrow$	$f_{\rm qu}(\alpha,\alpha^*) = \frac{1}{\pi^2}\int g_1(\xi,\xi^*) e^{i(\xi\alpha+\xi^*\alpha^*)} d^2\xi$ $\hat{a}\,\vert\alpha\rangle = \alpha\,\vert\alpha\rangle$: coherent basis

Mapping to classical description

adiabatic processes

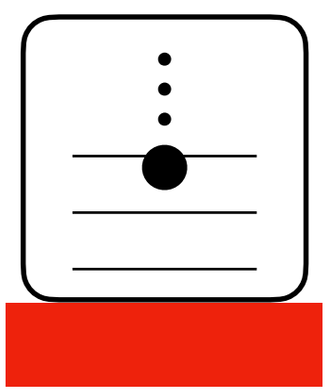


$$\frac{d}{dt}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H}(t), \hat{\rho}(t)] \longrightarrow \dot{\alpha}(t) = -i\omega(t)\alpha(t) + \frac{\dot{\omega}(t)}{2\omega(t)}\alpha^*(t)$$

$$\frac{dE(t)}{dt} = \mathrm{tr}\left(\hat{\rho}(t)\frac{d\hat{H}(t)}{dt}\right) = \dot{W}(t) \longrightarrow \dot{W}(t) = \frac{\hbar\dot{\omega}(t)}{2}\langle(\alpha + \alpha^*)^2\rangle$$

$$S(t) = -\int f(\alpha, \alpha^*, t)\ln f(\alpha, \alpha^*, t)d^2\alpha, \quad \frac{dS(t)}{dt} = 0$$

