Date: 2024-07-24

Effect of activity on target search with resetting in thermal environment

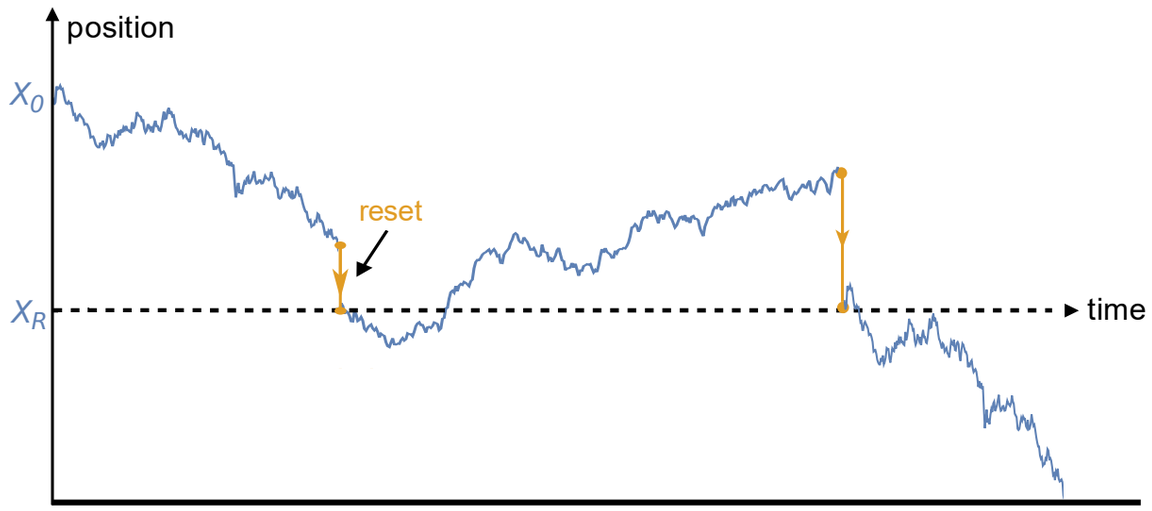
Priyo Shankar Pal

Korea Institute For Advanced Study

The 10th KIAS Conference on Statistical Physics
NONEQUILIBRIUM STATISTICAL PHYSICS OF COMPLEX SYSTEMS
(July 22 - July 25, 2024, KIAS)

Introduction

Stochastic resetting is a renewal process where the dynamics of a system is interrupted at random times and reset to a predetermined configuration.



$$x(t+dt) = x_R \quad \text{prob. } rdt$$
$$= x(t) + \xi(t)dt \quad \text{prob. } (1-rdt)$$

M. R. Evans and S. N. Majumdar, PRL 106, 160601 (2011)

M. R. Evans, S. N. Majumdar and G. Schehr, J. Phys. A: Math. Theor. 53 (2020) 193001

Introduction

Stochastic Resetting with Passive particle

Overdamped Brownian particle

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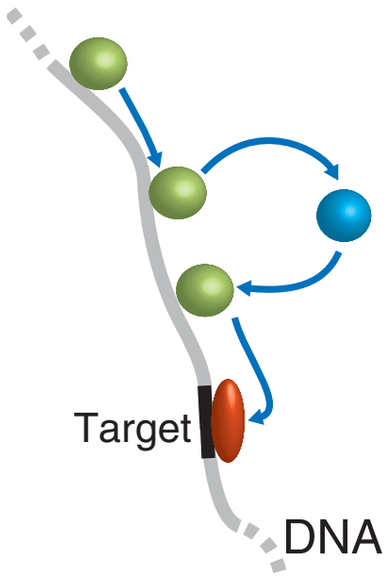
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Target search by proteins on DNA (Schematic diagram)



*O. Bénichou, Y. Kafri, M. Sheinman, and R. Voituriez, PRL **103**, 138102 (2009)*

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Stochastic Resetting with Active particle

Run and Tumble particle $\quad \dfrac{dx}{dt} = \sigma(t) f$

*M. R. Evans and S. N. Majumdar, J. Phys. A: Math. Theor. **51** (2018) 475003*

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*M. R. Evans and S. N. Majumdar, J. Phys. A: Math. Theor. **51** (2018) 475003*

GOAL: Study the combined effects of the active motion and thermal noise on search dynamics

Is active motion beneficial for target search with resetting in a thermal environment?, P. S. Pal, J. M. Park, A. Pal, H.Park and J. S. Lee, arXiv: 2407.03762