isochoric & isothermal processes

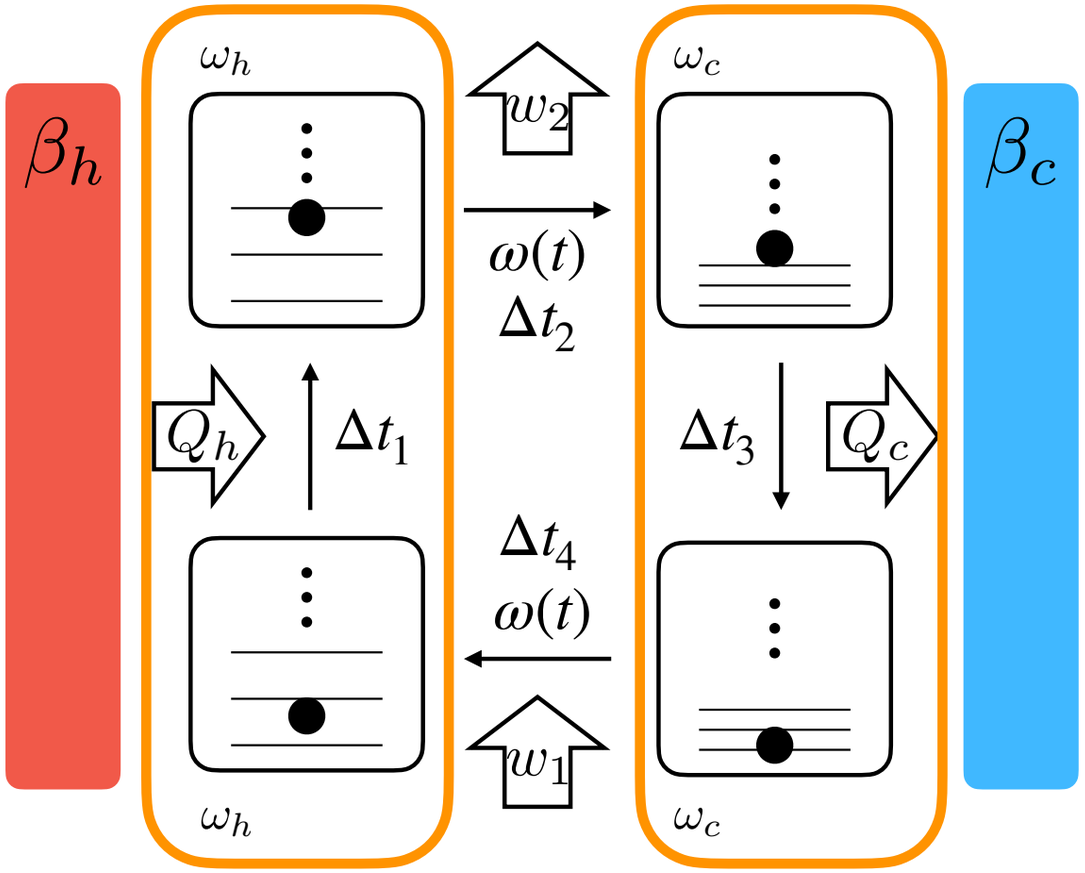


$$\frac{d}{dt}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H}(t), \hat{\rho}(t)] + \mathscr{D}\left(\hat{\rho}(t)\right) \longrightarrow \dot{\alpha}(t) = -\left(\frac{\gamma}{2} + i\omega\right)\alpha(t) + \sqrt{\frac{\gamma\hbar\omega}{\tilde{\beta}}}\xi(t)$$

$$\frac{dE(t)}{dt} = \mathrm{tr}\left(\hat{H}\frac{d\hat{\rho}(t)}{dt}\right) = \dot{Q}(t) \longrightarrow \dot{Q}(t) = \gamma\left(\frac{1}{\tilde{\beta}} - \hbar\omega\langle|\alpha|^2\rangle\right)$$

$$\frac{dS(t)}{dt} = \underbrace{\dot{\Pi}}_{\geq 0} - \tilde{\beta}\dot{Q} \qquad \text{where } \tilde{\beta}^{-1} = \frac{\hbar\omega}{2}\coth\left(\frac{\beta\hbar\omega}{2}\right)$$

Power-efficiency trade-off



$* \; w_2 - w_1 = W_{\text{ext}}$

$$Q_h = \hbar\omega \int dt \int d^2\alpha \left(\alpha^* J(\alpha, \alpha^*, t) + \alpha J^*(\alpha, \alpha^*, t)\right)$$

↓ Cauchy-Schwarz inequality

$$Q_h^2 \leq \frac{\Pi_h}{\tilde{\beta}_h}\left(\frac{\gamma \Delta t_1}{\tilde{\beta}_h} - Q_h\right)$$

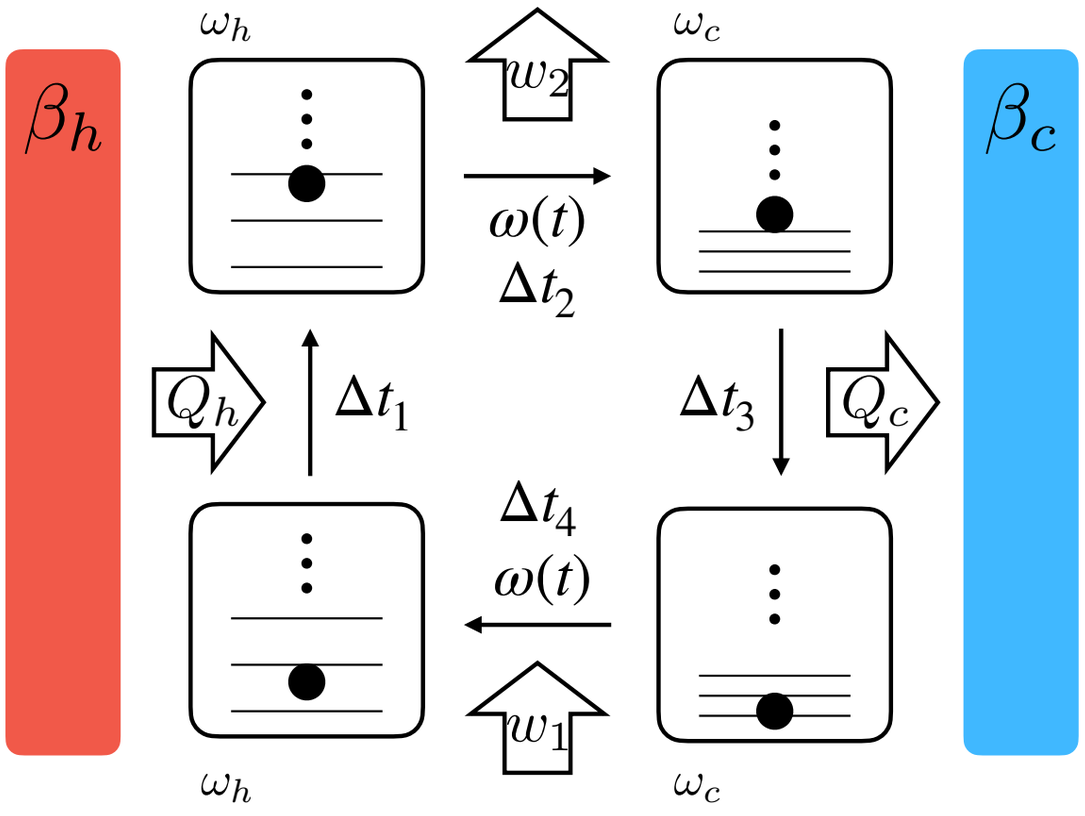
↓ $\Pi_h \leq \Pi_{\text{tot}} = (\tilde{\beta}_c - \tilde{\beta}_h)Q_h - \tilde{\beta}_c W_{\text{ext}}$: 2nd-law-like inequality