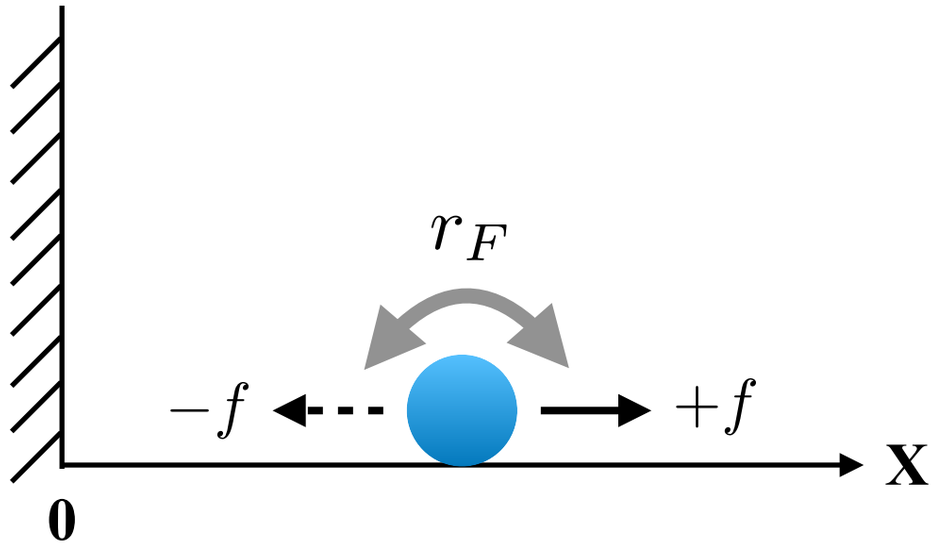
Stochastic Resetting of RTP in thermal bath

Langevin dynamics of 1D Run and Tumble particle:

$$\dot{x} = \sigma(t) f + \xi(t)$$

Telegraphic Noise $\sigma(t) = +1 \;\underset{r_F}{\rightleftharpoons}\; -1$

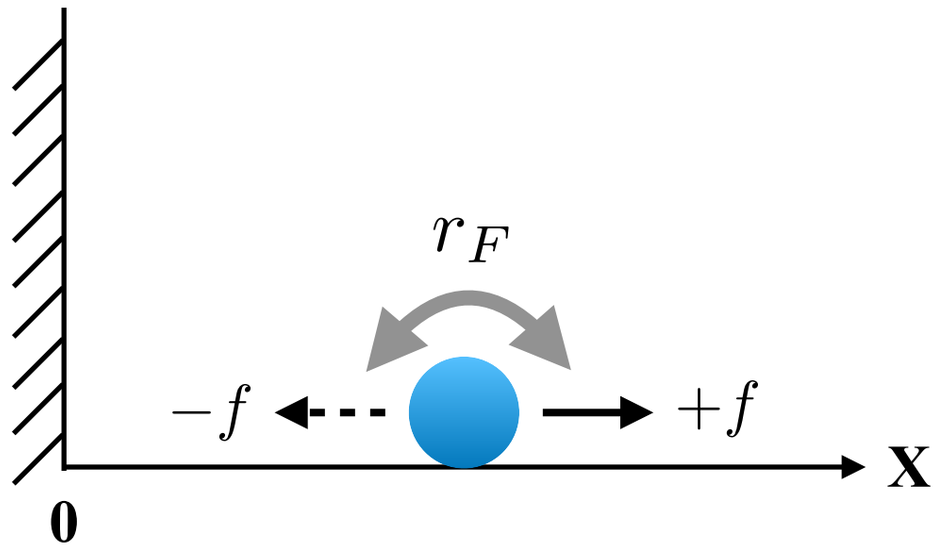
$$\langle \xi(t)\xi(t') \rangle = 2D\delta(t-t')$$



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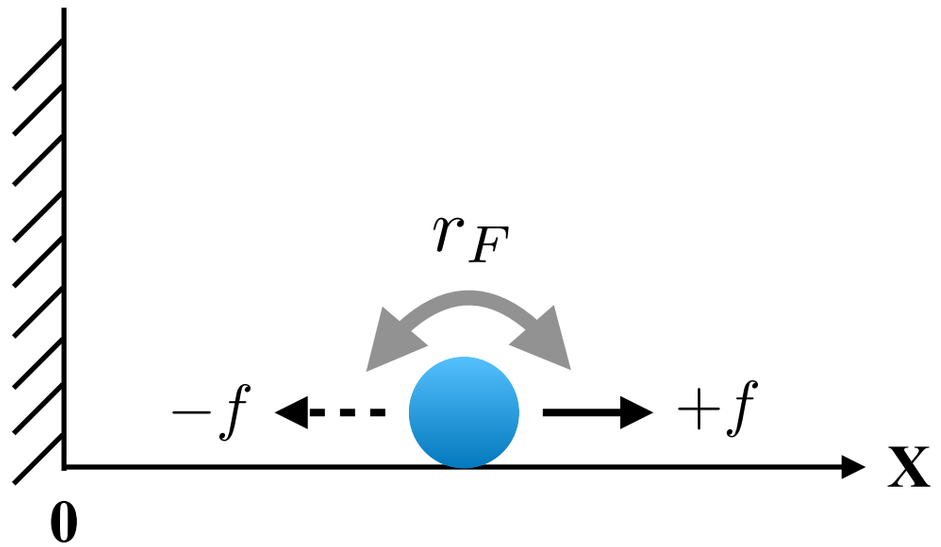
Resetting Mechanism: Instantaneous resetting with rate r_R