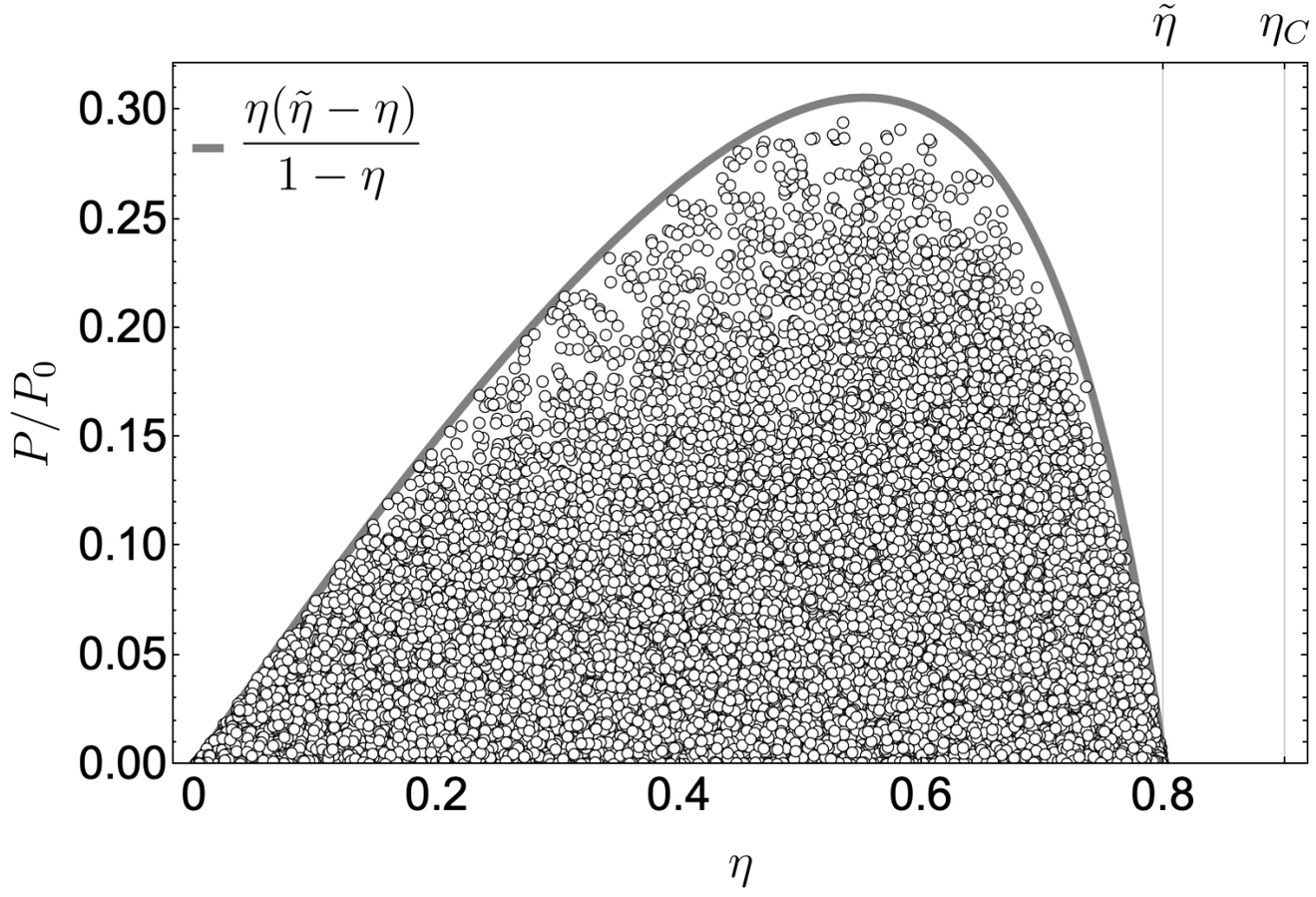
$$P \leq \frac{\gamma \Delta t_1}{\tilde{\beta}_h \tau_{\text{cyc}}} \frac{\eta(\tilde{\eta} - \eta)}{1 - \eta} \qquad \text{where } \tilde{\eta} = 1 - \frac{\tilde{\beta}_h}{\tilde{\beta}_c}$$

↓

$$P \leq \underbrace{\frac{\gamma \min\{\Delta t_1, \Delta t_3\}}{\tilde{\beta}_h \tau_{\text{cyc}}}}_{:=P_0} \frac{\eta(\tilde{\eta} - \eta)}{1 - \eta}$$

Illustration





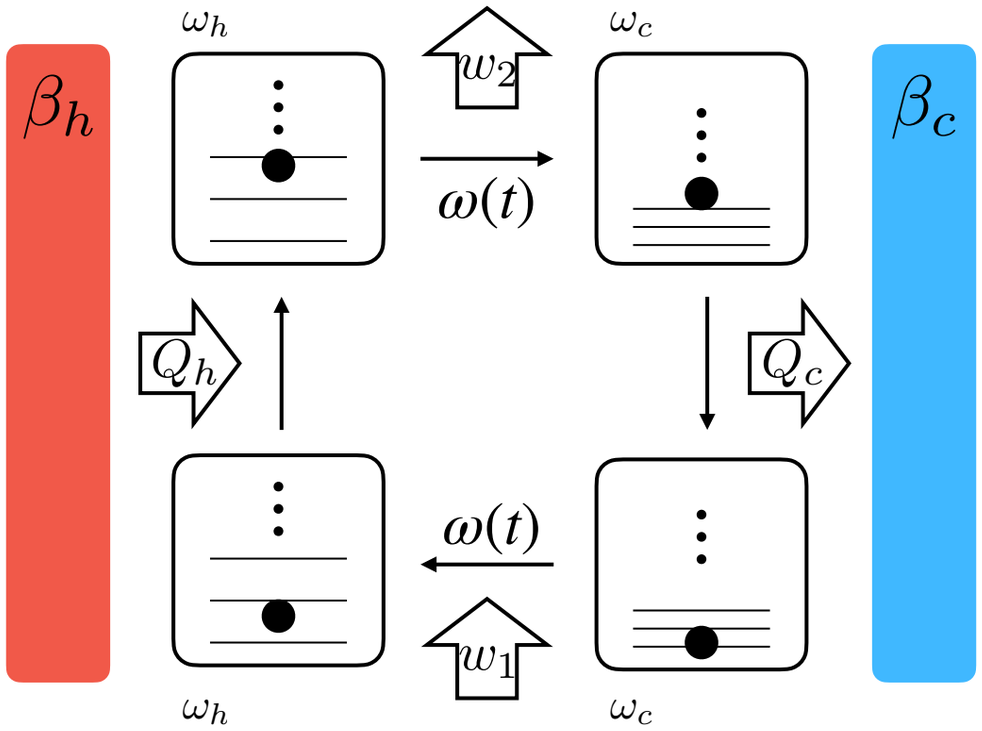
$$\dot{\omega}(t) \propto \omega^2(t)$$

$$\Delta t_1,\ \Delta t_2,\ \Delta t_3,\ \Delta t_4 \in [0,10]$$

$$\gamma, \hbar\omega_h, \beta_h \in [0,10]$$

$$\eta_C = 1 - \frac{\beta_h}{\beta_c} = 0.9, \quad \tilde{\eta} = 1 - \frac{\tilde{\beta}_h}{\tilde{\beta}_c} = 0.8$$

Summary

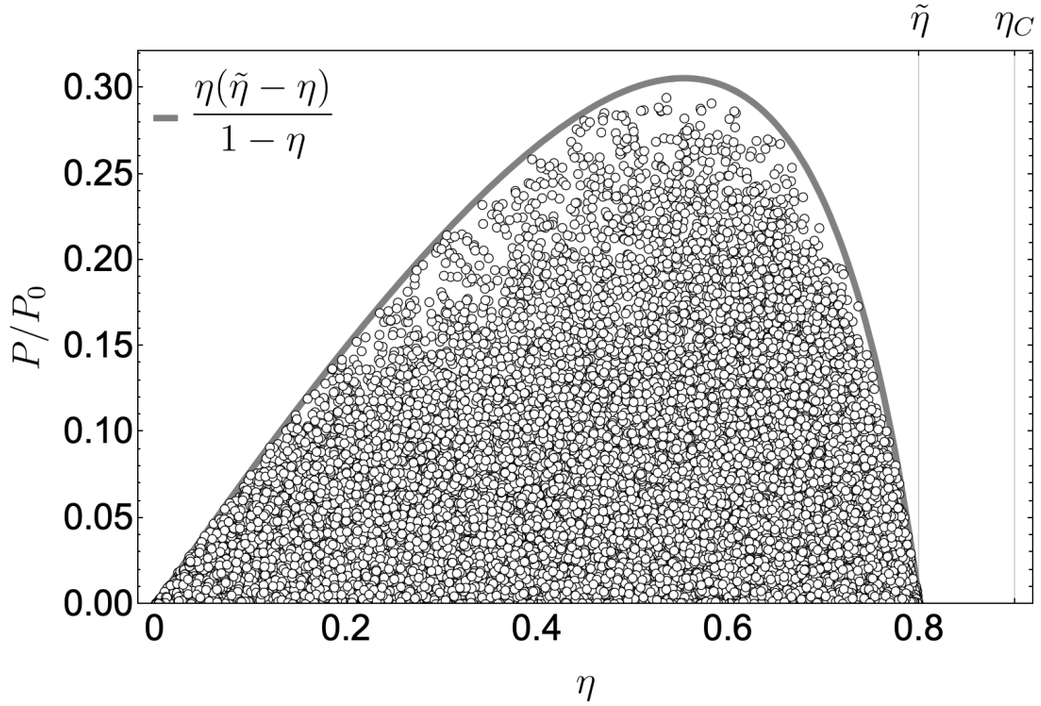


$$\frac{d}{dt}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H}(t), \hat{\rho}(t)]$$

$$\frac{d}{dt}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H}(t), \hat{\rho}(t)] + \mathscr{D}\left(\hat{\rho}(t)\right)$$

mapping

$$f(\alpha, \alpha^*) = \frac{1}{\pi^2}\int \mathrm{tr}\{\hat{\rho}e^{i(\xi\hat{a}+\xi^*\hat{a}^\dagger)}\}e^{i(\xi\alpha+\xi^*\alpha^*)}d^2\xi$$



$$\dot{\alpha}(t) = -i\omega(t)\alpha(t) + \frac{\dot{\omega}(t)}{2\omega(t)}\alpha^*(t)$$