- Position reset to x_{R}
- Direction flip with probability η

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Resetting Mechanism: Instantaneous resetting with rate r_R

- Position reset to x_{R}
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If $\sigma(t) = +1$ at any instant of time t, the position update in the next instant

$$x(t+\Delta t) = \begin{cases} x_{\mathrm{R}} \quad \& \quad f \to -f, & \text{with prob. } r_{\mathrm{R}}\eta\Delta t \\ x_{\mathrm{R}} \quad \& \quad f \to f, & \text{with prob. } r_{\mathrm{R}}(1-\eta)\Delta t \\ x(t) + f\Delta t + \xi(t)\Delta t, & \text{with prob. } (1-r_{\mathrm{R}}\Delta t)(1-r_{\mathrm{F}}\Delta t) \\ x(t) - f\Delta t + \xi(t)\Delta t, & \text{with prob. } (1-r_{\mathrm{R}}\Delta t) r_{\mathrm{F}}\Delta t \ . \end{cases}$$

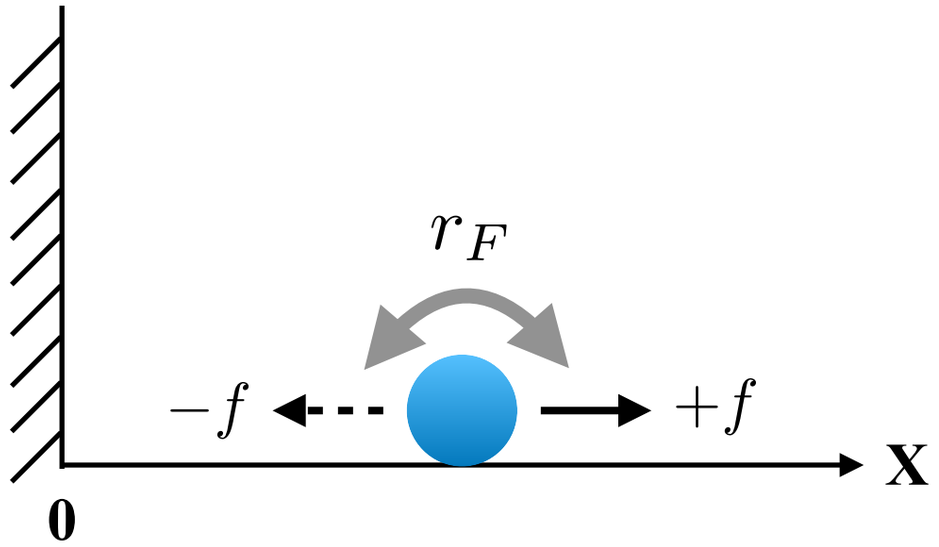
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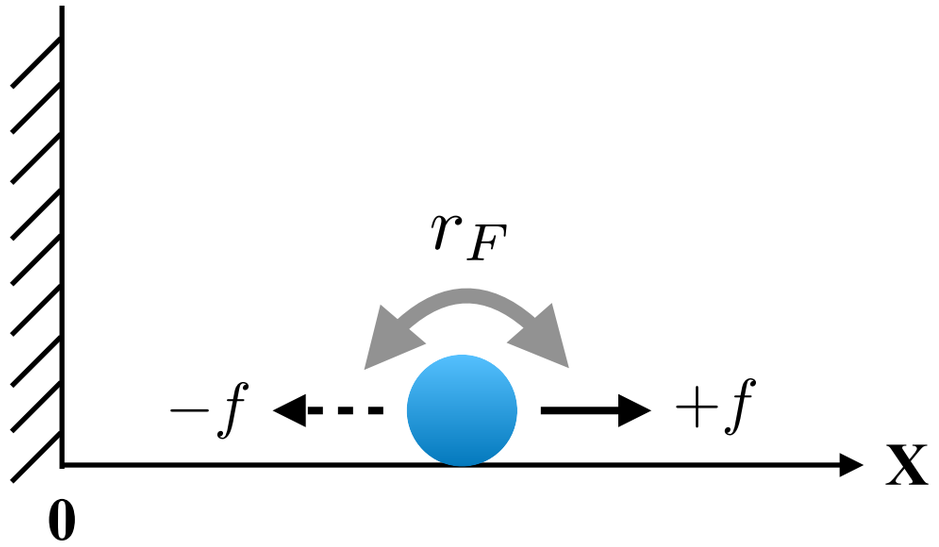
Aim: Calculation of the mean First Passage Time

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Aim: Calculation of the mean First Passage Time