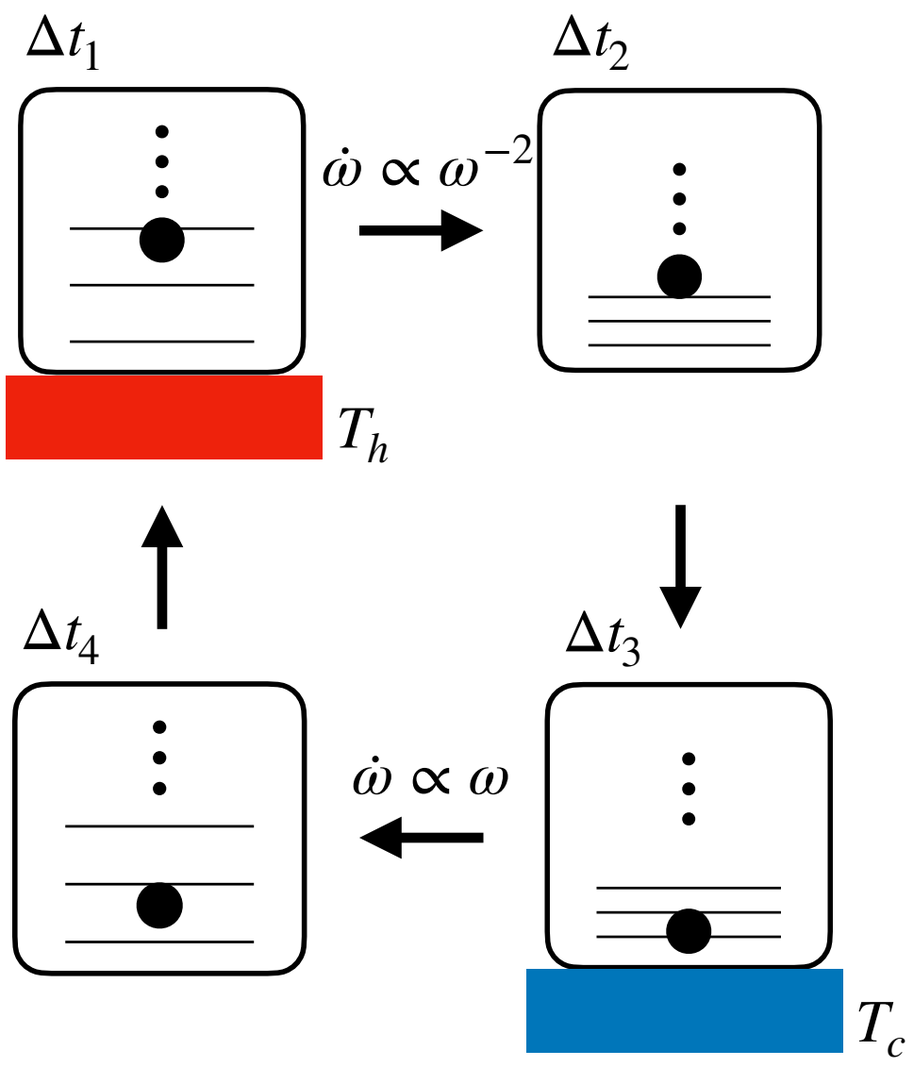
$$\dot{\alpha}(t) = -\left(\frac{\gamma}{2} + i\omega\right)\alpha(t) + \sqrt{\frac{\gamma\hbar\omega}{\tilde{\beta}}}\xi(t)$$



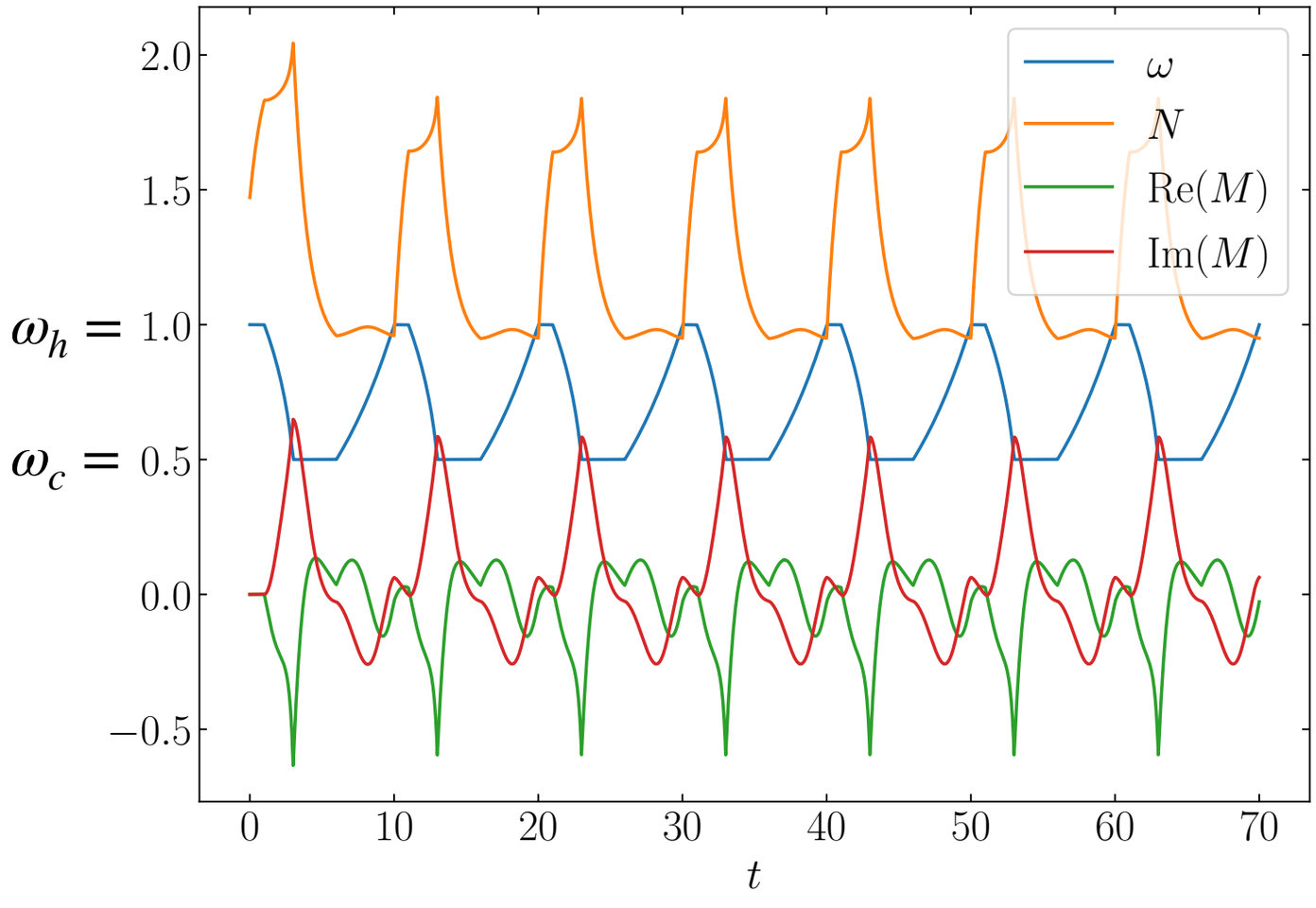
Thank you for your attention!