Mean First Passage time will depend on the initial direction of the active speed

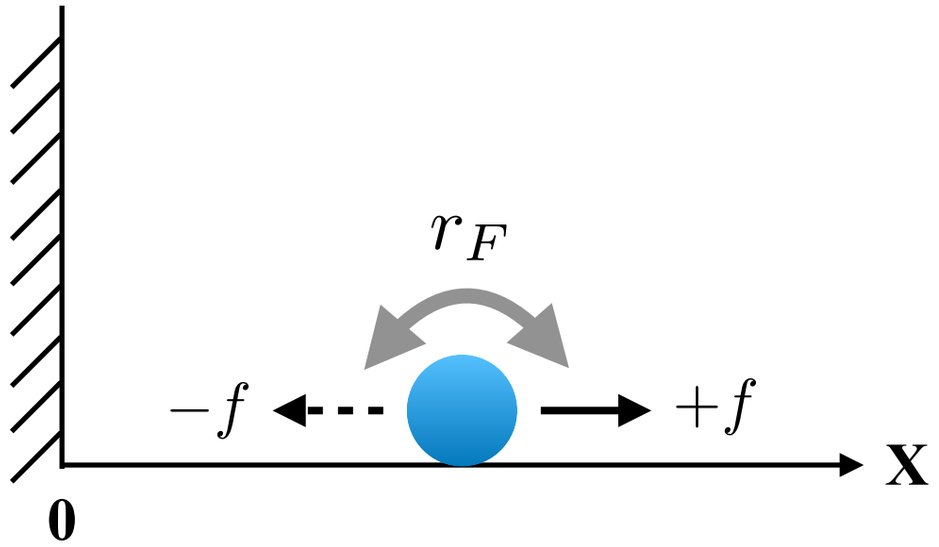
$p_\pm \longrightarrow$ Probabilities that $\quad \sigma(t=0) = \pm 1$

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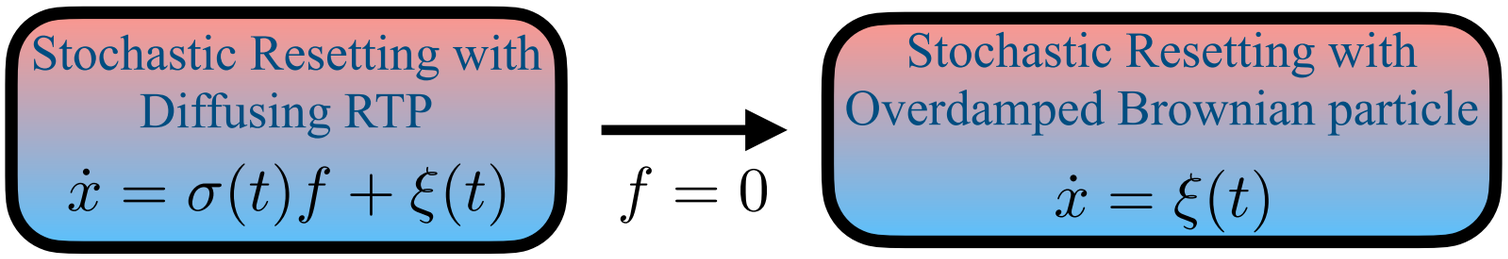
$\langle t_{\rm F}^{\pm} \rangle \longrightarrow$ Conditional Mean First Passage Time

Unconditional Mean First Passage Time: $\langle t_{\rm F} \rangle = p_+ \langle t_{\rm F}^+ \rangle + p_- \langle t_{\rm F}^- \rangle$

$p_\pm = 1/2 \longrightarrow$ Unbiased RTP

Is active motion beneficial for target search with resetting in a thermal environment?, P. S. Pal, J. M. Park, A. Pal, H.Park and J. S. Lee, arXiv: 2407.03762

Results: Unconditional Mean First Passage Time



*PRL **106**, 160601 (2011)*

$$\langle t_{\rm F} \rangle \to \langle t_{\rm F}^{f=0} \rangle = \frac{1}{r_{\rm R}} \left(e^{x_{\rm R}\sqrt{r_{\rm R}/D}} - 1 \right)$$

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Stochastic Resetting with Diffusing RTP

$$\dot{x} = \sigma(t) f + \xi(t)$$

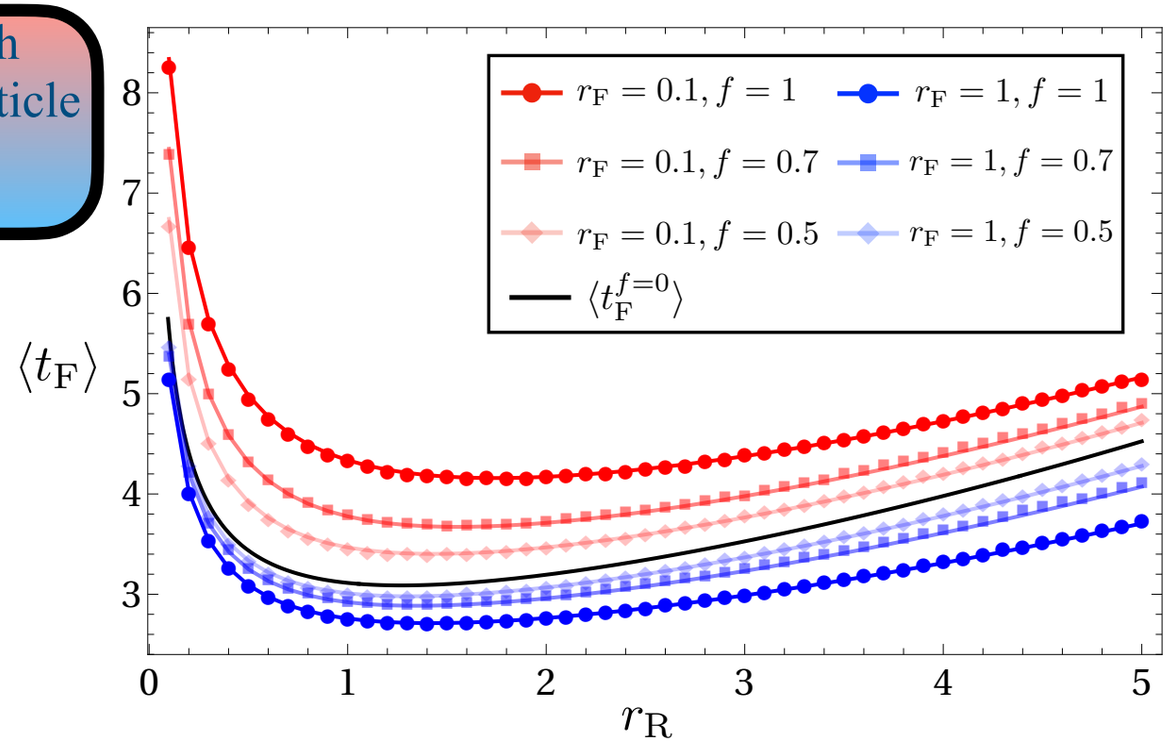
$f = 0$

Stochastic Resetting with Overdamped Brownian particle

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*PRL **106**, 160601 (2011)*

$$\langle t_{\rm F} \rangle \to \langle t_{\rm F}^{f=0} \rangle = \frac{1}{r_{\rm R}} \left(e^{x_{\rm R}\sqrt{r_{\rm R}/D}} - 1 \right)$$



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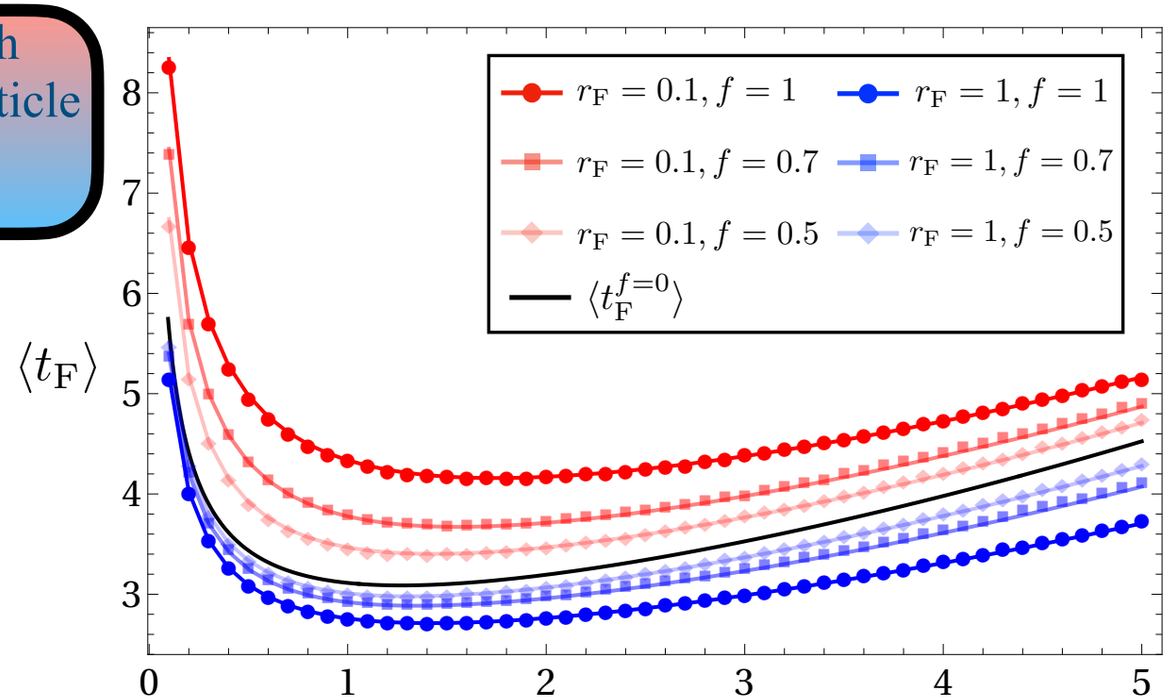
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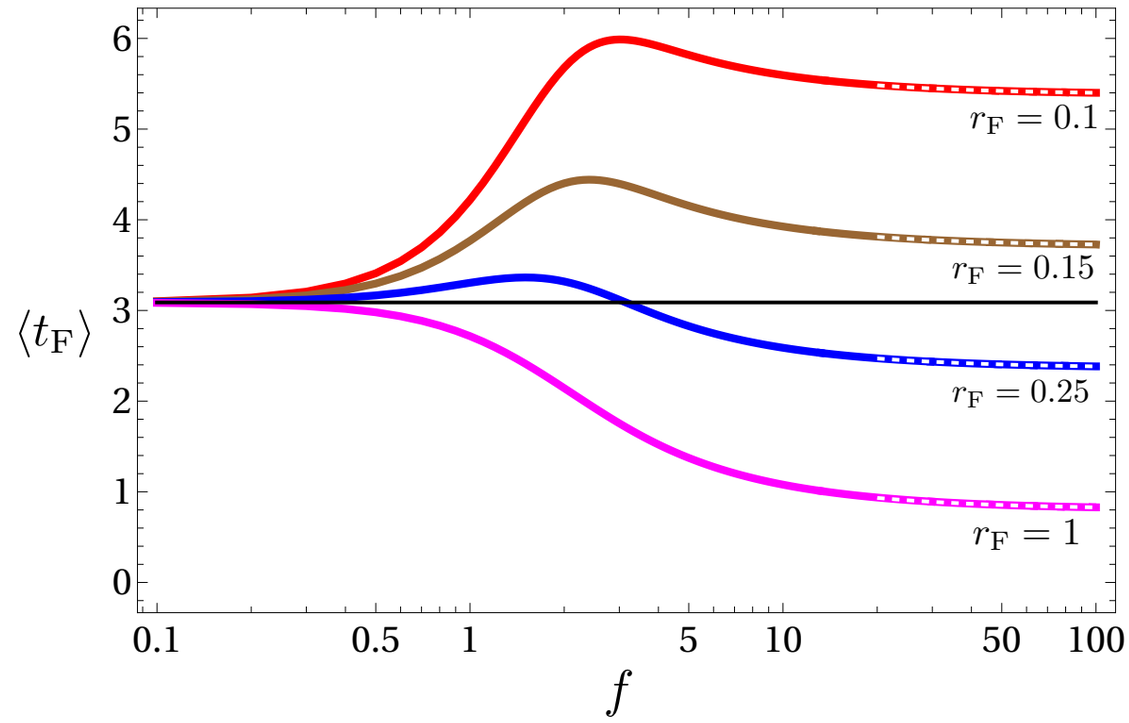
$$\langle t_{\rm F} \rangle \to \langle t_{\rm F}^{f=0} \rangle = \frac{1}{r_{\rm R}} \left(e^{x_{\rm R}\sqrt{r_{\rm R}/D}} - 1 \right)$$