Appendices

Operation of heat engine

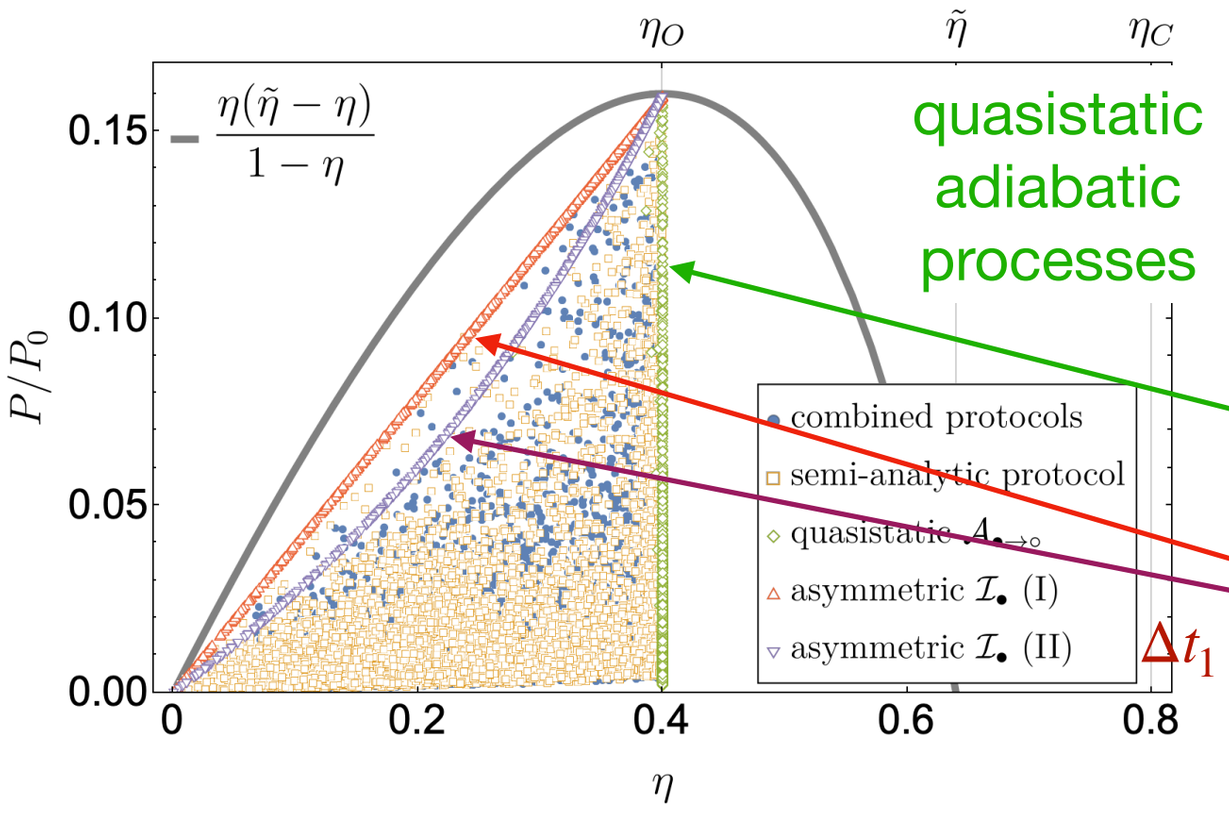


$$N(t) = \langle |\alpha|^2(t) \rangle, \quad M(t) = \langle \alpha^2(t) \rangle$$



$\Delta t_1,\ \Delta t_2,\ \Delta t_3,\ \Delta t_4 \in [0,10]$

Saturation conditions



$$Q_h = \hbar\omega \int dt \int d^2\alpha \left(\alpha^* J(\alpha,\alpha^*,t) + \alpha J^*(\alpha,\alpha^*,t)\right)$$

↓ Cauchy-Schwarz inequality

$$Q_h^2 \leq \frac{\Pi_h}{\tilde{\beta}_h}\left(\frac{\gamma\Delta t_1}{\tilde{\beta}_h} - Q_h\right)$$

$\Delta t_1 \ll \Delta t_3$

$\Delta t_1 \gg \Delta t_3$

$$\Pi_h \leq \Pi_{\text{tot}} = (\tilde{\beta}_c - \tilde{\beta}_h)Q_h - \tilde{\beta}_c W_{\text{ext}}$$

asymmetric isochoric processes

: 2nd-law-like inequality

$$\frac{P}{P_0} \leq \frac{\eta(\tilde{\eta}-\eta)}{1-\eta}$$

$\dot{\omega}(t) \propto \omega^m(t)$ with $m \in \{-2,-1,0,1,2\}$ for combined protocols

$\dot{\omega}(t) \propto \omega^2(t)$ for semi-analytic protocols