Limiting Expressions:

$$\langle t_{\rm F} \rangle = \langle t_{\rm F}^{f=0} \rangle + O(f^2)$$

$$\langle t_{\rm F} \rangle = \frac{\sqrt{1 + \frac{2r_{\rm F}}{r_{\rm R}}}}{2(r_{\rm F} + \eta r_{\rm R})} + x_{\rm R}\left(1 + \sqrt{1 + \frac{2r_{\rm F}}{r_{\rm R}}}\right)\frac{1}{f} + O(f^{-2})$$



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$\xrightarrow{D = 0}$

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$$\dot{x} = \sigma(t) f$$

J. Phys. A: Math. Theor. ***51*** *(2018) 475003*

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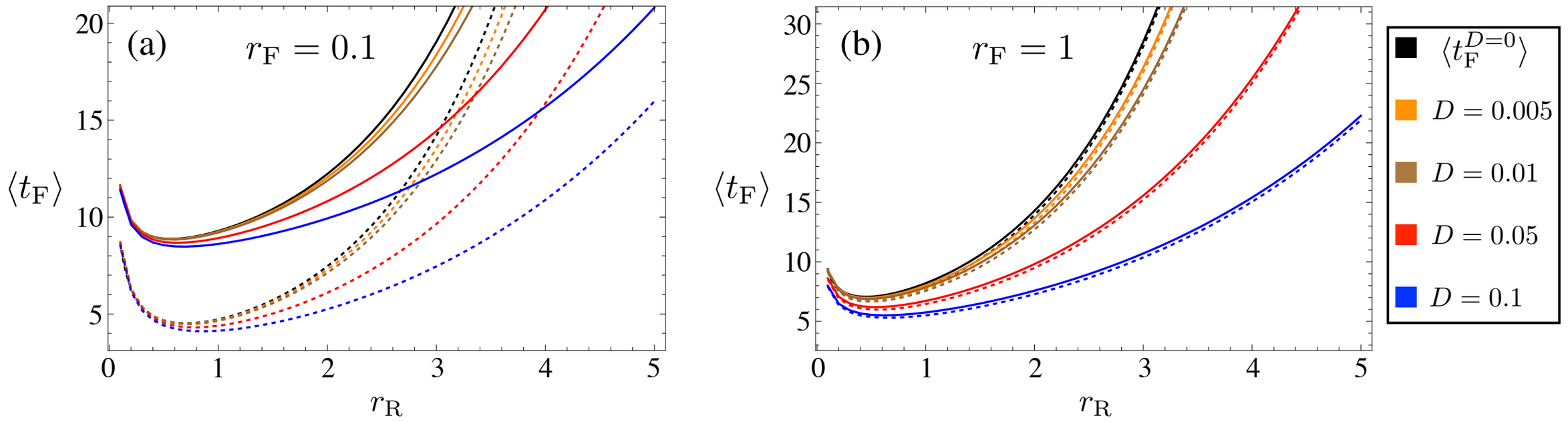
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J. Phys. A: Math. Theor. ***51*** *(2018) 475003*

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Solid Curves: $\eta = 0$ Dashed Curves: $\eta = 1/2$

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Results: Beneficiality of active motion

imagine the impossible

KIAS KOREA INSTITUTE FOR ADVANCED STUDY

Stochastic Resetting with Overdamped Brownian particle

$$\dot{x} = \xi(t)$$

$$\langle t_{\mathrm{F}} \rangle > \langle t_{\mathrm{F}}^{f=0} \rangle \longrightarrow$$ Overdamped Brownian particle is useful

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Beneficiality

$$\mathcal{B} = \mathcal{B}(D, f, r_{\mathrm{F}})$$

$$= \log \left[\frac{\langle t_{\mathrm{F}} \rangle|_{r_{\mathrm{R}}=r_{\mathrm{R}}^*}}{\langle t_{\mathrm{F}}^{f=0} \rangle|_{r_{\mathrm{R}}=r_{\mathrm{R},f=0}^*}} \right]$$

$\mathcal{B} > 0 \longrightarrow$ No benefit from activity

$\mathcal{B} < 0 \longrightarrow$ Active particle is beneficial

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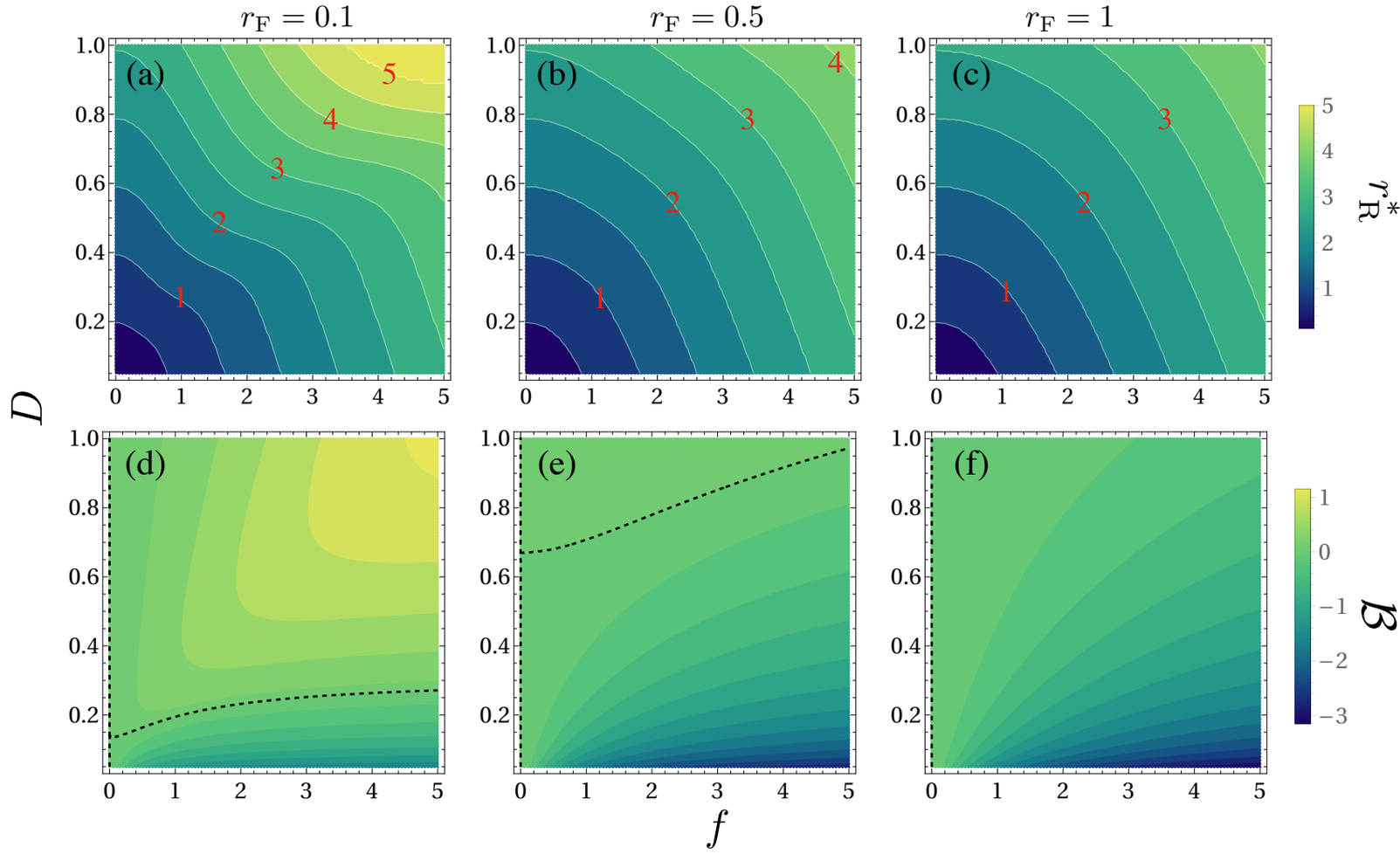
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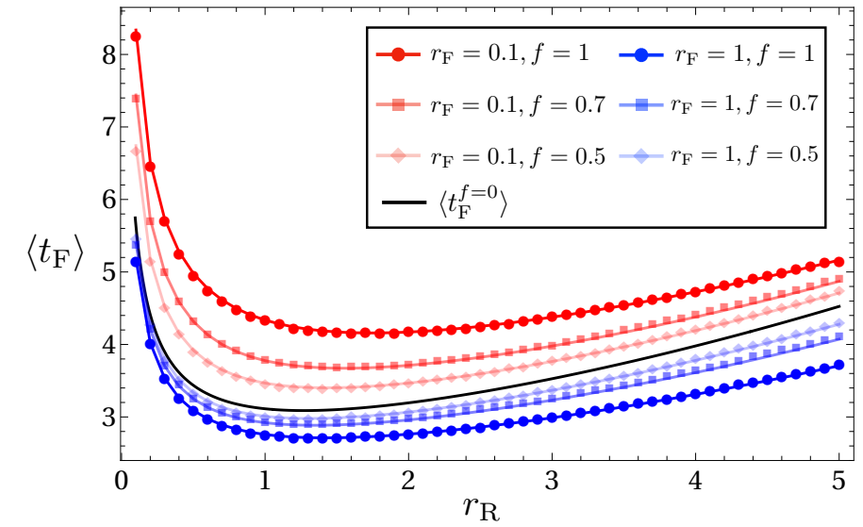
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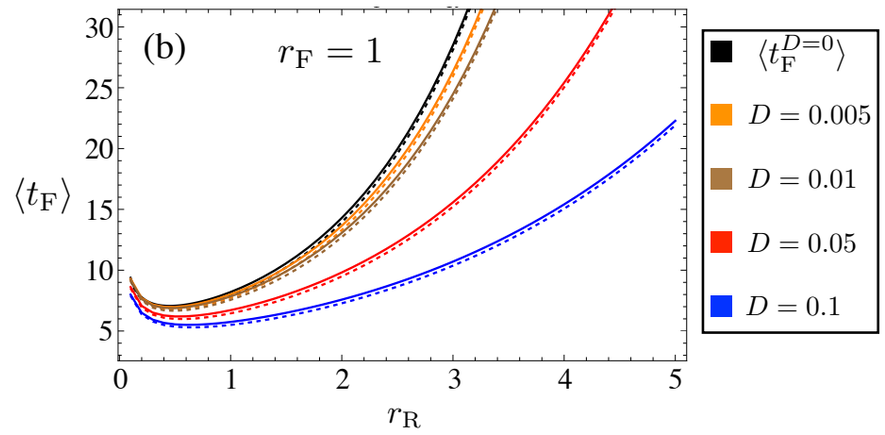


Take-Home Message

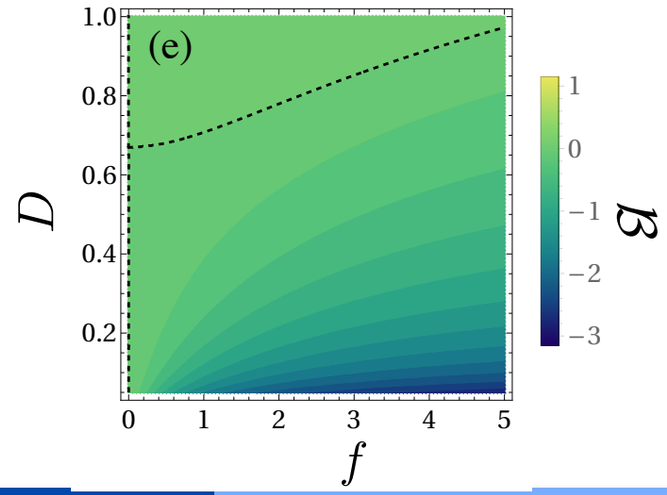
- Active motion significantly modifies the search dynamics either in reducing or prolonging the mean FPT.



- Thermal environment plays a significant role in reducing mean FPT.



- The function `beneficiality' has been introduced as a measure for discerning the degree of benefit of using an active particle for searching a target.



Collaborators



Jong-Min Park



Arnab Pal

THANK YOU



JaeSung Lee



Hyunggyu Park

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